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Statistical testing of attribute significance is not possible in conjoint studies that use nonmetric algorithms to analyze respondent ranks of multiattribute product profiles. Procedures that test attribute significance at an aggregate (e.g., segment) level but maintain individual differences in preferences can be used (1) to confirm differences among benefits sought by hypothesized segments of respondents, (2) to eliminate insignificant attributes, reducing the time and cost of conjoint choice simulations, and (3) to design subsequent conjoint studies for the same product class. The author presents two tests of attribute significance in conjoint analysis. One is appropriate when consumer preferences for attribute levels can be ordered *a priori* and the other can be used when such ordering is not permissible. Each test permits different levels of an attribute to appear in different numbers of product profiles. The proposed tests assess attribute significance across multiple respondents with idiosyncratic preferences. Because they use rank order data, the testing procedures are not limited to a specific scaling algorithm. A Monté Carlo simulation indicates that eliminating insignificant attributes does not affect share-of-choices predictions for new product concepts if the number of insignificant attributes is not very large. Otherwise, the usual tradeoff between parsimony and predictive accuracy is necessary.

Assessing Attribute Significance in Conjoint Analysis: Nonparametric Tests and Empirical Validation

Several developments have occurred in conjoint analysis since its introduction to marketing in the early 1970s (Green and Rao 1971; Johnson 1974). Among other approaches, choice-set experiments (Louviere and Woodworth 1983) and choice-set explosion of rank order data (Chapman and Staelin 1982) have been proposed. Un-

like traditional conjoint analysis, these methods employ the multinomial logit model to estimate utility function parameters at the group level and therefore permit statistical testing of attribute significance. Statistical testing of parameter estimates is also possible in Functional Measurement (Anderson 1981, 1982; Lynch 1985), which uses profile ratings as an interval-scaled dependent variable in OLS regression. A related development in the context of von Neumann-Morgenstern utility theory is described by Eliashberg and Hauser (1985). They propose an error theory that permits statistical testing of risk parameters in idiosyncratic, multiattribute utility functions for constant proportional risk aversion and constant absolute risk aversion.

Statistical testing methods have not been developed for the traditional nonmetric approaches to conjoint analysis, perhaps for two reasons. First, nonmetric scaling algorithms such as LINMAP (Shocker and Srinivasan

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1977), MONANOVA (Kruskal 1965), and PREFMAP (Carroll 1972) employ goodness-of-fit measures not related to an error theory. As a result, they provide no theoretical basis for testing attribute significance. OLS regression also is used to estimate utility function parameters with preference rank serving as the dependent variable. The parameter estimates obtained from this analysis are known to be robust to violations of the assumption of an interval-scaled dependent measure (Carmone, Green, and Jain 1978; Wittink and Cattin 1981). However, studies that estimate parameters in this way do not go so far as to test the significance of the attributes used, because it is not known whether the normal-theory-based tests are robust when ordinal responses are used to parameterize a regression model.

Second, because the major application of conjoint analysis is in product design settings, it is useful to assess attribute significance at a segment level rather than separately for each individual. However, it is not desirable to base the testing on "average" segment preferences, which ignore the information on preference heterogeneity *within* a segment. The difficulty of developing testing procedures that retain idiosyncrasies in preferences, yet test for attribute significance across consumers, has been the second major impediment in the development of significance testing methods for conjoint analysis.

Identifying which attributes in a conjoint study are significant and which are insignificant is important for at least three reasons. First, a nonmetric scaling of consumer preferences often is followed by a clustering of respondents in terms of part-worths similarities. Unlike Functional Measurement and choice-set experiments, statistical methods are not available for testing whether two segments differ in terms of the benefits they seek from a product. A statistical procedure identifying the significant attributes for each segment can be useful for validating hypothesized differences in the benefits sought by consumer segments: two segments seek different benefits if they have different, possibly overlapping, sets of significant attributes. Of course, identifying the same set of significant attributes need not necessarily imply that two hypothesized benefit segments are not distinct, because preferences for the *levels* of a significant attribute (e.g., the sizes of cars) also can differ across segments.

Second, insignificant attributes can be eliminated from simulations of new product performance if they have an insignificant effect on measures used in conjoint simulators (e.g., share of choices). Alternatively, a user may want to base the simulation not merely on prediction, but also on an understanding of which attributes significantly affect preferences. In this case, an insignificant attribute can be eliminated even if it has a significant effect on the predictive accuracy of a simulation model. Either way, reducing the number of attributes used can reduce the time and cost of the simulation—an important practical benefit when the simulation is performed to identify an "optimal" product concept (e.g., using the

QUALIN program of the POSSE method developed by Green et al. 1981). In contrast to conjoint choice experiments that derive a single, closed-form model to summarize an aggregate choice response surface for a set of product concepts (Louviere and Woodworth 1983, p. 360–1), explicit simulation methods are computationally demanding for problems of large size (Kohli and Krishnamurti 1987). For example, a simulation of all possible product concepts for a problem involving 200 respondents and eight attributes, each at five levels, takes approximately one hour on an FPS computer (which runs approximately 10 times faster than a DEC-10 computer and approximately twice as fast as a VAX-8600 computer). If two attributes are eliminated, the simulation time is reduced to approximately two minutes on an FPS computer. This computational benefit is enhanced if the simulation is repeated to validate predictions of product-concept performance (e.g., by perturbing individual utility functions or by comparing the performance of product concepts for randomly selected subsets of respondents) and to study the effect of competitive actions and reactions when a new product is introduced.[1]

Finally, knowledge of the significant attributes for a relevant population can guide data collection in subsequent conjoint studies for the same product class if the results are generalizable over time. Because insignificant attributes can be eliminated, one can include previously ignored but possibly important attributes. Alternatively, more reliable data can be collected by describing product profiles in terms of only the significant attributes, reducing the possibility of information overload on respondents (Green and Srinivasan 1978). The reliability of individual evaluations can be improved further if eliminating insignificant attributes permits the use of an experimental design with fewer treatments, so that each respondent also evaluates fewer product profiles.

Two procedures for testing attribute significance in conjoint analysis are presented here. Because they use only the preference ranks, the testing procedures are not allied with, and hence not limited to, a specific scaling algorithm. Both tests employ ordinal preferences of multiple respondents with heterogeneous preferences. No constraints are placed on the number of times an attribute level appears in product profiles. This feature is an important consideration in the present instance, because in collecting conjoint data one often uses experimental plans in which different levels of an attribute appear in different numbers of product profiles. It is also worth noting that though the proposed testing procedures are developed in the context of conjoint analysis, they can be used more generally to assess factor significance in any multifactor experiment in which n (≥ 1) observations are

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[1]Computational time is not a significant issue if the simulation is restricted to a few product profiles preselected by the user, or if a heuristic is employed to identify near-optimal product profiles (e.g., Kohli and Krishnamurti 1987).

obtained for each treatment condition and the response variable is measured on at least an ordinal scale.

The proposed procedures are applicable when a part-worths function is used to model individual preferences and when data are collected according to the full-profile approach (Green and Srinivasan 1978). Briefly, the part-worths model is applicable when each attribute is described at a finite number of levels. The full-profile method of data collection employs these attributes as factors in an experimental design. Treatments of the design describe multiattribute product profiles, which respondents rank in preference order.

The tests differ in terms of whether or not preferences for an attribute's levels can be ordered *a priori* for all respondents. For example, a preference ordering can be specified *a priori* for the attribute "cholesterol content," because higher cholesterol content is not expected to be preferred to lower cholesterol content by any respondent. The more frequently a respondent's preferences violate the ordering condition, the less likely it becomes that the attribute is significant. Thus when a preference ordering of attribute levels is feasible, a test of attribute significance should consider how frequently respondents' preferences violate the ordering condition.[2]

Such *a priori* ordering is not permissible if the levels of an attribute are only nominally comparable (e.g., alternative package designs). It is also not permissible if the underlying attribute is continuous and the sampled levels belong to a range in which an individual's utility function can be increasing, decreasing, or U-shaped/inverted U-shaped (e.g., levels of "sweetness" of a dessert). The significance of these attributes must be assessed via procedures that do not require information on individuals' attribute-level preference orderings.

The following section first describes the test when *a priori* ordering of attribute levels is not permissible and then the test when such ordering is permitted. The first test is related closely to Kruskal and Wallis' (1952) test for identical distribution functions. The second test is related closely to Friedman's (1937) test for random blocks. However, neither test is related directly to any measures of product performance used in conjoint choice simulators. A Monté Carlo simulation therefore is performed to test the effect of eliminating insignificant attributes on share of choices, a frequently used measure of product performance in conjoint choice simulators (Cattin and Wittink 1982).

PROCEDURES FOR TESTING ATTRIBUTE SIGNIFICANCE

Testing Attribute Significance Without Constraining Preferences for Attribute Levels

We begin by specifying notation. Let I denote the number of respondents in a segment. Let n denote the

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[2]Using an *a priori* ordering of the levels in *estimating* individual utility functions is described by Srinivasan, Jain, and Malhotra (1983).

number of product profiles evaluated by each respondent. Consider an attribute with m levels, level j appearing in n_j product profiles. Let x_{jk}, $k = 1, 2, \ldots, n_j$, denote the n_j product profiles in which level j of the attribute appears. For example, consider an attribute with two levels ($m = 2$). Let each level ($j = 1, 2$) appear in four product profiles ($n_1 = n_2 = 4$; $n = 8$). Then x_{11}, x_{12}, x_{13}, x_{14} denote the four product profiles in which level 1 appears and x_{21}, x_{22}, x_{23}, x_{24} denote the four product profiles in which level 2 appears.

Let "1" denote the rank of an individual's most-preferred product profile and let "n" denote the rank of an individual's least-preferred product profile. Let $r_i(x_{jk})$ denote the rank individual i associates with product profile x_{jk}. Let r_{ij} denote the sum of individual i's ranks for the n_j product profiles in which level j appears; that is,

$$(1) \qquad r_{ij} = \sum_{k=1}^{n_j} r_i(x_{jk}).$$

Assume that respondent i's preferences do not differ across the m attribute levels. Then a random association should occur between the attribute's levels and the rank ordering of the product profiles. Thus, under a random-association hypothesis, each product profile is assigned any one of the n ranks with probability $1/n$. The associated expected rank of product profile x_{jk} is

$$(2) \qquad E\{r_i(x_{jk})\} = \sum_{j=1}^{m} \sum_{k=1}^{n_j} (1/n) r_i(x_{jk})$$

$$= (1/n) \sum_{j=1}^{m} \sum_{k=1}^{n_j} r_i(x_{jk}).$$

Because individual i assigns a unique rank to each product profile, the sum of ranks across product profiles is

$$(3) \qquad \sum_{j=1}^{m} \sum_{k=1}^{n_j} r_i(x_{jk}) = (1 + 2 + \ldots + n)$$

$$= n(n + 1) \div 2.$$

Substituting equation 3 into the right side of equation 2 yields

$$(4) \qquad E\{r_i(x_{jk})\} = (n + 1) \div 2.$$

The variance of $r_i(x_{jk})$ is

$$(5) \quad \text{var}\{r_i(x_{jk})\} = E\{r_i(x_{jk})\}^2 - [E\{r_i(x_{jk})\}]^2$$

$$= E\{r_i(x_{jk})\}^2 - \{(n + 1) \div 2\}^2$$

$$= \left[\sum_{j=1}^{m} \sum_{k=1}^{n_j} (1/n)\{r_i(x_{jk})\}^2 \right] - \{(n + 1)^2 \div 4\}$$

$$= \{(n + 1)(2n + 1) \div 6\} - \{(n + 1)^2 \div 4\}$$

$$= (n + 1)(n - 1) \div 12$$

where:

(6) $\displaystyle\sum_{j=1}^{m}\sum_{k=1}^{n_j} (1/n)\{r_i(x_{jk})\}^2 = (1/n)(1^2 + 2^2 + \ldots + n^2)$

$$= (n + 1)(2n + 1) \div 6.$$

The covariance of the ranks associated with two distinct product profiles x_{jk} and x'_{jk} is

(7) $\quad \text{cov}\{r_i(x_{jk}), r_i(x'_{jk})\} = E[\{r_i(x_{jk}) - E(r_i(x_{jk}))\}$

$$\cdot \{r_i(x'_{jk}) - E(r_i(x'_{jk}))\}].$$

Substituting $E\{r_i(x_{jk})\} = (n + 1) \div 2$ from equation 4 into equation 7 yields

(8) $\quad \text{cov}\{r_i(x_{jk}), r_i(x'_{jk})\} = \displaystyle\sum_{t=1}^{n}\sum_{\substack{s=1 \\ t \neq s}}^{n} \left\{ t - \frac{(n + 1)}{2} \right\}$

$$\cdot \left\{ s - \frac{(n + 1)}{2} \right\} \left\{ \frac{1}{n(n - 1)} \right\}$$

where $1/n(n - 1)$ is the joint probability that product profiles x_{jk} and x'_{jk} are assigned ranks $r_i(x_{jk})$ and $r_i(x'_{jk})$, respectively. The summation in equation 8 extends over all t and s from 1 to n, except that t does not equal s because profiles x_{jk} and x'_{jk} cannot have the same rank. Rewrite equation 8 by adding and subtracting the terms for $t = s$:

(9) $\quad \text{cov}\{r_i(x_{jk}), r_i(x'_{jk})\} = \left[\displaystyle\sum_{t=1}^{n}\sum_{s=1}^{n} \left\{ t - \frac{(n + 1)}{2} \right\} \right.$

$$\cdot \left\{ s - \frac{(n + 1)}{2} \right\} \left\{ \frac{1}{n(n - 1)} \right\} \Bigg]$$

$$- \left\{ t - \frac{(n + 1)}{2} \right\}^2 \left\{ \frac{1}{n(n - 1)} \right\}$$

$$= \frac{1}{n(n - 1)} \left[\displaystyle\sum_{t=1}^{n} \left\{ t - \frac{(n + 1)}{2} \right\} \right]$$

$$\cdot \left[\displaystyle\sum_{s=1}^{n} \left\{ s - \frac{(n + 1)}{2} \right\} \right]$$

$$- \frac{1}{(n - 1)}$$

$$\cdot \displaystyle\sum_{t=1}^{n} \left\{ t - \frac{(n + 1)}{2} \right\}^2 \{1/n\}.$$

To simplify equation 9, note that

(10) $\quad \displaystyle\sum_{t=1}^{n} \left\{ t - \frac{(n + 1)}{2} \right\} = \displaystyle\sum_{t=1}^{n} t - \displaystyle\sum_{t=1}^{n} \frac{(n + 1)}{2}$

$$= \frac{n(n + 1)}{2} - \frac{n(n + 1)}{2} = 0.$$

Therefore the first term in equation 9 equals zero. Also,

$$\displaystyle\sum_{t=1}^{n} \left\{ t - \frac{(n + 1)}{2} \right\}^2 (1/n)$$

is the expression for the variance of $r_i(x_{jk})$, which from equation 5 equals $(n + 1)(n - 1) \div 12$. Hence expression 10 simplifies to

(11) $\quad \text{cov}\{r_i(x_{jk}), r_i(x'_{jk})\} = 0 - \dfrac{1}{(n - 1)}$

$$\cdot \{(n + 1)(n - 1) \div 12\}$$

$$= -(n + 1) \div 12.$$

Under the null hypothesis, r_{ij} is the sum of n_j ranks selected at random and without replacement from the ranks 1 to n. The associated mean of r_{ij} is

(12) $\quad E(r_{ij}) = E\left\{ \displaystyle\sum_{k=1}^{n_j} r_i(x_{jk}) \right\} = \displaystyle\sum_{k=1}^{n_j} E\{r_i(x_{jk})\}.$

Substituting for $E\{r_i(x_{jk})\}$ in equation 12 from equation 4 yields

(13) $\quad E(r_{ij}) = \displaystyle\sum_{k=1}^{n_j} (n + 1) \div 2 = n_j(n + 1) \div 2.$

The variance of r_{ij} is

(14) $\quad \text{var}(r_{ij}) = \displaystyle\sum_{k=1}^{n_j} \text{var}\left\{ r_i(x_{jk}) \right\} + \displaystyle\sum_{\substack{x_{jk}, x_{jk} \\ x_{jk} \neq x_{jk}}} \text{cov}\{r_i(x_{jk}), r_i(x'_{jk})\}$

where the summation over the covariance term extends over all *distinct* pairs of product profiles x_{jk}. The various terms in equation 14 are given by equations 5 and 11. The variance term appears n_j times and the covariance term appears $n_j(n_j - 1)$ times in equation 14. Hence

(15) $\quad \text{var}(r_{ij}) = \{n_j(n + 1)(n - 1) \div 12\}$

$$+ [\{n_j(n_j - 1)\}$$

$$\cdot \{ -(n + 1) \div 12\}]$$

$$= n_j(n + 1)(n - n_j) \div 12.$$

Now r_{ij} is the sum of n_j random variables. Hence the Wald-Wolfowitz theorem[3] (Noether 1967) implies that as n_j increases, the distribution of r_{ij} asymptotically approaches the normal distribution (the properties of the proposed test statistic for small n_j are discussed subsequently). Thus when there is no association between the attribute levels and individual i's rank ordering of product profiles, the *standardized* r_{ij} have an asymptotic normal distribution with zero mean and unit variance; that is, the statistic

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[3]The Wald-Wolfowitz theorem states that the distribution of the sum of n independent, identically distributed random *ranks* asymptotically approaches the normal distribution. It is the nonparametric equivalent of the central limit theorem. For a proof, see Fraser (1957).

(16) $\{r_{ij} - E(r_{ij})\} \div \sqrt{\mathrm{var}(r_{ij})}$

asymptotically approaches the standard normal distribution with increasing n_j. The square of a random variable with a standard normal distribution has a chi square distribution with 1 d.f. (Searle 1971, p. 47). Consequently, the square of statistic 16,

(17) $c_{ij} = \{r_{ij} - E(r_{ij})\}^2 \div \mathrm{var}(r_{ij})$,

has an asymptotic chi square distribution with 1 d.f. Also, the sum of m independent chi square random variables, each with 1 d.f., is chi square distributed with m degrees of freedom (Rao 1973, p. 166). Therefore if the c_{ij} ($j = 1, 2, \ldots, m$) were independent statistics, the sum of the c_{ij}

(18) $c_i' = \sum_{j=1}^{m} c_{ij}$

would have an asymptotic chi square distribution with m degrees of freedom. However, the c_{ij}'s are *not* independent because the sum of the r_{ij}'s is constant ($= n(n + 1) \div 2$). Kruskal (1952) has shown that if the c_{ij} are multiplied by $(n - n_j) \div n$, the resulting statistic

(19) $c_i = \sum_{j=1}^{m} \{(n - n_j) \div n\} c_{ij}$

$= \dfrac{12}{n(n + 1)} \sum_{j=1}^{m} \{r_{ij} - (1/2)\,n_j(n + 1)\}^2 \div n_j$

has a limiting chi square distribution with $(m - 1)$ degrees of freedom. The limiting chi square distribution is approached as all $n_j \to \infty$ simultaneously. Expression 19 is the Kruskal and Wallis (1952) statistic for testing the identity of m population distribution functions.

The preceding results are valid for large values of n and n_j. However, these values are typically small in conjoint analysis. It is therefore important to examine how well the chi square distribution approximates the exact distribution of c_i when n and n_j are small. At least four studies have examined the small-sample properties of expression 19 using Monté Carlo simulation. Kruskal and Wallis (1952) found that for small values of n_j and α levels less than .10, the chi square approximation furnishes a conservative test, the true significance level of c_i being smaller than the level of significance indicated by the chi square approximation. McSweeney and Penfield (1969) analyzed data from normal and uniform distributions. They employed three factor levels (attribute levels) and 5, 6, 8, 10, and 12 observations (product profiles) per level in a Monté Carlo simulation. The Kruskal-Wallis test was conservative in terms of type 1 error. Also, the goodness of fit of the chi square distribution was not impaired by using small sizes or by sampling from a uniform distribution rather than a normal distribution. Feir-Walsh and Toothaker (1974) studied the behavior of the Kruskal-Wallis statistic using data from a normal distribution and two exponential distri-

butions for $n = 28$. The Kruskal-Wallis test gave a good approximation to the type 1 error for the normal data. It was conservative (i.e., indicated a higher α value) for data from both exponential distributions. Gabriel and Lachenbruch (1969) also found the chi square approximation to be good for small values of n and n_j. The results of these studies therefore suggest that the chi square approximation of c_i should be appropriate for conjoint analysis.

Observe that c_i is computed with only the preference data for individual i. Hence c_i can be used to test attribute significance at the individual level. If all attributes are insignificant for an individual, one can conclude that the preference ranking of product profiles is random and exclude the individual from subsequent analysis.

Because preference data are collected from multiple respondents in conjoint analysis, it is also useful to assess the significance of an attribute *across* respondents. To this end, a test based on the statistic

(20) $c = \sum_{i=1}^{I} c_i$,

where I is the number of respondents, is proposed. Because c is the sum of I asymptotically chi square random variables, its distribution is also asymptotically chi square (Rao 1973, p. 166). Further, because each c_i has ($m - 1$) degrees of freedom, c has $I(m - 1)$ degrees of freedom.

Note that different individual preferences result in different values of r_{ij}, and hence the c_{ij}. However, c_i is based on the *square* of the difference between the observed and expected sum of ranks. Therefore summing the c_i in equation 20 does not cause individually significant attributes to appear insignificant as a result of the aggregation of heterogeneous preferences. In other words, idiosyncratic preferences are not "averaged out" in the test. Rather, preferences are aggregated in such a way that an attribute's significance depends on the extent to which *each respondent's* preference ranking deviates from a random ranking of the product profiles. Consequently, the test based on equation 20 permits segment-level testing of attribute significance without assuming that preferences for all respondents in a segment are identical.

Power of test. The most common index for comparing nonparametric tests with parametric tests is asymptotic relative efficiency (ARE). This index compares the power of one test with the power of the other by using mathematical computations based on extremely large sample sizes and extremely small location differences. In fact, sample size is permitted to approach infinity while at the same time location differences approach zero.

The Kruskal-Wallis test in comparison with the parametric (ANOVA) F-test has an ARE of .95 for the normal distribution and a lower bound ARE of .864 (Noether 1967, p. 89). Thus, asymptotically, the test based on equation 19 is 95% as powerful as the F-test when the "true" part worths are from a normal distribution and it

can never be less than 86% as powerful when the "true" part worths are from non-normal distributions.

Because ARE is computed for unrealistically large sample sizes with miniscule differences in measures of location, it is important to know the small-sample performance of the Kruskal-Wallis test. To our knowledge, only Feir-Walsh and Toothaker (1974) have investigated the empirical small-sample power of the test. Using Monté Carlo simulation, they compared the power of the Kruskal-Wallis test with the power of the parametric F-test for different sample sizes. The data used were generated from a normal distribution and from two exponential distributions. For small sample sizes and data from the normal distribution, the power of the Kruskal-Wallis test was at most 5% less than the power of the F-test. For the two exponential distributions, the power of the Kruskal-Wallis test was always higher than the power of the F-test. These limited results suggest that in comparison with the ANOVA F-test, the proposed test should have adequate power for applications to conjoint analysis.

Testing Attribute Significance by Constraining Preferences for Attribute Levels

The preceding test uses no information about a possible *a priori* ordering of individual preferences for an attribute's levels. If available, such information should be incorporated in a test of attribute significance. A procedure appropriate in this instance is described next. It is based on an extension of Friedman's (1937) rank test for random blocks.

As before, let $r_i(x_{jk})$ denote individual i's ranking of product profile x_{jk}. Let r_{ij} denote the sum of the ranks individual i associates with the n_j product profiles in which level j appears. Let

$$(21) \qquad r_j = \sum_{i=1}^{I} r_{ij}$$

where I is again the number of individuals across whom the significance of the attribute is being assessed. The proposed test is based on r_j because all individuals have the same *a priori* preference ordering and can differ only in the *degree* to which they prefer one attribute level to another. In an experimental design context, respondents are like "blocks" across whom the significance of an attribute is tested.

Under the hypothesis of no association between the attribute levels and the rank order preferences, the $r_i(x_{jk})$ are randomly selected integers between 1 and $n = \sum_{j=1}^{m} n_j$. Therefore

$$(22) \quad E(r_j) = E\left\{\sum_{i=1}^{I} r_{ij}\right\} = I E(r_{ij}) = I n_j(n + 1) \div 2$$

and

$$(23) \qquad \mathrm{var}(r_j) = \sum_{i=1}^{I} \mathrm{var}(r_{ij})$$

$$= I n_j(n + 1)(n - n_j) \div 12$$

where $E(r_{ij}) = n_j(n + 1) \div 2$ and $\mathrm{var}(r_{ij}) = n_j(n + 1)(n - n_j) \div 12$ are derived in expressions 13 and 15, respectively.

Now r_j is the sum of In_j random variables. Hence the Wald-Wolfowitz theorem (see footnote 3) implies that r_j has an asymptotic normal distribution with mean and variance given by equations 22 and 23, respectively. The asymptotic normal distribution is attained for increasing In_j. In commercial conjoint studies, In_j is large because of the large sample sizes employed. It follows that the standardized r_j has an asymptotic normal distribution with zero mean and unit variance; that is, the statistic

$$(24) \qquad h_j = \{r_j - E(r_j)\} \div \sqrt{\mathrm{var}(r_j)}$$

has an asymptotic standard normal distribution. If the r_j, and hence h_j, were independent observations from a standard normal distribution, h_j^2 would have a chi square distribution with 1 d.f. (Searle 1971, p. 47). Further,

$$(25) \qquad h' = \sum_{j=1}^{m} h_j^2$$

would be the sum of m independent chi square random variables and therefore would also have a chi square distribution with m degrees of freedom (Rao 1973, p. 166). However, the r_j are *not* independent because their sum is constant $(= In(n + 1) \div 2)$. Wilks (in Friedman's 1937 paper) has shown that the statistic

$$(26) \quad h = \sum_{j=1}^{m} h_j^2 (m - 1) \div m$$

$$= \frac{12(m - 1)}{Im(n - 1)} \sum_{j=1}^{m} [\{r_j - (1/2) In_j(n + 1)\}^2$$

$$\div n_j(n - n_j)]$$

is corrected for this dependence and that it is asymptotically chi square distributed with $m - 1$ degrees of freedom. If $n_j = 1$ for all $j = 1, 2, \ldots, m$, the preceding test is identical with Friedman's test for random blocks. If $n_j = n \div m$ for all $j = 1, 2, \ldots, m$ (i.e., each attribute appears in an equal number of product profiles), the test reduces to a generalization of Friedman's test proposed by Conover (1980, p. 307).

Power of test. Noether (1967, p. 90) discusses the asymptotic relative efficiency (ARE) of the Friedman test in comparison with that of the parametric (ANOVA) F-test. The ARE for the Friedman test depends on m, the number of levels of the attribute. It equals $(.955)$ $m/(m + 1)$ if the "true" part worths are from a normal distribution, $m/(m + 1)$ if the "true" part worths are from a uniform distribution, and $3m/2(m + 1)$ if the "true" part worths are from a double exponential distribution. The ARE of the Friedman test cannot fall below $(.864)m/(m + 1)$. The empirical power of the Friedman test has not, to our knowledge, been studied. However, the number of individuals (I), and hence the value of In_j, is large in conjoint analysis. Therefore, in the present context, the

ARE results are more relevant than small-sample power of the Friedman test.

AN EXAMPLE

The preceding tests are illustrated by a small hypothetical example involving two attributes, each at two levels (i.e., $m = 2$ for both attributes). Four product profiles are specified by combining each level of each attribute. Table 1 displays the rank order preferences of each of two respondents ($I = 2$) over the four product profiles.

Assume that no *a priori* preference ordering is specified for the levels of attribute 1. We should expect attribute 1 to be significant, because both individuals always prefer the two product profiles in which one level appears (i.e., level 2 for individual 1, level 1 for individual 2) to the two product profiles in which the other level appears. Now assume that an *a priori* ordering of the levels is specified for attribute 1. Attribute 1 should not be significant because individual 1's preference ranks are as strongly in agreement with the *a priori* ordering as individual 2's preference ranks are in disagreement with the *a priori* ordering.

For attribute 2, the sum of the ranks is the same (= 5) for each level and for both individuals. This fact suggests that attribute 2 should be insignificant either at the individual level or across individuals, regardless of the *a priori* ordering of the attribute levels.

Testing Significance With No A Priori Preference Ordering

The significance of attribute 1 is tested for each respondent separately, then across respondents. The values of r_{ij} (i.e., the sum of the ranks for product profiles in which level j of attribute 1 appears) are

$$(27) \qquad r_{11} = 1 + 2 = 3$$

$$(28) \qquad r_{12} = 4 + 3 = 7$$

for individual 1 and

$$(29) \qquad r_{21} = 4 + 3 = 7$$

$$(30) \qquad r_{22} = 1 + 2 = 3$$

Table 1
DATA FOR HYPOTHETICAL EXAMPLE ILLUSTRATING THE PROPOSED TESTING PROCEDURES

	Attribute		Profile Ranking	
	1	2		
Profile	Level	Level	Individual 1	Individual 2
1	1	1	1	4
2	1	2	2	3
3	2	1	4	1
4	2	2	3	2

for individual 2. Computing c_i from equation 19 yields

$$(31) \qquad c_i = \frac{12}{n(n+1)} \sum_{j=1}^{2} \{r_{1j} - (1/2)n_j\,(n+1)\}^2 \div n_j$$

$$= \frac{12}{4(5)} [\{3 - (1/2)(2)(5)\}^2 \div 2$$

$$+ \{7 - (1/2)(2)(5)\}^2 \div 2]$$

$$= 2.40$$

and

$$(32) \qquad c_2 = \frac{12}{n(n+1)} \sum_{j=1}^{2} \{r_{2j} - (1/2)n_j(n+1)\}^2 \div n_j$$

$$= \frac{12}{4(5)} [\{7 - (1/2)(2)(5)\}^2 \div 2$$

$$+ \{3 - (1/2)(2)(5)\}^2 \div 2]$$

$$= 2.40.$$

It follows that

$$(33) \qquad c = c_1 + c_2 = 9.60.$$

Both c_1 and c_2 are chi square distributed with $(m - 1)$ = $(2 - 1)$ = 1 d.f. Also, $c = c_1 + c_2$ is chi square distributed with 2 d.f. Hence, attribute 1 is insignificant at the individual level and is marginally significant ($p <$ .10) *across* individuals.

The significance of attribute 2 is assessed similarly. The r_{ij} (i.e., the sum of the ranks for product profiles in which level j of attribute 2 appears) are

$$(34) \qquad r_{11} = 1 + 4 = 5$$

$$(35) \qquad r_{12} = 2 + 3 = 5$$

for individual 1 and

$$(36) \qquad r_{21} = 4 + 1 = 5$$

$$(37) \qquad r_{22} = 3 + 2 = 5$$

for individual 2. Computing c_i from equation 19 for each respondent yields

$$(38) \qquad c_1 = \frac{12}{n(n+1)} \sum_{j=1}^{2} \{r_{1j} - (1/2)n_j(n+1)\}^2 \div n_j$$

$$= \frac{12}{4(5)} [\{5 - (1/2)(2)(5)\}^2 \div 2$$

$$+ \{5 - (1/2)(2)(5)\}^2 \div 2]$$

$$= 0$$

and

$$(39) \quad c_2 = \frac{12}{n(n+1)} \sum_{j=1}^{2} \{r_{2j} - (1/2)n_j(n+1)\}^2 \div n_j$$

$$= \frac{12}{4(5)} [\{5 - (1/2)(2)(5)\}^2 \div 2$$

$$+ \{5 - (1/2)(2)(5)\}^2 \div 2]$$

$$= 0.$$

It follows that

$$(40) \quad c = c_1 + c_2 = 0.$$

Thus, for each respondent, the chi square statistic has a zero value. These values indicate that attribute 2 is insignificant for each respondent and also across respondents.

Testing Significance With A Priori Preference Ordering

Now assume that level 1 is expected *a priori* to be preferred to level 2 for both attributes. Respondent 1's preferences suggest as strong a disagreement with this ordering as respondent 2's preferences suggest an agreement. Hence attribute 1 should be expected to be insignificant across the respondents. The Friedman test is applied to these data to determine whether this is in fact the case. First, compute the values of

$$(41) \quad r_j = \sum_{i=1}^{2} r_{ij}$$

for levels $j = 1, 2$ of attribute 1. Expressions 27 and 29 yield

$$(42) \quad r_1 = r_{11} + r_{21} = 3 + 7 = 10.$$

Similarly, expressions 28 and 30 yield

$$(43) \quad r_2 = r_{12} + r_{22} = 7 + 3 = 10.$$

Substituting these values of r_1 and r_2 and setting $I = 2$, $m = 2$, $n = 4$, $n_1 = n_2 = 2$ in expression 26 for the test statistic h yields

$$(44) \quad h = \frac{12(2-1)}{2 \times 2(4-1)} \sum_{j=1}^{2}$$

$$[\{r_j - (1/2)(2)(2)(5)\}^2 \div 2(4-2)]$$

$$= 0.$$

Hence attribute 1 is insignificant.

For attribute 2, expressions 34 and 36 yield

$$(45) \quad r_1 = r_{11} + r_{21} = 5 + 5 = 10.$$

Similarly, expressions 35 and 37 yield

$$(46) \quad r_2 = r_{12} + r_{22} = 5 + 5 = 10.$$

Substituting these values of r_1 and r_2 and setting $I = 2$, $m = 2$, $n = 4$, $n_1 = n_2 = 2$ in expression 14 for the test statistic h yields

$$(47) \quad h = \frac{12(2-1)}{2 \times 2(4-1)} \sum_{j=1}^{2}$$

$$[\{r_j - (1/2)(2)(2)(5)\}^2 \div 2(4-2)]$$

$$= 0$$

so that attribute 2 is also insignificant.

ASSESSING THE EFFECT OF DELETING INSIGNIFICANT ATTRIBUTES ON SHARE-OF-CHOICES PREDICTIONS

A major use of conjoint analysis is in predicting the share of choices of new product concepts. However, the proposed testing procedures are not directly related to this, or any other, measure of product performance. A Monté Carlo simulation therefore was conducted to examine the effect of deleting insignificant attributes on share of choices. The study began with the selection of a set of part worths for eight attributes, each at four levels. The idiosyncratic part worths were obtained from an actual conjoint study involving 187 respondents.

Individual preference rankings for 32 product profiles were simulated from the part worths data. The rankings were used to test the significance of the attributes. A main effects plan (Addelman 1962) was utilized to generate the 32 product profiles. Attributes were associated randomly with the factors of the experimental plan.

Five segments were indicated by a K-means clustering (Hartigan 1975) of the individual part worths. The number of respondents in segments 1 through 5 was 36, 37, 29, 47, and 38, respectively. The preceding tests were used to test the significance of the attributes for each segment. A preference ordering of attribute levels, based on *a priori* expectations, was specified for attributes 1, 4, 6, and 7, but not for attributes 2, 3, 5, and 8. The sets of significant attributes identified by each testing procedure are reported in Table 2. Only attributes 1, 2, and 5 are significant for all segments. The number of

Table 2

SEGMENT-LEVEL ANALYSIS OF ATTRIBUTE SIGNIFICANCE BY PROPOSED TESTING PROCEDURES[a]

Segment	Significant[b] attributes	Insignificant[c] attributes
1	1, 2, 4, 5, 6, 8	3, 7
2	1, 2, 4, 5	3, 6, 7, 8
3	1, 2, 5, 6, 7, 8	3, 4
4	1, 2, 3, 4, 5, 7, 8	6
5	1, 2, 3, 4, 5, 6, 7, 8	

[a]In all segments, an *a priori* preference ordering of attribute levels is specified for testing the significance of attributes 1, 4, 6, and 7. An *a priori* preference ordering of attribute levels is not permissible, and is therefore not specified, for testing the significance of attributes 2, 3, 5, and 8.
[b]$p < .05$.
[c]$p > .10$.

significant attributes ranges from four (segment 2) to eight (segment 5).

The effect of eliminating insignificant attributes on share of choices is examined for segments 1 through 4 but not for segment 5, which has no insignificant attributes. Status quo product profiles were specified randomly for the respondents in segments 1 through 4. The segment share of choices was simulated for 31 randomly selected product profiles under three conditions: (1) using all eight attributes, (2) eliminating each attribute one at a time, and (3) simultaneously deleting all attributes identified to be insignificant by the proposed testing procedures. Both the original part worths and those obtained from an OLS rescaling of input ranks were employed in the share-of-choices simulation.

Observe that the share of choices estimates the probability of a randomly selected individual selecting a product profile over status quo. Consequently, its variance depends on the actual probability value. To stabilize variance, an arcsin transformation of the share of choices was performed (Rao 1973, p. 427). "Product profiles" and "attributes deleted" were used as factors in an ANOVA in which the transformed share of choices was the dependent measure.

The main effect of "attributes deleted" was significant

for each segment. Tukey's multiple range tests therefore were used to examine which attribute's deletion leads to a significant change in share-of-choices predictions. Table 3 is a summary of results for segments 1 through 4. Across segments and testing procedures, using estimated part worths instead of "true" part worths does not change the sets of attributes that have statistically significant (insignificant) effects on share of choices. Consequently, the following conclusions are valid regardless of whether "true" or estimated part worths are employed to perform share-of-choices simulations.

1. Eliminating an attribute identified to be insignificant by the tests has an insignificant effect on share of choices.
2. Attributes that have a significant effect on share of choices are always identified to be significant by the tests.
3. In many cases, eliminating attributes identified to be significant by the tests has an insignificant effect on share of choices.
4. In two of the three segments with more than one insignificant attribute (i.e., segments 1, 2, and 3), simultaneously eliminating all insignificant attributes has an insignificant effect on share of choices; however, eliminating all four attributes identified as insignificant for segment 2 has a statistically significant effect on share of choices.

Table 3

EFFECT OF DELETING SIGNIFICANT AND INSIGNIFICANT ATTRIBUTES ON SHARE OF CHOICES FOR 31 RANDOMLY SELECTED PRODUCT PROFILES: TUKEY'S PAIRWISE COMPARISON TESTS USING "TRUE" AND ESTIMATED PART WORTHS[a]

Row	Attributes deleted	Segment 1 part worths		Segment 2 part worths		Segment 3 part worths		Segment 4 part worths	
		True	Est.	True	Est.	True	Est.	True	Est.
1	1	8.77 (<.01)	9.1 (<.01)	4.49 (<.05)	4.29 (<.06)	5.58 (<.01)	7.96 (<.01)	1.66 (ns)	3.52 (ns)
2	2	−11.85 (<.01)	−9.17 (<.01)	−9.80 (<.01)	−8.23 (<.01)	−11.59 (<.01)	−9.92 (<.01)	1.51 (ns)	.45 (ns)
3	3	−.37 (ns)	−1.20 (ns)	1.03 (ns)	.41 (ns)	−1.10 (ns)	−2.08 (ns)	1.14 (ns)	1.93 (ns)
4	4	.64 (ns)	.33 (ns)	−.52 (ns)	−1.21 (ns)	1.23 (ns)	1.99 (ns)	−5.90 (<.01)	−5.99 (<.01)
5	5	.83 (ns)	1.53 (ns)	9.67 (<.01)	8.15 (<.01)	3.14 (ns)	4.75 (<.05)	2.14 (ns)	2.41 (ns)
6	6	−.31 (ns)	−1.13 (ns)	.66 (ns)	−.032 (ns)	3.77 (ns)	−4.85 (<.05)	−2.60 (ns)	−3.22 (ns)
7	7	−.33 (ns)	−1.41 (ns)	−2.00 (ns)	−1.78 (ns)	−1.90 (ns)	−1.96 (ns)	.18 (ns)	.24 (ns)
8	8	2.56 (ns)	4.54 (<.05)	−.56 (ns)	−2.15 (ns)	.32 (ns)	.92 (ns)	1.72 (ns)	1.23 (ns)
9	All insignificant attributes	−1.20 (ns)	−2.58 (ns)	5.64 (<.01)	5.35 (<.01)	.59 (ns)	.11 (ns)		

[a]Cell entries denote values of Tukey's studentized q-statistic

$$q = (S_j - S_0) \div (\sqrt{MSE/31})$$

where:

S_j = arcsin-transformed share of choices when one or more attributes corresponding to row j are deleted and
S_0 = arcsin-transformed share of choices when no attribute is deleted.

Values in parentheses denote the joint significance level (p-value) for the multiple comparisons using Tukey's studentized-range values. Underlined values correspond to attributes identified to be insignificant by the proposed testing procedures.

The first three findings suggest that the proposed testing procedures are conservative in identifying attributes that have an insignificant effect on share-of-choices predictions. The reason is that attributes having an insignificant effect on share of choices sometimes are identified to be significant by the proposed tests, though attributes with a significant effect on share of choices are always identified to be significant by the proposed tests.

The fourth conclusion is a result often encountered in linear models: factors may be individually insignificant, but the variance explained by a collection of insignificant factors can be significant. However, note that in the simulation only the simultaneous elimination of four insignificant attributes of a total of eight attributes affected share of choices. Even then the change was smaller than when two significant attributes (2 and 5) were excluded *singly*. This result suggests that when multiple attributes are found insignificant by the proposed testing procedures, a user must make a tradeoff between parsimony (in the sense of identifying the attributes that have a significant effect on segment preferences) and predictive accuracy (in a share-of-choices sense) in selecting attributes for a simulation model.

CONCLUSION AND FUTURE RESEARCH

The preceding Monté Carlo simulation results are particularly useful given the importance of prediction in conjoint analysis (Green and Srinivasan 1978). The proposed tests can be used to confirm differences among benefit segments, to improve the computational efficiency of conjoint choice simulators, and to design subsequent conjoint studies in the same product class.

The proposed procedures also can be used to test for significant differences among preferences for *subsets* of attribute levels. Product profiles in which the reduced set of attribute levels appear are identified first. The relative rankings of these product profiles are used to compute the relevant test statistic. However, such testing should be restricted to comparisons among levels of attributes already identified to be significant. Even then the attribute-level subsets investigated should be restricted to a small number, because repeated tests increase the overall probability of type 1 error.

The tests developed here are applicable to main effects plans. They also assume that rank order data are collected according to the full-profile approach. For the two-factor-at-a-time method of data collection (Johnson 1974), for models permitting estimates of interaction effects, and for hybrid conjoint models combining ratings and rankings data (Green 1984; Green and Goldberg 1981; Green, Goldberg, and Montemayor 1981), alternative testing procedures are needed and should be pursued in future research.

Methods for constructing confidence intervals for part worths and for assessing the goodness of fit of nonmetric scaling algorithms also should be developed in future research. For example, nonparametric methods could be developed to estimate confidence intervals for part worths. The bootstrap and jackknife resampling procedures (Efron and Gong 1983; Green, Carmone, and Vankudre 1983) may be relevant to this line of research. Assume that a respondent has provided preference rankings for n product profiles. The jackknife can be used to specify n subsets, each consisting of $(n - 1)$ product profiles that are used to estimate part worths. For each part worth, an empirical distribution of the subset estimates can be constructed and used as a basis for obtaining a confidence interval.

Nonparametric methods also can be utilized to test the significance of the goodness of fit of a nonmetric scaling procedure. An example of this approach is provided by Mullet and Karson (1986). Using randomly generated preference data, they develop preference distributions for the LINMAP index of fit (Shocker and Srinivasan 1977) for several different main effects plans. Similar developments should be useful for MONANOVA (Kruskal 1965) and PREFMAP (Carroll 1972), the two other nonmetric scaling methods often used in conjoint analysis.

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