Correcting expected utility for comparisons between alternative outcomes: A unified parameterization of regret and disappointment

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Abstract A unified parameterization of an expected utility model corrected for regret and disappointment effects is presented, constrained to conform to a well-known choice pattern, the common consequence effect, a special case of the Allais paradox. For choices subject to regret and disappointment effects to be consistent with this choice pattern, the function that corrects the utility of the obtained outcome has to have a positive second derivative for its regret component and a negative second derivative for its disappointment component. These hypothesized functional forms make predictions about the relative effect that small vs. large differences between obtained and alternative outcomes should have on people's experiences of regret or disappointment.

Keywords Regret · Disappointment · Alternative outcomes

JEL Classification D8 · D81

The search for a general class of objective functions that risky decision makers seek to maximize has a long history (Machina 1987; Starmer 2000; Schoemaker 1982; Glimcher and Rustichini 2004). For centuries, decision researchers have compared the predictions of theoretical objective functions against observed patterns of preference. Deviations between model predictions and observed behavior have been typically communicated under the label of paradoxes. An early example of a deviation of observed preferences from the model of expected value maximization was provided by the St. Petersburg paradox (Bernoulli 1738), which resulted in the

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consideration of a more general model, namely expected utility (EU) maximization. Limitations to the predictive accuracy of EU maximization were, in due course, provided by the paradoxes of Allais (1953) and Ellsberg (1961), the common consequence effect, the common ratio effect, the preference reversal phenomenon (Lichtenstein and Slovic 1971), and framing effects (Tversky and Kahneman 1981), to name just a few. Those problems for expected utility theory generated new proposals for yet more general objective functions. The most influential alternative theories have been prospect theory by Kahneman and Tversky (1979), regret theory (RT) proposed first by Bell (1982) and Loomes and Sugden (1982) and later in axiomatic form by Sugden (1993), and disappointment theory, proposed in different versions by Bell (1985), Loomes and Sugden (1986), and more recently by Gul (1991), Skiadas (1997a, b) and Grant and Kajii (1998). A comparative analysis of the different models is provided by Grant, Kajii and Polak (2000). Prospect theory, regret theory, and disappointment theory were designed to preserve the basic structure of EU theory, but to also account for behavior not predicted by that theory.

In this paper, we take a closer look at regret and disappointment theories. This paper has two goals: (1) to provide a unified conceptualization of these two theories that both assume that decision makers, after experiencing an outcome, make counterfactual comparisons between their obtained outcome and other possible outcomes, which are in turn assumed to be anticipated and considered in the initial choice of action; (2) to provide a behavioral constraint for the functional form and parameter values for the regret and disappointment formalization. Since the Allais paradox was instrumental in the development of regret theory and has been used as a behavioral constraint in the evaluation of other choice models (e.g., Birnbaum 2004), we examine which combination of parameter values for regret and disappointment allow us to account for its Common Consequence Effect.

The paper is organized as follows. Section 1 reviews the Common Consequence Effect (CCE). Section 2 reviews different regret theory models and proposes a formulation that applies to a discrete set of possible states of the world and to decisions with more than two choice options. Like other formulations, our model corrects the utility of an obtained outcome by subtracting a nonlinear regret component. In Section 3, we show the conditions that allow our regret model to account for the CCE preference pattern. In particular, after proposing a specific functional form for the regret function, we specify the set of parameter values that allows the regret formalization to be consistent with the CCE preference pattern. Section 4 reviews the main aspects of disappointment theory and proposes a new formalization that unifies it with the formalization of regret theory, making it also applicable to decision situations with more than two choice options and a discrete set of states of the world, a very common phenomenon in real-world applications. One consequence of generalizing previous regret and disappointment models to a version that can be applied to decisions with more than two choice options is that the counterfactual comparison of obtained outcome to other outcomes (either as a function of choice selection in the case of regret, or as a function of state of the world in the case of disappointment) is always with respect to the best possible outcome. This allows for only the negative experiences of regret and disappointment, not for their positive equivalents of rejoicing and elation. In Section 5, a specific functional form for the new formulation of the disappointment function is proposed and the range of

parameter values that make it consistent with the CCE choice pattern is specified. Section 6 presents our conclusions and summarizes our accomplishment. Formalizing both regret and disappointment, two similar though distinct psychological reactions and phenomena, into utility adjustments that have the same mathematical structure allows for the combined consideration and modeling of both types of effects. Constraining our model formulations to provide a resolution of the Allais paradox, provides an upper and lower bound for the disappointment model parameter k_D, and a lower bound for the regret model parameter k_R. Finally, our model formulations make some predictions that can be experimentally falsified. In particular, they predict that while regret and disappointment both monotonically increase with increases in the discrepancy between obtained outcome and considered counterfactual outcome, regret effects will increase in an accelerating fashion, whereas disappointment effects will increase in a decelerating fashion.

1 The common consequence effect

Observed behavior of human decision makers often contradicts the axioms of expected utility theory (e.g., Starmer 2000). One historically very influential example was provided by Maurice Allais (1953) in the form of an informal experiment:

People were asked to choose between the following two prospects: $s_1 = (\$1M, 1)$ and $s_2 = (\$5M, 0.1; \$1M, 0.89; \$0, 0.01)$. The first option offers one million dollars for sure, i.e., with probability 1. The second option offers five million dollars with a probability of 0.1, one million dollars with probability 0.89, and nothing with probability 0.01.

Most people preferred the first option over the second option:

$$s_1 \succ s_2. \tag{1.1}$$

The same individuals were then asked to choose between the following two prospects: $s_1^{'} = (\$1M, 0.11; \$0, 0.89)$ and $s_2^{'} = (\$5M, 0.1; \$0, 0.9)$.

For this choice, the majority of respondents preferred the second option:

$$s_2^{'} \succ s_1^{'}. \tag{1.2}$$

It is easy to show that this pattern of preferences is not admissible for a decision maker who abides by the axioms of expected utility theory.

Machina (1987) showed that the Allais paradox is a special case of a more general set of risky decision problems, known as the common consequence effect (CCE). We can represent the CCE pattern for generic prospects s_1, s_2, $s_1^{'}$, $s_2^{'}$, in the following form:

$$s_1 = (X_1, p_1; X_3, 1 - p_1)$$

$$s_2 = (X_2, p_1; X_1, p_2; X_0, 1 - p_1 - p_2)$$

$$\begin{cases} s_1^{'} = (X_1, 1 - p_2; X_0, p_2) \\ s_2^{'} = (X_2, p_1; X_0, 1 - p_1) \end{cases}, \tag{1.3}$$

with $X_2 > X_1 > X_0$ and $X_3 > X_0$. It is easy to see that by setting $X_1 = X_3 = \$1M$, $X_0 = 0$, $X_2 = \$5M$, $p_1 = 0.1$ and $p_2 = 0.89$, we obtain the Allais paradox described above. The pattern of preferences associated with the Allais paradox is as follows:

$$s_1 \succ s_2 \text{ and } s_2' \succ s_1'. \tag{1.4}$$

To show the well-known result that the preference pattern of Eq. 1.4 implies a contradiction for expected utility (EU) maximization, let us define function EU(.) in the usual way:

$$EU(s_j) = \sum_{i=1}^{n} p_{ij} u(w_{ij}), \tag{1.5}$$

where u(.) is the von Neumann–Morgenstern (1957) utility function, $s_j = (w_{1j}, p_{1j}; ...; w_{nj}, p_{nj})$ is the prospect "j", $w_{1j}, ..., w_{nj}$ are the possible outcomes associated with prospect s_j, and $p_{1j}...p_{nj}$ are the probabilities with which outcomes w_{1j} can be expected to occur.

EU theory specifies that the function EU(.) for two arbitrary prospects s_1 and s_2, must satisfy

$$EU(s_1) \geq EU(s_2) \Leftrightarrow s_1 \succ (or =) s_2. \tag{1.6}$$

We briefly review the result that Eqs. 1.3, 1.4, 1.5 and 1.6, as in the Allais paradox, cannot all hold. In order to show the inconsistency between them, we will calculate, by means of Eq. 1.5, the expected utility for the generic prospects, defined by Eqs. 1.3, s_1, s_2, s_1', s_2'.[1] For simplicity, without excluding the Allais problem, let us choose $X_1 = X_3$ and $X_0 = 0$, and assume the reasonable simplification that the utility of a zero outcome is zero, i.e. $u(0) = 0$ (see Machina 1987). We know that with the preferences shown in the Eqs. 1.4 and with expected utility satisfying Eq. 1.6, the following inequalities should be satisfied

$$\begin{aligned} EU(s_1) &> EU(s_2) \\ EU(s_2') &> EU(s_1') \end{aligned}. \tag{1.7}$$

Calling $u_1 \equiv u(X_1)$, $u_2 \equiv u(X_2)$ setting $u(0) = 0$, and calculating EU by Eq. 1.5, we obtain

$$\begin{aligned} EU(s_1) &= u_1 \\ EU(s_2) &= p_1 u_2 + p_2 u_1 \\ EU(s_1') &= (1 - p_2) u_1 \\ EU(s_2') &= p_1 u_2 \end{aligned}. \tag{1.8}$$

Then, replacing Eqs. (1.8) in Eqs. (1.7), we obtain the inequalities

$$\begin{aligned} u_1 &> p_1 u_2 + p_2 u_1 \Rightarrow (1 - p_2) u_1 > p_1 u_2 \\ p_1 u_2 &> (1 - p_2) u_1 \end{aligned}. \tag{1.9}$$

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[1] We show this explicitly because our set of choice options differs from the original Allais paradox and the choice set of the CCE employed by Machina (1987).

Equations 1.9 are in contradiction, which describes the essence of the Allais paradox. In Section 3 we describe the conditions under which regret theory can avoid the apparent contradiction in the pattern of behavior exhibited in this family of CCE cases.

2 Regret theory

Loomes and Sugden (1982) and Bell (1982) present regret theory as an alternative to expected utility that is simpler than prospect theory. Regret theory introduces a correction to the utility calculation of expected utility theory. The correction describes the change in the utility assessment of an outcome as the result of comparing the obtained outcome with the outcome one could have obtained under the realized state of the world, had one chosen a different option.

Braun and Muermann (2004) proposed a formulation of regret theory that is consistent with the axioms provided by Sugden (1993) and Quiggin (1994) and can be applied to decisions that have more than two possible prospects. Many real world applications of the experience of regret contain more than two choice options. One example are cultivation decisions made by farmers, where a large number of different crops can be grown in different ways, and where the outcomes of cultivation decisions different from one's own under the realized state of the world (e.g., experienced geophysical and/or economic conditions: weather and prices) often are apparent because they have been taken by neighboring farmers. For cases like this and many others with more than two choice alternatives, Braun and Muermann's (2004) formulation deserves to be more widely known and applied. Their objective function corrects the traditional von Neumann–Morgenstern utility function u with a regret functional component g. The new function is called Regret-Theoretical Expected Utility (RTEU), and for continuous states of the world is represented by

$$\text{RTEU}(s) = \int \left\{ u(w_\theta(s)) - k_R \cdot g_R \left[u\left(w_\theta^{\max}\right) - u(w_\theta(s)) \right] \right\} dF(\theta), \qquad (2.1)$$

where $F(\theta)$ is the cumulative distribution function of the states of the world represented by θ. $w_\theta(s)$ is the outcome in state of world θ when action s is selected. $w_\theta^{\max}$ is the outcome in state of the world θ that would have provided the decision maker with the maximum in utility. The function g_R transforms the amount by which obtained outcome $w_\theta(s)$ falls short of maximally obtainable outcome $w_\theta^{\max}$ into a regret component, for each state of the world θ. The parameter k_R is a measure of the influence of the regret component on the decision, with $k_R \geq 0$. When k=0, RTEU reduces to EU.

2.1 Conditions on function g_R

The function $g_R[.]$ needs to be determined. Braun and Muermann (2004) propose the following desirable properties for the function:

$$g_R(0) = 0 \qquad (2.2)$$

$$g'_R(u) > 0 \quad \forall u \tag{2.3}$$

$$g''_R(u) > 0 \quad \forall u \tag{2.4}$$

where the prime "'" notation denotes the derivative d/du. The first condition establishes that when the difference between obtained and optimum value is zero, there is no regret. The second condition tells us that g ought to be an increasing function, i.e., when the difference between the optimum option and the chosen option increases, regret also increases. The third condition was not clearly justified. Conditions 2.2 to 2.4 were used by Braun and Muermann to analyze the possible effect of regret in decisions to buy insurance and the implications of regret theory for optimal insurance purchases. The stylized empirical fact reported by the authors that people tend to prefer low deductibles when buying personal insurance are in general agreement with their postulates for regret theory. We will show in Section 3 that condition 2.4 can be justified by the requirement that regret-influenced decisions should be able to account for the preference pattern of the CCE without any contradiction or paradox.

2.2 Formulation of regret theory for a discrete set of states of the world

The Allais-like paradoxes described in Section 1 involve lotteries with a discrete set of states of the world. In order to apply regret theory to Allais choice problems, it is necessary to express Eq. 2.1 in discrete form. Using the notation of Eq. 1.5, the RTEU for a risky option with a discrete set of outcomes is:

$$\text{RTEU}(s_j) = \sum_{i=1}^{n} p_{ij} \left\{ u(w_{ij}) - k_R \cdot g_R \left[\Delta^R u_{ij} \right] \right\}, \tag{2.5}$$

where $\Delta^R u_{ij} = u(w_i^R) - u(w_{ij})$ and $w_i^R = \max \{w_{i1}, \ldots, w_{im}\}$ is the maximum outcome for the "i" event when "m" possible actions are considered. As a consequence of this definition, the difference between the utility of the optimal outcome for each state of the world and the utility of the evaluated outcome is always positive or zero, i.e., $\Delta^R u_{ij} \geq 0$.

Our formulation, which is a discrete version of the formalization of Braun and Muermann (2004), is similar to the one by Loomes and Sugden (1982) in the following respects: (a) it is a discrete formulation, (b) it provides a non-linear correction to the utility, and (c) the correction is a function that operates on a difference of outcomes. Since Loomes and Sugden (1982) compare the difference of outcomes obtained by only a pair of actions under the same state of the world, their difference in outcomes can not only be positive, but also negative, in which case the corrected utility will be bigger than the utility without correction, resulting in an effect described as rejoicing. Since in our formalization the difference between obtained outcome and a counterfactual other outcome is always made with respect to the best action that could have been taken under the realized state of the world, the difference is always positive (or zero), resulting in only the experience of regret, and no rejoicing. Loomes and Sugden's (1982) regret model only results in choice

predictions different from expected utility theory to the extent that the effects of regret far exceed the effects of rejoicing. Inman, Dyer, and Jia (1997) provide some empirical support for the notion that regret effects are far more important than the effects of rejoicing.

3 Conditions on regret theory to model the CCE choice pattern

Our goal in this section is to produce the pattern of preference described by Eqs. 1.4 observed for the Allais choice problems by using the regret-corrected utility RTEU, i.e., to model the observed preferences without the contradictions that appear when expected utility is assumed to underlie people's preferences. The use of the Allais paradox preference pattern to put constraints on the regret function of regret theory is motivated by numerous suggestions that anticipated regret may lie at the root of people's choices in this historic choice situation (e.g., Schick 1991, p. 125). In order for regret theory to describe the CCE choice pattern, decision makers' regret-corrected utility given by Eq. 2.5 must satisfy:

$$\text{RTEU}(s_1) > \text{RTEU}(s_2) \tag{3.1}$$

$$\text{RTEU}\left(s_2'\right) > \text{RTEU}\left(s_1'\right) \tag{3.2}$$

In order to calculate the expressions in Eqs. 3.1 and 3.2, and in particular the regret components, we need to make explicit which outcomes are obtained for both choice options under the different states of the world. We therefore rewrite Eqs. 1.3 in the following equivalent way:

$$
\begin{cases}
s_1 = (X_1, p_1; X_1, p_2; X_1, 1 - p_1 - p_2) \\
s_2 = (X_2, p_1;\ X_1, p_2;\ \ 0,\ 1 - p_1 - p_2) \\
s_1' = (\ 0, p_1; X_1, 1 - p_2; 0, p_2 - p_1) \\
s_2' = (X_2, p_1;\ \ 0, 1 - p_2; 0, p_2 - p_1)
\end{cases}.
\tag{3.3}
$$

Using Eq. 2.5 on the prospects in Eq. 3.3 and calling $\Delta u = u_2 - u_1$, we obtain

$$
\begin{aligned}
\text{RTEU}(s_1) &= p_1 \cdot [u_1 - k \cdot g_R(\Delta u)] + p_2 \cdot u_1 + (1 - p_1 - p_2) \cdot u_1 \\
\text{RTEU}(s_2) &= p_1 \cdot u_2 + p_2 \cdot u_1 - (1 - p_1 - p_2) \cdot k \cdot g_R(u_1) \\
\text{RTEU}(s_1') &= -p_1 \cdot k \cdot g_R(u_2) + (1 - p_2) \cdot u_1 \\
\text{RTEU}(s_2') &= p_1 \cdot u_2 - (1 - p_2) \cdot k \cdot g_R(u_1).
\end{aligned}
\tag{3.4}
$$

Imposing the inequalities specified by Eqs. 3.1 and 3.2, we obtain:

$$(1 - p_2) \cdot u_1 - p_1 \cdot u_2 > p_1 \cdot k \cdot g_R(\Delta u) - (1 - p_1 - p_2) \cdot k \cdot g_R(u_1), \tag{3.5}$$

$$p_1 \cdot u_2 - (1 - p_2) \cdot u_1 > (1 - p_2) \cdot k \cdot g_R(u_1) - p_1 \cdot k \cdot g_R(u_2). \tag{3.6}$$

By summing inequalities 3.5 and 3.6 and simplifying the sum, we obtain the following inequality for the function g_R:

$$g_R(u_2) - g_R(u_1) > g_R(\Delta u). \tag{3.7}$$

Using the fact that $u_2 = u_1 + \Delta u$ and also that $g_R(0)=0$ we can rewrite Eq. 3.7 as

$$\frac{g_R(u_1 + \Delta u) - g_R(u_1)}{\Delta u} > \frac{g_R(\Delta u) - g_R(0)}{\Delta u}. \tag{3.8}$$

If we can assume Δu to be an arbitrary increment in utility,[2] then inequality 3.8 implies that the first derivative of function g_R must be an increasing function, which means that the second derivative must be positive:

$$g_R'' > 0. \tag{3.9}$$

This is the same as condition 2.4 given by Braun and Muermann (2004) without any justification. We now have a justification for that condition, in the sense that it allows us to account for the preference pattern described as the CCE.

3.1 Proposal for a specific regret function

We propose the following possible regret function g_R as satisfying the properties required for regret-corrected utility (RTEU) to predict the preference pattern described as the Allais paradox [namely $g_R(0)=0$, $g_R' > 0$ and $g_R'' > 0$]:

$$g_R[u] = (\beta_R)^u - 1, \tag{3.10}$$

with $\beta_R>1$. This function's first derivative, $g_R' = (\ln \beta_R) \cdot (\beta_R)^u$, is greater than zero ($g_R' > 0$, as $\ln\beta_R>0$) and its second derivative, $g_R'' = (\ln \beta_R)^2 \cdot (\beta_R)^u$, is clearly greater than zero ($g_R'' > 0$).

The functional form of g shown in Eq. 3.10 is similar to the regret-rejoice function used by Loomes and Sugden (1982) in one of their application examples. One difference is that the function developed above has no rejoice component, since in the generalization proposed by Braun and Muermann (2004) the obtained outcome can never be better than the maximum outcome (i.e., $\Delta u \geq 0$).

There are other differences, as well. Loomes and Sugden's (1982) regret function is $R(\xi) = 1 - 0.8^\xi$ with $\xi = -\Delta u$, which means that $R(\Delta u) = 1 - 1.25^{\Delta u}$. Their overall regret-corrected utility is defined as $\widetilde{u} = u + R(\Delta u)$, where the regret component is added to the outcome utility, rather than subtracted, as in our model. When corrected for the difference in sign, Loomes and Sugden's regret function corresponds to the following g function in our model: $g_{LS} = 1.25^{\Delta u} - 1$. This equation has the same functional form as Eq. 3.10 with $\beta_R=1.25$.

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[2] There are two reasons to consider Δu to be an arbitrary increment: (1) there is a broad set of hypothetical preference patterns we consider (most cases of the CCE); (2) while Δu depends on the monetary value of different options, it also depends on personal characteristics of the decision makers, e.g., on his or her level of risk aversion.

3.2 Range for the k_R parameter

Equations 3.5 and 3.6 constitute a system of inequalities that impose conditions on the values of k_R and β_R that are compatible with the pattern of preferences of the CCE. We now compute what the allowed range of values of k_R and β_R are. It is convenient to define:

$$G = p_1 \cdot u_2 - (1 - p_2) \cdot u_1 \tag{3.11}$$

$$G_1 = (1 - p_1 - p_2) \cdot g_R(u_1) - p_1 \cdot g_R(\Delta u) \tag{3.12}$$

$$G_2 = (1 - p_2) \cdot g_R(u_1) - p_1 \cdot g_R(u_2). \tag{3.13}$$

Since G_1 and G_2 are functions of β_R, we can rewrite Eqs. 3.5 and 3.6 in the form:

$$G < G_1(\beta_R) \cdot k_R \tag{3.14}$$

$$G > G_2(\beta_R) \cdot k_R, \tag{3.15}$$

which allows us to obtain conditions that relate β_R to k_R.

We assume a specific utility function, namely the functional form proposed by Pratt (1964), with a moderate degree of risk aversion. This choice gets some empirical justification from the fact that decision makers with strong risk attitudes do not seem to produce the Allais paradox, as shown by Finkelshtain and Feinerman (1997) in an experiment with 180 farmers. The 112 strongly risk averse farmers and the 22 risk seeking farmers did not violate utility theory and thus did not produce the paradoxical preference pattern, but 46 respondents with moderate levels of risk aversion did. A risk seeker will choose the options s_2 and s_2' because these are the options with the highest outcomes, while a strongly risk averse decision maker will choose s_1 and s_1' because these are the options with less variance. We thus use the utility function proposed by Pratt (1964) with a commonly observed moderate level of risk aversion ($r=1$), regularized so that $u(0)=0$, to obtain:

$$u(w) = \ln(w + 1) \tag{3.16}$$

Using the outcome and probability values of the Allais problem:

$$p_1 = 0.1, p_2 = 0.89, u(\$5M) = u_2, u(\$1M) = u_1, \tag{3.17}$$

and applying Eqs. 3.17 and 3.16 to 3.11–3.13, we obtain G, $G_1(\beta)$ and $G_2(\beta)$, observing the following properties: G, $G_1 > 0$ and $G_2 < 0$. The system of inequalities 3.14 and 3.15 then provides us with the minimum value that k can take[3]:

$$\frac{G}{G_1} < k_R, \tag{3.18}$$

we will call $k_{min} = G/G_1$.

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[3] The other equation provides a negative minimum.

Fig. 1 Regret function parameters: maximum and minimum k_R for $\beta_R \in [1.15, 1.35]$. The boundaries of k_R common to the whole interval of β_R are shown as two horizontal lines
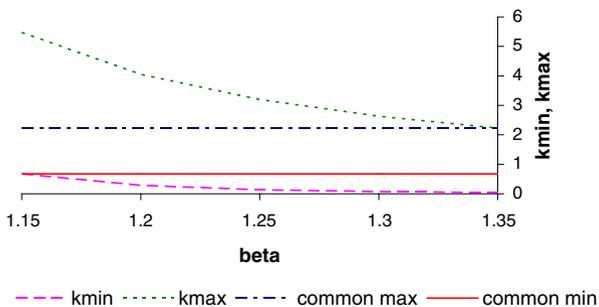
To obtain a plausible maximum value for k, we make the somewhat arbitrary assumption that the correction for regret should not be more than 10% of the amount of utility that it corrects[4], i.e.:

$$k_{\max} \ ; \ 0.1 \cdot u_1 / g_R(\Delta u). \tag{3.19}$$

Calculating $k_{\min}$ and $k_{\max}$ from Eqs. 3.18 and 3.19, respectively, we have the condition:

$$0.68 \leq k_R \leq 2.22. \tag{3.20}$$

The range for k_R given by Eq. 3.20 is compatible with the following range for β_R:

$$1.15 \leq \beta_R \leq 1.35. \tag{3.21}$$

It should be noted that the central value in inequality 3.21 corresponds to the value used by Loomes and Sugden (1982). Figure 1 shows the dependence between $k_{\min}$ and $k_{\max}$ and β_R for the range of values of k that provide a regret function that satisfies conditions 3.18 and 3.19.

4 Disappointment theory

Disappointment is caused by comparing an obtained outcome with a better or the best possible outcome of the action or option that was chosen. Bell (1985) introduced a simple model of disappointment in which the obtained outcome was compared not to a different possible outcome for the same choice, but to choice option's expected value (EV), and obtained outcomes are not transformed into subjective utilities. For obtained outcomes worse than the option's EV disappointment was postulated, proportional to the negative discrepancy between the two values, and for obtained outcomes better than the option's EV elation was assumed to result, proportional to the positive discrepancy. Both possible reactions to different choice outcomes are assumed to be anticipated by the decision maker at the time of decision and integrated ex-ante into the decision calculus. When both comparisons get equal consideration (i.e., when disappointment and elation get equal weight), the

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[4] Different assumptions about the relative amount of correction obviously lead to different estimates of plausible upper bounds on k.

resulting utility function is risk neutral. When disappointment gets greater consideration than elation, the assumed normal state of affairs, the resulting utility function is concave (resulting in risk averse behavior). If elation gets more weight than disappointment, the resulting utility function is convex (risk seeking).

Loomes and Sugden (1986) propose a formulation similar in goals to that of Bell (1985) and similar in details to ours, described below, with a non-linear correction to the utility of the obtained outcome. The main difference to our model is that in their model the counterfactual comparison between obtained outcome and alternative outcome is with respect to the option's EV, allowing for both disappointment and elation, whereas in our formulation the counterfactual comparison is with respect to the best possible outcome the chosen action could have had (under a different state of the world), allowing for only the experience of disappointment. This seems to be reasonable simplification of the model in light of the empirical results of Inman, Dyer, and Jia (1997), who showed that disappointment effects strongly overshadowed elation effects. Comparing their model to that of Bell (1985), Loomes and Sugden (1986) showed that a linear disappointment function may be consistent with some observations of the common consequence effect as well as the common ratio effect (CRE), but that it cannot explain such choice patterns when individuals are choosing between options with equal expected value, whereas a non-linear disappointment function is consistent with such behavior. Gul's (1991) model of disappointment is similar to Bell's (1985) in that an outcome causes disappointment (elation) if it is worse (better) than the risky option's certainty equivalent but, like our model, assumes that the obtained outcome (aside from its disappointment or elation effect) is evaluated on the decision maker's possibly risk averse utility function.

Below, we propose a different way to introduce disappointment that preserves the expected utility formulation in its usual form, but adds a corrective term for any discrepancy between the obtained outcome and the best possible outcome for the action that was chosen. In our approach, the disappointment-related utility correction also has the effect of producing some (additional) risk aversion, but is not the main cause of risk aversion that it is in Bell's (1985) model. Both approaches have the goal to account for preferences patterns for which the substitution principle is violated, as observed with the CCE. Our approach has the advantage of providing a unified conceptualization of the psychologically related feelings of regret and disappointment and their ex-ante effect on choice.

4.1 Proposed formulation

In a form analogous to Eq. 2.5, we introduce the Disappointment-Theoretical Expected Utility (DTEU), for a discrete set of n states of the world, with the following expression:

$$\text{DTEU}(s_j) = \sum_{i=1}^{n} p_{ij} \left\{ u(w_{ij}) - k_D \cdot g_D \left[\Delta^D u_{ij} \right] \right\}, \tag{4.1}$$

where, in this case, $\Delta^D u_{ij} = u\left(w_j^D\right) - u(w_{ij})$ and $w_j^D = \max\left\{w_{1j}, \ldots, w_{nj}\right\}$ is the maximum outcome for the "j" action, when "n" states of the world are considered. The index "i" represents the state of world.

5 Conditions on disappointment theory to model the preference pattern of the CCE

To describe the CCE choice pattern, the DTEU function must satisfy the following conditions, analogous to inequalities 3.1 and 3.2:

$$\text{DTEU}(s_1) > \text{DTEU}(s_2), \tag{5.1}$$

$$\text{DTEU}\left(s_2^{'}\right) > \text{DTEU}\left(s_1^{'}\right), \tag{5.2}$$

where s_1, s_2, $s_1^{'}$ and $s_2^{'}$ are the prospects given by 1.3. By applying the function DTEU (.) to Eqs. 1.3, we obtain:

$$
\begin{aligned}
&\text{DTEU}(s_2) = u_1 \\
&\text{DTEU}(s_2) = p_1 \cdot u_2 + p_2 \cdot [u_1 - k_D \cdot g(\Delta u)] - (1 - p_1 - p_2) \cdot k_D \cdot g_D(u_2) \\
&\text{DTEU}\left(s_1^{'}\right) = (1 - p_2) \cdot u_1 - p_2 \cdot k_D \cdot g_D(u_1) \\
&\text{DTEU}\left(s_2^{'}\right) = p_1 \cdot u_2 - (1 - p_1) \cdot k_D \cdot g_D(u_2)
\end{aligned} \tag{5.3}
$$

As before for the regret correction, we substitute the expressions in Eqs. 5.3 into the CCE preference pattern described by inequalities 5.1 and 5.2:

$$(1 - p_2) \cdot u_1 - p_1 \cdot u_2 > -p_2 \cdot k_D \cdot g_D(\Delta u) - (1 - p_1 - p_2) \cdot k_D \cdot g_D(u_2), \tag{5.4}$$

$$p_1 \cdot u_2 - (1 - p_2) \cdot u_1 > -p_2 \cdot k_D \cdot g_D(u_1) + (1 - p_1) \cdot k_D \cdot g_D(u_2). \tag{5.5}$$

By summing the two inequalities 5.4 and 5.5 and simplifying the resulting expression, we obtain the following inequality for the function g_D:

$$g_D(u_2) - g_D(u_1) < g_D(\Delta u), \tag{5.6}$$

This condition is the opposite of inequality 3.7 for the regret function g_D. Using the same reasoning as above, inequality 5.6 provides the following condition for the second derivative of the disappointment function:

$$g_D^{''} < 0. \tag{5.7}$$

The other conditions that we impose on the disappointment function are the same as Eqs. 2.2 and 2.3 for the regret function, namely $g_D(0)=0$ and $g_D^{'} > 0$. The first condition tells us that disappointment only occurs when the difference between the obtained and the best outcome of the chosen option or action is not zero. The second condition stipulates that the correction is an increasing function of the difference in outcomes.

The disappointment function in Loomes and Sugden's (1986) disappointment model satisfies the same properties as ours, i.e., it is zero when the difference between outcomes is zero, the first derivative is greater than zero, and the second derivative is less than zero, resulting in a concave disappointment function. Our formulation of disappointment and the imposed constraints thus support Loomes and Sugden's (1986) intuition that the intensity of disappointment and elation decreases at the margin.

5.1 Proposal for a specific disappointment function

As we saw previously, the necessary conditions for a disappointment function that will ensure that DTEU will accommodate the preference pattern of the CCE without any paradox are:

$$g_D(0) = 0, \tag{5.8}$$

$$g_D'(u) > 0, \quad \forall u, \tag{5.9}$$

$$g_D''(u) < 0, \quad \forall u. \tag{5.10}$$

For simplicity and in analogy to the regret model, we propose the following function:

$$g_D(u) = 1 - \beta_D^u, \tag{5.11}$$

with $\beta_D < 1$. It is easy to see that g_D satisfies conditions 5.8–5.10. Satisfaction of condition 5.8 is evident. Since the second derivative of g_D is $g_D' = -\beta_D^u \ln \beta_D$, condition 5.9 is satisfied for $\beta_D < 1$. Finally, condition 5.10 holds because the second derivative of g_D is $g_D'' = -\beta_D^u (\ln \beta_D)^2 < 0$.

5.2 Range for the k_D parameter

In analogy to Section 3.2, we will employ inequalities 5.4 and 5.5 in order to determine the range for parameters k_D and β_D that is compatible with the pattern of preferences of the CCE. As in Section 3.2, it is convenient to reexpress the choice options in the following way:

$$M = p_1 \cdot u_2 - (1 - p_2) \cdot u_1, \tag{5.12}$$

$$M_1 = p_2 \cdot g_D(\Delta u) + (1 - p_1 - p_2) \cdot g_D(u_2), \tag{5.13}$$

$$M_2 = (1 - p_1) \cdot g_D(u_2) - p_2 \cdot g_D(u_1). \tag{5.14}$$

Replacing the last equations in 5.4 and 5.5 we have:

$$M_1(\beta_D) \cdot k_D > M, \tag{5.15}$$

$$M > M_2(\beta_D) \cdot k_D. \tag{5.16}$$

The special case of the Allais example discussed above allows us to get numerical values for the parameters. Using the outcome and probability values of Eqs. 3.17 and the utility function of Eq. 3.16, and observing that M, M_1, $M_2 > 0$, we can define $k_{\min}$ and $k_{\max}$ from inequalities 5.15 and 5.16 as follows :

$$k_{\min} = \frac{M}{M_1}, \tag{5.17}$$

$$k_{\max} = \frac{M}{M_2}, \tag{5.18}$$

Fig. 2 Disappointment function parameters: maximum and minimum k_D for $\beta_D \in [0.7, 0.9]$. The boundaries of k_D common to the whole interval of β_D are shown as two horizontal lines
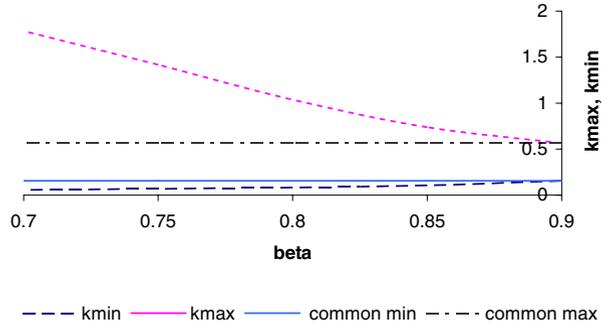
so that,

$$k_{\min} < k_D < k_{\max}, \tag{5.19}$$

where $k_{\min} = k_{\min}(\beta_D)$ and $k_{\max} = k_{\max}(\beta_D)$.

Bounds for the parameter k_D can then be calculated for a given range of values of the parameter β_D, with the results shown in Figure 2. Figure 2 shows that the range of values of k_D that provide a disappointment function that satisfies condition 5.19 gets narrower as β_D gets closer to 1, the point at which DTEU reduces to EU. The CCE preference pattern compatible ranges for k_D and β_D are:

$$0.16 \leq k_D \leq 0.56, \tag{5.20}$$

and

$$0.7 \leq \beta_D \leq 0.9. \tag{5.21}$$

Different to the regret function case, for the disappointment function it is not necessary to introduce an arbitrary additional condition to find an upper bound for the parameter k_D.

6 Conclusions and comments

This paper provides an explicit version of regret- and disappointment-corrected utility which can produce the pattern of preferences known as the common consequence effect, without posing a paradox.

Even though the two anticipated effects of counterfactual comparisons, regret and disappointment, have been considered separately in this paper, they can be included in the same expression, by simply adding their correction terms, i.e., $u - k_R \cdot g_R(\Delta^R u) - k_D \cdot g_D(\Delta^D u)$. This is an advantage that results from using the same mathematical structure. Inman, Dyer, and Jia (1997) also proposed a unified treatment of disappointment and regret. Their model, however, is a post-choice valuation model, which is intended to provide a basis for applying disappointment and regret to consumer choice. It is a linear model, and disappointment and regret are introduced as possible contributors to risk aversion, and as ways to categorize consumers into different categories by the degree to which they are susceptible to post-decision disappointment or regret. The model of Inman, Dyer, and Jia (1997) is

problematic if one were to apply it to find the optimal choice for a disappointment- or regret-susceptible decision maker ex ante, because the first derivative of the corrected utility does not depend on the derivation variable.

By imposing the requirement that regret- and disappointment-adjusted expected utility should give rise to the pattern of preference described by the CCE without being paradoxical, we found that the second derivative of regret function g_k must be positive, in agreement with Braun and Muermann (2004), but now with a justification. For the disappointment function g_D, the second derivative must be negative. The difference in the sign of the second derivative of the regret and disappointment function suggests that while both regret and disappointment increase with increases in the discrepancy between obtained outcome and considered counterfactual outcome, regret effects should increase in an accelerating fashion, whereas disappointment effects should increase in a decelerating fashion. We are not aware of data that could be examined to test the prediction made by our model, but consider it advantageous that the model makes predictions that are empirically falsifiable.

Another difference between the regret and disappointment components of our model has to do with the determinants for lower and upper boundaries for the weighting parameters k_R and k_D. In the case of k_R, the lower boundary was provided by the Allais paradox condition, but the upper boundary needed to be established by a somewhat arbitrary additional assumption. However for k_D, both parameter boundaries were provided by the Allais paradox condition.

One limitation of our models is related to another paradoxical pattern of responses, known as the common ratio effect, namely the change of preferences produced when the probabilities of economic outcomes that are being compared are multiplied by the same constant (see Machina 1987). Adding a regret (or disappointment) correction does not avoid the paradoxical nature of such choices because RTEU has the same linear dependence on probabilities that EU has. Solutions to the common ratio effect problem are provided by replacing the probability p_i with $\Omega(p_i)$, where $\Omega(\cdot)$ may be a weighting function like the one used in prospect theory. This solution is, of course, not incompatible with the regret or disappointment correction of our and other models. As described in Weber and Kirsner (1996), the rank-dependent weighting of prospect theory arises from a different set of psychological concerns that are orthogonal to concerns related to regret or disappointment. In ongoing work, we are exploring such combinations of models.

To summarize, we believe that the conceptualization and parameterization of regret and disappointment made in this paper makes several important contributions. The first one, of both theoretical and of practical value, is the unified treatment of the two psychological effects, which makes it possible to compare their relative influence in specific risky choice contexts. Other contributions are of a more practical nature, making these types of models possible and easier to apply to explain real world choice patterns. The explicit functional form for the calculation of the regret and the disappointment of our paper also provides a criterion to fix the values of the model parameters. Another important practical contribution is that our model provides a unified treatment of the two effects for cases with more than two possible states of the world. Since most real-world risky choice situations involve a much larger number of states of the world, our conceptualization makes regret and disappointment modeling available to this large segment of real-world data.

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