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THE SENSITIVITY OF STRATEGIC AND CORRECTIVE R & D POLICY IN BATTLES FOR MONOPOLY*

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We characterize the role for R & D subsidies in export markets where R & D is an uncertain process and the winner of the R & D competition monopolizes the market. Investments are assumed to induce either first order or mean-preserving second order shifts in the distribution of a firm's costs or, reinterpreting the model as a patent race, a firm's discovery dates. We show that, regardless of which form the uncertainty takes, a national strategic incentive to subsidize R & D exists, but must be balanced against a national corrective incentive to tax R & D whenever a country has multiple firms involved in the R & D competition.

1. INTRODUCTION

The effects of trade policy on the strategic interaction among firms has been an active area of research in international trade since the pioneering work of Spencer and Brander (1983) and Brander and Spencer (1985). The latter paper explores the strategic role of export subsidies, and its conclusions have been the subject of intense scrutiny. Perhaps the most central criticism comes from Eaton and Grossman (1986), who show that the sign of the appropriate strategic export policy depends on whether firms choose prices or quantities. An additional concern, pointed out by Dixit (1984), Eaton and Grossman (1986), and Krishna and Thursby (1988), is that a corrective incentive to tax exports arises when there is more than one domestic firm, clouding the case for export subsidies still further.

Much less attention has been given to the sensitivity of the conclusions of Spencer and Brander (1983), who establish that R & D subsidies can also perform a strategic role.[2] Nevertheless, since the Subsidies Code of the General Agreement on Tariffs and Trade (GATT) explicitly prohibits developed countries from directly subsidizing exports while it explicitly provides for certain kinds of R & D subsidies, R & D subsidies may well be the more relevant case. In any event, their widespread use is well-documented.[3]

The R & D model that Spencer and Brander (1983) consider has two exporting countries, each with a single firm, and one importing country. Firms engage in quantity competition in the product market after R & D leads in a deterministic

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[2] Exceptions include a companion paper, Bagwell and Staiger (1989a), as well as Cheng (1987) and Dixit (1988).

[3] See, for example, the discussion in Hufbauer and Erb (1984), and Komiya, Okuno, and Suzumura (1988).

fashion to lower production costs. With the assumption of a single firm per country ruling out corrective issues at the national level, the appropriate strategic policy is shown to be an R & D subsidy.

There are two fundamental assumptions in the Spencer and Brander (1983) analysis which we relax in this paper. First, R & D is an inherently uncertain process, and it is important to assess the robustness of any R & D policy prescription to the existence of uncertainty. The natural stochastic analogue to the deterministic model has R & D lowering the mean of the firm's cost distribution in the particular sense of a first order stochastic shift. This is not, however, the only plausible description of R & D activity. An additional possibility which we also consider is that R & D preserves the mean but alters the "risk" properties of the cost distribution. Such a shift in the cost distribution could correspond to R & D that commits the firm to a potentially more efficient but increasingly inflexible production process, prior to the resolution of uncertainty in the external environment. More generally, it is likely that R & D investments contain both mean-altering and risk-altering characteristics, and it thus seems important to explore these two "pure" cases.

Second, Spencer and Brander (1983) assume an oligopolistic product market. We consider an alternative assumption about product market behavior, namely, that the "winner" of the R & D competition monopolizes the market. Thus, we have in mind a situation in which firms undertake R & D to affect the distribution of their costs, knowing that only the lowest cost firm will actually operate in the final (production) stage. We argue that this "winner-take-all" feature is plausible when the product market is subject to sufficiently large scale economies, entering either on the supply side through large fixed costs of production or on the demand side through the existence of network externalities. Alternatively, this set up may also be plausible in the context of an international patent race. Here an investment outlay at time zero induces a distribution over dates for the discovery of an innovation. A firm wishes to discover the innovation first, because the first discovery can be patented.

When firms of different countries are engaged in a battle for monopoly, we find that the strategic incentive to subsidize R & D remains whether R & D reduces the mean of a firm's cost distribution or increases its riskiness. Thus, in a "high stakes" R & D competition where the winner takes all, the case for strategic R & D subsidies is not particularly sensitive to the way in which uncertainty enters the R & D process. Moreover, when there is more than one domestic firm (at the R & D stage), we find a corrective incentive to tax R & D, again regardless of the particular way in which R & D is modeled. Hence, the appropriate R & D policy when firms battle for monopoly depends mainly on the number of firms engaged in R & D competition.

These results are of direct interest, but are also noteworthy in relation to our findings in a companion paper (Bagwell and Staiger 1989a) in which we assume that the product market is oligopolistic. In this case, as we discuss further in the conclusion, the form of uncertainty plays a key role in the design of optimal R & D policy.

The rest of the paper is organized as follows. The basic model is described in

Section 2. Mean-reducing and mean-preserving investments are considered in Sections 3 and 4, respectively. Section 5 considers the introduction of multiple domestic and foreign firms. Our model is reinterpreted as a patent race in Section 6. Section 7 concludes.

2. THE BASIC MODEL

We consider a simple model in which there are two exporting countries and a third importing country. The export markets are imperfectly competitive; for now, we follow Spencer and Brander (1983) and assume each exporting country has a single exporting firm. One exporting country will be referred to as the home country, while the other is called the foreign country. Asterisks "*" denote foreign country variables.

The basic game has three stages. In the initial stage, the two exporting governments simultaneously choose the effective unit costs of investment, r and r^*, to their respective firms. In both countries, the social cost of investment is $\bar{r}$. Thus, for example, $r < \bar{r}$ indicates a home country subsidy on investment by the home firm. As all consumption takes place in a third country, the goal of each exporting country is to maximize its firm's expected profit less subsidy costs.

The second stage follows after both firms observe the policy choices of both exporting governments. The two firms then simultaneously choose nonnegative investment levels, I and I^*. Here, we depart from Spencer and Brander (1983) in modeling investment as a parameter in the random determination of production costs. Thus, $f(c|I)$ is the density of possible constant costs c, given the investment level I. $f^*(c^*|I^*)$ is then defined analogously. We maintain the assumptions that $f(c|I) \equiv f^*(c|I)$, $f(c|I) > 0$, and $f(c|I)$ is continuously differentiable in c and I, for every $c \in [\underline{c}, \bar{c}]$. The two investment technologies are thus symmetric and well-behaved. The goal of any one firm at this stage is to maximize its expected profit, given its cost of investment.

In the third and final stage, each firm observes its realized production cost as well as that of its rival. The two firms then simultaneously choose whether to enter the production stage of the market.

We assume that scale economies in the product market are sufficiently important that the market can only support the entry of one firm. These scale economies could enter on the supply side as a sufficiently large fixed cost in production, or they could enter on the demand side in the form of network externalities. When there is a large fixed cost, the entry game admits three classes of equilibria.[4] The most efficient firm may enter with probability one, the least efficient firm may enter with probability one, or the firms may play mixed entry strategies and earn zero expected gross profit. The latter case is least interesting, since if firms play in this fashion no investment will occur. Of the two former cases, the efficient class of equilibria seems focal. A similar logic applies when there are network externalities and the firms offer new and incompatible products of similar, intrinsic value. Here, if both

[4] Fixed costs in production need not be "large" in an absolute sense. For example, in the case of a Bertrand final stage with homogeneous products, fixed costs of production need only be positive.

firms enter, consumers may coordinate on the efficient firm or the inefficient firm or worse, divide up between the two firms. The most efficient equilibrium for the subgame in which both firms enter entails consumers going to the lowest cost firm. If the efficient equilibrium is anticipated and if there is some slight fixed cost to entry, then the efficient firm will enter with probability one and monopolize the market. Thus, while multiple equilibria to this entry game exist whether these scale economies enter on the supply or the demand side, the efficient equilibrium will have the low cost firm enter and monopolize the market. We take this equilibrium to the entry game as focal, and concentrate on it in what follows. We also note that the issue of multiple entry equilibria is sidestepped when, as in Section 6, our model is interpreted in the context of an international patent race. In this case, the legal rights to monopolization at the production stage are contained in a patent granted to the winner.

Throughout, we use the subgame perfect equilibrium concept (Selten 1975). We thus solve the final stage first, the second stage second and the first stage last.

We close this section by defining national welfare and discussing at a general level the determinants of appropriate R & D policy. In the absence of domestic consumption, home welfare can be defined as simply expected profits less subsidy costs, or

$$W(r, r^*) \equiv E\pi(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r) - (\bar{r} - r) \cdot \hat{I}(r, r^*). \tag{1}$$

Here, $E\pi(\cdot)$ is expected profits of the home firm (net of investment costs), while $\hat{I}(\cdot)$ and $\hat{I}^*(\cdot)$ denote equilibrium domestic and foreign investment levels, respectively. $W^*(r, r^*)$ is defined analogously for the symmetric foreign expected profit function, $E\pi^*(\cdot)$. To see the domestic welfare effects of a policy-induced change in the domestic cost of R & D, r, we differentiate (1) and use the envelope theorem to obtain

$$W_r(r, r^*) = E\pi_{I^*}(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r) \cdot \hat{I}^*_r(r, r^*) - (\bar{r} - r) \cdot \hat{I}_r(r, r^*) \tag{2}$$

where subscripts denote derivatives.[5] The domestic welfare effect of a change in r is composed of the difference between two terms. The first term captures the way in which a change in r alters equilibrium foreign investment, and through this channel, expected domestic profits. The second term simply gives the way in which a change in r alters equilibrium domestic investment, and through this, domestic subsidy payments.

In general, to determine the sign of $W_r(r, r^*)$ requires knowledge of the signs of $E\pi_{I^*}(\cdot)$, $\hat{I}^*_r(\cdot)$, and $\hat{I}_r(\cdot)$. The sign of $E\pi_{I^*}(\cdot)$ tells how greater investment abroad affects the expected profit at home. In the language of Fudenberg and Tirole (1984), $E\pi_{I^*}(\cdot) < 0$ would imply that investment (or a low cost of investment which leads to greater investment) makes a country "tough." The signs of $\hat{I}^*_r(\cdot)$ and $\hat{I}_r(\cdot)$ determine the direction of the equilibrium response of foreign and domestic investment, respectively, to a change in r.

To see what determines the signs of $\hat{I}^*_r(\cdot)$ and $\hat{I}_r(\cdot)$, we let $I(I^*, r)$ and $I^*(I, r^*)$

[5] This calculation employs the fact that r only directly enters into $E\pi(\cdot)$ through the total cost of investment, $r\hat{I}(\cdot)$.

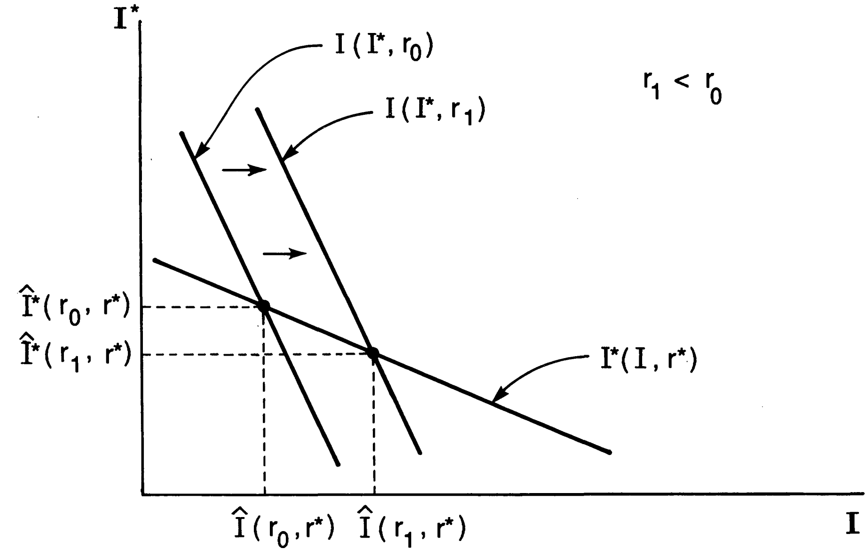


FIGURE 1

represent the firms' respective investment reaction functions. Provided that second order conditions are met, these functions satisfy $E\pi_I(I, I^*, r) = 0$ and $E\pi^*_{I^*}(I, I^*, r^*) = 0$, respectively. We will assume throughout the paper that the determinant of the Jacobian, J, associated with these first order conditions is positive,

$$(3)\qquad |J| \equiv E\pi_{II}(I, I^*, r) \cdot E\pi^*_{I^*I^*}(I, I^*, r^*) - E\pi_{I^*I}(I, I^*, r) \cdot E\pi^*_{II^*}(I, I^*, r^*) > 0,$$

and that this stability condition holds globally. With this condition, the reaction functions have at most one intersection, which we assume exists. $\hat{I}(r, r^*)$ and $\hat{I}^*(r, r^*)$ denote this solution, where

$$(4)\qquad \hat{I}_r(r, r^*) = \frac{E\pi^*_{I^*I^*}(I, I^*, r)}{|J|}; \qquad \hat{I}^*_r(r, r^*) = \frac{-E\pi^*_{II^*}(I, I^*, r)}{|J|}.$$

Thus, a home subsidy will raise home investment provided that $E\pi^*_{I^*I^*}(\cdot) < 0$, which is simply a second order condition. And a home subsidy will reduce foreign investment if and only if $E\pi^*_{II^*}(\cdot) < 0$. Since, provided second order conditions hold, the sign of $E\pi^*_{II^*}(\cdot)$ determines the sign of the investment reaction curve slopes, this last condition amounts to establishing whether investment reaction curves are negatively sloped.

Drawing all this together and using (2), it is evident that, provided second order conditions hold ($E\pi^*_{I^*I^*}(\cdot) < 0$) a small R & D subsidy by the home government to commit its firm to a larger investment will increase domestic welfare ($W_r(r, r^*) < 0$) if investment makes a country "tough" ($E\pi_{I^*}(\cdot) < 0$) and reaction curves are negatively sloped ($E\pi^*_{II^*}(\cdot) < 0$). This is illustrated in Figure 1, where the depicted relationship between I and I^* is negative, and where a lower r (a subsidy) shifts

$I(I^*, r)$ out in a parallel fashion, thereby raising the equilibrium level of I and lowering that of I^*, with the latter effect benefiting the domestic country if it prefers lower foreign investment. The focus of the remaining sections is to explore the generality with which these conditions hold and, in so doing, to explore the generality of the case for R & D subsidies in the context of "winner-take-all" international R & D rivalry.

3. MEAN-REDUCING INVESTMENTS

1. *Basic Assumptions.* As noted above, we focus on the efficient equilibrium of the final stage entry game, so that only the lowest cost firm will choose to "open." Formally, let $\pi(c, c^*)$ represent the home firm's profit (gross of investment costs) in the third stage if home costs are c and foreign costs are c^*. $\pi^*(c, c^*)$ is the symmetric function for the foreign firm. Then, if $\pi_m(c)$ is the profit from monopolizing the third country, we have that

$$\pi(c, c^*) \equiv \begin{cases} \pi_m(c), & c < c^* \\ 0, & c \geq c^*. \end{cases}$$ [6]

We assume only that $\pi_m(c) > 0$ and $\pi'_m(c) < 0$, where the prime refers to a derivative, for every $c \in [\underline{c}, \bar{c}]$.[7] We must also make a distributional assumption to convey the cost-reducing nature of investment. Let $f_I(c|I)$ denote the partial derivative of $f(c|I)$ with respect to I.

ASSUMPTION A. *For every I, there exists $\hat{c} \in (\underline{c}, \bar{c})$ such that $f_I(\hat{c}|I) = 0$, $f_I(c|I) < 0$ for $c \in (\hat{c}, \bar{c}]$, and $f_I(c|I) > 0$ for $c \in [\underline{c}, \hat{c})$.*

As shown in Figure 2, Assumption A simply amounts to the existence of a critical cost level such that a given raise in investment increases the density of lower costs and drops the density of higher costs. Defining $F(c|I) \equiv \int_{\underline{c}}^{c} f(c|I)\, dc$, it is straightforward to show that Assumption A implies $F_I(c|I) > 0$ for all $c \in (\underline{c}, \bar{c})$, which is the usual first order stochastic dominance condition (Hadar and Russell 1969). Assumption A also requires a "single crossing" of $f_I(c|I)$ through the c-axis.[8]

Thus, an increase in investment shifts density to lower costs. We next assume that this shifting process occurs at a decreasing rate as investment increases. The role of this assumption is to make the firm's investment problem a concave program.[9]

[6] Ties are actually irrelevant for our analysis, since they occur with zero probability.

[7] Our analysis is thus more general than the constant cost assumption we employ. Note also that $\pi_m(c)$ may embody a fixed cost component.

[8] Assumption A holds for the normal and exponential distributions, provided that greater investment lowers the mean, with $\hat{c}$ equal to the mean. (Our analysis carries through where c is distributed on open intervals.)

[9] The examples in footnote 8 are all consistent with Assumption B, provided in the normal and exponential cases that the mean's dependence on investment is not too concave. See Rogerson (1985) for related assumptions in a principal-agent framework.

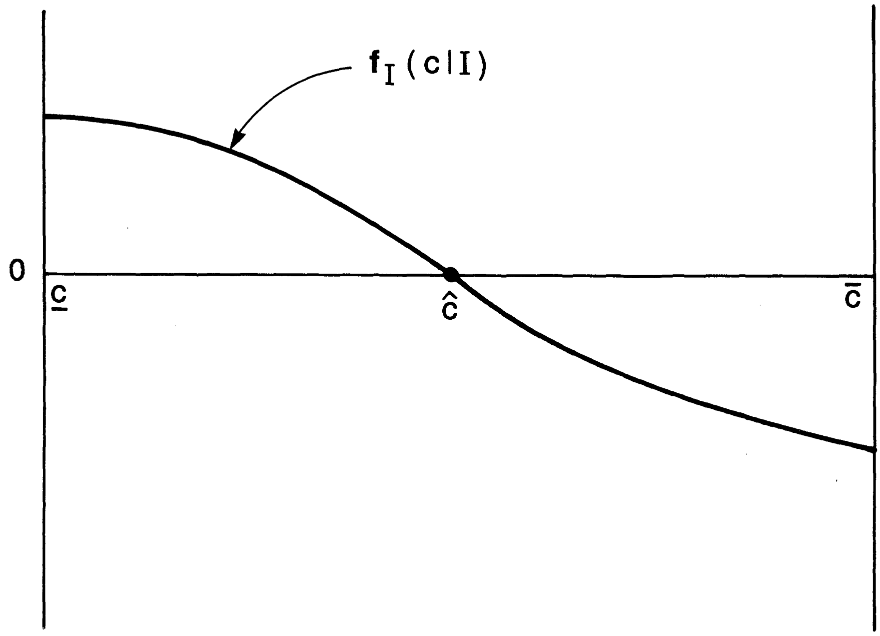


FIGURE 2

ASSUMPTION B. *For every I and $c \in [\underline{c}, \bar{c}]$, $F_{II}(c|I) \leq 0$, with a strict inequality holding over some positive measure of costs.*

2. *The Investment Stage.* We now fix r and r^* and consider the choice of investment levels. The home country will want to choose its investment level to maximize its expected profit, which is given by

$$(5) \qquad E\pi(I, I^*, r) \equiv \int_{\underline{c}}^{\bar{c}}\int_{\underline{c}}^{\bar{c}} f(c|I)f^*(c^*|I^*)\pi(c, c^*)\, dc^*\, dc - rI.$$

The first order condition is then

$$(6) \qquad E\pi_I(I, I^*, r) = \int_{\underline{c}}^{\bar{c}}\int_{\underline{c}}^{\bar{c}} f_I(c|I)f^*(c^*|I^*)\pi(c, c^*)\, dc^*\, dc - r = 0.$$

We assume a solution to this equation, so that a maximum is obtained if the second order condition holds. The solution corresponds to a reaction curve, $I = I(I^*, r)$. Symmetric arguments apply for the foreign country.

We begin with the following lemma stating that the second order condition does indeed hold.

LEMMA 3.1. *For all I, I^* and r, $E\pi_{II}(I, I^*, r) < 0$.*

PROOF. Observe using (6) that

(7) $$E\pi_{II}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} f_{II}(c|I)K(c|I^*)\, dc$$

where

(8) $$K(c|I^*) \equiv \int_{\underline{c}}^{\bar{c}} f^*(c^*|I^*)\pi(c, c^*)\, dc^* = \int_{c}^{\bar{c}} f^*(c^*|I^*)\pi_m(c)\, dc^* > 0$$

for all $c < \bar{c}$. $K(c|I^*)$ is simply expected domestic profit given a domestic cost realization of c if the foreign investment level is I^*. We also have that, with $\pi'_m(c) < 0$,

(9) $$K_c(c|I^*) = \int_{c}^{\bar{c}} f^*(c^*|I^*)\pi'_m(c)\, dc^* - f^*(c|I^*)\pi_m(c) < 0$$

for all $c \in [\underline{c}, \bar{c}]$. Thus, integrating by parts, we obtain

(10) $$E\pi_{II}(I, I^*, r) = -\int_{\underline{c}}^{\bar{c}} F_{II}(c|I)K_c(c|I^*)\, dc < 0.$$

Q.E.D.

The reaction functions, $I = I(I^*, r)$ and $I^* = I^*(I, r)$, are thus well-defined. Moreover, by Lemma 3.1, (3), and (4), $\hat{I}_r(r, r^*) < 0$: a domestic R & D subsidy will raise domestic investment.

We next show that investment makes a country "tough," i.e., greater investment by one country decreases the expected profit of the other country.

LEMMA 3.2. *For all I, I^* and r, $E\pi_{I^*}(I, I^*, r) < 0$.*

PROOF. Observe that

(11) $$E\pi_{I^*}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}}\int_{\underline{c}}^{\bar{c}} f(c|I) f^*_{I^*}(c^*|I^*)\pi(c, c^*)\, dc^*\, dc$$

which, using (8), can be rewritten as

(12) $$E\pi_{I^*}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} f(c|I)K_{I^*}(c|I^*)\, dc.$$

But it is straightforward to show that for all $c \in (\underline{c}, \bar{c})$,

(13) $$K_{I^*}(c|I^*) = -\pi_m(c)F^*_{I^*}(c|I^*) < 0.$$

Q.E.D.

Intuitively, as I^* increases, the home country is less likely to win and its expected profits decrease.

Finally, we must establish that reaction curves are negatively sloped. Our result is contained in the next lemma.

LEMMA 3.3. *For all* I, I^* *and* r, *if* $I = I^*$, *then* $E\pi_{I^*I}(I, I^*, r) < 0$.

PROOF. Observe that

$$E\pi_{I^*I}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}}\int_{\underline{c}}^{\bar{c}} f_I(c|I) f_{I^*}^*(c^*|I^*)\pi(c, c^*)\, dc^*\, dc \tag{14}$$

which with (8) simplifies to

$$E\pi_{I^*I}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} f_I(c|I) K_{I^*}(c|I^*)\, dc. \tag{15}$$

Unfortunately, $K_{I^*c}(c|I^*)$ is not of one global sign, so a simple application of Assumption A does not suffice to sign (15). Using (13), however, expression (15) can be rewritten as

$$E\pi_{I^*I}(I, I^*, r) = -\int_{\underline{c}}^{\bar{c}} f_I(c|I) F_{I^*}^*(c|I^*)\pi_m(c)\, dc. \tag{16}$$

Moreover, starting from symmetric initial investments ($I = I^*$) implies that

$$\int_{\underline{c}}^{\bar{c}} f_I(c|I) F_{I^*}^*(c|I^*)\, dc = \int_{\underline{c}}^{\bar{c}} f_I(c|I) F_I(c|I)\, dc = \int_{F_I(\underline{c}|I)}^{F_I(\bar{c}|I)} F_I\, dF_I = 0,$$

since $F_I(\bar{c}|I) = F_I(\underline{c}|I) = 0$. Using this result, the single crossing property of $f_I(c|I)$, and the monotonicity of $\pi_m(c)$, we then have

$$\int_{\underline{c}}^{\bar{c}} f_I(c|I) F_{I^*}^*(c|I^*)\pi_m(c)\, dc > \pi_m(\hat{c}) \int_{\underline{c}}^{\bar{c}} f_I(c|I) F_{I^*}^*(c|I^*)\, dc = 0,$$

where $\hat{c} \in (\underline{c}, \bar{c})$ is as defined in Assumption A. Q.E.D.

The intuition for this result centers around the monotonicity of $\pi_m(c)$, and can be seen as follows. Suppose for the moment that $\pi_m'(c) \equiv 0$, so that monopoly profits conditional on winning, π_m, are independent of costs. Then using (12), (13), and (16), we could write

$$E\pi_{I^*}(I, I^*, r) = -\pi_m \int_{\underline{c}}^{\bar{c}} f(c|I) F_{I^*}^*(c|I^*)\, dc$$

$$E\pi_{I^*I}(I, I^*, r) = -\pi_m \int_{\underline{c}}^{\bar{c}} f_I(c|I)F^*_{I^*}(c|I^*)\, dc.$$

The integral term in the first equation is simply the effect of an increase in foreign investment on the foreign firm's probability of winning. Thus, $E\pi_{I^*}(I, I^*, r)$ would be proportional to this probability effect if $\pi'_m(c) \equiv 0$. The integral term in the second equation is then the effect of an increase in home investment on the foreign firm's probability of winning when there is also an independent increase in foreign investment. Intuitively, given the symmetry of the investment technologies, this effect must be zero if the home and foreign investment levels are initially identical. Thus, were $\pi'_m(c) \equiv 0$, we would have $E\pi_{I^*I}(I, I^*, r) = 0$.[10] When $\pi_m(c)$ is monotonically decreasing in c, however, an increase in the probability with which the foreign firm wins is more costly in terms of lost profits to the home firm the lower is the home firm's expected costs (the higher is I). Thus, with $\pi'_m(c) < 0$, we have $E\pi_{I^*I}(I, I^*, r) < 0$.

Thus, about any symmetric equilibrium where $I = I^*$, reaction curves are negatively sloped. This implies by (4) that $\hat{I}^*_r(r, r^*) > 0$ for a symmetric equilibrium. A home subsidy lowers foreign investment.

3. *The Policy Stage.* In the previous section, we showed that the illustration in Figure 1 is locally correct. About any symmetric equilibrium, a home subsidy will increase I and decrease I^*. Further, the decrease in I^* is desirable for the home country, since foreign investment hurts the domestic country. We develop in this section implications that follow from this reasoning.

PROPOSITION 1. *From free trade ($r = r^* = \bar{r}$), a slight R & D subsidy improves that country's welfare while a slight R & D tax decreases that country's welfare. Further, the R & D subsidy has a "beggar thy neighbor" nature, as it decreases the welfare of the rival country.*

PROOF. Given the symmetric beginning at $r = r^* = \bar{r}$, $\hat{I}(\bar{r}, \bar{r}) = \hat{I}^*(\bar{r}, \bar{r})$ follows from country symmetry. Then, Lemma 3.2, Lemma 3.3, (1) and (2) give

$$W_r(\bar{r}, \bar{r}) = E\pi_{I^*}(\hat{I}(\bar{r}, \bar{r}), \hat{I}^*(\bar{r}, \bar{r}), \bar{r}) \cdot \hat{I}^*_r(\bar{r}, \bar{r}) < 0$$

$$W^*_r(\bar{r}, \bar{r}) = E\pi^*_I(\hat{I}(\bar{r}, \bar{r}), \hat{I}^*(\bar{r}, \bar{r}), \bar{r}) \cdot \hat{I}_r(\bar{r}, \bar{r}) > 0.$$

Hence, a slight R & D subsidy (tax) improves (decreases) home welfare, and a small R & D subsidy hurts the rival country. Q.E.D.

Our results here are not as strong as those developed by Spencer and Brander (1983). In their model, a country's best response to the selection of free trade by the rival country is a subsidy. In contrast, we are only able to establish a negative slope for reaction curves about symmetric investments (Lemma 3.3); thus, while it seems

[10] This finding is reported in a different context by Dixit (1987), who shows that reaction curves are flat about symmetric equilibria in contests, where the defining characteristic of a contest is that the value of the "prize" is constant (i.e., in our setting, $\pi'_m(c) \equiv 0$).

likely that the optimal response to free trade is a subsidy, we can only say that a slight subsidy is preferred to a passive, free trade response and that the rival country experiences a welfare loss when this subsidy is imposed. The same local feature of our model also prevents us from characterizing the optimal subsidy/tax strategies when governments choose policies sequentially.[11]

Like Spencer and Brander (1983), however, we can argue that a subsidy is generally optimal when countries choose policies simultaneously. We say a *symmetric, interior Nash equilibrium* occurs at $(\hat{r}, \hat{r}^*)$ if

(i) $\hat{r} = \hat{r}^*$
(ii) $W_r(\hat{r}, \hat{r}^*) = 0 = W^*_{r^*}(\hat{r}, \hat{r}^*)$
(iii) $W(\hat{r}, \hat{r}^*) \geq W(r, \hat{r}^*)$, for all $r \neq \hat{r}$, and $W^*(\hat{r}, \hat{r}^*) \geq W^*(\hat{r}, r^*)$ for all $r^* \neq \hat{r}^*$.

PROPOSITION 2. *In any symmetric, interior Nash equilibrium,* $\bar{r} > \hat{r} = \hat{r}^*$. *Both countries subsidize R & D.*

PROOF. Since $\hat{r} = \hat{r}^*$, $\hat{I}(\hat{r}, \hat{r}^*) = \hat{I}^*(\hat{r}, \hat{r}^*)$. Then, as before,

$$E\pi_{I^*}(\hat{I}(\hat{r}, \hat{r}^*), \hat{I}^*(\hat{r}, \hat{r}^*), \hat{r}) \cdot \hat{I}^*_r(\hat{r}, \hat{r}^*) < 0.$$

Further, $\hat{I}_r(\hat{r}, \hat{r}^*) < 0$. Thus, from (2), $W_r(\hat{r}, \hat{r}^*) = 0$ requires $\bar{r} > \hat{r}$, with symmetric arguments applying to the foreign country. Q.E.D.

The natural equilibrium to the three-stage game therefore involves subsidies on investments. Finally, we address the normative issue of whether the welfare of the exporting countries is higher or lower at the symmetric, interior equilibrium or at free trade. Not surprisingly, welfare is lower when countries subsidize than when they do not.

PROPOSITION 3. $W(\bar{r}, \bar{r}) = W^*(\bar{r}, \bar{r}) > W(\hat{r}, \hat{r}^*) = W^*(\hat{r}, \hat{r}^*)$. *Welfare is higher at free trade than at the symmetric, interior Nash equilibrium.*

PROOF. Totally differentiating $W(r, r^*)$ with respect to r and r^*, using the envelope theorem, and setting $r = r^*$ and $dr = dr^*$, we find that

$$dW(r, r^*) = [E\pi_{I^*}(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r) - (\bar{r} - r)][\hat{I}^*_r(r, r^*) + \hat{I}^*_{r^*}(r, r^*)]\ dr.$$

Now consider beginning with $r = r^* < \bar{r}$ and then increasing r and r^* together until $r = r^* = \bar{r}$. Then $dr > 0$ along this path. The first term in the above expression is

[11] The local nature of our results motivates the investigation of a specific example. One tractable example combines an exponential density, $f(c|I) = h(I)e^{-h(I)c}$, with an exponential profit function, $\pi_m(c) = V + ke^{-c}$. This density satisfies Assumptions A and B provided $(h(I), h'(I), V, k) > 0 \geq h''(I)$, with $h'(0)$ very large and $h'(\infty)$ very small. As long as r and r^* are not too different, this example admits a unique (and stable) Nash equilibrium in the investment subgame. The example also generates a *positively* sloped foreign investment reaction curve when I^* is sufficiently greater than I. Accordingly, it is difficult to make general arguments regarding globally-optimal R & D policy. As Dixit (1987) notes, this restriction to local analysis is also necessary even when $k = 0$, in which case the example corresponds to a fixed-prize contest model.

clearly negative. Finally, it can be shown that the second term is also negative, given that $\hat{I}(r, r^*) = \hat{I}^*(r, r^*)$ along this path and $|J| > 0$. It follows that $dW(r, r^*) > 0$ along this path when $r = r^* < \tilde{r}$. The proposition then follows from $(\tilde{r}, \tilde{r}) > (\hat{r}, \hat{r}^*)$ and $\hat{r} = \hat{r}^*$, and symmetric arguments for the foreign country. Q.E.D.

The subsidy game has a definite prisoners-dilemma character. A unilateral subsidy can improve welfare over free trade, but bilateral subsidies are self-forcing and inferior to free trade.

4. RISK-ALTERING INVESTMENTS

In the previous section we focused on investments which lowered expected cost. However, the effect of R & D on the distribution of costs could take on any of several plausible forms. To explore the sensitivity of our results to the way R & D is modeled, we now consider the case in which investment in R & D affects the riskiness of the distribution of cost outcomes in a mean-preserving way.

1. *Basic Assumptions.* We maintain the "winner take all" assumption of the previous section, so that $\pi(c, c^*)$ and $\pi^*(c, c^*)$ continue to be defined as above. However, in addition to the property that $\pi'_m(c) < 0$, we now require that $\pi''_m(c) > 0$ as well. This second derivative property will hold provided that the constant-cost monopoly problem is well-behaved.[12] Finally, we alter the way in which investment affects the distribution of costs to capture mean-preserving changes in risk.

ASSUMPTION C. *For every I, $(d/dI)[\int_{\underline{c}}^{\bar{c}} cf(c|I)\, dc] = 0$, and there exists $\hat{c} \in (\underline{c}, \bar{c})$, $c_1 \in (\underline{c}, \hat{c})$ and $c_2 \in (\hat{c}, \bar{c})$ such that $F_I(\hat{c}|I) = 0$, and*

$$f_I(c|I) \begin{cases} = 0 & \text{for} \quad c \in \{c_1, c_2\} \\ > 0 & \text{for} \quad c \in [\underline{c}, c_1) \cup (c_2, \bar{c}] \\ < 0 & \text{for} \quad c \in (c_1, c_2). \end{cases}$$

Figure 3 illustrates Assumption C. As depicted, there is a critical cost level associated with any given investment level such that this investment raises the value of the distribution function evaluated at lower costs and reduces the value of the distribution function evaluated at higher costs. It follows that higher levels of investment yield cost distributions which second-order stochastically dominate (Rothschild and Stiglitz 1970) those associated with lower levels of investment. Thus, Assumption C implies that higher levels of investment yield cost distributions which are more risky. Assumption C also requires the additional condition of a "single crossing" of $F_I(c|I)$ through the c-axis and of $f_I(c|I)$ on each side of this point.

[12] For example, this condition is met in the case of constant costs if the demand curve slopes downward and profits are concave in price.

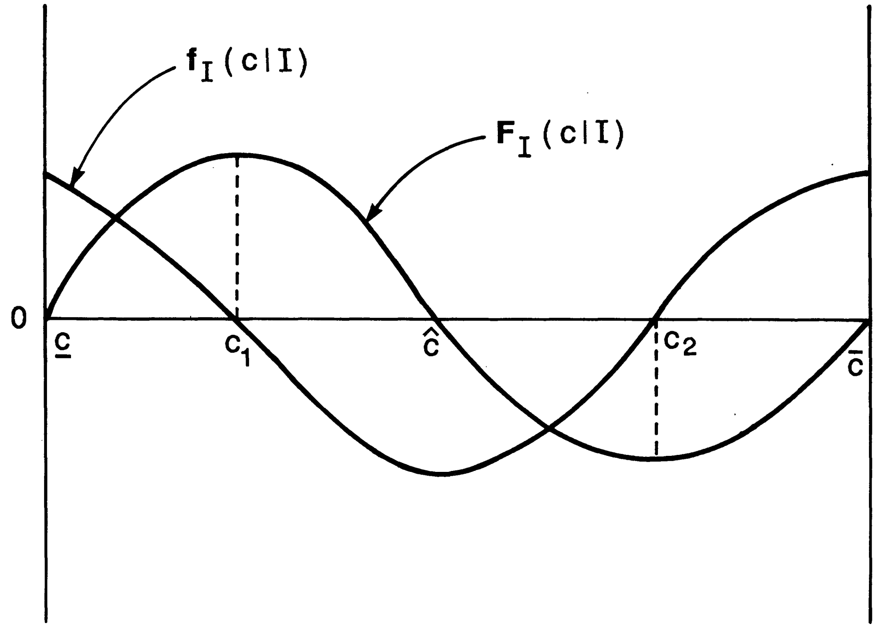


FIGURE 3

As before we will also require a regularity assumption on F_{II}. This ensures that the firm's investment problem is a concave program.

ASSUMPTION D. *For every I, there exists a $\tilde{c} \in (\underline{c}, \bar{c})$ such that $F_{II}(\tilde{c}|I) = 0$, with $F_{II}(c|I) \leq 0$ for $c \in [\underline{c}, \tilde{c})$, $F_{II}(c|I) \geq 0$ for $c \in (\tilde{c}, \bar{c}]$, and $F_{II}(c|I) \neq 0$ for some positive measure of $c \in (\underline{c}, \bar{c})$.*

Finally we assume that one of the following two conditions on the distribution of costs holds.

ASSUMPTION E. *Either* (1) *c is distributed symmetrically about its mean for every I or* (2) *$f(c|I)$ is nonincreasing in c for all $c \in [\underline{c}, \bar{c}]$ and every I.*[13]

2. *The Investment Stage.* As before, we fix r and r^* and consider the investment decisions of each firm.[14] With home firm expected profits still given by (5), it is straightforward to argue that firms find risk-increasing R & D attractive.

LEMMA 4.0. *For all I and I^*, $E\pi_I(I, I^*, 0) > 0$.*

PROOF. (See Bagwell and Staiger 1989b.)

[13] Examples of distributions that satisfy this assumption are the normal and the exponential.

[14] For ease, we continue to assume that greater investment is costly; our basic results, however, extend to more general contexts.

Lemma 4.0 establishes that firms desire risk-increasing (mean preserving) R & D.[15] This result comes from the fact that firm monopoly profits are a nonincreasing and convex function of costs. Thus, an investment which shifts probability weight from intermediate costs to very low costs and to very high costs will increase expected profits for two reasons: (i) since profits are nonincreasing in costs, it increases the chance of winning when winning is the most profitable (low costs) and decreases the chance of winning when winning is less profitable (higher costs), and (ii) since profits are convex in costs, it shifts probability weight to lower costs over a range (low costs) where profits fall steeply in costs and shifts probability weight to higher costs over a range (high costs) where profits are less sensitive to costs.

With this lemma in place, we now characterize the home firm's optimal investment level. Integration by parts indicates that the first and second order conditions for a maximum are given by

$$E\pi_I(I, I^*, r) = -\left[\int_{\underline{c}}^{\bar{c}} F_I(c|I)K_c(c|I^*)\, dc\right] - r = 0$$

and

$$E\pi_{II}(I, I^*, r) = -\left[\int_{\underline{c}}^{\bar{c}} F_{II}(c|I)K_c(c|I^*)\, dc\right] < 0.$$

We assume that a solution to the first order condition holds (the term in square brackets is negative for every I according to Lemma 4.0), so that a maximum is obtained if the second order condition is met. The next lemma verifies that the second order condition holds.

LEMMA 4.1. *For all I, I^*, and r, $E\pi_{II}(I, I^*, r) < 0$.*

PROOF. Suppose, first that Assumption E(1) holds, so that the distribution of costs is symmetric. Then using (9) and (10), we have that

$$E\pi_{II}(I, I^*, r) = -\int_{\underline{c}}^{\bar{c}} F_{II}(c|I)\left[\int_{c}^{\bar{c}} \pi'_m(c)f^*(c^*|I^*)\, dc^*\right] dc$$
$$+ \int_{\underline{c}}^{\hat{c}} F_{II}(c|I)f^*(c|I^*)\pi_m(c)\, dc + \int_{\hat{c}}^{\bar{c}} F_{II}(c|I)f^*(c|I^*)\pi_m(c)\, dc$$

where $\hat{c}$ is again the mean of c. Now, according to Assumptions C and E(1), $\tilde{c} = \hat{c}$ and $F_{II}(c|I)$ is negative for c below $\hat{c}$ and positive for c above $\hat{c}$. Also, since the mean is fixed $\int_{\underline{c}}^{\bar{c}} F_{II}(c|I)\, dc = 0$. Thus, since $[\int_{c}^{\bar{c}} \pi'_m(c)f^*(c^*|I^*)\, dc^*]$ is negative and increasing (toward zero) in c, the first term in the expression above is negative. Moreover, with a symmetric distribution, $f^*(c|I^*)$ is symmetric and $F_{II}(c|I)$ is

[15] Given Lemma 4.0, firms will never invest in risk-reducing R & D, and so we ignore that possibility in this paper.

antisymmetric about $\hat{c}$, so that the second and third terms above would sum to zero absent $\pi_m(c)$ which, since it is positive and decreasing in c, turns the sum of these two terms negative.

Finally, if E(2) holds instead, then differentiation of (9) gives that $K_{cc}(c|I^*)$ is nonnegative. With $K_c(c|I^*)$ negative and increasing in c for all $c \in [\underline{c}, \bar{c}]$, the term $[\int_{\underline{c}}^{\bar{c}} F_{II}(c|I)K_c(c|I^*)\, dc]$ will be positive, so that by (10) $E\pi_{II}(I, I^*, r) < 0$.

Q.E.D.

Thus, reaction functions are well-defined and a domestic R & D subsidy will raise domestic investment ($\hat{I}_r(r, r^*) < 0$). The next lemma shows that expected domestic profits are decreasing in foreign investment.

LEMMA 4.2. *For all I, I^*, and r, $E\pi_{I^*}(I, I^*, r) < 0$.*

PROOF. Using (11), (12) and (13), we have

$$E\pi_{I^*}(I, I^*, r) = -\int_{\underline{c}}^{\bar{c}} F_{I^*}^*(c|I^*)f(c|I)\pi_m(c)\, dc$$
$$= -\left[\int_{\underline{c}}^{\hat{c}} F_{I^*}^*(c|I^*)f(c|I)\pi_m(c)\, dc + \int_{\hat{c}}^{\bar{c}} F_{I^*}^*(c|I^*)f(c|I)\pi_m(c)\, dc\right].$$

Suppose first that Assumption E(1) holds so that the distribution of costs is symmetric. Then $\hat{c}$ is the mean of c, and absent $\pi_m(c)$ the two terms above would sum to zero. Since $F_{I^*}^*(c|I^*)$ is positive below $\hat{c}$ and negative above it by Assumption C, and since $\pi_m(c)$ is decreasing in c, $E\pi_{I^*}(I, I^*, r) < 0$ in this case. Alternatively, if Assumption E(2) holds, then $E\pi_{I^*}(I, I^*, r) < 0$, since $f(c|I)\pi_m(c)$ is a positive and decreasing function of c. Q.E.D.

Thus, the home country earns greater expected profits the lower is foreign investment. This is because, with lower foreign investment, the distribution of foreign costs is less risky. As a result, the domestic firm wins less often when winning doesn't mean much (when its own costs are high) and wins more often when winning means a great deal (when its own costs are low). As such, expected domestic profits rise as foreign investment falls.

Finally, we turn to the question of whether or not reaction curves are negatively sloped. The next lemma establishes that they are, at least starting from symmetric investment levels.

LEMMA 4.3. *For all I, I^*, and r, if $I = I^*$, then $E\pi_{I^*I}(I, I^*, r) < 0$.*

PROOF. Using (16) and imposing the condition that $I = I^*$ gives

$$E\pi_{I^*I}(I, I^*, r) = -\int_{\underline{c}}^{\bar{c}} F_I(c|I) f_I(c|I) \pi_m(c)\, dc.$$

Note also that

$$\int_{\underline{c}}^{\hat{c}} F_I(c|I) f_I(c|I)\, dc = \int_{F_I(\underline{c}|I)}^{F_I(\hat{c}|I)} F_I\, dF_I = 0,$$

$$\int_{\hat{c}}^{\bar{c}} F_I(c|I) f_I(c|I)\, dc = \int_{F_I(\hat{c}|I)}^{F_I(\bar{c}|I)} F_I\, dF_I = 0.$$

Finally, we rewrite $E\pi_{I^*I}(I, I^*, r)$ in four parts as

$$E\pi_{I^*I}(I, I^*, r) = -\left\{\left[\int_{\underline{c}}^{c_1} F_I(c|I) f_I(c|I) \pi_m(c)\, dc + \int_{c_1}^{\hat{c}} F_I(c|I) f_I(c|I) \pi_m(c)\, dc\right] + \left[\int_{\hat{c}}^{c_2} F_I(c|I) f_I(c|I) \pi_m(c)\, dc + \int_{c_2}^{\bar{c}} F_I(c|I) f_I(c|I) \pi_m(c)\, dc\right]\right\}.$$

To sign this expression, note that Assumption D implies

$$F_I(c|I) f_I(c|I) \begin{cases} \geq 0 & \text{for } c \in [\underline{c}, c_1] \cup [\hat{c}, c_2] \\ \leq 0 & \text{for } c \in [c_1, \hat{c}] \cup [c_2, \bar{c}] \end{cases}.$$

Using this and the fact that $\pi_m(c)$ is decreasing in c implies

$$E\pi_{I^*I}(I, I^*, r) < -\left[\pi_m(c_1) \int_{\underline{c}}^{\hat{c}} F_I(c|I) f_I(c|I)\, dc + \pi_m(c_2) \int_{\hat{c}}^{\bar{c}} F_I(c|I) f_I(c|I)\, dc\right] = 0.$$

Q.E.D.

The intuition for this result is analogous to that for Lemma 3.3, applied over the regions $c \in [\underline{c}, \hat{c}]$ and $c \in [\hat{c}, \bar{c}]$. Thus, around any symmetric equilibrium with $I = I^*$, reaction curves are negatively sloped. Consequently, starting from a position of symmetric equilibrium investment levels, $\hat{I}_r^*(r, r^*) > 0$ and a home subsidy lowers foreign investment.

3. *The Policy Stage.* With the lemmas of the previous subsection, each proposition in Section 3.3 holds. Thus, when investment yields a mean preserving increase in the riskiness of the distribution of costs as depicted above, (i) a slight subsidy increases that country's welfare and reduces welfare in the rival country, (ii) in any symmetric interior Nash equilibrium, both countries subsidize, and (iii) welfare of each exporting country is higher under free trade than in the symmetric interior Nash equilibrium.

5. GREATER NUMBER OF FIRMS: STRATEGIC AND CORRECTIVE ISSUES

In the previous sections, there is no incentive for corrective policy, since each country has but a single firm and thus no externalities within the country. All that remains is the possibility of a strategic policy, which results in a subsidy as shown in Proposition 2. Suppose now that a country has more than one firm entering the R & D competition. Certainly, there remains a role for strategic subsidies. As before, by committing its firms to greater investment, a country's welfare is improved by the associated reduction of investment in the rival country. On the other hand, there is now a role for a corrective policy as well. A negative externality to R & D arises among the firms of a country, as each firm's marginal investment lowers the probability that all other firms—including those in the same country—will win the R & D competition. This negative externality within a country will tend to induce excessive R & D in any one country. A corrective role for an R & D tax is thus provided.

To explore the tradeoff between strategic and corrective incentives, we suppose that there are H home firms and F foreign firms, and allow R & D investment to affect either the mean or the riskiness of the cost distribution. Defining domestic welfare as

$$W(r, r^*) \equiv \sum_{i=1}^{H} E\pi^i(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r) - (\bar{r} - r) \sum_{i=1}^{H} \hat{I}^i(r, r^*)$$

where $\hat{I}(r, r^*) \equiv \hat{I}^1(r, r^*) \cdots \hat{I}^H(r, r^*)$; $\hat{I}^*(r, r^*) \equiv \hat{I}^{*1}(r, r^*) \cdots \hat{I}^{*F}(r, r^*)$, we differentiate $W(r, r^*)$ with respect to r and impose symmetry among domestic and among foreign firms to obtain

$$W_r(r, r^*) = H \cdot \{F \cdot E\pi^i_{I^{*j}}(\cdot)\hat{I}^{*j}_r(r, r^*) + (H - 1)E\pi^i_{Ij}(\cdot)\hat{I}^j_r(r, r^*) - (\bar{r} - r)\hat{I}^i_r(r, r^*)\}.$$

Restricting our analysis to small subsidies starting from free trade, so that $r = r^* = \bar{r}$, we then have

$$W_r(r, r^*) = H \cdot E\pi^i_{Ij}(\cdot) \cdot \{F \cdot \hat{I}^{*j}_r(r, r^*) + (H - 1) \cdot \hat{I}^j_r(r, r^*)\}.$$

In analogy with our preceding analysis, it can be shown that $E\pi^i_{Ij}(\cdot) < 0$, $\hat{I}^j_r(r, r^*) < 0$ and $\hat{I}^{*j}_r(r, r^*) > 0$ provided that the second order and stability conditions are met. Thus, starting from free trade,

(17) $$\text{sign}\,(W_r(r, r^*)) = -\text{sign}\,(F \cdot \hat{I}_r^{*j}(r, r^*) + (H-1) \cdot \hat{I}_r^j(r, r^*)).$$

Hence, in the presence of more than one domestic firm, the sign of $W_r(r, r^*)$ is in general indeterminant. The term $F \cdot \hat{I}_r^{*j}(r, r^*)$ is positive, and captures the strategic rent-shifting effect of a change in r; a higher domestic interest rate increases foreign investment $(\hat{I}_r^{j*}(r, r^*) > 0)$ thereby shifting expected rents toward the foreign country and lowering domestic welfare. However, the term $(H-1)\hat{I}_r^j(r, r^*)$ is negative, and captures the impact of a change in r on the national externality problem; with more than one domestic firm $(H > 1)$, a higher domestic interest rate reduces domestic investment $(\hat{I}_r^j(r, r^*) < 0)$ thereby mitigating against excessive investment and thus increasing domestic welfare. Hence, whether a country's welfare improves with a small R & D tax or a small R & D subsidy depends in general on the relative importance of these two effects.[16]

To establish conditions under which $W_r(r, r^*)$ is negative and thus a small R & D subsidy starting from free trade improves national welfare, we use (17) and note that $W_r(r, r^*) < 0$ whenever

(18) $$-\left(\frac{\hat{I}_r^{*j}(r, r^*)}{\hat{I}_r^j(r, r^*)}\right) > \frac{H-1}{F}.$$

We find that (18) reduces to

(19) $$R > 1/[1 + F/(H-1)],$$

where $R \equiv (E\pi^i_{IiIj}(\cdot)/E\pi^i_{IiIi}(\cdot))$ denotes the negative of the slope of the investment reaction curve of a representative firm, starting from free trade. Since all (foreign and domestic) firms are symmetric starting from free trade, R is invariant to the *ratio* of foreign to domestic firms. Thus, holding the total number $(H + F)$ of firms fixed, condition (19) is more likely to be satisfied, and thus a small R & D subsidy is more likely to improve national welfare starting at free trade, the greater the number of foreign relative to home firms. Moreover, for any fixed ratio of foreign to domestic firms, condition (19) is more likely to be met the steeper are investment reaction curves, which suggests that a smaller total (foreign plus domestic) number of firms will also increase the likelihood of national welfare improvements from small R & D subsidies.[17]

6. COMPARISON WITH THE PATENT LITERATURE

We now emphasize that the model developed above can be reinterpreted as a patent race. To see this, note that (5) can be written as

$$E\pi(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} f(c|I) \int_{c}^{\bar{c}} f^*(c^*|I^*)\pi_m(c)\, dc^*\, dc - rI.$$

[16] This indeterminacy is reminiscent of Dixit (1988) who raises similar concerns in a competitive (free entry) setting in the context of a patent race.

[17] We have been unable to verify this last result at a general level. However, the result does hold for the exponential case if the hazard function is not too concave and $\pi_m(c)$ is exponential in c.

A simple change of variables gives

$$E\pi(I, I^*, r) = \int_0^{\bar{t}} f(t|I) \int_t^{\bar{t}} f^*(t^*|I^*)\pi_m(t)\, dt^*\, dt - rI.$$

In this latter formulation, an investment outlay at time zero induces a distribution over dates for the discovery of an innovation. A firm wishes to discover the innovation first, because the first discovery can be patented. Finally, $\pi_m(t)$ is the monopoly profit associated with the discovery. Assuming the market demand is invariant through time, $\pi_m(t) = e^{-rt}\pi_m$ where π_m is the flow profit from the innovation. To see that the two formulations are equivalent, recall that the only assumptions we placed on $\pi_m(c)$ were $\pi_m(c) > 0$, $\pi'_m(c) < 0$, and $\pi''_m(c) > 0$. Clearly, these assumptions are also satisfied by $\pi_m(t)$.

Patent race models in which investment reduces the mean time to discovery were first studied by Loury (1979) and Dasgupta and Stiglitz (1980). These authors were primarily interested in whether the optimal amount of investment occurs. Consequently, while the negative externality between firms and the corresponding corrective issues that we discuss have also been identified in this literature, our analysis of investment reaction functions and the associated strategic issues is novel.[18]

Bhattacharya and Mookherjee (1986), Dasgupta and Maskin (1987), Dasgupta and Stiglitz (1980), and Klette and De Meza (1986) have considered the possibility that investment affects the riskiness of discovery dates in patent race models. Under distributional assumptions similar to our own, this literature has identified a tendency for firms to invest more in risk than is collectively optimal, due to the negative externality that one firm's investment has on the expected profit of other firms. Previous work does not, however, analyze investment reaction functions and the corresponding strategic issues.

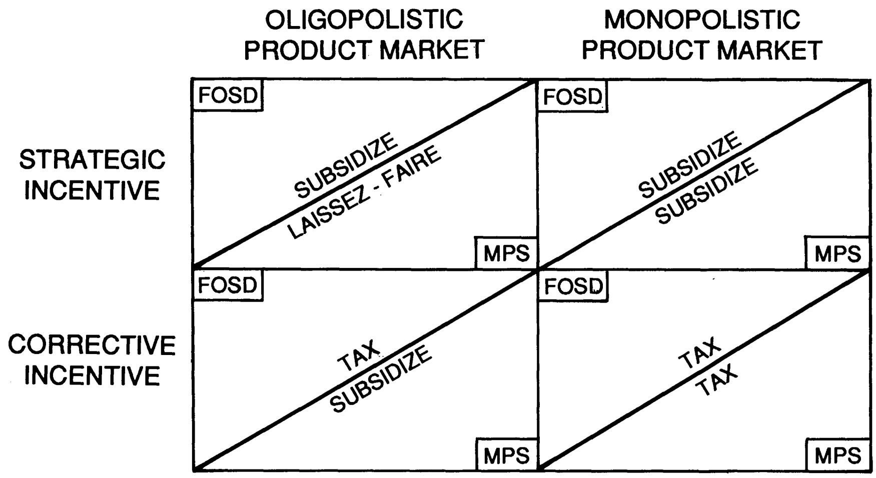
Thus, our basic results can be interpreted in terms of a patent race. This interpretation requires some qualification, however. As shown by Lee and Wilde (1980), if the costs of investment are not completely borne (or committed to) at time zero, then reaction curves may slope upward, in which case in our setting strategic *taxes* would occur.[19] In the context of patent races, our results thus are certain to apply only if investment costs are invariant with respect to the realized date of discovery.

7. CONCLUSION

We have analyzed the desirability of R & D subsidies for domestic firms involved in "winner-take-all" international rivalries. Our goal has been to examine the robustness of the conclusion drawn by Spencer and Brander (1983) that an R & D subsidy can play a positive strategic role. We have found that the strategic role

[18] We note that this previous work used an exponential (constant hazard rate) density function. The exponential function is a particular distribution that satisfies our assumptions.

[19] See Dixit (1988) for a recent model that also makes this point and which attempts to synthesize the strategic patent literature.



FOSD = First Order Stochastic Dominance
MPS = Mean Preserving Spread

FIGURE 4

played by an R & D subsidy in this setting remains attractive under several alternative specifications of the way uncertainty enters the R & D process.

This result contrasts sharply with our findings in Bagwell and Staiger (1989a). There, scale economies are sufficiently small that all competitors in the R & D stage also choose to enter and compete as oligopolists at the production stage. The way in which uncertainty enters the R & D process then turns out to be a crucial determinant of the nature of appropriate R & D intervention.[20] Together, as illustrated in Figure 4, the two papers provide a structural basis from which to determine optimal R & D policy. A strategic R & D subsidy is likely to be attractive in markets in which scale economies are sufficiently large that firms battle for the eventual monopoly position, whether R & D affects the riskiness or the mean of the cost distribution. Similarly, in oligopolistic product markets with associated smaller scale economies in which R & D is thought to lower expected costs without substantially increasing the riskiness of the distribution, a strategic R & D subsidy is likely to be appropriate. In either of these cases, a corrective incentive to tax R & D emerges if the number of domestic firms undertaking R & D exceeds one. On balance, an R & D subsidy is more likely to be preferred the smaller the number of domestic firms. However, if the product market is oligopolistic and R & D affects the riskiness of the cost distribution, then an R & D subsidy offers no strategic

[20] The difference between the two papers arises from the fact that, in the setting here where firms battle for the monopoly position, firms care only about rival costs in so far as these costs determine whether or not they "win" (have lowest costs) and not by how much they win. In contrast, when all firms engaged in R & D will also choose to produce as in Bagwell and Staiger (1989a), the amount by which rival costs differ from own costs becomes important. In the latter setting, the importance of "winning" is replaced by the curvature properties of the profit function, and the way in which R & D affects the distribution of costs then becomes pivotal.

advantage, but does constitute an attractive corrective policy when there are multiple domestic firms.

In concluding, a final remark is in order concerning our assumption that firms are risk-neutral. Under this assumption, we have shown that there are characteristics of the profit function which guarantee that risk-neutral firms will behave as if they are risk-loving when considering the attractiveness of risk-characteristics of R & D investments in a winner-take-all setting. An assumption that firms are inherently risk-averse would tend to offset the natural tendency toward risk-taking R & D investment in this environment, and could alter our results commensurately.

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