Credit Chains and Failure Propagation in Production Networks

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Abstract

We present a simple model of a production network in which firms are linked by supply-customer relationships involving extension of trade-credit. Our aim is to identify the minimal set of mechanisms which reproduce qualitatively the main facts of firm demography (like firm size distribution and aggregate output time series) and, at the same time, the spatio-temporal correlation of output, growth and failures among firms in the same neighborhood of the network. In previous multi-agent models investigating the emergence of macro-economic structures, firms interact only indirectly through the price or the interest rate. On the contrary, in this model it is possible to relate the aggregate behaviour of the economy to the direct firm-firm interdependence of output and profit. In this paper we characterize the behavior of a first version of the model in which the number of firms is constant, the network has a periodic static structure and the price dynamic is very simple. However the same framework can be easily extended to investigate which network structures are more robust against domino effects or, if the network evolves in time, which structures emerge spontaneously, depending on the individual strategies for orders and delivery.

1 Introduction

The basic paradigm in mainstream economic theory, i.e. the Arrow-Debreu general equilibrium framework, rests on the assumption that individuals take decisions in isolation using only the information contained in a general market signal such as the price vector: Interaction and co-ordination occur only indirectly through prices. Communication, imitation and learning through direct interaction among agents are ruled out by construction and the failures of co-ordination which are likely to arise in this context are simply defined away (Leijonhuvfud,1992,1993; Hahn and Solow 1995). The role of prices is undoubtedly important, but the price mechanism alone can work well only if information is perfect and markets are complete. If information is imperfect and markets are incomplete, i.e. the future is truly uncertain, we cannot ignore the role of co-ordination mechanisms in space – i.e. direct interaction among agents – and over time, i.e. the emergence of credit relationship. Firms interact also with banks on the credit market. In other words, there is a network of credit relationships among firms and between firms and banks which lends credit to the industrial sector (Stiglitz and Greenwald, 2003).

Heterogeneous agents interaction implies that the structure of aggregate behaviour at the macro level emerges from the interaction between the agents at the micro level. In other words, statistical regularities emerge as a self-organised process at the aggregate level: complex patterns of interacting individual behaviour may generate a certain regularity at the aggregate level, a notion known since the early '90 (Bak et al., 1993).

In this paper we take interaction and coordination seriously and we explore the features of a networked economy in which firms interact with each other directly for two reasons: (i) the output of some firms (sub-contractors) are input for some other firms and (ii) some firms extend trade credit to other firms. If firms are unable to reimburse debt, they run the risk of bankruptcy. In a world in which financing constraints are crucial, unexpected shocks to future revenues may affect bank-firm relationships as well as inter-firms credit links. In this context, the failure in fulfilling debt commitments may lead to a chain of similar failures and in extreme cases result in bankruptcy chains.

We move from the idea that describing the dynamics of the output of a large set of firms neglecting their interdependency may lead to miss important properties related to fluctuations of the output and in general to the stability of the economy.

We present a novel framework in which several specific models of production networks can be developed, in order to investigate how the business cycle is linked to the spreading of bankruptcies and to assess which types of network structure are more robust to external shocks. While in this paper, we discuss in detail a first simple model where the network structure is static, more articulated variants will be reported in future studies.

In this framework, failures can propagate based on direct firm-firm interaction: in other words the failure of a firm (in fulfilling its debt or in delivering) increases the probability of failure in connected firms, giving raise to clustered fluctuations of the number of failed firms. It is worth noting that this scenario of firm inter-linkages is different from that in which some firms go bankrupt at each time period because of unexpected price fluctuations with no systemic repercussions. In particular, previous models of financial fragility have been able to reproduce stylized facts of firm demography such as firm size distribution and temporal correlation of bankruptcies. However, in such models the accumulation of capital and the risk of bankruptcy do not depend on local interactions with other firms, although some models (Delli Gatti et al., 2005) include an indirect interaction that takes place through the endogenous determination of the interest rate on bank loans.

Such an approach is similar to the "mean field approximation" in Statistical Physics, which consists in supposing that each unit of the system is interacting with an average unit, thus neglecting fluctuations. The mean field approximation yields useful predictions when units interact in an all-to-all fashion and are not too heterogeneous, otherwise the dynamics of the system may be qualitatively different from the mean field prediction. Indeed, in a production network interactions are local and units are highly heterogeneous.

Interestingly, in the above mentioned models the evolution of the net worth of firms turns out to be reminiscent of a set of Multiplicative Stochastic Processes (MSP). The results on the firm's size distribution are then consistent with the well known fact that power law distributions can be obtained by combining an MSP with a specific additional ingredient – either a reflecting barrier or additive noise or a stochastic resetting of the size (Kesten, 1973; Biham et al, 1998; Sornette, 1998; Nirei and Souma, 2003). However, very little has been done on systems in which these processes are coupled, as in the model presented in this paper.

Overall, we proceed by identifying a minimal set of mechanisms that allow to reproduce qualitatively the main facts of firm demography (like firm size distribution and aggregate output time series) but at the same time the emergence of spatio-temporal patterns for output, growth and failures. The paper is organized as follows. In section2 we first describe a framework in which a class of models can be developed. We then provide a detailed description of a specific model, pointing out limitations and simplifications. In section3 we discuss the mechanisms of failure propagation and in section 4 we report the results of our computer simulations. Conclusion are presented in section 5

2 The Model

2.1 Organization of the Economy and Temporal Structure

The economy consists of N firms organized in M production layers. We will denote firms with indices $i, j, k, l, ...$ and levels with indices $J, K, L, ...$ We adopt the convention that production take place along the vertical axis in downwards direction. The structure of the connections defines the production network as in the example shown in figure 1, in which arrows represent supply (supply proceeds downwards, while payments move upwards).

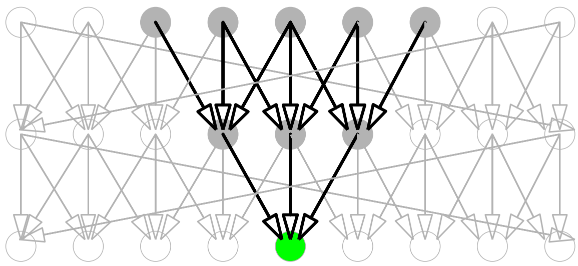

Figure 1: Example of structure for the production network. The sense of production is from top to bottom. Each firms in a layer is supplied by a subset of (3 in this case) firms from the upper layer. The top layer represents primary producers. Layer 3 (from the top) represents firms that sell on the consumer market. We have highlighted in black the set of all suppliers uphill to a given retailer (in green)

Each firm in a layer K is supplied by a subset of firms in the upper layer $K-1$ and in turn supplies a subset of the firms in the lower layer $K+1$. The bottom layer $K = M$ represents firms that sell on the consumer market (retailers). The top layer $K = 1$ represents primary producers. Firms are connected to each other through two mechanisms:

1. a firm requires the input from the supplier firms in order to produce its output
2. a firm requires the payment from the customer firms in order to realize its profit

The output of each layer is produced by processing the input from the previous layer and the output is qualitatively different from the input. For the sake of simplicity we assume the following linear production function:

$$Y_i = \sum_{j \in S_i} Q_{ij} Y_j \tag{1}$$

where Y_i is the output of firm i, S_i is the set of suppliers of firm i, and Q_{ij} represents the fraction of the total output of firm j that firm i uses to produce its own output. In other words, Q is the input-output matrix and it holds:

$$\sum_i Q_{ij} = 1 \tag{2}$$

Concerning the temporal structure, we assume that time is discrete and divided into periods. In a single period (or time step) t the following series of events takes place. All firms in the bottom layer M determine their desired output, based on the demand they face on the market and their production capacity, and then send orders to the upper layer $M-1$. Afterwards, all firms in layer $M-1$ determine their desired output based on the demand they face from their customers firms in layer M. One after the other all layers do the same, up to layer 1 (primary producers). At this point production starts in layer 1 and proceeds one layer by one in the lower layers, as each firm needs the input from the suppliers in order to produce. The output produced by each firm is fully delivered to customers and we exclude the possibility of inventories accumulation. When production reaches the bottom layer, products are fully sold on the consumer market and a sequence of payments start and proceed upwards up to the primary producers. At each layer, each firm is paid for its product and only then pays the cost of supply. If cost exceeds payment the firm goes bankrupt and does not pay the suppliers. In this case, the firm stops production and after a few periods is replaced by a new firm with an assigned entry level of production capacity.

The structure of the connections remains static during the process. This means that when a firm goes bankrupt, its customers do not create new links with other suppliers. This could correspond to a market in which is very costly for the firm to establish relations with new suppliers.

So far we have described a general framework, while the mechanisms involved can be specified in several ways (for example, we have to specify the dynamics on price, on profit and net worth). However, some of the results presented in this paper do not depend on the specification of such mechanisms. Therefore the structure presented so far, is a candidate for a class of model sharing similar behavior. In particular, the conditions for the presence of avalanches of failures are analyzed in section 3

In the following we provide a detailed description of a simple version of the model and a discussion of its limitations. In any period t each firm i is endowed with a level of real net worth $A_i(t)$, defined as the stock of firm's assets in real terms, that has been financed only through net profits (we assume complete equity rationing).

2.2 Desired Output

Any firm i at layer K determines its desired production at time t, $Y_i^d(t)$, based on the orders received from layer $K+1$, with the constraint of its **production capacity** that we assume to be proportional to the net worth of the firm:

$$Y_i^{(d,K)}(t) = min\{\theta A_i^{(K)}(t), \sum_{\text{j} \in V_i^C} O_{ij}^{(K,K+1)}(t) Y_j^{(d,K)}(t)\} \tag{3}$$

In the equation above V_i^C is the set of customers of firm i, $O^{(K,K+1)}$ is the order matrix describing the orders from layer $K+1$ to K, and in particular $O_{ij}^{(K,K+1)}$ is the fraction of the total supply needed by firm j that firm j orders to firm i. In matrix notation we can write:

$$Y^{(d,K)}(t) = min\{\theta A^{(K)}(t), O^{(K,K+1)} Y^{(d,K+1)}(t)\} \tag{4}$$

For level M, we assume that at each time step the consumer market absorbs the whole production and therefore:

$$Y^{(d,M)}(t) = \theta A^{(K)}(t) \tag{5}$$

2.3 Effective Output

A supplier firm may not be able to face an order either because of its production capacity or because it has gone bankrupt. In this version of the model the network is static and firms have a fixed set of suppliers. The way firms decide to place orders to their suppliers, in other words, the way $O_{ij}^{(K,K+1)}$ are determined, is discussed later on, and plays of course an important role. However, as a result, supply can be smaller than the ordered quantity and therefore the actual output Y_i can be smaller than the desired one $Y_i^{(d)}$. The production function of firms is assumed to be linear so that the output of a firm in layer K is a linear combination of the input actually received from the suppliers in layer $K-1$. In equations it reads:

$$Y_i^{(K)}(t) = \sum_{\text{j} \in V_i^S} Q_{ij}^{(K,K-1)}(t) Y_j^{(K-1)}(t) \tag{6}$$

$$\tag{7}$$

In the equation above, V_i^S is the set of suppliers of firm i, $Q^{(K,K-1)}$ is the input-output matrix describing the transformation of input from layer $K-1$ into the output of layer

K. Each entry $Q_{ij}^{(K,K-1)}$ represents the fraction of the total output of firm j that firm i uses to produce its own output. Firms in layer 1 are primary producers and do not need any supply, therefore $Y_i^{(1)} = Y_i^{(d,1)}$. In matrix notation the output of any layer can be expressed as a function of the output of the first layer as follows:

$$Y^{(K)}(t) = Q^{(K,K-1)}(t)Y^{(K-1)}(t) = Q^{(K,K-1)}(t) \cdot ... \cdot Q^{(2,1)}(t)Y^{(1)}(t) \tag{8}$$

2.4 Production Costs

The output produced by each firm is fully sold to the customer at the price $P_i(t)$ (no inventories accumulation). The price of firm's output in layer K is $P_i(t) = P^{(K)}(t)u_i(t)$ where $P^{(K)}(t)$ is the general price level at layer K and $u_i(t)$ is the relative price for the output of the single firm. We assume that $u_i(t)$ is a random variable, uniformly distributed in $[1-\delta_P \quad 1+\delta_P]$ and independent on $P^{(K)}(t)$. Similarly it happens for the other layer and therefore firm i incurs in the following costs for its supply from layer $K-1$:

$$\tilde{C}_i^{(s,K)}(t) = \sum_{\mathrm{j} \in V_i^S} Q_{ij}^{(K,K-1)} P^{(K-1)}(t)u_j(t)Y_j^{(K-1)} \tag{9}$$

Real supply costs are obtained normalizing supply costs by the level of prices in the layer K of the firm i in question:

$$C_i^{(s,K)}(t) = \frac{P^{(K-1)}(t)}{P^{(K)}(t)} \sum_{\mathrm{j} \in V_i^S} Q_{ij}^{(K,K-1)}(t)u_j(t)Y_j^{(K-1)}(t) = \tag{10}$$

$$= c_s \sum_{\mathrm{j} \in V_i^S} Q_{ij}^{(K,K-1)}(t)u_j(t)Y_j^{(K-1)} \tag{11}$$

where we define c_s as the ratio of the price levels at layer $K-1$ and K and we assume it to be the same for all K.

We also want to model the fact that when a supplier goes bankrupt and unexpectedly fails to deliver the agreed amount of input, the labour and premises a firm had allocated to process such input remain, at least in part, unused. We assume that in such an un unexpected event the firm will run a cost proportional to the agreed amount of input. In other words, a firm places an order to a supplier which agrees on a given amount depending on its production capacity. This amount is the expected output of the supplier which can be written formally as follows:

$$Y_j^{(e,K)}(t) = Y_j^{(K)}(t)(1 - S_j(t)) \tag{12}$$

where $S_j(t)$ is defined as the activity state of firm i ($S_j(t) = 1$ if firm i is active, $S_j(t) = 0$ if firm i is bankrupt).

In conclusion, unless specified, production costs are:

$$C_i^{(s,K)}(t) = c_s \sum_{\text{j} \in V_i^S} Q_{ij}^{(K,K-1)}(t) u_j(t) Y_j^{(e,K-1)} \tag{13}$$

2.5 Profit

At each period, when production is sold on the consumer market and payments start, for some firms sales revenue may be smaller than their supply costs. If this negative difference is high enough or if it accumulates, firms may go bankrupt and do not pay their suppliers.

Therefore we distinguish between the output delivered by firm i to its customers, $Y_i(t)$, and the output $Y_i^s(t)$ that is actually paid to firm i by its customers. Real profit equals the difference between payments and supply costs in real terms (normalized by the level of prices in the layer of the firm in question):

$$\pi_i^{(K)}(t) = u_i(t) Y_i^{(s,K)}(t) - C_i^{(s,K)}(t) \tag{14}$$

2.6 Bankruptcy

Overall, at each period, given the actual output, the actual payments and the cost, the firms make a profit (negative or positive) $\Pi_i(t)$ that increments its real net worth:

$$A_i(t+1) = \rho A_i(t) + \pi_i(t) \tag{15}$$

$$\tag{16}$$

where ρ is the depreciation rate. Firms go bankrupt when net worth falls below a threshold value $A < A_{fail}$.

If a firm goes bankrupt at time t it stops supplying customers and paying suppliers for a number of time steps τ called "refractory time". During the refractory time neighboring firms cannot look for other customers or suppliers and rewire their links. Once the refractory time elapsed, the firm is replaced by a new firm with the same links as its predecessor and $A(t + \tau + 1) = A_{entry}$.

2.7 Strategies for placing orders

Although the network is static in this version of the model, and the set of suppliers of a firm is fixed, still there are many possible ways to allocate orders to the suppliers. Consistently with our bonded rationality framework, we have considered two simple strategies:

1. The firm places orders proportionally to the production capacity of the suppliers:

$$O_{ij}^{(K,K-1)}(t) = \frac{A_j(t)}{\sum_{\mathrm{k} \in V_i^S} A_k(t)} \tag{17}$$

2. The firm distributes orders evenly:

$$O_{ij}^{(K,K-1)}(t) = \frac{1}{|V_i^S|} \tag{18}$$

   where $|V_i^S|$ is the cardinality of the set of suppliers of firm i.

Roughly speaking, we can expect strategy 1 to be more efficient in the short run as it increases the chances for the firm to be delivered the requested quantity. However, it induces an autocatalytic growth of the suppliers(the largest grows most) so that in the long run firms tend to have only one main supplier, which is risky against supplier failures. Strategy 2 is expected to be less efficient in the short run but to stimulate the growth of all the suppliers of a firm and allow to keep a balanced portfolio of suppliers.

2.8 Strategies for delivery when production do not satisfy orders

In our simple setting it may occur that a supplier cannot fulfill the demand of its customer because of production capacity constraint. In this case several ways of allocating delivery are possible. Again in line with our bonded rationality framework, we have considered two simple strategies:

1. The supplier delivers to each customer a quantity proportional to the order
2. Orders are sorted by quantity in ascending order and the supplier try to fulfill them in such order until some output is left.

Strategy 1 is expected to favor the growth of the largest customer and in the long run firms tend to have only one main customer, which is risky against customer failures. Strategy 2 is expected to stimulate the growth of all the customer of a firm.

3 Analysis of the Model

3.1 Propagation of Failures and Avalanches

In this section we want to make clear the distinction between the effect of a bankruptcy on the output of suppliers/customers and its effect on the probability of bankruptcy of these firms. A bankruptcy, or simply the change in output of a firm, affects the output of its customers (because they need its input to produce). However their probability of going bankrupt is not necessarily increased. Let us look closer at the concept of failure avalanche.

By *avalanche of events* B one usually means a process in which one event of type B at time t determines at least one or more other events of the same type. This implies that after a few time steps a number a of events B has occurred (a is also called the size of the avalanche).

The production network modelled in this paper is represented by an oriented graph in which the direction of the edges follows the direction of production. However failures can propagate in the same direction of production or in the opposite one. We will speak of *downward* or *upward propagation*, respectively. If failures can propagate only downward/upward, when they reach the bottom/top layer they stop, as shown in figure 2 (a-b). In this case the only firms affected are those in the cone downhill/uphill the initial failure.

If failures can propagate simultaneously in both direction, then, and only then, they are "reflected" diagonally at each layer and the result is a net horizontal propagation, that is perpendicular to the direction of production (figure 2, c-d). In the following we will speak of *horizontal failure propagation* to mean the situation in which failures can propagate potentially to the whole network and not only to the downhill/uphill cone of firms.

In the model bankruptcies occur when the net worth becomes smaller than a threshold, $A_i(t+1) < A_{failure}$. This requires that:

$$\Pi_i(t) < A_{fail} - \rho A_i(t) \tag{19}$$

If the real price of sale for firm i is a stochastic variable with probability distribution function $\mu(u_i)$, we can write the probability $\mathcal{P}_i^B$ of bankruptcy for firm i as:

$$\mathcal{P}_i^B = \mathcal{P}\{\pi_i(t) < A_{fail} - \rho A_i(t)\} = \tag{20}$$

$$= \mathcal{P}\{P_i(t)Y_i^s(t) - C_i^{(s)}(t) < A_{fail} - \rho A_i(t)\} = \int_{1-\delta}^{u_i^*(t)} \mu(u) du_i \tag{21}$$

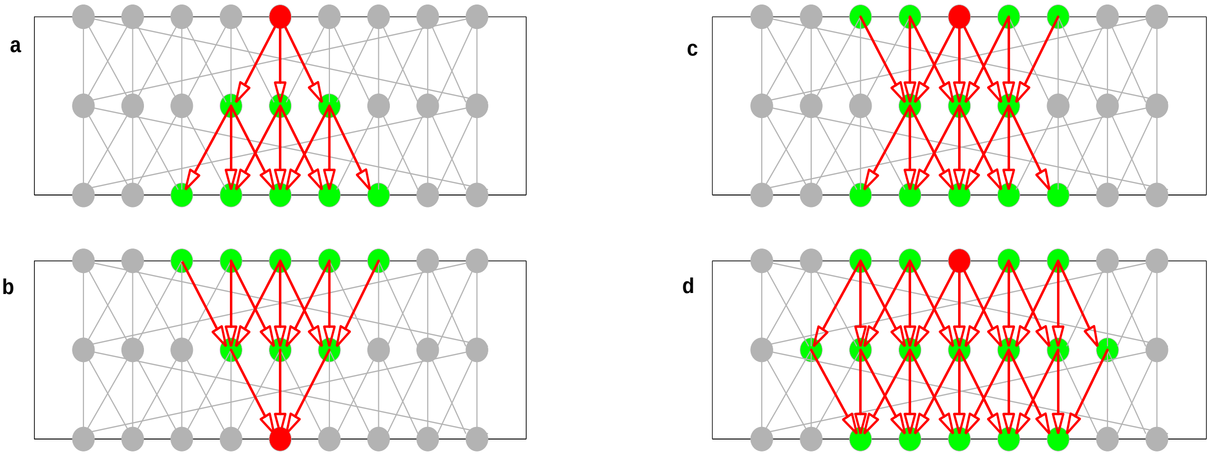

Figure 2: Different modalities of failure propagation (the edges in red) triggered by an initial failure (the firm in red). a)-b) Downward and upward propagation of failures. c)-d) Horizontal propagation occurs when each layer transmits downward but also reflects upwards. In c) failures have propagated up to two degrees of separation from the initial firm, in d) up to three degrees.

where

$$u_i^*(t) = \frac{A_{fail} - \rho A_i(t) + C_i^{(s)}(t)}{Y_i^s(t)} \tag{22}$$

If $\mu(u_i)$ is for instance a uniform distribution in $[1-\delta_P \quad 1+\delta_P]$, equation 20 yields:

$$\mathcal{P}_i^B = \frac{u_i^*(t) + \delta_P - 1}{2\delta_P} \tag{23}$$

As expected by construction, $\mathcal{P}_i^B = 0$ if $u* < 1 - \delta$ and $\mathcal{P}_i^B$ increases when cost $C_i^{(s)}$ increases, or when net worth A_i decreases, or finally when paid output Y_i^s decreases.

Suppose for a moment that production costs of firm i are just proportional to the input received from the suppliers (i.e. $Y_j^{(e)}(t) = Y_j(t)$) and output is paid at delivery ($Y_i^s(t) = Y_i(t)$), then neglecting fluctuations of supply prices we can write:

$$u_i^*(t) = \frac{A_{fail} - \rho A_i(t) + C_i^{(s)}(t)}{Y_i(t)} \simeq \frac{A_{fail} - \rho Y_i(t) + c^{(s)} Y_i(t)}{Y_i(t)} = \frac{A_{fail}}{Y_i(t)} - \rho\theta + c^{(s)} \tag{24}$$

Bankruptcy threshold A_{fail} will be set in the simulations to be proportional to the average value of net worth, for instance: $A_{failure} = 10^{-2} < A >$ (see section 4) and hence:

$$u_i^*(t) = \frac{10^{-2}\theta < Y >}{Y_i(t)} - \rho\theta + c^{(s)} \simeq -\rho\theta + c^{(s)} \tag{25}$$

It follows that in this situation firm i is not sensitive to a bankruptcy among the suppliers, unless it is already very small with respect to the average and in fact close to the failure threshold. We can say that there is no propagation of failures.

If instead payments are delayed, firm i is sensitive to the ratio between the produced and paid output, and therefore to the bankruptcies of customers.

$$u_i^*(t) \simeq (-\rho\theta + c^{(s))})\frac{Y_i(t)}{Y_i^s(t)} \tag{26}$$

If, on the other hand, production costs include costs associated with supply failure, firm i is sensitive to the ratio between expected and produced output, and therefore to the bankruptcies of suppliers.

$$u_i^*(t) \simeq (-\rho\theta + c^{(s))})\frac{Y_i^e(t)}{Y_i(t)} \tag{27}$$

Finally, if payments are delayed and production costs include costs associated with supply failure, firm i is sensitive to the ratio between expected and paid output, and therefore to the bankruptcies of both customers and suppliers.

$$u_i^*(t) \simeq (-\rho\theta + c^{(s))})\frac{Y_i^e(t)}{Y_i^s(t)} \tag{28}$$

In conclusion we can distinguish the following situations for failure propagation:

1. Supply failure imply a cost for the firm but output is paid at delivery. Only downward failure propagation occurs.

2. Supply failure does not imply cost for the firm but output is paid with delay. Only upward failure propagation occurs.

3. Supply failure does imply cost for the firm AND output is paid with delay. Both upward and downward propagation and therefore horizontal propagation of failures occurs.

4 Analysis of Simulations Results

We present now the results of computer simulations of the model described above with the following specifications valid for all the simulations, unless mentioned. The structure of the network is the one illustrated in figure 1 with three suppliers and three customers per firm. Price distribution is uniform in the interval $[1 - \delta_P \quad 1 + \delta_P]$. We are interested in a regime in which firms' profit is sensitive to unexpected variations of input from the suppliers. For this to be, profit variation due to stochastic price fluctuation must not be larger than those due to the variation of input, that is δ_P must be relatively small. However, in this regime bankruptcies solely due to price fluctuations are very rare. Therefore, we also assume that with small probability q, prices become zero, $P(t) = 0$. In this way we intend to model occasional failures during delivery that imply loss of the sales revenue for the firm. These failures are un-correlated and their probability is fixed during the simulation. Aggregate output is measured as total production before delivery and therefore regardless of these failures.

4.1 Parameter settings

The parameters of the model can be grouped as follows.

1. Size of the network and time span: number of firms N, number of layers M in the economy, number of time steps T. We test M between 3 and 5, a range close to real production networks. We have performed simulations with N of the order of few hundreds per layer and T between up to few thousands.

2. Prices and costs: price interval width δ_P, sale failure probability q, supply cost factor c_S. Unless specified, results reported in this paper are obtained with $\delta_P = 0.2$, $q = 0.05$ $c_S = 0.85$, which together ensure an aggregate growth of about 1% per period.

3. Bankruptcy and rebirth: Refractory time is set as $\tau = 3$. Firms are initially all endowed with the same value of net worth $A_i(0) = A_{init} = 1 \forall i$ and output $Y_i(0) = Y_{init} = 1 \forall i$. Because the total level of net worth is not constant (the economy grows or recedes depending on the parameters at point 2), bankruptcy threshold A_{fail} and entry level of net worth A_{entry} are set to be proportional to the average value of net worth: $A_{failure} = 10^{-2} < A >$, $A_{entry} = 10^{-1} < A >$. Depreciation rate is set $\rho = 0.03$

In a first set of simulations firms adopt strategy 1 for orders and strategy 1 for delivery. In this case we can say that not only the links but also the intensity of the connections is static. $N = 600$, $M = 3$, $T = 800$. The other parameters are set as in section 4.1. The model displays the following features:

1. spatio-temporal correlation of output growth and bankruptcies
2. exponential growth (around 1% per period)
3. oscillations of de-trended aggregate output (around $\%2 - 3$)
4. heterogeneous firm size (power law tail with exponent $\gamma = \ 2$)
5. exponential probability distribution of aggregate growth (right side)

In figure 3 the evolution of output in the production network is shown at 20 different time steps sampled over the duration of the simulation. Starting from a uniform distribution of output, soon regions of high output emerge while other collapse.

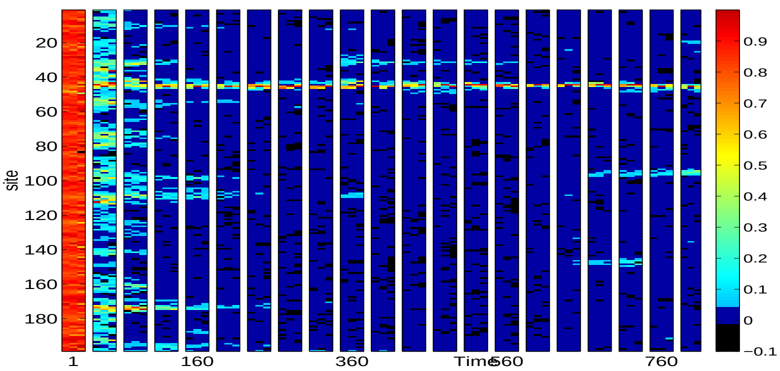

Figure 3: Evolution of output in the production network sampled over the whole time duration of the simulation. Each frame represents the production network at a given time step. Primary producers are in the leftmost layer of each frame. Retailers on the consumer market are in the rightmost layer of each frame. Output is normalized by the maximum value of output at each time step in order to emphasize the relative spatial distribution of output. Output is represented with a color code as specified by the color bar. Dark blue represent output close to 0 and black represent bankruptcy.

A zoom in time is shown in figure 4, with a different color scale to highlight smaller differences in output. From one frame to the other it is possible to observe the propagation of failures. Similarly, figure 5 shows a zoom of the time evolution of growth, measured as a fraction of net worth, $\frac{A_i((t+1) - A_i((t)}{A_i((t)}$. It is possible to observe the correlation of growth both in space and time.

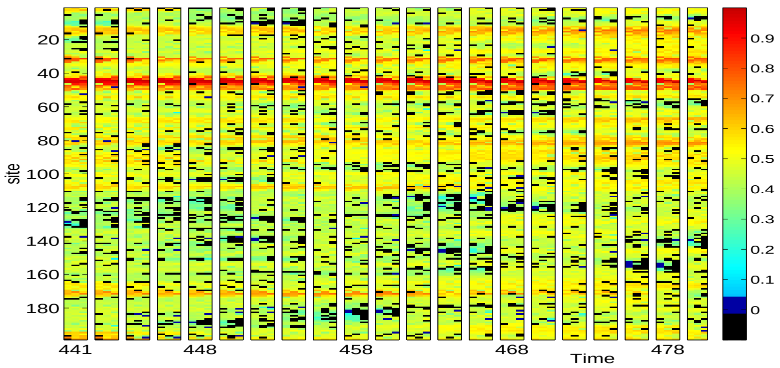

Figure 4: Evolution of output in the production network over time. Zoom in a time interval. The figure is obtained as described for figure 3. Propagation of failures is visible.

Exponential growth of the aggregate output is shown in figure 6 (slope of the $log_{10}(Y_{tot})$ vs time is 0.01).The de-trended aggregate output is obtained as difference between the aggregate output in log and its linear fit: $Y_{detr}(t) = log_{10}(Y_{tot}(t)) - bt + a$ where b and a are the coefficients obtained with the linear fit. Figure 7 shows irregular oscillations of the order of 2%.

Growth probability distribution is shown in figure 8. Finally, the heterogeneity of the output across firms is shown in figure 9 by its complementary cumulative distribution.

In a second set of simulations firms adopt strategy 2 for orders and strategy 2 for delivery. Although the structure of the network is still static, the intensity of the links is changing over time because orders are allocated proportionally to production capacity. The model displays the same general features as before. The effect of the new order and delivery strategies is that production tends now to be concentrated in thinner regions and the economy is characterized by slightly higher size heterogeneity.

5 Conclusions

We have presented a quite general framework for investigating the relation between local interactions and aggregate behavior in models of production networks. We have shown that in such framework only the simultaneous presence of delayed payments and supply failures costs may induce the propagation of bankruptcies. in direction transversal to production. In most previous models, propagation of failures is based on mechanisms which involve a global coupling (the more firms fail, the higher the interest rate for all,

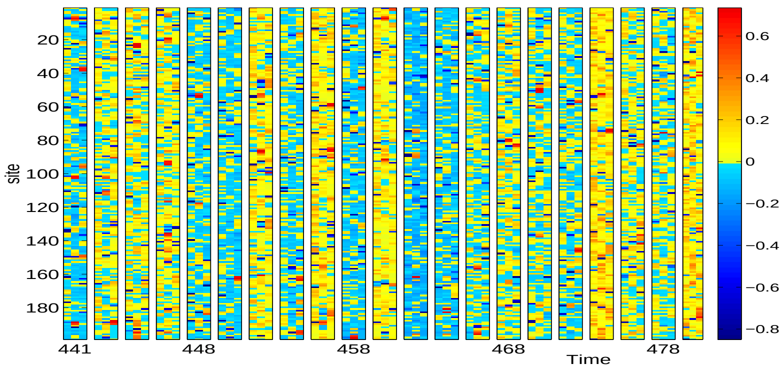

Figure 5: Evolution of growth in the production network over time. Zoom in a time interval. The figure is obtained as described for figure 3.

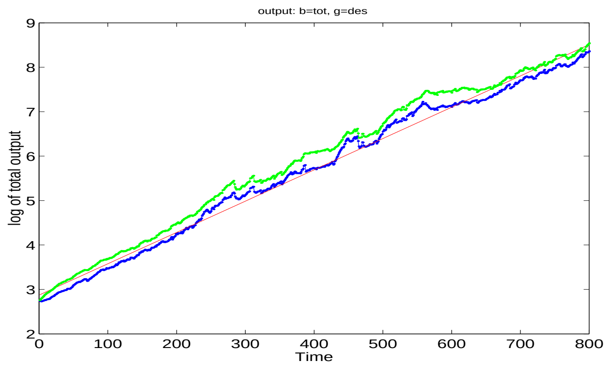

Figure 6: Log_{10} of aggregate output

hence the more they fail) and not on local interactions. Therefore this work offer a novel and alternative and bottom-up mechanism for failure propagation and in this sense it fills an important gap in the literature.

Within the proposed framework, we have implemented a specific model in which correlated fluctuations emerge in the aggregate output from spontaneous local spatio-temporal evolution. While it is relatively easy to reproduce output oscillations in macro-economic models within the representative agent approach, the value of this work is to root the emergence of fluctuations in the local interaction among firms. The model also exhibits exponential growth over time while the tent-shape for growth distribution is not reproduce and only the right side of the distribution can be fitted with an exponential. This discrepancy will be analysed in further investigations.

Tuning the assumptions of the model one can investigate the role of the main factors

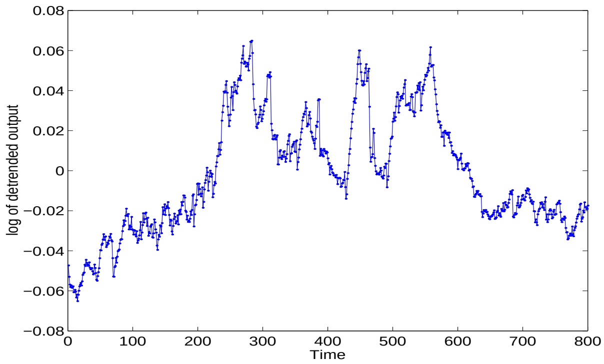

Figure 7: Log_{10} of de-trended aggregate output

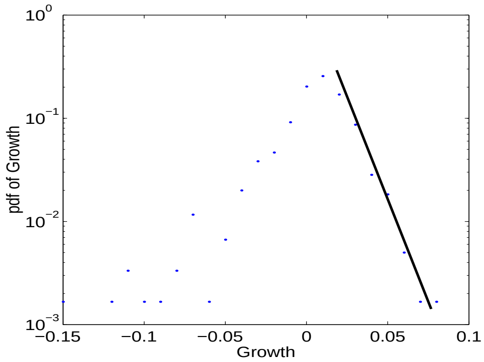

Figure 8: Probability distribution of growth of aggregate output over time

involved in economic fragility and address the following issues:

1) the role of trade-credit relationship and inertial costs in bankruptcies

2) the role of interest rate and policies to prevent large avalanches to occur

3) the role the structure of the network of interactions and policies to make such structure more robust against large avalanches

In conclusion we believe that this model opens up the way to a novel class of models for endogenous business fluctuations based on firm-firm interaction and trade-credit relationships.

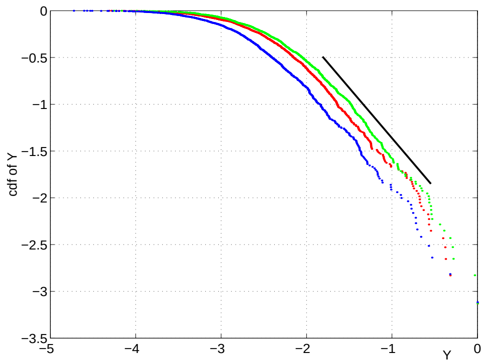

Figure 9: Complementary cumulative probability distribution of output across firms in layer 1,2,3 (blue, green, red, respectively)

6 Bibliography

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