

# Advertising and Coordination

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When market information such as price is difficult to communicate, consumers and firms may be unable to take advantage of mutually beneficial scale economies, so that *coordination failures* arise. Ostensibly uninformative advertising expenditures can be used to eliminate coordination failures, by allowing an efficient firm to communicate implicitly that it offers a low price. This provides a theoretical explanation for Benham's (1972) empirical association of the ability to advertise with lower prices and larger scale. Advertising becomes necessary for optimal coordination when the identity of the efficient firm is uncertain. An application to loss-leader pricing is developed.

## 1 INTRODUCTION

Sellers often find it difficult to transmit relevant information to their prospective customers. Satisfactory communication of price information, in particular, is hampered by the need to communicate a large number of prices (e.g. multi-product retail stores) or a complex pricing structure (e.g. long-distance telephone service). Price advertising may even be illegal (historic examples in the U.S. include retail eyeglass, liquor and prescription drug markets). Similarly, it is seldom easy to provide useful hard information about product qualities or the variety of product offerings (e.g. automobile or rug dealers).

In their efforts to communicate such information, sellers frequently resort to advertisements featuring vague and unverifiable claims to "low prices, high quality and great selection," as opposed to hard information. The price information that is available typically consists of "loss-leader" items making up a small fraction of a seller's total product line, which leaves buyers to wonder about the pricing policy for the remaining items. Assuming that firms seek to maximize profits in placing these kinds of ads, it becomes a challenge for economists to reconcile the prevalence of such advertising with the hypothesis of consumer rationality.

Empirical studies have uncovered a deeper puzzle. In his study of the retail eyeglass industry, Benham (1972) found that retailers selected lower prices and operated at larger scale in states that allowed advertising, even when state law did not permit advertisements to contain explicit price information. Similar relationships have been reported in analyses of other industries as well.<sup>1</sup> At a general level, these findings are consistent with the notion

1 See Cady (1976) and Luksetich and Lofgreen (1976) for studies of the retail prescription drug and Minnesota liquor industries, respectively. It is interesting to note that Benham also found prices in states that allowed price advertising to be comparable with prices in states that permitted only non-price advertising, this similarity seems best explained by Benham's observation that sellers provided little price information even when price advertising was legal.

that advertising allows a seller better to exploit scale economies. The surprising aspect of Benham's study, however, is that even apparently uninformative "non-price" advertising can have a profound effect on market conduct and structure.

In this paper we propose a theoretical explanation for these phenomena, based on the fundamental idea that buyers and sellers can mutually benefit by exploiting scale economies. Our concept of scale economies differs, however, from the traditional notion of declining average cost. We posit instead the following two properties. The *better profit property* states simply that a firm profits from an expansion of its market share. The centrepiece of our analysis is a less familiar effect, called the *better deal property*, which asserts that a firm offers consumers a better deal when it expects more business. The latter property hinges on the cost and demand conditions facing sellers, and it may or may not be related to the presence of declining average cost. We argue below that conditions giving rise to the better deal property are ubiquitous, particularly in retail markets.

Under these two properties, buyers and active sellers both prefer business to be concentrated among fewer firms. When information about price or product variety is difficult to communicate, however, then these coordination gains may go unrealized. Such *coordination failures* take two basic forms: an efficient firm may be unable to capture any market share, or firms may split the market and thus operate at inefficiently small scale.

In this setting, we show that ostensibly uninformative advertising can play a key role. By expending resources on advertising, an efficient firm communicates that it intends to capture large market share, as it would be unable to recoup its advertising expense if market share were small. As a consequence of the better deal property, consumers then infer that they will obtain lower prices from the advertising firm. In particular, consumers draw this inference if they believe that the firm does not choose advertising-price combinations that are *equilibrium dominated*, i.e. that are incapable of exceeding the profits earned in a conjectured equilibrium. In this way, advertising allows the efficient firm to communicate its better deal, and thereby eliminates the possibility of coordination failures.

With incomplete information as to the identity of the efficient firm, advertising plays an even more important role in bringing about coordination: in the absence of advertising consumers have no means of identifying which firm is most efficient, and thus optimal coordination is *impossible* to achieve. When advertising is allowed, however, there exists a focal separating equilibrium outcome that gives fully-efficient coordination. In this case, coordination is achieved at a cost, as the most efficient firm must choose a strictly positive advertising level in order to communicate its identity.<sup>2</sup> Thus incomplete information gives rise to a welfare tradeoff between advertising and coordination: consumers prefer the fully-coordinated outcome, but industry profits and social surplus may be greater when advertising is banned.

Our theory of advertising and coordination sheds light on the phenomenon of *loss-leader pricing* by multiproduct firms, wherein prices are communicated for only part of the product line. We show that loss leaders can suffice to eliminate coordination failures under certain conditions, but more generally it is necessary to accompany loss leaders with dissipative expenditures.

This paper contributes to the burgeoning theoretical literature on coordination issues. Past work has focussed on coordination failures in the contexts of entry and technology adoption (Farrell (1987), Farrell and Saloner (1985), Katz and Shapiro (1985)) and macroeconomic activity (Cooper and John (1988), Diamond (1982), Heller (1986)), and our

2 In a model without coordination economies Bagwell (1987) considers an alternate signaling channel whereby the efficient firm uses a low first-period price to convince consumers to visit it in the second period.

paper may be viewed as emphasizing the salience of coordination failures for product markets with imperfect information. Our approach relates most directly to recent game-theoretic treatments of equilibrium selection, in particular, Ben-Porath and Dekel (1992), Glazer and Weiss (1990), Kohlberg and Mertens (1986) and van Damme (1989) base their analyses on forward induction-type refinements, as we do in this paper.

A variety of papers have explored the role of advertising in signaling product quality.<sup>3</sup> This work models quality as an experience attribute, most commonly associated with particular brands (e.g. Coke). Advertising becomes a useful signal when high quality is associated with low marginal production cost or high returns from repeat business. By contrast, we emphasize the role of advertising in communicating search attributes such as price, and we link the usefulness of advertising to coordination economies.

The plan of the paper is as follows. Section 2 explains the better profit and better deal properties and discusses motivating examples. Section 3 explores the coordination role of advertising in a simple one-firm context, while Section 4 develops the full duopoly model and incorporates incomplete information. Section 5 discusses loss-leader pricing and Section 6 concludes.

## 2 THE BETTER PROFIT AND BETTER DEAL PROPERTIES

We consider a market for a single homogeneous good in which there are a large number of identical consumers, being uniformly distributed on the interval  $[0, 1]$  with unit mass. Consumers acquire price information through costly search. Thus their decisions whether or not to visit a particular firm are based on the price they *expect* the firm to charge, whereas purchase decisions are based on the *actual* price at the visited firm.

Let  $\Pi(p, m)$  denote the profit function of a firm that charges price  $p$  and is visited by mass  $m$  of the consumers. The dichotomy between consumers' visitation and purchase decisions is reflected by the separate dependence of  $\Pi$  on  $m$  and  $p$ . Assume that  $\Pi$  is a continuous function. For  $m > 0$ , let  $\Pi$  be strictly quasiconcave in  $p$ , with unique maximizer  $p^*(m)$  and maximized value  $\Pi^*(m)$ , thus profits are strictly increasing in  $p$  for  $p < p^*(m)$ , and strictly decreasing for  $p > p^*(m)$ . Profits are zero if  $m = 0$ .

Our focus in this section is on the following comparative statics question: if a firm expects a greater number of buyers, what happens to its maximized profits and profit-maximizing price? We will impose a particular answer to this question, and then verify the answer in a number of plausible models. Our answer is given by two properties:

*Better Profit Property*  $\Pi^*(m)$  is strictly increasing in  $m$

*Better Deal Property*  $p^*(m)$  is strictly decreasing in  $m$

The better profit property states that a firm benefits when it is visited by a larger number of consumers, given that it can adjust its price in a profit-maximizing way. This unsurprising property is valid in a variety of models, including all of those considered below, since it holds as long as the profit-maximizing price exceeds marginal cost. The less-familiar better deal property states that the firm's customers also benefit when it expands its market share, since the profit-maximizing price is reduced.

The better deal property may seem unintuitive at first glance, but there are in fact many situations in which it is quite plausible.

<sup>3</sup> See Kihlstrom and Riordan (1984), Klein and Leffler (1981), Matthews and Fertig (1990), Milgrom and Roberts (1986), Nelson (1970, 1974), Ramey (1987) and Rogerson (1986).

1 *Declining Marginal Cost* The customary assumption of convex costs may be inappropriate for a number of reasons, which make declining marginal cost the more plausible hypothesis. Consider the following examples: (i) *Manufacturer quantity discounts* lead to declining marginal cost for retail sellers: as more units are sold, the retailer qualifies for lower wholesale prices, which translate into lower marginal costs.<sup>4</sup> (ii) *Learning effects* are often important: as a seller expands its output, personnel become better acquainted with operating procedures, and coordination of managers and workers improves. (iii) *Technology choice* might give rise to declining marginal cost: as scale expands, the benefits from adopting low-marginal-cost technologies increase.

It is straightforward to verify that the assumption of declining marginal cost gives rise to the better deal property. Let  $Q(p)$  and  $C(q)$  represent the consumers' individual demand functions and the firm's cost function respectively, where  $q$  indicates the firm's output, and assume that  $Q' < 0 < C'$ . The profit function becomes

$$\Pi(p, m) = pmQ(p) - C(mQ(p))$$

From this we have

$$\frac{dp^*}{dm} = \frac{C''QmQ'}{\Delta_p}$$

where  $\Delta_p$  represents the second derivative of  $\Pi$  with respect to  $p$ , evaluated at  $p^*$ , which we assume is strictly negative. It follows that  $dp^*/dm < 0$  if and only if  $C' < 0$ .

Since declining marginal cost implies declining average cost, the better deal property might be regarded as a strengthened version of the usual notion of scale economies. This is not correct, however, as one can readily conceive of situations in which the better deal property holds even when marginal cost is non-decreasing. We mention two examples that are particularly relevant for retail establishments.

2 *Consumer Heterogeneity* The above framework may be modified by allowing consumers to differ in their preferences for the firm's product. Suppose that consumers having weaker preference for the product also have more elastic demand. As a greater number of customers visit the firm, these weaker-preference consumers enter the customer base, and the firm's total demand curve becomes more elastic. This leads the firm to reduce its price as market share grows, even if it has constant unit cost.<sup>5</sup>

3 *Product Variety* Suppose that the firm stocks a range of products, and can expand the range by incurring a greater stocking cost. Moreover, consumer utility is increasing in the range of products stocked. The additional net revenue from stocking a new product is then increasing in the number of consumers served. Thus the firm will be willing to incur the added cost of stocking extra products when it expects more customers, and consumers thereby obtain a better deal. If unit sales cost is constant for each product

4 Empirical evidence strongly suggests that manufacturer quantity discounts are important in many industries. Cady (1976) describes a significant role for quantity discounts in the retail prescription drug industry; the U.S. Federal Trade Commission (1980, p. 44) cites evidence that quantity discounts are prevalent in the retail eyeglass industry; and Brown and Medoff (1990) give further evidence that quantity discounts are widespread and important.

5 Let the utility from purchasing  $q$  units at price  $p$  from a given firm be  $\theta U(q) - pq$ , where  $U, U' > 0 > U''$ , with the population of  $\theta$  uniformly distributed on  $[0, 1]$ . Let the firm's cost function be  $C(q) = cq$  with  $0 < c < U'(0)$ . Maximized consumer utility at any  $p < \theta U'(0)$  is strictly increasing in  $\theta$ , so that the firm attracts the sub-interval  $[1-m, 1]$  of consumers when its market share is  $m$ . The firm's demand function is obtained by integrating the individual demand functions over  $[1-m, 1]$ . As long as  $U'''$  is not too positive, the elasticity of the firm's demand function increases as  $m$  rises, and thus the profit-maximizing price falls. Note that since the firm's market share  $m$  is independent of its actual price choice  $p$ , there is no difficulty for the second-order conditions with respect to the firm's profit-maximizing price.

stocked, then this variety-based version of the better deal property holds despite the fact that the price of each stocked product does not depend on the firm's market share<sup>6</sup>

Still other motivations for the better deal property may be given, including cost complementarities across product lines and product quality choice, the latter two are considered in Section 5. These examples demonstrate that the better profit and better deal properties encompass many market environments that might not fit the conventional definition of scale economies, and that these environments would seem to be quite common, particularly on the retail level<sup>7</sup>

### 3 THE COORDINATION ROLE OF ADVERTISING

In the presence of the better profit and better deal properties, a firm and its customers benefit jointly from concentration of sales at the firm. Since price information is difficult to communicate, however, these coordination economies may go unrealized. Ostensibly uninformative advertising expenditures then play a critical role in bringing about coordination, in that they allow the firm to communicate implicitly its better deal. In this section we will illustrate this idea in a simple one-firm context, our full duopoly model, which is more complete and detailed, will be considered in Section 4.

Specifically, we consider here the following simple search game.

*Stage 1* A single firm chooses its price  $p$ .

*Stage 2* Without observing the firm's price choice, consumers decide whether or not to visit the firm. If they visit, they purchase their desired number of units at the price  $p$ . If they do not visit, it is assumed that they purchase elsewhere at some pre-specified price  $\bar{p}$ .

The firm's payoff function is given by  $\Pi(p, m)$ , where  $m$  gives the mass of consumers that choose to visit the firm in Stage 2. Consumers desire simply to purchase at the lowest possible price. Here  $\bar{p}$  represents the opportunity cost to the consumers of visiting the firm, which may reflect consumers' beliefs about the pricing behaviour of other firms in the market. Alternatively,  $\bar{p}$  could represent the price that just offsets consumers' search cost in this market. We make the important assumption that the firm is willing to offer consumers a price below  $\bar{p}$ , but *only* if it expects to capture a large enough market share.

$$p^*(1) < \bar{p} < \lim_{m \downarrow 0} p^*(m) \tag{1}$$

Let  $\hat{p}$  denote the equilibrium price strategy of the firm, and let  $\hat{m}$  denote the mass of consumers that visit the firm in equilibrium. Given (1), it is easy to see that there exists a sequential equilibrium outcome of the search game in which *all* consumers visit the firm.

6. Let consumers be identical and suppose a given firm sells a range of products indexed by  $\theta$ , possible products are uniformly distributed on  $[0, 1]$ . Consumer utility from purchasing  $q_\theta$  units of product  $\theta$  at price  $p_\theta$  is  $U_\theta(q_\theta) - p_\theta q_\theta$ , where  $U_\theta, U'_\theta > 0 > U''_\theta$ . Total utility is obtained by integrating across purchased products. Let the firm's cost function be  $C_\theta(q_\theta) = c_\theta q_\theta$  with  $0 < c_\theta < U'_\theta(0)$ , and let  $R_\theta$  give the maximized per-consumer profit rate for product  $\theta$ , total profits are obtained by integrating across the product line. Let us reorder the products so that  $R_\theta$  is non-decreasing in  $\theta$ . Finally, let  $S(k)$  with  $S'' > 0 = S'(0)$  and  $S'(1) = \infty$  give the fixed stocking cost when a product line of mass  $k$  is stocked. It is straightforward to verify that the profit-maximizing level of  $k$  is strictly increasing in the total mass  $m$  of consumers and further that consumer utility is strictly increasing in  $k$ . Product variety is considered in greater detail in Bagwell and Ramey (1992) where complementarities between selling technology and variety are also analysed.

7. Network externalities of the sort considered by Farrell and Saloner (1985) and Katz and Shapiro (1985) give a source of coordination economies wherein consumers are directly concerned with firms' market shares. In contrast, we assume that consumers care only about firms' choices of price and variety. Our theory of advertising does not apply to the case of network externalities because a firm can use advertising only to signal its own choices, and not the choices of other consumers that are the source of network benefits.

given that all consumers are expected to visit the firm maximizes profits by choosing  $\hat{p} = p^*(1)$ , since consumers expect the firm to charge  $p^*(1) < \bar{p}$  they maximize utility by visiting the firm. Further, this outcome is favourable for both the firm and consumers: the firm's profits are greatest when it captures all the consumers, by the better profit property, while consumers are able to purchase at a price below their opportunity level  $\bar{p}$ . Since all agents benefit, we call this the *optimally-coordinated outcome*.

There also exist other, less-attractive sequential equilibria. In particular, consider the following equilibrium, in which *no* consumer visits the firm: given that the firm expects to receive zero market share, its profits are zero irrespective of price, so that we may specify  $\hat{p} > \bar{p}$  as a profit-maximizing price strategy, given that consumers expect  $\hat{p} > \bar{p}$  they maximize utility by not visiting the firm.<sup>8</sup> We refer to this outcome as a *coordination failure*, in that both the firm and consumers are strictly worse off relative to the optimally-coordinated outcome.

Coordination failure arises in this game because the firm is unable to communicate its willingness to offer a better deal, due to consumers' inability to observe price prior to search. Observable dissipative advertising can resolve this problem, by serving as a communication channel for the firm. To consider this possibility, let us augment the search game by allowing the firm to choose an advertising level  $A$  together with price in Stage 1. The advertising level represents simply an observable expenditure, which provides no direct utility to consumers. Consumers may condition their visitation decisions on observed advertising, however. Payoffs for the firm are now given by  $\Pi(p, m) - A$ , and the firm's equilibrium advertising strategy is denoted  $\hat{A}$ .

The mere presence of advertising does not eliminate the coordination-failure outcome described above, since consumers are free to simply ignore advertising in making their visitation decisions. This brings us to the final ingredient in our theory of advertising: for advertising to play a meaningful role, there must be some degree of sophistication in the way consumers draw inferences from observed advertising. In particular, consumers should recognize the implications of the better profit and better deal properties for the firm's incentives to choose advertising and price. Thus, when the firm deviates from an equilibrium by choosing a large advertising level, consumers implicitly receive the message "I am expending these resources only because I anticipate capturing a large market share. Given this, you know I will be offering a better deal, so you should visit me." If consumers understand this message and accept its logic, then advertising becomes a tool through which the firm can bring about optimal coordination.

To formalize this idea, we introduce the following definition: an advertising-price pair  $(A, p)$  is said to be *equilibrium dominated* relative to a given equilibrium if

$$\max_m \Pi(p, m) - A < \Pi^*(\hat{m}) - \hat{A}$$

That is, no matter what market share the firm obtains as a result of choosing  $(A, p)$ , it would do strictly better by sticking with its equilibrium strategy. We now impose the following restriction on consumer beliefs:

*No Equilibrium Dominated Conjectures.* Consumers never infer that a firm has chosen an equilibrium dominated strategy, if it is possible to infer that some non-equilibrium dominated strategy was selected.<sup>9</sup>

8 We can specify, for example, that  $\hat{p} = \lim_{\epsilon \rightarrow 0} p^*(m) > \bar{p}$  by positing that the firm expects to capture a vanishingly-small number of consumers who "tremble" in their visitation decisions.

9 This criterion derives from Kohlberg and Mertens' (1987) concept of strategic stability for finite normal form games. Cho and Kreps (1987) have adapted this idea to signaling games with continuous strategy spaces. Our use of equilibrium dominance can be viewed as a direct application of Cho and Kreps' *intuitive criterion* to a coordination game.

When consumers' inferences are restricted to satisfy no equilibrium dominated conjectures, the firm is able to use advertising as a communication channel, through which it brings about optimal coordination of consumer purchase behaviour. The first step in proving this result is contained in the following lemma (proofs are given in the Appendix)

**Lemma.** *If  $p > p' \geq p^*(1)$ , then*

$$\text{Max}_m \Pi(p, m) < \text{Max}_m \Pi(p', m)$$

The Lemma states that, for prices above the "best deal" price  $p^*(1)$ , higher prices imply a strictly lower upper bound to profits, where the upper bound is taken over all possible market shares. The proof involves a straightforward combination of the quasiconcavity assumption with the better profit property.

We now demonstrate that the outcome in which no consumer visits the firm is not supportable as an equilibrium when the inference restriction is imposed. Observe first that the firm chooses  $\hat{A} = 0$  in any equilibrium supporting the zero-market-share outcome (clearly  $\hat{A} > 0$  would not maximize profits, since the firm has nothing to lose by reducing advertising), and so equilibrium profits are zero in such an equilibrium. Choose any price  $p' \in (p^*(1), \bar{p})$ , and define the advertising level  $A'$  by

$$\text{Max}_m \Pi(p', m) - A' = 0 \tag{2}$$

Under our assumptions we have  $A' > 0$ <sup>10</sup>

Now suppose the firm deviates by choosing  $A'$  instead of  $\hat{A}$ . It follows from (2) and the Lemma that for any  $p > p'$ , the strategy  $(A', p)$  is equilibrium dominated. The strategy  $(A', p^*(1))$  is not equilibrium dominated, however, since by the Lemma

$$\text{Max}_m \Pi(p', m) < \text{Max}_m \Pi(p^*(1), m) = \Pi^*(1),$$

where the equality follows readily from the better-profit property. Under the inference restriction, therefore, consumers must infer that the firm charges  $p \leq p'$  after they have observed the deviation to  $A'$ , and since  $p' < \bar{p}$ , sequential equilibrium requires that consumers visit the firm. The firm's equilibrium strategy of  $\hat{A} = 0$  and  $\hat{p} > \bar{p}$  is then no longer a best response, as deviating to  $(A', p^*(1))$  captures all the consumers and gives the firm strictly positive profits of  $\Pi^*(1) - A'$ . Thus the zero-market-share outcome can no longer be supported as an equilibrium, and the coordination failure is thereby eliminated.

This result establishes an important coordination role for dissipative advertising: if consumers are sophisticated in drawing inferences from observed advertising, then the firm can use advertising to communicate that it offers a lower price. It follows that advertising gives rise to an implicit form of price communication that makes it possible to eliminate the coordination failure. In the next section we will show that this implicit communication eliminates all outcomes save the optimally-coordinated outcome, thus advertising ensures that optimal coordination is achieved.

One may object that our formalization of sophisticated consumer inference is unrealistic, either because consumers are unable to evaluate the implicit message due to computational or informational deficiencies, or because consumers and the firm must have very strong prior agreement on an equilibrium before equilibrium dominance can be checked. These considerations, however, should be balanced against the mutual desire of consumers

<sup>10</sup> To see that  $A' > 0$ , note that (1) implies  $p' = p^*(m')$  for some  $m' \in (0, 1)$ , and using (2) and the better-profit property we have  $A' = \text{Max}_m \Pi(p', m) \geq \Pi^*(m') > \lim_{m \rightarrow 1} \Pi^*(m) = 0$

and the firm to communicate. Our results establish that these agents will have good reason to look to advertising as a means of resolving their communication problems.

Moreover, advertising seems especially well suited to this role since it allows explanation of the implicit message as part of the advertisement. This point is illustrated by a recent advertising campaign of the Builders' Square hardware chain in which the slogan pointed explicitly to coordination benefits: "The more we sell, the lower the price, the lower the price, the more we sell." Direct communication such as this gives advertising a special capability to establish focal outcomes.

#### 4 ADVERTISING AND COORDINATION IN A DUOPOLY

##### 4.1 *Duopoly Model with Imperfect Price Information*

Thus far we have studied coordination between consumers and a single firm, given an exogenously-specified price that consumers expect to pay if they do not visit the firm. We complete the theory by introducing a second firm, whose price choice endogenizes the consumers' opportunity cost. In addition, the duopoly model further clarifies the nature of the coordination economies that are obtainable via advertising.

Let us modify the preceding framework by assuming that there are two firms in the market, called Firms 1 and 2. Firm  $i$ 's profit function, profit-maximizing price and maximized profit level are written  $\Pi_i(p_i, m_i)$ ,  $p_i^*(m_i)$  and  $\Pi_i^*(m_i)$ , respectively, where  $p_i$  is Firm  $i$ 's price choice, and  $m_i$  gives the mass of consumers that visit Firm  $i$ . The above assumptions, including the better profit and better deal properties, hold for both firms. We add the assumption that the firms choose in Stage 1 whether or not to enter the market, thus the two-stage search game without advertising is now given by

*Stage 1* Firms 1 and 2 simultaneously decide whether or not to enter the market, and if a firm decides to enter it also chooses its price at this time.

*Stage 2* Consumers observe the firms' entry decisions, but not their prices. Consumers then choose one of the firms to visit, and they purchase the desired number of units at the price set by the visited firm.

If a firm enters, it is assumed to incur an added sunk entry cost of  $F \in (0, \Pi_1^*(1))$ , and so its payoff function is  $\Pi_i(p_i, m_i) - F$ . The payoff from staying out is zero.<sup>11</sup> We assume further that Firm 1 is known to be the more efficient supplier of the good, in the sense that Firm 1 offers a lower price than would Firm 2 if it expects to capture the market:

$$p_1^*(1) < p_2^*(1) < \lim_{m_1 \rightarrow 0} p_1^*(m_1) \quad (3)$$

The second inequality in (3) ensures that Firm 2 is a viable competitor if Firm 1's market share is sufficiently small. In Section 4.2 we will consider the more complex case in which consumers do not know which firm is more efficient.

Let  $\hat{p}_i$  denote Firm  $i$ 's equilibrium price if it enters and let  $\hat{m}_i$  denote the mass of consumers that visit Firm  $i$ . The search game has three sequential equilibrium outcomes:

*Outcome 1: Firm 1 captures the market.* Firm 1 enters and chooses  $\hat{p}_1 = p_1^*(1)$ , while Firm 2 stays out. Consumers have only one firm to visit, so Firm 1 captures all of them. Should Firm 2 unexpectedly enter the market, consumers would continue to visit Firm 1,

<sup>11</sup> The sunk entry cost is useful for ruling out situations in which the price choice of a firm that captures zero market share gives consumers a credible threat against the other firm, given that there is a positive sunk entry cost, no firm actually would be willing to offer such an opportunity to consumers while making zero sales.

based on the inference that Firm 2 must be charging some price above  $p_1^*(1)$ . Thus Firm 2 chooses to stay out to save the entry cost  $F$ .

*Outcome 2 Firm 2 captures the market* This is like Outcome 1, except that the roles of the firms are reversed. Now the more efficient firm is unable to capture any market share from the less efficient firm, as consumers believe that the more efficient firm charges a high price if it enters.

*Outcome 3 The firms split the market* Equilibrium market shares  $\hat{m}_1$  and  $\hat{m}_2$  are uniquely determined by  $p_1^*(\hat{m}_1) = p_2^*(\hat{m}_2)$  and  $\hat{m}_1 + \hat{m}_2 = 1$ . Given that  $\hat{m}_1$  and  $\hat{m}_2$  are anticipated, the firms maximize profits by choosing these prices. Consumers are then indifferent about which firm to visit, and  $\hat{m}_1$  and  $\hat{m}_2$  are consistent with utility-maximizing behaviour. Existence of this kind of equilibrium requires that  $F$  is small enough to sustain both firms in the market, i.e.  $\Pi_i^*(\hat{m}_i) \geq F$ .

These outcomes can be ranked in terms of the benefits provided to consumers and active firms. Consumers prefer Outcome 1 to Outcome 2, since they receive a lower price from the efficient firm, observe that Outcome 2 corresponds to the coordination-failure outcome considered in the preceding section, where  $p_2^*(1)$ , now plays the role of  $\bar{p}$ . Since equilibrium prices in Outcome 3 exceed  $p_2^*(1)$ , market-splitting by the firms represents an even worse coordination failure from consumers' point of view. Each of the firms prefers the outcome in which it captures the market to the market-splitting outcome. Thus imperfect price information leads to two kinds of coordination failures: either (i) the inefficient firm captures the market, or (ii) the firms split the market, and consequently are unable to exploit the coordination economies implied by the better profit and better deal properties. In either case, mutual benefits would be obtained if consumers could better coordinate their purchase activities.

We now augment the duopoly search game by allowing the firms to choose dissipative advertising expenditures along with prices if they enter in Stage 1. Consumers observe the advertising choices of both firms before they make their Stage 2 visitation decisions.<sup>12</sup>

The inference restriction from above may be applied directly to the duopoly game, and the arguments of the preceding section are readily extended to establish that Firm 1 can use advertising to communicate that its price is close to  $p_1^*(1)$ , this overturns Outcomes 2 and 3. Outcome 1 survives the inference restriction, however, and so we have

**Proposition 1.** *Outcome 1 uniquely survives the inference restriction, i.e. in every sequential equilibrium satisfying no equilibrium dominated conjectures, the more efficient firm captures the market and chooses zero advertising.*<sup>13</sup>

Since the efficient firm chooses zero advertising in the surviving outcome, positive advertising appears only "off the equilibrium path," to upset inefficient outcomes.<sup>14</sup> We show below, however, that if the efficient firm must also communicate its identity

<sup>12</sup> In defining sequential equilibrium outcomes for this case, we impose Kreps and Wilson's (1982) criterion of consistent beliefs in the following way: consumers must form *independent price conjectures*, meaning that beliefs about a firm's price can depend only on that firm's advertising level, and not on the advertising level of the other firm. This seems sensible, as firms do not know each others' prices when they choose advertising.

<sup>13</sup> Our model can be extended to the case of multi-good sellers, and Proposition 1 may be derived using arguments similar to those of the single-good case. In this setting, the efficient firm uses advertising to communicate that *all* of its prices lie close to the levels that are optimal when it captures the market, see Bagwell and Ramey (1990, Appendix B).

<sup>14</sup> Indeed, positive advertising can arise only in sequential equilibria with market-splitting, if one firm captures the market in a sequential equilibrium, the rival firm chooses to stay out, and thus the entering firm cannot be deterred from choosing zero advertising.

to consumers, then positive equilibrium advertising does arise under the inference restriction

With the duopoly model in hand, we are now able to provide a rationale for Benham's empirical association of non-price advertising with lower market prices and larger scale firms. Imperfect price observability together with the better deal and better profit properties lead to coordination failures, in which inefficient suppliers serve the market, or firms split the market and fail to exploit scale economies. Coordination failures may be difficult to eliminate where advertising is banned. Where advertising is allowed, however, implicit communication via advertising causes coordination failures to be overturned, and this corresponds directly to lower prices and larger-scale firms.

#### 4.2 Incomplete Information as to the Identity of the Efficient Firm

In the preceding analysis, coordination failures may occur in the absence of advertising yet equilibria that give optimal coordination might also arise, thus advertising is sufficient for coordination, but not necessary. Further, our assumption that consumers are aware of the identity of the efficient firm may be untrue in practice. Finally, while we have identified a coordination role for advertising in overturning inefficient outcomes, our theory does not explain how positive advertising might arise in robust equilibria.

In this subsection we address these issues by positing that consumers do not know *a priori* the identity of the efficient firm. With incomplete information, the optimally-coordinated outcome cannot be supported by *any* equilibrium in the absence of advertising. Full coordination can be obtained when advertising is allowed, but only at the added cost of strictly-positive advertising expenditures by the efficient firm.

Suppose for simplicity that there is uncertainty concerning the costs of Firm 2. Thus Firm 1's costs are known by all, but consumers and Firm 1 are uncertain as to whether Firm 2 is more or less efficient than Firm 1. Firm 2 is said to be of a "high type," or type H, if it is less efficient than Firm 1, this is the situation considered in Section 4.1. If Firm 2 is of a "low type," or type L, then Firm 2 is the more efficient firm. We refer to the high and low types of Firm 2 as Firm 2H and Firm 2L, respectively.

Consumers and Firm 1 believe that with probability  $\rho \in (0, 1)$  they are competing against Firm 2H, just as above, but with probability  $1 - \rho$  they are competing against Firm 2L. The earlier profit-function assumptions, including  $t < \Pi_1^*(1)$ , hold for  $t = 1, 2H, 2L$ . Assumption (3) is extended as follows

$$p_{2L}^*(1) < p_1^*(1) < \lim_{m_2 \downarrow 0} p_{2H}^*(m_2) \quad \text{and} \quad p_1^*(1) < p_{2H}^*(1) < \lim_{m_1 \downarrow 0} p_1^*(m_1)$$

We add a condition that makes it possible for advertising credibly to signal costs

*Sorting Condition* For  $m_2 > m_2'$

$$\Pi_{2H}^*(m_2) - \Pi_{2H}^*(m_2') < \Pi_{2L}^*(m_2) - \Pi_{2L}^*(m_2')$$

That is, Firm 2H gains strictly less than does Firm 2L from any given increase in market share. The sorting condition can be derived from the assumption that Firm 2H has strictly greater marginal cost than does Firm 2L at every output level.<sup>15</sup>

The two-stage search game proceeds as above, augmented by a "Stage 0" in which "Nature" determines whether Firm 2's position is to be taken by Firm 2H or Firm 2L.

15 Let the cost function be  $C(q, \alpha)$  where  $C_{qq} < 0$  and  $C_{q\alpha} > 0$ . The sorting condition holds when Firm 2's types are identified with values  $\alpha_1'$  and  $\alpha_1$  of  $\alpha$  with  $\alpha_1' > \alpha_1$ .

In the absence of advertising, three sequential equilibrium outcomes are possible. Firm 1 may capture the market, or Firms 2H and 2L may capture the market, or the firms may split the market. In the latter case the equilibrium market shares are determined by the condition that consumers are indifferent as to which firm to visit. Further, letting  $\hat{m}_i$  give the equilibrium market share of Firm  $i$ , we must have  $\hat{m}_{2H} = \hat{m}_{2L}$  if Firms 2H and 2L both enter, since the consumers cannot tell whether Firm 2 is the high or low type.

These outcomes exhibit each of the coordination failures described above: the less efficient firm serves the market when Firm 2H captures the market from Firm 1, and coordination economies are unrealized when the market is split. Note however that efficiency is no longer guaranteed when Firm 1 captures the market, since Firm 2 is more efficient when it is the low type. Thus in the absence of advertising, it becomes impossible to obtain equilibrium outcomes that give optimal coordination for both of Firm 2's possible cost levels.<sup>16</sup> The difficulty is that each of the three outcomes involves "pooling" by Firm 2's types, and so consumers are unable to discriminate between the situations in which Firm 2 is the more or less efficient firm.

If firms are allowed to advertise and consumers are sophisticated in interpreting the advertising, then advertising becomes a channel by which Firm 2 can establish its identity. Let us henceforth denote the decision to stay out by  $A_i = -1$ , thus  $A_i \geq 0$  means that Firm  $i$  enters, and  $A_i = -1$  means that Firm  $i$  stays out. Let  $\hat{A}_i$  denote the equilibrium entry/advertising strategy of Firm  $i$ , and  $m_i(A_1, A_2)$  give the proportion of consumers that visit Firm  $i$ ,  $i = 1, 2$ , when the strategies  $A_1$  and  $A_2$  are observed, we necessarily have  $\hat{m}_1(\hat{A}_1, -1) = 1$  for  $\hat{A}_1 \geq 0$  and  $\hat{m}_2(-1, \hat{A}_2) = 1$  for  $\hat{A}_2 \geq 0$ . The following proposition demonstrates that under the inference restriction, advertising ensures that the most efficient firm, which is Firm 2L, is always able to capture the market.<sup>17</sup>

**Proposition 2.** *In any sequential equilibrium satisfying no equilibrium dominated conjectures,  $\hat{A}_{2L} \geq 0$  and  $\hat{m}_2(\hat{A}_1, \hat{A}_{2L}) = 1$ , i.e. Firm 2L must enter and capture the market.*

As before, advertising allows the efficient firm to communicate that its price is low, but now Firm 2L must also communicate that it is the efficient firm. This is made possible by the sorting condition: since Firm 2L differentially prefers capturing the market, there are profitable advertising deviations for Firm 2L that would never be profitable for Firm 2H, and so consumers do not associate such deviations with Firm 2H.

In view of Proposition 2, it follows that an equilibrium must be "separating" in order for Firm 1 to capture the market from Firm 2H, i.e. consumers must be able to infer Firm 2's type from its advertising level. The following proposition establishes existence of a separating equilibrium that gives optimal coordination, and moreover the proposition shows that this is the only possible separating outcome under the inference restriction.

**Proposition 3.** *There is at most one separating sequential equilibrium outcome that satisfies no equilibrium dominated conjectures. This outcome is characterized by  $\hat{A}_1 = 0$ ,  $\hat{A}_{2H} = -1$ ,  $\hat{A}_{2L} = \Pi_{2H}^*(1) - F > 0$ , and  $\hat{m}_2(\hat{A}_1, \hat{A}_{2L}) = 1$ . Moreover, such an equilibrium exists if and only if  $\rho \Pi_1^*(1) - F \geq 0$ .*

<sup>16</sup> It is conceivable that Firms 1 and 2L enter, Firm 2H stays out, and consumers visit Firm 2 if both firms enter, thus optimal coordination is achieved. This outcome cannot be supported as an equilibrium, however, since  $F < \Pi_{2H}^*(1)$  implies that Firm 2H would choose to enter and mimic Firm 2L.

<sup>17</sup> The inference restriction extends to the new game in the natural way. In particular, if  $(A_2, p_2)$  is equilibrium dominated for both Firms 2L and 2H, while some  $(A_2, p_2)$  is not, then consumers do not conjecture that  $p_2$  accompanies  $A_2$ .

In the surviving separating equilibrium outcome, only Firms 1 and 2L enter, and only Firm 2L advertises at a positive level, the level of  $\hat{A}_{2L}$  is the smallest that discourages deviation by Firm 2H. Further, the most efficient firm captures the market in each contingency, since Firm 2H stays out while Firm 2L captures the market from Firm 1. For the separating equilibrium to exist, Firm 1 must desire to enter given that it has only probability  $\rho$  of capturing the market, thus  $\rho$  must be sufficiently large. Here non-existence of separating equilibrium is equivalent to infeasibility of the optimally-coordinated outcome, since coordination requires Firm 1 to commit to the entry cost before it knows whether or not it is the most efficient firm.<sup>18</sup>

Note that Firm 2L must choose strictly positive advertising in the separating equilibrium, in order to establish credibly that it is not Firm 2H. Thus with incomplete information, positive advertising on the equilibrium path is required in order to achieve optimal coordination. The need for positive advertising introduces a potential welfare tradeoff between advertising and coordination: consumers strictly prefer the optimally-coordinated outcome, but because of the advertising expense, industry profits may well be higher in one or more of the outcomes in which advertising is banned. For example, it is easy to show that if  $\Pi_1^*(1) > \Pi_{2L}^*(1) - \Pi_{2H}^*(1)$ , then the outcome in which advertising is banned and Firm 1 captures the market gives higher industry profits than the separating equilibrium. If coordination leads to small gains in consumer surplus, then social surplus may well be greater in the absence of advertising.

While it has been established that advertising makes optimal coordination possible, it is also true that advertising does not completely eliminate the possibility of coordination failure, contrary to the complete information case. This is because the pooling equilibrium in which Firm 2 captures the market can survive the inference restriction. Letting  $V(p)$  denote consumers' utility from purchasing at price  $p$ , we have

**Proposition 4.** *There is at most one pooling sequential equilibrium outcome that satisfies no equilibrium dominated conjectures. This outcome is characterized by  $\hat{A}_1 = -1$  and  $\hat{A}_{2H} = \hat{A}_{2L} = 0$ . Moreover, such an equilibrium exists if and only if*

$$\rho V(p_{2H}^*(1)) + (1 - \rho)V(p_{2L}^*(1)) \geq V(p_1^*(1)) \quad (4)$$

Only Firms 2L and 2H enter in this equilibrium and they do not advertise since Firm 1 does not enter. If (4) were to fail, Firm 1 could overturn the equilibrium by entering and advertising to communicate a low price, thereby capturing the market. When (4) holds, meaning that  $\rho$  is sufficiently small, we can think of the pooling equilibrium as giving optimal coordination in an *ex ante* sense, in that Firm 2 is more efficient on average.<sup>19</sup>

18 It is straightforward to extend the model to give Firm 1 the option of delaying its entry decision in order to observe Firm 2's decision. The analysis of this case is given in Bagwell and Ramey (1990, Appendix D). Separating equilibria that give optimal coordination exist for all values of  $\rho$ , and Proposition 2 is unaltered. Propositions 3 and 4 hold when  $\rho$  is sufficiently close to unity and the inference restriction is modified as in Section 4.3 below. For small  $\rho$ , however, equilibrium dominance may be insufficient to rule out market-splitting outcomes, for the same reasons as discussed in Section 4.3.

19 Propositions 1-4 continue to hold when mixed strategies are allowed, given that the inference restriction and sorting condition are extended in the natural way. In the incomplete-information model, there arises a new class of mixed equilibria, where Firms 1 and 2H split the market with positive probability, the inference restriction eliminates these if and only if  $\rho$  is sufficiently large. See Bagwell and Ramey (1990, notes 15 and 25).

Finally, Propositions 2 and 3 together indicate that inefficient market-splitting outcomes are possible only for markets in which advertising is banned. The scope for coordination failures in such markets is again consistent with Benham's association of the ability to advertise with larger-scale firms and lower prices.

#### 4.3 Coordination when Prices are More Flexible than Advertising

We have assumed that the firms commit to their prices at the same time as advertising levels are chosen, but there are many situations in which prices might be more easily adjusted than advertising. The strategic situation is then changed, as firms are able to alter their prices in response to each others' observed advertising.

The duopoly game may be modified to reflect this possibility, by allowing firms to adjust prices after observing each others' entry and advertising decisions, but prior to consumers' visitation decisions. Under an appropriate strengthening of the inference restriction, Propositions 1 and 2 hold as stated.<sup>20</sup> Proposition 3 is weakened, however, in that for small  $\rho$ , inefficient separating equilibria in which Firms 1 and 2H split the market may survive the inference restriction.<sup>21</sup> Further, condition (4) of Proposition 4 is altered, but the pooling outcome again survives the inference restriction only for small  $\rho$ .

The price flexibility case, along with other permutations of the order of moves, is discussed in detail in Bagwell and Ramey (1990). The key conclusion is that optimal coordination remains the unique separating equilibrium outcome if and only if  $\rho$  is sufficiently large, intuitively, Firm 1 is willing and able to use advertising to capture the market if and only if it is sufficiently likely *ex ante* to be the most efficient firm.

## 5 LOSS-LEADER PRICING

In many retail markets, sellers are able to communicate price information for some, but not all, of the products in their product lines. In this section we investigate whether such "loss-leader" advertising is by itself sufficient to eliminate coordination failures. When consumers are sophisticated in interpreting observed prices, it is indeed possible under certain conditions for loss-leader advertising to be used to communicate low prices for the rest of the product line. Under more general conditions, however, coordination failures are eliminated only if loss-leaders are accompanied by dissipative expenditures.<sup>22</sup>

Let us modify the framework of Section 4.1 by supposing the firms supply two goods,  $q$  and  $s$ . Price communication is possible only for good  $s$ , which serves as the loss-leader item. Let  $p_i$  and  $r_i$  denote the prices charged by Firm  $i$  for goods  $q$  and  $s$ , respectively.

20 The strengthened inference restriction states that the strategy  $(A_i, p_i(A_j))$  is equilibrium dominated for Firm  $i$  if  $\text{Max}_m \Pi_i(p_i(A_j), m_i) - A_i - F < W_i$ , where  $W_i$  is Firm  $i$ 's equilibrium payoff. Thus equilibrium dominance applies only to that part of a firm's strategy that is relevant given the other firm's equilibrium strategy. This criterion is implied by the *elimination of never weak best responses* criterion (Kohlberg and Mertens (1986), Cho and Kreps (1987)). Our criterion is a bit weaker, in that dominance is assessed against the entire range of possible consumer responses, rather than the consumers' equilibrium response, although both criteria produce identical results in our game.

21 Specifically, consumers can conjecture that Firm 1 shifts its price to  $p_1^*(1)$  when Firm 2H deviates, so that Firm 2H cannot capture the market in the inefficient separating equilibria that remain after Proposition 2, in contrast to the proof of Proposition 3. These equilibria are eliminated if Firm 1 uses advertising to capture the market from Firm 2H, but not Firm 2L, by choosing  $A_1$  large enough to convince consumers that  $p_1(A_{2H})$  is close to  $p_1^*(1)$ . Firm 1 desires to do this if and only if  $\rho$  is sufficiently large.

22 Lal and Matutes (1991) emphasize a different role for loss-leaders, whereby a low loss-leader price guarantees sufficient consumer surplus to justify costly consumer search, even though unadvertised goods are priced at consumers' reservation value.

The profit function is now written  $\Pi_i(p_i, r_i, m_i)$ , where  $\Pi_i$  is continuous and strictly quasi-concave in  $p_i$  for each  $r_i, m_i$ . Let  $p_i^*(r_i, m_i)$  denote the profit-maximizing choice of  $p_i$  when  $r_i$  and  $m_i$  are fixed, and let  $\Pi_i^*(r_i, m_i)$  denote the corresponding maximized profit level. Further, let  $r_i^*$  denote the level of  $r_i$  that maximizes  $\Pi_i^*(r_i, 1)$ , i.e.  $r_i^*$  gives the full-market monopoly price for good  $s$ . We assume  $F < \Pi_i^*(r_i^*, 1)$  for both  $i$ .

Since firms cannot raise  $r_i$  to exploit higher market share the better profit property must be modified, as follows

*Better Profit Property* Fix  $m_i$  and  $m_i'$  with  $m_i > m_i'$

(a) If  $\Pi_i^*(r_i, m_i) > 0$ , then  $\Pi_i^*(r_i, m_i) > \Pi_i^*(r_i, m_i')$

(b) If  $\Pi_i^*(r_i, m_i) \leq 0$ , then  $\Pi_i^*(r_i, m_i') \leq 0$

This assumption states that profits are increasing in market share if  $r_i$  is set high enough to give positive profits. If  $r_i$  is so low that profits are non-positive, then profits are non-positive at smaller market shares as well. This version of the better profit property is implied by non-increasing ray average cost for example (Sharkey (1982), Ch 4). Further

*Better Deal Property*  $p_i^*(r_i, m_i)$  is strictly decreasing in  $m_i$  for each  $r_i$

The better deal property could again be motivated by positing declining marginal cost for good  $q$ , but in the present case cost complementarities across the two products can generate the better deal property even if costs are convex in good  $q$ .

Let  $V(p_i, r_i)$  give the consumer utility obtained from purchasing at prices  $p_i, r_i$ .  $V$  is assumed to be continuous and strictly decreasing in its arguments, and also

$$V(p_1^*(r_1, 1), r_1) > V(p_2^*(r_2, 1), r_2) > V\left(\lim_{m_i \downarrow 0} p_i^*(r_i, m_i), r_i\right) \text{ for all } r_1 \text{ and } r_2 \quad (5)$$

$$\lim_{r_i \rightarrow -\infty} \Pi_i^*(r_i, 1) - F < 0 \quad (6)$$

Assumption (5) is a very strong form of (3), wherein Firm 1 offers greater utility when it captures the market, irrespective of the loss-leader prices chosen by *either* firm. Assumption (6) ensures that a firm loses money when it captures the market via a very low loss-leader price, despite profit-maximizing pricing on the remainder of the product line.

Outcomes 1, 2 and 3 of Section 4.1 can all be supported as sequential equilibria in the loss-leader case, since consumers are free to make unfavourable inferences about  $p_i$  when they observe deviations in  $r_i$  (supporting Outcome 3 again requires  $F$  to be sufficiently small). Thus competition in the loss-leader price does not by itself eliminate the possibility of coordination failures. When the inference restriction is imposed, however, loss-leader advertising is sufficient to bring about optimal coordination.

**Proposition 5.** *Outcome 1 uniquely satisfies no equilibrium dominated conjectures when firms are allowed to use only loss-leader advertising.*

The proof of Proposition 5 closely follows that of Proposition 1, except that here Firm 1 communicates via a low loss-leader price, rather than direct expenditures.

While it is conceivable that loss-leader advertising suffices to eliminate coordination failures, the preceding result relies on assumptions that seem contradictory. (5) essentially means that the loss leader represents a small percentage of consumers' total purchases

but (6) requires the loss leader to be important in the firm's profit function. Moreover, Proposition 5 may break down if the inconsistency is resolved in either direction. On the one hand, if the loss leader forms a significant percentage of total sales, then deep cuts in the loss-leader price may induce the firm to make large increases in the prices of the remaining products (this occurs, for example, if the goods are substitutes, costs are strongly concave in  $q$  and cost complementarities are weak). We may then have  $V(p_1^*(r_1, 1), r_1) < V(\hat{p}_2, \hat{r}_2)$  for levels of  $r_1$  that are low enough to establish  $p_1$  close to  $p_1^*(r_1, 1)$ , where  $\hat{p}_2$  and  $\hat{r}_2$  give Firm 2's equilibrium price choices, it follows that Firm 1 cannot induce consumers to leave Firm 2.

On the other hand, if the loss leader is a small percentage of sales, then it may be reasonable to suppose that  $\Pi_1^*(r_1, 1) - F > 0$  for all  $r_1$ , i.e. profits from capturing the market are positive for all  $r_1$ . The inference restriction then allows consumers to associate any given  $r_1$  with a range of possible values of  $p_1$ , and Outcome 2 will not be overturned if the range includes high levels of  $p_1$  and good  $s$  is relatively unimportant to consumers. This difficulty is exacerbated if firms can ration supplies of the loss leader, e.g. through "rain checks" that serve to delay delivery of the good.

In the latter cases, loss-leader advertising and sophisticated consumer inference do not rule out the possibility of coordination failures. Coordination failures are eliminated, however, when loss-leaders can be combined with dissipative advertising expenditures. In particular, the following proposition establishes that loss leaders and dissipative expenditures ensure optimal coordination under conditions much weaker than (5) and (6).

**Proposition 6.** *Let (5) be weakened to*

- (a)  $p_1^*(r, 1) < p_2^*(r, 1) < \lim_{m_i \rightarrow 0} p_1^*(r, m_i)$  for all  $r$ , and
- (b)  $V(p_1^*(r_1^*, 1), r_1^*) > V(p_2^*(r_2, 1), r_2)$  for all  $r_2$

*Also let (6) be relaxed. Then Outcome 1 uniquely satisfies no equilibrium dominated conjectures when firms choose dissipative advertising expenditures together with loss-leader prices.*

The proof directly parallels the proof of Proposition 1 and is omitted. Condition (a) gives a sufficient condition for existence of coordination failures. Condition (b) ensures that there is a profitable market-capturing strategy for Firm 1, in which Firm 1 accompanies the loss-leader price  $r_1^*$  with a large advertising level, thereby convincing consumers that  $p_1$  is close to  $p_1^*(r_1^*, 1)$ . The condition also rules out the possibility that Firm 2 overturns Outcome 1 through direct competition in the loss-leader price. It follows that dissipative advertising may be essential for eliminating coordination failures, even in the presence of loss-leader advertising. The incomplete-information results may be extended in a similar way to allow for loss-leader pricing and dissipative advertising, given an appropriate extension of the sorting condition.<sup>23</sup>

Finally, the loss-leader model can be immediately re-interpreted as a model of price and advertising as signals of product quality. In this case there is a single good, with  $r_i$  denoting Firm  $i$ 's price and  $p_i$  its quality choice, lower  $p_i$  now corresponds to higher quality. Here quality is a *search attribute*, i.e. consumers can observe quality prior to purchase, but not prior to search. As discussed in Bagwell and Ramey (1990), the better

23. One important added feature is that the loss leader price of Firm 2L in the efficient separating equilibrium will not be uniquely determined unless a sufficiently strong form of the sorting condition is specified.

profit and better deal properties are present for an important class of cases in the product-quality context, and in particular a firm's profit-maximizing quality choice is strictly increasing in its market share, thus coordination failures may arise despite the fact that price is observable prior to search. Assumptions (5) and (6) may be directly applied to obtain a quality-based version of Proposition 5 in any sequential equilibrium that satisfies the inference restriction, the high-quality firm captures the market when firms are allowed to communicate only with price and not dissipative advertising. As in the loss-leader case, optimal coordination can be assured under conditions much weaker than (5) and (6) if the high-quality firm is able to accompany its price signal with dissipative advertising expenditures.

### 6 CONCLUSION

We have provided a theory of advertising that explains the prevalence of "vague" retail advertisements, as well as Benham's association of the ability to advertise with lower prices and larger scale. The analysis hinges on three key assumptions: sellers find it difficult to communicate relevant information (e.g. price, quality, selection), buyers and active sellers mutually benefit when buyers concentrate their purchases among fewer firms and consumers are sophisticated in interpreting advertising messages.

An intriguing area for future research concerns the transition path to equilibria. Implicit in our story is an unmodelled dynamic in which positive advertising is used to "break" inefficient equilibria. While our analysis has considered explicitly only the steady-states of such a dynamic, it does suggest a transition path along which advertising is expanded in the short-run, and then decreased gradually, in the course of establishing a long-run outcome.

### APPENDIX

*Proof of Lemma* For  $p' \geq \lim_{m \rightarrow 0} p^*(m)$ , the result follows directly from strict quasiconcavity. Otherwise we have  $p' = p^*(m')$  for some  $m' > m \geq m'$  implies

$$\Pi(p, m) < \Pi(p', m) \leq \text{Max}_m \Pi(p', m)$$

where the first inequality follows from quasiconcavity. Thus if the result does not hold we contradict the better profit property, since for some  $\tilde{m} < m'$

$$\Pi^*(m') \leq \text{Max}_m \Pi(p', m) \leq \text{Max}_m \Pi(p, m) = \Pi(p, \tilde{m}) \leq \Pi^*(\tilde{m}) \quad \parallel$$

*Proof of Proposition 1* Note first that the equilibrium payoff of Firm  $i$  is given by

$$W_i = \begin{cases} \Pi_i^*(\hat{m}_i) - \hat{A}_i - F, & \text{if Firm } i \text{ enters,} \\ 0, & \text{if Firm } i \text{ stays out} \end{cases}$$

where  $\hat{A}_i$  gives the equilibrium advertising strategy of Firm  $i$  if it chooses to enter. The advertising-price pair  $(A_i, p_i)$  is equilibrium dominated for Firm  $i$  if

$$\text{Max}_m \Pi(p_i, m_i) - A_i - F < W_i$$

Using these extended definitions the inference restriction of the text may be applied directly to the duopoly search game.

Consider Outcome 2, in which Firm 2 enters and chooses  $(\hat{p}_2, \hat{A}_2) = (p_2^*(1), 0)$  and Firm 1 stays out. Fix  $p'_1 \in (p_1^*(1), p_2^*(1))$  and define  $A'_1$  by

$$\text{Max}_{m_1} \Pi(p'_1, m_1) - A'_1 - F = 0$$

The assumption  $F < \Pi_1^*(1)$  ensures that  $A'_1 > 0$  if  $p'_1$  is chosen sufficiently close to  $p_1^*(1)$ . Using the Lemma, it follows that any  $(A'_1, p_1)$  with  $p_1 > p'_1$  is equilibrium dominated for Firm 1, while  $(A'_1, p_1^*(1))$  is not equilibrium dominated. Thus the inference restriction requires that consumers must infer  $p_1 \leq p'_1$  if they observe Firm 1 entering and choosing  $A'_1$ ; they must continue to infer  $p_2 = p_2^*(1)$  following Firm 1's deviation, according to the independent price conjectures restriction (see note 12), thus by sequential rationality all consumers must visit Firm 1 following the deviation, and in fact Firm 1 strictly prefers to deviate. Similar arguments are used to eliminate Outcome 3 together with all positive-advertising sequential equilibria with market-splitting, in this case the deviant advertising level  $A'_1$  is defined by

$$\text{Max}_{m_1} \Pi(p'_1, m_1) - A'_1 - F = \Pi_1^*(\hat{m}_1) - \hat{A}_1 - F$$

It remains to show that Outcome 1 survives the inference restriction. Since  $W_1 = \Pi_1^*(1) - F$  in any equilibrium supporting Outcome 1, any  $(A_1, p_1)$  with  $A_1 > 0$  is equilibrium dominated, thus we may specify that consumers conjecture  $p_1 = p_1^*(1)$  for any  $A_1$  they observe. As for Firm 2, suppose that  $(A_2, p_2)$  is not equilibrium dominated. Since  $\text{Max}_{m_2} \Pi_2(p_2, m_2) \leq \Pi_2^*(1)$ , it follows that  $(A_2, p_2^*(1))$  is not equilibrium dominated. This allows us to specify that consumers conjecture  $p_2 = p_2^*(1)$  for any observed  $A_2$ , and by sequential rationality all consumers visit Firm 1 for any observed profile  $(A_1, A_2)$ .      ||

*Proof of Proposition 2* Suppose Firm 2L chooses  $\hat{A}_{2L} \geq 0$  and has equilibrium market share  $\hat{m}_{2L}$  with  $\hat{m}_{2L} < 1$ . Note that

$$\lim_{p_2, p_{2L}^*(1)} \text{Max}_{m_2} \Pi_{2L}(p_2, m_2) = \Pi_{2L}^*(1)$$

Thus using the sorting condition, we may choose  $p'_2 \in (p_{2L}^*(1), p_1^*(1))$  sufficiently close to  $p_{2L}^*(1)$  to give

$$\text{Max}_{m_2} \Pi_{2L}(p'_2, m_2) - \Pi_{2L}^*(\hat{m}_{2L}) > \Pi_{2H}^*(1) - \Pi_{2H}^*(\hat{m}_{2L}) > 0 \tag{A1}$$

Now define  $A'_2$  by

$$\text{Max}_{m_2} \Pi_{2L}(p'_2, m_2) - A'_2 - F = W_{2L} \tag{A2}$$

By (A1) we have  $A'_2 > \hat{A}_{2L}$ , and further the Lemma implies that for all  $p_2 > p'_2$ ,  $(A'_2, p_2)$  is equilibrium dominated for Firm 2L. Moreover, combining (A1) and (A2) gives

$$\Pi_{2H}^*(1) - A'_2 - F < \Pi_{2H}^*(\hat{m}_{2L}) - \hat{A}_{2L} - F \leq W_{2H}$$

where the weak inequality follows from the equilibrium conditions (Firm 2H cannot strictly prefer  $\hat{A}_{2L}$ ). Thus for all  $p_2$ ,  $(A'_2, p_2)$  is equilibrium dominated for Firm 2H. It is apparent that  $(A'_2, p_{2L}^*(1))$  is not equilibrium dominated for Firm 2L, and further that capturing the market with  $(A'_2, p_{2L}^*(1))$  gives Firm 2L profits strictly greater than  $W_{2L}$ . Thus when consumers observe Firm 2 deviating to  $A'_2$ , they must conclude that Firm 2 is actually Firm 2L and that it charges  $p_2 < p'_2$ . Since  $p'_2 < p_1^*(1) \leq \hat{p}_1$ , Firm 2L captures the market with such a deviation. The argument is easily modified to rule out  $\hat{A}_{2L} = -1$ .      ||

*Proof of Proposition 3* Suppose first that

$$\hat{A}_{2L} > \Pi_{2H}^*(1) - F - W_{2H} \tag{A3}$$

Define  $A'_2$  by (A2) for  $p'_2 \in (p_{2L}^*(1), p_1^*(1))$ . Proposition 2 implies that  $A'_2 < \hat{A}_{2L}$  and the Lemma implies that for all  $p_2 > p'_2$ ,  $(A'_2, p_2)$  is equilibrium dominated for Firm 2L. Further, by choosing  $p'_2$  sufficiently close to  $p_{2L}^*(1)$  we have

$$\hat{A}_{2L} > A'_2 > \Pi_{2H}^*(1) - F - W_{2H}$$

from which it follows that for all  $p_2$ ,  $(A'_2, p_2)$  is equilibrium dominated for Firm 2H. Clearly  $(A'_2, p_{2L}^*(1))$  is not equilibrium dominated for Firm 2L, and it gives profits greater than  $W_{2L}$  when the market is captured. Thus consumers visit Firm 2 when they observe a deviation to  $A'_2$ , and so Firm 2L deviates. The equilibrium conditions rule out the reverse strict inequality to (A3), and so equality must hold. Since  $W_{2H} = 0$  in any equilibrium that supports the efficient outcome, the proposition gives the unique level of  $\hat{A}_{2L}$  in any such equilibrium.

Now consider an equilibrium in which Firms 1 and 2H split the market. Since the equilibrium is separating market-splitting implies that the equilibrium prices of Firms 1 and 2H satisfy  $p_1^*(1 - \hat{m}_{2H}) = p_{2H}^*(\hat{m}_{2H})$ , where  $\hat{m}_{2H}$  is the equilibrium market share of Firm 2H. Choose  $p_2' \in (p_1^*(1 - \hat{m}_{2H}), p_{2H}^*(\hat{m}_{2H}))$  and define the advertising level  $A_2'$  by

$$A_2' = \text{Max} \left\{ \text{Max}_{m_2} \Pi_{2H}(p_2', m_2) - F - W_{2H}, \text{Max}_n \Pi_{2L}(p_2', m_2) - F - W_{2L} \right\}$$

(A3) with equality implies that  $0 < A_2' < \hat{A}_{2L}$  if  $p_2'$  is chosen sufficiently close to  $p_{2H}^*(1)$ . Using the Lemma we know that for all  $p_2 > p_2'$ ,  $(A_2', p_2)$  is equilibrium dominated for both Firm 2H and Firm 2L. If the first term in braces gives the maximum, then Firm 2H strictly prefers  $(A_2', p_{2H}^*(1))$  and capturing the market to its equilibrium payoff, while Firm 2L strictly prefers  $(A_2', p_{2H}^*(1))$  and capturing the market if the second term gives the maximum. In either case consumers infer  $p_2 \leq p_2' < p_1^*(1 - \hat{m}_{2H})$  and visit Firm 2 when they observe a deviation to  $A_2'$ , and so at least one of Firm 2's types will deviate.

Finally, an equilibrium in which Firm 2L captures the market. Firm 1 captures the market from Firm 2H, and  $\hat{A}_{2L} = \Pi_{2H}^*(1) - F$  satisfies the inference restriction. Suppose that  $(A_1, p_1)$  with  $A_1 > 0$  is not equilibrium dominated for Firm 1. Then for some  $m_1'$

$$W_1 \leq \Pi_1(p_1, m_1') - A_1 - F \leq \Pi_1^*(1) - A_1 - F$$

so  $(A_1, p_1^*(1))$  is not equilibrium dominated. Thus consumers may conjecture  $p_1 = p_1^*(1)$  and  $p_2 = p_{2L}^*(1)$  upon observing  $(A_1, \hat{A}_{2L})$  and visit Firm 2, and this deters deviation by Firm 1. Similarly, consumers may conjecture  $p_1 = p_1^*(1)$  and  $p_2 = p_{2H}^*(1)$  upon observing  $(\hat{A}_1, A_2)$  with  $A_2 \in (0, \hat{A}_{2L})$ , since  $(A_2, p_{2H}^*(1))$  is not equilibrium dominated for Firm 2H. Of course,  $A_2 > \hat{A}_{2L}$  is equilibrium dominated in conjunction with any  $p_2$  for both types of Firm 2. ||

*Proof of Proposition 4* Proposition 2 implies that Firm 2L must capture the market in any pooling equilibrium that satisfies the inference restriction. In such an equilibrium Firm 1 stays out and Firms 2H and 2L choose zero advertising. Firm 1 can use positive advertising to communicate that it chooses a price close to  $p_1^*(1)$ , and this deviation captures the market if and only if (4) is violated. ||

*Proof of Proposition 5* Note first that under the modified version of the better profit property, the Lemma is altered as follows. Fix  $r$ , and  $p_i > p_i' \geq p_i^*(r, 1)$ . Then  $\text{Max}_m \Pi_i(p_i', r, m_i) > 0$  implies  $\text{Max}_m \Pi_i(p_i', r, m_i) > \text{Max}_m \Pi_i(p_i, r, m_i)$ , while  $\text{Max}_m \Pi_i(p_i', r, m_i) \leq 0$  implies  $\text{Max}_m \Pi_i(p_i, r, m_i) \leq 0$  (the proof of this claim is a straightforward adaptation of the proof of the Lemma).

Next, let  $\underline{r}_i$  be defined by

$$\underline{r}_i = \text{Max} \{ r_i' \mid \Pi_i^*(r_i, 1) \leq F \text{ for all } r_i' \leq r_i' \}$$

Existence of  $\underline{r}_i$  is assured by (6). For any  $p_i' > p_i^*(\underline{r}_i, 1)$ , we have

$$\text{Max}_{m_i} \Pi_i(p_i', \underline{r}_i, m_i) < \Pi_i^*(\underline{r}_i, 1) = F$$

By continuity of  $\Pi_i$  in  $r_i$ , we may choose  $r_i' > r_i$  sufficiently close to  $\underline{r}_i$  to give  $p_i' > p_i^*(r_i', 1)$  and

$$\text{Max}_{m_i} \Pi_i(p_i', r_i', m_i) < F < \Pi_i^*(r_i', 1) \tag{A4}$$

Now consider Outcome 2. By choosing  $p_i' > p_i^*(\underline{r}_i, 1)$  sufficiently close to  $p_i^*(\underline{r}_i, 1)$  and  $r_i' > \underline{r}_i$  sufficiently close to  $\underline{r}_i$ , we have (A4) and

$$V(p_i', r_i') > V(p_2^*(\hat{r}_2, 1), \hat{r}_2) \tag{A5}$$

where the latter inequality follows from (5), and  $\hat{r}_2$  gives Firm 2's equilibrium loss-leader price. (A4) implies that for all  $p_i > p_i'$ ,  $(p_i, r_i')$  is equilibrium dominated for Firm 1, while Firm 1 strictly prefers  $(p_i^*(r_i', 1), r_i')$  and capturing the market to its equilibrium payoff of zero. (A5) ensures that this deviation captures the market for Firm 1. Outcome 3 is ruled out by a similar argument, and by (5) Outcome 1 survives the inference restriction. ||

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