Product Improvement and Technological Tying in a Winner-Take-All Market

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Abstract

In a winner-take-all duopoly market for systems in which firms invest to improve their products, a monopoly supplier of an essential system component may have an incentive to advantage itself by technological tying; that is, by designing the component to work better in its own system. If the vertically integrated firm is prevented from technologically tying, then there is a pure strategy equilibrium in which the more efficient firm invests and serves the entire market. Other equilibria may exist, however, including a pure strategy equilibrium in which the less efficient firm invests and captures the market, and mixed strategy equilibria in which each firm captures the market with positive probability. In contrast, if the vertically integrated firm is able to degrade the quality of its rival's system with a technological tie, and if the wholesale price of the essential component is insufficiently remunerative, then there is a unique equilibrium outcome in which the supplier of the essential component invests alone and forecloses a more efficient rival with an actual, or merely threatened, technological tie. A comparison of these equilibria for the two game forms demonstrates that a prohibition of technological tying can either increase or decrease social welfare depending on equilibrium selection.

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1 Introduction

During the "browser war" between Microsoft and Netscape, Microsoft and its defenders argued that its Internet Explorer browser gained market acceptance because it was a superior product (see e.g., Liebowitz and Margolis, 1999). Microsoft's critics responded that Internet Explorer benefited from Microsoft's exclusionary practices associated with the distribution and use of its ubiquitous Windows operating system. In this paper we examine strategic competition in markets for systems that combine one or more component inputs to produce a final output, when one of the components is controlled by a monopoly. Examples of such markets can be found in telecommunications, electricity service, and other industries in addition to information technologies. In these markets a single supplier often controls an essential component of the system such as an operating system, a local telephone exchange network, or an electricity transmission grid. We examine competition under the assumption that the firm that supplies the system with the lowest quality-adjusted price wins the entire market. When system suppliers have unfettered access to the essential component, a firm can prevail by investing to improve the quality of its system. We show that even on this level playing field, the firm that is the most efficient supplier of systems need not emerge as the market leader. We also consider the incentives of the monopolist to tip the scales of competition by granting itself superior technological access to the essential component. This could be accomplished by designing the essential component to work better with its own system, thereby degrading the performance of rival systems relative to its own. Such a "technological tie" can give the monopoly component supplier a greater incentive to innovate and is an additional reason why the market structure *ex post* need not reflect the most capable supplier of systems *ex ante*.[2]

The monopolist confronts a trade-off in considering the merits of a technological tie. On the one hand, by limiting rivals' access to an essential component, the monopolist profits by curtailing competition in the market for systems. On the other hand, the technological tie reduces the monopolist's ability to extract rents from more efficient rivals through sales of the essential component. If rivals have a superior ability to innovate, or produce systems that appeal to a large number of consumers, then by providing access the monopolist profits from sales of the component. The technological tying trade-off depends on the price of the component. A high component price encourages the monopolist to provide efficient access to the essential component, while a low price encourages the monopolist to limit access with a technological tie. Therefore technological tying is likely to be an attractive business strategy for the monopolist only if sales of the upstream component are insufficiently remunerative.

The traditional "Chicago School" emphasizes that there is no incentive for technological tying (other than for efficiency reasons) if a monopoly profit for the tying product can be extracted by charging a profit-maximizing price.[3] There are reasons, however, why

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[2] A technological tie refers to the physical integration of a product with another product, in a manner that makes it costly for rivals to sell similar integrated products. A technological tie may also be accomplished by designing an interface or withholding technical information to impede the interoperability of a complementary product. See Lessig (2000) for a review of the case law on tying and its applicability to U.S. v. Microsoft.

[3] Bowman (1957) and Bork (1978), among others, maintain that the owner of an essential input that is used in fixed proportions with another competitively supplied good has no incentive to bundle the input and the complementary good or to tie purchase of the complementary good to the essential input. The

a monopolist may have limited flexibility to charge rivals a monopoly price for an essential component. One possibility is that the component has other uses that consumers value differently, and a non-discriminatory profit-maximizing price fails fully to extract the monopoly profit from consumers who demand a system. Alternatively, regulation (including antitrust scrutiny) constrains the price that the monopolist charges for the essential input. In such circumstances, the monopolist may opt for a mixed bundling strategy, selling the component both bundled in a system and on a stand-alone basis, while imposing a technological tie to prevent arbitrage between the two offerings.

In light of these considerations our analysis focuses on tying incentives conditional on the price of the tying product. Consistent with the traditional view, we find that the ability to technological tie does not affect market outcomes when the price of the tying good is sufficiently high. Otherwise, a technological tie, or even the threat of a technological tie, can significantly impact market structure, prices, and innovation. In some cases, these impacts have negative welfare consequences; in other cases, the ability to impose a technological tie increases social welfare. The ambiguity is due to multiple equilibria, as we discuss shortly.

Economides (1998) shows that a price-regulated upstream monopolist participating in a downstream Cournot (quantity-setting) oligopoly has an incentive for non-price discrimination.[4] Our analysis develops this theme by analyzing the incentives for and consequences of technological tying for product improvement in a downstream systems market where firms compete on quality and price.[5] In this paper, we consider the case of homogeneous consumer preferences over vertically differentiated products. We defer consideration of heterogeneous consumers and horizontal product differentiation for future research.

More specifically, we study markets for systems as duopoly games. Two firms market to consumers systems comprised of two components: A and B. Firm 1 offers a system comprised of one unit of component A and one unit of its version of component B. Firm 2 purchases component A from Firm 1 at a price w and offers consumers a system that consists of component A and its version of component B.[6] Firm 2 is assumed to have an initial quality advantage. In our basic "product improvement game," given the wholesale price of component A, rival firms invest in quality improvements of component B and subsequently

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argument is that there is a single monopoly profit, which the owner of the essential input can capture by charging a monopoly price.

[4] In line with Chicago School reasoning, there is no incentive for non-price discrimination if the input monopolist is a less efficient supplier of systems in the downstream market and the upstream monopoly is unregulated. See Sibley and Weisman (1998), Bergman (2000), and Economides (2000). As note above, the Chicago School solution implictly assumes either that there are no other uses for the monopoly input, or that effective third-degree price discrimination is feasible across uses.

[5] The new vertical foreclosure literature, which includes papers by Salop and Scheffman (1983), Krattenmaker and Salop (1986), Ordover, Saloner and Salop (1990), Riordan and Salop (1995), Hart and Tirole (1990), and Bolton and Whinston (1991) and Rey and Tirole (1997), identifies incentives for a firm that operates in both upstream and downstream markets to use price and exclusionary contracts to influence downstream competition and to "raise rivals' costs". Our analysis can be interpreted as an exploration of technological tying as a raising rivals' costs strategy.

[6] While we assume that Firms 1 and 2 supply systems comprised of component A and firm-specific versions of component B, our analysis would be unchanged if consumers were to purchase component A from Firm 1 and combine A with component B from Firm 1 or Firm 2. In this case component A can be an operating system (e.g. Microsoft Windows), and component B can be an application program, such as Microsoft Word or Wordperfect.

compete on the price of systems. In the companion "technological tying game" there is also an intermediate stage in which Firm 1 can act to degrade the quality of Firm 2's system.

Given that consumers have homogeneous preferences, the market has a winner-take-all character and multiple equilibria of the product improvement game are possible. There always exists an efficient pure strategy equilibrium of the product improvement game in which Firm 2 improves its product optimally and captures the entire market. If the initial quality advantage of Firm 2 is sufficiently small, then there also exists an inefficient pure strategy equilibrium in which Firm 1 invests in product improvement and captures the market. In these equilibria, consumers enjoy a positive surplus as long as the losing firm exerts some competitive price pressure on the winner. There also can exist mixed strategy equilibria of the product improvement game in which each of the firms invests with positive probability. Possible mixed strategy equilibrium outcomes include one or the other firm investing alone, duplicative investments, and a complete failure to invest.

The technological tying game, in contrast, has a unique equilibrium outcome if the wholesale price of the essential component is not too large. Firm 1 forecloses competition with an actual, or perhaps merely threatened, technological tie,[7] improves its product efficiently, and sets a monopoly price that fully extracts consumer surplus. In this case, technological tying distorts market structure and reduces consumer welfare by eliminating competition from Firm 2. Technological tying can improve social welfare compared to a mixed strategy equilibrium of the product improvement game, even though consumers are worse off. The technological tying game also has a unique equilibrium outcome if the wholesale price is sufficiently close to the monopoly level. Firm 1 prefers the role of supplier to Firm 2 and declines to compete for the retail market. For an intermediate range of wholesale prices, the technological tying game surprisingly can have multiple equilibria, including a mixed strategy equilibrium in which both firms invest with some probability, but Firm 1 forecloses its rival when both firms invest in product improvement.

Farrell and Katz (2000) also study the incentives for product innovation and technological tying in a market for vertically differentiated systems. In their model, a monopoly supplies an essential component and competes with others to supply a complementary component to consumers who assemble a system. By "overinvesting" in product R&D for the complementary component, the integrated firm squeezes the rents of rival suppliers and is able to charge consumers more for the essential component.[8] Moreover, the integrated firm has no incentive to disadvantage rivals with a technological tie, due the their assumption that the monopolist prices the essential component to extract all available surplus after observing the qualities and prices of the competitively-supplied component. Thus Farrell and Katz (2000) extend the Chicago School insight that there is no reason for technological tying if the monopolist can effectively extract rents by adjusting the price of the monopoly good. In contrast, we assume that the wholesale price of the essential component is de-

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[7] When a merely threatened technological tie does the job, Firm 2 is foreclosed by a price squeeze, meaning that the wholesale price is prohibitive compared to Firm 2's equilibrium quality. Firm 2 declines to invest in product quality because it rationally believes that Firm 1 would foreclose a competitive product with a technological tie.

[8] See Bolton and Whinston (1993) and Kranton and Minehart (2002) for related models of strategic overinvestment.

termined prior to systems market competition, and focus on cases in which the wholesale price fails to fully extract monopoly rents from the systems market.

Choi and Stefanadis (2001) analyze a model of a systems market that is similar in structure to Farrell and Katz (2000). In their model, an incumbent monopolist sells two complementary components and potential entrants invest to introduce new lower-cost versions of one or both of the components, which they may combine with a component owned by the incumbent. Choi and Stefanadis (2001) compare investment incentives with and without tying, and show that tying deters low-cost entry in both component markets and also strengthens the incumbent's investment incentives. Choi, Lee and Stefanadis (2002) explore a similar model with discrete investment. In contrast, our analysis focuses on the incentive for technological tying by an integrated monopolist to foreclose a competitor who requires access to an essential component of the system.

Section 2 describes the structure of the market for systems and the technology for product improvement. This section introduces the assumptions that Firm 2 is the higher quality supplier of systems when neither firm invests and that, by investing, Firm 1 can leapfrog Firm 2's quality advantage. These assumptions frame the policy issue by defining an environment in which Firm 1 can use its control over access to the essential component to influence investment incentives and thereby distort market outcomes. Section 3 introduces the product improvement and technological tying games, identifies their pure strategy equilibria, and examines the welfare properties of the pure strategy equilibrium outcomes. Section 4 considers possible mixed strategy equilibria of these games and Section 5 concludes. Throughout the discussion we consider the competitive consequences of several variations on our basic theme, such as assigning the initial quality advantage to Firm 1, or assuming that a firm exits the market if it cannot make any sales.

2 Vertical Product Differentiation

There are two firms, indexed $i = 1, 2$. Both firms produce a system that is offered to final consumers. Firm 1 is also the monopoly producer of an intermediate good that is an essential component of systems, which it supplies at zero marginal cost. Firm 2 must have access to the essential component to compete in the systems market. As explained further below, the essential component can be sold also on a stand-alone basis separately from systems.

Systems are differentiated in quality, which is partly exogenous and partly endogenous. Each of M identical consumers demands a single system. A consumer's willingness-to-pay for a system consisting of components A and B from firm i is

$$q_i = \gamma_i + q(r_i), \tag{1}$$

where γ_i is an exogenous quality parameter specific to systems sold by Firm i and the endogenous variable r_i is Firm i's investment in R&D to improve the quality of its system (or, equivalently, the quality of its component B). For analytical convenience, we assume that there are no additional variable costs of producing systems.

It is convenient to reinterpret (1) as Firm i choosing a level of quality improvement

$$z_i = q_i - \gamma_i$$

by incurring an R&D cost

$$r_i = r(z_i).$$

We maintain several assumptions.

A1: The symmetric R&D cost function $r(z)$ is increasing, strictly convex, twice differentiable, and satisfies $r(0) = r'(0) = 0$.

The first assumption implies that there is a unique z^M that maximizes the net benefits from quality improvement $zM - r(z)$ and is the solution to $r'(z^M) = M$. Thus z^M is the efficient level of quality improvement for a firm selling to the entire market, yielding a net return

$$\pi^M \equiv z^M M - r(z^M) > 0.$$

A2.: $\Gamma \equiv \gamma_2 - \gamma_1 > 0$.

The second assumption implies that Firm 2 is the more efficient supplier of systems for any level of investment in quality improvement.[9] The maximum social surplus in this market is $\gamma_2 M + \pi^M$, which corresponds to investment of z^M by Firm 2.[10] This surplus is fully extracted by Firm 1 with a wholesale price equal to

$$\bar{w} \equiv \gamma_2 + \pi^M / M.$$

We make extensive use of $\bar{w}$ below.

A3: $\pi^M > \Gamma M$.

The third assumption implies that Firm 1 can profitably leapfrog Firm 2's initial quality advantage by investing efficiently in product improvement; i.e., $(\gamma_1 + z^M)M - r(z^M) > \gamma_2 M$. Although this assumption is not necessary for some of our results, it describes an environment in which investment effects can dominate firm-specific efficiencies, which is the focus of our analysis. For future reference, note that Assumption 3 implies that $\bar{w} - \Gamma > \gamma_2$.

The monopoly good that is an essential component of systems also has a separate standalone use that is valuable to a different group of consumers. While M consumers are willing to pay q_i for a system, another M' consumers are willing to pay w for the standalone product. The monopolist is unable to charge different prices to these two groups of

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[9] This assumption serves to emphasize the potential social costs of strategic conduct by a vertically integrated supplier. We discuss the implications for the analysis if $\Gamma \leq 0$.

[10] More formally, let xM be the allocation of consumers to Firm 1 and $(1-x)M$ the allocation to Firm 2, and let z_1 and z_2 be the firms' investments in quality improvement. The social planner chooses (x, z_1, z_2) to maximize

$$W(x, z_1, z_2) = M\left[x\left(\gamma_1 + z_1\right) + (1-x)\left(\gamma_2 + z_2\right)\right] - r(z_1) - r(z_2).$$

Clearly $W(0, 0, z^M) > W(1, z^M, 0)$. Furthermore, the convexity of $r(z)$ implies that $W(0, 0, z^m) > W(x, z_1, z_2)$ for $x \in (0, 1)$. Therefore, the welfare optimum has $x = 0$, $z_1 = 0$, and $z_2 = z^M$.

consumers, and sets a profit-maximizing price equal to w if M' is sufficiently large relative to M. Moreover, if $w < \bar{w}$, then the profit-maximizing price fails to extract monopoly profits from the M consumers who demand systems. We make these additional assumptions, and hereafter treat w as the predetermined wholesale price of the monopoly component.

A4: $\bar{w} \geq w$ and $M' > (\frac{\bar{w}-w}{w})M > 0$.

The fourth assumption states that the monopoly price of component A when used in a system ($\bar{w}$) weakly exceeds the monopoly price of the component when sold on a stand-alone basis (w), and that the profit from selling in both submarkets exceeds the monopoly profit from the systems market alone. This implies that Firm 1 optimally commits to a wholesale price equal to w,[11] earning a profit wM' from stand-alone sales. The profit from stand-alone sales of the component plays no significant role in our analysis, and is ignored hereafter.

We compare the subgame perfect Nash equilibria of two different game forms. In the "product improvement game," competition proceeds in two stages. In the first stage, the firms simultaneously and independently choose quality improvements z_i at cost $r(z_i)$. In the second stage, the firms simultaneously and independently set prices $P_i \geq w$ after observing each other's quality. The "technological tying game" amends the product improvement game by allowing the upstream monopolist to degrade the quality of its rival's system. In the first stage, the firms choose costly quality improvements z_i as in the product improvement game. In the second stage, Firm 1 can degrade Firm 2's quality by a fixed amount $\delta > 0$. A technological tie always forecloses competition if δ is sufficiently large, which we assume to be the case. In the final stage, the firms set prices.

The price subgame is the same in both game forms. Consumers observe prices and qualities and choose the product that offers the greatest net utility. Consumers have identical preferences, so Firm i makes sales to all M consumers if $q_i - P_i > \max(q_j - P_j, 0)$. When both products offer the same net utility, consumers are assumed to choose the higher quality product, and if both firms also have the same quality, then consumers are assumed to choose Firm 1.

Equilibrium prices and sales in the market for systems depend on the level of w and product qualities. In an equilibrium of the price subgame, only one firm sells to the entire market. If $q_1 > q_2$ and $q_2 \geq w$, then Firm 2 sets price $P_2 = w$ and Firm 1 wins the market at price $P_1 = q_1 - q_2 + w$. A similar result holds for Firm 2 if $q_2 > q_1$ and $q_1 \geq w$, because w is an opportunity cost of sales for Firm 1 and a direct cost for Firm 2.[12] If $q_2 < w$, then

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[11] As discussed in the introduction, we assume that Firm 1 commits to the wholesale price w before firms invest in product improvement. Without such a commitment, Firm 2 would not invest if Firm 1 is able to "hold up" Firm 2 by raising the wholesale price after Firm 2 has invested in product improvement. Farrell and Katz (2000) reach this conclusion for their price leadership model in which the integrated firm prices the essential component after observing the realized qualities.

[12] Let xM be Firm 1's sales of systems, with $x \in [0,1]$. Firm 2's sales are $(1-x)M$. If w is the price of component A, then the respective profits of the two firms are $\pi_1 = (P_1 x + w(1-x))M - r(z_1) = (P_1 - w)xM + wM - r(z_1)$ and $\pi_2 = (P_2 - w)(1-x)M - r(z_2)$. The wholesale price of component A (w), is an opportunity cost of system sales for Firm 1 as well as a direct marginal cost for Firm 2.

Firm 1 wins the market at price $P_1 = q_1$ even if $q_1 < q_2$. Summarizing, Firm 1 sells to all M customers at a price $P_1 = q_1 - \max(q_2 - w, 0)$ if $q_1 \geq q_2$ or if $w \geq q_2$, and Firm 2 sells to all M customers at a price $P_2 = q_2 - \max(q_1 - w, 0)$ if $q_2 > q_1$ and $w < q_2$. The losing firm serves as a competitive check when the quality of its system is above its opportunity cost.

We analyze both pure and mixed strategy equilibria of the product improvement and technological tying games. Pure strategy equilibria are natural outcomes in a winner-take-all market when firms are able to coordinate their investment decisions. In contrast, mixed strategy equilibria describe industry conduct when firms are uncertain about the investment decisions of their rivals. The equilibria of the product improvement game illustrate the ability and incentives of a firm that controls an essential input to improve its system in order to gain a competitive advantage over downstream rivals. The upstream monopolist may win the market with a better product even when it would be more efficient for a downstream rival to engage in product improvement. Thus, an *ex post* measure of product superiority can be a misleading indicator of market performance.

A technological tie can effectively foreclose a more efficient downstream rival from participating in the market for systems. When technological tying is feasible and the price of the upstream good is insufficiently remunerative, the upstream monopolist has an incentive to foreclose rivals and substitute its own innovative efforts. Technological tying is potentially costly because it facilitates product improvement and market dominance by a less efficient firm. Nonetheless, technological tying is also potentially beneficial because it avoids the inefficiencies from the low and redundant investments that can occur in mixed strategy equilibria of the product improvement game.

3 Pure strategies

3.1 The Product Improvement Game

We begin with pure strategy equilibria of the product improvement game. In stage 1, the firms invest in quality improvement anticipating the Bertrand-Nash equilibrium of the price subgame discussed above. It is immediate that both firms cannot make positive investments in a pure strategy equilibrium. One of the firms will capture the entire market, leaving the other firm better off not investing. Given our maintained assumptions, there always exists an equilibrium in which the more efficient Firm 2 captures the entire market. A second equilibrium exists if the cost of efficient quality improvement is high relative to Firm 2's initial quality advantage.

Proposition 1 *In the product improvement game:*

(i) There exists a pure strategy equilibrium in which Firm 2 invests z^M and Firm 1 does not invest in quality improvement. Firm 2 sets a price equal to $P_2 = \gamma_2 + z^M - \max(\gamma_1 - w, 0)$, sells systems to all M customers, and earns $\pi_2 = [(\bar{w} - w) - \max(\gamma_1 - w, 0)]M \geq 0$. Firm 1 sets price $P_1 = w$, sells M units of component A and no systems, and earns $\pi_1 = wM$.[13] *This equilibrium is efficient given Firm 2's assumed initial quality advantage.*

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[13] This is only Firm 1's profit from systems, including components sold to Firm 2. As noted earlier, Firm 1 also earns a profit from stand-alone component sales that, given A4, has no significant role in our analysis.

(ii) There exists a second pure strategy equilibrium in which Firm 1 invests $z_1 = z^M$ *and Firm 2 does not invest if and only if* $r(z^M) \geq \Gamma M$. *Firm 1 sets price* $P_1 = \gamma_1 + z^M - \max(\gamma_2 - w, 0)$, *sells systems to all* M *customers, and earns* $\pi_1 = [\bar{w} - \Gamma - \max(\gamma_2 - w, 0)]M \geq 0$. *Firm 2 sets price* $P_2 = w$, *sells no systems, and earns* $\pi_2 = 0$.

(iii) There are no other pure strategy equilibria.

The payoffs to each firm depend on their investment levels and the price of component A. The payoff matrix is shown in Table 1 below when the firms choose either to invest at the efficient level z^M or not at all and the price of component A is less than γ_2. The proof, which is in the Appendix, shows that neither firm can profit by choosing an investment level other than 0 or z^M. There are two possible equilibria, corresponding to investment by Firm 1 (and not by Firm 2) and investment by Firm 2 (and not by Firm 1). The first equilibrium exists if and only if $r(z^M) \geq \Gamma M$, otherwise Firm 2 would leapfrog the less efficient Firm 1 even when Firm 1 invests. If $r(z^M) \geq \Gamma M$, the differential quality Γ is not large enough to compensate Firm 2 for the cost of the quality improvement. If the component price exceeds γ_2, the payoffs in Table 1 change in only one respect. Firm 2 is foreclosed by the high component price when neither firm invests, so in this event Firm 1 earns $\gamma_1 M$ and Firm 2 earns zero. As in the case where $w \leq \gamma_2$, the efficient equilibrium in which Firm 2 invests is unique if $r(z^M) < \Gamma M$.

Table 1. Firm payoffs when $w \leq \gamma_2$.

	$z_2 = z^M$	$z_2 = 0$
$z_1 = z^M$	$wM - r(z^M); \Gamma M - r(z^M)$	$[\bar{w} - \Gamma - (\gamma_2 - w)]M; 0$
$z_1 = 0$	$wM; [\bar{w} - w - \max(\gamma_1 - w, 0)]M$	$wM; [\gamma_2 - w - \max(\gamma_1 - w, 0)]M$

In the equilibrium in which only Firm 1 invests, corresponding to $r(z^M) \geq \Gamma M$, Firm 2 makes no sales even though it is able to produce systems more efficiently *ex ante*. By improving its system, Firm 1 endogenously becomes such a formidable competitor that Firm 2 cannot effectively compete, once Firm 1's R&D expenditures are sunk. For illustration, Figure 1 shows the payoffs to Firm 1 in each of the possible pure strategy equilibria as a function of w.[14]

The pure strategy equilibria of the product improvement game show that one cannot rely on *ex post* market structure to infer which firm is the more efficient supplier. Product superiority is endogenous and in our model either firm can invest to become the market leader. In particular, the firm that has a quality disadvantage can overcome its disadvantage by investing in quality. Having done so, the more efficient firm cannot profitably invest to win the market even though it can supply a better product than its rival. Thus the "wrong" firm can emerge as the market leader. This indeterminacy exists for any value of the component price, w, even when the vertically integrated firm does not engage in

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[14] We have assumed that $\Gamma > 0$. If $\Gamma \leq 0$, then an equilibrium exists in which the more efficient Firm 1 invests z^M for any $0 \leq w \leq \bar{w} - \Gamma$. There is a second equilibrium in which the less efficient Firm 2 invests z^M if $0 \leq w \leq \bar{w}$ and $r(z^M) > -\Gamma M$. The component price would foreclose Firm 2 if $\bar{w} < w \leq \bar{w} - \Gamma$. Firm 1 would leapfrog Firm 2's post-investment quality if $r(z^M) \leq -\Gamma M$.

technological tying to impede access by its more efficient rival. The next section explores equilibrium outcomes when such strategies are feasible.

Our analysis assumed that a firm can be a competitive threat even if it does not win the market. By "waiting in the wings", a lower-quality firm can discipline the price charged by a higher-quality firm, provided that it is not otherwise foreclosed from competing. An alternative assumption is that a firm exits the market if it cannot make sales.[15] This implies that Firm 2 exits the market and is not a competitive constraint if Firm 1 invests and Firm 2 does not, or if it is foreclosed by a high component price. Similarly, Firm 1 does not constrain the prices charged by Firm 2 if Firm 2 invests and wins the market. Under this alternative exit assumption the payoff matrices for the product improvement game would appear as shown in Table 2 below for $w \leq \gamma_2$. In this case, the efficient equilibrium in which Firm 2 invests z^M and Firm 1 does not invest is the unique equilibrium of the product improvement game. Firm 2 sets a price equal to $P_2 = \gamma_2 + z^M$, sells systems to all M customers, and earns $\pi_2 = [\bar{w} - w]M \geq 0$. Firm 1 sets price $P_1 = w$, sells M units of component A and no systems, and earns $\pi_1 = wM$.

Table 2. Firm payoffs when $w \leq \gamma_2$ and the losing firm exits.

	$z_2 = z^M$	$z_2 = 0$
$z_1 = z^M$	$wM - r(z^M); [\bar{w} - w]M$	$[\bar{w} - \Gamma]M; 0$
$z_1 = 0$	$wM; [\bar{w} - w]M$	$wM; [\gamma_2 - w]M$

If $\gamma_2 < w < \bar{w}$, the component price would foreclose Firm 2 if it did not invest, allowing Firm 1 to earn its stand-alone profit when neither firm invests. For these values of w, Table 2 would change only in the cell corresponding to no investment by either firm, with corresponding payoffs $[\gamma_1 M; 0]$. Thus, if the losing firm exits the market, the efficient equilibrium in which Firm 2 invests z^M and Firm 1 does not invest is the unique equilibrium of the product improvement game for all $w < \bar{w}$.

The analysis is little changed if Firm 1 has the initial quality advantage. Suppose $\gamma_1 > \gamma_2 \geq 0$ and the lower quality firm continues to discipline the price charged by the higher quality firm. Define $\bar{w} \equiv \gamma_1 + \pi^M/M$ and $\Gamma \equiv \gamma_1 - \gamma_2$. The payoffs are shown below for $w \leq \min(\gamma_2 + z^M, \bar{w})$. There is an efficient equilibrium in which Firm 1 invests z^M and Firm 2 does not invest. This is the unique equilibrium if $r(z^M) < \Gamma M$. There is a second equilibrium in which Firm 2 invests z^M and Firm 1 does not invest if and only if $r(z^M) \geq \Gamma M$. Note that $\bar{w} \leq \gamma_2 + z^M$ if and only if $r(z^M) \geq \Gamma M$. Consequently, Table 3 describes the outcomes for all feasible w when $r(z^M) \geq \Gamma M$. If $r(z^M) < \Gamma M$, then there are outcomes in which Firm 2 is foreclosed by a high component price when both firms invest and $\gamma_2 + z^M \leq w < \bar{w}$. The efficient equilibrium in which Firm 1 invests is unique in this case.

Table 3. Firm payoffs when $\gamma_1 > \gamma_2 > 0$ and $w \leq \min(\gamma_2 + z^M, \bar{w})$.

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[15] Formally, we introduce an an additional stage, prior to price competition in which firms can exit the systems market and recover $\varepsilon > 0$. We consider the limiting case as $\varepsilon \to 0$.

	$z_2 = z^M$	$z_2 = 0$
$z_1 = z^M$	$\Gamma M - r(z^M) + wM; -r(z^M)$	$[\bar{w} - \max(\gamma_2 - w)]M; 0$
$z_1 = 0$	$wM; [\bar{w} - w - \Gamma - \max(\gamma_1 - w, 0)]M$	$[\gamma_1 - \max(\gamma_2 - w)]M; 0$

If we assume instead that a firm exits if it does not win the market, then the payoffs become as shown in Table 4 below. Again, under this alternative exit assumption, the efficient equilibrium (in this case the equilibrium in which Firm 1 invests z^M and Firm 2 does not invest, is the unique equilibrium of the product improvement game.

Table 4. Firm payoffs when $\gamma_1 > \gamma_2 > 0$.and the losing firm exits

	$z_2 = z^M$	$z_2 = 0$
$z_1 = z^M$	$\bar{w}M; -r(z^M)$	$\bar{w}M; 0$
$z_1 = 0$	$wM; (\bar{w} - w - \Gamma)M$	$\gamma_1 M; 0$

3.2 The Technological Tying Game

Firm 1 can avoid competition from Firm 2 by foreclosing Firm 2's access to component A, which we assume is available only from Firm 1. It conceivably might do this by contractually conditioning the purchase of A on the purchase of its component B, by selling a system consisting of components A and B and refraining from selling A separately (a pure bundling strategy), by charging a price for component A that is so high that Firm 2 cannot compete (or, equivalently, refusing to sell component A), or by designing component A so that a system performs worse when used with Firm 2's component B. This last strategy is a technological tie. A technological tie lowers the quality of a system made with component B from Firm 2 by a fixed amount δ. That is, $q_2 = \gamma_2 + z_2 - \delta$ and we assume that q_1 is unchanged by the tie at $\gamma_1 + z_1$. The parameter δ is sufficiently large that technological tying always forecloses competition.

Foreclosure strategies that are based on a contractual tie, pure system sales, or refusals to deal in the upstream product may be ineffective if there is a separate demand for component A that the upstream monopolist wishes to serve. In contrast, a technological tie that obstructs the ability of Firm 2 to offer a competitive system, or makes it expensive for consumers to assemble a system using component B from Firm 2, does not limit the ability of the upstream monopolist to pursue a mixed bundling strategy in which the firm both sells systems and makes separate sales of component A in a different market. For simplicity, we assume that technological tying is costless for Firm 1 (other than the indirect cost of lost revenues from sales of component A to Firm 2) and consider only Firm 1's incentives to engage in this activity. Foreclosure is clearly inefficient because it eliminates competition from a more efficient producer. Nonetheless, Firm 1 may profit by foreclosing production by Firm 2 under some circumstances.

Consider the following three-stage "technological tying game," which amends the basic product improvement game studied in the previous subsection. In stage one, the firms choose costly quality improvements z_i as before. In stage two, Firm 1 is able to impose a technological tie that degrades Firm 2's quality by an amount $\delta \geq 0$. In stage three, the firms set prices $P_i \geq w$. Because the tie is costless to Firm 1, the game can have multiple equilibria when Firm 1 is indifferent to imposing a tie in the second stage. To avoid these trivial outcomes, we assume that Firm 1 does not impose a tie when it is indifferent, which would be the case if technological tying incurred an arbitrarily small cost.

With vertical product differentiation, Firm 1 may profit by degrading Firm 2's quality only if it wins the system competition; i.e., only if $q_1 > q_2 - \delta$. Thus, it is sufficient to focus on technological tying strategies that foreclose Firm 2 from the market. The following proposition establishes the existence of a unique pure strategy equilibrium outcome of the technological tying game for sufficiently low values of the component price. When $w < \bar{w} - \Gamma$, Firm 1 invests z^M and Firm 2 is foreclosed from the systems market by either an actual or threatened technological tie. For low values of w ($w < \gamma_2$), Firm 1 would foreclose Firm 2 with a technological tie. For intermediate values of w, Firm 1 has no need to impose a technological tie in equilibrium because Firm 2 poses no competitive threat unless it invests and Firm 2 is deterred from investing by the credible threat of a technological tie if it were to leapfrog Firm 1. Although the threat of a technological tie is critical to the equilibrium outcome, whether or not Firm 1 actually imposes a technological tie in equilibrium in this case has no effect on profits or welfare.

Proposition 2 *In the technological tying game:*

(i) If and only if $w < \gamma_2$, there exists a pure strategy equilibrium in which Firm 1 invests z^M, Firm 2 does not invest, and Firm 1 forecloses Firm 2 with a technological tie. In this equilibrium, Firm 1 sets $P_1 = \gamma_1 + z^M$, sells systems to the entire market, and earns $\pi_1 = [\bar{w} - \Gamma]M$. Firm 2 sets $P_2 = w$ and earns $\pi_2 = 0$.

(ii) If $\gamma_2 \leq w < \gamma_1 + z^M$, there exists a pure strategy equilibrium in which Firm 1 invests z^M and Firm 2 does not invest. Firm 1 does not impose a technological tie, sets $P_1 = \gamma_1 + z^M$, sells systems to the entire market, and earns $\pi_1 = [\bar{w} - \Gamma]M$. Firm 2 sets $P_2 = w$ and earns $\pi_2 = 0$.

(iii) If $w \geq \bar{w} - \Gamma$, there exists a pure strategy equilibrium in which Firm 2 invests z^M and Firm 1 does not invest. Firm 1 does not impose a technological tie. Firm 2 sets $P_2 = \gamma_2 + z^M$, sells systems to the entire market, and earns $\pi_2 = [\bar{w} - w]M$. Firm 1 sets $P_1 = w$ and earns $\pi_1 = wM$.

(iv) There are no other pure strategy equilibrium outcomes.

The formal proof is in the Appendix, where we consider arbitrary levels of investments by each firm and show that they choose to invest either z^M or nothing in equilibrium. Here we restrict their investment choices to z^M or zero. When $w < \gamma_2$, Firm 1 prefers to invest and impose a tie. The tie eliminates Firm 2 as a potential competitor and allows Firm 1 to set $P_1 = \gamma_1 + z^M$ and earn its stand-alone value $\pi_1 = [\bar{w} - \Gamma]M$. Firm 2 cannot leapfrog Firm 1's quality because it is foreclosed by the tie, and given that Firm 2 does not invest, Firm 1 is better off with the tie. If Firm 1 chose instead to sell the component to Firm 2, the most it could charge is γ_2, and by Assumption A.3, $[\bar{w} - \Gamma]M > \gamma_2 M$. Investing and foreclosing Firm 2 with a technological tie is the unique equilibrium strategy for Firm 1if $w < \gamma_2$.

Assumption A.3 also implies that $\gamma_2 < \gamma_1 + z^M$. For $\gamma_2 \leq w < \gamma_1 + z^M$, there is an equilibrium in which Firm 1 invests z^M, Firm 2 does not invest, and Firm 1 does not impose a technological tie. The tie is unnecessary for these parameter values because the cost of component A forecloses competition from Firm 2 when it does not invest, and Firm 1 would strictly prefer to impose a tie if Firm 2 invests. Given that Firm 1 has already invested z^M at the first stage of the game, in the second stage Firm 1 is better off selling systems at a price $P_1 = \gamma_1 + z^M$ rather than selling the component to Firm 2 at $w < \gamma_1 + z^M$. Firm 1's credible threat to impose a tie in the second stage when $w < \gamma_1 + z^M$ is sufficient to deter investment by Firm 2. Firm 1 invests in an efficient level of R&D and provides a superior system to Firm 2. In this case, no actual anticompetitive behavior is ever observed. Yet market structure is distorted, even in the case where $\Gamma M > r(z^M)$ and efficient investment by Firm 2 is the unique equilibrium of the product improvement game. The more efficient supplier of systems is effectively foreclosed by the ability of the vertically integrated firm to impose a technological tie and its incentive to do so off the equilibrium path.

For $w \geq \bar{w} - \Gamma$, a third equilibrium exists in which Firm 2 invests z^M, Firm 1 does not invest and does not impose a technological tie. In the second stage, Firm 1 would earn $\gamma_1 M$ if it imposes a tie, which is less than it would earn by selling the component to Firm 2. In this equilibrium Firm 2 sets $P_2 = \gamma_2 + z^M$, sells systems to the entire market, and earns $\pi_2 = [\bar{w} - w]M$. Firm 1 sets $P_1 = w$ and earns $\pi_1 = wM$. This case corresponds closely to standard Chicago School argument that a firm does not benefit from a technological tie if it can charge the monopoly price for an essential upstream input.

Multiple equilibria exist when $\bar{w} - \Gamma \leq w < \gamma_1 + z^M$. The equilibrium in which Firm 2 invests efficiently Pareto-dominates the equilibrium in which Firm 1 invests while threatening Firm 2 with a technological tie. In this case, the ability to impose a technological tie is a trap that the tying firm would prefer to avoid, because the ability to tie can result in a "bad equilibrium" in which Firm 2 is discouraged from investing to improve its product. When $w \geq \bar{w} - \Gamma$, Firm 1 has no profitable use for a technological tie, either threatened or actual, and would be better off relinquishing its ability to impose a tie.

The standard Chicago School argument is that the owner of an essential input has no incentive to foreclose access to the input if it can charge the monopoly price. Since it is possible that $\gamma_1 + z^M > \bar{w}$, this argument must be qualified in the presence of endogenous innovation by the possibility of multiple equilibria even when the wholesale price is set at the monopoly level ($\bar{w}$). Although the integrated monopolist does not have an incentive to impose a technological tie when $w \geq \bar{w} - \Gamma$, its mere ability to tie can deter investment by the more efficient Firm 2 and make it impossible to fully extract the monopoly profit. Thus, in this case, there is a policy rationale for prohibiting technological tying. Moreover, $\gamma_1 + z^M > \bar{w}$ implies $\Gamma M > r(z^M)$, and from Proposition 1 this implies that, when tying is not possible, efficient investment is the unique equilibrium of the product improvement game.[16]

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[16] AT&T's voluntary divestiture of Western Electric was explained in part by the costs of being both a supplier to downstream firms and a competitor of those firms. Our analysis confirms that the ability to distort downstream competition can indeed be liability that a monopoly supplier of an essential component would want to relinquish.

One purpose of tying in this model is to eliminate Firm 2 as a competitive threat when Firm 1 invests and $w < \gamma_2$. Without a tie, consumers could choose Firm 2's unimproved product at a lower price, and this would discipline the price that Firm 1 could charge for its improved product. The tie would be unnecessary if Firm 2 would exit the market if it could not make sales. There are only two pure strategy equilibria of the technological tying game if the firm with the lower quality system would exit the market. If $w \leq \gamma_1 + z^M$, there is an equilibrium in which Firm 1 invests z^M and Firm 2 does not invest. Firm 1 does not impose a technological tie, sets $P_1 = \gamma_1 + z^M$, sells systems to the entire market, and earns $\pi_1 = [\bar{w} - \Gamma]M$. Firm 2 sets $P_2 = w$ and earns $\pi_2 = 0$. In this equilibrium, the mere threat of a technological tie is sufficient to deter investment by Firm 2. As in the previous model, no actual anticompetitive behavior is observed in this equilibrium, even though the outcome is inefficient and even if the product improvement game without tying has a unique equilibrium in which the more efficient Firm 2 invests. If $w \geq \bar{w} - \Gamma$, there is a second pure strategy equilibrium in which Firm 2 invests z^M, Firm 1 does not invest, and does not impose a technological tie. Firm 2 sets $P_2 = \gamma_2 + z^M$, sells systems to the entire market, and earns $\pi_2 = [\bar{w} - w]M$. Firm 1 sets $P_1 = w$ and earns $\pi_1 = wM$. Either equilibrium outcome could occur if $\bar{w} - \Gamma \leq w \leq \gamma_1 + z^M$.

Some of our results flow from the assumption that Firm 1 can impose a technological tie in the second stage of the game, after the firms invest, which makes the threat of a tie an important element of the firms' investment behavior. This is a reasonable assumption for some competitive environments. For example, a manufacturer of mainframe computers could change the interface protocols that determine how components communicate with the central processor after the manufacturer invests in most of the attributes of the mainframe system. In other environments, it is more reasonable to assume that the technological tying decision is made at the same time or before the firm invests to improve the characteristics of its system. Microsoft, for example, maintained that the integration of the Internet Explorer browser in its Windows 98 operating system was a feature of the improved operating system, and not an *ex post* decision to offer both functionalities in a single product. How would our results change if we reverse stages one and two of the technological tying game? In this modification of the technological tying game, Firm 1 chooses whether to impose the tie in the first stage. In the second stage both firms make their investment decisions, and they choose prices in the third stage.

The modified technological tying game has two pure strategy equilibrium outcomes. For $w < \bar{w} - \Gamma$, Firm 1 would impose a tie in the first stage and invest z^M in the second stage. Firm 2 would not invest. Firm 1 sells systems at price $P_1 = \gamma_1 + z^M$ and earns $\pi_1 = [\bar{w} - \Gamma]M$. Firm 2 sets $P_2 = w$ and earns zero. Firm 1 would not impose a tie in the first stage if $w \geq \bar{w} - \Gamma$. When Firm 1 chooses not to impose a tie in the first of the game, the possible equilibrium outcomes are the same as in the product improvement game without tying. If $R(z^M) \geq \Gamma M$, there are two pure strategy equilibria of the technological tying game when $w \geq \bar{w} - \Gamma$, corresponding to investment by Firm 2 or by Firm 1. If Firm 2 invests, it earns $\pi_2 = [\bar{w} - w]M$ and Firm 1 earns $\pi_1 = wM$. If Firm 1 invests, it earns $\pi_1 = [\bar{w} - \Gamma]M$; Firm 2 sets $P_2 = w$ and earns zero. Only the efficient equilibrium in which Firm 2 invests would survive if either $r(z^M) < \Gamma M$ or if a firm would exit the market when it does not make sales.

The tying game assumes that the owner of the essential component is initially the less efficient supplier of systems. Suppose we reverse this assumption and give Firm 1 the initial quality advantage. Define $\bar{w} \equiv \gamma_1 + \pi^M/M$ and $\Gamma \equiv \gamma_1 - \gamma_2$. As in the original tying game, we assume that Firm 1 chooses whether to impose a tie in the second stage of the game. Then, for $w < \gamma_2$, there is an equilibrium in which Firm 1 invests z^M, Firm 2 does not invest, and Firm 1 forecloses Firm 2 with a technological tie. Firm 1 sets $P_1 = \gamma_1 + z^M$, sells systems to the entire market, and earns $\pi_1 = \bar{w}M$. Firm 2 sets $P_2 = w$ and earns $\pi_2 = 0$. For $w \geq \gamma_2$, there is a second equilibrium in which Firm 1 invests z^M, Firm 2 does not invest, and Firm 1 does not impose a technological tie. The component price eliminates Firm 2 as a potential competitor when it does not invest, and if Firm 2 were to invest, then Firm 1 would have a credible threat to impose a tie. Payoffs are the same as in the first equilibrium. These are the only pure strategy equilibrium outcomes of the tying game when Firm 1 is the more efficient supplier of systems. For example, if Firm 2 invested z^M, Firm 1 could earn no more than $[\bar{w} - \Gamma]M$ by selling the component to Firm 2; but by investing and imposing a technological tie in the second stage of the game it would earn $\bar{w}M$. When Firm 1 has the initial quality advantage, the technological tie has the beneficial result of selecting the efficient equilibrium of the product improvement game.

If the more efficient Firm 1 could impose a tie in the first stage of the game, it would do so if $w < \gamma_2$ or if $r(z^M) \geq \Gamma M$. In the first case, the tie eliminates costly potential competition from Firm 2 when Firm 1 invests. In the second case, the tie eliminates the possibility of an inefficient equilibrium in which Firm 2 invests. A tie would be unnecessary if the more efficient Firm 1 could impose a tie in the first stage of the game and the less efficient firm would exit the market. In this case the efficient equilibrium in which Firm 1 invests and earns the maximum profit is the unique equilibrium of the product improvement game, so the tie is of no benefit.

In the next sections we examine the welfare implications and possible mixed strategy equilibria of the product improvement and tying games. We focus our attention on the game forms in which Firm 2 is initially the higher quality firm, a firm remains in the market even if it does not make sales, and the tying decision takes place in the second stage of the game after the firms invest in R&D.

3.3 Welfare

We now consider the welfare implications of the pure strategy equilibria of the product improvement and the technological tying games. Social welfare is obviously at a maximum in the efficient pure strategy equilibrium of the product improvement game. Nonetheless, consumers (weakly) prefer the inefficient pure strategy equilibrium to the efficient one. With Bertrand competition, the equilibrium price is the quality level of the investing firm less the margin between quality and cost for the rival firm, provided this margin is positive. This margin determines consumer surplus. The margin is (weakly) larger for Firm 2 because Firm 2's quality level exceeds Firm 1's when neither firm invests. Firm 2 is a greater competitive threat to Firm 1 in the inefficient equilibrium than Firm 1 is to Firm 2 in the efficient equilibrium. Consumers benefit directly from the greater competitive threat of Firm 2 in the inefficient equilibrium.

Proposition 3 *Consumer surplus is weakly higher in the inefficient pure strategy equilibrium of the product improvement game than in the efficient equilibrium, and strictly higher when* $w < \gamma_2$.

Proof. The surplus that each consumer enjoys from the purchase of system i is $CS_i = q_i - P_i$. The equilibrium price of system i when $q_i > q_j$ is $P_i = q_i - \max(q_j - w; 0)$. Therefore, consumer surplus when Firm i wins the market is $CS_i = \max(q_j - w, 0)$. It follows that $CS_1 \geq CS_2$, with a strict inequality when $\gamma_2 > w$. ■

The ability of Firm 1 to impose a technological tie obviously threatens social welfare whenever the product improvement game yields an equilibrium in which the more efficient Firm 2 invests. Equilibria with actual or threatened tying exist for all values of $w < \gamma_1 + z^M$, and for $w < \bar{w} - \Gamma$ all of the equilibria of the tying game involve an actual or threatened tie should Firm 2 invest. Furthermore, when $r(z^M) < \Gamma M$, investment by Firm 2 is the unique equilibrium of the product improvement game and is the efficient market structure, but technological tying destroys the possibility of an efficient market structure when $w < \bar{w} - \Gamma$.

Corollary 1 If $r(z^M) < \Gamma M$, then technological tying reduces social welfare relative to the pure strategy equilibrium of the product improvement game.

If $r(z^M) \geq \Gamma M$, there are multiple pure strategy equilibria of the product improvement game. Technological tying does not improve total welfare in this case and can reduce welfare by preventing an efficient pure strategy equilibrium.

Corollary 2 If $r(z^M) \geq \Gamma M$, then technological tying weakly reduces social welfare relative to pure strategy equilibria of the product improvement game. If the efficient equilibrium is focal, then technological tying strictly reduces social welfare. Otherwise, the ability of Firm 1 to technologically tie is irrelevant for market structure and social welfare (but not for consumer welfare).

Technological tying is never in the interest of consumers. It is evident from Proposition 2 that consumer surplus is zero for all values of w in the technological tying game. Tying eliminates Firm 2 as a potential competitor when Firm 1 invests, and the high component price eliminates Firm 1 as a potential competitor when Firm 2 invests and wins the market. Absent any effective competition, consumer surplus is fully extracted.

Corollary 3 *Consumer welfare is weakly lower in the technological tying game than in the product improvement game. If* $w < \gamma_2$, *then consumer surplus is strictly lower in the technological tying game relative to the inefficient pure strategy equilibrium of the product improvement game. If* $w < \gamma_1$, *then consumer surplus also is strictly lower relative to the efficient equilibrium of the product improvement game.*

An important caveat is in order. The above welfare analysis is premised on the assumption that the wholesale price is the same in the product improvement game and the corresponding technological tying game. The assumption is most plausible when the wholesale price is determined by regulation. The assumption also makes sense for our specific model in which the wholesale price is a market price, and technological tying allows the upstream monopolist to price discriminate between different uses for the component. More generally, the non-discriminatory profit-maximizing price that the upstream monopolist would charge without technological tying is a compromise between the profit-maximizing prices for each possible use. Technological tying allows the firm effectively to set a higher price for the component when it is used as part of a system, and a lower price for other uses. When such a mixed bundling strategy is profitable, a prohibition against technological tying could cause the price for alternative uses to increase, depending on price elasticities. This possibility tempers the case against technological tying.

4 Mixed strategies

4.1 The Product Improvement Game

The product improvement game lacks pure strategy equilibria in which both firms invest (Proposition 1). Moreover, firms' preferences over alternative pure strategy equilibria disagree if the wholesale price of the component is low. When $w < \bar{w} - \Gamma$, Firm 1 strictly prefers the inefficient pure strategy equilibrium in which it invests and forecloses Firm 2. Firm 2, of course, prefers the efficient equilibrium in which it wins the market. There would be a compelling case to restrict our analysis to the pure strategy equilibria of the games if the firms could coordinate their R&D investments. This could come about because, for example, one of the firms observably moves first in its choice of R&D expenditure. However, the characteristics of R&D investment do not suggest that such coordination would be easy. Firms often invest with limited information about their rivals' investments and there is often no natural first mover in R&D. Absent coordination on a pure strategy equilibrium, mixed strategy equilibria in which both firms invest with some probability are plausible. The possibilities of wasteful investments in product improvement, deficient product improvement, or no investments at all, are all realistic outcomes when firms are unsure of each other's incentives and must form beliefs about what the other will do.[17]

In general, there exist multiple mixed strategy equilibria. The Appendix shows that in any mixed strategy equilibrium, at least one firm's strategy must have at least one discrete component.[18] To keep things simple we only consider mixed strategy equilibria in which each firm randomizes over a binary support. That is, Firm 1 randomizes between z_1^L and z_1^H with $z_1^H > z_1^L$ and Firm 2 randomizes between z_2^L and $z_2^H > z_2^L$. While equilibrium mixed strategies involving three or more discrete levels of investment may exist, our focus on binary mixed strategy equilibria is sufficient to demonstrate that welfare conclusions depend on equilibrium selection and that consumers can be better off when the firms play mixed strategies.

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[17] Mixed strategy equilibria can be interpreted in terms of each player's beliefs about the actions of others. Alternatively, it is possible to "purify" mixed strategy equilibria along the lines of Harsanyi (1973) as the limit of a Perfect Bayesian Equilibrium of a corresponding game of incomplete information in which the two firms are unsure of each other's incentive for product improvement. Thus the mixed strategy equilibrium has a realistic interpretation as rational conduct in a strategically uncertain market environment. Following a different approach, Cheng and Zhu (1995) show that if agents have quadratic utility, mixed strategy equilibria exist with unique best-reply probabilities for each agent.

[18] This assumes $\Gamma > 0$. A symmetric continuous mixed strategy equilibrium may exist if $\Gamma = 0$.

There are two possible binary mixed strategy equilibria to consider, depending on which firm wins the market when both firms invest at high levels. The first candidate mixed strategy equilibrium features $z_2^H \geq z_1^H - \Gamma$. The more efficient Firm 2 is sure to win the market by investing high and does so efficiently by choosing $z_2^H = z^M$. This being the case, Firm 1 only has an incentive to invest if it wins when Firm 2 invests low, i.e. $z_1^H \geq z_2^L + \Gamma$. Firm 2 also has an incentive for a lower level of investment that wins the market when Firm 1 invests low. As there are no other scenarios for profitable investment, Firm 1's low level of investment is zero.

Proposition 4 *For* Γ *sufficiently small, a mixed strategy equilibrium exists in which Firm 2 randomizes between* z^M *and* z_2^L *and Firm 1 randomizes between* z_1^H *and* 0*, with* $z^M \geq z_1^H - \Gamma > z_2^L > 0$ *if and only if:*

$$w \leq \gamma_2 + z_2^L \tag{2}$$

$$r'(z_1^H)(z_1^H - z_2^L - \Gamma) = r(z_1^H) \tag{3}$$

$$r'(z_2^L)(z_2^L + \Gamma) - r(z_2^L) = \pi^M - (z_1^H - \Gamma)M + r'(z_2^L)z_1^H. \tag{4}$$

Firm 1 invests z_1^H *with probability*

$$\alpha = 1 - \frac{r'(z_2^L)}{M} \tag{5}$$

and Firm 2 invests z^M *with probability*

$$\beta = 1 - \frac{r'(z_1^H)}{M}. \tag{6}$$

The equilibrium exists if $\Gamma M < r(z^M)$*.*

The proof is in the Appendix. Firm 1 must be indifferent between not investing and investing z_1^H given Firm 2's strategy. Furthermore, z_1^H must be locally optimal. Together, these conditions imply equations (3) and (6). Similarly, Firm 2 must be indifferent between investing z^M and z_2^L given Firm 1's strategy, and z_2^L must be locally optimal. These conditions imply equations (4) and (5). Moreover, w must not be so large as to foreclose either z_1^H or z_2^L. Since $z_1^H + \gamma_1 > z_2^L + \gamma_2$, the binding condition is $w \leq z_2^L + \gamma_2$. The assumed mixed strategy equilibrium exists only if Γ is sufficiently small; otherwise equations (3)-(4) do not have an appropriate solution.

As an example, consider the quadratic case: $r(z) = \frac{1}{2}kz^2$ and define $m \equiv \frac{M}{k}$. Then

$$z_1^H = \frac{2}{3}m(1 + \frac{\Gamma}{m}); \quad z_2^L = \frac{1}{3}m(1 - 2\frac{\Gamma}{m})$$

and

$$\alpha = \frac{2}{3}(1 + \frac{\Gamma}{m}); \quad \beta = \frac{1}{3}(1 - 2\frac{\Gamma}{m}).$$

The mixed strategy equilibrium exists if $w \leq \gamma_2 + z_2^L$ and $\Gamma < \frac{1}{2}m$.

Under some conditions there is another mixed strategy equilibrium in which Firm 1 randomizes between z^M and z_1^L and Firm 2 randomizes between z_2^H and 0, with $z^M \geq z_2^H + \Gamma > z_1^L \geq 0$. These two equilibria have similar properties when Γ is small. For the quadratic case,

$$z_1^L = \frac{1}{3}m(1 + 2\frac{\Gamma}{m}); \quad z_2^H = \frac{2}{3}m(1 - \frac{\Gamma}{m})$$

and

$$\alpha = \frac{1}{3}(1 + 2\frac{\Gamma}{m}); \quad \beta = \frac{2}{3}(1 - \frac{\Gamma}{m}).$$

The Appendix shows that this equilibrium exists if $w \leq \gamma_1 + z_1^L$ and $\Gamma < \frac{1}{4}m$.

We assumed that the lower quality firm remains in the industry and disciplines the price charged by the higher quality firm unless it is otherwise foreclosed. Investing is a dominant strategy for Firm 2 in the product improvement game when the weaker firm exits the industry. Therefore the game does not have a mixed strategy equilibrium if the weaker firm exits the industry. Finally, allowing Firm1 to have the initial quality advantage does not qualitatively change the results when the firms play mixed strategies.

4.2 The Technological Tying Game

Mixed strategy equilibria also can exist for the technological tying game, but only for a relatively high range of w. Tying is a weakly dominant strategy when $w < \bar{w} - \Gamma$, hence mixed strategies can exist only for higher values of w. An implication is that only the pure strategy equilibrium exists in the limiting technological tying game as $\Gamma \to 0$. A mixed strategy equilibrium of the technological tying game exists under particular conditions when $w \geq \overline{w} - \Gamma$.

Proposition 5 *In the technological tying game, a mixed strategy equilibrium exists in which Firm 1 randomizes between z^M and z_1^L and Firm 2 randomizes between z_2^H and 0 with $z^M > z_2^H + \Gamma > z_1^L$ if and only if*[19]

$$\bar{w} - \Gamma \leq w < \gamma_1 + z^M - \frac{r(z^M - \Gamma)}{r'(z^M - \Gamma)}, \tag{7}$$

$$(M - r'(z_1^L))(w - \gamma_1) = \pi^M - \pi(z_1^L), \tag{8}$$

$$\pi(z_2^H) + r'(z_2^H)(\gamma_2 - w) = 0, \tag{9}$$

and $\Gamma < z^M$. Firm 1 invests z^M with probability

$$\alpha = 1 - \frac{r'(z_2^L)}{M} \tag{10}$$

and Firm 2 invests z_2^H with probability

$$\beta = 1 - \frac{r'(z_1^H)}{M}. \tag{11}$$

Firm 1 imposes a technological tie whenever $\gamma_1 + z_1 > w$.

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[19] The first condition requires $\bar{w} - \Gamma \leq \gamma_1 + z^M - \frac{r(z^M-\Gamma)}{r'(z^M-\Gamma)}$, or $\frac{r(z^M-\Gamma)}{r'(z^M-\Gamma)} \leq \frac{r(z^M)}{r'(z^M)}$, which is satisfied for convex $r(\cdot)$.

The proof is in the appendix. The conditions of proposition are necessary and sufficient conditions to satisfy the local optimality and indifference, requirements of a mixed strategy equilibrium.

Using these results, direct calculation proves the following special case.

Corollary 4 Suppose $r(z) = \frac{1}{2}kz^2$ and define $m \equiv \frac{M}{k}$. A binary mixed strategy of the technological tying game exists in which Firm 1 randomizes over z^M and $z_1^L = 2[w - (\bar{w} - \Gamma)]$ and Firm 2 randomizes over $z_2^H = 2(w - \gamma_2)$ and 0 if and only if

$$\bar{w} - \Gamma < w < \bar{w} - \frac{1}{2}\Gamma.$$

Firm 1 and Firm 2 both prefer the pure strategy equilibrium in which Firm 2 captures the market. In the mixed strategy equilibrium, however, the firms fail to coordinate their beliefs on this outcome. Firm 1 invests in product improvement with positive probability out of a concern that Firm 2 might decline to improve its product. And having invested, Firm 1's finds it in its own self interest to impose a technological tie even when Firm 2 does invest. Firm 2, for its part, becomes hesitant to invest out of fear that Firm 1 will also invest and impose a technological tie.

We focus the rest of our analysis of mixed strategies on the binary equilibria described in Propositions 4 and 5 for the product improvement game and the technological tying game, respectively. We implicitly assume that other mixed strategy equilibria are not selected. This approach is sufficient to demonstrate the implications of equilibrium selection for evaluating technological tying.

4.3 Welfare

How do consumers fare if the firms play a mixed strategy equilibrium? The next proposition demonstrates that consumers do better in the mixed strategy equilibrium of the product improvement game than in the efficient pure strategy equilibrium. They may do better or worse than in the inefficient pure strategy equilibrium (which yields $max(\gamma_2 - w, 0)$), depending on parameters.

Proposition 6 *Expected consumer surplus is higher in the mixed strategy equilibrium of the product improvement game (of Proposition 5) than in the efficient pure strategy equilibrium.*

Proof. There are two cases to consider.

Case 1: $w \leq \gamma_1$. Consumer surplus (for each of the M consumers) in the efficient pure strategy equilibrium is equal to $\gamma_1 - w$ with probability one. Expected consumer surplus in the mixed strategy equilibrium is

$$CS_{mixed} = \gamma_1 - w + \alpha\left(1 - \beta\right)\left(z_2^L + \Gamma\right) + \alpha\beta z_1^H > \gamma_1 - w.$$

Case 2: $\gamma_1 < w \leq \gamma_2 + z_2^L$.[20] Consumers get zero surplus in the efficient pure strategy equilibrium. Expected consumer surplus in the mixed strategy equilibrium is

$$\begin{aligned} CS_{mixed} &= \alpha\,(1-\beta)\,(z_2^L + \Gamma) + \alpha\beta z_1^H + \alpha(\gamma_1 - w) \\ &\geq \alpha\beta(z_1^H - z_2^L - \Gamma) > 0. \end{aligned}$$

■

Corollary 5 Expected consumer surplus is a decreasing function of the wholesale price w.

This last result follows from the fact that competitive pressure is greater when the wholesale price is less.

Social welfare (measured by social surplus) obviously is highest in the efficient pure strategy equilibrium. But how do the mixed strategy and the inefficient pure strategy equilibria of the product improvement game compare from a social welfare perspective? The answer is not obvious.

Total welfare in the mixed strategy equilibrium of the product improvement game is equal to

$$\begin{aligned} W_{mixed} &= \alpha\,(1-\beta)\left[\gamma_1 M + \pi(z_1^H) - r(z_2^L)\right] \\ &\quad + (1-\alpha)\,\beta\left[\gamma_2 M + \pi^M\right] \\ &\quad + \alpha\beta\left[\gamma_2 M + \pi^M - r(z_1^H)\right] \\ &\quad + (1-\alpha)\,(1-\beta)\,[\gamma_2 M + \pi(z_2^L)]. \end{aligned}$$

where $\pi(z) \equiv zM - r(z)$. Note that $\pi(z) < \pi^M$ if $z < z^M$. The level of welfare in the efficient pure strategy equilibrium is $W_{pure_+} = \gamma_2 M + \pi^M$. Welfare is lower in the mixed strategy equilibrium compared to the efficient pure strategy equilibrium for three reasons. First, market structure is distorted because the less efficient Firm 1 sometimes wins the market in the mixed strategy equilibrium. Second, there is wasteful investment in product improvement when both firms invest. Third, product improvement is deficient when neither firm invests at the efficient level z^M.

The comparison with the inefficient pure strategy equilibrium is less clear on first inspection. The level of welfare in the inefficient pure strategy equilibrium is $W_{pure_-} = \gamma_1 M + \pi^M$. Therefore, the difference in welfare between the efficient pure strategy equilibrium and the mixed strategy equilibrium is

$$\begin{aligned} W_{mixed} - W_{pure_-} &= \alpha\,(1-\beta)\left[\pi(z_1^H) - \pi^M - r(z_2^L)\right] \\ &\quad + (1-\alpha)\,\beta[\Gamma M] \\ &\quad + \alpha\beta\left[\Gamma M - r(z_1^H)\right] \\ &\quad + (1-\alpha)\,(1-\beta)\left[\Gamma M - \pi^M + \pi(z_2^L)\right]. \end{aligned} \tag{12}$$

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[20] Recall that the mixed strategy equilibrium of the product improvement game does not exist for $w > \gamma_2 + z_2^L$ (Proposition 5).

The first term is negative, because Firm 1 has deficient investment incentives in the mixed strategy equilibrium and there is redundant investment by Firm 2. The second term is positive because the mixed strategy equilibrium sometimes selects an efficient market structure. The third term is ambiguous because, even though Firm 2 is more efficient, Firm 1's investment in product improvement is wasteful. The fourth term is also ambiguous because $\Gamma M < \pi^M$ from assumption A3 (Firm 1 can profitably leapfrog Firm 2) and $\pi(z_2) \geq 0$. Thus, on the one hand, the mixed strategy equilibrium sometimes beneficially achieves a more efficient market structure. On the other hand, a pure strategy equilibrium eliminates wasteful investment and improves investment incentives. It appears that welfare may be higher or lower in the mixed strategy equilibrium, depending on the strength of these various effects. It is, however, possible to resolve this ambiguity when Firm 2's efficiency advantage is small.

Proposition 7 *Expected welfare is lower in the mixed strategy equilibrium than in an inefficient pure strategy equilibrium of the product improvement game if* Γ *is sufficiently small.*

Proof. >From equation (12), as $\Gamma \to 0$,

$$\begin{aligned} W_{mixed} - W_{pure_-} \quad \to \quad & \alpha(1-\beta)[\pi(z_1^H) - \pi^M - r(z_2^L)] \\ & -\alpha\beta r(z_1^H) + (1-\alpha)(1-\beta)[-\pi^M + \pi(z_2^L)]. \end{aligned} \tag{13}$$

Each term on the right-hand-side of (13) is negative. ■

The result that expected welfare is lower in the mixed strategy equilibrium than in the pure strategy equilibria of the product improvement game as $\Gamma \to 0$ is intuitive. The mixed strategy equilibria incur unnecessary costs when both firms invest and, in the limit as $\Gamma \to 0$, provide no benefit when Firm 2 invests instead of Firm 1. Figure 2 compares expected total welfare in the mixed and pure strategy equilibria when the R&D function has the form $r(z) = kz^2$ for different values of Γ. Note that the mixed strategy equilibrium yields slightly greater expected welfare than the inefficient pure strategy for some values of Γ.

A welfare analysis of technological tying is more complicated when there exists a mixed strategy equilibrium of the product improvement game. When $w < \bar{w} - \Gamma$, technological tying accomplishes the same market structure as the inefficient pure strategy equilibrium. As observed in Proposition 7, expected social welfare in the mixed strategy equilibrium is lower than social welfare in an inefficient pure strategy equilibrium when Γ is sufficiently small. Therefore, technological tying can increase social welfare when a mixed strategy equilibrium of the product improvement game is focal.

Corollary 6 *If* $w < \bar{w} - \Gamma$ *and* Γ *is sufficiently small, technological tying increases social welfare relative to the mixed strategy equilibrium of the product improvement game.*

Allowing for mixed strategy equilibria does not change the conclusion that technological tying is not in the interest of consumers. Consumers earn no surplus in the mixed strategy equilibria of the tying game because tying eliminates Firm 2 as a potential competitor when Firm 1 wins the market and the high component price eliminates Firm 1 as a potential competitor when Firm 2 wins the market. However, mixed strategy equilibria of the tying game exist only for $w \geq \bar{w} - \Gamma > \gamma_2$ and consumers earn no surplus in the pure strategy equilibria of the product improvement game when $w \geq \gamma_2$. Thus, consumers are no worse off in a mixed strategy tying equilibrium than they are in the pure strategy equilibria of the product improvement game, but they are better off compared to a mixed strategy equilibrium of the product improvement game. Thus, we can generalize Corollary 3.

Corollary 7 *Consumer welfare is weakly lower in pure and mixed strategy equilibria of the technological tying game compared to pure and mixed strategy equilibria of the product improvement game.*

5 Some Extensions

Although our basic model is very stylized, the basic results extend to more realistic situations. For example, we could extend the model to allow for n firms that supply vertically differentiated systems, each with exogenous quality γ_j for $j = 1, ..., n$. Assume that Firm 1 owns the essential component and suppose that $\gamma_1 < \gamma_2 < ... < \gamma_n$. Define $\Gamma_j = \gamma_j - \gamma_1$ and assume that $\pi^M > \Gamma_n M$, so that Firm 1 can leapfrog the exogenous quality advantage of the most efficient firm. The pure strategy equilibria of the product improvement game are similar to the equilibria described in Section 3. In particular, there exists an equilibrium in which Firm 1 invests in quality improvement and wins the market if $r(z^M) \geq \Gamma_n M$. Of course if this condition holds, then there exist other pure strategy equilibria in which other firms invest and win the market, including the most efficient firm. The pure strategy equilibria of the technological tying game with many firms are also similar to the equilibria for the duopoly model.

Another extension to the model is to allow for network effects. For example, suppose that the perceived quality of the j^{th} system is $q_j = \gamma_j + r(z_j) + v(y_j)$, where y_j is the network of consumers that choose technology j and $v(y_j)$ is a positive network externality with $v(0) = 0$ and $v'(y) \geq 0$. If the networks are fully compatible, and if all consumers choose a system, then $v(y_j) = v(M)$ for all $j = 1, ..., n$. With compatibility, the network externality is common to all systems and raises each system's quality by the same level. As a consequence, the relative payoffs from product improvement are unchanged, assuming that unsuccessful firms remain in the market as potential competitors. Under the assumption of full compatibility, network effects do not change the predictions in the basic models introduced in section 3. However, network effects may offer a further justification for the winner-take-all assumption.

If networks are incompatible, multiple equilibria may arise because the values that consumers place on each system depend on the number of consumers who are expected to adopt the system. Thus, in the absence of investments in quality improvement, Firm 1 may win the market because consumers expect it to win and $\gamma_1 + v(M) > \gamma_2$. Investments in quality improvement can reinforce consumer expectations and lead to inefficient market outcomes. For example, the less efficient Firm 1 could embark on an R&D program to improve the quality of its system. If this is communicated to consumers, they may reason that Firm 1 will become the more efficient firm, which would justify purchase of its system. If systems are incompatible, these expectations reinforce can each other. Firm 1 may become the system of choice because others expect it to be superior, even if the firm's R&D program is ultimately unsuccessful.

6 Conclusions

We have examined the causes and consequences of technological tying in a winner-take-all model of a market for systems. In this model, a vertically integrated upstream monopolist supplies an essential component to a more efficient independent competitor in the downstream systems market. The two firms compete on the price and quality for sales to consumers with homogeneous preferences over these vertically differentiated products. If the wholesale price of the essential component is insufficiently remunerative, then the upstream monopolist has an incentive to foreclose rival systems, either by selling only systems, contractually tying components, or designing an essential component so that it works better with its own systems (technological tying). A technological tying strategy has the advantage of facilitating price discrimination for alternative uses of the essential component. The equilibrium market structure is inefficient in this case. A technological tie, or even in some cases the mere threat of technological tie, can reduce social welfare by distorting market structure.

The ambiguity regarding the welfare effects of technological tying has to do with the nature of equilibrium when technological tying is infeasible for the vertically-integrated firm, e.g. due to antitrust enforcement. In some cases, a mixed strategy equilibrium can emerge when technological tying is infeasible. If a mixed strategy equilibrium exists and is focal, then the prevention of technological tying reduces social welfare under some conditions. If instead an efficient pure strategy equilibrium is focal, then preventing technological tying increases social welfare. If the inefficient pure strategy equilibrium is focal, then technological tying is irrelevant for social welfare.

If the wholesale price of the essential component is sufficiently near the monopoly price, then the upstream monopolist and independent downstream firm both prefer the pure strategy equilibrium in which the upstream monopolist supplies the component efficiently and the independent firm wins the downstream market. Surprisingly, in this case there can also exist a mixed strategy equilibrium in which the vertically integrated firm invests with positive probability to improve its product and forecloses the more efficient downstream firm with a technological tie. This unfortunate coordination failure would be prevented by a ban on technological tying.

The simple vertical differentiation model does not admit pure strategy equilibria in which both firms invest in product improvement. We plan in future work to allow for systems that are both vertically and horizontally product differentiated, so that some consumers prefer the system sold by Firm 1 and others prefer Firm 2's system, even when each has the same (vertical) quality and is sold at the same price. In this richer model, both firms may have an incentive to improve their products in a pure strategy equilibrium, and the ability of Firm 1 to technological tie might reduce Firm 2's market share short of complete foreclosure. The welfare effects of technological tying are more subtle in this case.

References

Bergman, Mats A. (2000), "A Note on N. Economides: 'The Incentive for Non-Price Discrimination by an Input Monopolist'," *International Journal of Industrial Organization*, vol. 18, pp. 985-988.

Bolton, Patrick and Michael Whinston (1991), "The 'Foreclosure' Effects of Vertical Mergers," *Journal of Institutional and Theoretical Economics*, pp. 207-26.

Bolton, Patrick and Michael Whinston (1993), "Incomplete Contracts, Vertical Integration, and Supply Assurance," *Review of Economic Studies*, pp. 121-48.

Bork, Robert (1978), *The Antitrust Paradox*, New York: Basic Books.

Bowman, Ward (1957), "Tying Arrangements and the Leverage Problem," *Yale Law Journal*, 19.

Cheng, Leonard and Min Zhu (1995), "Mixed-Strategy Nash Equilibrium Based upon Expected Utility and Quadratic Utility," *Games and Economic Behavior*, vol 9, pp. 139-150.

Choi, Jay Pil, Gwanghoon Lee, and Christodoulis Stefanadis (2001), "The Effects of Integration on R&D Incentives in Systems Markets," *Netnomics*, forthcoming.

Choi, Jay Pil and Christodoulis Stefanadis (2001), "Tying, Investment, and the Dynamic Leverage Theory," *Rand Journal of Economics*, vol. 32, no.1, pp. 52-74.

Economides, Nicholas, (1998), "The Incentive for Non-Price Discrimination by an Input Monopolist," *International Journal of Industrial Organization*, vol. 16, pp. 271-284.

Economides, Nicholas (2000), "Comment on 'A Note on N. Economides: The Incentive for Non-Price Discrimination by an Input Monopolist'," *International Journal of Industrial Organization*, vol. 18, pp. 989-991.

Farrell, Joseph and Michael Katz (2000), "Innovation, Rent Extraction, and Integration in Systems Markets," *Journal of Industrial Economics*, vol. 48, no. 4, pp. 413-432.

Harsanyi, John (1973), "Games with Randomly Disturbed Payoffs: A New Rationale for Mixed Strategy Equilibrium Points," *International Journal of Game Theory*, 2, 1-23.

Hart, Oliver and Jean Tirole (1990), "Vertical Integration and Market Foreclosure," *Brookings Papers on Economic Activity: Microeconomics,* pp. 205-285.

Kranton, Rachel and Deborah Minehart (2002), "Vertical Merger and Specific Investments: A Tale of the Second Best," University of Maryland working paper.

Krattenmaker, Thomas and Steven Salop (1986), "Anticompetitive Exclusion: Raising Rivals' Costs to Achieve Power Over Price," *Yale Law Journal*, vol. 96, pp. 209-93.

Lessig, Lawrence (2000), "Brief of Professor Lawrence Lessig as Amicus Curiae," U.S. v. Microsoft, Civil Action No. 98-1233 (TPJ), U.S. Dist. Ct. for the District of Columbia.

Liebowitz, Stan and Stephen E. Margolis (1999), *Winners, Losers, & Microsoft, Competition and Antitrust in High Technology*, The Independent Institute, Oakland, CA.

Ordover, Janusz, Garth Saloner and Steven Salop (1990), "Equilibrium Vertical Foreclosure," *American Economic Review*, pp. 127-42.

Rey, Patrick and Jean Tirole (1997), "A Primer on Foreclosure," *Handbook of Industrial Organization*, forthcoming.

Riordan Michael H. and Steven Salop (1995), "Evaluating Vertical Mergers: A Post-Chicago Approach," *Antitrust Law Journal*, vol. 63, no. 2, pp. 513-568.

Salop, Stephen and David Scheffman (1983), "Rising Rivals' Costs", *The American Economic Review*, vol. 73, pp.267-271.

Sibley, David, and Dennis Weisman (1998),"Raising Rivals' Costs: Entry of an Upstream Monopolist into Downstream Markets," *Information Economics and Policy*, vol. 10, no. 4, December, pp. 551-570.

Appendix

A.1. Proof of Proposition 1. Pure strategy equilibria of the product improvement game.

The text shows that if the firms' choice sets are to invest z^M or zero, there is an equilibrium in which Firm 2 invests z^M and Firm 1 invests zero. In addition, if and only if $r(z^M) \geq \Gamma M$, then there is another equilibrium in which Firm 1 invests z^M and Firm 2 invests zero. There are no other profitable investments. Investment by Firm 1 cannot be profitable unless it can win the market from Firm 2, which requires $z_1 \geq z^M + \Gamma$ when Firm 2 invests. The best investment for Firm 1 maximizes $\pi_1 = \left(z_1 - z^M - \Gamma + w\right) M - r(z_1)$ subject to this constraint. Convexity of $r(z)$ implies that the constraint binds and Firm 1's maximum deviation profit is $\pi_1 = wM - r(z^M + \Gamma) < wM$. Thus Firm 1 earns less profit by deviating from $z_1 = 0$ when Firm 2 invests, and given that Firm 1 chooses $z_1 = 0$, the profit-maximizing investment for Firm 2 is $z_2 = z^M$. If $z_1 = z^M$ and $z_2 = 0$, Firm 1 has no incentive to deviate and earn $\pi_1 = wM$ by choosing $z_1 = 0$, and has no incentive to choose any other level of quality improvement. If it is profitable for Firm 2 to deviate, then Firm 2 would choose z^M and earn $\pi_2 = \Gamma M - r(z^M)$. Therefore, Firm 2 has no incentive to deviate if and only if $r(z^M) \geq \Gamma M$.

A.2. Proof of Proposition 2. Pure strategy equilibria of the technological tying game.

The text identifies the equilibria of the technological tying game under the assumption that the firms invest z^M or nothing. Consider any investment levels (z_1, z_2). In the second stage of the game, after the firms invest, Firm 1 would foreclose Firm 2 with a technological tie (unless Firm 2 is already foreclosed by a high component price) if $w < \gamma_1 + z_1$, because it can earn $(\gamma_1 + z_1)M$ on its own, ignoring sunk investment costs, and only wM by selling the component to Firm 2. Given that $w < \gamma_1 + z_1$ and Firm 2 is foreclosed, Firm 1 earns $\pi_1(z) = (\gamma_1 + z)M - r(z)$. This is a maximum at $z = z^M$, for which $\pi_1(z^M) = [\bar{w} - \Gamma]M$. Thus, for $w < \gamma_1 + z^M$, there is an equilibrium in which Firm 1 invests z^M and forecloses Firm 2 with a technological tie. Any investment level other than z^M would be less profitable for Firm 1, and any positive investment by Firm 2 that threatened Firm 1's profit would be undone by the technological tie and therefore would not be profitable. If $\gamma_2 \leq w < \gamma_1 + z^M$, Firm 1 does not need to impose a tie because the price of component A forecloses Firm 2 if it does not invest. If $w < \gamma_2$, Firm 1 imposes a technological tie to foreclose competition from Firm 2 when it does not invest.

If $w \geq \gamma_1 + z_1$, Firm 1 would not foreclose Firm 2 in the second stage of the game. Firm 2 would earn $\pi_2(z) = (\gamma_2 + z - w - \max(\gamma_1 + z_1 - w, 0))M - r(z)$, which is a maximum at $z = z^M$. However, Firm 1 can guarantee itself a profit of $\pi_1(z) = (\gamma_1 + z)M - r(z)$ by imposing a technological tie. This is a maximum at $z = z^M$, for which $\pi_1(z^M) = [\bar{w} - \Gamma]M$. If $w < \bar{w} - \Gamma$, Firm 1 is better off when it invests for any level of investment by Firm 2. Note that $\bar{w} - \Gamma < \gamma_1 + z^M$ Therefore, if $w < \bar{w} - \Gamma$, then a fortieri, $w < \gamma_1 + z^M$, so Firm 1 would foreclose competition from Firm 2 when $w < \bar{w} - \Gamma$ and it invests. Therefore $(z_1 = z^M, z_2 = 0)$ is the unique equilibrium when $w < \bar{w} - \Gamma$, with Firm 1 imposing a technological tie if $w < \gamma_2$. If $w \geq \gamma_1 + z^M$, Firm 1 is better off selling the component to Firm 2, regardless of its own level of investment. Therefore, $(z_1 = 0, z_2 = z^M)$ is the unique equilibrium when $w \geq \gamma_1 + z^M$. For $\bar{w} - \Gamma < w < \gamma_1 + z^M$, the technological tying game has multiple equilibria, corresponding to investment z^M by Firm 1 or Firm 2. In this range of component prices it is an equilibrium for Firm 2 to invest z^M and for Firm 1 to not invest. A second equilibrium exists in which Firm 1 invests z^M because Firm 1 would foreclose Firm 2 in the second stage of the game, even if Firm 2 invests. Knowing this, Firm 2 would not invest, and consequently it would be rational for Firm 1 to invest z^M. Therefore, in this range of component prices, it is also an equilibrium for Firm 1 to invest z^M and for Firm 2 not to invest. In this range of component prices for which there are multiple equilibria, both firms are better off in the equilibrium in which Firm 2 invests.

Summarizing, if $w < \bar{w} - \Gamma$, there is a unique equilibrium outcome of the technological tying game in which $z_1 = z^M$ and $z_2 = 0$. Firm 1 would foreclose Firm 2 with a technological tie if $w < \gamma_2$. The threat of foreclosure is sufficient to deter investment by Firm 2 if $\gamma_2 \leq w < \bar{w} - \Gamma$. Note that Assumption A.3 implies that $\gamma_2 < \bar{w} - \Gamma$. If $w \geq \gamma_1 + z^M$, there is a unique equilibrium outcome in which $z_1 = 0$ and $z_2 = z^M$. For $\bar{w} - \Gamma < w < \gamma_1 + z^M$ there are multiple equilibria corresponding to investment of z^M by Firm 1 or Firm 2, however both firms are better off in the second equilibrium.

A.3 Proof that in a mixed strategy equilibrium, at least one firm's strategy must have a discrete component

Let S_i denote the support of Firm i's mixed strategy, and define $\overline{z}_i = \inf S_i$. We prove by contradiction that either $\overline{z}_1$ is a discrete element of S_1, or $\overline{z}_2$ is a discrete element of S_2. Let S_i denote the support and Ψ_i the c.d.f. of Firm i's mixed strategy for $i = 1, 2$. The assumption (A1) that $r(z)$ is convex increasing requires that the S_i are bounded. Define the maximal investment levels $\bar{z}_i = \sup S_i$. Obviously we must have $\Psi_i(\bar{z}_i) = 1$. Given quality choices the (expected) profit of Firm 1 is

$$\pi_1(z_1) = M\left[w + \int_0^{z_1 - \Gamma} (z_1 - \Gamma - \max\{z_2, w - \gamma_2\})\, d\Psi_2(z_2)\right] - r(z_1),$$

and of Firm 2 is

$$\pi_2(z_2) = M \int_0^{z_2 + \Gamma} (z_2 + \Gamma - \max\{z_1, w - \gamma_1\})\, d\Psi_1(z_1) - r(z_2).$$

Suppose that both firms' strategies are continuous over a region that includes the upper bounds of S_1 and S_2. Thus $[z', \bar{z}_1] \subseteq S_1$ and $[z'', \bar{z}_2] \subseteq S_2$ for $z' < \bar{z}_1$ and $z'' < \bar{z}_2$. Firm 1's indifference condition, $\pi_1(z_1) = \pi_1(\bar{z}_1)$ for all $z_1 \in S_1$, requires $\Psi_2(\bar{z}_2) = \frac{r'(\bar{z}_2 + \Gamma)}{M}$ for $z_2 \in [z' - \Gamma, \bar{z}_1 - \Gamma) \subset S_2$. Therefore, $\bar{z}_1 - \Gamma \leq \bar{z}_2$. Similarly, Firm 2's indifference condition, $\pi_2(z_2) = \pi_2(\bar{z}_2)$ for all $z_2 \in S_2$, requires $\Psi_1(\bar{z}_1) = \frac{r'(\bar{z}_1 - \Gamma)}{M}$ for $z_1 \in [z'' + \Gamma, \bar{z}_2 + \Gamma) \subset S_1$. Therefore, $\bar{z}_1 - \Gamma = \bar{z}_2$. Furthermore, $\Psi_1(\bar{z}_1) = \Psi_1(\bar{z}_2 + \Gamma) \geq \frac{r'(\bar{z}_2)}{M}$ and $\Psi_2(\bar{z}_2) = \Psi_2(\bar{z}_1 - \Gamma) \geq \frac{r'(\bar{z}_1)}{M}$.

It must also be that $\bar{z}_1 = \bar{z}_2 + \Gamma = z^M$. Clearly, $\bar{z}_1 \geq z^M$; otherwise, Firm 1 could more profitably choose $z_1 = z^M$. But then $\Psi_2(\bar{z}_2) \geq \frac{r'(\bar{z}_1)}{M} \geq \frac{r'(z^M)}{M} = 1$ implies $\bar{z}_1 = z^M$.

Finally $\bar{z}_2 = z^M - \Gamma$ implies that Firm 2 could increase its profit by choosing $z_2 = z^M$. This contradicts the definition of equilibrium. Therefore, either $\bar{z}_1$ is a discrete element of S_1 or $\bar{z}_2$ is a discrete element of S_2.

A.4 Proof of Proposition 4: Mixed strategy equilibrium of the product improvement game.

In the assumed equilibrium, Firm 1 randomizes between z_1^H and 0 and Firm 2 randomizes between z^M and z_2^L, with $z^M \geq z_1^H - \Gamma > z_2^L > 0$.

Necessity: In the candidate mixed strategy equilibrium, Firm 1 must be indifferent between investing z_1^H and investing zero. This requires

$$(1-\beta)[\gamma_1 + z_1^H - \max(\gamma_2 + z_2^L - w, 0)]M + \beta w M - r(z_1^H) = wM$$

Local optimality requires

$$(1-\beta)M = r'(z_1^H).$$

If $w > z_2^L + \gamma_2$, then optimality and indifference for Firm 1 require

$$r'(z_1^H)[z_1^H + \gamma_1] = r(z_1^H),$$

which is impossible because $r(\cdot)$ is strictly convex. Given $w \leq z_2^L + \gamma_2$, indifference and local optimality require

$$r'(z_1^H)(z_1^H - z_2^L - \Gamma) = r(z_1^H).$$

Similarly, Firm 2 must be indifferent between investing z^M and z_2^L. This requires

$$\begin{aligned} & (1-\alpha)[\gamma_2 + z^M - \max(\gamma_1 - w, 0) - w]M + \alpha[\gamma_2 + z^M \\ & - \max(\gamma_1 + z_1^H - w, 0) - w]M - r(z^M) \\ = \; & (1-\alpha)[\gamma_2 + z_2^L - \max(\gamma_1 - w, 0) - w]M - r(z_2^L). \end{aligned}$$

Local optimality requires

$$(1-\alpha)M = r'(z_2^L).$$

If $w \leq z_2^L + \gamma_2 < z_1^H + \gamma_1$, indifference and local optimality for Firm 2 requires

$$r'(z_2^L)(z_2^L + \Gamma - z_1^H) = r(z_2^L) + (z^M - z_1^H)M + (\Gamma M - r(z^M)).$$

Therefore equations (2)-(6) in Proposition 4 are necessary for the mixed strategy equilibrium. Note that the left-hand side of the equation above is negative and the first two terms on the right are positive. Therefore, $\Gamma M < r(z^M)$ is also necessary.

Sufficiency: Firm 1 would not deviate from the assumed equilibrium by investing $0 < z_1 < z_2^L + \Gamma$, because it would have no sales. For $z_2^L + \Gamma \leq z_1 < z^M + \Gamma$, Firm 1 wins the market with probability $1-\beta$ and, by convexity, $z_1 \neq z_1^H$ is not profitable in this interval. If Firm 1 invests $z_1 \geq z^M + \Gamma$, it earns no more than $r'(z_1^H)(z^M - z_2^L) - r(z^M + \Gamma) + wM$. This is less than $r'(z_1^H)(z_1^H - z_2^L - \Gamma) - r(z_1^H) + wM$, its payoff when it invests z_1^H. Hence Firm 1 would not deviate from the assumed equilibrium. Firm 2 could deviate by not investing. This is unprofitable if $(1-\alpha)(z_2^L + \Gamma)M - r(z_2^L) > (1-\alpha)\Gamma M$, or if $r'(z_2^L)z_2^L > r(z_2^L)$, which is true by convexity. Finally, Equation (3) directly implies $z_1^H > z_2^L + \Gamma$, while equation (4) and the convexity of $r(\cdot)$ imply $z^M > z_1^H - \Gamma$. Therefore equations (2)-(6) are sufficient for the mixed strategy equilibrium.

Existence: Assume $w \leq \gamma_2 + z_2^L$. Equation (3) implicitly defines

$$z_2 = \frac{1}{r'(z_1)}[z_1 r'(z_1) - r(z_1)] - \Gamma \equiv \omega(z_1)$$

with the properties $\omega(0) \to -\Gamma$, $\omega'(z_1) = \frac{r''(z_1)r(z_1)}{r'(z_1)^2} \geq 0$ for $z_1 > 0$, $\omega(z^M) = \frac{\pi^M}{M} - \Gamma$, and $\lim_{\Gamma \to 0} \omega(z_1) > 0$ for $z_1 \in (0, z^M]$. Equation (4) implicitly defines

$$z_1 = \frac{\pi^M - [r'(z_2)z_2 - r(z_2)]}{M - r'(z_2)} + \Gamma \equiv \varphi(z_2)$$

with the properties $\varphi(0) = \frac{\pi^M}{M} + \Gamma$, $\varphi'(z_2) = \frac{r''(z_2)[\pi^M - (z_2 M - r(z_2)]}{[M - r'(z_2)]^2} > 0$, and $\varphi(z_2) \to z^M + \Gamma$ as $z_2 \to z^M$. Now define $\hat{z}_1$ by $\omega(\hat{z}_1) = 0$ and $\tilde{z}_2$ by $\varphi(\tilde{z}_2) = z^M$. Sufficient conditions for a solution $(z_1^H, z_2^L) \in (0, z^M)^2$ are $\hat{z}_1 < \frac{\pi^M}{M} + \Gamma$ and $\tilde{z}_2 > \frac{\pi^M}{M} - \Gamma$. (See Figure 3, which shows a fixed point of $\omega(z_1)$ and $\varphi(z_2)$). By continuity, if Γ is sufficiently small, then $\omega(\frac{\pi^M}{M} + \Gamma) > 0$. Furthermore, $\omega(\Gamma) < 0$. These inequalities imply $\hat{z}_1 < \frac{\pi^M}{M} + \Gamma$. Note that $\tilde{z}_2 \to z^M$ as $\Gamma \to 0$. Thus $\tilde{z}_2 > \frac{\pi^M}{M} - \Gamma$ if Γ is sufficiently small.

A.5 Mixed strategy equilibrium of the product improvement game with quadratic R&D costs.

Assume R&D costs are quadratic, $r(z) = \frac{1}{2}kz^2$, and let $m \equiv M/k$. In the assumed equilibrium, Firm 1 randomizes between z^M and z_1^L and Firm 2 randomizes between z_2^H and zero, with $z^M > z_2^H + \Gamma > z_1^L \geq 0$. Furthermore, equilibrium requires $z_1^L > \Gamma$ if $w < \gamma_2$. A mixed strategy equilibrium cannot exist if $w \geq \gamma_2 + z_2^H$, because then Firm 2 would not invest. Therefore, without loss of generality we assume $w < \gamma_2 + z_2^H$. There are two cases to consider, corresponding to $w \leq \gamma_1 + z_1^L$ and $w > \gamma_1 + z_1^L$.

Case (i): $w \leq \gamma_1 + z_1^L$.

Necessity: In the candidate mixed strategy equilibrium, Firm 2 must be indifferent between investing z_2^H and investing zero. The indifference condition for Firm 2 is

$$\begin{aligned} \pi_2(z_2^H) &= (1-\alpha)(\gamma_2 + z_2^H - \max(\gamma_1 + z_1^L - w, 0) - w)M - r(z_2^H) \\ &= (1-\alpha)(z_2^H - z_1^L + \Gamma)M - r(z_2^H) = 0. \end{aligned}$$

Local optimality of z_2^H requires:

$$r'(z_2^H) = (1-\alpha)M. \tag{14}$$

Equation (14) gives for quadratic R&D costs

$$z_2^H = 2(z_1^L - \Gamma). \tag{15}$$

Similarly, assuming $z_1^L > \Gamma$, the indifference condition for Firm 1 is

$$\begin{aligned}
\pi_1(z_1^L) &= (1-\beta)(\gamma_1 + z_1^L - \max(\gamma_2 - w, 0))M + \beta w M - r(z_1^L) = \\
\pi_1(z^M) &= (1-\beta)(\gamma_1 + z^M - \max(\gamma_2 - w, 0))M + \beta(\gamma_1 + z^M - \max(\gamma_2 + z_2^H - w, 0))M - r(z^M)
\end{aligned}$$

or

$$(1-\beta)z_1^L M - r(z_1^L) = \pi^M - \beta(z_2^H + \Gamma)M.$$

The local optimality condition is

$$r'(z_1^L) = (1-\beta)M. \tag{16}$$

Equation (16) gives for quadratic R&D costs

$$z_2^H = \frac{1}{2}(m + z_1^L) - \Gamma. \tag{17}$$

Equations (15) and (17) imply

$$\begin{aligned}
z_1^L &= \frac{1}{3}(m + 2\Gamma) \\
z_2^H &= \frac{2}{3}(m - \Gamma).
\end{aligned}$$

Equilibrium also requires $m > z_2^H + \Gamma$, or $m > \Gamma$. This condition also implies $z_2^H + \Gamma > z_1^L > \Gamma$. Note that $w \le \gamma_1 + z_1^L$ requires $w < \gamma_1 + \frac{1}{3}(m + 2\Gamma)$. The investment probabilities (α, β) follow directly from the local optimality conditions (14) and (16).

Sufficiency: If Firm 2 deviates from the assumed equilibrium and invests z^M, it would earn $(1-\alpha)(z^M + \Gamma - z_1^L)M + \alpha\Gamma M - r(z^M)$, which cannot dominate Firm 2's profit at $z_2 = 0$. This requires

$$r(z^M) > r'(z_2^H)(z^M - z_1^L) + M\Gamma$$

or, for the case of quadratic R&D costs, $\Gamma < \frac{1}{4}m$.

Firm 1 could deviate from the assumed equilibrium by investing $z_1 < z_1^L$, which also must be unprofitable. If $z_1 \ge \Gamma$, this requires $\pi_1(z) = (1-\beta)(\gamma_1 + z - \max(\gamma_2 - w, 0))M + \beta w M - r(z) \le (1-\beta)(\gamma_1 + z_1^L - \max(\gamma_2 - w, 0))M + \beta w M - r(z_1^L)$ for $z < z_1^L$. This is satisfied because$(1-\beta)zM - r(z) > 0$ is increasing in z for $z < z_1^L$. If $z_1 < \Gamma$, then Firm 1 would make no sales and have no incentive to invest unless Firm 2 is foreclosed by w. The assumption that $w \le \gamma_1 + z_1^L$ implies that $w < \gamma_2$. Hence the corresponding condition is $wM \le (1-\beta)(z_1^L - \Gamma + w)M + \beta w M - r(z_1^L)$ for $z < z_1^L$. For quadratic R&D costs, this is satisfied if $\Gamma < \frac{1}{4}m$.

Finally, the firms have no incentives to deviate from prescribed strategies. By construction, Firm 2 is indifferent between investments z_2^H and 0, and Firm 1 is indifferent between z^M and z_1^L. And we have already argued that Firm 1 has no incentive to deviate to 0 and Firm 2 has no incentive to deviate to z^M. Other possible deviations are unprofitable by the concavity of profit functions over relevant ranges. Hence, we conclude that the assumed equilibrium exists in case (i) if $\Gamma < \frac{1}{4}m$ and $w < \gamma_1 + \frac{1}{3}(m + 2\Gamma)$.

Case (ii): $w > \gamma_1 + z_1^L$.

Necessity: The assumed equilibrium cannot exist if $\gamma_1 + z_1^L < w < \gamma_2$, because Firm 2 would win the market when Firm 1 invests z_1^L, so Firm 1 would not invest. Suppose instead that $w \geq \gamma_2$. Then Firm 2 is foreclosed if it does not invest, even if $0 \leq z_1^L \leq \Gamma$. The indifference condition for Firm 2 is

$$\begin{aligned}\pi_2(z_2^H) &= (1-\alpha)(\gamma_2 + z_2^H - \max(\gamma_1 + z_1^L - w, 0) - w)M - r(z_2^H) \\ &= (1-\alpha)(\gamma_2 + z_2^H - w)M - r(z_2^H) = 0.\end{aligned}$$

Using local optimality of z_2^H (equation (14)) and assuming quadratic costs gives

$$z_2^H = 2(w - \gamma_2). \tag{18}$$

The indifference condition for Firm 1 is $\pi_1(z_1^L) = \pi_1(z^M)$,or

$$\begin{aligned}&(1-\beta)(\gamma_1 + z_1^L)M + \beta w M - r(z_1^L) \\ = \; &(1-\beta)(\gamma_1 + z^M)M + \beta(z^M - \Gamma - z_2^H + w)M - r(z^M).\end{aligned}$$

Using the local optimality condition (16) and assuming quadratic R&D costs gives

$$z_1^L = 2(z_2^H + \Gamma) - m,$$

and substituting (18) gives

$$z_1^L = 2(2w - (\gamma_1 + \gamma_2)) - m. \tag{19}$$

The assumption that $w > \gamma_1 + z_1^L$ along with equation (19), $z_1^L \geq 0$, and $w \geq \gamma_2$ imply that

$$\max[\gamma_2, \gamma_1 + \frac{1}{4}(m + 2\Gamma)] \leq w < \gamma_1 + \frac{1}{3}(m + 2\Gamma). \tag{20}$$

This in turn implies that a necessary condition for equilibrium is $m > \Gamma$. This also implies $w < \gamma_1 + z^M$.

Sufficiency: As in case (i), Firm 1 would not profit by deviating to 0 and has no incentive to invest at levels other than z_1^L or z^M. If Firm 2 deviates by investing z^M, it would earn

$$\begin{aligned}\pi_2(z^M) &= (1-\alpha)(\gamma_2 + z^M - w)M + \alpha(\gamma_2 + z^M - \max(\gamma_1 + z^M - w, 0) - w)M - r(z^M) \\ &= \pi^M - (1-\alpha)(w - \gamma_2)M - \alpha(z^M - \Gamma)M,\end{aligned}$$

which has to be less than the zero payoff when it does not invest. Substituting the local optimality condition (14) and assuming quadratic R&D costs, this requires

$$\pi^M - r'(z_2^H)(w - \gamma_2) - (M - r'(z_2^H))(z^M - \Gamma) \leq 0.$$

For the case of quadratic R&D, using (18), sufficiency requires

$$(w - \gamma_2)^2 - (w - \gamma_2)(m - \Gamma) + \frac{1}{4}m(m - 2\Gamma) \geq 0.$$

This, in turn, implies either $w \leq \gamma_1 + \frac{1}{2}m$ or $w \geq \gamma_2 + \frac{1}{2}m$. However, the necessary conditions for an equilibrium require $\max[\gamma_2, \gamma_1 + \frac{1}{4}(m + 2\Gamma)] < w < \gamma_1 + \frac{1}{3}(m + 2\Gamma)$. Notice that $w < \gamma_1 + \frac{1}{3}(m + 2\Gamma)$ contradicts $w \geq \gamma_2 + \frac{1}{2}m$. Therefore, an equilibrium exists in this case (ii) only if

$$\max[\gamma_2, \gamma_1 + \frac{1}{4}(m + 2\Gamma)] \leq w \leq \min(\gamma_1 + \frac{1}{2}m, \gamma_1 + \frac{1}{3}(m + 2\Gamma)).$$

No equilibrium exists when $w \geq \gamma_2$ if $m < 2\Gamma$. Furthermore, $\gamma_2 \geq \gamma_1 + \frac{1}{4}(m + 2\Gamma)$ when $m \geq 2\Gamma$. Thus equilibrium requires

$$\gamma_2 \leq w \leq \min(\gamma_1 + \frac{1}{2}m, \gamma_1 + \frac{1}{3}(m + 2\Gamma))$$

and $m \geq 2\Gamma$. As $\Gamma \to 0$, this condition becomes

$$\gamma_2 \leq w \leq \gamma_1 + \frac{m}{3}.$$

The equilibrium exists for an intermediate range of w if Γ is sufficiently small.

A.6 Proof of Proposition 5 and Corollary 4: Mixed strategy equilibrium of the technological tying game.

Consider a candidate mixed strategy equilibrium with $z_1^H = z^M > z_2^H + \Gamma > z_1^L > z_2^L = 0$. Assume $\gamma_1 + z^M > w \geq \bar{w} - \Gamma \geq \gamma_2$. A mixed strategy equilibrium of the tying game does not exist if $w < \bar{w} - \Gamma$ or if $w > \gamma_1 + z^M$. In the former case, Firm 1 will always foreclose. In the latter case, Firm 1 would always prefer to sell the component. Define $\pi(z) = r'(z)z - r(z)$. Suppose Firm 1 invests $z_1^H = z^M$. The component price would foreclose Firm 2 if it does not invest because $w \geq \gamma_2$. If it does invest, Firm 1 would impose a tie because $\gamma_1 + z^M > w$. Thus, when Firm 1 invests z^M, it earns

$$\pi_1(z^M) = \gamma_1 M + \pi^M.$$

Suppose Firm 1 invests z_1^L. As before, the component price forecloses Firm 2 if it does not invest. If Firm 2 invests, then Firm 1 would not foreclose if $\gamma_1 + z_1^L < w$. Then

$$\pi_1(z_1^L) = \beta w M + (1 - \beta)(\gamma_1 + z_1^L)M - r(z_1^L).$$

Local optimality requires

$$r'(z_1^L) = (1 - \beta)M$$

and strategy indifference requires

$$(M - r'(z_1^L))(w - \gamma_1) = \pi^M - \pi(z_1^L).$$

Thus conditions (8) and (11) are necessary for a mixed strategy equilibrium. Define

$$\psi(z) = (M - r'(z))(w - \gamma_1) - \pi^M + \pi(z).$$

We have $\psi(z_1^L) = 0$. At $z = 0$, $\psi(0) = M(w - \gamma_1) - \pi^M \geq 0$ if and only if $w \geq \bar{w} - \Gamma$, and $\psi(z^M) = 0$. Note that

$$\psi'(z) = (z - (w - \gamma_1))r''(z)$$

and $\psi'(z)$ has the same sign as $(z - w + \gamma_1)$. Furthermore, $\psi'(z^M) > 0$ if and only if $w < \gamma_1 + z^M$ and $\psi'(0) < 0$ if and only if $w > \gamma_1$. Therefore, if $\bar{w} - \Gamma \leq w < \gamma_1 + z^M$, then there exists $z^M > z_1^L \geq 0$ such that $\psi(z_1^L) = 0$. (See Figure 4.) Moreover, as necessary, $z_1^L < w - \gamma_1$. Thus, under the conditions of the proposition, there a exists an appropriate z_1^L.

Now consider the incentive of Firm 2. This firm earns a profit only if it invests $z_2 = z_2^H$ and if $z_1 = z_1^L$ and $w \leq \gamma_2 + z_2^H$. Then

$$\pi_2(z_2^H) = (1-\alpha)[\gamma_2 + z_2^H - w]M - r(z_2^H)$$

Local optimality requires condition (10). Thus we can write

$$\pi_2(z_2^H) = \pi(z_2^H) + r'(z_2^H)(\gamma_2 - w).$$

Existence of a mixed strategy equilibrium requires $\pi_2(z_2^H) = 0$, which is equivalent to (9) Now define

$$\phi(z) = \pi(z) + r'(z)(\gamma_2 - w).$$

Note that

$$\phi'(z) = (z - (w - \gamma_2))r''(z)$$

and $\phi'(z)$ has the same sign as $(z - w + \gamma_2)$. Therefore, if $\phi(0) \leq 0$ and $\phi(z^M - \Gamma) > 0$, then there exists a $z_2^H < z^M - \Gamma$ such that $\phi(z_2^H) = 0$ and $z_2^H > w - \gamma_2$. (See Figure 5.) Now $\phi(0) \leq 0$ if $w > \gamma_2$. Assume $z^M > \Gamma$. Then $\phi(z^M - \Gamma) = r'(z^M - \Gamma)(z^M + \gamma_1 - w) - r(z^M - \Gamma) \geq 0$ if and only if

$$w \leq \gamma_1 + z^M - \frac{r(z^M - \Gamma)}{r'(z^M - \Gamma)},$$

which is the second part of condition (7). Furthermore, $z_2^H > w - \gamma_2$, hence $z_2^H + \Gamma > w - \gamma_1 > z_1^L$. Thus, under the conditions of the proposition, there a exists an appropriate z_2^H.

Summarizing, Proposition 5 provides a statement of the necessary and sufficient conditions for existence of a mixed strategy equilibrium of the tying game with $z_1^H = z^M > z_2^H + \Gamma > z_1^L > z_2^L = 0$. As an example, consider the quadratic case: $r(z) = \frac{1}{2}kz^2$ and let $m \equiv M/k$. Then $\psi(z_1^L) = 0$ implies

$$z_1^L = 2(w - \gamma_1) - m,$$

and $\phi(z_2^H) = 0$ implies

$$z_2^H = 2(w - \gamma_2).$$

A mixed strategy equilibrium exists with $z_1^H = z^M > z_2^H + \Gamma > z_1^L > z_2^L = 0$ if

$$\bar{w} - \Gamma \leq w < \bar{w} - \frac{1}{2}\Gamma,$$

and $\Gamma < m$.

Figure 1. Firm 1 profit in pure strategy equilibria of the product improvement game

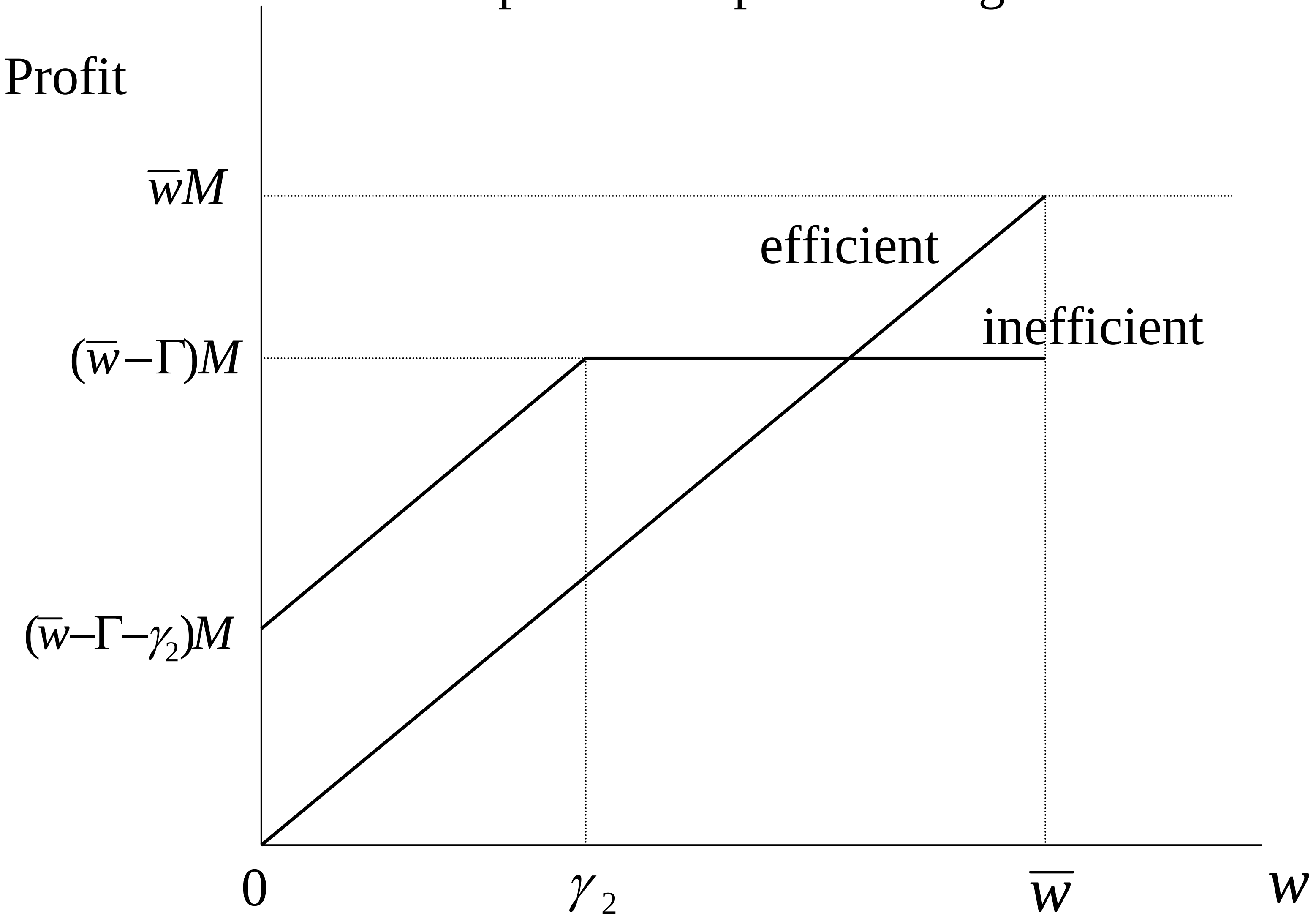

Figure 2. Expected welfare with pure and mixed strategy equilibria

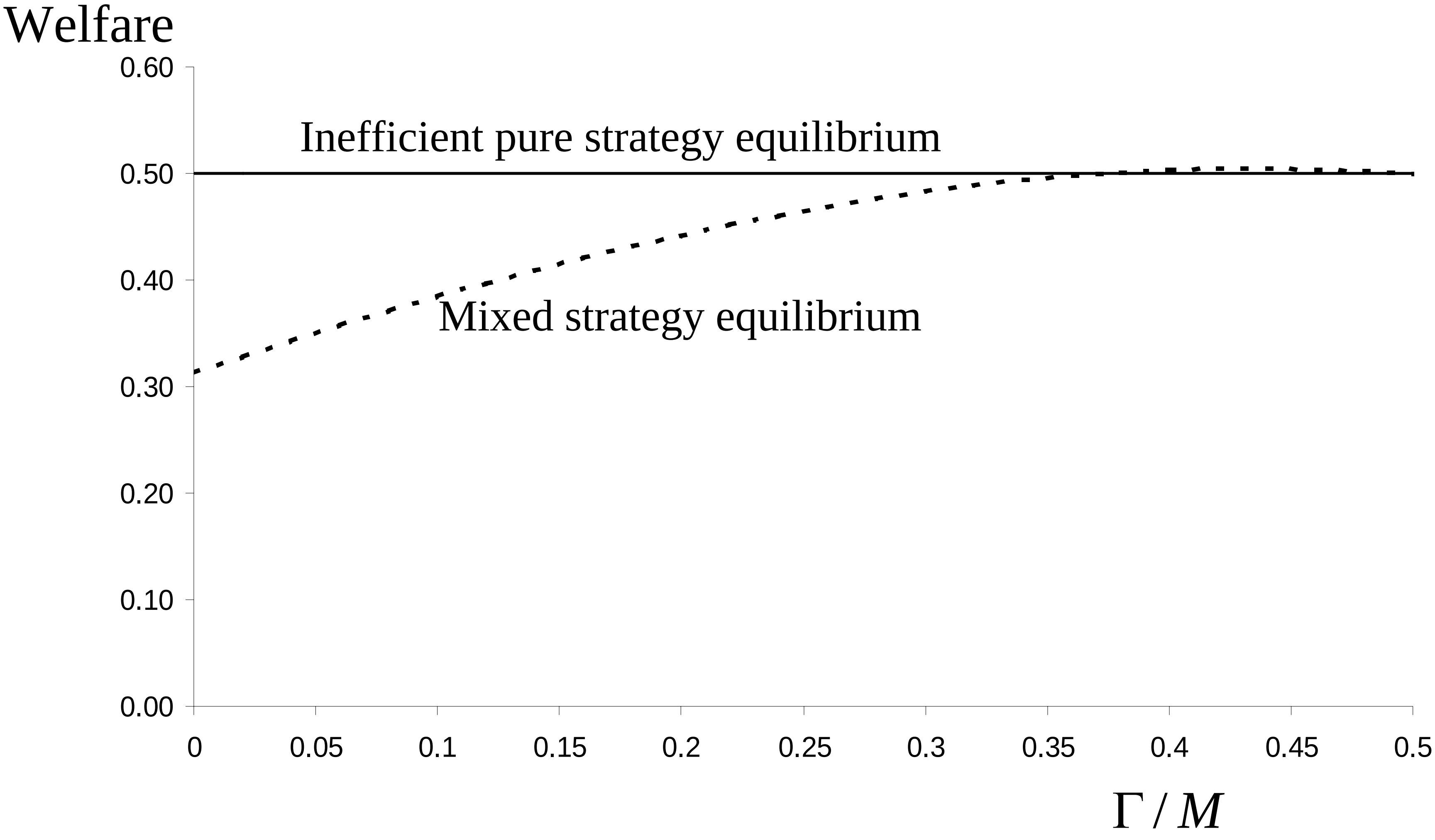

Figure 3. Illustration of fixed point for z_1^H .

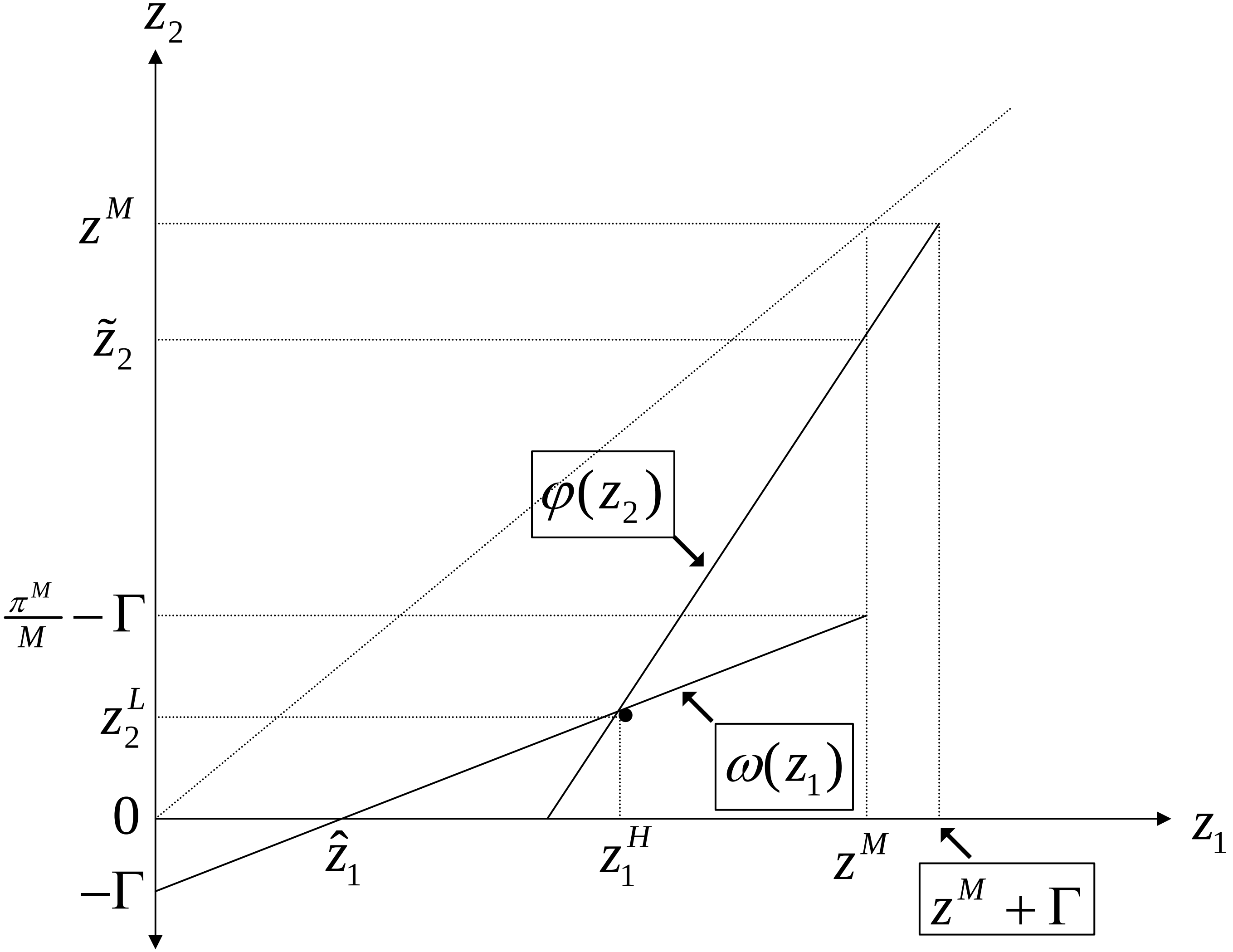

Figure 4. Demonstration of $z_1^L < w - \gamma_1$.

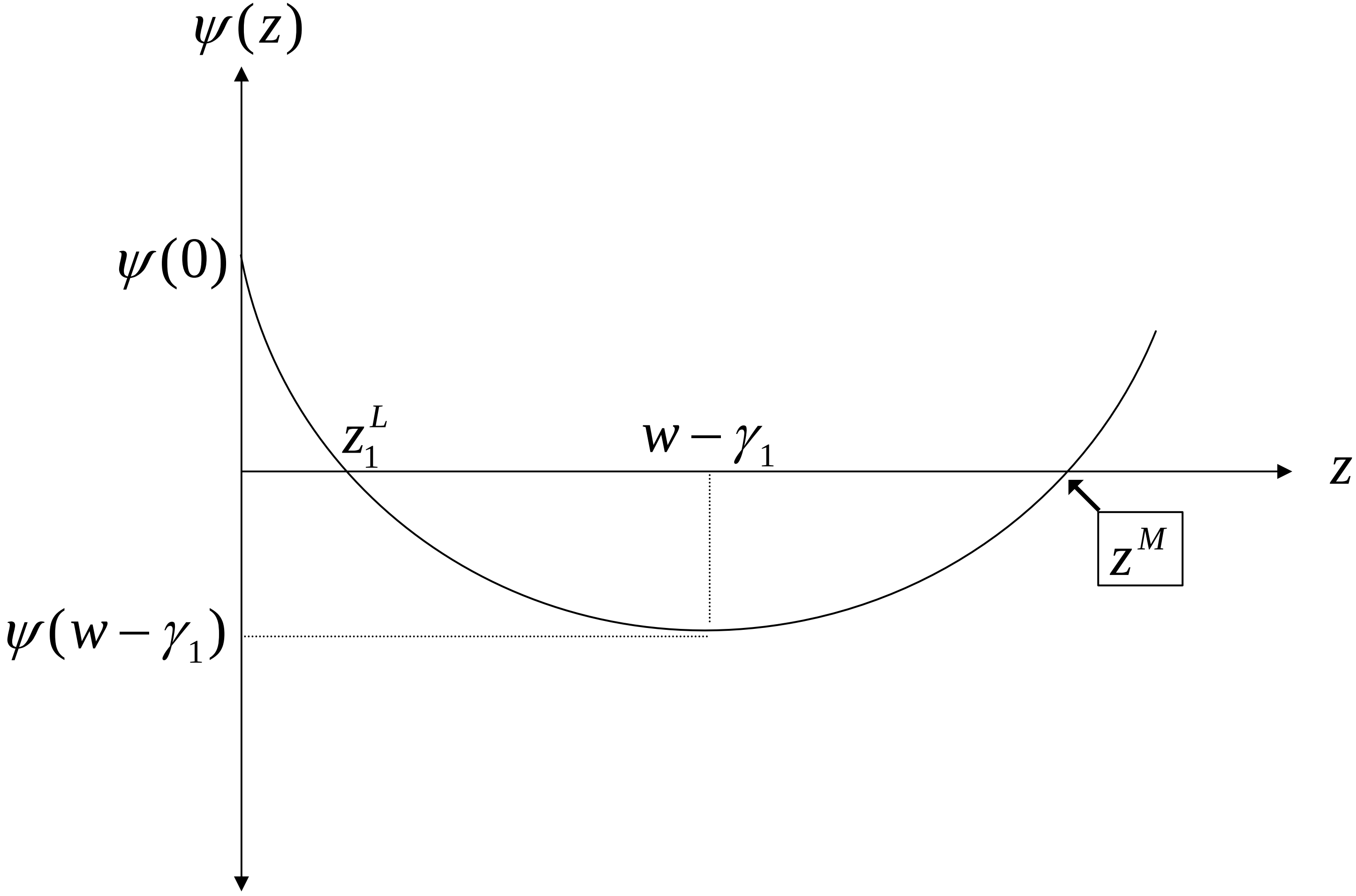

Figure 5. Demonstration of $z_2^H > w - \gamma_2$.