Macro Risks and the Term Structure of Interest Rates*

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Abstract

We use non-Gaussian features in U.S. macroeconomic data to identify aggregate supply and demand shocks while imposing minimal economic assumptions. Recessions in the 1970s and 1980s were driven primarily by supply shocks; later recessions by demand shocks. We estimate macro risk factors that drive "bad" (negatively skewed) and "good" (positively skewed) variation for supply and demand shocks. We document that macro risks significantly contribute to the variation of yields, risk premiums and return variances for nominal bonds. While overall bond risk premiums are counter-cyclical, an increase in aggregate demand variance significantly lowers risk premiums.

Keywords: macroeconomic volatility, business cycles, bond return predictability, term premium, Great Moderation

JEL codes: E31, E32, E43, E44, G12, G13

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1 Introduction

Distinguishing supply shocks from demand shocks has long been a goal of empirical macroeconomics (e.g., Shapiro and Watson, 1988, Blanchard and Quah, 1989, or Gali, 1992), in part because the appropriate monetary and fiscal policy responses may be quite different for adverse demand versus supply shocks. In the field of asset pricing, supply shocks may prompt quite different responses in nominal bond prices than do demand shocks. It follows that variation in the magnitude of supply versus demand shocks may have important effects on the risk profile of nominal bonds and other asset prices.

We extract aggregate supply and demand shocks for the US economy from data on inflation, real GDP growth, core inflation and the unemployment gap. We begin by defining aggregate supply shocks as shocks that move inflation and real activity in the opposite direction. Similarly, demand shocks are defined as innovations that move inflation and real activity in the same direction. This identification scheme is motivated by Blanchard (1989), who finds empirically that the joint behavior of output, unemployment, prices, wages and nominal money in the U.S. is consistent with this structure.

Defining supply and demand shocks as above presents an identification problem. We resolve this issue without further economic assumptions, but instead using a novel approach exploiting unconditional higher-order moments in the data, which we show to be highly statistically significant. Despite this economically agnostic approach, we show that the structural shocks that we identify exhibit some intuitive properties. For example, in a classic paper, Blanchard and Quah (1989) use a vector-autoregressive dynamic structure to identify "demand-like" shocks as shocks that affect output temporarily, whereas supply disturbances have a permanent effect on output, with neither having a long-run effect on the unemployment rate. The shocks that we estimate also exhibit these dynamic properties even though we do not impose them ex ante.

Next, we define *macro risks* as the variables that govern the time-varying variance, skewness and higher-order moments of supply and demand shocks. To model the time variation in these risk factors, we use the Bad Environment-Good Environment model (Bekaert and Engstrom, 2017), which we motivate by showing that it fits the data well relative to extant models, and because it offers a straightforward economic interpretation. In the model, the macro risk factors drive "good-type" (positively skewed) and "bad-type" (negatively skewed) variances of the structural demand and supply shocks. As the good-type variance increases, the distribution for the shock becomes more positively skewed. Increases in bad-type variance may pull skewness into negative territory.

The time-variation in the macro risks allows for the covariance between inflation and real activity to potentially change through time. Theoretically, the sign and magnitude of this covariance are important determinants of the risk premium for nominal bonds. When supply (demand) shocks dominate, real activity and inflation are negatively (positively) correlated, and bonds are a poor (good) hedge against macroeconomic fluctuations, presumably leading to relatively higher (lower) nominal term and risk premiums. This economic intuition has surfaced before (see, e.g., Fama, 1981; Piazzesi and Swanson, 2008; Campbell, Sunderam and Viceira, 2017), but has not been empirically explored.

Our key results for macroeconomic data are as follows. First, we find that the variance of supply shocks was high during the 1970s and again during the Great Recession. Supply shocks do not show much skewness but are leptokurtic. In contrast, macroeconomic variation in the 1980s and 1990s, particularly during recessions, was more strongly dominated by demand shocks, which tend to be substantially negatively skewed. Second, our analysis suggests that the Great Moderation - a reduction in the volatility of many macroeconomic variables since the mid-1980s - is attributed largely to a decrease in good-type demand variance. Meanwhile, the bad-type variance risk factors for both supply and demand shocks have not experienced any secular decline. As a result the frequency and severity of recessions, which are associated with elevated bad-type volatility, have not changed much over our sample. These results offer a refinement to the work of Jurado, Ludvigson and Ng (2015), who find a strong counter-cyclical component to aggregate volatility. Our macro uncertainty measures have a structural "demand" versus "supply" interpretation and generate different higher order (> 2) moments depending on being primarily "good" or "bad". Third, we offer a characterization of the Great Recession of 2008-2009. Some researchers suggest that the Great Recession of 2008-2009 was accompanied by a rather large negative aggregate demand shock (see, e.g., Bils, Klenow, and Malin, 2012, or Mian and Sufi, 2014), but there is little consensus on this issue (see, e.g., Ireland, 2011, or Mulligan, 2012, arguing for the importance of supply shocks). We find that negative demand and supply shocks contributed approximately equally to the Great Recession.

We make several contributions to the asset pricing literature on bond returns. Our first key result is that macro risks are economically and statistically significant predictors of excess bond returns. This is important because Bauer and Hamilton (2017) recently have cast doubt on the statistical predictive power of macro factors for excess bond returns (as originally demonstrated by Ludvigson and Ng, 2009) due to statistical biases. We establish our predictability results using Bauer and Hamilton's conservative bootstrap. Theoretically, although many asset pricing paradigms (e.g., the habit model of Buraschi and Jiltsov, 2007, or the long-run risk model of Bansal and Schaliastovich, 2013) predict that the bond risk premium should be a function of expected second and higher order moments of macroeconomic fundamentals, the vast majority of the empirical literature has surprisingly focused on explaining expected bond returns with the expectations of the level of macroeconomic variables or, even more simply, actual realized macroeconomic data (see, e.g., Ludvigson and Ng, 2009). Notable exceptions are Wright (2011) and Bansal and Shaliastovich (2013). Wright (2011) links term premiums to inflation uncertainty, whereas Bansal and Shaliastovich (2013) link bond risk premiums to consumption and inflation volatility.[1] Compared to these papers, our contribution is twofold. First, we show the importance of decomposing macroeconomic variation into components due to the variance of supply and demand shocks, and into the good and bad types of variance. We find that the time-variation in the macro risk factors for supply and demand implies that the covariance between inflation and real activity changes through time and sometimes switches sign. Our analysis links this time-variation to bond risk premiums by showing that demand (supply) variance negatively (positively) predicts bond excess returns. We also show that while overall the expected excess bond returns are counter-cyclical, an increase in demand (supply) variance is associated with lower (higher) expected returns. However, these cyclicality results are statistically weak. Second, we quantify the relative importance of first and higher order macroeconomic moments for the standard term structure factors (level, slope and curvature). Finally, our novel macro risk factors prove to be statistically significant predictors of future realized bond return variances, and are relatively more important predictors than are level macro factors and factors extracted from the term structure. There is a well-established literature linking equity return variances to macro factors (e.g., Engle, Ghysels and Sohn, 2013), but less

[1] In the equity return predictability literature, there are several papers demonstrating that higher order moments of macro factors predict stock returns (see, e.g., Anderson, Ghysels and Juergens, 2009; Colacito, Ghysels, Meng and Siwasarit, 2016). There is also work on the effect of inflation disagreement embedded in surveys on the term structure (see, e.g., Hong, Sraer, and Yu, 2017, or Ehling et.al., 2018), and heteroskedasticity and disagreement are likely positively correlated.

work on bond return variances.[2]

The remainder of the paper is organized as follows. In section 2, we describe how we theoretically identify aggregate supply and aggregate demand shocks and how we model macro risk factors. Section 3 describes the econometric methodology that we use to extract the structural shocks and the macro risk factors. In Section 4, we provide empirical estimates for the US economy from 1962 to 2016 and a structural interpretation of the macro data using our identification scheme. In Section 5, we link the macro risk factors to term structure data. We also assess whether they have predictive power for excess bond returns and explain term premium behavior. A final section summarizes our key results and sets out an agenda for future research.

2 Modeling Macro Risks

2.1 Aggregate supply and demand shocks in a simplified model

Consider a bivariate system in real GDP Growth (g_t) and inflation (π_t):

$$\begin{aligned} g_t &= E_{t-1}[g_t] + u_t^g, \\ \pi_t &= E_{t-1}[\pi_t] + u_t^\pi, \end{aligned} \tag{1}$$

where E_{t-1} denotes the conditional expectation operator. In a first departure from standard macroeconomic modeling, the shocks to output growth and inflation are a function of two structural shocks, u_t^s and u_t^d:

$$\begin{aligned} u_t^\pi &= -\sigma_{\pi s} u_t^s + \sigma_{\pi d} u_t^d, \\ u_t^g &= \sigma_{gs} u_t^s + \sigma_{gd} u_t^d, \\ &\sigma_{\pi s} > 0, \sigma_{\pi d} > 0, \sigma_{gs} > 0, \sigma_{gd} > 0, \\ &Cov(u_t^d, u_t^s) = 0, Var(u_t^d) = Var(u_t^s) = 1. \end{aligned} \tag{2}$$

The first fundamental economic shock, u_t^s, is an aggregate supply shock, defined so that it moves GDP growth and inflation in opposite directions, as happens, for instance, in episodes of stagflation. The second fundamental shock, u_t^d , is an aggregate demand shock, defined so that it moves GDP growth and inflation in the same direction as would

[2]Baele, Bekaert and Inghelbrecht (2010) is an exception but their focus is on comovements between bond and stock returns.

be the case in a typical economic boom or recession. Supply and demand shocks are assumed to be uncorrelated.

Note that the sample covariance matrix of the shocks from the bivariate system in (1) only yields three unique moments, but we need to identify four coefficients in equation (2) to extract the supply and demand shocks. Hence, absent additional assumptions, a system with Gaussian shocks would be underidentified. Fortunately, it has been well established that macroeconomic data exhibit substantial non-Gaussian features (see, e.g., Evans and Wachtel (1993) for inflation, and Hamilton (1989) for GDP growth). Our second departure from standard macroeconomic modeling is to assume that the demand and supply shocks are potentially non-Gaussian in that they may have non-zero unconditional skewness and excess kurtosis. For example, there are four available unconditional skewness and co-skewness moments for GDP growth and inflation. These four moments, in conjunction with the three available second moments, could in principle be used to identify the four $\sigma_{\pi/g,s/d}$ parameters (and two requisite unconditional skewness coefficients for the supply and demand shocks).

While econometrically it is clear that non-Gaussianity achieves identification (see Lanne, Meitz, and Saikkonen, 2017, for a theoretical paper on obtaining identification through higher-order moments in a VAR), it is useful to clarify the economic sources of identification. Co-skewness moments, for example, are informative. Suppose that demand and supply shocks are negatively skewed (if they are differentially skewed, that information also helps identification). Consider first co-skewness moments, that is, for example in unscaled form, the expectation of the inflation shock squared times the GDP growth shock or vice versa. Such moments only depend on the shock sensitivities and the third moments of supply and demand shocks and thus would be zero under Gaussianity. In particular,

$$\begin{aligned} E[u_t^g (u_t^\pi)^2] &= \sigma_{gd}\sigma_{\pi d}^2 E[(u_t^d)^3] + \sigma_{gs}\sigma_{\pi s}^2 E[(u_t^s)^3], \\ E[(u_t^g)^2 u_t^\pi] &= \sigma_{gd}^2\sigma_{\pi d} E[(u_t^d)^3] - \sigma_{gs}^2\sigma_{\pi s} E[(u_t^s)^3]. \end{aligned} \tag{3}$$

Suppose the skewness of demand and supply shocks is similar (and, recall, negative). In this case, the $E[u_t^g (u_t^\pi)^2)]$-moment has a negative contribution coming from both supply shocks (as the movements of inflation and GDP growth in opposite directions are cancelled) and demand shocks. However, the $E[(u_t^g)^2 u_t^\pi]$ moment retains its negative contribution from demand shocks but obtains a positive contribution from supply shocks (as the negative skewness is multiplied by shock exposures of opposite sign). Therefore, skewed structural shocks should result in different magnitudes of these two co-skewness moments, with the inflation squared moment much more negative than the GDP growth squared moment. The exact relative magnitude of these two moments then reveals information about the sensitivity of the macro shocks to the structural shocks.

A particularly intuitive case would be one where the supply shocks are relatively Gaussian (zero skewness) and the demand shock relatively non-Gaussian (and negatively skewed). Suppose for ease of exposition that the skewness of supply shocks is literally zero (which, as we will see, is not far from the truth). Then, given the value of demand skewness, the two co-skewness moments would admit identification of $\sigma_{\pi d}$ and σ_{gd}. If $E[u_t^g(u_t^\pi)^2]$, the "inflation squared" moment, is much more negative than $E[(u_t^g)^2 u_t^\pi]$, the "GDP growth squared" moment, inflation must be more sensitive to demand shocks than are GDP growth shocks and vice versa.

Of course, the variance of demand and supply shocks is likely to be time-varying. In this case, the model also implies that the conditional variance between inflation and GDP growth shocks is time-varying and can switch signs:

$$Cov_{t-1}[u_t^g, u_t^\pi] = -\sigma_{\pi s}\sigma_{gs} Var_{t-1} u_t^s + \sigma_{\pi d}\sigma_{gd} Var_{t-1} u_t^d, \tag{4}$$

where the subscripts on the Cov and Var operators denote that they may vary over time. Thus, when demand shocks dominate the covariance is positive but when supply shocks dominate it is negative.

2.2 The Interpretation of the Macro Shocks

The main advantage of the supply and demand shocks definition above is that it carries minimal theoretical restrictions (only a sign restriction)[3]. However, these supply and demand shocks definitions do not necessarily correspond to demand and supply shocks in, say, a New Keynesian framework (see e.g. Woodford, 2003) or identified VARs in the

[3]The idea to impose a minimal set of sign restrictions to achieve identification is reminiscent of Uhlig's (2005) identification scheme for monetary policy shocks. Gali (1992) uses sign restrictions similar to ours in a VAR setting but does not obtain identification through non-Gaussianity.

Sims tradition (Sims, 1980).[4] The classic Blanchard and Quah (1989) paper famously identifies "demand like" shocks as those that affect output only temporarily whereas supply disturbances have a permanent effect on output, with neither having a long run effect on unemployment rate. However, Blanchard (1989) notes that these short- and long-run effects of supply and demand shocks are consistent with responses to shocks in the context of standard Keynesian models. For instance, supply shocks include productivity shocks which tend to have a longer run effect on output. We reverse the identification strategy here, by first exploiting the sign restrictions to identify the shocks, and then verifying their long-run impact on inflation and real activity in subsequent analysis. Furthermore, in this paper we abstract from further economic interpretation of demand and supply shocks and their sources. Such analysis would be of great economic interest, but would require an advanced general equilibrium model which tends to be highly stylized and can not accommodate meaningful time variation in higher order moments (see, e.g., van Binsbergen et.al., 2012).

While we include interest rates in our main specification (mostly to help predict macro variables), we do not formally define a monetary policy shock and such shocks may be correlated with or even help drive our macro shocks.[5] Given that our foremost goal is to differentiate shocks that make real activity and inflation move in either the same or opposite directions, criticisms of the interpretation of standard New Keynesian models when interest rates are at or near the zero lower bound do not apply here either (see Eggertsson, Ferrero and Raffo, 2014).

2.3 Modeling Macro Risks

We define macro risk factors as the time-varying determinants of the second and higher-order moments of supply and demand shocks. We parameterize the distribution of supply and demand shocks using a model that accommodates conditionally non-Gaussian distributions, the Bad Environment-Good Environment (BEGE) model (Bekaert and Engstrom, 2017).

[4]Furthermore, in some models the "supply" shocks might move real activity and inflation in the same direction: see, for instance, news shocks in Cochrane (1994).

[5]We also therefore do not take a stand on potential monetary policy regime changes (see Bekaert et al, 2015). Our empirical strategy assumes stationarity and uses reduced form VARMA estimation to deliver the conditional mean dynamics, which is appropriate given the Wold theorem.

2.3.1 Bad Environment - Good Environment Model

Following a BEGE structure, demand and supply shocks are component models of two independent distributions:

$$u_t^s = \sigma_p^s \omega_{p,t}^s - \sigma_n^s \omega_{n,t}^s,$$
$$u_t^d = \sigma_p^d \omega_{p,t}^d - \sigma_n^d \omega_{n,t}^d, \tag{5}$$

where t is a time index, and σ_p^s, σ_n^s, σ_p^d, and σ_n^d are positive constants. We use the notation:

$$\begin{aligned}
\omega_{p,t+1}^d &\sim \tilde{\Gamma}(p_t^d, 1), \\
\omega_{n,t+1}^d &\sim \tilde{\Gamma}(n_t^d, 1), \\
\omega_{p,t+1}^s &\sim \tilde{\Gamma}(p_t^s, 1), \\
\omega_{n,t+1}^s &\sim \tilde{\Gamma}(n_t^s, 1),
\end{aligned} \tag{6}$$

to denote that $\omega_{p,t}^d$ follows a centered gamma distribution with shape parameter p_t^d and a unit scale parameter. The corresponding probability density function, $\phi(\omega_{p,t}^d)$, is given by:

$$\phi(\omega_{p,t+1}^d) = \frac{1}{\Gamma(p_t^d)} (\omega_{p,t+1}^d + p_t^d)^{p_t^d - 1} exp(-\omega_{p,t+1}^d - p_t^d),$$

for $\omega_{p,t+1}^d > -p_t^d$; with $\Gamma(\cdot)$ representing the gamma function. Similar definitions apply to $\omega_{n,t+1}^d$, $\omega_{p,t+1}^s$, and $\omega_{n,t+1}^s$. Unlike the standard gamma distribution, the centered gamma distribution has mean zero. For such a distribution, the shape parameter equals the variance of the random variable.

The top panel of Figure 1 illustrates that the probability density function of $\sigma_p^d \omega_{p,t}^d$ (the "good" component of the demand shock) is bounded from the left and has a right tail. Similarly, the middle panel of Figure 1 shows that the probability density function of $-\sigma_n^d \omega_{n,t}^d$ (the "bad" component) is bounded from the right and has a left tail. Finally, the bottom panel of Figure 1 plots the component model of these two components which has both tails. The components of u_t^s have the same distributional properties. Hence, we define a "good" ("bad") shape parameter as one associated with a ω_p (ω_n)-shock.

The good (p_t^d, p_t^s) and bad (n_t^d, n_t^s) shape parameters of our macro shocks are assumed to vary through time in an autoregressive fashion as in Gourieroux and Jasiak (2006):

$$
\begin{aligned}
p_t^d &= \bar{p}^d(1-\phi_p^d) + \phi_p^d p_{t-1}^d + \sigma_p^d \omega_{p,t}^d, \\
p_t^s &= \bar{p}^d(1-\phi_p^s) + \phi_p^s p_{t-1}^s + \sigma_p^s \omega_{p,t}^s, \\
n_t^d &= \bar{n}^d(1-\phi_n^d) + \phi_n^d n_{t-1}^d + \sigma_n^d \omega_{n,t}^d, \\
n_t^s &= \bar{n}^s(1-\phi_n^s) + \phi_n^s p_{t-1}^s + \sigma_n^s \omega_{n,t}^s.
\end{aligned} \tag{7}
$$

Note that positive $\omega_{p,t}^d$ shocks drive up GDP growth, as do the $\omega_{p,t}^s$ shocks, and those shocks are associated with an increase in both p_t^d and p_t^s. We call this "good volatility" because it induces more positive skewness in GDP growth. Conversely, positive realizations of $\omega_{n,t}^d$ and $\omega_{n,t}^s$ shocks drive down GDP growth and they are associated with an increase in "bad" volatility and more negative skewness. This explains the "BEGE" moniker.

Using the demand shock as an example, Figure 2 illustrates possible conditional distributions of demand shocks which could arise as a result of the time variation in shape parameters in equation (7). In particular, the probability density function in the top panel of Figure 2 characterizes the situation where good volatility is relatively large and the component distribution has a pronounced right tail, while the probability density function in the bottom panel of Figure 2 corresponds to the case where bad volatility is relatively large and the component distribution exhibits a pronounced left tail.

2.3.2 Conditional Moments under the Bad Environment-Good Environment Model

At this point, we have set out an economy with four shocks ($\omega_{p,t}^d$, $\omega_{n,t}^d$, $\omega_{p,t}^s$, and $\omega_{n,t}^s$) and four state variables, which we collect in $X_t^{mr} = [p_t^s, n_t^s, p_t^d, n_t^d]'$. These four state variables summarize the macroeconomic risks in the economy. Using the properties of the centered gamma distribution, we have, for example:

$$
\begin{aligned}
& E_{t-1}[u_t^s] = 0, \\
& E_{t-1}[(u_t^s)^2] = (\sigma_p^s)^2 p_t^s + (\sigma_n^s)^2 n_t^s, \\
& E_{t-1}[(u_t^s)^3] = 2(\sigma_p^s)^3 p_t^s - 2(\sigma_n^s)^3 n_t^s, \\
& E_{t-1}[(u_t^s)^4] - 3(E_{t-1}[(u_t^s)^2])^2 = 6(\sigma_p^s)^4 p_t^s + 6(\sigma_n^s)^4 n_t^s.
\end{aligned} \tag{8}
$$

And analogously for u_t^d.

Thus, the BEGE structure implies that the conditional variance of inflation and output vary through time, with the time-variation potentially coming from either demand or supply shocks, and either bad or good volatility. In addition, the distribution of inflation and output shocks is conditionally non-Gaussian, with time variation in the higher order moments driven by variation in X_t^{mr}.

2.4 The Full Model

A model with only two macroeconomic variables such as the one presented above would be too narrow for our purposes and our estimates of supply and demand shocks are based on a more extensive model of the macroeconomy. First, we consider a four variable macro model, rather than a two variable system, adding core inflation and the unemployment gap. Core inflation, which strips out components of overall inflation that are particularly volatile such as energy and food prices, is, of course, a variable that is closely followed by monetary policy makers. Core inflation has been shown to be useful in forecasting future inflation. Ajello, Benzoni and Chyhruk (2012) in fact claim that adding core inflation to a macro system results in inflation forecasts that are as accurate as forecasts based on survey data (see Ang, Bekaert and Wei, 2007, for more on the accuracy of survey based inflation forecasts). This is relevant, because we use quarterly data starting in 1962 and thus cannot easily use survey forecasts (for instance, the quarterly Survey of Professional Forecasters started in 1969). Analogously, for many practitioners, the unemployment rate gap is preferred to GDP growth as an indicator of economic activity. Moreover, as Bauer and Rudebusch (2016) demonstrate, this variable is in fact little correlated with GDP growth and contains useful alternative information about real economic activity.

Because we want to identify shocks to these four variables, it is important that we specify their conditional means carefully. Bond yields have well-established predictive power for economic variables (see Harvey, 1988, and many others, for the predictive ability of the term spread for GDP growth, for example) prompting us to add yields to our set of state variables. Specifically, the vector X_t consists of the 4 macro variables, and the one quarter and 10-year Treasury yields.

We use a VARMA model to extract macroeconomic shocks from X_t:

$$X_t = B(L)X_{t-1} + C(L)u_t. \tag{9}$$

Furthermore:

$$u_t = \Sigma u_t^m + \Omega e_t \tag{10}$$

where $u_t^m = [u_t^s, u_t^d]$, the structural shocks, and Σ is a 6x2 matrix containing the exposures of macroeconomic and yield shocks to AS/AD shocks. In particular, we also impose the sign restrictions discussed earlier on Σ where the left 4x2 block is:

$$\begin{bmatrix} -\sigma_{\pi s} & \sigma_{\pi d} \\ \sigma_{gs} & \sigma_{gd} \\ -\sigma_{\pi^c s} & \sigma_{\pi^c d} \\ -\sigma_{u^e s} & -\sigma_{u^e d} \end{bmatrix},$$

and all $\sigma_{i,j}$ coefficients are positive. The vector e_t represents shocks uncorrelated with u_t, with mean zero, unit variance and zero skewness and excess kurtosis and Ω is diagonal except for the interest rate block.[6] It is necessary to add these uncorrelated innovations to the macro series to avoid having a singularity in their covariance matrix. We assume that these orthogonal shocks have zero skewness and excess kurtosis mostly for convenience, but this assumption also aids in the identification of the supply and demand shocks. That is, all the excess skewness and kurtosis among the macro variables must solely arise from the structural shocks. Note that the orthogonal shocks may not just represent measurement error (as, e.g., in Wilcox, 1992). They may also represent important variation, not modeled in our framework, such as that arising from monetary policy shocks, stressed, e.g., in Campbell, Pflueger and Viceira (2015).

3 Identifying Macro Risks in the US economy

While there are multiple ways to estimate the system in equations (2), (5), (7), (9), and (10), the presence of the gamma distributed shocks makes the exercise nontrivial. We therefore split the problem into three manageable steps. First, we use standard

[6]This interest rate block will only be relevant in the impulse response analysis described below.

techniques to estimate the VAR model and determine its order. Second, we filter the demand and supply shocks from the system in equation (10) by estimating a GMM system that includes higher-order unconditional moments of the macroeconomic variables. The use of third- and fourth-order moments is essential to achieve identification in our framework and has a strong economic motivation as well. Third, once the demand and supply shocks are filtered, we can estimate univariate BEGE systems on supply and demand shocks (exploiting the identifying assumption that they are independent) using approximate maximum likelihood as in Bates (2006). Importantly, the three steps are internally consistent.

A disadvantage of using a multi-step estimation process is that statistical inference is complicated by the fact that all steps after the first one use pre-estimated coefficients or filtered variables that are subject to sampling error. To account for these errors, we also execute the entire multi-step estimation process using data bootstrapped under the estimated parameters. The bootstrap procedure is described in Appendix A. Theoretically, our model could be estimated in one step using Bayesian methods. However, given the high dimensionality of the parameter space, it is not computationally feasible without tight priors. We begin by describing the data we use.

3.1 Data

The data are quarterly from 1962:Q2 to 2016:Q4 (219 quarters). Potentially, we could have included data back to 1947:Q1 (the starting date for GDP data). The later start date is chosen to exclude a period when there was higher measurement error in the GDP data (Bureau of Economic Analysis, 1993). Moreover, US long-term rates were pegged by the Federal Reserve prior to the Treasury Accord of 1951. For inflation (core inflation) we use 100 times log changes in the headline CPI index (CPI excluding food and energy) measured for the last month of each quarter, from the Bureau of Labor Statistics (BLS). Real GDP growth is 100 times the log difference in real GDP (in chained 2009 dollars) from the Bureau of Economic Analysis. The unemployment rate gap is the difference between the unemployment rate (in percent) from the last month of each quarter from the BLS, and the estimated level of the natural rate of unemployment published by the Congressional Budget Office.

Interest rate data consists of yields, prices and returns for nominal U.S. Treasury securities. For maturities of length 1 quarter and 1, 2, 3, 4 and 5 years, estimated yields for zero-coupon securities are taken from the Fama-Bliss (1987) data set (part of the CRSP). For yields of maturity 10 years, data from 1962:Q2 through 1971:Q1 are from the McCullough-Kwon (1993) data set. From 1971:Q1-2016:Q4, data for 10-year yields are from Gürkaynak, Sack, and Wright (2010). Yields at maturities other than those discussed above are estimated by linear interpolation. We use continuously compounded yields, expressed as annualized percentages.

3.2 Estimating VAR(p) and VARMA (p, q) models

To estimate the time series model for X_t, including inflation, real GDP growth, core inflation, the unemployment rate gap and short- and long-term interest rates, we first de-mean the variables. We then choose from a set of time series models, in particular, VARMA(i,j) for $i = 1, 2, 3$ and $j = 0, 1, 2, 3$, using standard information criteria. We only consider diagonal ("own lag") specifications for the MA components. As emphasized, for instance, by Dufour and Pelletier (2014), any identified VARMA model can be represented by using full (unrestricted) VAR specifications together with a sufficient number of diagonal MA terms.

Because some of these models are heavily parameterized (the highest-order ones have over 100 parameters), in our estimation we employ a two-step projection-based procedure that was proposed by Hannan and Rissanen (1982) rather than attempting to maximize a likelihood function. Specifically, we first estimate by OLS a vector-autoregression with a large number of lags. We use 6 lags, but that choice does not appear material for the results. We then recover the estimated residuals from this step, $\hat{u}_t$. These residuals serve as a "plug-in" estimator of lagged shocks for the VARMA model, and then we estimate the VARMA model by OLS. We again recover the residuals from this step, providing new estimates of $\hat{u}_t$. This procedure is repeated until all of the estimated parameters of the VARMA and all of the estimated residuals converge, which we define as changing by less than 1e-6.

Model selection criteria are reported in Table 1. We use the standard Bayesian information criterion (BIC), but the Akaike information criterion (AIC) is modified to correct for small sample biases (Sugiura, 1978; Burnham and Anderson, 2004). The AIC model identifies the VAR(2) model as optimal. The BIC criterion identifies the VAR(1) model as optimal, but the VAR(2) comes in second place. We proceed by using the VAR(2) specification to identify shocks to the macro variables.

3.3 Identifying supply and demand shocks

3.3.1 Methodology

The VAR(2) model delivers time series observations on u_t. Theoretically, it is possible to estimate the system defined by equations (2), (5), (7), (9), and (10) in one step, but computationally this is a very tall order. There are 4 unobserved state variables (the X_t^{mr} vector) which have non-Gaussian innovations. However, note that if we can identify the coefficients in Σ in equation (10), we can filter the supply and demand shocks from the original macro shocks u_t. With these structural shocks in hand, we can estimate univariate BEGE systems on each of demand and supply shocks separately.

We use information in 2^{nd}, 3^{rd} and 4^{th} order unconditional moments of the reduced-form macroeconomic shocks to identify their loadings onto supply and demand shocks in a classical minimum distance (CMD) estimation framework (see, e.g., Wooldridge, 2002, pp. 445-446). Specifically, we calculate 48 statistics using the four macroeconomic shocks. These are the unconditional standard deviations (4), correlations (6), univariate (scaled) skewness and excess kurtosis (8), selected co-skewness (12), and selected co-excess kurtosis measures (18).[7]

With 48 moments to match and many fewer parameters in the structural model of equation (10), our system is substantially overidentified, thus requiring a weighting matrix. To generate a weighting matrix, we begin with the covariance matrix of the sampling error for the statistics. To calculate the covariance matrix, we use a block bootstrapping routine. Specifically, we sample, with replacement, blocks of length 20 quarters of the 4 variable - vector of macroeconomic shocks, to build up a synthetic sample of length equal to that of our data. We calculate the same set of 2^{nd}, 3^{rd}, and 4^{th} order statistics for each of 10,000 synthetic samples. We then calculate the covariance matrix of these statistics across bootstrap samples. In principle, the inverse of this covariance matrix should be a good candidate as a weighting matrix for our CMD system. However, inspecting the bootstrapped covariance matrix, we found that the sampling errors for some statistics are highly correlated, leading to ill-conditioning of the covariance matrix. We therefore used a diagonal weighting matrix with the inverses of the bootstrapped variances of the

[7]We exclude third and fourth order moments that involve more than two different shocks such as $E(x_1 \times x_2 \times x_3)$.

moments on the diagonal and zero elsewhere.[8]

Table 2 reports the higher-order moments we use in the estimation. Not surprisingly, all volatility statistics are statistically significantly different from zero, but so are the coefficients of excess kurtosis. However, among the skewness coefficients, only the positive skewness of shocks to the unemployment gap is statistically significant while 4 of 12 co-skewness coefficients are significant. Over half of the co-kurtosis measures are statistically significant. The p-value for the joint significance of all the 3^{rd} and 4^{th} order moments is < 0.0001, which we interpret as a strong rejection of the hypothesis that the data are distributed unconditionally according to a multivariate Gaussian distribution.

We next use the information in these higher order moments to identify the loadings on our supply and demand shocks. We estimate a total of 13 parameters using our 48 estimated statistics. These can be grouped into three sets:

- The loadings of four macro shocks onto supply and demand shocks (8 parameters) in the matrix Σ in (10), imposing the sign restrictions described above.
- The share of variation of the macro shocks that comes from idiosyncratic variation or measurement error, that is the matrix Ω in (10)). We assume this share is constant across the four variables (1 parameter). We do this to impose a prior that all 4 series contribute (jointly) to demand and supply shocks. If we do not impose this restriction, the system tends to drive the variance of idiosyncratic factors to zero for the less noisy macro series, in which case the noisier macro series (such as real GDP growth) do not contribute much to the identification of supply and demand shocks.
- The skewness and kurtosis of the supply and demand shocks (4 parameters). Note that we do not assume a parametric model for the distribution of supply and demand shocks at this stage: we simply estimate their skewness/kurtosis coefficients as free parameters.

[8]This weighting matrix is not asymptotically efficient and it also does not reflect sampling error associated with the VAR(2) parameters that were used to identify the macroeconomic shocks, but it ensures that all moments receive an easily interpretable positive weight in the objective function.

3.3.2 Economic intuition behind higher-order moments

The economic intuition behind using unconditional skewness moments was emphasized in Section 2. However, asymmetric co-kurtosis moments (e.g., the expectation of the inflation shock to the third power times the GDP growth shock or the analogous reverse moment) are also very informative about the coefficients. Under Gaussianity, these moments in scaled form only depend on the correlation and therefore contain no new information about the distribution. Consequently, we compute "excess" co-kurtosis which is zero under Gaussianity but non-zero under a non-Gaussian distribution. In particular, in unscaled form:

$$\begin{aligned} E[u_t^g(u_t^\pi)^3] - 3E[u_t^g u_t^\pi]E[(u_t^\pi)^2] &= \sigma_{gd}\sigma_{\pi d}^3(E[(u_t^d)^4] - 3) - \sigma_{gs}\sigma_{\pi s}^3(E[(u_t^s)^4] - 3), \\ E[(u_t^g)^3 u_t^\pi] - 3E[u_t^g u_t^\pi]E[(u_t^g)^2] &= \sigma_{gd}^3\sigma_{\pi d}(E[(u_t^d)^4] - 3) - \sigma_{gs}^3\sigma_{\pi s}(E[(u_t^s)^4] - 3), \end{aligned} \tag{11}$$

where constants occur due to the expectations of the structural shocks' second moments being equal to 1.[9] Because a large negative supply shock increases inflation and decreases GDP growth, supply kurtosis decreases both the asymmetric kurtosis moments, but demand kurtosis with inflation and GDP growth moving in the same direction increases them. The relative magnitude of these two co-kurtosis moments thus is very informative about the sensitivity of the macro shocks to the structural shocks. Suppose the kurtosis in supply and demand shocks is similar, then a relative high inflation sensitivity to supply shocks relative to its sensitivity to demand shocks lowers $E[u_t^g(u_t^\pi)^3]$, the co-kurtosis moment with inflation to the third power, much more than $E[(u_t^g)^3u_t^\pi]$, the moment with GDP growth to the third power, and vice versa, all else equal. Since we have already established that there are highly significant co-skewness and co-kurtosis moments, identification is assured.

3.3.3 Empirical results

Table 2 shows that our CMD estimation misses only one moment by more than 1.96 standard errors (the fitted value for real GDP growth skewness is negative, whereas the sample value is positive, though not significantly so, a miss of 2.03 standard errors). Nevertheless, the test of the overidentifying restrictions does reject at the 10 percent level (p-value of 8.63 percent), showing that higher order moments indeed have statistical

[9]Analogously, for scaled symmetric co-kurtosis moments, we subtract $(1 + 2\rho)$, where ρ is the correlation between inflation and GDP growth, to obtain a moment which is zero under Gaussianity.

"bite".

In Table 3, Panel A, we report the supply and demand loadings for the various macro variables. These are generally quite precisely estimated. Our estimates suggest that demand shocks contribute more to the unconditional variance of inflation shocks than supply shocks. Real GDP growth, core inflation, and the unemployment gap all load roughly evenly on supply and demand shocks. We estimate the share of idiosyncratic variation for the four series to be relatively high at 44 percent.

Based on these loadings, we invert the supply and demand shocks from the macro shocks using a constant linear filter:

$$\begin{aligned} u_t^m &= K u_t, \\ K &= \Sigma'_{4\times 2}(\Sigma_{4\times 2}\Sigma'_{4\times 2} + \Omega_{4\times 4}\Omega'_{4\times 4})^{-1}, \end{aligned} \tag{12}$$

where u_t and u_t^m are the vectors of macro and structural shocks, respectively, as in (10), Σ is the 4×2 loading of the macro shocks onto the supply and demand factors, and Ω is a diagonal 4×4 matrix of loadings onto the idiosyncratic shocks (corresponding to the 4 top rows of the matrices Σ and Ω in equation (10)). These loadings are what we would obtain under, for instance the Kalman filter, which generates minimum root mean squared error (RMSE) estimates among linear filters with constant gain. Table 3, Panel B, reports Kalman gain coefficients, which are all of the intuitive sign.

In Panel C of Table 3, we show a variance decomposition illustrating how much of the demand/supply shock variance is accounted for by the four macro variables. That is, we compute, for example, $\frac{Cov(u_t^d, K_{d,\pi}\pi_t)}{Var(u_t^d)}$, where $K_{d,\pi}$ is the Kalman gain coefficient on inflation for the demand shock. By construction, these variables add up to one. The results show that the four different series all contribute nontrivially to the structural shocks. Inflation shocks contribute substantially more to the identification of demand shocks than they do for supply shocks, with the other reduced-form shocks contributing more evenly across supply and demand.

Finally, in Panel D of Table 3, we report the skewness and kurtosis of the filtered supply and demand shocks. Both shocks are leptokurtic but the demand shock is negatively skewed whereas the supply shock has essentially zero skewness. The departure from the Gaussian distribution of the demand shocks is clearly more pronounced than that of the supply shock. Yet, a standard (small sample corrected) Jarque-Bera test rejects the null of normality with p-values 0.015 and < 0.001, respectively for supply and demand shocks.

3.4 Estimating Macro Risk Factors

Note that the identification scheme for structural shocks described above is completely model-free, making our methodology applicable with any statistical model which can accommodate non-Gaussian unconditional moments in the data. Given the structural shocks, we are left to identify the BEGE model parameters. We use an estimation and filtering apparatus due to Bates (2006). The methodology is similar in spirit to that of the Kalman filter, but the Bates routine is able to accommodate non-Gaussian shocks. The details of the estimation are in Appendix C. Before describing the BEGE estimation results in detail, we compare the performance of the BEGE model to that of more well-known stochastic processes that can also generate unconditional distributions that exhibit departures from Gaussianity.

3.4.1 Model Comparision

We test the performance of the BEGE model to examine whether it fits the estimated supply and demand shocks as well as more well-known models that also feature time-varying second- and higher-order moments. Specifically, we look at the performance of the BEGE model relative to regime-switching models of the Hamilton (1989)-type, and a model of Gaussian stochastic volatility. To evaluate the relative performance of the models, we use standard BIC and AIC (with the usual small sample correction) criteria.[10] These results are presented in Table 4. Within the class of regime switching models, a two-state model performs better than three and four state models, so we only show results for the two-state model. Similarly, for BEGE models, the model with all shape parameters time-varying outperforms the simpler models, so we focus on the full BEGE model in the table. As shown in the top panel, for the supply shock, the BEGE model performs worst using either AIC or BIC criteria. However, for demand shocks, as reported in the middle panel, the BEGE model outperforms both models on both criteria. When examining the performance jointly across supply and demand shocks (recalling that the two shocks are modeled as independent, so the joint log likelihood of the bivariate process is just the sum of the two univariate log likelihoods), the BEGE

[10]Because the BEGE and the Gaussian stochastic volatility models are estimated using approximate maximum likelihood as in Bates (2006), the comparison of these models to the regime switching models, which are estimated using exact maximum likelihood, is only informal.

model outperforms the other two models using the AIC, but the stochastic volatility model, which is very parsimonious, wins when using the BIC criterion. We conclude that the BEGE model generally performs well in this competition, and we carry forward its conditional estimates of good and bad volatility for the subsequent analysis.

3.4.2 Parameter Estimates

The parameter estimates for the BEGE model are reported in Table 4. For the demand shock, the parameters governing the "good environment" state variable, p_t, generate behavior similar to that of a Gaussian stochastic volatility model. The unconditional mean of the process, $\bar{p}$, hits an upper bound fixed at 20. Recall that p_t is the shape parameter for one of the two component gamma distributions for demand shocks. With the shape parameter of over 10, the gamma distribution appears nearly Gaussian and further increases in the shape parameter do not substantially change the shape of the distribution. Our filtered values for p_t^d do vary substantially over time, but rarely does the process dip much below 10, suggesting that good variance for supply is nearly always close to Gaussian. That said, there is substantial variation in the level of the process over time and strong autocorrelation, with a persistence parameter of nearly 0.94. The properties of the bad environment state variable for demand shocks, n_t, contrasts sharply with those of p_t. The unconditional mean of n_t is just 0.34. This implies that the bad environment variable is very non-Gaussian. In particular, its unconditional skewness is $\frac{2}{\sqrt{\bar{n}^d}}$, or 3.45 and its kurtosis is $\frac{6}{\bar{n}^d}$ or 17.86. (Recall that because demand shocks load negatively onto the bad-environment shocks by construction, this generates substantial negative skewness for demand shocks.) The bad environment shape parameter is also less persistent than the good environment variable, therefore capturing rather short-lived recessionary bursts (0.72 versus 0.94 autocorrelation).

The BEGE parameter estimates for supply shocks are broadly similar to those for demand shocks. The mean of p_t hits the upper bound of 20, and the filtered values for the process rarely dip below 10, suggesting nearly Gaussian innovations, albeit with substantial variation in volatility. Good supply variances are very persistent with an autoregressive coefficient of nearly 0.99. The supply bad-environment distribution is substantially non-Gaussian with the unconditional mean of the shape parameter equal to 4.00. This implies unconditional skewness of 1.00. The shock has similar persistence to the bad environment demand shock, suggesting that supply driven recessions may have similar duration to demand driven recessions.[11]

The bootstrap procedure in Appendix A allows us to verify the small sample properties of our estimation approach. In unreported results, we find that average estimates over the bootstrap samples are essentially unbiased. Bootstrap standard errors in Panel A of Table 3 and Table 5 confirm that the parameter variation across the samples is reasonable.

4 Macro Risks in the US Economy

Having estimated macroeconomic dynamics, we can now use our model as a lense to interpret the history of key U.S. macroeconomic data over the 1962-2015 period. We begin by characterizing the long-run effects of supply and demand shocks; we subsequently analyze the nature of recessions within our framework, followed by examining the time series and cyclical behavior of the macro risk factors themselves.

4.1 Impulse responses to aggregate supply and demand shocks

Our identification of supply and demand shocks utilizes a set of minimal linear sign restrictions and information in higher order moments. These sign restrictions are present in other classic papers as well, such as Gali (1992) and Shapiro and Watson (1988) but are typically accompanied by a set of additional economic restrictions (e.g., that demand shocks have no long run effect on the level of GDP as in the classic Blanchard and Quah (1989) paper) which we do not need. In this section, we characterize the long run effects of the structural shocks using standard impulse response analysis.

For the purposes of calculating impulse response functions for the macro data, we use our estimated VAR(2) parameters. To compute the response of the four macroeconomic series at various horizons to the supply and demand shocks, we need the contemporaneous response of all the variables to supply and demand shocks. For the four macroeconomic series, these responses are the row elements of the Σ matrix corresponding to macro data in equation (10). For the two yield variables, we extract the time series for reduced-form shocks from the VAR(2)-estimation and simply regress these shocks onto the filtered supply and demand shocks. The responses of the six endogenous variables to the two structural shocks, supply and demand, of unit size at horizon h, are given by the expres-

[11] The astute reader will notice that seven parameters are reported for the supply and demand processes, but there are only six independent parameters required for the estimation, because the unconditional variance of demand and supply shocks is restricted to equal 1. However, $\bar{n}$-parameters can be expressed as functions of the other model parameters. Their standard errors are calculated using the delta method.

sion:

$$IR(h) = (A_1^h + A_2^{\max(h-1,0)})\Sigma, \tag{13}$$

where A_1 and A_2 are lag 1 and 2 AR matrices from the VAR(2)-model. Note that the standard error for the impulse response coefficients must account not only for the estimation of the VAR(2) parameters but also for the error incurred in identifying supply and demand shocks, which involves the higher order moments of VAR residuals. To this end, we use a bootstrap procedure, which is described in detail in Appendix A. As a robustness check, in Appendix B we also calculate "model-free" impulse responses following Jorda (2005).

Table 6 contains the results, with the effects of demand (supply) shocks on the left (right) (recall that these shocks have unit variance by construction). The effects are consistent with the standard Keynesian interpretation. Demand shocks have large short run effects on real GDP growth (with the initial shock being 0.40 percent) but their cumulative effect on output is small (0.09 percent) and insignificantly different from zero. Supply shocks generate smaller short run GDP growth effects but their cumulative effect is 0.52 percent which is significantly different from zero. Demand and supply shocks have very different effects on the price level, with the cumulative effects close to +2 percent in the case of demand shocks, but the supply shock effect peters out to zero. In sum, our identification scheme yields shocks whose long-run effects are consistent with a well-established macroeconomic literature.

4.2 Characterizing recessions using aggregate supply and demand shocks

Our identification of supply and demand shocks allows us to characterize recessions as either supply or demand driven (or a combination of both). Figure 3 graphs the filtered demand and supply shocks with NBER recessions shaded: it is apparent that many recessions are accompanied by a negative supply shock, but this appears more prevalent in the seventies. A large negative demand shock is very apparent for the Great Recession, but the recessions in the early eighties were also accompanied by large negative demand shocks.

Table 7 quantifies the visual impression by simply adding up the (net) demand and supply shocks over the recession period (that is, positive and negative shocks can cancel each other out). The 1973-75 recession did not feature negative demand shocks but all the other recessions did, with the 1981-82 and Great Recession featuring the largest negative demand shocks. All recessions featured negative supply shocks, with the largest negative shocks occurring in the 1973-1975 recession and the Great Recession. For the 1981-82 recession, the cumulative supply effect is quite small however. On a relative basis, the first three recessions were predominantly supply driven whereas three of last four were more demand driven (the exception being the 1990-91 recession). For the first five recessions, these results are broadly consistent with Gali's (1992) results, who also characterizes the 1973-75 recession as mostly supply driven and the 1981-82 recession as mostly demand driven. Our results for the Great Recession assign a perhaps surprisingly large role to supply shocks, but this is not inconsistent with the results in Ireland (2011) or Mulligan (2012), for example. At the same time, recent work by Bils, Klenow and Malin (2012) and Mian and Sufi (2014) using micro data stresses lower aggregate demand as the main cause of the steep drop in employment during the Great Recession.

The Great Recession of 2008-2009 stimulated much research on the effects of macroeconomic uncertainty on the economy (see, e.g., Ludvigson, Ma, and Ng, 2016; Carriero, Clark and Marcellino, 2016). The BEGE structure implies that shocks to supply and demand are correlated with changes in the macroeconomic risk factors. For example, the shock to the bad volatility risk factor is perfectly conditionally correlated with the bad demand shock (see equations (5) and (7)), so that uncertainty shocks affect the levels of macroeconomic variables by assumption. We therefore also investigate the behavior of the macroeconomic risk factors during recessions. Our model implies that the total conditional variance of demand and supply shocks are the sum of the good and bad components. These are plotted in Figure 4. The good demand variance (see Panel A) was relatively high in the 70s and the early 80s, and then decreased to low levels consistent with the Great Moderation (a further discussion of the Great Moderation is below). The bad demand variance shows much less pronounced low frequency variation but increases in most recessions with notable peaks in the 1981-82, 2001, and the recent Great Recession. It also shows short-lived peaks twice in the decade between 2000 and the beginning of the Great Recession.

Panel B of Figure 4 performs the same exercise for supply variances. The level of good variance does not show much time-variation but is more elevated up until mid-

1980s after which it appears to trend down. The bad supply variance appears higher in the stagflationary episodes of the 1970s, but it peaks in most recessions. Its increase in the Great Recession is extreme, starting towards the end of the period and exceeding its unconditional average level of 0.46 until 2012Q1.[12] The secular decline that one might associate with the Great Moderation appears to come from the good variances of both supply and demand shocks.

Panel C of Figure 4 plots together the conditional variances of demand and supply shocks. Given that both supply and demand shocks have unit variance, the graph immediately gives a sense of which variance dominates. In terms of "variance" peaks, the 1981-82, and Great Recession are dominated by demand variances, the other recessions by supply variance peaks.

One novel feature of our model is that it accommodates and provides estimates of the non-Gaussian features of the shocks. In particular, in environments dominated by elevated levels of bad supply variance, we would expect high-inflation scares and positive inflation skewness, whereas in aggregate demand environments, we may witness negative inflation skewness (deflation scares). For the real activity variables, recessions, being riskier macro environments, should be naturally accompanied by negative skewness for real GDP growth and positive skewness for the unemployment gap. Figure 5 graphs the (scaled) conditional skewness for our 4 macro variables. For real GDP growth and the unemployment gap, it is indeed the case that in recessions, there generally is a local trough in the skewness of GDP growth and a local peak in the skewness of the unemployment rate gap. The movements are largest in the recent Great Recession. For the inflation variables, positive spikes are less pronounced. Yet, core inflation exhibits small positive spikes in the first three recessions. Since then, the measures of inflation skewness have generally remained negative and generally spike down during recessions. The main economic reason is that inflation is mostly driven by demand shocks.

4.3 Time variation in conditional macro variances and the Great Moderation

Because our model generates time variation in the conditional variance of the macro variables, it can potentially inform the debate on the Great Moderation. The literature

[12] Campbell, Pflueger, and Viceira (2015) suggest that supply shock volatility decreases after 1980 but its decrease may have been masked by changes in monetary policy, at least until 2000.

has mostly focused on output volatility and puts a "break point" for output volatility in the first quarter of 1984 (see McConnell and Perez-Quiros, 2000; Stock, Watson, Gali and Hall, 2002). For inflation, Baele et. al. (2015) suggest a later date, the first quarter of 1990. Whereas most of the discussion in the literature has tried to attribute the decreased volatility to either good luck or improvements in monetary policy (see e.g. Cogley and Sargent, 2005; Benati and Surico, 2009; Sims and Zha, 2006, and Baele et. al., 2015, and the references therein), our model offers an alternative perspective. First, Figure 4 Panel C shows little visual evidence of a break in supply variances apart from a slight and slow decline during the 80s. However, supply variances peak in recessions and so the recession-intensive 70s and 80s naturally feature higher supply variances than the period thereafter. While it is possible that monetary policy lowered the incidence of recessions, it is not obvious how monetary policy would stave off the volatility associated with supply shocks, and indeed does not appear to have done so in the 1990 and recent Great Recession. Second, for the demand variance, it is obvious that the more benign "good" variance process shows a distinct break in the mid-eighties, but the more pernicious "bad" variance continues to peak in recessions as it did before. This result is reminiscent of a recent finding in Gadea, Gomez Loscos and Perez-Quiros (2014), who, after examining a very long historical period, also conclude that declines in output volatility are associated with expansionary not recessionary periods.

We next test more formally whether inflation and real GDP growth have seen declines in volatility such as that suggested by the Great Moderation, and if so, what variance types (good/bad, demand/supply) explain the shift. Table 8 reports simple dummy regressions for inflation (Panel A) and GDP growth (Panel B). The first three columns report the constant and slope of a regression of the conditional variance of either inflation or GDP growth on a constant and a dummy. We use conservative standard errors correcting for heteroscedasticity and serial correlation using 20 Newey-West lags, but we do not account for sampling error in the filtered macro risk factors.

The rest of the table then splits up the conditional variance in their demand and supply components, and in their respective good and bad demand components. To facilitate comparisons with the literature, we focus mainly on changes in volatility from an initial period spanning 1962 to 1990 compared to the later period spanning from 1990 to 2000. For inflation, there is strong evidence of a decrease in variance after 1990, with the variance decreasing by about 1/3 of its magnitude before the break and the break being statistically significant at the 1 percent level. The additional tests reveal that the break is entirely due to decreases in good variance of both demand and supply components, with most of the effect attributable to a decline in the good demand variance. In other words, the Great Moderation may not imply smaller inflation volatility in future recessions.

In Panel B of Table 7, the same analysis is performed for real GDP growth volatility. The GDP variance also decreases at the break point by about 25 percent of its pre-break value, with the change significant at the 5 percent level. The decomposition analysis is quite similar to the inflation case, with a slightly more important role for the good supply variance. The decline in demand variance is only significant at the 10% level.

Lengthening our sample period to the present could increase the power of our tests, and also enables us to address a more recent question: "Is the Great Moderation over?" Baele et. al. (2015) use a macro-regime switching model suggesting that the Great Moderation for both inflation and output has ended, even before (for inflation) or just with the onset (for output) of the Great Recession. However, Gadea, Gomez Loscos and Perez-Quiros (2015) argue, based on a pure statistical analysis of GDP growth volatility, that the Great Moderation is alive and well, despite the Great Recession experience. To test these claims using our estimates of the conditional variance of inflation and output, we examine ending the sample in the fourth quarter of 2006 (just before the Great Recession) or the final quarter in 2015 (using the full sample).

For inflation, when the data from the Great Recession are ignored, the Great Moderation result and its decomposition we documented before is maintained. When we extend the sample to the end of 2015, the decline in the inflation variance weakens slightly but is still statistically significant at the 5% level. It is still the case though that the (good) demand variances become significantly lower post 1990. Clearly, if more non-recession data accumulate, we may well find that the Great Moderation for inflation holds up. For real GDP growth, the results are robust to extending the sample to both 2006 and 2015.

We conclude that there is evidence that the "good" demand and supply variances have decreased over time, but there is no strong evidence that either "bad" demand or supply variances have declined. Our analysis of the structural sources of recessions suggest that we therefore should not expect them to be less variable in the future than they were in the past.

One advantage of our framework is that we can also look at changes in skewness of the structural shocks. In unreported results, we find that good demand skewness significantly increases after 1984 (by about 0.2). The increase in good supply skewness is more modest and turns negative (and significant) when the sample is extended to 2016, incorporating the Great Moderation. There is no significant change in "bad" structural skewness, whether demand or supply driven. Therefore, there is no evidence that recession shocks will look different following the Great Moderation.

4.4 Conditional Covariances between Macroeconomic Time Series

From the perspective of theoretical asset pricing, an important implication of our structural framework regards the covariance between inflation and real activity. From Equation 4, it is evident that in an environment where demand (supply) variances dominate, the conditional covariance between inflation and real activity is positive (negative). To the extent that variances are persistent, changes in this covariance may have important ramifications for term and bond return premiums, which we examine in Section 5. Surprisingly, to our knowledge, sign-switching macro-correlations have so far only been documented for consumption growth and aggregate inflation (Hasseltoft and Burkhardt, 2012, Ermolov, 2015, Boons, Duarte, de Roon, and Szymanowska, 2017, and Song, 2017).

Figure 6 graphs the conditional covariance between, respectively, inflation and real GDP growth and also between core inflation and the unemployment gap (where the aforementioned signs are reversed). Overall, the covariance is mostly positive (over 90 percent of the time), which is driven by the important contribution of (good) demand variances to all macro variables. For the inflation-GDP growth covariance, there is substantial time variation, but the covariance rarely becomes negative. Early in the Great Recession, demand shocks generate a local peak in the covariance but subsequent large supply shocks then bring the covariance down. A mirror image of this happens for the core inflation-unemployment gap covariance. There, we see more frequent sign switches and the covariance remains positive until 1975, in a supply shocks driven macro-environment.

An overall covariance of near zero can in fact hide some strong structural non-zero sources of comovement from structural risk factors. To see this more clearly, we also show the good and bad supply and demand covariance components of the total covariance. For example, the near-zero correlation between real GDP and inflation from 2000 up to the onset of the Great Recession (with occasional peaks) is the sum of a sizable positive covariance driven by good and bad demand shocks and a sizable negative covariance driven by supply shocks. In the Great Recession, the conditional bad variance of both kinds of shocks shoots up, with the bad demand shock first ratcheting the covariance upwards, and bad supply variance later bringing it down substantially. Similar movements happen for the core inflation-unemployment covariance with the covariance actually switching signs.

5 Macro Risks and the Term Structure

In this section, we explore the interaction of macro factors with the term structure of interest rates. In the preceding sections, we have identified four novel macro-risk factors (p_t^d , n_t^d, p_t^s and n_t^s). These variables can be interpreted as "good" or "bad" conditional volatilities of demand and supply shocks, but their time variation also changes the entire conditional distribution of these shocks. For comparison with the existing literature on explaining bond yields and returns using macro data, we also examine the performance of "level" macro factors, which include expected inflation, expected core inflation and expected real GDP growth (we use the previously described VAR(2) system to compute these expectations). We also use the unemployment gap as a macro level factor. Thus, there are a total of 8 macro-factors we consider.

We address four questions. First, we ask whether macroeconomic factors help explain the yield curve. Second, we investigate the predictive power of our new macro risk factors for bond excess returns. Third, we also explore how the macro risk factors affect term premiums. Finally, we examine the predictive power of the macro factors for realized bond variances.

5.1 Macro Risks and the Yield Curve

We start by computing the classic yield curve financial factors. The "level" factor is the equally weighted average of all yields (from the one year to the 10 year maturity); the "slope" factor is the difference between the 10 year yield and one quarter yield; and finally, the "curvature" factor subtracts twice the two-year rate from the sum of the one quarter rate and the 10 year yield. Taken together, these three factors span the overwhelming majority of variation in yields at all maturities. Thus, to operationalize our test of whether macro factors explain yields, we test whether the macro factors explain variation in these three factors. To assess whether macro factors are important determinants of these three financial factors, Table 9 reports R^2 statistics from regressions of the financial factors onto macro factors. We report the parameter coefficients in the Online Appendix. For yields, the coefficients are difficult to interpret. For example, bad supply volatility should increase the term premium and thus increase yields, but may also decrease yields through a precautionary savings mechanism.

Panel A reports results regarding the macro level factors and the macro risk factors. First, the explanatory power of the macro level factors alone for the financial factors is substantial, with the adjusted R^2's about 70, 60, and 30 percent respectively for the level, slope, and curvature factors. Second, we proceed to determine the adjusted R^2 increment coming from the macro risks and its statistical significance. We use the bootstrap test of Bauer and Hamilton (2017) to determine the statistical significance. The null is that macro risks are unrelated to financial factors. We simulate 5,000 samples of historical length under the null and compute the p-value as the proportion of samples where the adjusted R^2 increases by at least as much as in the data after the inclusion of by construction unrelated macro risks. We find that the macro risks contribute in a statistically significant fashion to all factors, but the statistical and economic significance is much larger for the level (an adjusted R^2 increase of 7.5%) and curvature (an adjusted R^2 increase of 12.5%) factors.

As a robustness check, in Panel B we check whether the boost in explanatory power due to the macro risk factors survives the inclusion of a second set of contemporaneous macro level factors in the regression, those constructed by Ang and Piazzesi (2003). The increase in R^2's due to the macro risk factors is essentially unaffected, but becomes insignificant for the slope factor. Appendix D reports the results using realizations (instead of expectations) of macro level factors, in which case the relative contribution of the macro risk factors is more substantive.

5.2 Macro Risks and Bond Return Predictability

The literature on bond return predictability is voluminous, but mostly focuses on using information extracted from the yield curve to predict future holding period returns (e.g. Cochrane and Piazzesi, 2005). Ludvigson and Ng (2009) find that "real" and "inflation" factors, extracted from a large number of macroeconomic time series, have significant forecasting power for future excess returns on nominal bonds and that this predictability is above and beyond the predictive power contained in forward rates and yield spreads.

Also, the bond risk premia implied by these regressions have a marked countercyclical component. Bansal and Shaliastovich (2013) show that consumption growth and inflation volatility predict excess bond returns. Cieslak and Pavola (2015) uncover short-lived predictability in bond returns by controlling for a persistent component in inflation expectations. Barillas (2011) shows that the predictability due to macro factors for excess bond returns is economically significant.

In Tables 10 and 11, we explore the link between future bond returns and our macro factors. We focus on excess one-quarter holding period returns relative to the one quarter yield. This avoids the use of overlapping data which can spuriously increase R^2's in predictability regressions due to the high autocorrelation (Bauer and Hamilton, 2017). Nonetheless, all statistical inference is calculated using the small-sample bootstrap of Bauer and Hamilton (2017). To delve into the economic mechanism by which macro risks forecast future bond returns, Table 10 presents the coefficients from forecasting regressions that include both level macro and macro risk factors.[13] Individually, there are few significant coefficients. Of the macro level factors, expected core inflation enters with a positive sign, while expected aggregate inflation enters with a negative significant coefficient of similar magnitude, and is highly significant at all maturities. The significance of expected inflation in such regressions is consistent with the results of Cieslak and Pavola (2015) (but their regression also includes yields). Of the macro risk factors, the bad demand variance has a negative significant coefficient and the bad supply variance a positive (albeit mostly insignificant) coefficient. Therefore, consistent with intuition, being in a risky (that is volatile) demand environment, where bonds are good hedges against general macroeconomic risks, reduces the risk premium on bonds, and the reverse is true in the case of a supply environment. The effect of bad demand variance is economically large: for example, for the 10 year maturity a one standard deviation increase in the bad demand factor decreases the expected annualized excess bond return by 3.38 percentage points (the risk factors were standardized to a unit variance). The corresponding coefficients increase with bond maturity. The coefficients on the "good" demand risk factors are also negative and significantly different from zero, with coefficients that are

[13] Including financial factors (level, slope, and curvature) in the regressions does not materially change macro factors and macro risks signs except that the p_t^s-signs switches from being insignificantly positive to being insignificantly negative.

even larger than for the bad demand variance factor.[14]

Table 11 mimics the regression set up in Ludvigson and Ng (2009) and Bauer and Hamilton (2009). The adjusted R^2's produced by the financial factors alone are significantly boosted by including both macro level factors and macro risk factors. For maturities from 1 to 10 years, the R^2's from regressions including only financial factors are around 7 percent. Macro level factors only increase R^2 by 2 to 3% at short horizons with the increase only significant at the 10% level. Macro risks further increase the R^2s by about 4-5% for short maturities and by about 3% at the longer maturities. Macro risks alone increase the adjusted R^2 by 6-7% at short maturities and 3-4% at long maturities compared to the specification where only financial factors are included. These increases in explanatory power are statistically significant under the Bauer and Hamilton (2017) bootstrap, for testing the null of no predictability coming from macro risks. This is important given that Bauer and Hamilton (2017) have shown that the additional predictive power coming from macro factors over financial factors, as, e.g., in Ludvigson and Ng (2009) and Joslin, Priebsch, and Singleton (2014), often does not survive when p-values are computed using bootstrap procedure. Additionally, while macro risks significantly increase explanatory power for the specification which includes financial and macro level factors, the increase in adjusted R^2 from macro level factors for the specification which already includes financial factors and macro risks is economically small and statistically insignificant. Appendix D reports the return predictability of our macro level and risk factors over Ang-Piazzesi factors, showing an increase in the adjusted R^2's by about 4%.

Given that previous studies have considered macroeconomic "level" and "risk" factors in isolation and that factors measuring macroeconomic risk have received scant attention in such investigations, the relative predictive power of risk factors is of interest. Table 10 indicates that the adjusted R^2 from macro level factors alone in excess return regressions is around 4-5% with macro risk factors contributing an additional 2%. Ludvigson and Ng (2009) found the bond risk premiums implied by their predictive regressions, which included both yield variables and macro-factors, to be counter-cyclical. It is not difficult

[14]To further elaborate on the risk premium intuition, we also added the contemporaneous demand and supply shocks (u^d_{t+1} and u^s_{t+1}) to the bond return regressions. In unreported results, we find that the supply shocks carry positive but economically small and statically non-significant coefficients and the demand shocks carry negative coefficients that are significant at the 5% (short maturities) and 1% (long maturities) levels and become larger in magnitude with maturity. That is, realized bond excess returns are high if a negative demand shock occurred during the holding period.

to obtain counter-cyclical real bond risk premiums in economic models, e.g., in habit models with counter-cyclical prices of risk (see, e.g., Wachter, 2006). Our framework suggests that not all recessions are equal in this respect. Our predictive regressions indicate that risk premiums are, everything else equal, lower when the macro-environment is primarily demand driven. To check on the cyclicality of bond risk premiums that are implied by our regressions, we use the fitted values of the predictive regressions[15] as an estimate of the risk premium and regress it on a NBER dummy, the ratio of the aggregate demand variance, including the good and bad variances, to the corresponding aggregate supply variance, and the interaction of the two. We rescale the demand/supply ratio variable to have a standard deviation of one. Table 12 reports the results. First, coefficients on the NBER dummy are positive and increase with maturity. Economically, the effect is rather large: an NBER recession increases the annualized expected excess return on a 10-year bond on average by 1.95 percentage points. However, after computing Newey-West standard errors with 20 lags, we find that the coefficients are not statistically significantly different from zero, so we find only weak statistical evidence of counter-cyclical risk premiums. Second, the demand/supply ratio is indeed negatively associated with risk premiums, and especially so for the 5 and 10 year bonds. Again, these effects are economically very large for the longer maturities and highly statistically significant. For example, for the 10 year bond, if the demand/supply ratio were to increase by 1 standard deviation, the annualized bond risk premium would not increase by 1.95 percentage points in a recession, but decrease by 1.86 (1.95-3.95+0.15) percentage points. Of course, it is important to recall that supply variances spike up as well in most recessions.

5.3 Macro Risks and Term Premiums

As we indicated before, most of the literature examining the link between the macroeconomy and bond risk premiums has focused on macro level factors.[16] One important exception is Wright (2011), who does not examine excess holding period returns, but an important and closely related component of bond yields, the term premium. Wright (2011) shows that term premiums are countercyclical and strongly affected by inflation

[15] Including financial factors (level, slope, and curvature) to construct the expected excess bond returns does not materially change any of the results.

[16] An exception is Wachter (2006), where the risk premium depends on the surplus ratio, essentially a weighted average of past consumption shocks. However, the more recent theoretical literature (e.g., Buraschi and Jiltsov, 2007; Bansal and Shaliastovich, 2013) suggests that focusing on second and higher order moments is more logical.

uncertainty in a panel of countries.[17] We compute term premiums for the 5 year and 10 year maturity as the yield for each maturity minus the average of expected future short-term rates over the life of the bond. To measure the expected short yield, we use Blue Chip survey, which is available semi-annually from 1986Q2.[18]

Results from this exercise are reported in Table 13. They are somewhat similar to the results in Table 10 on excess holding period returns. Expected core inflation, expected inflation and expected GDP growth significantly affect term premiums with the same signs as in the excess holding period return regressions. Whereas the bad demand variance risk factor negatively affects the term premium, consistent with the idea that in such an environment bonds act as a good hedge, the effect is statistically insignificant for the 5 year bond and marginally significant for 10 year bond. Instead, increases in the good demand variance significantly decrease term premiums. We also find that the good supply variance affects term premiums positively. The adjusted R^2 is 69 percent for the 5 and 10 year bonds. The macro risk factors' addition to the explanatory power of the macro level variables is marginally significant.

In Table 14, we examine the cyclicality of the term premiums. In line with Wright (2011) and Bauer, Rudebusch, and Wu (2014), we find that the term premium increases in recessions, by 0.55 percentage points (0.53 percentage points) for the 5-year (10-year) bond. These numbers are economically significant but not statistically significant. The term premium is smaller in demand environments, but the effect is also not significant. The interaction effect with the NBER dummy has a negative sign but also fails to be significant. The demand environment effects are substantive; a one standard deviation increase in the demand/supply variance ratio decreases the term premium in a recession by about 56 basis points for the 5 year bond and about 52 basis points for the 10-year bond. Therefore, "demand effects" of this magnitude completely offset the usual countercyclical term premium increase in recessions.

[17] Bauer, Rudebusch, and Wu (2014) re-examine Wright's empirical evidence correcting for small sample bias in the VAR he runs to compute the term premium, but his main empirical conclusions remain robust.

[18] Our results are similar if we employ the expected short yield computed using Bauer, Rudebusch, and Wu (2014) small-sample adjusted VAR(1) including 1 quarter, 1 year, and 10 year yields as the state variables. The correlations between the survey and statistical term premia are 0.7578 and 0.7964 for the 5 and 10 year term premia, respectively.

5.4 Macro Risks and the Bond Return Variance

Consider a model of the term structure of interest rates in which macroeconomic factors help to determine the levels of bond yields (e.g., habit of Wachter, 2006, or long-run risk of Bansal and Shaliastovich, 2013). Then the conditional variance of the macroeconomic factors, which is captured by our macroeconomic risk factors, should help to determine the conditional variance of bond returns. In the context of a forecasting regression, the macro risk factors should help forecast ex-post bond return variance. In Table 15, we present empirical evidence that such a link between the variance of bond returns and the macro risk factors is indeed present in the data. Specifically, we regress the quarterly realized variance of returns for the 10-year bond[19] on the lagged values of the macro risk factors and/or as other controls. In panel A, we report the adjusted R^2 statistics from such regressions. By themselves, the macro risk factors span about 35 percent of the variation in the ex-post realized variance. In contrast, the macro level factors span only about 19 percent, and the financial factors span less than 14 percent. Further, the macro risk factors always significantly add to the explanatory power of regressions which already use the macro level factors or financial factors as explanatory variables. In contrast, the macro level factors do no significantly add to the explanatory power of regressions that already use the macro risk factors and financial factors as explanatory variables, nor do the financial factors significantly add to the explanatory power of regressions that already use the macro risk factors and the macro level factors. We conclude that the macro risk factors are quite powerful predictors of bond return variance.

Panel B shows the pattern of regression coefficients for one such regression that includes macro level factors and macro risk factors as explanatory variables. The most statistically significant explanatory variable is the bad variance component of demand, which positively affects bond return variance, as expected. Moreover, the coefficients for three out of the four macro risk factors are of the expected positive sign. Among the macro level factors, expected aggregate and core inflation are significant at the 10 and 5 percent level, respectively.

Figure 7 shows the historical pattern of realized variance for bond returns (the blue

[19]We compute the quarterly realized variances as the sum of squared daily returns inside the quarter. The realized daily returns are computed under the assumption that the 10 year - 1 day zero coupon yield is equal to the 10 year zero coupon yield.

line), and the fitted values from two of the forecasting regressions described above. The regression which uses the macro level factors and macro risk factors shown by the red/circle symbols, captures some of the most prominent features of realized variance, especially the high levels seen in the 1980s and during the 2008-2009 financial crisis. As shown by the line with green/triangle symbols, adding the financial factors to this regression does not significantly alter the patterns of the fitted return variance.

6 Conclusion

In this article, we provide three main contributions. First, we develop a new identification methodology to decompose macroeconomic shocks into "demand" shocks which move inflation and GDP growth in the same direction and "supply shocks" which move inflation and GDP growth in opposite directions. The identification relies on non-Gaussianities in the macro data. We find aggregate demand shocks to be distinctly negatively skewed and leptokurtic, whereas supply shocks unconditionally show little skewness but are also leptokurtic. Despite this alternative identification, the long-run effects of the aggregate demand and supply shocks conform to standard intuition as in the seminal work of Blanchard and Quah (1989). Investigating the various recessions in our sample, we find the three recessions in the 1970s and 1980 to be predominantly supply driven, whereas of the last four, three were more demand driven (the exception being the 1990-91 recession). The Great Recession featured both large negative demand and supply shocks.

Second, we develop a new dynamic model for real economic activity and inflation, where the shocks are drawn from a Bad Environment - Good Environment model, which accommodates time-varying non-Gaussian features with "good" and "bad" volatility. We extract four macro-risk factors, bad and good volatilities for respectively aggregate demand and supply shocks. Until about the mid-seventies conditional supply variances appear to dominate macroeconomic volatility, while afterwards demand variances are more important until the mid-eighties: afterward there are roughly equal contributions of both. However, supply shocks variances invariably peak in recessions. The "good" demand variance has decreased markedly over time, but there is no strong evidence that either "bad" demand variances or supply variances have declined. Importantly, recessions continue to be accompanied by temporarily high bad demand and supply variances. We also provide new insights about the Great Moderation in that it appears to reflect primarily a decline in good demand variance, with a small contribution of a secular decrease in good supply variance. Finally, we find that the conditional correlation between inflation and real activity varies through time with occasional sign switches, as the relative importance of demand and supply risk factors varies over time.

Third, we link the macro factors extracted from the dynamic macro model, expected GDP growth, the unemployment gap, and expected (core) inflation and the macro risk variables represented by the conditional variances (shape parameters) of the demand and supply shocks, to the term structure. The macro variables explain 79 percent of the variation in the levels of yields. While the contribution of the macro risk factors to this R^2 is modest, it is nonetheless statistically significant. When we run predictive regressions of excess bond returns onto the macro variables, the R^2 is around 6 percent, with the macro risk factors contributing one third of the explanatory power. Our macro risk factors resurrect the statistical importance of macro factors for return predictability regressions. We find that increases in both good and bad aggregate demand variance significantly reduce bond risk premiums; the former also significantly decreases term premiums. Macro risks also significantly predict realized bond return variances.

It would be useful to be elucidate how variation in risk premiums is accounted for by the various macro risk factors and to decompose risk premiums into real and inflation components. To accomplish this, a term structure model is necessary. In future work, we plan to build a non-Gaussian term structure model where the set of state variables includes both financial factors (as in Feldhütter, Heyerdahl-Larsen, and Illeditsch, 2018) and macro variables (level and risk factors). Despite the non-Gaussianities in their dynamics, the BEGE structure has the advantage that bond prices nonetheless remain affine in the state variables. Lastly, future work should verify whether the predictive power of our macro risk factors survives when real time data are used instead of the final revised data (see Ghysels, Horan, Moench, 2018).

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Figure 1 – Components of Bad Environment - Good Environment Distribution.

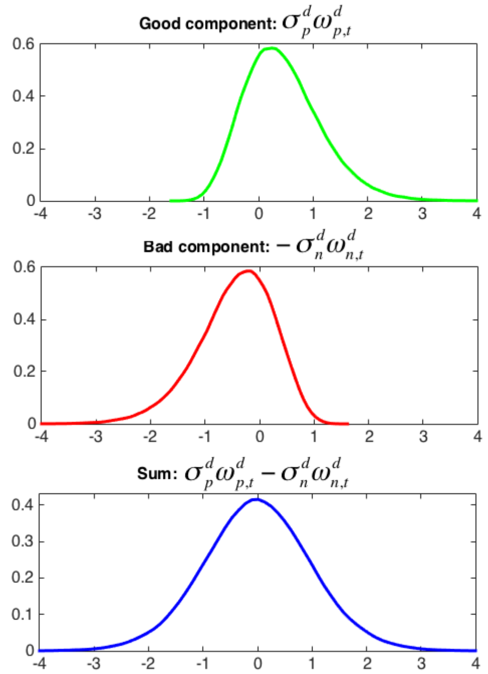


Figure 2 – Time-varying Shape Parameters of Bad Environment - Good Environment Distribution.

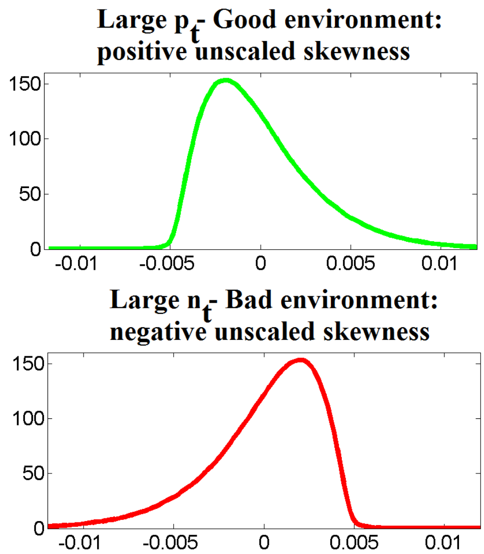


Figure 3 – Filtered Quarterly Demand and Supply Shocks. Shading corresponds to NBER Recessions.

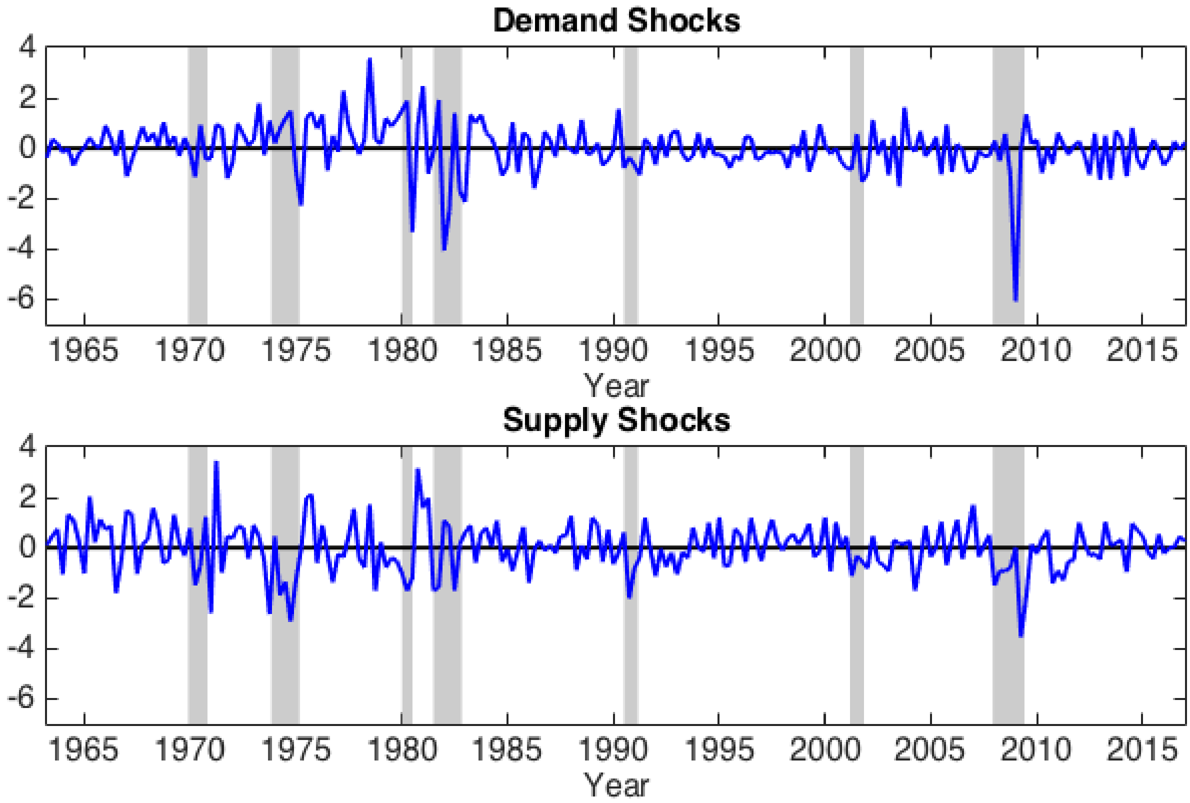


Figure 4 – Filtered Quarterly Demand and Supply Variances. Shading corresponds to NBER Recessions.

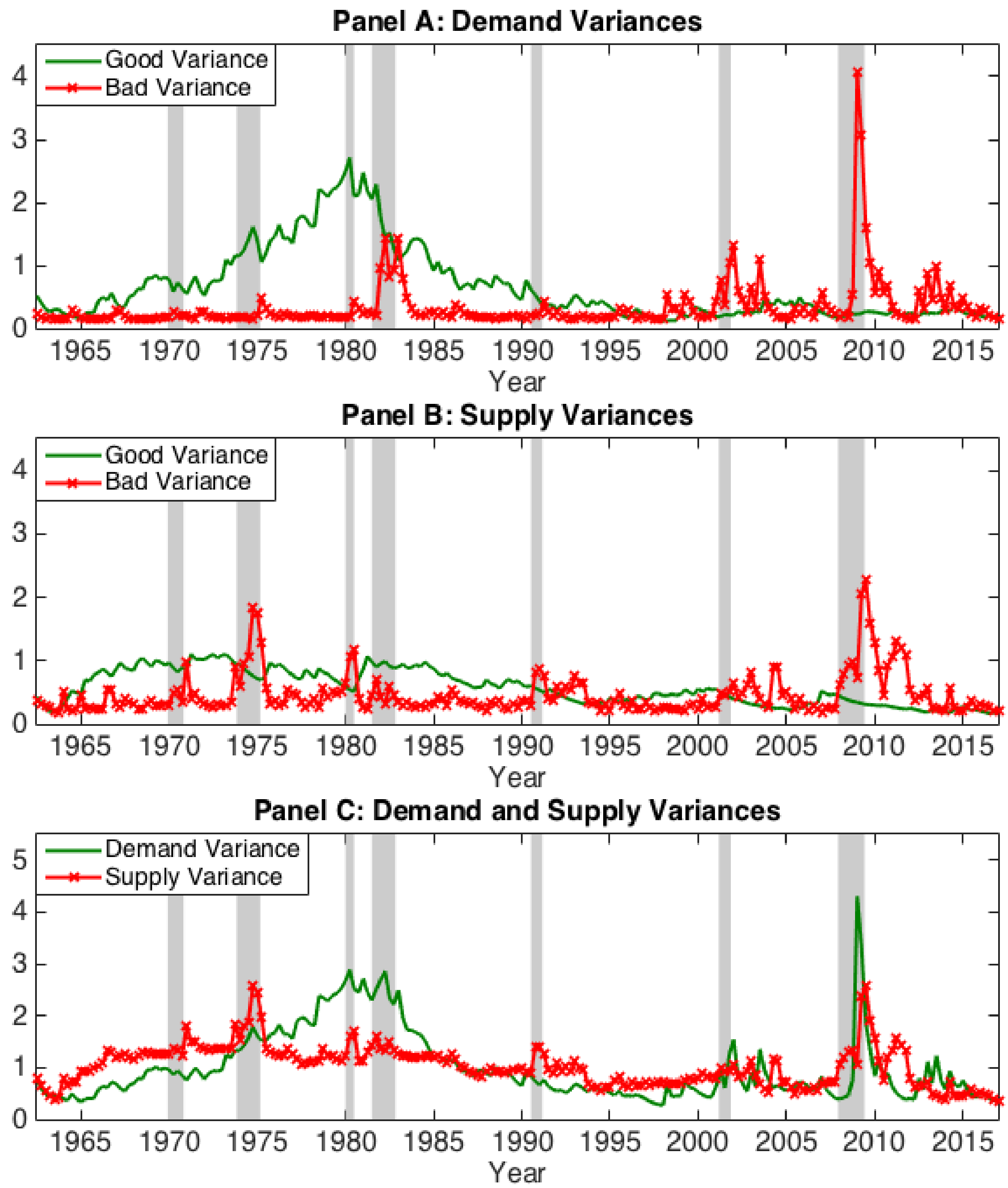


Figure 5 – Quarterly Conditional Skewness of Macroeconomic Variables. Shading corresponds to NBER Recessions.

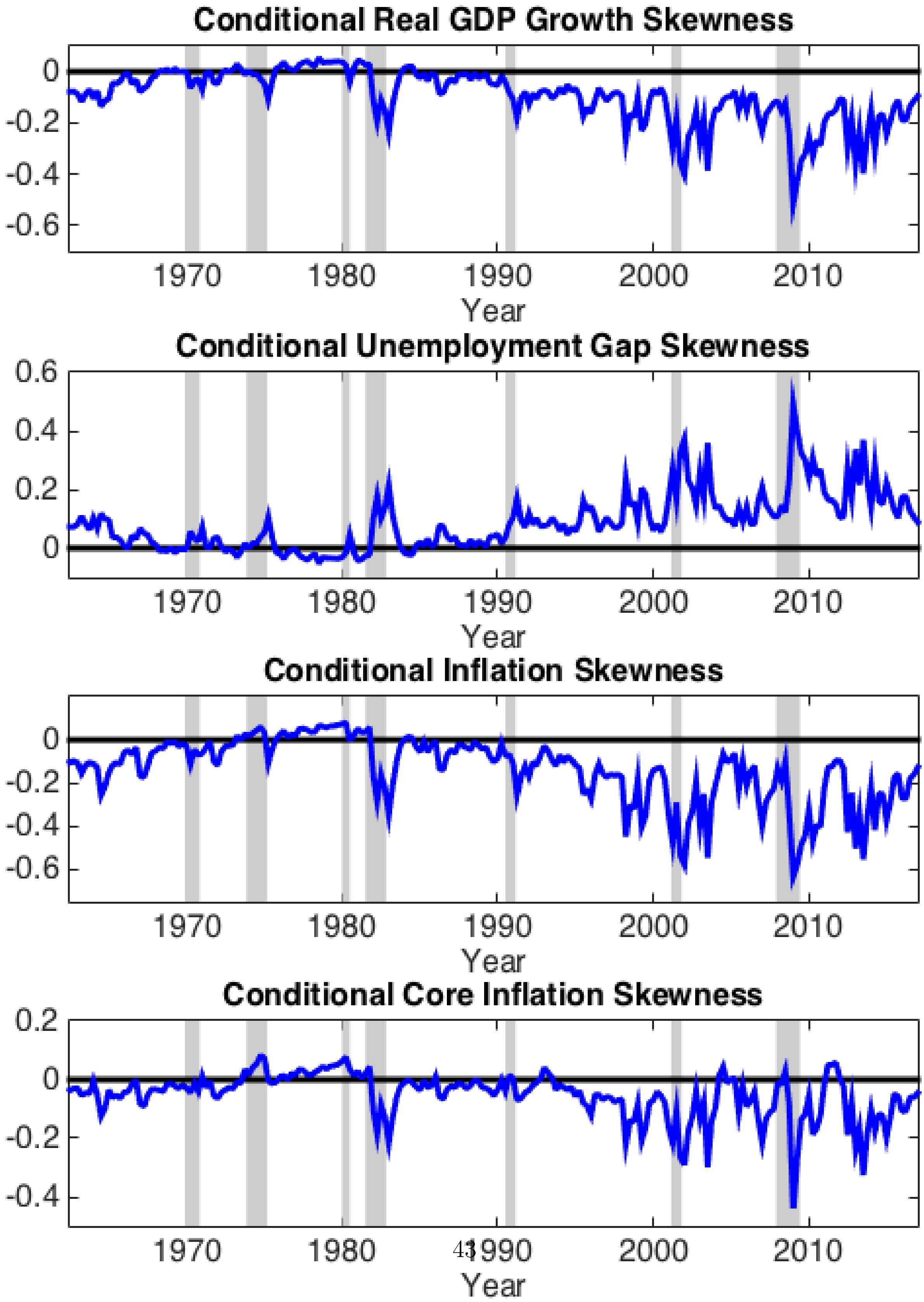
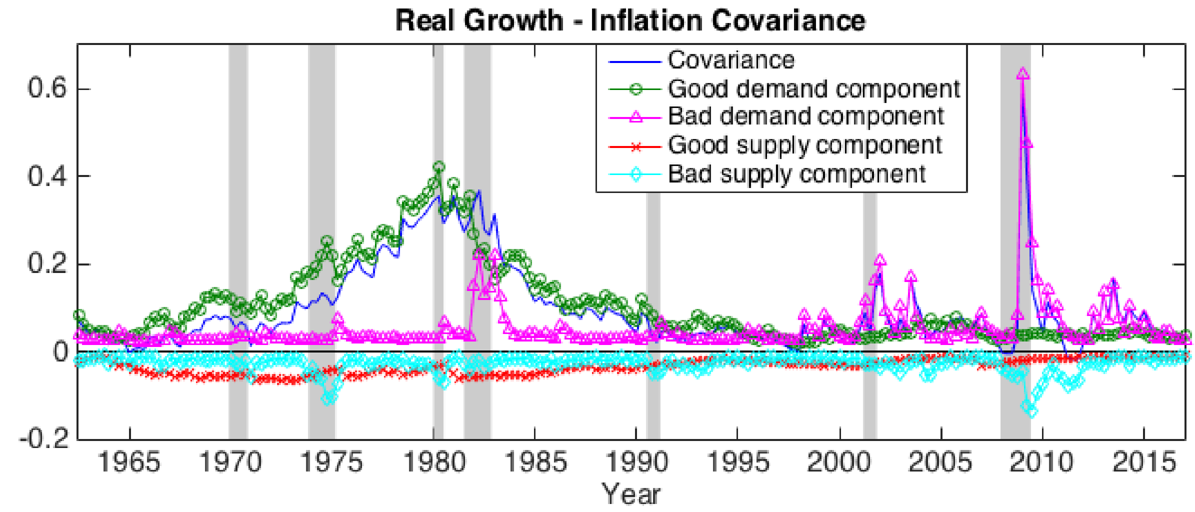


Figure 6 – Quarterly Conditional Covariance between Macroeconomic Variables. Shading corresponds to NBER Recessions.



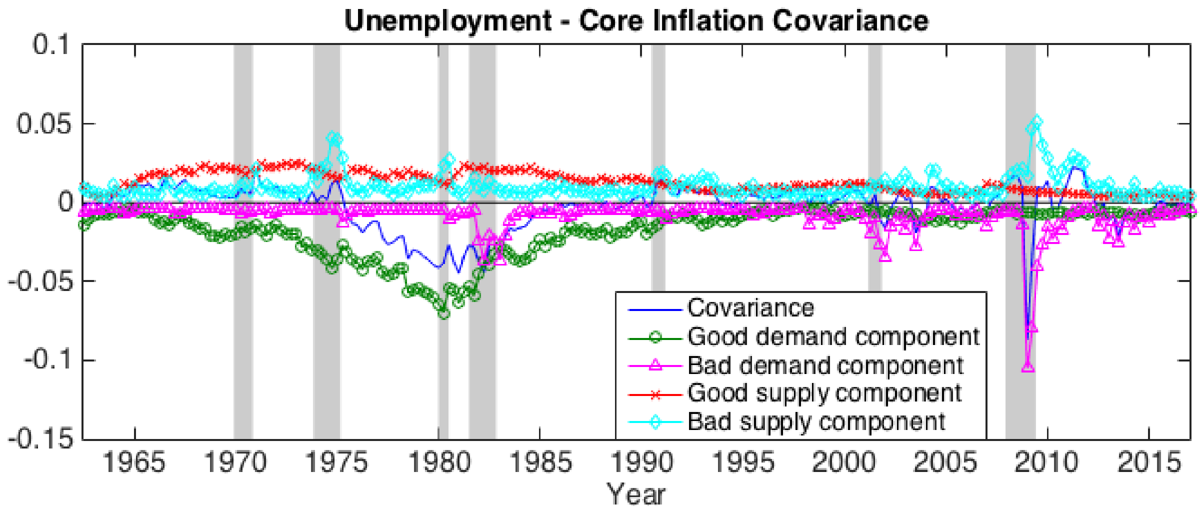


Figure 7 – Explaining Realized 10 Year Bond Return Variance with Macroeconomic and Financial Factors. Realized variances are computed as the sums of squared daily bond returns inside the quarter. The fit is from OLS regressions. Financial factors are the level, slope, and curvature factors. The level factor is the average over 1-10 year yields. The slope factor is the 10 year yield minus the 1 quarter yield. The curvature factor is 10 year yield plus 1 quarter yield minus 2 times the 2 year yield. The macroeconomic level factors are expected inflation, expected core inflation, expected GDP growth and unemployment gap.

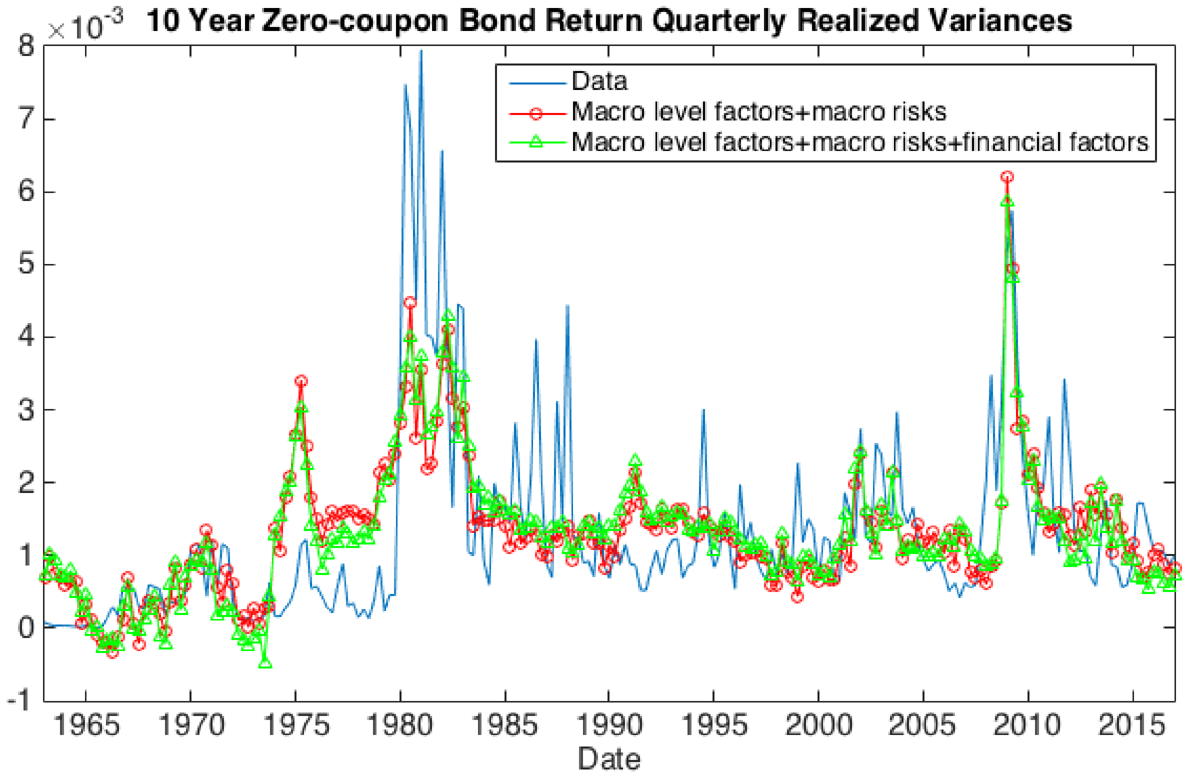


Table 1 – Model Selection for Expectations of Macro Variables. The sample is quarterly from 1962Q4 to 2016Q4. Dependent variables are the log-difference of the CPI, log real GDP growth, the log difference of core CPI, and the unemployment rate gap. The predictive variables are the macro variables mentioned above and the 90-day T-bill and the 10-year zero-coupon Treasury yield. AIC and BIC are Akaike and Bayesian information criteria, respectively. The models are sorted by AIC.

Model	Number of parameters	Log-likelihood	AIC	BIC
VAR(2)	93	-798.9	1801.8	2097.7
VAR(3)	129	-755.5	1802.8	2204.5
VARMA(2,2)	105	-785.9	1804.5	2136.3
VARMA(2,1)	99	-794.7	1807.6	2121.5
VARMA(3,1)	135	-752.9	1812.7	2231.4
VARMA(2,3)	111	-787.2	1821.5	2171.0
VARMA(3,2)	141	-749.9	1822.1	2257.7
VARMA(3,3)	147	-743.8	1825.5	2277.9
VARMA(1,3)	75	-856.8	1875.6	2116.8
VARMA(1,1)	63	-879.5	1893.8	2097.7
VAR(1)	57	-888.6	1898.4	2083.5
VARMA(1,2)	69	-875.7	1899.7	2122.3

Table 2 – Higher Order Moments of Macroeconomic Shocks Used for Classical Minimum Distance Estimation. u_t^g, u_t^π, $u_t^{\pi core}$, and u_t^u are the shocks to real GDP growth, aggregate inflation, core inflation and unemployment gap, respectively. The data is quarterly from 1962Q4 to 2015Q2. The covariance matrix for moments is a diagonal matrix calculated via a block-bootstrap with a block length of 20 quarters. Asterisks, *, **, and **** correspond to statistical significance of individual moments at the 10, 5, and 1 percent levels, respectively.

			Volatility			
	u_t^π	u_t^g	$u_t^{\pi^c}$	u_t^{ue}		
data	0.5655***	0.7078***	0.3252***	0.2658***		
standard error	(0.0867)	(0.0781)	(0.0531)	(0.0228)		
fitted	0.5655	0.7078	0.3252	0.2658		
			Skewness			
	u_t^π	u_t^g	$u_t^{\pi^c}$	u_t^{ue}		
data	-1.3570	0.4956	0.1144	0.3745**		
standard error	(1.0067)	(0.3714)	(0.3808)	(0.1879)		
fitted	-0.4456	-0.2585	-0.2264	0.2308		
			Excess kurtosis			
	u_t^π	u_t^g	$u_t^{\pi^c}$	u_t^{ue}		
data	11.2751**	2.5052**	2.0640**	1.0528***		
standard error	(5.7197)	(1.0656)	(0.8233)	(0.4056)		
fitted	1.9051	1.1046	0.9798	1.0160		
			Correlations			
	$u_t^\pi u_t^g$	$u_t^\pi u_t^{\pi^c}$	$u_t^\pi u_t^{ue}$	$u_t^g u_t^{\pi^c}$	$u_t^g u_t^{ue}$	$u_t^{\pi^c u_t^{ue}}$
data	0.1392	0.5400***	-0.2058***	0.0626	-0.5615***	-0.1630*
standard error	(0.1197)	(0.0726)	(0.0733)	(0.1281)	(0.0534)	(0.0969)
fitted	0.2415	0.5274	-0.2204	0.0604	-0.5587	-0.0372
			Co-skewness			
	$(u_t^\pi)^2 u_t^g$	$(u_t^\pi)^2 u_t^{\pi^c}$	$(u_t^\pi)^2 u_t^{ue}$	$(u_t^g)^2 u_t^\pi$	$(u_t^g)^2 u_t^{\pi^c}$	$(u_t^g)^2 u_t^{ue}$
data	-0.9790*	-0.4251	0.9978*	-0.2876	-0.1337	-0.1683
standard error	(0.5588)	(0.3519)	(0.5623)	(0.3977)	(0.2386)	(0.3941)
fitted	-0.3714	-0.3544	0.3579	-0.3144	-0.2514	0.2489
	$(u_t^{\pi^c})^2 u_t^\pi$	$(u_t^{\pi^c})^2 u_t^g$	$(u_t^{\pi^c})^2 u_t^{ue}$	$(u_t^{ue})^2 u_t^\pi$	$(u_t^{ue})^2 u_t^g$	$(u_t^{ue})^2 u_t^{\pi^{core}}$
data	-0.0814	-0.2427	0.2308	-0.4526*	-0.0987	-0.2621**
standard error	(0.2620)	(0.1813)	(0.1901)	(0.2513)	(0.3258)	(0.1180)
fitted	-0.2826	-0.2311	0.2225	-0.2926	-0.2397	-0.2342
			Excess co-kurtosis			
	$(u_t^\pi)^2(u_t^g)^2$	$(u_t^\pi)^2(u_t^{\pi^c})^2$	$(u_t^\pi)^2(u_t^{ue})^2$	$(u_t^g)^2(u_t^{\pi^c})^2$	$(u_t^g)^2(u_t^{ue})^2$	$(u_t^{\pi^c})^2(u_t^{ue})^2$
data	2.8288*	0.9001**	2.5459	0.8804***	1.1683**	0.7172**
standard error	(1.7353)	(0.4307)	(1.7067)	(0.2841)	(0.5452)	(0.2931)
fitted	1.3899	1.2650	1.3041	1.0355	1.0571	0.9972
	$(u_t^\pi)^3 u_t^g$	$(u_t^\pi)^3 u_t^{\pi^c}$	$(u_t^\pi)^3 u_t^{ue}$	$(u_t^g)^3 u_t^\pi$	$(u_t^g)^3 u_t^{\pi^c}$	$(u_t^g)^3 u_t^{ue}$
data	5.4690*	2.3743	-5.3776*	1.6048*	0.9830	-1.6559*
standard error	(3.3311)	(1.6502)	(3.1267)	(0.9644)	(0.7055)	(0.6289)
fitted	1.5255	1.5383	-1.4667	0.9894	0.6839	-1.0801
	$(u_t^{\pi^c})^3 u_t^\pi$	$(u_t^{\pi^c})^3 u_t^g$	$(u_t^{\pi^c})^3 u_t^{ue}$	$(u_t^{ue})^3 u_t^\pi$	$(u_t^{ue})^3 u_t^g$	$(u_t^{ue})^3 u_t^{\pi^c}$
data	1.0483	0.5848	-0.7485**	-1.1668	-0.9086*	-0.3166
standard error	(0.4346)	(0.5241)	(0.3655)	(0.7724)	(0.5445)	(0.2325)
fitted	1.0780	0.5661	-0.5272	-0.8572	-1.0357	-0.5635
J-stat	29.6525					
p-value	(0.0819)					
Joint significance of 3^{rd} and 4^{th} order moments	299.43					
p-value	(<0.0001)					

Table 3 – Loadings of Macroeconomic Shocks on Demand and Supply Shocks. The coefficients are from Classical Minimum Distance estimation matching unconditional higher order moments of 4 macroeconomic shocks time series: real GDP growth (u_t^g), aggregate (u_t^π) and core inflation ($u_t^{\pi core}$) and unemployment gap (u_t^u). Standard errors in parentheses account for sampling error in the higher-order moments and the VAR(2) parameters.

Panel A: Loadings of Macro Shocks on Supply and Demand Shocks

Shock	Supply loading	Demand Loading
u_t^π	-0.1736	0.3856
	(0.0555)	(0.1012)
u_t^g	0.3414	0.4044
	(0.0888)	(0.0950)
$u_t^{\pi^c}$	-0.1678	0.1760
	(0.0438)	(0.0678)
u_t^{ue}	-0.1344	-0.1464
	(0.0334)	(0.0264)
idiosyncratic variance share	0.4408	
	(0.0473)	

Panel B: Kalman Gain of Macro Shocks for Supply and Demand

Shock	u_t^π	u_t^g	$u_t^{\pi^c}$	u_t^{ue}
Supply	-0.4553	0.5069	-1.2790	-1.4202
	(0.1744)	(0.1038)	(0.3453)	(0.2772)
Demand	0.6758	0.4233	0.9561	-1.0825
	(0.1312)	(0.1066)	(0.2000)	(0.2848)

Panel C: Variance Decomposition for Demand and Supply Shocks

Shock	u_t^π	u_t^g	$u_t^{\pi^c}$	u_t^{ue}
Supply	12.02%	26.32%	32.64%	29.02%
Demand	34.35%	22.57%	22.18%	20.89%

Panel D: Unconditional moments of supply and demand

Shock	Skewness	Excess Kurtosis
Supply	0.0289	3.3186
	(0.8770)	(1.7417)
Demand	-1.4030	8.6770
	(0.9987)	(4.8979)

Table 4 – Model Comparision for Aggregate Demand and Aggregate Supply Shocks. AIC refers to Akaike information criterion and BIC refers to Bayesian information criterion. The models are sorted by AIC. Regime-switching model refers to the 2 state regime-switching model. BEGE is the full BEGE (with both p- and n-tails being time-varying) for both demand and supply shocks.

Panel A: Supply Shock				
Model	Log-likelihood	Number of parameters	AIC	BIC
Regime-switching	-297.0011	5	604.0022	620.9476
Gaussian stochastic volatility	-300.5985	2	605.1970	611.9751
BEGE	-297.8910	6	607.7820	628.1164
Panel B: Demand Shock				
Model	Log-likelihood	Number of parameters	AIC	BIC
BEGE	-266.7475	6	545.4950	565.8294
Regime-switching	-270.1899	5	550.3798	567.3252
Gaussian stochastic volatility	-278.8772	2	561.7544	568.5325
Panel C: Demand and Supply Shocks				
Model	Log-likelihood	Number of parameters	AIC	BIC
BEGE	-564.6385	12	1153.2770	1193.9459
Regime-switching	-567.1910	10	1154.3820	1188.2727
Gaussian stochastic volatility	-579.4757	4	1166.9514	1180.5077

Table 5 – Bad Environment - Good Environment Parameter Estimates for Demand and Supply Processes. Parameter estimates are obtained using Bates (2006) approximate maximum likelihood methodology. Standard errors in parentheses are approximate maximum likelihood asymptotic standard errors. As demand and supply shocks are assumed to have variances exactly equal to 1, $\bar{n}$-parameters can be solved as functions of other model parameters, and their standard errors are calculated using the delta method.

	Supply shock	Demand shock
$\bar{p}$	20.0000	20.0000
	–	–
$\bar{n}$	4.0030	0.3359
	(7.1293)	(0.2177)
σ_p	0.1644	0.1801
	(0.0193)	(0.0107)
σ_n	0.3389	1.0229
	(0.2879)	(0.3271)
ρ_p	0.9881	0.9392
	(0.0177)	(0.0279)
ρ_n	0.6737	0.7243
	(0.2046)	(0.1551)
σ_{pp}	0.5524	0.9834
	(0.4162)	(0.3434)
σ_{nn}	1.2502	0.5723
	(1.1114)	(0.3905)

Table 6 – VAR(2) Impulse Responses of Real GDP and Aggregate Price Level to One Standard Deviation Demand and Supply Shocks. The cumulative impulse responses include the quarter 0 (where the shocks happened) responses. Standard errors in parentheses are bootstrap standard errors.

Panel A: Contemporaneous (Quarter 0) Responses		
	Demand Shock	Supply Shock
Real GDP level	0.40%	0.34%
	(0.10%)	(0.08%)
Price level	0.39%	-0.17%
	(0.10%)	(0.06%)
Panel B: Cumulative (20 Quarters) Responses		
	Demand Shock	Supply Shock
Real GDP level	0.09%	0.52%
	(0.27%)	(0.27%)
Price level	2.15%	-0.05%
	(0.66%)	(0.54%)

Table 7 – Decomposition of Real GDP Growth during NBER Recessions into Demand and Supply Components. Aggregate demand component of the GDP growth is computed as σ_{gd} multiplied by the sum of aggregate demand shocks over the period of the recession. Aggregate supply component of the GDP growth is computed as σ_{gs} multiplied by the sum of aggregate supply shocks over the period of the recession.

NBER Recession	GDP Growth: Demand Component	GDP Growth: Supply Component
1969Q4-1970Q4	-0.46%	-0.94%
1973Q4-1975Q1	0.23%	-2.40%
1980Q1-1980Q2	-0.57%	-0.99%
1981Q3-1982Q4	-2.85%	-0.20%
1990Q3-1991Q1	-0.84%	-1.10%
2001Q1-2001Q4	-1.04%	-0.95%
2008Q1-2009Q2	-2.88%	-2.09%

Table 8 – Decomposing the Great Moderation into Changes in Demand and Supply Volatility. Coefficients in Panel A are OLS regression coefficients from regressing the dependent variable on a constant equal to 1 and a dummy variable which is 0 before 1990Q4 and 1 between 1991Q1 and 2000Q4 for the sample of 1962Q4-2000Q4 (specification Data till 2000), 0 before 1990Q4 and 1 between 1991Q1 and 2006Q4 for the sample of 1962Q4-2006Q4 (specification Data till 2006) , and 0 before 1990Q4 and 1 between 1991Q1 and 2016Q4 for the sample of 1962Q4-2016Q4 (specification Data till 2016). Coefficients in Panel B are OLS regression coefficients from regressing the dependent variable on a constant equal to 1 and a dummy variable which is 0 before 1983Q4 and 1 between 1984Q1 and 2000Q4 for the sample of 1962Q4-2000Q4 (specification Data till 2000), 0 before 1983Q4 and 1 between 1984Q1 and 2006Q4 for the sample of 1962Q4-2006Q4 (specification Data till 2006), and 0 before 1983Q4 and 1 between 1984Q1 and 2016Q4 for the sample of 1962Q4-2016Q4 (specification Data till 2016). Standard errors in parentheses are Newey-West (1987) standard errors computed with 40 lags. The standard errors for the constant are from the regression using only data up to 2000Q4. The standard errors for the constant for samples spanning until 2006Q4 and 2016Q4 are slightly different, but these differences are economically and statistically negligible. The asterisks, *, **, and *** correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

Panel A: Aggregate Inflation				
Dependent variable	Constant	Data till 2000	Data till 2006	Data till 2016
Aggregate variance	0.3668***	-0.1243***	-0.1098**	-0.0965**
	(0.0442)	(0.0450)	(0.0444)	(0.0457)
Supply variance	0.0364***	-0.0126***	-0.0127***	-0.0113***
	(0.0029)	(0.0033)	(0.0031)	(0.0035)
Good supply variance	0.0234***	-0.0104***	-0.0116***	-0.0130***
	(0.0016)	(0.0016)	(0.0018)	(0.0021)
Bad supply variance	0.0130***	-0.0022	-0.0011	0.0017
	(0.0015)	(0.0023)	(0.0018)	(0.0029)
Demand variance	0.1895***	-0.1117***	-0.0971**	-0.0853*
	(0.0426)	(0.0431)	(0.0428)	(0.0437)
Good demand variance	0.1519***	-0.1098***	-0.1073***	-0.1108***
	(0.0378)	(0.0388)	(0.0383)	(0.0381)
Bad demand variance	0.0377***	-0.0019	0.0102	0.0254*
	(0.0064)	(0.0066)	(0.0098)	(0.0140)
Panel B: Real GDP Growth				
Dependent variable	Constant	Data till 2000	Data till 2006	Data till 2016
Aggregate variance	0.5907	-0.1479**	-0.1485**	-0.1425**
	(0.0705)	(0.0686)	(0.0674)	(0.0688)
Supply variance	0.1465***	-0.0418***	-0.0453***	-0.0439***
	(0.0131)	(0.0158)	(0.0149)	(0.0147)
Good supply variance	0.0939***	-0.0312***	-0.0377***	-0.0452***
	(0.0072)	(0.0103)	(0.0101)	(0.0102)
Bad supply variance	0.0526***	-0.0105	-0.0076	0.0013
	(0.0054)	(0.0078)	(0.0076)	(0.0107)
Demand variance	0.2233***	-0.1062*	-0.1032*	-0.0986
	(0.0623)	(0.0591)	(0.0592)	(0.0604)
Good demand variance	0.1800	-0.1004*	-0.1072**	-0.1173**
	(0.0545)	(0.0532)	(0.0528)	(0.0524)
Bad demand variance	0.0434***	-0.0058	0.0039	0.0187
	(0.0101)	(0.0105)	(0.0142)	(0.0186)

Table 9 – Explanatory Power (Adjusted R^2) of Macro Risk Factors for Yield Curve Factors. The sample is quarterly from 1962Q4 to 2016Q4. Ang-Piazzesi factors are contemporaneous Ang and Piazzesi (2003) real and nominal factors. Macro level factors are expected real GDP growth, expected aggregate and core inflation, and unemployment gap. Financial factors are the level, slope, and curvature factors. The level factor is the average over 1-10 year yields. The slope factor is the 10 year yield minus the 1 quarter yield. The curvature factor is 10 year yield plus 1 quarter yield minus 2 times the 2 year yield. The increase in adjusted R^2 significance, which is always tested over the specification in the previous row, is Bauer and Hamilton (2017) small-sample adjusted significance using 5000 bootstrap runs. The asterisks, *, **, and ***, correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

Panel A: Macro Level Factors and Macro Risks			
	Level	Slope	Curvature
Macro level factors	0.7146	0.5713	0.2808
Macro level factors+macro risks	0.7902***	0.5975*	0.4072***
Panel B: Ang-Piazzesi Factors, Macro Level Factors, and Macro Risks			
	Level	Slope	Curvature
Ang-Piazzesi (2003) factors	0.2555	0.3126	0.1229
Ang-Piazzesi (2003) factors + macro level factors	0.7122***	0.5906***	0.2918***
Ang-Piazzesi (2003) factors + macro level factors + macro risks	0.7974***	0.6078	0.4086***

Table 10 – Explaining Quarterly Excess Bond Returns with Macro Factors. The sample is quarterly from 1962Q4 to 2016Q4. The excess returns are annualized 1 quarter holding period returns on zero coupon US Treasuries. Macro risks (p_t^d, n_t^d, p_t^s and n_t^s) are scaled to have unit variance. The value in parentheses is the proportion out of 5,000 Bauer and Hamilton (2017) bootstrap runs where the t-stat for the coefficient is smaller than in data. The asterisks, * , **, and *** correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

	1 year bond	2 year bond	5 year bond	10 year bond
Constant	0.0533	0.7436	2.3547	5.1106
	(0.0698)	(0.3742)	(0.5058)	(0.5942)
$E_t\pi_{t+1}^{core}$	5.5115	11.6445	22.5331	38.2388
	(0.6818)	(0.6766)	(0.7170)	(0.7468)
$E_t\pi_{t+1}$	-5.1162***	-11.0131***	-21.7026***	-36.4865***
	(0.0014)	(0.0016)	(0.0016)	(0.0018)
$E_t g_{t+1}$	0.7092	0.9958	3.0505	7.5204
	(0.6442)	(0.5416)	(0.5672)	(0.5918)
$ugap_t$	0.2131	0.5477	1.2754	2.1034
	(0.6058)	(0.6228)	(0.7056)	(0.6346)
p_t^d	-0.8742***	-1.5057***	-3.1487***	-5.2105***
	(0.0020)	(0.0014)	(0.0014)	(0.0016)
n_t^d	-0.2270***	-0.6327***	-1.6587***	-3.3794***
	(0.0008)	(0.0010)	(0.0010)	(0.0008)
p_t^s	0.3998	0.5255	0.8686	0.7653
	(0.8600)	(0.6622)	(0.5794)	(0.3338)
n_t^s	0.3359	0.6965	1.4538	2.9693*
	(0.8668)	(0.8844)	(0.9296)	(0.9514)
Adjusted R^2 without macro risks	0.0416	0.0475	0.0471	0.0469
Adjusted R^2 with macro risks	0.0604	0.0610	0.0613	0.0685

Table 11 – Explanatory Power (Adjusted R^2) of Macro Risk Factors for Quarterly Excess Bond Returns over Macro Level and Financial Factors. The sample is quarterly from 1962Q4 to 2016Q4. Macro level factors are expected real GDP growth, expected aggregate and core inflation, and unemployment gap. Financial factors are the level, slope, and curvature factors. The level factor is the average over 1-10 year yields. The slope factor is the 10 year yield minus the 1 quarter yield. The curvature factor is the 10 year yield plus the 1 quarter yield minus 2 times the 2 year yield. The increase in adjusted R^2 significance, which is tested over the specification without the last set of factors (e.g., "3 financial factors+macro level factors+macro risks" row tests the incremental contribution of macro risks for the specification already including 3 financial factors and macro level factors), is Bauer and Hamilton (2017) adjusted significance using 5000 bootstrap runs. The asterisks, *, **, and ***, correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

	1 year bond	2 year bond	5 year bond	10 year bond
3 financial factors	0.0666	0.0657	0.0708	0.0796
3 financial factors+macro level factors	0.0962*	0.0932*	0.0774	0.0749
3 financial factors+macro risks	0.1338***	0.1292***	0.1101**	0.1164*
3 financial factors+macro level factors+macro risks	0.1429**	0.1370**	0.1065*	0.1051*
3 financial factors+macro risks+macro level factors	0.1429	0.1370	0.1065	0.1051

Table 12 – Cyclicality of Expected Excess Bond Returns. The sample is quarterly 1962Q4-2016Q4. The dependent variable is the expected annualized quarterly excess return computed from the OLS regressions of realized annualized quarterly excess returns on 4 macro level factors (expected aggregate and core inflations, expected real GDP growth, and unemployment gap) and 3 macro risks (good and bad demand variance and bad supply variance). NBER recession is a dummy equal to 1 if there is a recession in that quarter. Demand/supply-ratio is the ratio of aggregate demand variance (good+bad) to aggregate supply variance (good+bad). Demand/supply-ratio is scaled to have the standard deviation of 1. Standard errors are Newey-West standard errors computed with 20 lags.

	1 year bond	5 year bond	10 year bond
constant	1.1468***	5.2244***	9.4250***
	(0.2923)	(1.6030)	(2.9254)
NBER-dummy	0.1436	1.3425	1.9486
	(0.3594)	(2.0213)	(3.4851)
demand-supply ratio	-0.5601***	-2.2523***	-3.9519***
	(0.1389)	(0.7079)	(1.2366)
NBER-dummy × demand-supply ratio	0.2666	0.4973	0.1476
	(0.1816)	(0.7413)	(1.2374)
Adjusted R^2	0.3600	0.2909	0.2933

Table 13 – Explanatory Power (Adjusted R^2) of Macro Factors for Term Premiums. The dependent variable is annualized term premium computed as the observed US Treasury long yield minus the expected 1 quarter US Treasury yield over the life of the long yield. The expectations of 1 quarter yield over the life of the long yield are from Blue Chip survey and are available semi-annually. The sample is 1986Q2-2016Q4. The standard deviation of each macro risk factor is scaled to 1. The value in parentheses is the proportion out of 5,000 Bauer and Hamilton (2017) bootstrap runs where the t-stat for the coefficient is smaller than in data. The significance of the increase in adjusted R^2 is computed using 5,000 bootstrap runs of Bauer and Hamilton (2017) bootstrap. The asterisks, * , **, and *** correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

	5 year bond	10 year bond
constant	0.1852	0.5254*
	(0.9370)	(0.9604)
$E_t\pi_{t+1}^{core}$	6.7811***	8.0065***
	(0.9978)	(0.9994)
$E_t\pi_{t+1}$	-5.0956***	-6.5618***
	(0.0026)	(0.0008)
$E_t g_{t+1}$	0.8876*	1.0378*
	(0.9720)	(0.9608)
$ugap_t$	0.0769	0.1164
	(0.5100)	(0.6018)
p_t^d	-0.0236*	-0.1107**
	(0.0412)	(0.0206)
n_t^d	-0.0121	-0.0887*
	(0.2678)	(0.0318)
p_t^s	0.5720***	0.6415***
	(0.9998)	(0.9996)
n_t^s	-0.2629	-0.1723
	(0.2614)	(0.2928)
Adjusted R^2 without macro risks	0.6513	0.6543
Adjusted R^2 with macro risks	0.6914*	0.6941*

Table 14 – Cyclicality of the Term Premium. The dependent variable is annualized term premium computed as the observed US Treasury long yield minus the expected 1 quarter US Treasury yield over the life of the long yield. The expectations of 1 quarter yield over the life of the long yield are from Blue Chip survey and are available semi-annually. The sample is 1986Q2-2016Q4. NBER recession is a dummy equal to 1 if there is a recession in that quarter. Demand/supply-ratio is the ratio of aggregate demand variance (good+bad) to aggregate supply variance (good+bad). Demand/supply- ratio is scaled to have the standard deviation of 1. Standard errors are Newey-West standard errors computed with 20 lags. The asterisks, *, **, and *** correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

	5 year	10 year
constant	0.5171	1.1815*
	(0.6445)	(0.6211)
NBER-dummy	0.5473	0.5262
	(0.5155)	(0.4156)
Demand-supply ratio	-0.1508	-0.2368
	(0.2394)	(0.2229)
NBER-dummy×demand-supply ratio	-0.4121	-0.2855
	(0.3979)	(0.3398)
Adjusted R^2	0.0228	0.0267

Table 15 – Explanatory Power of Macro Factors for Realized 10 Year Bond Return Variances. The sample is quarterly from 1962Q4 to 2016Q4. Realized variances are computed as the sums of squared daily bond returns inside the quarter. The standard deviation of each macro risk factor is scaled to 1. Financial factors are the level, slope, and curvature factors. The level factor is the average over 1-10 year yields. The slope factor is the 10 year yield minus the 1 quarter yield. The curvature factor is 10 year yield plus 1 quarter yield minus 2 times the 2 year yield. The signs in Panel B are from the OLS regression. The value in parentheses is the proportion out of 5,000 Bauer and Hamilton (2017) bootstrap runs where the t-stat for the coefficient is smaller than in data. The increase in adjusted R^2 significance, which is tested over the specification without the last set of factors (e.g., "3 financial factors+macro level factors+macro risks" row tests the incremental contribution of macro risks for the specification already including 3 financial factors and macro level factors), is Bauer and Hamilton (2017) small-sample adjusted significance using 5000 bootstrap runs. The asterisks, * , **, and *** correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

Panel A: Adjusted R^2's	
Macro risks	0.3473
Macro level factors	0.1890
3 financial factors	0.1390
Macro level factors + macro risks	0.4200***
3 financial factors +macro risks	0.4267***
3 financial factors+macro level factors	0.2937***
3 financial factors+macro level factors+macro risks	0.4408***
3 financial factors+macro risks+macro level factors	0.4408
macro risks+macro level factors+financial factors	0.4408
Panel B: Regression coefficients	
constant	0.0015*
	(0.9574)
$E_t\pi_{t+1}^{core}$	0.0026**
	(0.9822)
$E_t\pi_{t+1}$	-0.0016*
	(0.0348)
$E_t g_{t+1}$	3.14E-05
	(0.5970)
$ugap_t$	1.61E-04
	(0.8098)
p_t^d	8.48E-05
	(0.6284)
n_t^d	4.92E-04***
	(0.9998)
p_t^s	-3.50E-04*
	(0.9592)
n_t^s	3.15E-05
	(0.5452)

INTERNET APPENDIX

Appendix A - Bootstrapping standard errors for the impulse responses

The VAR(2) parameters and the resulting reduced-form shocks are estimated with error, and so are the higher-order moments of the reduced-form shocks (and their covariance matrix). These sources of error affect the distribution of the sampling error of the loadings of the endogenous variables onto supply and demand shocks, the time series estimates of the supply and demand shocks, and the impulse response functions. To account for all of these sources of error, we use a bootstrapping routine.

We begin by sampling, with replacement, the reduced-form shocks from the estimated VAR(2) model. We assemble synthetic samples using 22 randomly chosen blocks of length 20 quarters. This results in synthetic samples of approximately the same length as our data (220 for bootstraps, 225 for the data). We use these shocks and the estimated VAR(2) parameters to build up synthetic samples of the endogenous variables. Note that we do not need any estimates of the covariance matrix of shocks to do this. Beginning from these synthetic samples, we follow the same procedures for each bootstrap sample that we do for the actual sample to calculate all the statistics of interest:

- Estimate VAR(2) parameters on the synthetic sample.
- Estimate higher-order moments of the reduced form shocks and their covariance matrix
- Estimate loadings of the macro variables onto supply and demand using the GMM procedure on the higher order moments
- Invert supply and demand shocks using the Kalman filter procedure
- Estimate the loadings of the yield variables onto the supply and demand shocks by OLS
- Estimate the impulse responses

Appendix B - Model-free Impulse Responses to Aggregate Demand and Supply Shocks

Following Jorda (2005), we calculate the model-free impulse responses using OLS regressions of the form:

$$Y_{t+h} = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \beta_3 \hat{u}^{supply}_{t-1} + \beta_4 \hat{u}^{demand}_{t-1} + \epsilon_{t+h},$$

where $\hat{u}^{supply}$ and $\hat{u}^{demand}$ are the inverted supply and demand shocks. Standard errors are computed as described in Appendix A.

The results are as follows:

Cumulative (20 Quarters)		
	Demand Shock	Supply Shock
Real GDP level	0.37%	0.81%
	(0.46%)	(0.42%)
Price level	2.15%	-0.05%
	(0.31%)	(0.30%)

Appendix C - Maximum likelihood estimation of demand and supply shock dynamics

We restrict attention to the demand shock estimation, as the supply shock estimation is identical. The system to estimate is:

$$\begin{aligned}
&u^d_{t+1} = \sigma^d_p \omega^d_{p,t+1} - \sigma^d_n \omega^d_{n,t+1}, \\
&\omega^d_{p,t+1} \sim \Gamma(p^d_t, 1) - p^d_t, \\
&\omega^d_{n,t+1} \sim \Gamma(n^d_t, 1) - n^d_t, \\
&p^d_{t+1} = \bar{p}^d + \rho^d_p (p^d_t - \bar{p}^d) + \sigma^d_{pp} \omega^d_{p,t+1}, \\
&n^d_{t+1} = \bar{n}^d + \rho^d_n (n^d_t - \bar{n}^d) + \sigma^d_{nn} \omega^d_{n,t+1},
\end{aligned} \tag{14}$$

where only the time series of demand shock realizations, $\{u^d_t\}^T_{t=1}$ is observed.

The following notation is defined:

$U^d_t \equiv \{u^d_1, ..., u^d_t\}$ is the sequence of observations up to time t.

$F(i\phi, i\psi^1, i\psi^2 | U^d_t) \equiv E(e^{i\phi u^d_{t+1} + i\psi^1 p^d_{t+1} + i\psi^2 n^d_{t+1}} | U^d_t)$ is the next period's joint conditional characteristic function of the observation and the state variables.

$G_{t|s}(i\psi^1, i\psi^2) \equiv E(e^{i\psi^1 p^d_t + i\psi^2 n^d_t} | U^d_s)$ is the characteristic function of the time t state variables conditioned on observing data up to time s.

The estimation procedure is an application of Bates (2006)'s algorithm for the component model of two gamma distributed variables and consists of the time 0 initialization and 3 steps repeated for each observation in $\{u^d_t\}^T_{t=1}$. At time 0, the characteristic function of the state variables $G_{0|0}(i\psi^1, i\psi^2)$ is initialized. The distribution of p^d_0 and n^d_0 is approximated with gamma distributions. Note that the unconditional mean and variance of p^d_t are $E(p^d_t) = \bar{p}^d$ and $Var(p^d_t) = \frac{\sigma^2_{pp}}{1-\rho^{d2}_p} \bar{p}^d$, respectively. The approximation by the gamma distribution with the shape parameter k_0 and the scale parameter σ^p_0 is done by matching the first two unconditional moments. Using the properties of the gamma distribution, $k^p_0 = \frac{E^2 p^d_t}{Var(p^d_t)}$ and $\theta^p_0 = \frac{Var(p^d_t)}{E(p^d_t)}$. Thus, p^d_0 is assumed to follow $\Gamma(k^p_0, \theta^p_0)$ and n^d_0 is assumed to follow $\Gamma(k^n_0, \theta^n_0)$, where k^n_0 and θ^n_0 are computed in the same way. Using the properties of the expectations of the gamma variables, $G_{0|0}(i\psi^1, i\psi^2) = e^{-k^p_0 \ln(1-\theta^p_0 i\psi^1) - k^n_0 \ln(1-\theta^n_0 i\psi^2)}$. Given $G_{0|0}(i\psi^1, i\psi^2)$, computing the likelihood of U^d_T is performed by repeating the steps 1-3 below for all subsequent values of t.

Step 1. Computing the next period's joint conditional characteristic function of the observation and the state variables:

$$
\begin{aligned}
F(i\Phi, i\psi^1, i\psi^2|U_t^d) &= E(E(e^{i\Phi(\sigma_p^d\omega_{p,t+1}^d-\sigma_n^d\omega_{n,t+1}^d)+i\psi^1(\bar{p}^d+\rho_p^d p_t^d+\sigma_{pp}^d\omega_{p,t+1}^d)+i\psi^2(\bar{n}^d(1-\rho_n^d)+\rho_n^d n_t^d+\sigma_{nn}^d\omega_{n,t+1}^d)}|U_t^d) \\
&= E(e^{i\psi^1\bar{p}^d(1-\rho_p^d)+i\psi^2\bar{n}^d(1-\rho_n^d)+(i\psi^1\rho_p^d-\ln(1-i\Phi\sigma_p^d-i\psi^1\sigma_{pp}^d)-i\Phi\sigma_p^d-i\psi^1\sigma_{pp}^d)p_t^d+(i\psi^2\rho_n^d-\ln(1+i\Phi\sigma_n^d-i\psi^2\sigma_{nn}^d)+i\Phi\sigma_n^d-i\psi^2\sigma_{nn}^d)n_t^d}|U_t^d) \\
&= e^{i\psi^1\bar{p}^d(1-\rho_p^d)+i\psi^2\bar{n}^d(1-\rho_n^d)}G_{t|t}(i\psi^1\rho_p^d-\ln(1-i\Phi\sigma_p^d-i\psi^1\sigma_{pp}^d)-i\Phi\sigma_p^d-i\psi^1\sigma_{pp}^d, i\psi^2\rho_n^d-\ln(1+i\Phi\sigma_n^d-i\psi^2\sigma_{nn}^d)+i\Phi\sigma_n^d-i\psi^2\sigma_{nn}^d).
\end{aligned}
$$

Step 2. Evaluating the conditional likelihood of the time $t+1$ observation:

$$
p(u_{t+1}^d|U_t^d) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(i\Phi, 0, 0|U_t^d)e^{-i\Phi u_{t+1}^d})d\Phi,
$$

where the function F is defined in step 1 and the integral is evaluated numerically.

Step 3. Computing the conditional characteristic function for the next period, $G_{t+1|t+1}(i\psi^1, i\psi^2)$:

$$
G_{t+1|t+1}(i\psi^1, i\psi^2) = \frac{\frac{1}{2\pi}\int_{-\infty}^{\infty} F(i\Phi, i\psi^1, i\psi^2|U_t^d)e^{-i\Phi u_{t+1}^d}d\Phi}{p(u_{t+1}^d|U_t^d)}.
$$

As above, the function $G_{t+1|t+1}(i\psi^1, i\psi^2)$ is also approximated with the gamma distribution via matching the first two moments of the distribution. The moments are obtained by taking the first and second partial derivatives of the joint characteristic function:

$$
\begin{aligned}
E_{t+1}p_{t+1}^d &= \frac{1}{2\pi p(u_{t+1}^d|U_t^d)}\int_{-\infty}^{\infty} F_{\psi^1}(i\Phi, 0, 0|U_t^d)e^{-i\Phi u_{t+1}^d}d\Phi, \\
Var_{t+1}p_{t+1}^d &= \frac{1}{2\pi p(u_{t+1}^d|U_t^d)}\int_{-\infty}^{\infty} F_{\psi^1\psi^1}(i\Phi, 0, 0|U_t^d)e^{-i\Phi u_{t+1}^d}d\Phi - E_{t+1}^2 p_{t+1}^d, \\
E_{t+1}n_{t+1}^d &= \frac{1}{2\pi p(u_{t+1}^d|U_t^d)}\int_{-\infty}^{\infty} F_{\psi^2}(i\Phi, 0, 0|U_t^d)e^{-i\Phi u_{t+1}^d}d\Phi, \\
Var_{t+1}n_{t+1}^d &= \frac{1}{2\pi p(u_{t+1}^d|U_t^d)}\int_{-\infty}^{\infty} F_{\psi^2\psi^2}(i\Phi, 0, 0|U_t^d)e^{-i\Phi u_{t+1}^d}d\Phi - E_{t+1}^2 n_{t+1}^d,
\end{aligned}
$$

where F_{ψ^i} denotes the derivative of F with respect to ψ^i. The expressions inside the integral are obtained in closed form by derivating the function $F(i\Phi, i\psi^1, i\psi^2|U_t^d)$ in step 1, and integrals are evaluated numerically. Using the properties of the gamma distribution, the values of the shape and the scale parameters are $k_{t+1}^p = \frac{E_{t+1}^2 p_{t+1}^d}{Var_{t+1}p_{t+1}^d}$ and

$\theta^p_{t+1} = \frac{Var_{t+1} p^d_{t+1}}{E_{t+1} p^d_{t+1}}$, respectively. The expressions for k^n_{t+1} and θ^n_{t+1} are similar.

The total likelihood of the time series is the sum of individual likelihoods from step 2: $L(Y_T) = \ln p(u^d_1 | k^p_0, \theta^p_0) + \sum_{t=2}^{T} \ln p(u^d_{t+1} | U^d_t)$.

Appendix D - Additional Results on Explanatory Power of Macro Risks

Explanatory Power (Adjusted R^2) of Macro Risk Factors for Yield Curve Factors over Realizations of Macroeconomic Time Series. The sample is quarterly from 1962Q4 to 2016Q4. Macro level factors are real GDP growth, aggregate and core inflation, and unemployment gap. Financial factors are the level, slope, and curvature factors. The level factor is the average over 1-10 year yields. The slope factor is the 10 year yield minus the 1 quarter yield. The curvature factor is 10 year yield plus 1 quarter yield minus 2 times the 2 year yield. The increase in adjusted R^2 significance, which is always tested over the specification in the previous row, is Bauer and Hamilton (2017) adjusted significance using 5000 bootstrap runs. The asterisks, *, **, and *** correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

Realizations of Macroeconomic Level Factors and Macro Risks			
	Level	Slope	Curvature
Realization of macroeconomic level factors	0.4795	0.5277	0.2168
Realization of macroeconomic level factors + macro risks	0.7151***	0.5675*	0.4038***

Explanatory Power (Adjusted R^2) of Macro Risk Factors for Quarterly Excess Bond Returns over Ang-Piazzesi (2003) and Financial Factors. The sample is quarterly from 1962Q4 to 2016Q4. Ang-Piazzesi factors are lag 1-12 Ang and Piazzesi (2003) real and nominal factors. Macro level factors are expected real GDP growth, expected aggregate and core inflation, and unemployment gap. Financial factors are the level, slope, and curvature factors. The level factor is the average over 1-10 year yields. The slope factor is the 10 year yield minus the 1 quarter yield. The curvature factor is the 10 year yield plus the 1 quarter yield minus 2 times the 2 year yield. The increase in adjusted R^2 significance, which is tested over the specification in the previous row, is Bauer and Hamilton (2017) adjusted significance using 5000 bootstrap runs. The asterisks, *, **, and *** correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

Predictors	1 year bond	2 year bond	5 year bond	10 year bond
3 financial factors	0.0663	0.0653	0.0638	0.0795
3 financial factors+Ang-Piazzesi	0.1549**	0.1415**	0.1325*	0.1295
3 financial factors+Ang-Piazzesi+macro level factors	0.1734	0.1537	0.1432	0.1471
3 financial factors+Ang-Piazzesi+macro level factors+macro risks	0.1903	0.1870**	0.1710*	0.1622