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# Collusion over the business cycle

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*We present a theory of collusive pricing for markets in which demand alternates stochastically between fast-growth (boom) and slow-growth (recession) phases. We show that (1) the most-collusive prices are weakly procyclical (countercyclical) when demand growth rates are positively (negatively) correlated through time, and (2) the amplitude of the collusive pricing cycle is larger when the expected duration of boom phases decreases and when the expected duration of recession phases increases. We also offer a generalization of Rotemberg and Saloner's (1986) model, interpreting their findings in terms of transitory demand shocks that occur within broader business cycle phases.*

## 1. Introduction

■ Collusion is a balancing act. Each colluding firm balances the short-term temptation to cut its price against the expected long-term cost of the price war that such an act might instigate. When the level of demand grows and fluctuates through time, as along a business cycle, the relationship between the short-term temptation to cheat on a collusive agreement and the expected long-term cost from doing so need not be constant, and maintaining a balance between the two may require periodic adjustments in the collusive price. In this way, it is possible to forge a link between the state of the business cycle and the price level of colluding firms.

In a pioneering article, Rotemberg and Saloner (1986) offer one such theory. Taking a simple but illustrative view of the business cycle, they assume that the level of market demand is determined in an identically and independently distributed fashion each period, so that the expected level of future demand—and thus the expected long-term cost from cheating—is independent of the current demand level. Today's demand

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level, however, does affect the short-term incentive to cheat, since a price cut is more attractive when many consumers are in the market. Associating a business-cycle boom (recession) with a period of high- (low-) demand realization, Rotemberg and Saloner then argue that collusion is most difficult in booms, when the incentive to cheat is greatest. Expanding on this insight, they conclude that for moderate values of the discount factor, collusive pricing is countercyclical, i.e., firms set a lower price in periods in which the level of demand is higher.[1]

As is well recognized, an important limitation of the Rotemberg-Saloner model is the assumption that demand follows an identically and independently distributed process. While analytically attractive, this assumption rules out the possibility that a relatively high-demand realization today might signal an "upturn" in business conditions that leads firms to expect further growth in future demand. This lack of persistence, or indeed of any notion whatsoever of an "expansionary phase," compromises the interpretation of their article as a theory of collusion over the business cycle.

With this limitation in mind, we develop here a theory of collusive pricing for markets in which demand movements are stochastic and persistent. Motivated by Hamilton's (1989) description of the U.S. business cycle, we assume that the level of market demand alternates stochastically between slow- and fast-growth states, where the transition from one state to the other is determined by a Markov process. We interpret a boom phase as a sequence of periods of fast growth in the level of market demand. A recession phase then corresponds to periods of slower growth, and we allow—but do not insist—that recessions entail negative growth. Given this representation of the business cycle, a definition of cyclical pricing in terms of the level of market demand is misguided, and we thus instead say that collusive prices are procyclical (countercyclical) when they are higher in fast- (slow-) growth periods, i.e., in boom (recession) phases.

Within this context, we provide a complete characterization of the collusive prices as functions of a rich set of parameters, and establish a new role for the parameters that determine the extent of correlation in demand growth rates through time and the expected duration of boom and recession phases, respectively. We have two main results: (1) collusive pricing is weakly procyclical (countercyclical) when market demand growth rates are positively (negatively) correlated through time, and (2) the amplitude of the collusive pricing cycle is larger when the expected duration of boom (recession) phases decreases (increases).

With these basic results in place, we next consider an extended model that allows as well for random demand fluctuations within given phases. The formulation that we explore entails a combination of our modelling approach with that of Rotemberg and Saloner. In particular, we assume that an identically and independently distributed process generates shocks to the level of market demand in each period, and that such shocks occur outside of the Markov-growth process for demand, in that a shock to current-period demand has no affect on future-period demand levels. We thus refer to shocks of this form as being transitory. We then show that a higher transitory shock to demand results in a (weakly) lower collusive price, regardless of whether the market is in a boom or a recession phase. In this extended model, therefore, Rotemberg and Saloner's theory of collusive pricing can be interpreted in terms of the response of collusive prices to transitory demand shocks that occur within broader business cycle

[1] Green and Porter (1984) consider an alternative theory of collusion, in which firms are unable to perfectly observe current demand conditions; in this case, low prices may be required during downturns. Staiger and Wolak (1992) reach an analogous conclusion by introducing capacity constraints into the Rotemberg-Saloner setup.

phases. We also demonstrate that our predicted association between procyclical (countercyclical) collusive prices and positively (negatively) correlated demand growth rates is robust to—and in fact strengthened by—the inclusion of transitory demand shocks.

Although we do not offer an empirical analysis of collusion in this article, we anticipate that the results developed here will prove useful in future empirical studies. At present there is empirical evidence supporting both pro- and countercyclical pricing, and on the whole the evidence appears to be somewhat mixed and inconclusive.[2] Our theory suggests that the key determinants of the cyclical behavior of collusive prices correspond to parameters that represent the correlation of demand growth rates through time and the expected duration of boom and recession phases. An empirical analysis that organizes industry data along these lines may thus offer a more satisfactory assessment of the cyclical properties of collusive prices. The theory developed here is potentially well suited for such an analysis, since we give a complete characterization of collusive prices in terms of these parameters. We also note that Hamilton and others have devised econometric techniques for estimating these same business-cycle parameters.

Finally, we compare our results with those derived in the interesting article by Haltiwanger and Harrington (1991). Under the assumption that the level of demand follows a deterministic cycle, Haltiwanger and Harrington show that collusive prices are higher when demand is rising, holding fixed the level of demand. The overall relationship between the collusive price and the level of current market demand is less clear, and Haltiwanger and Harrington are unable to provide complete analytic results on this matter. Simulations reveal that collusive prices are often procyclical; however, as the discount factor drops relative to the number of firms, collusive prices become increasingly countercyclical, much as Rotemberg and Saloner originally predicted.[3] A central difference between our model and that of Haltiwanger and Harrington is that we assume that turning points are unpredictable. With this assumption, we enhance the empirical plausibility of the business-cycle model while simplifying the analysis of collusive pricing.[4]

The article is organized as follows. Section 2 presents the basic assumptions of the oligopoly market and considers the benchmark case of a market demand level that grows at a stationary rate. In Section 3, the Markov-growth model is developed and the incentive constraints for collusion are derived. The most-collusive prices are characterized in Section 4, while Section 5 presents the extended model with transitory shocks to demand. Section 6 concludes.

## 2. The stationary benchmark

■ **Basic assumptions.** We analyze a Bertrand-pricing supergame, in which a fixed set of $n \geq 2$ firms sells the same nondurable good in each period $t \in \{1, \ldots, \infty\}$. The total mass or number of consumers in any period $t$ is $G_t$, which is also called the level

[2] The empirical literature includes the cross-sectional work of Domowitz, Hubbard, and Petersen (1986) and Rotemberg and Woodford (1992), as well as the industry studies of Borenstein and Shepard (1996), Chevalier and Scharfstein (1996), Ellison (1994), Porter (1983), and Rotemberg and Saloner (1986). See also Suslow (1988) for evidence that cartels are most likely to break down during recessions.

[3] Further support for this conclusion is found in Kandori's (1991) work. Countercyclical markups may also arise in customer markets, as Chevalier and Scharfstein (1996) and Klemperer (1995) demonstrate.

[4] See, e.g., Zarnowitz (1992) on the ample evidence that turning points are difficult to predict. While Haltiwanger and Harrington's deterministic approach is thus not ideal for analyzing collusive pricing over the business cycle, the assumption of a deterministic cycle is much more appealing for collusive markets that are subjected to seasonal demand fluctuations. Borenstein and Shepard, e.g., provide evidence in support of the Haltiwanger-Harrington model in the context of the retail gasoline market.

of market demand in period $t$. Within any period $t$, the firms select their prices simultaneously, and consumers then divide up evenly over the lowest-priced firms. A firm earns zero profit for the period if it is not a lowest-priced firm, and it earns a profit of $\pi(P) = (P - c)\, D(P)$ per consumer when its price $P$ is among the lowest, where $c \geq 0$ denotes unit costs and $D(P)$ is the consumers' common demand function. Assuming that $\pi$ has a unique maximizer, $P_m$, and that $\pi$ is strictly increasing and differentiable over $P \in [0, P_m]$, we note that the monopoly price $P_m$ is acyclic, since its value is independent of the number of consumers to whom the firm sells. For simplicity, firms select prices from the set $[0, P_m]$.

As is well known, in any Nash equilibrium of the Bertrand stage game all sales occur at the competitive price, $P = c$. Firms may be able to earn positive equilibrium profit in a dynamic model, however, as the short-term temptation to undercut rivals is then balanced against the long-term price war that such "cheating" might trigger. To model this idea, we assume that each firm observes all past prices, so that a firm's period-$t$ price is a function of the prices charged by other firms in periods $\tau \in \{1, \ldots, t-1\}$. We also introduce a common discount factor parameter, $\delta \in (0, 1)$, as a measure of firms' patience with regard to future profit. The tradeoff between short- and long-term profit is affected by the current and the projected future levels of market demand. We do not develop an endogenous business cycle model here; rather, we assess the implications of exogenously imposed business cycles for collusive pricing. In particular, we allow the state of the business cycle to determine the level of market demand, $G_t$. We assume further that each firm knows the current and all past levels of market demand when selecting current prices.

We are interested in subgame-perfect equilibria and select among the set of such equilibria with two additional requirements. First, we assume that firms adopt symmetric strategies. Second, we characterize the most-collusive prices, which we define as the highest prices that can be supported in a symmetric subgame-perfect equilibrium. Following the arguments of Abreu (1986), we find such prices by supposing that a deviation induces a maximal punishment, with firms reverting to competitive pricing in every period following a deviation by any firm.

□ **The stationary-growth game.** We begin with a simple and instructive game, in which the level of market demand grows according to a stationary growth rate. Specifically, in the stationary-growth game, $G_0 > 0$ and $G_{t+1} = gG_t$, where $0 < \delta g < 1$. Thus, the level of market demand expands (contracts) over time if $g > 1$ ($g < 1$), and the stationary-growth game includes the familiar case of stationary demand as a special case when $g = 1$. The assumption that $\delta g < 1$ ensures that the discounted growth is finite.

A firm now faces a tradeoff between cheating today and sacrificing future profits in a market that grows at rate $g$. Since the growth rate is stationary, this tradeoff is the same at every date, and so a single most-collusive price, $P^c$ will be charged in all periods of a most-collusive equilibrium. To characterize this price, let $\Omega(P) = \pi(P) - \pi(P)/n = \pi(P)(n - 1)/n$ denote a firm's per-consumer incentive to cheat from a collusive agreement specifying that all firms select the price $P$. Note that a firm captures the entire market if it deviates and selects a price just below $P$. Next, let $\omega(P) = \pi(P)/n - \pi(c)/n = \pi(P)/n$ give the per-period and per-consumer cost of a price war. When a firm defects, it sacrifices future profit as the subsequent price war forces the collusive price $P$ to be abandoned and replaced with the competitive price $c$.

With these definitions in place, the central incentive constraint that each firm faces in any period $t$ may be represented as

$$G_t\Omega(\mathrm{P}) \leq G_t \sum_{\tau=t+1}^{\infty} (\delta g)^{\tau-t}\omega(P),$$

which may be rewritten as $\Omega(P) \leq [\delta g/(1 - \delta g)]\omega(P)$. Since $\Omega$ and $\omega$ are both proportional to $\pi(P)$, the incentive constraint holds for any $P > c$ if and only if $\delta g \geq (n - 1)/n$. By contrast, when this inequality fails, the incentive constraint is violated for all $P > c$. We may summarize as follows: for the stationary-growth game, $P^c = P_m$ when $\delta g \geq (n - 1)/n$ and $P^c = c$ otherwise. Observe that collusion is easier when the growth rate of the market level of demand is higher; furthermore, the ability to collude is a discontinuous function of the model's parameters, with the key relationship being the sign of $\delta g - (n - 1)/n$.

## 3. The Markov-growth model

■ **Basic assumptions.** We now assume that the growth rate of the level of market demand is stochastic and determined by a Markov process. The purpose of this section is to formally define this Markov process and to derive and interpret the corresponding incentive constraints for collusion. A characterization of the most-collusive prices is deferred until the next section.

The level of market demand is assumed to grow at one of two possible rates. We say that period $t$ is a boom period if $G_t = bG_{t-1}$ and that period $t$ is a recession period if $G_t = rG_{t-1}$, where $1 > \delta b > \delta r > 0$. In other words, if $g_t$ denotes the period-$t$ growth rate, then period $t$ is a boom (recession) period if $g_t = b$ ($g_t = r$). The transition between boom and recession periods is governed by a Markov process, in which

$$\rho \equiv \mathrm{Prob}(g_t = r \mid g_{t-1} = b) \in [0,1]$$

$$\lambda \equiv \mathrm{Prob}(g_t = b \mid g_{t-1} = r) \in [0,1]$$

$$\mu \equiv \mathrm{Prob}(g_1 = b) \in [0,1].$$

Thus, $\rho$ is the transition probability associated with moving from a boom to a recession, while $\lambda$ is the transition probability corresponding to moves from recessions to booms. The parameter $\mu$ describes how the system begins. Assume further that $G_0 > 0$.

The parameters $\rho$ and $\lambda$ also may be interpreted in terms of the expected duration of boom and recession phases, respectively. Suppose that $g_{t-1} = r$ and $g_t = b$, so that a switch to a boom period occurs at period $t$, and define $t^* \equiv \min\{\tau > t | g_\tau = r\}$. We then define a boom phase as a sequence of boom periods, $\{t, \ldots, t^* - 1\}$, and the expected duration of a boom phase is given by

$$\sum_{z=1}^{\infty} z\rho(1 - \rho)^{z-1} = 1/\rho.$$

In the same manner, we may define a recession phase and derive that the expected duration of a recession phase is $1/\lambda$.

With the Markov-growth process now fully specified, we define the Markov-growth game as the Bertrand supergame for the case in which $G_t$ evolves in the implied manner. The Markov-growth game includes the stationary-growth game as a special case (e.g., $g = b$, $\mu = 1$, $\rho = 0$).

□ **The incentive constraints.** The next task is to find a tractable representation of the incentive constraints for collusion. The Markov structure is especially helpful here, since it implies that the incentives for collusion are the same in any boom period regardless of the specific date, and similarly for any recession period. The most-collusive prices thus now emerge as a pair, with $P_b^c$ and $P_r^c$ denoting the most-collusive prices in boom and recession periods, respectively. An additional benefit of the Markov structure is that it admits a simple recursive structure, once the appropriate definitions are put forth.

To begin, consider a candidate pair of prices, $P_b$ and $P_r$, for boom and recession periods, respectively. We may then define $\bar{\omega}_b(P_b, P_r)$ as the expected discounted profit per market consumer to a firm in period $t + 1$ and thereafter, if period $t + 1$ is a boom period and the prices $P_b$ and $P_r$ are charged in the future. Analogously, we may define $\bar{\omega}_r(P_b, P_r)$ when period $t + 1$ is a recession period. Observe that $\bar{\omega}_b(P_b, P_r)$ and $\bar{\omega}_r(P_b, P_r)$ also provide a measure of the cost of a price war, since firms earn zero profit once such a war commences. With these definitions in place, the incentive constraint for collusion when period $t$ is a boom period is

$$G_t\Omega(P_b) \leq \delta\{\rho(rG_t)\bar{\omega}_r(P_b, P_r) + (1 - \rho)(bG_t)\bar{\omega}_b(P_b, P_r)\},$$

since $G_{t+1} = rG_t$ with probability $\rho$ and $G_{t+1} = bG_t$ with probability $1 - \rho$. Observe that the current-period level of market demand, $G_t$, cancels, enabling us to write the incentive constraint as $\Omega(P_b) \leq \delta\{\rho r\bar{\omega}_r(P_b, P_r) + (1 - \rho)b\bar{\omega}_b(P_b, P_r)\}$. Intuitively, the future level of market demand is always proportional to the current level, so the current demand level is simply a scaling factor that is irrelevant for the incentive to collude.

The incentive constraint given above is clearly incomplete, both because the counterpart incentive constraint for recession periods is not presented and because explicit representations for the terms $\bar{\omega}_b(P_b, P_r)$ and $\bar{\omega}_r(P_b, P_r)$ are not given. Suppressing notation slightly, a complete system of incentive constraints is given in the following four inequalities:

$$\Omega(P_b) \leq \delta\{\rho r\bar{\omega}_r + (1 - \rho)b\bar{\omega}_b\} \tag{1}$$

$$\Omega(P_r) \leq \delta\{\lambda b\bar{\omega}_b + (1 - \lambda)r\bar{\omega}_r\}, \tag{2}$$

where

$$\bar{\omega}_b = \omega(P_b) + \delta\{\rho r\bar{\omega}_r + (1 - \rho)b\bar{\omega}_b\} \tag{3}$$

$$\bar{\omega}_r = \omega(P_r) + \delta\{\lambda b\bar{\omega}_b + (1 - \lambda)r\bar{\omega}_r\}. \tag{4}$$

Notice that (1) and (2) reflect the tension between the current incentive to cheat and the expected discounted future profit that cheating would sacrifice, while through (3) and (4) the recursive nature of the model may be exploited so as to explicitly calculate the cost of a price war.

Formally, solving (3) and (4) for $\bar{\omega}_b$ and $\bar{\omega}_r$, one obtains

$$\bar{\omega}_b = \{\omega(P_b)[1 - (1 - \lambda)\delta r]/\delta + \omega(P_r)\rho r\}\Delta \tag{5}$$

$$\bar{\omega}_r = \{\omega(P_r)[1 - (1 - \rho)\delta b]/\delta + \omega(P_b)\lambda b\}\Delta, \tag{6}$$

where

$$\Delta = \delta/\{[1 - (1 - \lambda)\delta r][1 - (1 - \rho)\delta b] - \delta^2\lambda b\rho r\}. \tag{7}$$

It is easy to show that $\Delta > 0$ and $\Delta$ increases strictly in $\delta$ for $\delta \in (0, 1/b)$.[5] Substituting (5), (6), and (7) back into (1) and (2), we are now able to write the two incentive constraints in terms of the known functions, $\Omega$ and $\omega$:

$$\Omega(P_b) \leq \{\omega(P_r)\rho r + \omega(P_b)b[1 - \rho - \delta r(1 - \lambda - \rho)]\}\Delta \tag{8}$$

$$\Omega(P_r) \leq \{\omega(P_b)\lambda b + \omega(P_r)r[1 - \lambda - \delta b(1 - \lambda - \rho)]\}\Delta. \tag{9}$$

Intuitively, inequality (8) indicates that the incentive to cheat in a boom period must be no greater than the expected discounted loss in future profit that would occur were a price war initiated, where this loss reflects the expected future duration in boom and recession periods given that the current period is a boom period. Inequality (9) may be interpreted similarly for recession periods.

As it will sometimes be more convenient to express these incentive constraints in terms of the underlying profit function, we note finally that (8) and (9) may be rewritten as:

$$\pi(P_b)B \leq \pi(P_r)\rho r\Delta \tag{10}$$

$$\pi(P_b)\lambda b\Delta \geq \pi(P_r)R, \tag{11}$$

where

$$B = n - 1 - b\Delta[1 - \rho - \delta r(1 - \lambda - \rho)] \tag{12}$$

$$R = n - 1 - r\Delta[1 - \lambda - \delta b(1 - \lambda - \rho)]. \tag{13}$$

□ **Correlation.** Although we now have the incentive constraints represented in a manageable form, the model still embodies several parameters ($n$, $\delta$, $b$, $r$, $\lambda$, and $\rho$). Before proceeding to a characterization of the most-collusive prices, we therefore first offer and interpret a partial organizational scheme.

An important ingredient in the stationary-growth game is the growth rate, $g$, of the level of market demand. Reasoning by analogy for the Markov-growth game, we might expect that the ability to collude in a given period would be influenced by the expected growth in the level of market demand in the following period. Since this expectation may be in turn sensitive to whether the current period is a boom or a recession, we perform the following calculation:

$$E(g_{t+1}|g_t = b) - E(g_{t+1}|g_t = r) = (1 - \lambda - \rho)(b - r). \tag{14}$$

Thus, the expected rate of growth in period $t + 1$, and hence the expected level of market demand in period $t + 1$, is higher (lower) when period $t$ is a boom as opposed to recession period if and only if $1 - \lambda - \rho > 0$ ($1 - \lambda - \rho < 0$). Intuitively, when $\lambda$ and $\rho$ are small, the current rate of growth is likely to persist into the next period, and thus the expected rate of growth for the subsequent period is higher if current growth is at the boom rate.

[5] Let $\Delta \equiv \delta/D(\delta)$, where $D$ is the denominator of (7). Calculations reveal that $D(0) = 1$, $D'(0) \leq 0$, $D(1/b) \geq 0$, and $\text{sign}\{D''(\delta)\} = \text{sign}\{1 - \lambda - \rho\}$. Thus, if $1 - \lambda - \rho \leq 0$, then $D''(\delta) \leq 0$ over $(0, 1/b)$ and so $D(\delta) > 0$ follows. If instead $1 - \lambda - \rho > 0$, then $D''(\delta) > 0$. Observe that $D(1/r) \leq 0$, where $1/r > 1/b$. Given the convexity of $D(\delta)$ and that $D(1/b) \geq 0$, it follows that $D'(\delta) < 0$ for $\delta \in [0, 1/b]$. This implies $D(\delta) > 0$ over $(0, 1/b)$.

We therefore organize the sequel around the following three cases: $1 - \lambda - \rho > 0$, $1 - \lambda - \rho < 0$, and $1 - \lambda - \rho = 0$. When $1 - \lambda - \rho > 0$, a higher value for the growth rate at period $t$ leads to a higher expected growth rate in period $t + 1$, and so we say that growth rates exhibit positive correlation. Likewise, growth rates exhibit negative correlation when $1 - \lambda - \rho < 0$, and finally there is zero correlation between the growth rates if $1 - \lambda - \rho = 0$.

## 4. The most-collusive prices

■ **Extreme cases.** Before exploring the implications of correlated growth rates for collusion, we first examine a pair of extreme cases for which the form of most-collusive pricing is clear. The identification of these cases will in turn indicate the interesting range for the parameters $n$, $\delta$, $b$, and $r$.

As faster growth rates have been linked to better collusion, it might be expected that if perfect collusion is possible in a stationary-growth game with the slow growth rate $r$, then perfect collusion would also occur in the Markov-growth game, where growth is sometimes even faster. Likewise, if the most-collusive price is the competitive price in a stationary-growth game with the fast rate of growth $b$, then competitive pricing also would be expected in the Markov-growth game, where growth is sometimes even slower. This intuition is confirmed in the following theorem.

*Theorem 1.* In the Markov-growth game,

(i) If $\delta r \geq (n - 1)/n$, then $P_b^c = P_r^c = P_m$.

(ii) If $\delta b < (n - 1)/n$, then $P_b^c = P_r^c = c$.

A proof of this theorem is found in the Appendix.[6]

It follows that the interesting case for the Markov-growth game is when

$$\delta b > (n - 1)/n > \delta r, \tag{15}$$

indicating that perfect collusion would be possible under stationary growth if and only if growth occurs at the fast rate, $b$. Henceforth, we therefore maintain the assumption that $n$, $\delta$, $b$, and $r$ are such that (15) holds.[7]

At this point, we have a complete organizational scheme for our parameters: if $n$, $\delta$, $b$, and $r$ fail (15), then the conclusions of Theorem 1 apply, whereas if, as we assume below, $n$, $\delta$, $b$, and $r$ are such that (15) is satisfied, then the three correlation cases for $\lambda$ and $\rho$ will be considered.

□ **Zero correlation.** Among the three kinds of correlation, the case of zero correlation ($1 - \lambda - \rho = 0$) is the most simple with which to begin. Under zero correlation, the expected future growth rate is independent of whether the current period is a boom or a recession; as a consequence, it is possible to think of the zero-correlation case in terms of a stationary rate of growth, $g$, that satisfies $\rho r + (1 - \rho)b = g = \lambda b + (1 - \lambda)r$. This parallel with the stationary-growth game in turn indicates that the most-collusive prices are the same in booms and recessions ($P_b^c = P_r^c$) when there is zero correlation.

Recalling that the stationary growth rate $g$ will support perfect collusion if and only if $\delta g \geq (n - 1)/n$, we have that $P_b^c = P_r^c = P_m$ when correlation is zero if and only if

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[6] In a similar way, it can be demonstrated that when $\delta b = (n - 1)/n$, we have $P_b^c = P_r^c = c$ if $\lambda < 1$ and $\rho > 0$.

[7] See, however, footnote 14 for a relaxation of this constraint in an extended model.

$$\delta[\rho r + (1 - \rho)b] \geq (n - 1)/n. \tag{16}$$

Letting $\rho^*$ solve (16) with equality, and putting $\lambda^* \equiv 1 - \rho^*$, we find that

$$\rho^* = [\delta b - (n - 1)/n]/[\delta(b - r)] \tag{17}$$

$$\lambda^* = [(n - 1)/n - \delta r]/[\delta(b - r)], \tag{18}$$

where $\rho^* \in (0, 1)$ and $\lambda^* \in (0, 1)$ under the maintained assumption (15). Since the left-hand side of (16) is decreasing in $\rho$, we conclude that perfect collusion is possible when there is zero correlation if and only if $\rho \leq \rho^*$, while competitive pricing occurs otherwise.

Our results for the zero-correlation case now may be summarized as follows.

*Theorem 2.* In the Markov-growth game with zero correlation,

(i) If $\rho \leq \rho^*$, then $P_b^c = P_r^c = P_m$.

(ii) If $\rho > \rho^*$, then $P_b^c = P_r^c = c$.

Intuitively, if the expected duration of a boom phase is sufficiently long, then the associated zero-correlation growth rate is high enough to support perfect collusion.

□ **Positive correlation.** The zero-correlation case serves to illustrate a relationship between the Markov-growth and stationary-growth games, but it does not deliver cyclical pricing. This possibility arises in the more interesting case of positive correlation ($1 - \lambda - \rho > 0$), since then the expected growth of future demand is sensitive to the current state of the business cycle. In particular, when the evolution of market demand is characterized by positive correlation, it may be especially difficult to collude in recessions, as the expected future growth in demand is then lower, implying that there is less to lose from a price war. This suggests that the most-collusive prices then might be procyclical (i.e., $P_b^c > P_r^c$), as firms reduce the collusive price in recessions so as to diminish the incentive to cheat and bring incentives back in line. We sketch here arguments supporting this conclusion, leaving a complete proof for the Appendix.

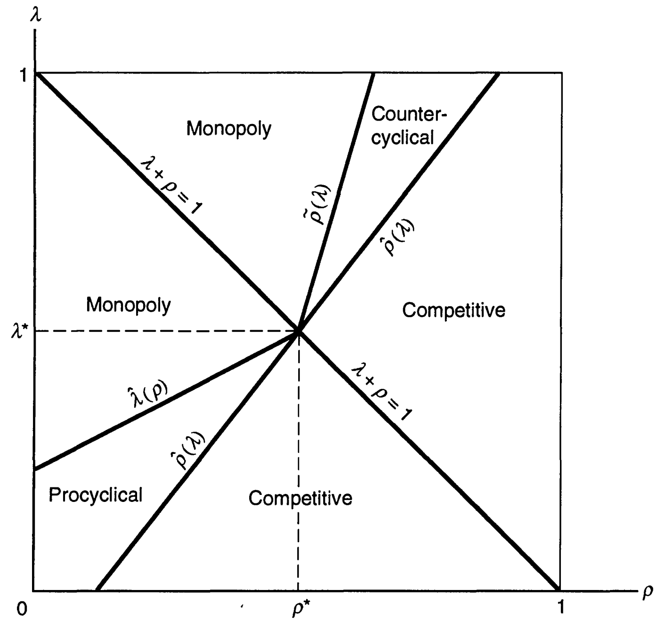
We begin by characterizing the parameter region for which perfect collusion ($P_b^c = P_r^c = P_m$) can be supported. To this end, let $\hat{\lambda}(\rho)$ satisfy $\lambda b \Delta = R$ when $1 - \lambda - \rho \geq 0$. Straightforward calculations then yield

$$\hat{\lambda}(\rho) = [1 - (1 - \rho)\delta b]/(1/\lambda^* - \delta b), \tag{19}$$

from which it is easily verified that $\hat{\lambda}$ is a linear function of $\rho$ with $\hat{\lambda}(0) > 0$, $\hat{\lambda}'(\rho) > 0$, and $\hat{\lambda}(\rho^*) = \lambda^*$, as Figure 1 illustrates. Under positive correlation, we find that $\lambda \geq \hat{\lambda}(\rho)$ implies $P_b^c = P_r^c = P_m$. Intuitively, for the positive-correlation case, collusion is especially difficult to support in recession periods, and $\hat{\lambda}(\rho)$ is defined from (11) so that perfect collusion is just possible in such a period. It follows that an even lower expected duration for the recession phase (i.e., $\lambda > \hat{\lambda}(\rho)$) ensures that perfect collusion can be maintained in both boom and recession periods with slack.

Suppose next that $\lambda < \hat{\lambda}(\rho)$, in which case the expected duration of a recession phase is large enough to preclude perfect collusion in a recession period. The incentive constraints associated with this case are represented in Figure 2. The constraints are upward sloping in the figure, with a higher price in one state raising the cost of a price war and thus enabling a higher price in the other state as well, and all prices to the southeast of (10) (northwest of (11)) satisfy the boom- (recession-) period incentive

FIGURE 1




constraint.[8] In the case depicted, (11) lies northwest of the 45° line, indicating that perfect collusion cannot be sustained in a recession period.

As Figure 2 illustrates, even if perfect collusion is infeasible, a region of profitable prices exists at which both incentive constraints are satisfied, provided (as depicted) that the boom-period incentive constraint lies on or northwest of that of the recession period. Moreover, the most-collusive prices are then procyclical, with $P_b^c = P_m > P_r^c = \overline{P}_r$. To characterize the boundary conditions that mark the existence of a procyclical region, we consider the circumstances under which the incentive constraints lie atop one another. Examining (10) and (11), we see that this occurs when the incentive constraints are redundant with $RB = (\Delta\lambda b)(\Delta\rho r)$, and we thus define a function $\hat{\rho}(\lambda)$ as the $\rho$ value for which this equation holds. Calculations reveal that

$$\hat{\rho}(\lambda) = [(b - r)/b]\rho^* + [\rho^* r/(b\lambda^*)]\lambda. \tag{20}$$

Note that $\hat{\rho}$ is a linear function of $\lambda$, with $\hat{\rho}(0) > 0$, $\hat{\rho}'(\lambda) > 0$, and $\hat{\rho}(\lambda^*) = \rho^*$. Further, and as Figure 1 illustrates, $\hat{\rho}(\hat{\lambda}(\rho)) > \rho$ for $\rho < \rho^*$ and so $\hat{\rho}(\lambda)$ lies below $\hat{\lambda}(\rho)$.

With the $\hat{\rho}$ now defined, it is possible to state a second result: under positive correlation, if $\lambda < \hat{\lambda}(\rho)$ and $\rho \leq \hat{\rho}(\lambda)$, then the most-collusive prices are procyclical with $P_b^c = P_m > P_r^c$. Intuitively, even if firms are unable to collude perfectly in recessions, they may be able to collude imperfectly by reducing the price in recession periods to below-monopoly levels; in this way, they reduce the incentive to cheat in recessions and thus make credible an imperfect collusive agreement.

The remaining possibility is that $\rho > \hat{\rho}(\lambda)$. In terms of Figure 2, this case corresponds to the situation in which (10) lies southeast of (11), indicating that both incentive constraints hold only when both prices are set at competitive levels. In fact, the following general result is easily confirmed: under positive correlation, if $\rho > \hat{\rho}(\lambda)$, then $P_b^c = P_r^c = c$. This result, too, rests on a simple intuition: if the expected duration

[8] It is possible that $B \leq 0$, in which case the boom-period incentive constraint (10) does not have positive slope; see the Appendix for this case.

of a boom phase is too short relative to the expected duration of a recession phase, then even imperfect collusion is impossible.

Our findings are summarized in Figure 1, which pinpoints the regions of monopoly, procyclical, and competitive pricing for the positive-correlation case. A remaining point of interest concerns the behavior of the recession-period collusive price over the procyclical region. We find that in the procyclical region, $P_r^c$ satisfies the following properties:

(i) $P_r^c$ is continuous, increasing in $\lambda$ and decreasing in $\rho$.

(ii) $P_r^c \to P_m$ as $\lambda \to \hat{\lambda}(\rho)$, and $P_r^c \to c$ as $\lambda \to 0$.

(iii) $P_r^c$ is increasing in $\lambda$ along $\hat{\rho}(\lambda)$, with $P_r^c = c$ when $\lambda = 0$ and $P_r^c \to P_m$ as $\lambda \to \lambda^*$.

With $P_b^c = P_m$ holding throughout the procyclical region, it follows from (i) that the amplitude (i.e., $|P_b^c - P_r^c|$) of the price cycle increases as the expected duration of a recession (boom) phase lengthens (shortens). Such changes make collusion more difficult, and so a breakdown of the collusive agreement in recession periods can be averted only if the recession-period price is depressed further. Notice also from (ii) that $P_r^c$ continuously climbs to the monopoly price as the monopoly region is approached. As in the stationary-growth game, the Bertrand nature of the stage game again generates a discontinuity for the most-collusive prices at the boundary of the competitive region. As (iii) reveals, $P_r^c$ varies along this boundary, increasing from the competitive to the monopoly price as $\lambda$ and $\rho$ rise along $\hat{\rho}(\lambda)$.

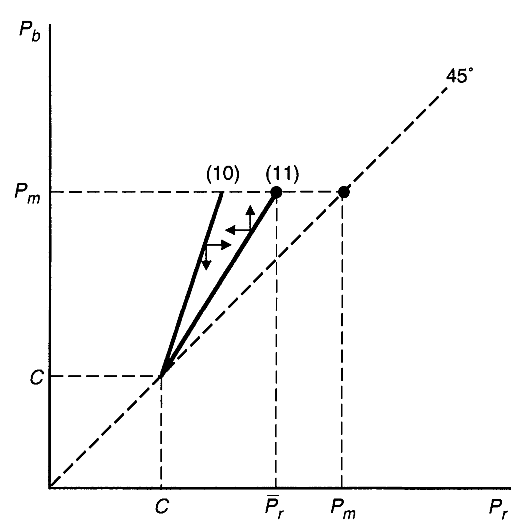
The main conclusions of the positive-correlation case pertain to the predictions of procyclical ($P_b^c > P_r^c$) versus countercyclical ($P_b^c < P_r^c$) pricing and to the determinants of the amplitude of pricing cycles. We therefore summarize this case as follows.

*Theorem 3.* In the Markov-growth game with positive correlation,

(i) The most-collusive prices are sometimes procyclical but never countercyclical.

(ii) When the most-collusive prices are procyclical, the amplitude of the price

FIGURE 2



cycle is increasing (decreasing) in the expected duration of a recession (boom) phase.

□ **Negative correlation.** We turn last to the case of negative correlation ($1 - \lambda - \rho < 0$). In this case, expected future market demand conditions are less favorable when the current period is a boom period, suggesting that collusion is now most difficult to maintain in boom periods. The implication is then that the most-collusive price must be depressed in boom periods, in order to reduce the incentive to defect, and so a prediction of countercyclical pricing is anticipated. The formal analysis of this case is analogous to that developed above, so we offer here only a sketch of the main ideas.

We begin by characterizing the parameter space over which perfect collusion is possible. To this end, we now analyze the boom-period incentive constraint (10) and define the function $\tilde{\rho}(\lambda)$ as the solution to $B = \Delta\rho r$ when $1 - \lambda - \rho < 0$. Calculations reveal that

$$\tilde{\rho}(\lambda) = [1 - (1 - \lambda)\delta r]/[1/\rho^* - \delta r], \tag{21}$$

from which it can be verified that $\tilde{\rho}$ is a linear function of $\lambda$ with $\tilde{\rho}'(\lambda) > 0$, $\tilde{\rho}(\lambda^*) = \rho^*$, and $\tilde{\rho}(\lambda) < \hat{\rho}(\lambda)$ over the negative-correlation range, as Figure 1 illustrates. The curve $\tilde{\rho}(\lambda)$ represents the combinations of parameters at which a firm is just indifferent between perfectly colluding and cheating when currently in a boom period. When the expected duration of a boom phase is greater, perfect collusion can be maintained over this phase with slack. Consistent with the idea that boom periods are the most difficult time to collude in the negative-correlation case, it is in fact true for this case that $\rho \leq \tilde{\rho}(\lambda)$ implies $P_b^c = P_r^c = P_m$.

Consider next the region that lies between $\tilde{\rho}(\lambda)$ and $\hat{\rho}(\lambda)$. Here, the expected duration of a boom phase is sufficiently short that perfect collusion cannot be supported over this phase. Collusion must be imperfect over this range, and it takes the form of countercyclical pricing: under negative correlation, if $\tilde{\rho}(\lambda) < \rho \leq \hat{\rho}(\lambda)$, then $P_r^c = P_m > P_b^c$.[9] As expected, the collusive price is depressed in booms in order to reduce the corresponding incentive to cheat. Finally, the last region has $\rho > \hat{\rho}(\lambda)$, and as before, prices above cost can no longer be found that satisfy both incentive constraints; consequently, we have $P_b^c = P_r^c = c$ when $\rho > \hat{\rho}(\lambda)$.

Over the countercyclical region, we find also that the boom-period collusive price satisfies the following:

(i) $P_b^c$ is continuous, increasing in $\lambda$ and decreasing in $\rho$.

(ii) $P_b^c \to P_m$ as $\rho \to \tilde{\rho}(\lambda)$.

(iii) $P_b^c$ is decreasing in $\lambda$ along $\hat{\rho}(\lambda)$, with $P_b^c \to P_m$ as $\lambda \to \lambda^*$.

The amplitude of the collusive pricing cycle is again increased by a lengthening (shortening) in the expected duration of a recession (boom) phase, since the cost of a price war is then reduced, forcing a greater drop in the boom-period collusive price. Note also that collusive prices move continuously between the monopoly and countercyclical regions, and a discontinuity remains once the competitive region is encountered.

With these results in place, we summarize the negative-correlation case as follows.

*Theorem 4.* In the Markov-growth game with negative correlation,

[9] In analogy with Figure 2, the incentive constraints now lie below the 45° line, with the boom-period incentive constraint (10) lying on or northwest of the recession-period incentive constraint (11).

(i) The most-collusive prices are sometimes countercyclical but never procyclical;

(ii) When the most-collusive prices are countercyclical, the amplitude of the price cycle is increasing (decreasing) in the expected duration of a recession (boom) phase.

The prediction of countercyclical pricing reverses that of the positive-correlation case.

Inspecting Figure 1, it is now clear that perfect collusion is sustainable if booms are sufficiently long and recessions are sufficiently short in expected duration, regardless of the precise nature of correlation. Similarly, whatever the level of correlation, competitive pricing is required whenever booms are sufficiently short and recessions are sufficiently long in expected duration. In the intermediate situation, however, monopoly pricing is sustainable in only one phase, and the collusive price in the other phase must be reduced, in order to thwart the incentive to cheat. The designation of the "weak" state then depends upon the sign of correlation, with procyclical (countercyclical) collusive pricing occurring under positive (negative) correlation.[10]

## 5. Collusion and within-phase demand fluctuations

■ **Basic assumptions.** In the business-cycle model adopted above, the demand growth rate is stochastic and follows a Markov process. The model also specifies that within any given phase, the level of market demand at any given period is a deterministic function of its value in the preceding period. The purpose of the present section is to analyze collusive pricing in an extended model of the business cycle, where the level of market demand also fluctuates randomly within a given phase.

In particular, we suppose that the process through which the market demand level evolves is described by $G_t = g_t(G_{t-1}/\epsilon_{t-1})\epsilon_t$, where $g_t = b$ ($g_t = r$) when period $t$ is a boom (recession) period, with the Markov transition probabilities as described in Section 3, and where $\epsilon_t$ is identically and independently distributed through time with full support over $[\underline{\epsilon}, \overline{\epsilon}]$ and $E\{\epsilon_t\} = 1 \in (\underline{\epsilon}, \overline{\epsilon})$. Thus, we have the basic Markov-growth process, except that now shocks to the level of market demand occur in each period. Observe moreover that these shocks are transitory, since future levels of market demand are completely independent of $\epsilon_t$.[11] Maintaining our assumption that firms observe all current-period demand conditions prior to selecting prices, we thus define the Markov-growth game with transitory shocks as the Bertrand supergame for the case in which $G_t$ evolves in the described way.

Transitory shocks pose interesting problems for collusive agreements, since they affect the short-term incentive to cheat but not the long-term cost of a price war. We now combine the methods presented above with those developed by Rotemberg and Saloner and characterize the most-collusive price at any period $t$ as a function of the business-cycle phase (boom or recession) *and* the within-phase shock ($\epsilon_t$) experienced in that period. In so doing, we generalize the analysis of Rotemberg and Saloner to a business-cycle specification that allows for multiple phases (i.e., a stochastic trend), and thereby offer a formal interpretation of their results in terms of the transitory shocks to demand that occur within broader business-cycle phases.

[10] Finally, using (17) and (18), it is easily verified that $\lambda^* \rightarrow 0$ as $\delta r \rightarrow (n-1)/n$ and that $\rho^* \rightarrow 0$ as $\delta b \rightarrow (n-1)/n$. The predicted most-collusive prices for the extreme parameter values described in Theorem 1 are thus approached continuously as parameters are varied over the region for which (15) holds.

[11] To see this more directly, observe that $G_t = G_0\Pi(g_\tau)\epsilon_t$, where the product is taken from $\tau = 1$ to $\tau = t$. In our discussion paper (Bagwell and Staiger, 1995), we also consider permanent within-phase shocks, represented by $G_t = g_tG_{t-1}\epsilon_t$, or $G_t = G_0\Pi(g_\tau\epsilon_\tau)$, where the product is taken from $\tau = 1$ to $\tau = t$. In this case, we establish that within-phase shocks have no effect on collusive pricing: the most-collusive prices derived for the Markov-growth game continue to apply.

□ **Solution method.** Our first step is to exploit the methods developed above in order to get the incentive constraints in a tractable form. Given that $\epsilon_t$ is identically and independently distributed, we now drop the time subscript and let $P_b(\epsilon)$ and $P_r(\epsilon)$ represent the prices charged in boom and recession periods, respectively, when the current-period within-phase demand shock is given by $\epsilon$. Let us next define $\bar{\omega}_b(P_b(\epsilon), P_r(\epsilon))$ as the expected discounted profit per market consumer to a firm in period $t + 1$ and thereafter, if period $t + 1$ is a boom period, $P_b(\epsilon)$ and $P_r(\epsilon)$ are the pricing functions, and the value for $\epsilon$ in period $t + 1$ is not yet determined. We define $\bar{\omega}_r(P_b(\epsilon), P_r(\epsilon))$ analogously when period $t + 1$ is a recession period.

Consider next the incentive constraint facing a firm in period $t$, when period $t$ is a boom period and the period-$t$ within-phase shock is given by $\epsilon_t = \epsilon$. Simplifying notation slightly, we may represent this incentive constraint as

$$G_t\Omega(P_b(\epsilon)) \leq \delta\{\rho(rG_t/\epsilon)\bar{\omega}_r + (1 - \rho)(bG_t/\epsilon)\bar{\omega}_b\},$$

or more simply, $\epsilon\Omega(P_b(\epsilon)) \leq \delta\{\rho r\bar{\omega}_r + (1 - \rho)b\bar{\omega}_b\}$. Thus, the current-period "base" level of demand, $G_t$, again cancels, since all future demand growth is proportional to this base, but the current-period within-phase shock, $\epsilon$, is not represented in future demand growth, and its value remains in the incentive constraint, with higher values for $\epsilon$ raising the incentive to cheat.

Building on these insights, we now represent the complete incentive system as

$$\epsilon\Omega(P_b(\epsilon)) \leq \delta\{\rho r\bar{\omega}_r + (1 - \rho)b\bar{\omega}_b\} \tag{22}$$

$$\epsilon\Omega(P_r(\epsilon)) \leq \delta\{\lambda b\bar{\omega}_b + (1 - \lambda)r\bar{\omega}_r\}, \tag{23}$$

where

$$\bar{\omega}_b = E\{\omega(P_b(\epsilon))\epsilon\} + \delta\{\rho r\bar{\omega}_r + (1 - \rho)b\bar{\omega}_b\} \tag{24}$$

$$\bar{\omega}_r = E\{\omega(P_r(\epsilon))\epsilon\} + \delta\{\lambda b\bar{\omega}_b + (1 - \lambda)r\bar{\omega}_r\}. \tag{25}$$

But this is the same incentive system as represented in (1)–(4), except that $\epsilon\Omega$ replaces $\Omega$ and $E\{\omega(\cdot)\epsilon\}$ replaces $\omega(\cdot)$. Proceeding as before, we may thus rewrite the incentive constraints in the more useful form,

$$\epsilon\Omega(P_b(\epsilon)) \leq E\{\omega(P_r(\epsilon))\epsilon\}\rho r\Delta + E\{\omega(P_b(\epsilon))\epsilon\}[(n - 1) - B] \tag{26}$$

$$\epsilon\Omega(P_r(\epsilon)) \leq E\{\omega(P_b(\epsilon))\epsilon\}\lambda b\Delta + E\{\omega(P_r(\epsilon))\epsilon\}[(n - 1) - R]. \tag{27}$$

Having derived the incentive constraints we next present a two-step solution process for the most-collusive prices, $P_b^c(\epsilon)$ and $P_r^c(\epsilon)$.[12] The initial step involves viewing the right-hand sides of (26) and (27) as fixed values, defined as

$$\tilde{\omega}_b \equiv E\{\omega(P_r(\epsilon))\epsilon\}\rho r\Delta + E\{\omega(P_b(\epsilon))\epsilon\}[(n - 1) - B] \tag{28}$$

$$\tilde{\omega}_r \equiv E\{\omega(P_b(\epsilon))\epsilon\}\lambda b\Delta + E\{\omega(P_r(\epsilon))\epsilon\}[(n - 1) - R]. \tag{29}$$

[12] Rotemberg and Saloner propose a similar two-step process. Our analysis is more involved, since we must confirm simultaneous satisfaction of boom- and recession-phase incentive constraints. While Rotemberg and Saloner allow transitory shocks to affect demand in a general fashion, we assume all demand fluctuations enter multiplicatively.

Using (26)–(29), the incentive constraints now appear as $\epsilon\Omega(P_b(\epsilon)) \leq \tilde{\omega}_b$ and $\epsilon\Omega(P_r(\epsilon)) \leq \tilde{\omega}_r$, and so, after substituting for $\Omega$, the incentive constraints may be rewritten as

$$\pi(P_b(\epsilon)) \leq [n/(n-1)]\tilde{\omega}_b/\epsilon \tag{30}$$

$$\pi(P_r(\epsilon)) \leq [n/(n-1)]\tilde{\omega}_r/\epsilon. \tag{31}$$

We may now define $P_b(\tilde{\omega}_b, \epsilon)$ and $P_r(\tilde{\omega}_r, \epsilon)$ as the most-collusive prices when $\tilde{\omega}_b$ and $\tilde{\omega}_r$ are taken as fixed values; i.e., $P_b(\tilde{\omega}_b, \epsilon)$ is the most-profitable price satisfying (30) and $P_r(\tilde{\omega}_r, \epsilon)$ is defined analogously for (31). These prices can be represented as follows:

$$P_b(\tilde{\omega}_b, \epsilon) = P^*(\tilde{\omega}_b/\epsilon) \tag{32}$$

$$P_r(\tilde{\omega}_r, \epsilon) = P^*(\tilde{\omega}_r/\epsilon), \tag{33}$$

where

$$P^*(\tilde{\omega}/\epsilon) \equiv P_m, \text{ if } \pi(P_m) \leq [n/(n-1)]\tilde{\omega}/\epsilon \tag{34}$$

$$P^*(\tilde{\omega}/\epsilon) \equiv \min\{P \mid \pi(P) = [n/(n-1)]\tilde{\omega}/\epsilon\}, \text{ if } \pi(P_m) > [n/(n-1)]\tilde{\omega}/\epsilon. \tag{35}$$

In short, each price is set as close to the monopoly price as possible, while still being consistent with the corresponding incentive constraint.

We now proceed to the next step in this process and present a fixed-point technique through which the most-collusive values for $\tilde{\omega}_b$ and $\tilde{\omega}_r$ may be endogenously determined. Specifically, consistency requires that the most-collusive values for $\tilde{\omega}_b$ and $\tilde{\omega}_r$ lead through (34) and (35) to prices that in turn generate through (28) and (29) the originally specified values for $\tilde{\omega}_b$ and $\tilde{\omega}_r$. This requirement is captured by the following two fixed-point equations:

$$\tilde{\omega}_b = E\{\omega(P^*(\tilde{\omega}_r/\epsilon))\epsilon\}\rho r\Delta + E\{\omega(P^*(\tilde{\omega}_b/\epsilon))\epsilon\}[(n-1) - B] \tag{36}$$

$$\tilde{\omega}_r = E\{\omega(P^*(\tilde{\omega}_b/\epsilon))\epsilon\}\lambda b\Delta + E\{\omega(P^*(\tilde{\omega}_r/\epsilon))\epsilon\}[(n-1) - R]. \tag{37}$$

It is straightforward to see that one consistent solution has $\tilde{\omega}_b = \tilde{\omega}_r = 0$, corresponding to competitive pricing in all states. We are interested instead in the most-collusive fixed-point solution, $(\hat{\omega}_b, \hat{\omega}_r)$, which represents the largest values for $(\tilde{\omega}_b, \tilde{\omega}_r)$ that satisfy (36) and (37). Once these values are determined, the most-collusive prices are then defined by

$$P_b^c(\epsilon) \equiv P^*(\hat{\omega}_b/\epsilon) \tag{38}$$

$$P_r^c(\epsilon) \equiv P^*(\hat{\omega}_r/\epsilon). \tag{39}$$

In this way, the problem of solving for the most-collusive price functions is reduced to the alternative task of solving for the most-collusive fixed-point values.[13]

[13] The approach pursued here thus presumes that the fixed-point solutions to (36) and (37) are pointwise ranked, so that a maximal solution can be unambiguously identified. The approach also presumes that the most-collusive prices are found by raising price as high as possible in each state, as is evident from (34) and (35) and (38) and (39). These presumptions are appropriate in the present model because incentive constraints are complementary, with more collusion in any one state fostering greater collusion in the other as well.

□ **Results.** The possibility of transitory within-phase demand shocks suggests that attempts to collude will be frustrated by high transitory shocks. It thus may be anticipated that although the comparison of collusive prices across boom and recession phases will still hinge upon the correlation of growth rates through time, higher transitory demand shocks should require lower collusive prices within any given phase. We argue below that this is indeed the case and, further, that the disruptive effect of transitory demand shocks serves to expand the region over which cyclical pricing occurs, since perfect collusion (for both phases and all within-phase shocks) is possible over a reduced range when large transitory demand shocks are possible.

Of course, perfect collusion is sure to fail if $\bar{\epsilon}$ is sufficiently big, as the temptation to cheat is then irresistible when the within-phase shock is near its upper bound. To create at least the possibility of perfect collusion, we thus restrict the size of $\bar{\epsilon}$ with the following assumption:

$$\delta b > \bar{\epsilon}(n - 1)/[1 + \bar{\epsilon}(n - 1)]. \tag{40}$$

This assumption ensures that even a maximal transitory shock would not disrupt perfect collusion, if the growth rate were stationary at the boom rate.

A complete analysis of the model is found in the Appendix. We show there that under the assumption given in (40), the parameter space can be carved into the nonempty regions depicted in Figure 3. We now illustrate some general themes and sketch the method of proof for the parameter region B in the case of positive correlation. Comparing Figures 1 and 3, we see that this region supports perfect collusion in the Markov-growth game, whereas we will argue that it gives procyclical collusive pricing in the Markov-growth game with transitory shocks.

It is useful to think of the fixed-point equations (36) and (37) as defining two respective implicit functions, with $\tilde{\omega}_b$ being a function of $\tilde{\omega}_r$ in each case. These functions are naturally expected to have upward-sloping regions, whereby better collusion in one phase complements collusive efforts in the other phase as well. Figure 4 depicts

FIGURE 3

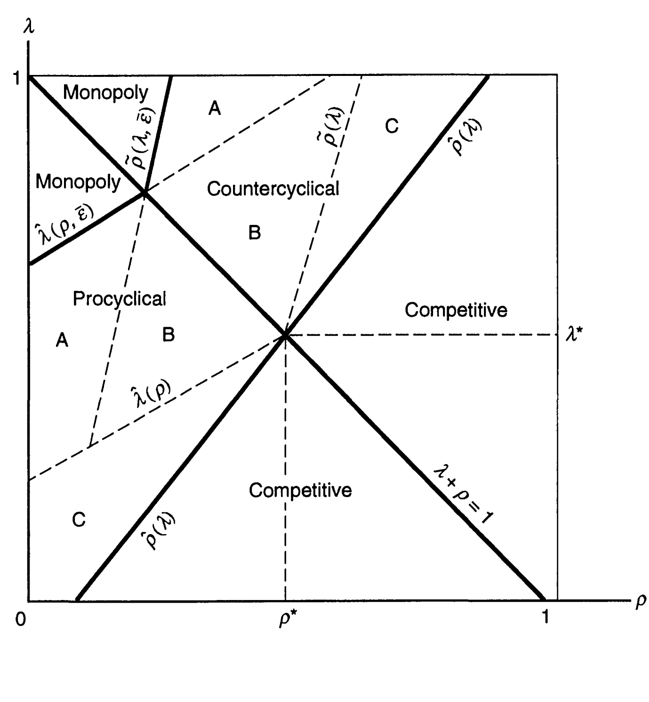
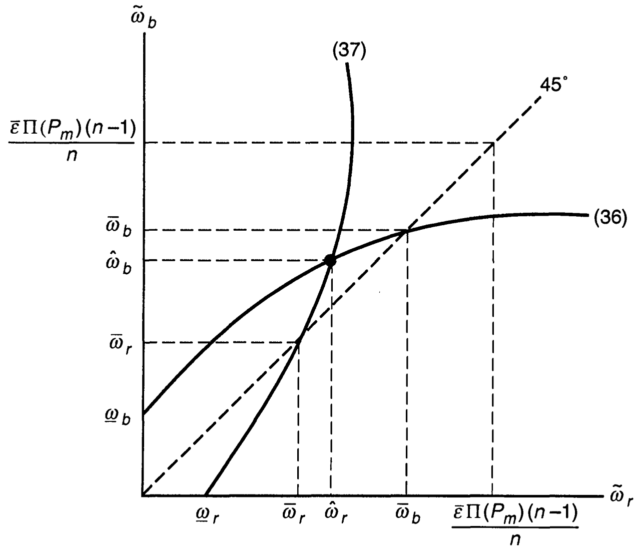


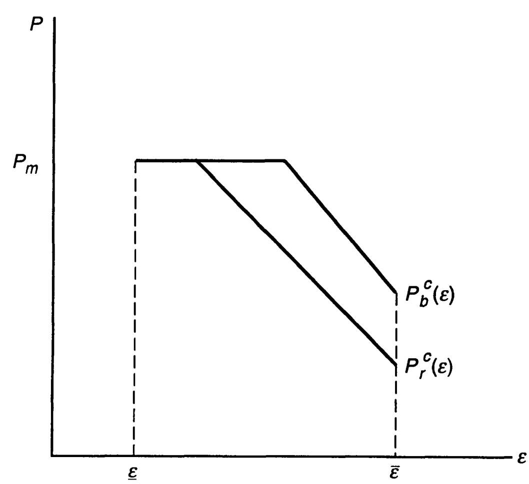
FIGURE 4




the two fixed-point equations for the parameter set corresponding to region B under positive correlation. The most-collusive fixed point solution, $(\hat{\omega}_b, \hat{\omega}_r)$, satisfies two interesting properties: $\hat{\omega}_b$ exceeds $\hat{\omega}_r$ under positive correlation, and both values fall between $\underline{\epsilon}\pi(P_m)(n - 1)/n$ and $\bar{\epsilon}\pi(P_m)(n - 1)/n$.

These properties may be interpreted in terms of the most-collusive prices as follows. Looking to (34)–(35) and (38)–(39), it is apparent that $P_b^c(\epsilon)$ and $P_r^c(\epsilon)$ are determined by the same monotonic pricing function, $P^*(\tilde{\omega}/\epsilon)$, and so $\hat{\omega}_b > \hat{\omega}_r$ implies $P_b^c(\epsilon) \geq P_r^c(\epsilon)$. (Equalities are possible at the monopoly price.) Thus, our earlier prediction that positive correlation yields procyclical pricing is preserved. Observe also from these equations that the indicated range for $(\hat{\omega}_b, \hat{\omega}_r)$ implies that, in both phases, monopoly pricing is possible when $\epsilon = \underline{\epsilon}$ but submonopoly prices must be charged when $\epsilon = \bar{\epsilon}$. The most-collusive price functions are illustrated in Figure 5.

FIGURE 5



Continuing with the positive-correlation case, consider next regions A and C. In both of these regions, it continues to be true that under positive correlation, $P_b^c(\epsilon) \geq P_r^c(\epsilon)$, with the inequality being strict for large shocks. A novel feature of region A not shared by region B is that the most-collusive fixed-point solution may entail $\hat{\omega}_b \geq \bar{\epsilon}\pi(P_m)(n - 1)/n$, indicating that the monopoly price can always be supported in the boom phase. At the border between region A and the region designated as the monopoly region, the monopoly price can just be supported in the recession phase when the transitory shock is at its most disruptive level, $\epsilon = \bar{\epsilon}$. Above this border, therefore, perfect collusion is sustainable. Region C describes a setting in which the prospects for collusion are less favorable, and it may be true here that $\hat{\omega}_r < \underline{\epsilon}\pi(P_m)(n - 1)/n$, meaning that the monopoly price cannot be sustained for any transitory shock in the recession phase. The lower boundary of region C is defined by the function $\hat{\rho}(\lambda)$, and along this border monopoly pricing can be supported in the boom phase only under the smallest transitory demand shock, $\epsilon = \underline{\epsilon}$. Below this border, monopoly pricing is thus infeasible even in this best-case scenario, and the most-collusive prices switch discontinuously to the competitive solution, $P_b^c(\epsilon) \equiv P_r^c(\epsilon) \equiv c$.

Exactly analogous findings occur in the case of negative correlation. Consider for example region B under negative correlation. In this case, the most-collusive fixed-point solution in Figure 4 now occurs below the 45° line, and the most-collusive prices appear as in Figure 5, except that the positions of the functions $P_b^c(\epsilon)$ and $P_r^c(\epsilon)$ are reversed. The prediction of countercyclical pricing is therefore preserved, and higher transitory shocks require (weakly) lower most-collusive prices in both phases. Similarly, with zero correlation, the most-collusive prices are acyclic, as the most-collusive fixed-point solution then rests on the 45° line, and higher transitory demand shocks necessitate lower most-collusive prices throughout region B.

Comparing Figures 1 and 3, we see that the monopoly region under Figure 3 is strictly smaller than that in Figure 1, while the regions with cyclical pricing are strictly larger. In particular, in the Markov-growth game, region C has cyclical pricing but regions A and B do not, whereas when transitory shocks are permitted, all three regions are characterized by cyclical pricing.[14]

Two main conclusions can be drawn from this analysis.

*Theorem 5.* In the Markov-growth game with transitory demand shocks,

(i) Under positive (negative) correlation, the most-collusive prices are sometimes procyclical (countercyclical) but never countercyclical (procyclical), and the range of cyclical pricing is larger than in the Markov-growth game.

(ii) Regardless of the nature of correlation, and in both boom and recession phases, a higher transitory demand shock results in a (weakly) lower most-collusive price.

We also note that point (ii) provides an interpretation of the Rotemberg-Saloner (1986) theory in terms of the transitory demand shocks that occur within broader business-cycle phases.

## 6. Conclusion

■ We develop a simple theory of collusive pricing over the business cycle. The most-collusive prices are completely characterized as functions of business-cycle parameters,

[14] Moreover, if (40) is relaxed, the monopoly region may disappear altogether. Note also that (15) now may be relaxed as well; e.g., if $\delta r \geq (n - 1)/n$, then cyclical pricing is impossible in the Markov-growth game, but it will be possible when transitory demand shocks are permitted, provided that sufficiently large shocks are allowed.

and a variety of specific predictions are offered. The most-collusive prices are weakly procyclical when growth rates are positively correlated through time, the amplitude of the collusive pricing cycle is increased when recession phases are longer and boom phases are shorter in expected duration, and transitory demand shocks have the effect of lowering the most-collusive price regardless of whether the market is in a boom or a recession phase.

An interesting direction for future empirical research would be to examine the cyclical properties of collusive prices with industry-level data. Our theory characterizes the most-collusive prices in an industry as functions of the business-cycle parameters that describe the evolution of demand in that industry. Given that Hamilton (1989) has developed econometric techniques for estimating these same business-cycle parameters, our predictions thus may be especially well suited for empirical analysis. Ideally, industry-specific price and sales data could be used to estimate for each industry the expected duration of boom and recession phases, so that the pricing predictions of our model could then be compared with industry pricing data.

Because an empirical analysis of this sort for industry-level data has not yet been performed, it is at this point premature to predict whether collusive prices in specific industries will be pro- or countercyclical. Nevertheless, it is instructive to refer to Hamilton's description of the aggregate data. Working with U.S. real GNP data, he finds that growth rates are positively correlated across quarters and that boom phases last longer in expectation than do recession phases. According to our theory, if these features are representative of the demand cycles in a given industry, then the prediction of procyclical collusive pricing appears most salient.

Our model is constructed to isolate the consequences of stochastic demand growth cycles for collusive pricing. Interesting future work might further assess the robustness of our conclusions. First, we assume throughout that marginal costs are acyclic. While this assumption simplifies the analysis considerably, it may be interesting to explore an extended model that also accounts for cyclical movements in marginal costs.[15] Second, we have assumed that firms can distinguish between transitory demand shocks and turning points in the business cycle. If this assumption is relaxed, opportunities for richer cyclical price movements may arise.[16] Finally, we take the business cycle as exogenous. Following Rotemberg and Woodford (1992), future research might embed the collusion model explored here in a dynamic general-equilibrium model. New countereffects may then appear. For example, discounting may be more severe in booms, and this could partially reverse the collusion-enhancing effect of persistent booms.

## Appendix

■ The Appendix contains proofs of the various propositions presented in the text. The propositions are proved through a series of lemmas.

*Lemma 1.* In the Markov-growth game,

(i) if $\min\{\Delta\lambda b - R, \Delta\rho r - B\} \geq 0$, then $P_b^c = P_r^c = P_m$.

(ii) if $\min\{\Delta\lambda b - R, \Delta\rho r - B\} < 0$ and $RB > (\Delta\lambda b)(\Delta\rho r)$,

[15] The related empirical literature seems somewhat inconclusive on this matter, as evidence exists that offers some support for acyclic (Miron and Zeldes, 1988) and also procyclical (Bils, 1987) marginal costs. Yet another possibility is that booms are caused by technology shocks, in which case marginal costs may be countercyclical.

[16] Richer price movements might also arise in other equilibria for the games that we analyze. To characterize the most-collusive equilibria, we assume that a deviation induces the maximal punishment (i.e., Bertrand pricing). There also exist less-collusive equilibria, in which less severe punishments follow a deviation. If the payoff associated with the punishment varies with the phase in which the deviation occurred, then the chosen punishment rule also affects the cyclical properties of equilibrium prices.

then $P_b^c = P_r^c = c$.

*Proof.* It is useful to first record that

$$[\Delta\rho r - B] - [\Delta\lambda b - R] = \Delta(b - r)(1 - \lambda - \rho) \tag{A1}$$

and so, e.g., the minimum value is $\Delta\lambda b - R$ under positive correlation. To prove (i), we have that $\Delta\lambda b - R \geq 0$ and $\Delta\rho r - B \geq 0$, and thus (10) and (11) are satisfied at $P_b^c = P_r^c = P_m$. To prove (ii), observe that $R > 0$ and $B > 0$. Suppose first that $\lambda = 0$ or $\rho = 0$. Then (10) and (11) require $P_b \leq c$ and $P_r \leq c$, and so $P_b^c = P_r^c = c$. If $\lambda > 0$ and $\rho > 0$, then $P_b > c$ can satisfy (10) and (11) only if $\pi(P_b) \geq \pi(P_r)R/(\Delta\lambda b) \geq \pi(P_b)RB/[(\Delta\lambda b)(\Delta\rho r)] > \pi(P_b)$, a contradiction. Thus, $P_b \leq c$ and so (11) implies $P_r \leq c$. Hence $P_b^c = P_r^c = c$. *Q.E.D.*

*Proof of Theorem 1.* For $G_t = 1$, define $\bar{G}_b$ ($\bar{G}_r$) as the expected discounted level of market demand in period $t$ and all future periods when period $t$ is a boom (recession) period. Then

$$\bar{G}_b = 1 + \delta[\rho r\bar{G}_r + (1 - \rho)b\bar{G}_b] \text{ and } \bar{G}_r = 1 + \delta[\lambda b\bar{G}_b + (1 - \lambda)r\bar{G}_r].$$

Solving these equations, one finds that $\bar{G}_b - \bar{G}_r = \Delta(b - r)(1 - \lambda - \rho)$. It is easy to confirm that

$$\bar{G}_b \leq 1/(1 - \delta b) \tag{A2}$$

$$\bar{G}_r \geq 1/(1 - \delta r), \tag{A3}$$

with the former inequality strict if $\rho > 0$ and the latter strict if $\lambda > 0$. We also have that

$$b\bar{G}_b - r\bar{G}_r = (b - r)\Delta/\delta > 0 \tag{A4}$$

$$\Delta\lambda b - R = \delta\{r(1 - \lambda)\bar{G}_r + b\lambda\bar{G}_b\} - (n - 1) \tag{A5}$$

$$\Delta\rho r - B = \delta\{b(1 - \rho)\bar{G}_b + r\rho\bar{G}_r\} - (n - 1). \tag{A6}$$

To prove part (i) of Theorem 1, assume $\delta r \geq (n - 1)/n$ (i.e., $\delta r/(1 - \delta r) \geq n - 1$). Using (A4), it is immediate that (A3), (A5), and (A6) then yield

$$\min\{\Delta\lambda b - R, \Delta\rho r - B\} \geq \delta r\bar{G}_r - (n - 1) \geq \delta r/(1 - \delta r) - (n - 1) \geq 0,$$

so Lemma 1 implies $P_b^c = P_r^c = P_m$. To prove part (ii), if $\delta b < (n - 1)/n$ (i.e., $\delta b/(1 - \delta b) < n - 1$), then using (A4), it is direct from (A2), (A5), and (A6) that

$$\max\{\Delta\lambda b - R, \Delta\rho r - B\} \leq \delta b\bar{G}_b - (n - 1) \leq \delta b/(1 - \delta b) - (n - 1) < 0$$

and so $\Delta\lambda b - R < 0$ and $\Delta\rho r - B < 0$. Lemma 1 thus applies, and hence $P_b^c = P_r^c = c$. *Q.E.D.*

*Lemma 2.* In the Markov-growth game, if

(i) $1 - \lambda - \rho \geq 0$ and $\lambda \geq \hat{\lambda}(\rho)$ or $1 - \lambda - \rho \leq 0$ and $\rho \leq \tilde{\rho}(\lambda)$, then $P_b^c = P_r^c = P_m$.

(ii) $\rho > \hat{\rho}(\lambda)$,

then $P_b^c = P_r^c = c$.

*Proof.* To prove (i), note that $\lambda \geq \hat{\lambda}(\rho)$ is equivalent to $\Delta\lambda b - R \geq 0$ and that $\rho \leq \tilde{\rho}(\lambda)$ is equivalent to $\Delta\rho r - B \geq 0$. Using (A1), Lemma 2 (i) is now an immediate consequence of Lemma 1. To prove (ii), note that $\rho > \hat{\rho}(\lambda)$ is equivalent to $RB > (\Delta\rho r)(\Delta\lambda b)$. Further, under positive correlation, $\hat{\rho}(\hat{\lambda}(\rho)) > \rho$ implies $\lambda < \hat{\lambda}(\rho)$ and so $\Delta\lambda b - R < 0$. Similarly, in the case of negative correlation, $\hat{\rho}(\lambda) > \tilde{\rho}(\lambda)$ implies $\Delta\rho r - B < 0$. The proof is direct from (A1) and Lemma 1. *Q.E.D.*

*Lemma 3.* In the Markov-growth game with positive correlation, if $\lambda < \hat{\lambda}(\rho)$ and $\rho \leq \hat{\rho}(\lambda)$, then

(i) $P_b^c = P_m > P_r^c$.

(ii) $P_r^c$ is continuous, increasing in $\lambda$ and decreasing in $\rho$.

(iii) $P_r^c \to P_m$ as $\lambda \to \hat{\lambda}(\rho)$ and $P_r^c \to c$ as $\lambda \to 0$.

(iv) $P_r^c$ is increasing in $\lambda$ along $\hat{\rho}(\lambda)$, with $P_r^c = c$ when $\lambda = 0$ and $P_r^c \to P_m$ as $\lambda \to \lambda^*$.

*Proof.* We begin with (i). Under the conditions of the lemma, $\Delta\lambda b < R$ and $RB \leq (\Delta\lambda b)(\Delta\rho r)$; thus, $R > 0$ and $\Delta\rho r > B$. When (10) and (11) hold with equality, we have

$$\frac{\partial P_b}{\partial P_r}\Big|_{(10)} = [\pi'(P_r)/\pi'(P_b)][\Delta\rho r/B] \tag{A7}$$

$$\frac{\partial P_b}{\partial P_r}\Big|_{(11)} = [\pi'(P_r)/\pi'(P_b)][R/\Delta\lambda b]. \tag{A8}$$

Since $\pi'(P) > 0$ and $R > 0$, the recession constraint (11) binds at the price pair $(c, c)$ and along an upward-sloping path of prices. Note that (11) fails when $P_b = P_r > c$, and so (11) when binding crosses the 45° line only at $(c, c)$. Finally, observe that (11) holds with slack for higher (lower) values of $P_b$ ($P_r$). Let $\bar{P}_r \in [c, P_m)$ denote the value for $P_r$ that solves (11) with equality when $P_b = P_m$. Figure 2 illustrates.

Consider now the boom constraint (10). Clearly, if $B \leq 0$, then any $P_b \geq c$ and $P_r \geq c$ satisfy (10), and so $(P_b^c, P_r^c) = (P_m, \bar{P}_r)$ and (i) holds. If $B > 0$, then (10) emanates from $(c, c)$ when binding and slopes upward, and (10) holds with slack for higher (lower) values of $P_r$ ($P_b$). Furthermore, we have from (A7) and (A8) that at any point of intersection,

$$\frac{\partial P_b}{\partial P_r}\Big|_{(10)} - \frac{\partial P_b}{\partial P_r}\Big|_{(11)} = [\pi'(P_r)/\pi'(P_b)][(\Delta\lambda b)(\Delta\rho r) - RB]/(\Delta\lambda bB), \tag{A9}$$

and so under the conditions of the lemma, either $RB < (\Delta\lambda b)(\Delta\rho r)$ and the binding constraints cross only at the competitive price with the binding (10) otherwise lying northwest of the binding (11), or $RB = (\Delta\lambda b)(\Delta\rho r)$ and the binding constraints overlap and traverse exactly the same prices. In either event, $(P_b^c, P_r^c) = (P_m, \bar{P}_r)$ and so (i) holds.

To prove (ii), assume $1 - \lambda - \rho > 0$, $\lambda < \hat{\lambda}(\rho)$, and $\rho < \hat{\rho}(\lambda)$, and consider small changes in $\rho$. Since (11) holds with equality over this region when $P_b = P_m$, we implicitly differentiate and, using $R > 0$, find that $\text{sign}\{\partial P_r^c/\partial\rho\} = \text{sign}\{\pi(P_m)\lambda b(\partial\Delta/\partial\rho) - \pi(P_r^c)(\partial R/\partial\rho)\}$. Deriving that

$$\frac{\partial\Delta}{\partial\rho} = -\Delta^2 b[1 - \delta r] < 0 \tag{A10}$$

and using $\pi(P_m) > \pi(P_r^c)$ we find that $\pi(P_m)\lambda b(\partial\Delta/\partial\rho) - \pi(P_r^c)(\partial R/\partial\rho) < \pi(P_r^c)\Delta^2 b\lambda(r - b) < 0$, from which it follows that $P_r^c$ decreases in $\rho$. After deriving that

$$\frac{\partial\Delta}{\partial\lambda} = -\Delta^2 r[1 - \delta b] < 0, \tag{A11}$$

we may argue in a similar manner to show that $P_r^c$ increases in $\lambda$.

Consider next (iii). As $\lambda \to \hat{\lambda}(\rho)$, we have that $\lambda\Delta b \to R$. The binding constraint (11) then approaches the 45° line, so $P_r^c = \bar{P}_r \to P_m$. Next, as $\lambda \to 0$, $P_r^c = \bar{P}_r \to c$, since as (A8) indicates, the binding constraint (11) then becomes vertical at $P_r^c = c$. Finally, we consider (iv). Again, (11) must bind, and so we differentiate (11) when $P_b = P_m$ and $\rho = \hat{\rho}(\lambda)$ to get that

$$\text{sign}\left\{\frac{\partial P_r^c}{\partial\lambda}\right\}\Big|_{\rho=\hat{\rho}(\lambda)} = \text{sign}\left\{\pi(P_m)b\left[\Delta + \lambda\frac{d\Delta}{d\lambda}\right] - \pi(P_r^c)\frac{dR}{d\lambda}\right\}.$$

The total derivatives reflect the dependence of $\hat{\rho}$ upon $\lambda$. After further calculations, we find that $\pi(P_m)b[\Delta + \lambda(d\Delta/d\lambda)] - \pi(P_r^c)(dR/d\lambda) = [1 - \delta b + \delta(b - r)\rho^*]\Delta^2\{\pi(P_m)b(1 - \delta r) - \pi(P_r^c)r[1 - \delta b]\}/\delta > 0$, where the inequality uses $b > r$ and $\pi(P_m) > \pi(P_r^c)$. Thus, $P_r^c$ increases in $\lambda$ along $\rho = \hat{\rho}(\lambda)$. *Q.E.D.*

*Proof of Theorem 3.* Follows immediately from Lemmas 2 and 3.

*Proof of Theorem 4.* Analogous to that for Theorem 3; see Bagwell and Staiger (1995).

□ **Section 5 definitions and facts.** Let us define $E(\tilde{\omega}) \equiv E\{\omega(P^*(\tilde{\omega}/\epsilon)\epsilon)\}$. Calculations and integration by parts reveals

$$E(\tilde{\omega}) = \tilde{\omega}/(n-1), \text{ if } \tilde{\omega} \leq \underline{\epsilon}\pi(P_m)(n-1)/n \tag{A12}$$

$$E(\tilde{\omega}) = \tilde{\omega}/(n-1) - \pi(P_m)/n \int_{\underline{\epsilon}}^{[n/(n-1)]\tilde{\omega}/\pi(P_m)} F(\epsilon)d\epsilon, \text{ if } \underline{\epsilon}\pi(P_m)(n-1)/n < \tilde{\omega} < \bar{\epsilon}\pi(P_m)(n-1)/n \tag{A13}$$

$$E(\tilde{\omega}) = \pi(P_m)/n, \text{ if } \tilde{\omega} \geq \bar{\epsilon}\pi(P_m)(n-1)/n, \tag{A14}$$

where $F(\epsilon)$ is the distribution function for $\epsilon$. It follows that $E(\tilde{\omega})$ is linear and increasing at the rate $1/(n-1)$, then increasing at a lower rate and (strictly) concave, and then constant at value $\pi(P_m)/n$. The boom- and recession-period fixed-point equations, (36) and (37), now may be rewritten as

$$0 = E(\tilde{\omega}_r)\Delta\rho r + E(\tilde{\omega}_b)[(n-1) - B] - \tilde{\omega}_b \tag{A15}$$

$$0 = E(\tilde{\omega}_b)\Delta\lambda b + E(\tilde{\omega}_r)[(n-1) - R] - \tilde{\omega}_r. \tag{A16}$$

Observe that $\tilde{\omega}_b = \tilde{\omega}_r = 0$ satisfies (A15) and (A16).

In correspondence with (A15) and (A16), when $\tilde{\omega}_b = \tilde{\omega}_r$, we may define

$$fb(\tilde{\omega}) \equiv E(\tilde{\omega})[\Delta\rho r + (n-1) - B] - \tilde{\omega} \tag{A17}$$

$$fr(\tilde{\omega}) \equiv E(\tilde{\omega})[\Delta\lambda b + (n-1) - R] - \tilde{\omega}. \tag{A18}$$

Thus, e.g., when $f_b(\tilde{\omega}) = 0$, (A15) is satisfied on the 45° line at $\tilde{\omega}_b = \tilde{\omega}_r = \tilde{\omega}$. Note that $f_b(\tilde{\omega})$ and $f_r(\tilde{\omega})$ are (weakly) concave.

We now differentiate the boom- and recession-period fixed-point equations, (A15) and (A16), respectively, to get

$$\frac{\partial\tilde{\omega}_b}{\partial\tilde{\omega}_r}\Big|_b = E'(\tilde{\omega}_r)\Delta\rho r/\{1 - E'(\tilde{\omega}_b)[(n-1) - B]\} = E'(\tilde{\omega}_r)\Delta\rho r/[E'(\tilde{\omega}_b)\Delta\rho r - f_b'(\tilde{\omega}_b)] \tag{A19}$$

$$\frac{\partial\tilde{\omega}_b}{\partial\tilde{\omega}_r}\Big|_r = \{1 - E'(\tilde{\omega}_r)[(n-1) - R]\}/[E'(\tilde{\omega}_b)\Delta\lambda b] = [E'(\tilde{\omega}_r)\Delta\lambda b - f_r'(\tilde{\omega}_r)]/[E'(\tilde{\omega}_b)\Delta\lambda b]. \tag{A20}$$

*Lemma 4.* In the Markov-growth game with transitory shocks, if

(i) $\min\{\Delta\lambda b - R, \Delta\rho r - B\} > 0$

(ii) $\max\{\Delta\lambda b - R, \Delta\rho r - B\} < (\bar{\epsilon} - 1)(n-1)$,

then $\underline{\epsilon}\pi(P_m)(n-1)/n < \hat{\omega}_b, \hat{\omega}_r < \bar{\epsilon}\pi(P_m)(n-1)/n$ and $\text{sign}\{\hat{\omega}_b - \hat{\omega}_r\} = \text{sign}\{1 - \lambda - \rho\}$.

*Proof.* Using (A12), (A17), and (A18), we have that (i) implies that $f_b'(\tilde{\omega}) > 0$ and $f_r'(\tilde{\omega}) > 0$ for all $\tilde{\omega} \in [0, \underline{\epsilon}\pi(P_m)(n-1)/n]$; furthermore, it is direct from (A14), (A17), and (A18) that (ii) implies that $f_b(\tilde{\omega}) < 0$ and $f_r(\tilde{\omega}) < 0$ for all $\tilde{\omega} \geq \bar{\epsilon}\pi(P_m)(n-1)/n$. Thus, unique positive roots, $\bar{\omega}_b$ and $\bar{\omega}_r$, exist at which $f_b(\bar{\omega}_b) = f_r(\bar{\omega}_r) = 0$, with $\underline{\epsilon}\pi(P_m)(n-1)/n < \bar{\omega}_b, \bar{\omega}_r < \bar{\epsilon}\pi(P_m)(n-1)/n$. Given the concavity of $f_b$ and $f_r$, we have also that $f_b'(\bar{\omega}_b) < 0$ and $f_r'(\bar{\omega}_r) < 0$. Finally, we have from (A1), (A17), and (A18) that

$$f_b(\bar{\omega}_r) = f_b(\bar{\omega}_r) - f_r(\bar{\omega}_r) = E(\bar{\omega}_r)\Delta(b - r)(1 - \lambda - \rho), \tag{A21}$$

so that $E(\bar{\omega}_r) > 0$ implies

$$\text{sign}\{\bar{\omega}_b - \bar{\omega}_r\} = \text{sign}\{1 - \lambda - \rho\}. \tag{A22}$$

Thus, under (i) and (ii), both fixed-point equations cross the 45° line, and the boom-period fixed-point equation crosses this line at a higher value under positive correlation.

Using (A19) and (A20), it is now a simple matter to see that

$$\frac{\partial\tilde{\omega}_b}{\partial\tilde{\omega}_r}\Big|_b \in [0,1) \text{ at } \tilde{\omega}_b = \tilde{\omega}_r = \bar{\omega}_b \tag{A23}$$

$$\frac{\partial\tilde{\omega}_b}{\partial\tilde{\omega}_r}\Big|_r > 1 \text{ at } \tilde{\omega}_b = \tilde{\omega}_r = \bar{\omega}_r. \tag{A24}$$

(A15) and (A16) both pass through the origin, and the signs of (A19) and (A20) at the origin depend on the respective signs of $B$ and $R$, but eventually the respective curves slope upward and cross the 45° line: there exist $\underline{\omega}_b$ and $\underline{\omega}_r$, with $0 \leq \underline{\omega}_b \leq \overline{\omega}_b$ and $0 \leq \underline{\omega}_r \leq \overline{\omega}_r$, such that (A15) is satisfied at $(\tilde{\omega}_b, \tilde{\omega}_r) = (\underline{\omega}_b, 0)$ and slopes upward from $(\underline{\omega}_b, 0)$ through $(\overline{\omega}_b, \overline{\omega}_b)$ and on, while (A16) holds at $(\tilde{\omega}_b, \tilde{\omega}_r) = (0, \underline{\omega}_r)$ and slopes upward from $(0, \underline{\omega}_r)$ through $(\overline{\omega}_r, \overline{\omega}_r)$ and on. Thus, (A15) crosses the 45° line at $(\overline{\omega}_b, \overline{\omega}_b)$ from above, while (A16) crosses the 45° line at $(\overline{\omega}_r, \overline{\omega}_r)$ from below. Neither crosses the 45° line elsewhere, except at the origin.

It can now be easily verified that $\underline{\epsilon}\pi(P_m)(n-1)/n < \min\{\overline{\omega}_b, \overline{\omega}_r\} \leq \hat{\omega}_b, \hat{\omega}_r \leq \max\{\overline{\omega}_b, \overline{\omega}_r\} < \overline{\epsilon}\pi(P_m)(n-1)/n$, where $\text{sign}\{\hat{\omega}_b - \hat{\omega}_r\} = \text{sign}\{\overline{\omega}_b - \overline{\omega}_r\}$. Combining this with (A22) then proves the lemma. Figure 4 illustrates the case where $1 - \lambda - \rho > 0$. *Q.E.D.*

*Lemma 5.* In the Markov-growth game with transitory shocks, if

(i) $\min\{\Delta\lambda b - R, \Delta\rho r - B\} \in (0, (\overline{\epsilon} - 1)(n - 1))$

(ii) $\max\{\Delta\lambda b - R, \Delta\rho r - B\} \geq (\overline{\epsilon} - 1)(n - 1)$,

then $\underline{\epsilon}\pi(P_m)(n-1)/n < \min\{\hat{\omega}_b, \hat{\omega}_r\} < \overline{\epsilon}\pi(P_m)(n-1)/n$ and $\text{sign}\{\hat{\omega}_b - \hat{\omega}_r\} = \text{sign}\{1 - \lambda - \rho\}$.

*Proof.* Suppose first that $1 - \lambda - \rho > 0$. Then (A1) and (i) and (ii) imply $\Delta\rho r - B \geq (\overline{\epsilon} - 1)(n - 1) > \Delta\lambda b - R > 0$, so (A12)–(A14) and (A17)–(A18) yield that $f_b'(\tilde{\omega}) > 0$ and $f_r'(\tilde{\omega}) > 0$ for all $\tilde{\omega} \in [0, \underline{\epsilon}\pi(P_m)(n-1)/n]$ and $f_b(\overline{\epsilon}\pi(P_m)(n-1)/n) \geq 0 > f_r(\overline{\epsilon}\pi(P_m)(n-1)/n)$. It then follows that $\overline{\omega}_r$ exists at which $f_r(\overline{\omega}_r) = 0 > f_r'(\overline{\omega}_r)$, and further that $\underline{\epsilon}\pi(P_m)(n-1)/n < \overline{\omega}_r < \overline{\epsilon}\pi(P_m)(n-1)/n$. There also exists a positive root at which $f_b(\overline{\omega}_b) = 0$, and this root satisfies $\overline{\omega}_b \geq \overline{\epsilon}\pi(P_m)(n-1)/n$ and $f_b'(\overline{\omega}_b) < 0$. Clearly, $\overline{\omega}_b > \overline{\omega}_r$. (A16) then takes the same form as described in the proof of Lemma 4 (for the positive correlation case) and as presented in Figure 4. (A15) is also described as before, except that now it is flat when it crosses the 45° line (since $\overline{\omega}_b \geq \overline{\epsilon}\pi(P_m)(n-1)/n$ and $E'(\overline{\omega}_b) = 0$ by (A14)). It follows that $\hat{\omega}_b > \hat{\omega}_r > \overline{\omega}_r > \underline{\epsilon}\pi(P_m)(n-1)/n$ and $\overline{\epsilon}\pi(P_m)(n-1)/n > \hat{\omega}_r$. Related arguments apply when $1 - \lambda - \rho \leq 0$. *Q.E.D.*

*Lemma 6.* In the Markov-growth game with transitory shocks, if

$$\min\{\Delta\lambda b - R, \Delta\rho r - B\} \geq (\overline{\epsilon} - 1)(n - 1),$$

then $\min\{\hat{\omega}_b, \hat{\omega}_r\} \geq \overline{\epsilon}\pi(P_m)(n-1)/n$.

*Proof.* In this case, we have from (A12)–(A14) and (A17)–(A18) that $f_b(\overline{\epsilon}\pi(P_m)(n-1)/n) \geq 0$ and $f_r(\overline{\epsilon}\pi(P_m)(n-1)/n) \geq 0$. Since $f_b$ and $f_r$ always have a positive root, it follows that these roots satisfy $\overline{\omega}_b \geq \overline{\epsilon}\pi(P_m)(n-1)/n > 0 = f_b(\overline{\omega}_b)$ and $\overline{\omega}_r \geq \overline{\epsilon}\pi(P_m)(n-1)/n > 0 = f_r(\overline{\omega}_r)$, respectively. Now use (A14) to see that $(\hat{\omega}_b, \hat{\omega}_r) = (\overline{\omega}_b, \overline{\omega}_r)$ satisfies (A15) and (A16). *Q.E.D.*

*Lemma 7.* In the Markov-growth game with transitory shocks, if

(i) $\min\{\Delta\lambda b - R, \Delta\rho r - B\} \leq 0$

(ii) $\max\{\Delta\lambda b - R, \Delta\rho r - B\} \leq (\overline{\epsilon} - 1)(n - 1)$

(iii) $\rho \leq \hat{\rho}(\lambda)$,

then $\underline{\epsilon}\pi(P_m)(n-1)/n \leq \max\{\hat{\omega}_b, \hat{\omega}_r\} \leq \overline{\epsilon}\pi(P_m)(n-1)/n$ and $\text{sign}\{\hat{\omega}_b - \hat{\omega}_r\} = \text{sign}\{1 - \lambda - \rho\}$.

*Proof.* Suppose that $1 - \lambda - \rho > 0$. Then (A1) and (i) and (ii) imply

$$\Delta\lambda b - R \leq 0 \tag{A25}$$

$$\Delta\rho r - B \leq (\overline{\epsilon} - 1)(n - 1), \tag{A26}$$

so $f_r'(\tilde{\omega}) \leq 0$ for all $\tilde{\omega} \in [0, \underline{\epsilon}\pi(P_m)(n-1)/n]$ and $f_b(\overline{\epsilon}\pi(P_m)(n-1)/n) \leq 0$. Next, (iii) and $1 - \lambda - \rho > 0$ imply that

$$\Delta\rho r - B > 0. \tag{A27}$$

Otherwise, we have $\Delta\rho r - B \leq 0$, or equivalently $\rho \geq \tilde{\rho}(\lambda) > 0$, where the definition of $\tilde{\rho}(\lambda)$ is extended into the positive correlation range. Using (A1), we also have that $\lambda b\Delta - R < 0$. It then would follow that $RB > (\lambda b\Delta)(\rho r\Delta)$, or equivalently $\rho > \hat{\rho}(\lambda)$, contradicting (iii); thus, (A27) must hold. This implies $f_b'(\tilde{\omega}) > 0$ for all $\tilde{\omega} \in [0, \underline{\epsilon}\pi(P_m)(n-1)/n]$. It follows that $\overline{\omega}_b$ exists at which $f_b(\overline{\omega}_b) = 0 > f_b'(\overline{\omega}_b)$, and further that $\underline{\epsilon}\pi(P_m)(n-1)/n < \overline{\omega}_b \leq \overline{\epsilon}\pi(P_m)(n-1)/n$. There are two cases for $f_r$. If (A25) holds strictly, then $f_r(\tilde{\omega}) < 0$ and $f_r'(\tilde{\omega}) < 0$ for all $\tilde{\omega} > 0$; while if (A25) holds with equality, then $f_r(\tilde{\omega}) = 0$ for all $\tilde{\omega} \in [0, \underline{\epsilon}\pi(P_m)(n-1)/n]$, with $f_r(\tilde{\omega}) < 0$ and $f_r'(\tilde{\omega}) < 0$ for all $\tilde{\omega} > \underline{\epsilon}\pi(P_m)(n-1)/n$.

Referring to (A19) and (A20), we thus have that (A15) eventually slopes upward, crossing the 45° line at $\overline{\omega}_b$, as in Figure 4; (A16) slopes upward and is everywhere above the 45° line (if (A25) holds strictly), or it runs along this line up until $\tilde{\omega}_r = \underline{\epsilon}\pi(P_m)(n-1)/n$ and then slopes upward and remains above the 45° line (if (A25) holds with equality). Moreover, for $\tilde{\omega} \in [0, \underline{\epsilon}\pi(P_m)(n-1)/n]$, the former takes slope $\Delta\rho r/B$ while the latter takes slope $R/(\Delta\lambda b) \geq 1$. (A15) thus departs the origin northwest of (A16) if $B \leq 0$, or if $B > 0$ and (iii) holds strictly. The curves necessarily intersect, and it must be that $\bar{\epsilon}\pi(P_m)(n-1)/n \geq \overline{\omega}_b > \hat{\omega}_b > \hat{\omega}_r$; in addition, since the curves are linear out of the origin, we have $\hat{\omega}_b \geq \underline{\epsilon}\pi(P_m)(n-1)/n$, with equality when $\rho = \hat{\rho}(\lambda)$ and the two curves are collinear initially.

Finally, the case of negative correlation may be solved analogously. When there is zero correlation, the conditions of the lemma require $\rho = \hat{\rho}(\lambda)$ and so $\hat{\omega}_b = \hat{\omega}_r = \underline{\epsilon}\pi(P_m)(n-1)/n$. *Q.E.D.*

*Lemma 8.* In the Markov-growth game with transitory shocks, if

(i) $\min\{\Delta\lambda b - R, \Delta\rho r - B\} \leq 0$

(ii) $\max\{\Delta\lambda b - R, \Delta\rho r - B\} > (\bar{\epsilon} - 1)(n-1)$

(iii) $\rho \leq \hat{\rho}(\lambda)$,

then $\underline{\epsilon}\pi(P_m)(n-1)/n \leq \max\{\hat{\omega}_b, \hat{\omega}_r\}$ and $\text{sign}\{\hat{\omega}_b - \hat{\omega}_r\} = \text{sign}\{1 - \lambda - \rho\}$.

*Proof.* Suppose $1 - \lambda - \rho > 0$. Then (A1) and (i) and (ii) imply that (A25) holds while (A26) now fails; hence, (A16) is as described in the proof of Lemma 7, but $f_b(\bar{\epsilon}\pi(P_m)(n-1)/n) > 0$ is now true, whence $f_b'(\tilde{\omega}_b) > 0$ for all $\tilde{\omega}_b \in [0, \underline{\epsilon}\pi(P_m)(n-1)/n]$. Thus, (A15) crosses the 45° line with zero slope at $\overline{\omega}_b > \bar{\epsilon}\pi(P_m)(n-1)/n$. As in the proof of Lemma 7, we then have that $\hat{\omega}_b > \hat{\omega}_r$ and $\hat{\omega}_b \geq \underline{\epsilon}\pi(P_m)(n-1)/n$; note, though, that $\hat{\omega}_b > \bar{\epsilon}\pi(P_m)(n-1)/n$ is now possible. Similar arguments apply when $1 - \lambda - \rho < 0$, and (i) and (ii) rule out $1 - \lambda - \rho = 0$. *Q.E.D.*

*Lemma 9.* In the Markov-growth game with transitory shocks, if $\rho > \hat{\rho}(\lambda)$, then $\hat{\omega}_b = \hat{\omega}_r = 0$.

*Proof.* As in the proof of Lemma 2, $\rho > \hat{\rho}(\lambda)$ is equivalent to $RB > (\Delta\rho r)(\Delta\lambda b)$ and implies that $\min\{\Delta\lambda b - R, \Delta\rho r - B\} < 0$; thus, it must be that $\min\{R, B\} > 0$. Consider now the range $\tilde{\omega}_r \in [0, \underline{\epsilon}\pi(P_m)(n-1)/n]$, over which (A12), (A19), and (A20) yield

$$\frac{\partial\tilde{\omega}_b}{\partial\tilde{\omega}_r}\Big|_r = R/(\Delta\lambda b) > (\Delta\rho r)/B = \frac{\partial\tilde{\omega}_b}{\partial\tilde{\omega}_r}\Big|_b,$$

so (A16) initially lies above (A15), with an intersection possible for the given range only at the origin. A subsequent intersection can occur only if (A15) crosses (A16) from below; this is impossible when $\tilde{\omega}_b \geq \bar{\epsilon}\pi(P_m)(n-1)/n$, since then (A16) takes infinite slope, as is evident from (A20) and (A14). Further, we have that

$$\frac{\partial^2\tilde{\omega}_b}{\partial\tilde{\omega}_r^2}\Big|_b = \frac{E''(\tilde{\omega}_r)\Delta\rho r + \left[\frac{\partial\tilde{\omega}_b}{\partial\tilde{\omega}_r}\Big|_b\right]^2[(n-1) - B]E''(\tilde{\omega}_b)}{1 - E'(\tilde{\omega}_b)[(n-1) - B]}$$

$$\frac{\partial^2\tilde{\omega}_b}{\partial\tilde{\omega}_r^2}\Big|_r = \frac{E''(\tilde{\omega}_r)[(n-1) - R] + \left[\frac{\partial\tilde{\omega}_b}{\partial\tilde{\omega}_r}\Big|_r\right]^2 E''(\tilde{\omega}_b)}{-E'(\tilde{\omega}_b)\Delta\lambda b}.$$

But since $(n-1) - B \geq 0$, $(n-1) - R \geq 0$, $E''(\tilde{\omega}) \leq 0$, and $E'(\tilde{\omega}) \leq 1/(n-1)$ are true generally, and noting that $E'(\tilde{\omega}_b) > 0$ for $\tilde{\omega}_b < \bar{\epsilon}\pi(P_m)(n-1)/n$, we have that (A15) defines a concave function while (A16) gives a convex function, so (A15) can never cross (A16) from below. Hence, the unique intersection occurs at the origin, where $\hat{\omega}_b = \hat{\omega}_r = 0$. *Q.E.D.*

*Proof of Theorem 5.* The lemmas give an exhaustive characterization of the most-collusive fixed-point solutions. We now define parameter relationships that delineate the regions in Figure 3 for which the various lemmas apply. Let $\hat{\lambda}(\rho, \bar{\epsilon})$ satisfy $\Delta\lambda b - R = (\bar{\epsilon} - 1)(n-1)$, and let $\tilde{\rho}(\lambda, \bar{\epsilon})$ satisfy $\Delta\rho r - B = (\bar{\epsilon} - 1)(n-1)$. Calculations reveal that $\hat{\lambda}(\rho, \bar{\epsilon}) = [1 - (1-\rho)\delta b]/(1/\lambda^*(\bar{\epsilon}) - \delta b)$ and $\tilde{\rho}(\lambda, \bar{\epsilon}) = [1 - (1-\lambda)\delta r]/[1/\rho^*(\bar{\epsilon}) - \delta r]$, where $\lambda^*(\bar{\epsilon}) = 1 - \rho^*(\bar{\epsilon})$ and $\lambda^*(\bar{\epsilon}) = \{\bar{\epsilon}(n-1)/[1 + \bar{\epsilon}(n-1)] - \delta r\}/[\delta(b-r)]$ and $\rho^*(\bar{\epsilon}) = \{\delta b - \bar{\epsilon}(n-1)/[1 + \bar{\epsilon}(n-1)]\}/[\delta(b-r)]$. Under assumption (40), we find that $\lambda^*(\bar{\epsilon}) \in (\lambda^*, 1)$ and $\rho^*(\bar{\epsilon}) \in (0, \rho^*)$, implying that $\hat{\lambda}(\rho, \bar{\epsilon}) > \hat{\lambda}(\rho)$ and $\tilde{\rho}(\lambda, \bar{\epsilon}) < \tilde{\rho}(\lambda)$, which corresponds to Figure 3. We note that $\tilde{\rho}(0, \bar{\epsilon}) > 0$ is true, so $\tilde{\rho}(\lambda, \bar{\epsilon})$ intersects $\hat{\lambda}(\rho)$; however, it may or may not occur that $\hat{\lambda}(\rho, \bar{\epsilon})$ intersects $\tilde{\rho}(\lambda)$.

Using (A1), and looking at Figure 3, it may now be confirmed that Lemma 4 describes region B, Lemma 5 corresponds to region A, Lemma 6 applies for the monopoly region, Lemma 7 represents that part of region C for which $\rho \geq \tilde{\rho}(\lambda, \bar{\epsilon})$, Lemma 8 covers the remainder of region C (including the origin), and Lemma 9 describes the competitive region. The lemmas confirm the cyclical features of the most-collusive prices stated in the text, once the prices are recovered from the most-collusive fixed-point solutions via (32)–(35) and (38)–(39). *Q.E.D.*

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