Lending Without Access to Collateral A Theory of Micro-Loan Borrowing Rates & Defaults

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March 13, 2007

Abstract

A model of micro loans is used to determine the equilibrium borrowing rates, and default probabilities. Monitoring by lenders is critical for an equilibrium to exist in our model if the maturity of the loan is long. With short maturity loans, monitoring is shown to be counter-productive. The manner in which the loan rates depend on the market structure, monitoring costs, joint-liability provisions and punishment technology is characterized when the borrowing group optimally chooses the timing of default. Designing the loan contract so that borrowers make higher payments in good states and lower payments in bad states is shown to be pareto improving.

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There are very large groups of society, especially in poor and developing parts of the world who do not have access to rudimentary financial services such as bank savings accounts, credit facilities, or insurances. Households in these sections of the society are typically poor and access credit in *informal credit markets.* Such informal credit markets include: a) local money-lenders, b) local shop-keepers, who provide trade credit, c) pawn-brokers, d) payday lenders, and e) **RO**tating **S**avings and **C**redit **AS**sociations (ROSCAS). A number of economists have examined these informal credit markets, and their potential linkages to more formal credit markets. A partial list of such research includes Besley, Coate, and Loury (1993), Braverman, and Guasch (1986), Varghese (2000, 2002), and Caskey (2005). It is well understood that the interest rates in such informal markets tend to be much higher than the borrowing rates that prevail in formal credit markets[1].

Micro-loan markets represent one of the more recent developments, which enable poor households to access credit. These are markets where very small (hence micro) loans are extended to poor households. Often, such loans are given only to women, and in groups. Borrowers in these markets have no meaningful physical collateral and are heavily credit constrained. Micro-loans are characterized by three essential features: a) loans are short-term in nature, relatively small amounts and consummated without physical collateral, but structured with social collateral[2]; b) loans are extended typically to a group, whose size can range from five (in the Grameen model) to twenty (in the Self-Help-Groups or SHG), where the group members are *jointly-liable* for default by any member of the group; and c) loans carry frequent interest payments (weekly in many cases) and carry significant administrative expenses that are incurred in order to ensure timely delivery of loans to remote villages and for the collection of payments[3].

To our understanding, no formal model has been developed for understanding the determina-

[1] A complete survey of research in informal credit markets and micro-finance is well beyond the scope of our paper. We refer to two excellent sources: 1) Armendariz, and Morduch (2005), and 2) Bolton and Rosenthal (2005).

[2] Defaulting borrowers often have to live in the same community in front of whom they defaulted; this leads to a sense of shame leading sometimes to extreme outcomes.

[3] See Shankar (2006) for an analysis of transactions costs in micro credit.

tion of borrowing rates and default rates in micro loans, and how they depend on various features of the micro-loan contracts [4]; nor has there been a treatment of the relationship between the market structures of loans (competitive or monopolistic) and the equilibrium borrowing rates and default rates. This is the primary objective of the paper. The paper is of a broader interest since it delivers a framework to think about enforcing loan contracts when the lender has no access to physical collateral.

Since 1976, micro-finance and micro-loans have emerged as a sector where poor households are able to accumulate savings and access credit. Additional financial services such as rainfall insurance, livestock insurance, and health insurance are also being provided increasingly through these channels[5]. Table 1 illustrates the size of the micro-loan market as of 2003, based on voluntary reporting of lending institutions to a centralized database maintained by MIX[6].

Insert Table 1 here.

The total size of the micro-loan market covering a little over 28 million borrowers as reported in Table 1 is about \$8.7 billions; this is potentially a very serious underestimate of the actual size of the market since many lenders do not report their activities to MIX. Another estimate found in Microcredit Summit (2003) reports that nearly 2500 lending institutions covered a total of 67 million borrowers as of 2002. Note that Latin America and East Asia are the regions that account for more than 50% of the loans, but South Asia accounts for more than 50% of active borrowers. The total number of active borrowers based on voluntarily reported data is in excess of 28 millions. Regardless of the estimates, what is clear is that the number of households in need of rudimentary financial services such as loans, savings, and insurance is considerably higher. For example, in India alone, the estimated number of people in need of rudimentary financial services is over 200 millions.

[4]See *The Economics of Microfinance* (Armendariz and Morduch), Forthcoming from the MIT Press, 2005, for a full discussion of this area.

[5]See Ananth, Barooah, Ruchismita, and Bhatnagar (2004) for an illuminating discussion on designing a framework for delivering financial services to the poor in India.

[6]MIX was incorporated in June 2002 as a not-for-profit private organization. The MIX (Microfinance Information eXchange) aims to promote information exchange in the micro-finance industry.

Morduch (1999) provides estimates of borrowing rates in this market, but no systematic evidence of defaults is available to our knowledge. In this section, we use the MIX data to estimate interest rates on micro-loans, ex-ante assessment of default exposure, and ex-post write-offs on loans.

Table 1 also provides estimates of borrowing rates by assuming that the net revenue reported by the institutions are comprised exclusively by loan interest income only. This is likely to be a somewhat noisy estimate because the income may include both principal and interest payments, as well as income from other sources such as investments made by the lending institutions. Hencem the estimates of interest rates reported in Table 1 should be interpreted with caution. The ex-ante assessment of default is captured by the Portfolio At Risk (PAR) measure reported by each lending institution. The ex-post measure is captured by the write-offs reported in the data set.

The average figures indicate that the micro-loan rates are in excess of 30%. The rates are significantly lower in South Asia, which is characterized by many borrowers who borrow very small amounts. Morduch (1999) has provided estimates of borrowing rates ranging from 20% in Grameen Bank in India to 55% in Indonesia. Our estimates, which reflect the bias of self-reporting institutions, range from 20% to 40%. The rates are much higher in Africa. The rates charged in micro-loans appear to be well below the rates that local money lenders tend to charge. If we think of local money lenders as the outside borrowing option available to the borrowers, then micro-loan rates may not look usurious. The portfolio at risk (PAR) is highest in South Asia, but the write-offs are higher in Africa and Latin America.

It is interesting to note that the PAR estimates are systematically higher than the actual write-offs. This suggests that, ex-ante, the lender believes that the probability of default is much higher than ex-post observed default probability.

Insert Table 2 here.

To get an appreciation of the composition of lenders, Table 2 provides a breakdown of the micro-loans across different lenders. Of the 613 lending institutions voluntarily reporting, banks account for more than 50% of the dollar value of loans, responsible for more than 31% of all active borrowers. In sharp contrast, NGOs account for just 17.3% of the dollar value of the loans, but spans more than 46% of all active borrowers. NGOs and non-bank financial institutions have a similar overall coverage globally with very important cross-sectional variations.

The small size of the loan and the presence of numerous borrowers make investment, in additional screening and monitoring efforts, an expensive proposition. This makes micro-loan portfolio unattractive for many big commercial banks. Due to these factors, micro-lending approaches focus on a) contractual arrangements, b) punishment conditional on default, c) peer group efforts to in effect substitute social collateral for physical collateral, and d) partnering with local financial institutions, which may possess informational advantages and thus may be able to monitor the loans better.

In Table 2, we also provide a breakdown of interest rates and default measures across the different organizational forms. Banks appear to charge the lowest interest rates. The rates charged by other lenders vary, ranging from 30% to 38%.

To summarize, the following facts emerge from our analysis: first, the micro-loan interest rates are rather high, ranging from 30% to nearly 40%, and the loan sizes are, by and large, relatively small. It is therefore reasonable to ask whether at such high rates, borrowers can eventually move up the economic ladder by scaling up their loan sizes and escape poverty. We will explore whether by a judicous choice of the loan contract and enforcement technology one can bring down the borrowing rates, without increasing the default rates. Second, the actual losses as conveyed by the write-off ratios are relatively small, ranging from 0.50% to 3.50%. The lenders tend to estimate the portfolio at risk at a much higher level, averaging around 7%. There is a cross-sectional variation in the borrower size and default rates depending on the organizational arrangement of the lender. This latter point is very interesting but one which we do not pursue in our paper.

In our numerical illustrations, we will attempt to calibrate the model so that loan rates roughly match these numbers. Then, we will ask how various terms of the loan contract can be used to bring the loan rates down without significantly increasing the default probabilities. In particular, we find that decreasing monitoring has the most significant impact.

The borrowing rates in micro-loan contracts must depend on the following important factors:

1. *Administrative and monitoring expenses*: these are needed to deliver the loans at the doorsteps of poor borrowers in rural areas. High administrative expenses (arguably) keep the default rates low, but render the borrowing costs very high. Note that these costs are borne by the lender, and are eventually passed on to the micro-borrower. CGAP(2003)[7] reports that the administrative expenses range from 18.9% in Asia to 38.2% in Africa on the loans. Consequently, micro-loan interest rates are rather high; this should, however, be put in the context of the fact that the other ”outside options” for the micro-borrowers are even more expensive. For example, CGAP (2003) reports money-lenders charging anywhere from 10% per month[8]. In Philippines, the estimated daily interest rates on loans made by local money lenders is 20% per day.

2. *Joint-Liability Arrangements*: when properly structured, groups of borrowers are formed through an assortative matching procedure to provide sufficient peer pressure and monitoring in order to keep the default rates of group members low[9]. This should reduce the costs of borrowing. Note that the peer monitoring costs are borne by the borrowers. Thus, there is a trade-off for the group: active peer monitoring reduces defaults and delinquencies but increases the efforts required. As noted by Stiglitz (1990) peer monitoring can help to mitigate, if not solve, the ex-ante moral hazard problem: it prevents any member of the group from taking risky projects because others in the group, who are jointly liable will attempt to prevent that from happening. Considerable research has focussed on joint-

[7]CGAP stands for Consultative Group to Assist the Poor.

[8]Making Sense of Microcredit Interest Rates, CGAP (January 20003), Donor brief.

[9]In assortatively matched groups, potential borrowers form their own group without any intervention from the lender.

liability contracting (See Ghatak (1999), and Ghatak and Guinnane (1999), for example.), where borrowers form groups and members of the groups agree to take responsibility for delinquencies and defaults by individual group members. Requiring each member of the group to be liable for the entire group's liabilities makes peer monitoring much more effective. In addition, the group's ability to obtain additional loans will be predicated on the entire group fulfulling the contractual obligations on existing loans.

3. *Credible Threat of Punishment Upon default*: the lender must be able to communicate credibly, ex-ante, that default by the group will lead to significant costs to the group, including the inability to access further loans. This can be done in two ways: first, the target group is chosen so that they have little or no access to formal credit markets, and second, institutional mechanisms such as credit bureaus be put in place so that lenders are able to share information about defaulters and collectively enforce punishment. Furthermore, the group-lending mechanism places a very high social cost on individual defaulters. Often, borrowers are promised additional loans if they successfully pay off existing loans. Rational borrowers know that their access to micro-loans in the future is conditional on not defaulting on existing loans. This provides strong incentives for the micro-loan borrowers not to default.

4. *Informational Asymmetry*: One aspect of the micro-loans is that banks with depth in lending ability do not necessarily possess informational advantages about many borrowers located in different corners of the country. They also lack the monitoring technology that is needed to enforce payment should joint-liability contracting fail to deliver acceptable recovery rates on loans. The informational advantages and monitoring tools in such a market are usually the domain of local lenders and Micro finance institutions (MFIs). Increasingly, micro-loans are structured with complementary participation by local lenders and bigger financial institutions (see Nini (2004), in the context of emerging market lending): big banks will lend requisite amount of capital to local MFIs at a certain rate of interest . Local MFIs will then re-lend the capital to groups of local borrowers under a joint-liability

scheme. The local MFI will assume the “first loss tranche” (say, the initial 10% of losses experienced by the loan portfolio), and the rest will pass through to the bank. In models used by certain banks in India, the loans remain on the books of the bank and not with the local MFI. This so-called partnership model is discussed in Ananth (2005) and Harper and Kirsten (2006).

The primary goal of the paper is to construct a model of borrowing and lending where the lender has no access to collateral and the borrower is severely credit constrained. We first wish to study the impact of market structure on borrowing rates and defaults. In addition, our model relates equilibrium borrowing rates and default probabilities to some salient features of the loan contracts we described earlier. While the model is applied to micro-loans, it is easy to explore other markets in which borrowers obtain loans from lenders who do not have access to collateral upon default. We wish to understand the relative importance of a) lender monitoring, b) punishment upon default, c) maturity structure of the loan, and d) peer monitoring on the loan rates. In particular, we explore whether a particular combination of these variables can keep the default risk low, and at the same time provide lower interest rates to the borrower. Such an outcome should be of great interest in the design of micro-loan contracts. We also wish to conduct a welfare analysis. To this end, we wish to characterize the borrower's value function as a function of these levers that the lender has at her disposal and as a function of the market structure. We also explore the value of designing the loan contract, which combines an insurance feature by collecting higher payments in good states and lower amounts in bad states.

In order to keep the problem tractable we have abstracted from the important questions of adverse selection and moral hazard problems and focus directly *after* the borrowers have formed a group. As a first approximation, we assume that lenders and borrowers have the same information about the cash flows generated by the technology of the borrowing group.

The paper is organized as follows: Section 2 formulates the basic model. In section 3, we characterize the equilibrium and its properties. In particular, we show monitoring does not work in equilibrium, and that the debt maturity can be used to control default probability. We also characterize a) default probabilities b) loan rates, and c) borrower's welfare for varying levels of monitoring, joint-liability efforts, and punishment technology. Section 5 concludes. We also include a technical appendix including the details of the derivations for interested readers.

1 Model Specification

Our model is directly specified at the level of the borrowing group. In doing so, we abstract from several interesting questions about how the group is formed, and the role that joint liability plays in the choice of the members of the group as well as the choice of the riskiness of projects by the members of the group. See Stiglitz (1991) and Ghatak (1999) for a treatment of these important issues. The salient features of the model are that a) the group cannot undertake any productive investment in the absence of the loan (they are heavily capital constrained) and b) the group is unable to post any meaningful physical collateral. Our goal then is to determine the equilibrium interest rates, where the lender must resort to a different approach for attempting to enforce the loan since there is no physical collateral or a bankruptcy code.

The following are key variables in our model. The aggregate loan for the entire group is denoted by L. A key assumption we make is that the group cannot engage in production in the absence of the loan. Hence, the loan is extremely attractive to the borrowers. Administrative expenses incurred by the lenders for monitoring the group is denoted by x per unit time. The members of the group are subject to joint-liability, with y denoting the associated expenses. This tends to reduce the value of the loan to the borrowers because of the efforts expended by the members of the group, but may improve the value of the loan due to improved performance from peer-monitoring efforts induced by joint-liability. We capture this trade-off explicitly in our model. These features have powerful ex-ante effects on how the group selects its members, and how the riskiness of the projects is chosen. By operating at the level of the group, our model will not be able to shed any insight on these ex-ante effects. As a special case, our model can also be used to examine micro-loans that are extended to individuals who are not part of a group.

We denote by $\delta(x, y)$ the proportion of wealth diverted away for consumption by the group. This is a decreasing function of the administrative expenses, x, incurred by the lender and the group efforts, y, to monitor each other. In practice, loans have a short maturity, which we denote by T. The lender is assumed to employ a punishment technology to deal with ex-post defaults. In reality, such costs take the following form. The lender might be able to make the cost of entry into credit markets in future very high for borrowers who default. This is certainly a credible threat and imposes a cost if the borrowers need repeated access to credit markets. In our model, we do not consider dynamic borrowing. Hence, we may think of the cost as the present value of the difference between the rates at which the defaulting borrower should borrow in informal credit markets and the (lower) rates that would have prevailed in micro-loan markets. Also, as noted earlier, defaults in the context of group borrowing may have a significant social cost to the defaulting borrower. Anecdotal evidence suggests that exiting borrowers often pay the loans rather than defaulting to avoid such social costs. Finally, institutions such as credit bureaus can be used to reduce the incentives of borrowers to default. Punishment technology, or the credible costs associated with default, takes the form of a lump sum cost of K. If K is too small, the borrowers will rationally take the loan and default promptly in an endogenous model of default. If it were too high, they will not borrow. It has to be high enough to induce payments of contractual obligations, which will increase the value of loans to the lender but not so large as to adversely reduce the participation of borrowers in the micro-loan programs. We will find these limits in our set up. The endogenously determined equilibrium loan rate R is one of the key objects our study.

The dynamics of wealth, C_t, generated by the investment for borrowing group is given by the equation shown below[10]:

$$dC_t = (\mu - \delta(x, y) - y)C_t dt + \sigma C_t dW(t) \tag{1}$$

It is assumed that $C(0) = L$ is the initial wealth of the borrowing group. The loan amount L is

[10] We assume that μ is the expected growth rate of wealth, and $\{W_t, t \geq 0\}$ is a Brownian motion process.

specified exogenously at $t = 0$. The equilibrium borrowing rate, R, is determined endogenously at $t = 0$. Group-specific risks are characterized by the constant drift and diffusive coefficients.

The specification of the process is for analytical convenience. It could be argued that it may not capture the lumpiness in income that micro-loan borrowers might face. We have solved the problem by accommodating lumpiness in output by modeling random jumps through a double exponential Poisson process, but our main results are qualitatively similar but the analytics are much more complicated. For this reason we decided to present our results for the case of the simple Brownian motion process as specified in (1).

It is useful to motivate the technology in the context of micro-loans. Micro-borrowers tend to invest their borrowing into activities such as a) livestock, b) kiosks, c) repair shops, d) paying high-interest loans to local money lenders, and e) consumption smoothing. These are typically small investments on which the rates of returns can be very high. CGAP reports estimates of rates of returns on micro-loans ranging from 40% to as high as 600%[11]. But returns must decrease with scale, and this raises questions about the existence of a threshold scale level of micro-loans beyond which they may be less effective as a development tool unless ways are found to reduce the interest rates charged on the loans. This makes the understanding of equilibrium borrowing rates very critical.

Requiring that $L = C(0)$ makes the loan extremely valuable to the borrower because without the loan, the borrower cannot access the technology and will have a utility of zero. It is easy to introduce an endowment of $x(0)$, which will lead to the requirement that $C(0) = x(0) + L$. We have noted in our data that some borrowers have prior indebtedness when they enter the first round of borrowing in the micro-loan markets, implying that $x(0) \leq 0$. This makes the loan even more attractive to the borrowers.

Note that joint-liability has several interesting effects on budget dynamics of the borrowing group. First, the peer monitoring activity is costly to the group, and this increases with the

[11] CGAP, January (2003).

number of members in the group. Second, when the wealth level is low, peer monitoring, which leads to a low $\delta(x, y)$, helps to overcome potential liquidity shortages facing the group. Third, the overall risk of the project portfolio of the group represented by σ is much lower than the risks of the projects of individual borrowers, when the group members ensure that only low risk borrowers join their group, recognizing the joint-liability feature of the loan.

Note that $\delta(x, y)$ is the "payout" or the amount diverted by the borrowing group for consumption purposes. The excess of wealth over $\delta(x, y)$, peer monitoring expenses and contractual payments is consumed in good states of the world. In bad states of the world, when the payout is less than the required payments, wealth must be liquidated to make the contractual payments.

2 Borrower's Problem and Endogenous Default

The objective of the borrowing group is to maximize the discounted payoffs from the loan and select the optimal default strategy as follows. Let $\tau = inf\{t \geq 0 : C_t \geq c^*\}$ be the first passage time of the wealth process, where c^* is the borrower's endogenously chosen optimal default trigger.

Borrower's Problem:

$$\begin{aligned} B(C_0) &= supE[\int_0^{\tau \wedge T} e^{-rs}(\delta(x, y)C_s - LR)ds] \\ &\quad + E[e^{-r\tau} J_B(C_\tau - K)1_{\{\tau \leq T\}}] - Le^{-rT}P(\tau > T) + E[J_B(C_T)]e^{-rT}P(\tau > T) \end{aligned} \tag{2}$$

where $J_B(\cdot)$ denotes the payoffs to the borrowing group upon optimally choosing to default and r denotes the risk-free rates. Here we consider two possibilities. First, default leads to a lump sum punishment K but the group continues to have access to technology in which the residual wealth can be invested. This is akin to saying that the tools of trade of borrowers may not be seized when default occurs. Certainly, with micro-loans the political and social costs of such an action by lenders are rather high. One interpretation of this punishment is the following: when the borrowing group defaults, it is precluded from entering the micro-loan markets again and is forced to borrow from local money lenders at a prohibitive cost to continue to run their business.

This way, the group is able to continue to have access to the technology but suffers a lump sum cost upon default. In this case, the payoff function upon default is[12]:

$$J_B(C_\tau - K) = \frac{\delta(x=0,y)(C_\tau - K)}{r + \delta(x=0,y) + y - \mu}$$

Alternatively, we can assume that default leads to a lack of access to the technology itself. This is a more severe punishment. The borrowing group receives a certain amount of wealth, which may be thought of as the liquidation value of their business net of punishment costs K, and they must consume out of that for the rest of their lives. Given their lack of access to savings, this will constitute a more severe punishment. In this case, the payoff upon default is simply the cash flow at time of default minus the punishment cost, namely:

$$J_B(C_\tau - K) = C_\tau - K$$

The maximization problem of the borrower leads to the following HJB equation:

$$0 = \max\left[-B_t - rB + \delta(x,y)c - LR + B_c(\mu - \delta(x,y) - y)c + B_{cc}\frac{1}{2}\sigma^2 c^2\right] \tag{3}$$

When the wealth level of the borrowing group reaches a threshold low level c^*, the group collectively defaults and receives a payoff as follows:

$$B(c \downarrow c^*, T) = J_B(c^* - K) \tag{4}$$

In order for the expected payoffs of the borrowing group to be finite, we need to impose a transversality condition.[13] To summarize, we have specified the optimal default strategy of the borrowing group. Next, we proceed to characterize the borrower's value from taking a micro-loan of size L and defaulting optimally. The finite maturity loan is a very complicated problem

[12] We assume that the group operates as a unit even after default and enjoy the benefits of peer monitoring after default. This can be relaxed to consider the case where default eliminates the benefits of peer monitoring.

[13] We will maintain the following assumption throughout our analysis:

$$r + \delta(x,y) + y - \mu > 0. \tag{5}$$

and does not have a closed-form solution. We provide a very accurate approximation of the finite maturity problem, which is given in the appendix, which also explains the approximation procedure[14].

2.1 Equilibrium & Endogenous Borrowing Rates

We now propose a specification to describe the lender's behavior. The lender will take into account the optimal default strategy of the borrower in valuing the loan as follows. If the loan were to have a finite maturity date T, the lender's problem is:

$$D(L) = E[\int_0^{\tau \wedge T} e^{-rs} L(R-x)ds] + E[e^{-rT} L 1_{\{\tau \geq T\}}] \tag{6}$$

A *competitive equilibrium* in this economy is an endogenously determined borrowing rate R for a loan of size L to the borrowing group such that:

1. Borrowing group's value is maximized.

2. Lender's required rate of return satisfies the fixed point requirement that $D(L) = L$.

Note that in a competitive equilibrium the lender is not getting any surplus and the market value of the loan is exactly the amount supplied to the borrower.

We also consider a market structure in which all the surplus is extracted by the lender by setting $B(L) = L$. We refer to this case as the monopolistic market structure, where the lender sets the interest rate R such that the borrower has no surplus. In what follows, we first characterize the competitive equilibrium.

Carrying out the integration and applying the lender's break even condition yields :

$$R = x + r\frac{1 - e^{-rT}P(\tau > T)}{1 - E[e^{-r\tau}1_{\{\tau \leq T\}}] - e^{-rT}P(\tau > T)} \tag{7}$$

[14]The idea of the approximation is to decompose the finite maturity loan contract into two parts: 1. the no early exercise loan contract and 2. the value of the early exercise option to the borrower. This approximation is particularly good for very short and very long maturities. This approximation is particular attractive in the context of non-collateral lending as micro-loans are usually short maturity, whereas sovereign loans are usually long maturity.

Note that the equilibrium interest rate must compensate for a) administrative expenses x borne by the lenders, b) the funding costs, which is assumed to be the risk-free rate, and c) the possibility of default by the borrowing group. Note that the probability of default will be influenced by the factors x, y and K, which we will characterize in the next section.

It is easy to verify that for a perpetual loan, the equilibrium interest rate will be as follows:

$$R = x + r\frac{1}{1 - E[e^{-r\tau}]} \tag{8}$$

where

$$E[e^{-r\tau}] = (\frac{c^*}{c})^{\beta 1} \tag{9}$$

and $\beta_1 = \frac{1}{2} - \frac{\mu - \delta(x,y) - y}{\sigma^2} + \sqrt{(\frac{\mu - \delta(x,y) - y}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}}$.

The intuition of the loan rate is straight forward. The first term is the monitoring cost expended by the lender. The second term is a risk premium demanded by the lender in order to take on the risky position. The proportionality factor for risk premium demanded is $1 - E[e^{-r\tau}]$, where $E[e^{-r\tau}]$ can be interpreted as the price of an Arrow-Debreu security which pays \$1 in the event of default. Note that for a 1% increase in the funding cost or the interest in the micro loan, the borrowing rate is magnified by the risk premium which is greater than 1.

The equilibrium interest rates on micro-loans is now completely characterized for given default boundary c^* and maturity T, and a risk structure of default premium can be obtained in our model. The borrowing rates under finite maturity differ from the perpetual loan rates. The key difference (in additional to different default boundaries) is in the proportionality factor for risk premium demanded. We first ignore the denominator and note that the numerator is now $1 - e^{-rT}P(\tau > T)$, which suggests that loan rates can be reduced. Economically, this implies the possibility of an increase in the range of equilibrium, suggesting that lower monitoring cost may be admissible in finite maturity. We explore this later and demonstrate that this is in fact the case.

Our model for the determination of loan rates differs sharply from how practitioners set the interest rate. A CGAP report indicates that the following is a standard formula for the interest rate charged in micro-finance markets:

$$R = \frac{AE + LL + CF + K - II}{1 - LL} \tag{10}$$

where, AE denotes the administrative expense/monitoring cost - namely, x in our model, LL denotes the loan losses, CF is the cost of funds - namely r in our model, K is the capitalization rate, and II denotes the Investment Income. This equation presents a *linear relationship* among monitoring effort, interest rate, and joint-liability (implicit in AE). However, our model suggests that this relationship should be highly nonlinear. The difference is mainly due to the fact that the practitioner's loan pricing equation ignores the fact that the loan loss rate (or defaults) depend on the borrowing rate. Furthermore, a given increase in AE, which is x in our model, will lead to a higher loan loss rate due to the increased probability of default. The intricate dependence of these variables and the necessary risk premium are not properly reflected in the loan pricing equation of practitioners.

2.2 Role of Lender Monitoring & Defaults

The model delivers several implications for the role of monitoring by lenders. We now state one of the main results of our paper.

Proposition 1 *In the absence of any collateral, when the borrowing group has access to technology upon default, there must be monitoring by the lender in order for there to exist an equilibrium for a perpetual debt.*[15].

The intuition behind proposition 1 is the following. When there is no monitoring by lenders, the consumption rate by the borrowing group is the same before or after default. But during the period that the loan is solvent, borrowers are forced to pay costly contractual interest payments. Immediate default allows the borrowing group to maintain its consumption and avoid paying the interest rates, which already reflect the ex-post costs of punishment. Using the parameters

[15] Proof is in the appendix.

in the numerical illustration section, we present here the equilibrium behavior of the borrower's valuation function as a function of monitoring cost x and punishment cost K for perpetual debt. In deriving this proposition, we assume that the group still operates as a unit after default. Were this not the case, the peer monitoring benefits may still induce the group not to default even in the absence of lender monitoring.

Insert Figure 1 here.

The left panel in Figure 1 illustrates our proposition. Equilibrium only exists after monitoring increases up to a certain threshold level. This is due to the continuity of our problem. If monitoring decreases too much (i.e. say, below $x = 0.2$), there can be no equilibrium anymore. This suggests how a minimum level of lender monitoring is essential to sustain an equilibrium in micro-loan markets. Monitoring, however, can potentially be a very inefficient method for the lender to enforce the loan. We note that for each monitoring cost, there exists a K_{max} such that after this point there will not be an equilibrium anymore. To the right of this region the borrowers realize that the costs of default are too excessive for them to take the loan. We further note that $K = 0$ can be sustained as an equilibrium in our model. This is due to the fact that the borrowers' consumption is specified as an exogenous process. As long as there is monitoring, this will increase the drift of the borrower's production process. The increase in the drift is big enough to stop the borrowers from immediate default.

We now examine the situation when the maturity of the loan is very short (3 months).

Proposition 2 *In the absence of any collateral, when the borrowing group has access to technology upon default, an equilibrium exists for a finite maturity debt even in the absence of monitoring*[16]*.*

The intuition for this result is simple: the impending balloon payment of the principal accelerates the punishment costs K, and hence the borrower does not immediately default even in the absence

[16] Proof is in the appendix.

of monitoring by lenders. This suggests the use of the debt maturity as a substitute for monitoring by lenders, as illustrated in the right panel of Figure 1. We will later show that debt maturity indeed outperforms monitoring in reducing defaults in equilibrium.

2.3 Impact of Access to Physical Collateral

In the absence of borrowing in micro-loan markets, we assume that the borrower has a utility of $\hat{U}$, which we assume to be zero for simplicity. This is the utility associated with borrowing from local money lenders, without the benefits of assortatively matched groups and peer monitoring. Once the group has access to micro-loans it is able to access the technology and create value for the borrowers. The value of the loan to the lenders is set equal to the present value of the payments promised by the borrowers, leaving the lenders with no surplus; namely $D(L) = L$. The value to the borrower is determined by the optimization problem described above. The value created by the loan to the borrower is summarized by the ratio $\frac{B(C)}{L}$. The lender can extract some of this surplus by requiring a higher rate of return on the loan. His ability to do so will depend on the market structure within which lenders operate.

In this section we explore the case wherein the lender receives a fraction $(1-\alpha)$ of the value conditional on default. We first modify the lender's break-even condition to accommodate the existence of collateral upon default in our model. Let c^* be the borrower's default strategy associated with a given loan size L, then the lender's valuation is[17]:

$$D(L) = E[\int_0^{\tau} e^{-rs} L(R-x)ds] + E[e^{-r\tau}(1-\alpha)c^*] \tag{11}$$

As before, we now invoke the break even condition $D(L) = L$ to get the equilibrium loan rate, as given by:

$$R = x + r\frac{1-(1-\alpha)\frac{c^*}{L}(\frac{c^*}{c})^{\beta_1}}{1-(\frac{c^*}{c})^{\beta_1}} \tag{12}$$

where $\beta_1 = \frac{1}{2} - \frac{\mu-\delta(x,y)-y}{\sigma^2} + \sqrt{(\frac{\mu-\delta(x,y)-y}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}}$.

[17] We explore the case of perpetuity here for simplicity.

Note that in the above expression, the term $1-(1-\alpha)\frac{c^*}{L}(\frac{c^*}{c})^{\beta_1}$ takes account of the existence of collateral. If we set $\alpha = 1$, we will recover our previous equation.

In our model, an increase in r typically increases the credit spreads. Also, our model would predict that in the absence of sufficient controls (as modeled by y, K and T), the defaults in micro-loans will occur sooner and spreads would be higher in micro-loans as compared to loans backed by physical collateral. To summarize,

1. Micro-loan rates are higher than loan rates backed by physical collateral.

2. Defaults in micro-loans will occur sooner than in loans backed by physical collateral, in the absence of peer monitoring, lender monitoring and punishments upon default.

In the next section, we focus on a competitive loan market and examine how various contractual features influence a) loan rates, b) borrower's welfare, and c) default probabilities. In other words, we assume that the loan rate is set such that

$$B(L) \quad = \quad L.$$

This is followed by a comparision of competitive equilibrium with a market in which there is a monopolistic lender. Finally, we explore the implications of designing a loan contract wherein the contractual interest payments are tied to the flow rate of output of borrower's technology.

2.4 Contractual Features

In order to obtain additional results, we impose the following structure on the $\delta(x, y)$ function. We need to ensure that the consumption function $\delta(x, y)$ is decreasing in monitoring and joint-liability. That is, more monitoring and peer-monitoring lead to less consumption for the borrowers. We also require that the cross derivative $\frac{\partial^2\delta(x,y)}{\partial x \partial y} > 0$. This ensures no effect dominates each other. A particular example of such function, which we will employ throughout our analysis, is[18]:

$$\begin{aligned}
\delta(x > 0, y < y^*) &= [(1 - e^{-\beta x})\underline{\delta} + e^{-\beta x}\hat{\delta}] \times e^{-b\frac{y}{y^*}} \\
\delta(x = \infty; y) &= \underline{\delta} \times e^{-b\frac{y}{y^*}} \\
\delta(x = 0; y = 0) &= \hat{\delta}
\end{aligned}$$

In the absence of peer monitoring and lender monitoring, the borrowing group will consume an amount denoted by $\hat{\delta}$. The presence of infinite peer monitoring will lead to a level consumption denoted by $\underline{\delta}e^{-b\frac{y}{y^*}}$. This level, $\underline{\delta}$, can be thought of as the level of consumption for the borrowing group in the absence of any peer monitoring. Once the peer monitoring is put in place, its effectiveness is governed by the parameter, b, and the peer monitoring effort, y, relative to a threshold level y^*, which is a scaling variable. For a given effort y, higher the parameter b is, greater is the effect of peer monitoring in controlling wasteful consumption. When $b = 0$, peer monitoring has no effect. A small y^*, and a high b, cetaris-paribus, may be thought of as a very effective peer monitoring outcome, which in reality may be due to an "assortative matching" process in which low-risk borrowers with no informational disadvantages identify other low-risk borrowers (and thereby exclude higher risk borrowers) in forming the group so that the pool in the group has low risk both in payment behavior and the riskiness of the projects undertaken by group members[19]. At any effort level y which is below the threshold contracting effort y^*, there is additional diversion of output for private consumption. A caveat is that we do not model the riskiness of the project as a function of this matching process.

With a monitoring effectiveness or efficiency parameter $\beta < 0$, lenders can in the limit approach this ideal benchmark level. In the absence of any monitoring, the existence of a punishment technology and peer monitoring through joint-liability contracting is assumed to lead to a diversion rate of $\hat{\delta} > \underline{\delta}$. Our specification assumes that a minimum level of monitoring x^* is needed even when the group is formed under ideal conditions. In the absence of any monitoring,

[18]Specifically, we choose $\beta = -0.5$, $\underline{\delta} = 0.06$, $\hat{\delta} = 0.10$, $b = 0.25$, and $y^* = 0.02$.

[19]See Bannerjee, Besley and Guinnane (1994)

the existence of a punishment technology and peer monitoring through joint-liability contracting is assumed to lead to a diversion rate of $\hat{\delta} > \underline{\delta}$.

Insert Figure 5 here.

The delta function used in this numerical illustration is plotted in Figure 5. Throughout this section we have assumed the following parameters. We set the interest rate, r, at 3%, the drift parameter, μ at 12%, the Volatility parameter, σ^2 at 20%, the Initial Lending Amount, L at 1000, the Punishment Cost, K, at 500 and the maturity, T at 3 months. These parameters are chosen to reflect the contractual parameters observed in practice. The average size of micro-loans are three months and are relatively small.

Insert Figure 6 here.

Figure 6 investigates the range of equilibrium and how different amount of monitoring cost and joint liability affects equilibrium. Note that with short maturity, excessive monitoring reduces the welfare of the borrower. First, default probability is increasing in monitoring. This is a consequence of the high loan rates charged by the lenders to compensate for the increased monitoring costs. With short maturity, the borrowers are now faced with higher repayment rates, and since the technology may not be able to produce enough cash flow for repayments in such short maturity, the borrowers are forced to default. Although the lender can transfer all the cost incurred to the borrower, the lender still has to suffer from lower repayment probabilities. This is due to the feedback effect of increased probability of default induced by the higher borrowing rates in our equilibrium analysis, which is absent in the practitioner model. Hence, practitioners believe lender monitoring can lead to lower defaults, whereas it is not necessarily the case in our model. This suggests that finite maturity is an useful substitute for monitoring. It reduces default probability while keeping loan rates feasible for the borrowers. Joint liability, on the other hand, serves as another substitute for monitoring. Default probability is concave in joint liability, suggesting that when joint liability is low, default probability increases. However, once the borrowers exert too much joint liability, they do not concentrate in the production process enough and causing more defaults in equilibrium. This effect translates to a concave borrower's valuation function. Loan rates are decreasing in joint liability as a very slow rate.

Figure 7 investigates the dynamics of the equilibrium quantities in terms of punishment cost K and joint liability y. We will analyze values joint-liability for $y = 0\%$ (solid line), $y = 0.5\%$ (dashed line), and $y = 1\%$ (dotted line). We will fix $x = 30\%$. Several aspects of these pictures are worthy of additional discussion: first, note that a higher level of peer monitoring leads to a lower probability of default at all levels of K. Second, at higher levels of peer monitoring as proxied by the variable y, the range of punishment costs K conditional on default is much higher: in other words, the default probability is a much flatter function of K at higher levels of y. Although the loan rates are lower with higher levels of y, the borrower's value function is declining in y as the borrowing group is forced to put in the peer monitoring effort.

Insert Figure 7 here.

Figure 8 explores how equilibrium changes as a function of punishment cost as we increase monitoring cost for $x = 20\%$ (solid line), $x = 25\%$ (dashed line), and $x = 30\%$ (dotted line). We will fix $y = 0.5\%$. As the monitoring effort x by the lenders increases, the borrowing costs increase as well. We note that monitoring increases default probability while raising loan rates. It also lowers borrower's value due to the increased defaults. Hence, in short maturity debt contracts, monitoring do not play an effective role.

Insert Figure 8 here.

Figure 9 explores the relationship between the borrower's and the lender's actual cost of lending. As before, we focus on the cases where an equilibrium exists. Namely, we have x=20% (solid line), x=25% (dashed line), and x=30% (dotted line). Default probability and loan rates are increasing in both the monitoring expense and the interest rate. Loan rate is increasing in interest rate, as the lender demands a higher risk premium to compensate for his increasing opportunity cost. Furthermore, higher administrative cost induces a higher equilibrium loan rate. As a result of the higher loan rates demanded by the lender, default probability is increasing in loan rates. These effects force the borrower's value to be decreasing in monitoring expense and interest rate. The intuition behind these results is that as interest rate goes up, loan rates increase much more significantly. This suggests that the cost of funding for the lender may be a key to the determination of micro-loan interest rates.

Insert Figure 9 here.

Insert Figure 10 here.

Figure 10 explores the term-structure implied by our model. The lines plot the term structure of normalized borrower's value, default probability, and equilibrium loan rates for $L = 1000$ (solid line), $L = 5000$ (dashed line), and $L = 10000$ (dotted line). Note that for fixed punishment cost K, our model requires large loans and long maturities. Small loans on the other hand require short maturities. Note that as K increases from 500 to 750, the short maturities become admissible for loan size of 1000. This suggests the use of punishment cost as a very powerful device for enlarging the range of equilibriums. Finally, as the loan size increases, the probability of default increases and the loan rates dramatically increase unless the maturity of the loans are increased.

We now present the trade off between monitoring and debt maturity and show the choice of debt maturity can be used as a substitute to monitoring. We focus on x=20% (solid line), x=25% (dashed line), and x=30% (dotted line). Figure 11 clearly show that debt maturity can be used to control defaults in equilibrium. Short maturity loans almost never default while long maturity loans are more inclined to default. On the other hand, increasing monitoring simply increases loan rates and default probability. This suggests that the use of debt maturity as a contracting device for the lenders instead of monitoring.

Insert Figure 11 here.

In summary, a prevalent effect in our model is that default probability and loan rates increase in monitoring. This suggests that when practitioners decide their micro-lending business strategy, monitoring costs should be given extra concern. If the group is properly formed (i.e., for a reasonable joint liability y), and a fixed opportunity cost r, micro-finance institutions may achieve a lower default rate by reducing monitoring. Furthermore, the micro-finance institution may substitute monitoring using maturity T of debt as a tool. Finally, punishment cost K can also be used to alter equilibrium risk structure.

2.5 Competition versus Monopoly

In this section, we explore the differences in the predictions of our model in two different loan market structures: competitive and monopoly. This is done by recognizing that the borrower will take the loan as long as $B(L) \geq L$. In the limit, the lender can extract all the rents from the borrower, by setting the rate R such that:

$$B(L) \quad = \quad L$$

The monopolistic debt lending is characterized by the interest rates R such that this holds, where we denote $\hat{R}$[20].

Figure 2 explores the differences between predictions of our models as punishment cost K varies. Note that, predictably, the borrower's welfare is far higher under competitive equilibrium and it increases with ex-post punishment costs K. The probability of default is lower and perhaps most importantly the equilibrium borrowing rates are significantly lower. Note as well that the borrowing rates are far more elastic to increases in ex-post punishment costs under a competitive market structure.

Insert Figure 2 here.

[20]Other market structure can be analyzed by setting interest rates:

$$R \quad = \quad \theta\bar{R} + (1-\theta)\hat{R}$$

.

Insert Figure 3 here.

Figure 3 explores the differences between predictions of our models as interest rate r varies. As the cost of funding r increases, the default probability increases under both market structures, but the increase is far less elastic under a competitive market structure. The loan rates are much less sensitive to increases in cost of funding r under the competitive market conditions. Note that under the monopolistic market structure, loan rates decline with increases in funding coats r. This is due to the fact that a monopolistic lender can lower the loan rate R and still extract surplus from the borrower, which is not possible under the competitive structure.

Insert Figure 4 here.

Figure 4 explores here the differences between predictions of our models as the loan maturity T varies. The important result here is that long maturity loans improve the welfare of the borrower, and keeps the interest rates low under a competitive market structure. On the other hand, the loan rates R increase with maturity in a monopolistic market structure.

2.6 Fixed Rate Loan vs. Floating Rate Loan

In this section, we explore the welfare consequences of designing the loan contract so that the interest payments are proportional to the flow rate of output from the technology of the borrowing group. We refer to this case as a floating rate loan. Essentially, with a floating rate loan, lenders permit borrowers to make higher payments in "good states" to insure against lower payments in "bad states" but keep the present value of payments equal to the loan amount L in a competitive market. A complete solution of the problem is given in the appendix. The contractual coupon payments at each instant are $\hat{\beta}C_t$, and the lender still enforces the break-even condition as before. Analysis of this case leads to the following important result:

- $c^*_{Fixed} \geq c^*_{Floating}$, meaning that in fixed rate specification, the borrower defaults earlier. This leads to the implication that the default probabilities are lower if the loan contract permits a variable interest payment schedule that is linked to the flow rate of output of the borrowing group.

In the floating rate loan specification, default decision is independent of the interest repayment rate, which is labeled as $\hat{\beta}$ in the solution given in the appendix. The intuition is that the lenders now bear all the risk with the borrowers. The range of admissible equilibrium is greater under fixed rate specification than under floating rate specification. Given that the lenders know the default strategy of the borrowers, lenders do not lend as much regardless of the rate they charge since they know it does not affect the borrower's decision.

3 Conclusion

We have presented a simple model of lending without collateral. The lender attempts to enforce the contract by relying on three things: a) monitoring to reduce the diversion of resources by the borrower from productive uses, b) peer monitoring by lending to a group, which is jointly-liable for the fulfillment of the contractual provisions, and c) a punishment technology that imposes a finite cost on defaulting group of borrowers. We show that peer monitoring combined with a limited amount of monitoring by lenders is sufficient to reduce default probability to acceptable levels, so long as there is a credible punishment cost. Excessive monitoring by lenders increases the cost of borrowing and this might lead to non-participation by borrowers. As the loan size increases, we show that the probability of default increases, and the loan rates dramatically increase, unless the maturity of the loans is increased.

We have extended our analysis to examine situations where the borrowers face low frequency jump risks. Episodes such as heavy monsoons or health epidemics could have dire consequences for borrowers in this market. Predictably, we found initial loan rates to be too prohibitive in the presence of adverse jump risks. An important limitation of our work is that we do not examine repeated borrowing and the discipline that may impose on the borrowing group.

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5 Appendix

In all our derivations, let $\tau = inf\{t > 0 : C_t > c^*\}$ be the first passage time of the cash flow process.

5.1 Fixed Rate Debt Contract

5.1.1 GBM Perpetual Loan Contract

Since most structural corporate debt models assume perpetual debt, we present here the borrower's valuation with perpetual loans:

$$B(c) = B(\alpha c - K) \tag{13}$$

for $c \geq c^*$:

$$B(c) = A_1(\frac{c^*}{c})^{\beta_1} + A_3 c + A_4 \tag{14}$$

where

$$\begin{aligned}
A_1 &= (\alpha B - A_3)c^* - (BK + A_4) \\
A_3 &= \frac{\delta(x, y)}{r + \delta(x, y) + y - \mu} \\
A_4 &= -\frac{LR}{r} \\
c^* &= \frac{(BK + A4)\beta_1}{(\alpha B - A3)(1 + \beta_1)} \\
B &= \frac{\delta(x = 0, y)}{r + \delta(x = 0, y) + y - \mu} \\
\beta_1 &= \frac{1}{2} - \frac{\mu - \delta(x, y) - y}{\sigma^2} + \sqrt{(\frac{\mu - \delta(x, y) - y}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}}
\end{aligned}$$

and B is defined above, which can take on two values depending whether we allow for access to technology or not upon default. The interpretation of the above formula straight forward. The first term is the risk neutral expectation of the investment technology net of punishment cost and default risk. The second term is the value of the technology up on default. The last term is the present value of the total cost exerted by the borrower.

Proof Let $x = log(C)$ and assume:

$$B(C) \;=\; A_1 e^{-x\beta_1} + A_3 e^x + A_4$$

We will suppress the dependencies of the δ for convenience. Substituting this into the HJB equation yields:

$$\begin{aligned} 0 \;=\; & A_1 e^{-x\beta_1}(-r + (\mu - \delta - y - \frac{1}{2}\sigma^2)(-\beta_1) + \frac{1}{2}\sigma^2\beta_1^2) \\ & + e^x((-r + \mu - \delta - y)A_3 + \delta) \\ & - LR - rA_4 \end{aligned}$$

Now, by the technique of matching the coefficient, we can solve for β_1, A_3 and A_4 in closed form. A_1 is obtained via the *Principal of Continuity*:

$$B\alpha e^{x_0} - BK \;=\; A_1 e^{-\beta_1 x_0} + A_3 e^{x_0} + A_4$$

where B is defined in the paper, referring to different values according to whether the borrower has access to technology or not after default.

Finally, the *Principal of Smoothing Pasting* gives the optimal default boundary:

$$B\alpha e^{x_0} \;=\; -A_1\beta_1 e^{-\beta_1 x_0} + A_3 e^{x_0}$$

We have 5 equations and 5 unknowns, giving us an identified system to solve for: β_1, A_1, A_3, A_4 and x_0. Now recognizing that $c^* = e^{x_0}$, the proof is complete.

5.1.2 GBM Finite Maturity Loan

The borrower's value function is given by: For $c \leq c^*$:

$$B(c, T) = B(c - K) \tag{15}$$

for $C \geq c^*$:

$$B(C_0, T) = EuB(C_0, T) + A_1(\frac{c^*}{c})^{\beta_1} + A_3 c + A_4 \tag{16}$$

where

$$
\begin{aligned}
A_1 &= (B - A_3 - Q)c^* - (BK + A_4) \\
A_3 &= \frac{\delta(x,y)/z}{r/z + \delta(x,y) + y - \mu} \\
A_4 &= -\frac{LR}{r} \\
c^*_{approx} &= \frac{(BK + A4)\beta_1}{(B - A3 - Q)(1 + \beta_1)} \\
Q &= Be^{(\mu - r - \delta(x,y) - y)T} \\
z &= 1 - e^{-rT} \\
\beta_1 &= \frac{1}{2} - \frac{\mu - \delta(x,y) - y}{\sigma^2} + \sqrt{(\frac{\mu - \delta(x,y) - y}{\sigma^2} - \frac{1}{2})^2 + \frac{2r/z}{\sigma^2}}
\end{aligned}
$$

The constant B is again chosen according to whether there is access to technology or not up on default. $EuB(C_0, T)$ represents the European version of the same debt contract, and is given by:

$$
\begin{aligned}
EuB(C_0) &= -Le^{-rT} + e^{-rT}E[J_B(C_T)] \\
&= -Le^{-rT} + BC_0e^{(\mu - r - \delta(x,y) - y))T} \\
&= -Le^{-rT} + QC_0
\end{aligned}
$$

where $B = \frac{\delta(x=0,y)}{r + \delta(x=0,y) + y - \mu}$.

Note that c^*_{approx} converges to c^* and the finite maturity approximation of the borrower's value converges to the perpetuity function as T goes to infinity, verifying the accuracy of our approximation scheme.

From the finite maturity approximation, we see that even with $x = 0$, there can be an equilibrium - a main difference between the finite maturity and the perpetual conclusion. The intuition is that finite maturity itself is also an additional tool both the lender and the borrower can use to screen out unwanted loan contracts. Although the consumption rate is still the same before and after default, with the additional constraint of finite maturity, the borrowers will choose not to immediate default as long as the amount of interest paid is less than the punishment cost of defaulting.

Proof In the following, we will suppress the dependencies of δ for convenience.

The borrower's value has to satisfy the following HJB equation: For $c \geq c^*$:

$$0 = \max\left[-B_t - rB + \delta c - LR + B_c(\mu - \delta - y)c + B_{cc}\frac{1}{2}\sigma^2 c^2\right] \tag{17}$$

and

$$B(c,T) = J_B(c - K) \tag{18}$$

By Feymann-Kac, we know that $EuB(c,T)$ solves the following partial differential equation for all c:

$$0 = \max\left[-EuB_t - rEuB + \delta c - LR + EuB_c(\mu - \delta - y)c + EuB_{cc}\frac{1}{2}\sigma^2 c^2\right] \tag{19}$$

Hence, the early exercise premium $\epsilon(c,T)$ must satisfy: For $c \geq c^*$:

$$-\epsilon_t - r\epsilon + (\mu - \delta - y)\epsilon_c + \frac{1}{2}\sigma^2\epsilon_{cc} + \delta(x,y)c - LR = 0 \tag{20}$$

and for $c \leq c^*$:

$$\epsilon(c,T) = J_B(c - K) - EuB(c,T) \tag{21}$$

Now by letting $z = 1 - e^{-rt}$ and $g(c,z) = \frac{\epsilon(c,T)}{z}$. It is easy to see that: $z_t = re^{-rt}$, $\epsilon_x = zg_x$, $\epsilon_{xx} = zg_{xx}$ and $\epsilon_t = z_t g + zg_z z_t$. Substitute this into the HJB and divide by z, the HJB becomes: For $c \geq c^*$:

$$-r(1-z)g_z - \frac{r}{z}g + \frac{1}{2}\sigma^2 zg_{cc} + (\mu - \delta - y)zg_c + \frac{\delta}{c}c - LR = 0 \tag{22}$$

We will assume that $(1-z)g_z = 0$ for the approximation. This approximation becomes very accurate for very short and very long maturity.

We are now in the position to solve this equation. We recognize this as the Euler's equation and hence assume:

$$\epsilon(c) \;=\; A_1\left((\frac{c^*}{c}\right)^{\beta_1} + A_3 c + A_4$$

By the method of matching coefficients, we get the following equations:

$$\begin{aligned} -r/z + (\mu - \delta - y - \frac{1}{2}\sigma^2)(-\beta_1) + \frac{1}{2}\sigma^2\beta_1^2 &= 0 \\ (-\frac{r}{z} + \mu - \delta(x,y) - y)A_3 + \frac{\delta}{z} &= 0 \\ rA_4 + LR &= 0 \end{aligned}$$

Imposing *continuity* at the boundary:

$$\epsilon(c^*, T) = B(c^* - K) - EuB(c^*, T) \tag{23}$$

allows us to solve for the coefficient A_1 as a function of optimal default boundary c^*. Imposing the *principal of smooth fit*:

$$\epsilon'(c^*) = Bc^* - \frac{\partial}{\partial x} EuB(e^x, T)|_{x=x_0} \tag{24}$$

gives the optimal default boundary.

We have a system of 5 equations and 5 unknowns and hence all the variables are identified. This completes the proof.

5.2 Proof of Proposition 1

Proof Step 1.

First, we will show that the value of continuation is always less than value of defaulting immediately, when $x = 0$.

Case 1. $c^* > L$

There is no equilibrium by definition.

Case 2. $c^* < L$

Suppose $c^* = cbar < L$, then by transversality condition, the borrower's value function is well defined and finite. Hence, $c*$ must satisfy the equation for the optimal default boundary:

$$c^* = \frac{(BK + A4)^{\frac{\beta_1\beta_2}{\eta_2}}}{(B - A3)^{\frac{(1+\beta_1)(1+\beta_2)}{(\eta_2+1)}}}$$

Note that the LHS is finite by assumption. The numerator of RHS depends on R, which we can calculate given default boundary using (8). However, the denominator is 0 and hence $c*$ is undefined, contradicting the fact that transversality condition (5) guarantees a well-defined value's function.

Case 3. $c^* = L$

When $c^* = L$, for any triplet (K, x, y) the value of continuation equals the value of immediate default by the *Principle of Smooth Fit* (i.e. the value function is continuous at c^*).

Step 2.

Let us now show that $c^* = L$ cannot be an equilibrium for any punishment cost K.

Case 1. $K < L$

If the punishment cost is less than initial loan amount, the borrower's optimal strategy is to default immediately. However, the lender knows it and hence she won't lend.

Case 2. $K > L$

If the punishment cost is higher than initial loan amount, the borrower's value function will always be negative since $J_B(L) < 0$. Hence, the borrower is better off not borrowing.

Case 3. $K = L$

The borrower has a value function exactly 0. Hence, she is indifferent between lending and borrowing.

5.3 Collateralized versus Micro-loan Rates

Proof We note that in Leland(1994)'s model, we can rewrite the default boundary for a collateralized loan as:

$$\begin{aligned} c^*_{coll} &= \frac{C}{r}\frac{\beta_{corp}}{1+\beta_{corp}} \\ &= \frac{LR_{corp}}{r}\frac{\beta_{corp}}{1+\beta_{corp}} \end{aligned}$$

Similarly, in the absence of access to technology upon default and the drift of the technology process restricted to r for the existence of a martingale measture, we have $A_3 = 1$ and $B = 1$. This gives:

$$\begin{aligned} c^* &= \frac{\frac{LR}{r}\frac{\beta_1}{1+\beta_1}}{1-(1-\alpha)} \\ &= \frac{\frac{LR}{r}\frac{\beta_1}{1+\beta_1}}{\alpha} \end{aligned}$$

The first part of the proposition follows immediately once we note that $\alpha \geq 1$ is a natural assumption. After rearranging for R, the same analysis leads to the second result.

5.4 Numerical Procedure

This section documents the numerical procedure in solving for the equilibrium (R, c^*). We solve for our equilibrium as follows:

1. In the equation for c^*, we plug in the equation for equilibrium loan rate R, which depends on c^* as well. Note that the equations for R in the finite maturity case involves the terms $P(\tau > T)$ and $E[e^{-r\tau}1_{\{\tau \leq T\}}]$, which we use the method of Laplace transforms to obtain. Specifically, we apply the Gaver–Stehfest inversion algorithm to the Laplace transforms of $P(\tau > T)$ and $E[e^{-r\tau}1_{\{\tau \leq T\}}]$.
2. We numerically vary c^* until the fixed point equation for c^* is satisfied.
3. We then use the solution for c^* to get our equilibrium loan rate R.
4. If c^* is within the admissible range $(0, L)$, then we check whether the borrower's valuation function is positive. If so, we have an equilibrium. Otherwise, there is no equilibrium.

5.5 Floating Rate Debt Contract

5.5.1 GBM Perpetual Loan Contract

We present here the borrower's problem with floating rate, by which we mean that the instantaneous repayment for the borrower is $\beta C_t dt$. The HJB equation is given by:

$$0 = \max\left[-B_t - rB + (\delta - \beta)c + B_c(\mu - \delta - y)c + B_{cc}\frac{1}{2}\sigma^2 c^2\right] \tag{25}$$

The solution of this equation with $B_t = 0$ is given by:

$$B(c) = B(c - K) \tag{26}$$

for $c \geq c^*$:

$$B(c) = A_1(\frac{c^*}{c})^{\beta_1} + A_3 c \tag{27}$$

where

$$\begin{aligned} A_1 &= (\alpha B - A_3)c^* - BK \\ A_3 &= \frac{\delta(x,y) - \beta}{r + \delta(x,y) + y - \mu} \\ c^* &= \frac{(BK\beta_1}{(\alpha B - A3)(1 + \beta_1)} \\ B &= \frac{\delta(x = 0, y)}{r + \delta(x = 0, y) + y - \mu} \\ \beta_1 &= \frac{1}{2} - \frac{\mu - \delta(x,y) - y}{\sigma^2} + \sqrt{(\frac{\mu - \delta(x,y) - y}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}} \end{aligned}$$

and B is defined above, which can take on two values depending whether we allow for access to technology or not upon default.

5.5.2 GBM Finite Maturity Loan

The borrower's value function is given by: For $c \leq c^*$:

$$B(c, T) = B(c - K) \tag{28}$$

for $C \geq c^*$:

$$B(c,T) = EuB(c,T) + A_1(\frac{c^*}{c})^{\beta_1} + A_3 c \tag{29}$$

where

$$\begin{aligned}
A_1 &= (B - A_3 - Q)c^* - BK \\
A_3 &= \frac{(\delta(x,y) - \beta)/z}{r/z + \delta(x,y) + y - \mu} \\
c^* &= \frac{BK\beta_1}{(B - A3 - Q)(1 + \beta_1)} \\
Q &= Be^{(\mu - r - \delta(x,y) - y)T} \\
z &= 1 - e^{-rT} \\
\beta_1 &= \frac{1}{2} - \frac{\mu - \delta(x,y) - y}{\sigma^2} + \sqrt{(\frac{\mu - \delta(x,y) - y}{\sigma^2} - \frac{1}{2})^2 + \frac{2r/z}{\sigma^2}}
\end{aligned}$$

The constant B is again chosen according to whether there is access to technology or not up on default. $EuB(c,T)$ represents the European version of the same debt contract, and is in the finite maturity section for the fixed rate section above.

5.5.3 Some Discussions

1. Note that $c^*_{Fixed} \geq c^*_{Floating}$, meaning that in fixed rate specification, the borrower defaults earlier.

2. In the floating rate specification, default decision is independent of the interest repayment rate β. The intuition is that the lenders now bear all the risk with the borrowers.

3. The range of admissible equilibrium is greater under fixed rate specification than under floating rate specification. Given that the lenders know the default strategy of the borrowers, lenders do not lend as much regardless of the rate they charge since they know it does not affect the borrower's decision.

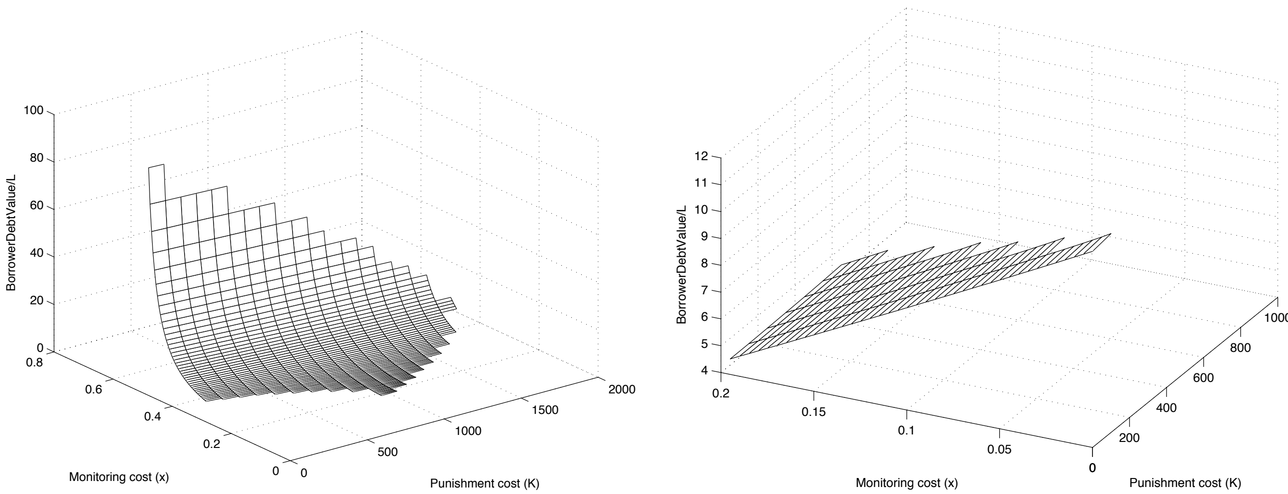

Figure 1: The left panel displays the borrower's value as a function of monitoring cost (x) and punishment cost (K) in the case of a perpetual loan. The right panel displays the borrower's value as a function of monitoring cost (x) and punishment cost (K) for finite maturity T = 3 months.

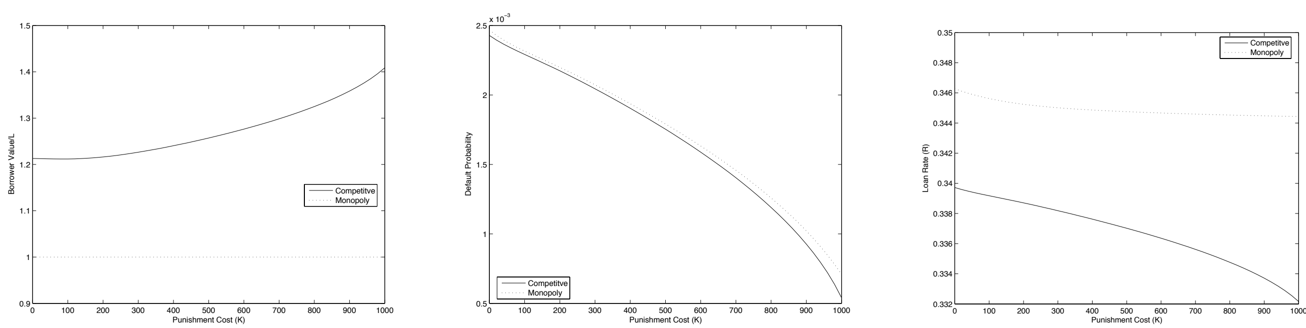

Figure 2: This set of figures analyze the effect of two market structures: monopolistic and competitive lending. The equilibrium behaviors as ex-post punishment cost (K) varies are displayed. The left panel displays the normalized borrower's value. The middle panel displays the default probability. The right panel displays the equilibrium loan rates.

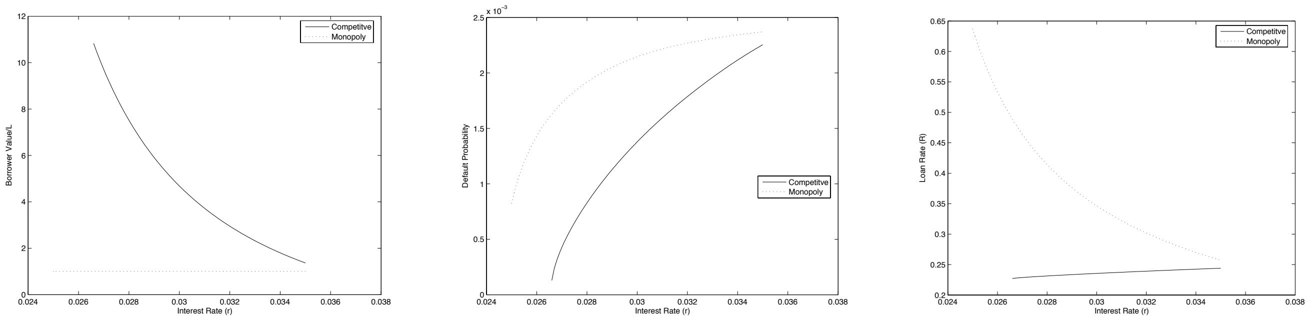

Figure 3: This set of figures analyze the effect of two market structures: monopolistic and competitive lending. The equilibrium behaviors as lender's cost of funding (r) varies are displayed. The left panel displays the normalized borrower's value. The middle panel displays the default probability. The right panel displays the equilibrium loan rates.

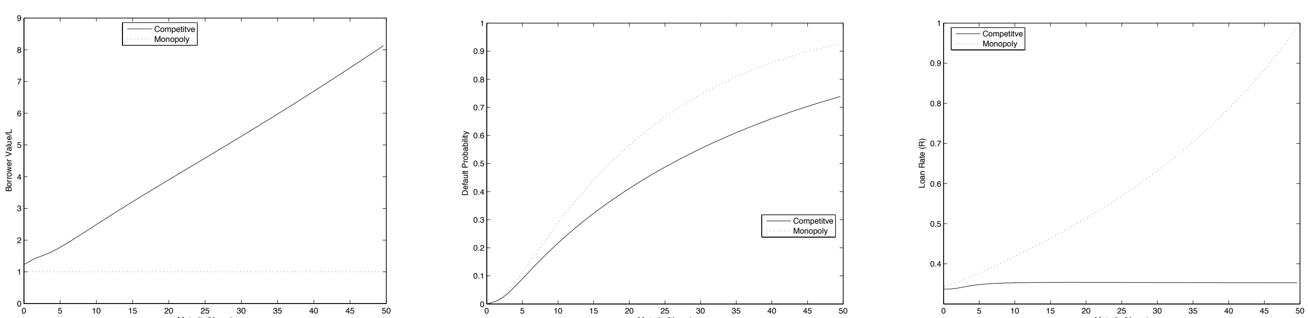

Figure 4: This set of figures analyze the effect of two market structures: monopolistic and competitive lending. The equilibrium behaviors as maturity (T) varies are displayed. The left panel displays the normalized borrower's value. The middle panel displays the default probability. The right panel displays the equilibrium loan rates.

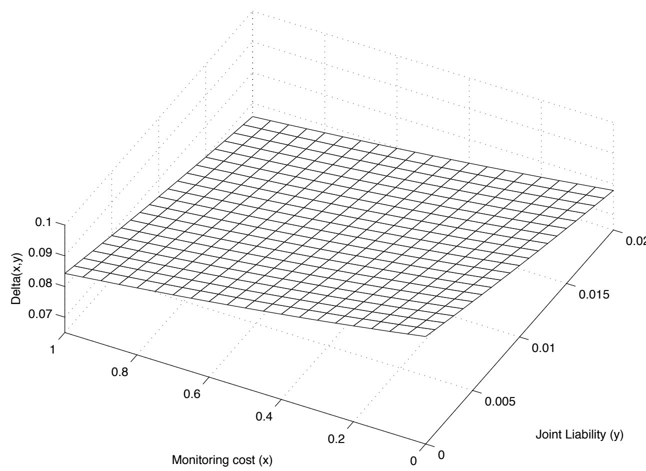

Figure 5: This illustrates our choice of the borrowers' consumption ratio, as captured by the $\delta(x, y)$ function, as a function of monitoring cost (x) and joint liability (y).

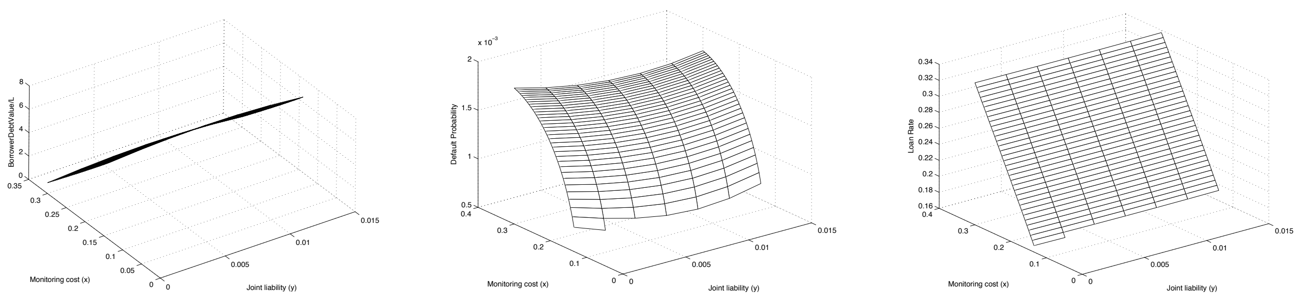

Figure 6: Equilibrium behavior as a function of administrative cost (x) and joint-liability (y) when there is access to technology upon default. The left panel displays the borrower's value normalized by the initial borrowing amount. The middle panel displays default probability. The right panel displays equilibrium loan rates.

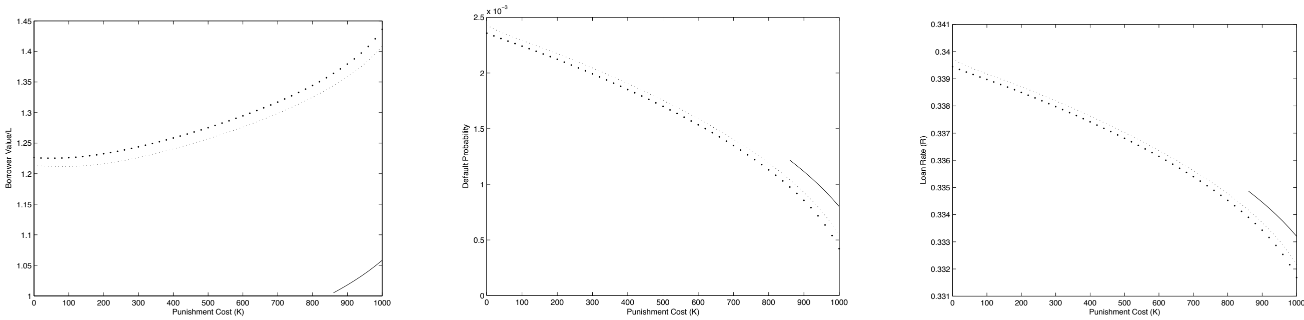

Figure 7: Equilibrium behavior as joint-liability (y) increases. The values analyzed are for joint-liability $y = 0\%$ (solid line), $y = 0.5\%$ (dashed line), and $y = 1\%$ (dotted line). Administrative cost is fixed at $x = 30\%$. The left panel displays the borrower's value normalized by the initial borrowing amount. The middle panel displays default probability. The right panel displays equilibrium loan rates.

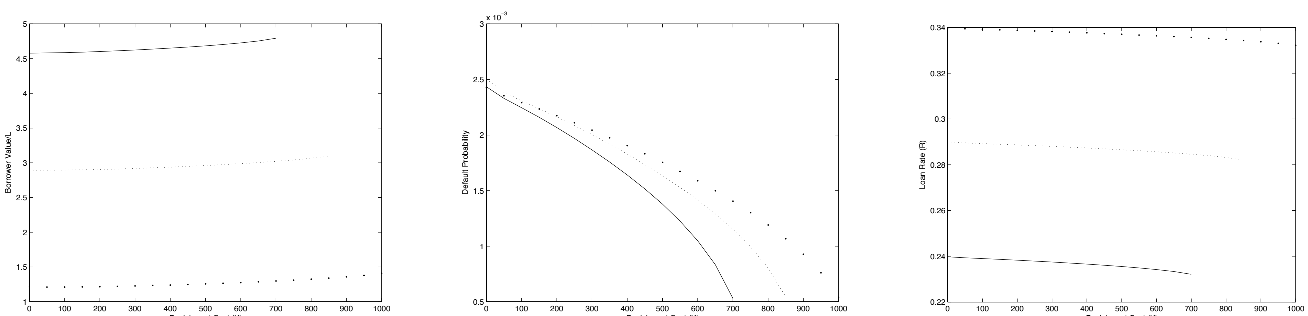

Figure 8: Equilibrium behavior as administrative cost (x) increases. The values examined are for administrative cost $x = 20\%$ (solid line), $x = 25\%$ (dashed line), and $x = 30\%$ (dotted line). Joint-liability is fixed at $y = 0.5\%$. The left panel displays the borrower's value normalized by the initial borrowing amount. The middle panel displays default probability. The right panel displays equilibrium loan rates.

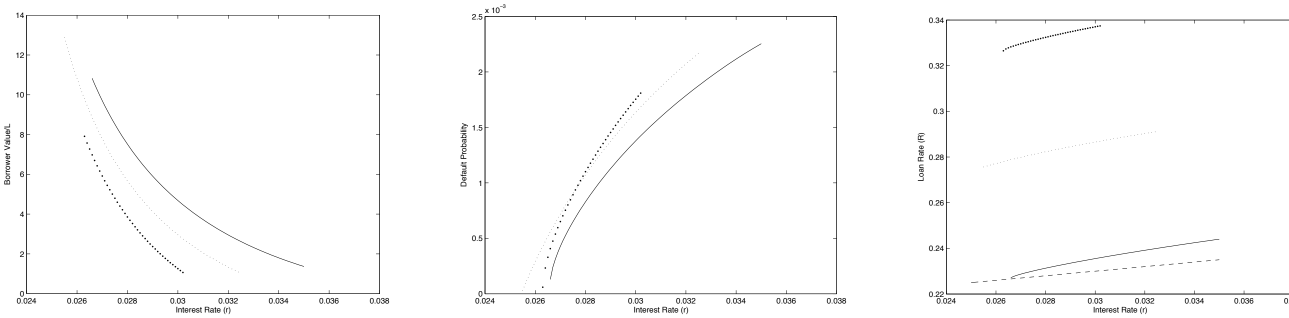

Figure 9: Equilibrium behavior as lender's cost of funding (r) increases. We fix adminsitrative costs to be x=20% (solid line), x=25% (dashed line), and x=30% (dotted line). The left panel displays the borrower's value normalized by the initial borrowing amount. The middle panel displays default probability. The right panel displays equilibrium loan rates.

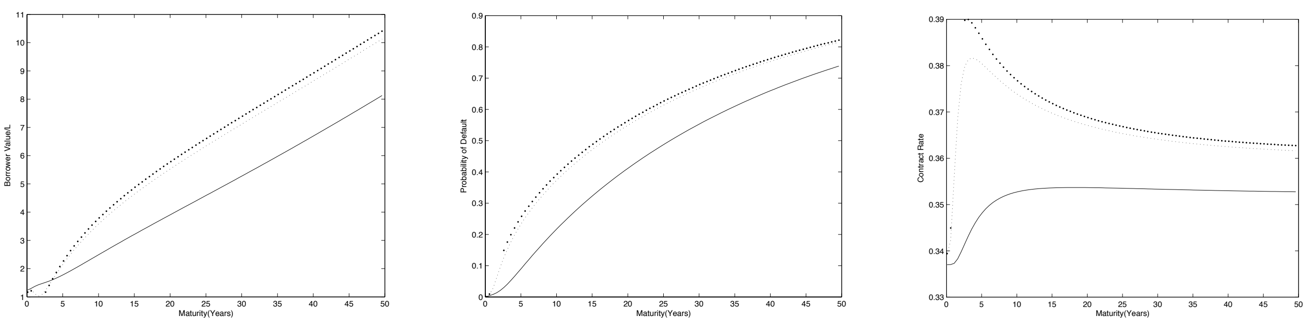

Figure 10: Equilibrium behavior as the maturity T increases. The monitoring cost examined are $x = 20\%$ (solid line), $x = 25\%$ (dashed line), and $x = 30\%$ (dotted line). We fix $y = 0.5\%$. The left panel displays the borrower's value normalized by the initial borrowing amount. The middle panel displays default probability. The right panel displays equilibrium loan rates.

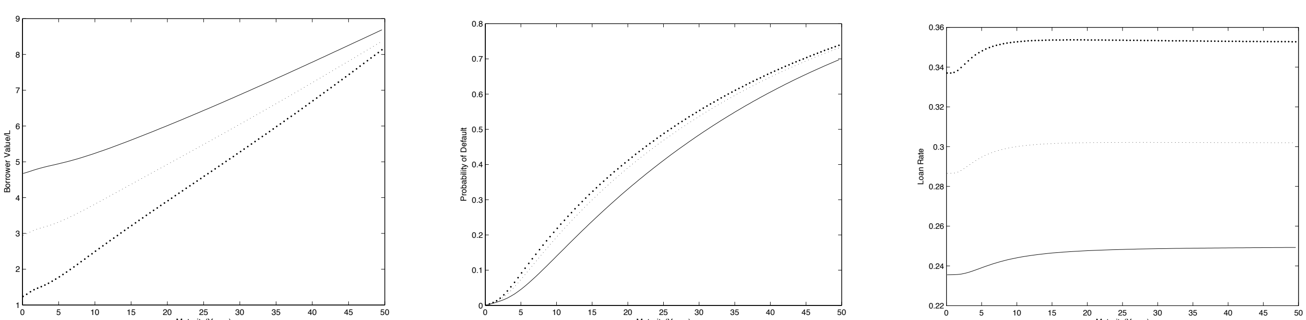

Figure 11: Equilibrium behavior as maturity T increases. We focus on x=20% (solid line), x=25% (dashed line), and x=30% (dotted line). We fix $y = 0.5\%$. The left panel displays the borrower's value normalized by the initial borrowing amount. The middle panel displays default probability. The right panel displays equilibrium loan rates.

REGION	Gross Loan Portfolio	Active Borrowers	Per Capita Loan Size	Loan Rates	Write-off Ratio	PAR $\geq$ 30 Days
	Size in US $	Number		Percent	Percent	Percent
Africa	1,010,088,380 (12%)	3,154,502 (11%)	320.21	39.84%	3.32%	9.15%
East Asia	1,983,635,418 (23%)	4,103,326 (15%)	483.42	39.10%	4.12%	7.83%
East Europe	1,449,653,047 (17%)	789,936 (3%)	1,835.15	28.86%	0.60%	1.78%
Latin America	2,740,536,803 (31%)	3,231,062 (11%)	848.18	35.82%	2.52%	6.62%
Middle East	208,032,901 (2%)	794,083 (3%)	261.98	34.61%	0.15%	2.60%
South Asia	1,327,858,980 (15%)	16,057,919 (57%)	82.69	20.29%	0.74%	10.44%
TOTAL:	8,719,805,529 (100%)	28,130,828 (100%)				
AVERAGE:				33.09%	1.91%	6.40%

Table 1: A decomposition of the Micro-Loan Market by regions. The data is collected using the MIX database for the year 2003. The numbers in the parentheses under Gross Loan Portfolio and Active Borrowers indicate their proportions in the total market. Per Capita Loan Size is reported in US dollars.

Lender	Gross Loan Portfolio	Active Borrowers	Per Capita Loan Size	Loan Rates	Write-off Ratio	PAR ≥ 30 Days
	Size in US $	Number		Percent	Percent	Percent
Banks (47)	4,502,900,041 (51.6%)	8,853,649 (31.5%)	509	24.39%	0.51%	9.61%
Cooperatives and Credit (98) Unions	701,205,569 (8.0%)	805,018 (2.9%)	871	30.92%	3.29%	8.58%
Non-bank financial (49) Institutions	1,627,919,395 (18.7%)	4,827,168 (17.2%)	132	37.80%	0.83%	4.71%
NGO (286)	1,509,926,890 (17.3%)	13,074,367 (46.5%)	115	35.75%	2.48%	7.52%
Others (32)	346,857,110 (4.0%)	425,679 (1.5%)	815	37.54%	1.51%	4.26%
Rural bank (15)	30,996,524 (0.4%)	144,947 (0.5%)	214			7.58%
TOTAL:	8,719,805,529 (100%)	28,130,828 (100%)				
AVERAGE:				33.28%	1.72%	7.04%

Table 2: A decomposition of the Micro-Loan Market by lenders. The data is collected using the MIX database for the year 2003. The numbers in the parentheses under the column labeled Lender is the number of voluntary reporting institutions for each category of lenders. The numbers the parentheses under Gross Loan Portfolio and Active Borrowers indicate their proportions in the total market. Per Capita Loan Size is reported in US dollars.