ADVERTISING AND PRICING TO DETER OR ACCOMMODATE ENTRY WHEN DEMAND IS UNKNOWN*

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We consider the advertising and pricing strategies of an incumbent firm who is concerned with deterring or accommodating entry and privately informed as to the level of market demand. Our fundamental result is that a demand-exaggerating distortion occurs: if the incumbent seeks to signal a low (high) demand, then he behaves as if there were complete information but demand were lower (higher) than it is. Pre-entry pricing and demand-enhancing advertising are therefore distorted downward (upward), as a consequence of signaling. Purely dissipative advertising is thus not employed as a signal. Refinements of sequential equilibrium are featured.

1. Introduction

It is natural to assume that an incumbent firm in a market possesses more information about market characteristics than does a firm contemplating entry. In particular, the incumbent may well have some private information concerning the nature of market demand. This information is, of course, potentially valuable to the entrant, since it would enable both a more informed entry decision and, in the event of entry, a more informed choice of post-entry variables. Consequently, the entrant has incentive to monitor the incumbent's pre-entry behavior so as to possibly infer the incumbent's private information. In turn, this inference process can lead the incumbent to distort his pre-entry behavior.

We analyze a particular instance of this theme. Specifically, we examine a two-period game in which a single incumbent initially monopolizes the market. The incumbent is privately informed as to 'size' of demand, which is either high or low. He then chooses his first-period price and advertising levels and earns a corresponding first-period profit. A single entrant observes

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the incumbent's first-period selections, but not the size of demand, and forms some belief as to the likelihood that demand is high. Having formed a belief about the size of demand, the entrant then takes some action. This action, for example, may be a decision not to enter, a decision to enter and produce a particular quantity, or a decision to enter and charge a particular price. We consider various interpretations below.

We initially assume that, regardless of the true nature of demand, the incumbent prefers to be believed to operate in a low-demand market. This would be the case, for instance, if the entrant were less likely to enter a low-demand market than a high-demand market. There is thus an *entry deterrence effect* which gives the incumbent an incentive to represent his market as having low demand. In addition, even if the incumbent perceives entry as a certainty, it may be that the incumbent accommodates entry most profitably when market demand is believed to be low. This *entry accommodation effect* arises, for example, if the post-entry game is a quantity game with incomplete information. In this setting, the incumbent would like the entrant to believe that demand is low, since that belief would lead the entrant to produce a small quantity.[1]

Of course, in such an environment, a low-demand incumbent has incentive to separate from his high-demand counterpart. That is, a low-demand incumbent will tend to choose price-advertising pairs that would be especially unattractive, were demand truly high. Consequently, the low-demand incumbent's choice will typically be distorted as compared to the choice he would make in a complete-information setting.

We find that a continuum of separating equilibria exist under the sequential equilibrium concept [Kreps and Wilson (1982)]. It is the lack of structure placed on disequilibrium beliefs that makes possible multiple equilibria. We thus refine the equilibrium set by placing 'plausibility' requirements on disequilibrium beliefs. Specifically, we do not allow the entrant to believe that the incumbent plays a dominated strategy. This refinement was initially proposed by Milgrom and Roberts (1986) and Moulin (1981). After eliminating dominated strategies, we show that at most one separating equilibrium can exist. Moreover, in this equilibrium, the low-demand incumbent acts as if there were complete information but demand were even lower. That is, the low-demand incumbent picks a price-

[1]An additional entry accommodation feature arises if the entrant chooses an advertising level as well as price or quantity. The incumbent might then have greater incentive to signal a low demand if advertising by the entrant decreased post-entry demand for the incumbent's product, since such a signal would induce the entrant to advertise less. Alternatively, if entrant advertising had a public good flavor and increased demand for both products, the incumbent might have incentive to signal a high demand, thereby encouraging the entrant to advertise more. Since the 'advertising effect' can cut either way, we ignore it below and interpret our results largely in terms of an entrant who chooses either price or quantity and not advertising. Our theorems, however, apply directly to the more general possibilities.

advertising pair that would maximize period-one profit if demand were even lower and there were no threat of entry. The fundamental distortion occurring in the undominated separating equilibrium is thus a demand-reducing distortion.

Interestingly, a related result occurs in pooling equilibria, where the incumbent's price-advertising selection is independent of the true demand state. Here, once we require beliefs to be intuitive as in Cho and Kreps (1987), we find conditions under which all remaining pooling equilibria are characterized by a demand-decreasing distortion, in that the incumbent chooses the complete-information optimal section for some level of demand no higher than the low demand.

In both separating and pooling equilibria, the fundamental distortion is demand-decreasing. It then follows that pre-entry price is distorted downward, demand-enhancing advertising is also distorted downward, and dissipative or wasteful advertising is not employed nor, therefore, distorted. 'Limit' prices and low advertising are used to signal a low demand.

We also consider the possibility that, regardless of the true nature of demand, the incumbent prefers to be believed to operate in a high-demand market. This preference might arise, for example, if the incumbent viewed entry as a certainty and expected to compete in prices with an entrant selling a substitute product. Entry is then accommodated most profitably if the entrant believes demand to be high, since that belief would lead the entrant to charge a high price. There is thus an *entry accommodation effect* that may inspire the incumbent to signal a high demand.

Our results for this possibility are natural extensions of the above reasoning. Again, a continuum of separating equilibria exist. But once dominated strategies are eliminated, there can exist at most one separating equilibrium and in this equilibrium the high-demand incumbent acts as if demand were higher than it truly is and then chooses the corresponding monopoly price and advertising selections. Likewise, conditions are given such that in any intuitive pooling equilibria, the incumbent chooses the complete-information optimal selection for some demand no lower than the high demand. In both separating and pooling equilibria, a demand-increasing distortion occurs. Thus, pre-entry price is distorted upward, demand-enhancing advertising is also distorted upward, and dissipative advertising is not done. High prices and advertising are used to signal a high demand.[2]

We now relate our ideas to previous research. Matthews and Mirman (1983) establish that limit pricing is optimal when an incumbent is attempt-

[2]Our general prediction of complete-information distortions extends to any finite number of signals. The prediction is trivial in the single signal case, since any relevant signal value will generally correspond to a complete-information choice for some type. The arguments, however, are more subtle with more than one signal, where 'most' of the feasible combinations of signal values do not correspond to a complete-information choice for any type.

ing to deter an entrant who is not informed of the state of demand. Their model differs from our's in two fundamental respects. First, they investigate an environment in which the entrant observes a 'noisy' signal of demand. Noise in the signal eliminates the possibility of disequilibrium events. Second, they focus on price as the sole signal, whereas we allow for multiple signals. This extension is important, as it allows us to investigate whether a firm will continue to distort the price signal when other signals are available. Our results are in fact consistent with their's, in that we find that limit pricing is again optimal in an entry deterrence setting, even when other signals are also employed.

Other authors have explored the strategic interaction of existing firms when demand characteristics are privately known. Roberts (1986) shows that an incumbent may 'overproduce' in order to depress price and signal that demand is low, so as to encourage a rival to exit or, at least, produce smaller quantities. Gal-Or (1987) analyzes an analogous model in the context of a Stackleberg leader-follower arrangement. Ramey (1987) considers a model in which the incumbent's price acts as a signal of demand where firms also select capacity. Finally, Riordan (1985) has shown that 'overproduction' may occur when firms can influence a jointly observed signal, even if no firm has private information about demand. An implication of previous work is that, when firms are quantity choosers, the incumbent would prefer the entrant to believe that demand is low. We differ from this research in that we treat the post-entry game quite generally, and focus instead on the employment of multiple signals in the pre-entry period.

The seminal article in this literature is by Milgrom and Roberts (1982), who considered a model in which the incumbent is privately informed about his costs of production. They show that the incumbent's effort to deter entry may lead to a downward distortion in pre-entry pricing, as the incumbent attempts to signal a low cost structure. Bagwell and Ramey (1987) enrich this model to allow for price and advertising signals, and establish a cost-reducing distortion. Interestingly, this paper differs from the present paper as to the distortion in advertising when limit pricing occurs: when cost information is incomplete, limit pricing is accompanied by high advertising, while if demand information is incomplete, limit pricing is accompanied by low advertising. Both Milgrom and Roberts and Bagwell and Ramey assume that the post-entry game has complete information: entry accommodation effects are not considered.[3] The work of Milgrom and Roberts has generated an immense literature on firm interaction when costs are privately

[3]Cho (1986) examines a model in which the incumbent pre-entry price is distorted in an attempt to deter entry and influence the post-entry game. Cho's primary goal is to investigate sufficient conditions for separation, when the incumbent can signal only with price and an incomplete information, post-entry quantity game is played.

known; work of particular interest includes a repeated oligopoly model by Mailath (1987) and a general survey by Roberts (1985).

A large literature has investigated the role of advertising. Bain (1956), Salop (1979), and Schmalensee (1983) have all discussed the direct influence of pre-entry advertising on post-entry profit. By contrast, we assume that the post-entry game is not directly dependent upon pre-entry choices and focus on advertising as a signal. Finally, Nelson (1970, 1974), Kihlstrom and Riordan (1984), and Milgrom and Roberts (1986) have each argued that dissipative advertising can signal product quality to consumers.

The paper is organized in five sections. We begin in section 2 with a description of the model. Next, in section 3, we consider separating equilibria. Section 4 then characterizes pooling equilibria, and section 5 concludes.

2. Model

We consider the following setting. There are two periods and two firms, an incumbent and an entrant. In the first period of the marklet, only the incumbent is active. Knowing whether demand is high or low, the incumbent chooses a first-period price and advertising level. The entrant observes these selections and forms some belief as to the likelihood that demand is high. Based upon this belief, some action is taken by the entrant and a post-entry profit is generated for the incumbent.

The market demand is represented as $\alpha^i + X(P,A)$, where $i=\mathrm{H}$ or L, $\alpha^{\mathrm{H}} > \alpha^{\mathrm{L}}$, and $(P,A) \geqq 0$ denotes the incumbent's first-period price-advertising selection. We assume that $X(P,A)$ is continuous, and we say that demand is high (low) if $i=\mathrm{H}(\mathrm{L})$. The incumbent knows the value of the shift parameter α^i when choosing (P,A). The incumbent's first-period profit can thus be written:

$$\Pi^i(P,A) = (P-c)(\alpha^i + X(P,A)) - A, \qquad i=\mathrm{H},\mathrm{L}.$$

Notice that we assume constant unit costs for convenience. Let (P^i, A^i) give the unique maximizer of $\Pi^i(P,A)$, where $P^i > 0$ and $A^i \geqq 0$.

The entrant initially holds some prior probability assessment on the event that $i=\mathrm{H}$. Let $\rho^0 \in (0,1)$ be the prior probability with which the entrant believes demand to be high. After observing (P,A), the entrant may wish to revise his beliefs. We represent the entrant's posterior belief function as $\rho(P,A)$.

After forming his posterior belief, the entrant takes some second-period action. This action may be entry and a choice of quantity, entry and a choice of price, or no entry, for example. We do not give an explicit representation of the entrant's behavior. We let $\tilde{\Pi}^i(\rho)$ be the second-period profit to an incumbent when demand is actually in state α^i and the entrant begins the second period with belief value ρ. Notice that post-entry profit is not directly dependent upon pre-entry choices. We make the natural assumption that $\Pi^i(P^i, A^i) \geqq \tilde{\Pi}^i(\rho) > 0$, for all $\rho \in [0,1]$ and $i = \mathrm{H}$ or L.

We consider pure-strategy sequential equilibria [Kreps and Wilson (1982)]. The collection $\{(\hat{P}^i, \hat{A}^i)_{i=\mathrm{H,L}};\ \hat{\rho}(P,A)\}$ gives an *equilibrium* if the following conditions are satisfied:

(A) *Optimality for the incumbent*: For $i = \mathrm{H,L}$, $(\hat{P}^i, \hat{A}^i) \in \operatorname{argmax}_{(P,A)} \{\Pi^i(P,A) + \delta\tilde{\Pi}^i(\hat{\rho}(P,A))\}$, where $\delta \in (0,1)$ is the incumbent's discount factor.

(B) *Bayes-consistency of beliefs*: If $(\hat{P}^\mathrm{L}, \hat{A}^\mathrm{L}) = (\hat{P}^\mathrm{H}, \hat{A}^\mathrm{H})$, then $\hat{\rho}(\hat{P}^\mathrm{L}, \hat{A}^\mathrm{L}) = \rho^0$. If $(\hat{P}^\mathrm{L}, \hat{A}^\mathrm{L}) \neq (\hat{P}^\mathrm{H}, \hat{A}^\mathrm{H})$, then $\hat{\rho}(\hat{P}^\mathrm{L}, \hat{A}^\mathrm{L}) = 0$ and $\hat{\rho}(\hat{P}^\mathrm{H}, \hat{A}^\mathrm{H}) = 1$.

When $(\hat{P}^\mathrm{L}, \hat{A}^\mathrm{L}) = (\hat{P}^\mathrm{H}, \hat{A}^\mathrm{H})$, information is pooled and nothing is learned by observing first-period choices. This possibility is called a *pooling equilibrium.* By contrast, the demand states are separated if $(\hat{P}^\mathrm{L}, \hat{A}^\mathrm{L}) \neq (\hat{P}^\mathrm{H}, \hat{A}^\mathrm{H})$. Here, first-period choices signal the type of demand, and a *separating equilibrium* is said to occur. Thus, (B) requires the entrant's beliefs to agree with Bayes' rule along the equilibrium path. Notice, though, that $\hat{\rho}(P,A)$ is unrestricted for (P,A) not played in equilibrium. As we will see below, this freedom in specifying off-the-equilibrium path beliefs leads to a continuum of equilibrium possibilities.

3. Separating equilibria

In this section, we characterize the set of separating equilibria and establish the uniqueness of undominated separating equilibria. We consider two cases. In the first, $\tilde{\Pi}^i(\rho)$ is assumed strictly decreasing. Here, regardless of the true demand state, the incumbent prefers the entrant to believe that demand is low. We next consider the opposite case, where $\tilde{\Pi}^i(\rho)$ is strictly increasing and the incumbent prefers that the market be believed to have a high demand.[4]

$\tilde{\Pi}^i(\rho)$ strictly decreasing

As mentioned above, there exist entry deterrence and entry accommo-

[4] Informal discussion of environments corresponding to these two cases is given in the text below. For a more precise understanding, consider a class of entry accommodation games in which the incumbent chooses a second-period action, a_I, simultaneously with the entrant's second-period choice, a_e. Write the incumbent's corresponding second-period equilibrium profits as $\tilde{\Pi}^i(a_I(\rho), a_e(\rho))$, and use the envelope theorem to get

$$\mathrm{d}\tilde{\Pi}^i(a_I(\rho), a_e(\rho))/\mathrm{d}\rho = \partial\tilde{\Pi}^i(a_I(\rho), a_e(\rho))/\partial a_e(\rho)\ \partial a_e(\rho)/\partial\rho.$$

Thinking of a_e as a price or quantity, it is certainly reasonable to assume a_e increases in ρ. If a_e is a quantity choice, higher a_e depresses price and thus $\tilde{\Pi}^i$, whence $\tilde{\Pi}^i$ decreases in ρ. Reverse conclusions hold if a_e is the price of a substitute good. Simple linear examples are easily constructed to confirm this general reasoning.

dation effects under which the incumbent would prefer the entrant to believe that demand is low. In particular, this preference would arise if the entrant were less likely to enter in a low-demand market and/or if entry were followed by quantity competition in an incomplete information environment. We thus now assume $\tilde{\Pi}^i(\rho)$ is a strictly decreasing function.

We begin with a characterization of the set of separating equilibria. Observe first that $(\hat{P}^{\mathrm{H}}, \hat{A}^{\mathrm{H}}) = (P^{\mathrm{H}}, A^{\mathrm{H}})$ in any separating equilibrium. Otherwise, the high-demand incumbent would earn $\Pi^{\mathrm{H}}(\hat{P}^{\mathrm{H}}, \hat{A}^{\mathrm{H}}) + \delta\tilde{\Pi}^{\mathrm{H}}(1) < \Pi^{\mathrm{H}}(P^{\mathrm{H}}, A^{\mathrm{H}}) + \delta\tilde{\Pi}^{\mathrm{H}}(1) \leqq \Pi^{\mathrm{H}}(P^{\mathrm{H}}, A^{\mathrm{H}}) + \delta\tilde{\Pi}(\hat{\rho}(P^{\mathrm{H}}, A^{\mathrm{H}}))$, which is a contradiction. Intuitively, in a separating equilibrium, all information is revealed and so the weak, high-demand incumbent simply maximizes period-one profit.

For a separating equilibrium to exist, it must also be true that the high-demand incumbent is unwilling to mimic the choice he would make if demand were truly low. That is, $(\hat{P}^{\mathrm{L}}, \hat{A}^{\mathrm{L}})$ must satisfy $\Pi^{\mathrm{H}}(\hat{P}^{\mathrm{L}}, \hat{A}^{\mathrm{L}}) + \delta\tilde{\Pi}^{\mathrm{H}}(0) \leqq \Pi^{\mathrm{H}}(P^{\mathrm{H}}, A^{\mathrm{H}}) + \delta\tilde{\Pi}^{\mathrm{H}}(1)$, or equivalently

$$\Pi^{\mathrm{H}}(\hat{P}^{\mathrm{L}}, \hat{A}^{\mathrm{L}}) \leqq \Pi^{\mathrm{H}}(P^{\mathrm{H}}, A^{\mathrm{H}}) + \delta[\tilde{\Pi}^{\mathrm{H}}(1) - \tilde{\Pi}^{\mathrm{H}}(0)] \equiv \bar{\Pi}^{\mathrm{H}}. \tag{1}$$

We will denote as $\bar{\mathrm{H}}$ the set of (P, A) satisfying $\Pi^{\mathrm{H}}(P, A) \leqq \bar{\Pi}^{\mathrm{H}}$. In addition, a separating equilibrium will exist only if the low-demand incumbent is optimizing in choosing $(\hat{P}^{\mathrm{L}}, \hat{A}^{\mathrm{L}})$. At the very least, it must be true that the low-demand incumbent does better by separating with $(\hat{P}^{\mathrm{L}}, \hat{A}^{\mathrm{L}})$ than by choosing $(P^{\mathrm{L}}, A^{\mathrm{L}})$ and facing the most unfavorable belief $(\hat{\rho} = 1)$. Otherwise, the low-demand incumbent would certainly deviate to $(P^{\mathrm{L}}, A^{\mathrm{L}})$ and generate the (perhaps more favorable) belief $\hat{\rho}(P^{\mathrm{L}}, A^{\mathrm{L}})$. Thus, we require

$$\Pi^{\mathrm{L}}(\hat{P}^{\mathrm{L}}, \hat{A}^{\mathrm{L}}) \geqq \Pi^{\mathrm{L}}(P^{\mathrm{L}}, A^{\mathrm{L}}) + \delta[\tilde{\Pi}^{\mathrm{L}}(1) - \tilde{\Pi}^{\mathrm{L}}(0)] \equiv \underline{\Pi}^{\mathrm{L}}. \tag{2}$$

The set of (P, A) satisfying $\Pi^{\mathrm{L}}(P, A) \geqq \underline{\Pi}^{\mathrm{L}}$ is denoted $\underline{\mathrm{L}}$.

We have established that $(\hat{P}^{\mathrm{H}}, \hat{A}^{\mathrm{H}}) = (P^{\mathrm{H}}, A^{\mathrm{H}})$ and $(\hat{P}^{\mathrm{L}}, \hat{A}^{\mathrm{L}}) \in \bar{H} \cap \underline{\mathrm{L}}$ in any separating equilibrium. We now argue that these conditions are also sufficient for a separating equilibrium. The argument is straightforward. Exploit the arbitrariness of disequilibrium beliefs and set $\hat{\rho}(P, A) = 1$, for all $(P, A) \neq (\hat{P}^{\mathrm{L}}, \hat{A}^{\mathrm{L}})$. Then, since $(\hat{P}^{\mathrm{L}}, \hat{A}^{\mathrm{L}}) \in \bar{\mathrm{H}} \cap \underline{\mathrm{L}}$, it is simple to verify from (1) and (2) that the incumbent is behaving optimally for each possible state demand.

As fig. 1 suggests, the set $\bar{\mathrm{H}} \cap \underline{\mathrm{L}}$ is typically extremely large, and the notion of a separating equilibrium generally offers little predictive power. Since it is the arbitrariness of disequilibrium beliefs that supports so many equilibria, predictive power might be restored with a standard of plausibility for beliefs off-the-equilibrium path. As Kohlberg and Mertens (1986), Milgrom and Roberts (1986) and Moulin (1981) have argued, a minimum standard on

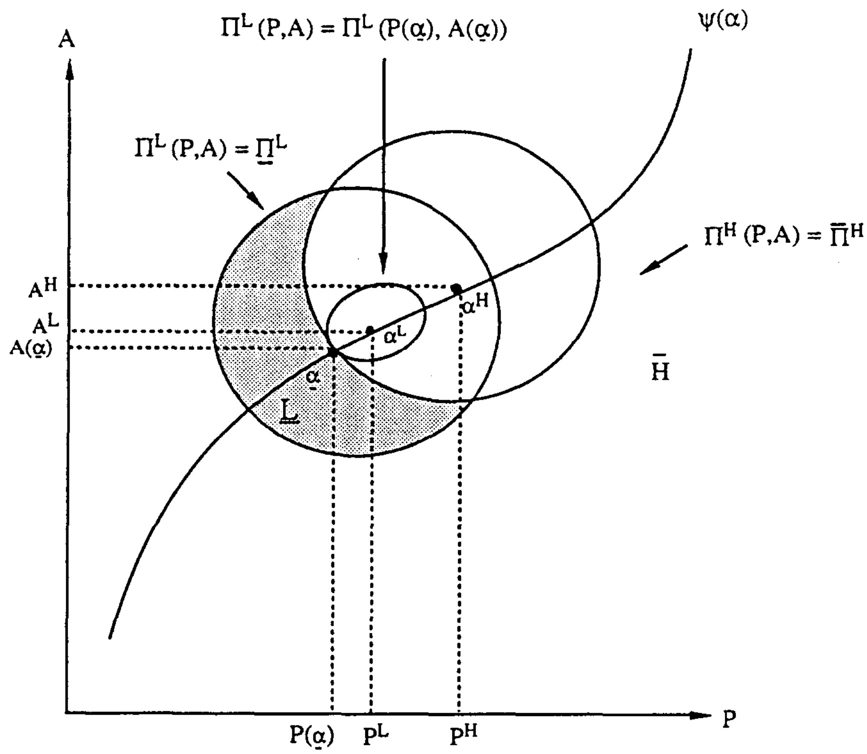

Fig. 1. $\bar{H} \cap \underline{L} \equiv$ shaded set.

beliefs is that the entrant does not believe that strictly dominated strategies are played. We will thus proceed by refining the set of separating equilibria through elimination of dominated strategies.

We say that the pair (P, A) is *dominated when demand is* α^i if

$$\Pi^i(P, A) + \delta\tilde{\Pi}^i(0) < \Pi^i(P^i, A^i) + \delta\tilde{\Pi}^i(1).$$

Thus, (P, A) is dominated when demand is α^i if it yields less profit under the most favorable belief than does (P^i, A^i) under the least favorable belief. An equilibrium will be called *undominated* if $\hat{\rho}(P, A) = 0$ whenever (P, A) is dominated when demand is α^H but not when demand is α^L.

Notice that $\bar{H} \cap \underline{L}$ is the set of (P, A) that are weakly dominated when demand is high and not dominated when demand is low. The definition of an undominated equilibrium allows the low-demand incumbent to choose freely over (P, A) dominated for the high-demand incumbent, without being mistaken for a high-demand incumbent. That is, in an undominated equilibrium, $\hat{\rho}(P,A)=0$ for points in $\bar{\mathrm{H}} \cap \underline{\mathrm{L}}$ such that $\Pi^{\mathrm{H}}(P,A)<\bar{\Pi}^{\mathrm{H}}$. It follows that $(\hat{P}^{\mathrm{L}},\hat{A}^{\mathrm{L}})$ must maximize the low-demand incumbent's pre-entry profit on $\bar{\mathrm{H}}$, since $\hat{\rho}(P,A)=0$ for (P,A) arbitrarily close to the maximizer. Consequently, if $(P^{\mathrm{L}},A^{\mathrm{L}})\in\bar{\mathrm{H}}$, then $(\hat{P}^{\mathrm{L}},\hat{A}^{\mathrm{L}})=(P^{\mathrm{L}},A^{\mathrm{L}})$ in the undominated separating equilibrium. This case is relatively uninteresting as the low-demand incumbent is able to separate at no cost to himself. We thus assume that $(P^{\mathrm{L}},A^{\mathrm{L}})\notin\bar{\mathrm{H}}$.

We seek now to find $(\hat{P}^{\mathrm{L}},\hat{A}^{\mathrm{L}})$ when $(P^{\mathrm{L}},A^{\mathrm{L}})\notin\bar{\mathrm{H}}$. To this end, define for *every* real number α

$$\Pi(P,A\,|\,\alpha)=(P-c)(\alpha+X(P,A))-A.$$

Let $\psi(\alpha)\equiv(P(\alpha),A(\alpha))$ give the price and advertising levels which uniquely maximize $\Pi(P,A\,|\,\alpha)$, for every α. Thus, as we vary α, $\psi(\alpha)$ gives the complete-information, monopoly price-advertising selection. We assume that $\psi(\alpha)$ is continuous. Note that $P(\alpha)=0$ for sufficiently small α. As far as our proofs are concerned, we do not require that $\psi(\alpha)$ be monotonic.[5] However, the natural assumption is that $P(\alpha)$ is strictly increasing, $A(\alpha)$ is strictly increasing when advertising enhances demand, and $A(\alpha)$ is constant at zero when advertising is purely a dissipative signal. For convenience, we assume $\psi(\alpha)$ is strictly monotonic in fig. 1. To ensure that $(\hat{P}^{\mathrm{L}},\hat{A}^{\mathrm{L}})\geqq 0$, we make the technical assumption that $\alpha<\alpha^{\mathrm{H}}$ exists such that $\psi(\alpha)\geqq 0$ and $\Pi(\psi(\alpha)\,|\,\alpha^{\mathrm{H}})<\bar{\Pi}^{\mathrm{H}}$. Thus, as α gets small, the high-demand incumbent eventually makes less than $\bar{\Pi}^{\mathrm{H}}$ by selecting $\psi(\alpha)$.

With this structure established, we prove in the appendix the following theorem:

Theorem 1. Suppose $\tilde{\Pi}^{i}(\rho)$ is strictly decreasing. If $(P^{\mathrm{L}},A^{\mathrm{L}})\notin\bar{\mathrm{H}}$, then there exists at most one undominated separating equilibrium, in which $(\hat{P}^{\mathrm{L}},\hat{A}^{\mathrm{L}})=(P(\underline{\alpha}),A(\underline{\alpha}))$, for some $\underline{\alpha}<\alpha^{\mathrm{L}}$. Moreover, the equilibrium exists if $\tilde{\Pi}^{\mathrm{L}}(0)-\tilde{\Pi}^{\mathrm{L}}(1)\geqq\tilde{\Pi}^{\mathrm{H}}(0)-\tilde{\Pi}^{\mathrm{H}}(1)$.

As shown in fig. 1, the element of $\bar{\mathrm{H}}$ which maximizes $\Pi^{\mathrm{L}}(P,A)$ is the point of intersection of $\psi(\alpha)$ and $\bar{\Pi}^{\mathrm{H}}$, that is, the point $(P(\underline{\alpha}),A(\underline{\alpha}))$. Thus, if an undominated separating equilibrium exists, then it is unique and characterized by a demand-reducing distortion in that the low-demand incumbent acts as if he were maximizing single-period profit and demand were in state $\alpha<\alpha^{\mathrm{L}}$. Furthermore, if the low-demand incumbent has more to gain from

[5] The assumption that $\psi(\alpha)$ exists is actually sufficient to establish that $P(\alpha)$ increases in α; see the proof of Theorem 1.

generating favorable beliefs than does the high-demand incumbent, then the low-demand incumbent has the 'resources' available with which to incur the financial sacrifice that signaling requires. $\tilde{\Pi}^{L}(0)-\tilde{\Pi}^{L}(1)\geqq\tilde{\Pi}^{H}(0)-\tilde{\Pi}^{H}(1)$ is thus a sufficient condition for the existence of the unique undominated separating equilibrium.[6]

Since the above equilibrium is fundamentally characterized by a demand-reducing distortion, it is immediate to see that the low-demand incumbent distorts both pre-entry pricing and pre-entry demand-enhancing advertising downward. If advertising is purely dissipative, it is not employed nor, therefore, distorted. Summarizing, when entry deterrence and accommodation effects are such that the incumbent wants the entrant to believe that demand is low, a demand-decreasing distortion occurs, which results in lower pre-entry prices and lower pre-entry demand-enhancing advertising.

$\tilde{\Pi}^{i}(\rho)$ strictly increasing

It is difficult to imagine that the incumbent would deter more entry if he signaled that demand is high. However, there may be an entry accommodation effect that would lead the incumbent to signal in this fashion. Specifically, if the incumbent expected to compete in prices with an entrant selling a substitute product, he would like the entrant to believe that demand is high, as this belief would encourage the entrant to price high. It follows that $\tilde{\Pi}^{i}(\rho)$ may be strictly increasing if, for example, entry is certain to occur and is followed by price competition between substitute products. We are thus led to investigate the distortions that occur in pre-entry variables when $\tilde{\Pi}^{i}(\rho)$ is strictly increasing.

As before, we begin with the set of separating equilibria. Observe first that $\Pi^{L}(\hat{P}^{L},\hat{A}^{L})+\delta\tilde{\Pi}^{L}(0)<\Pi^{L}(P^{L},A^{L})+\delta\tilde{\Pi}^{L}(\hat{\rho}(P^{L},A^{L}))$, if $(\hat{P}^{L},\hat{A}^{L})\neq(P^{L},A^{L})$. In any separating equilibrium, it must therefore be that $(\hat{P}^{L},\hat{A}^{L})=(P^{L},A^{L})$. We now characterize the set of possible values for $(\hat{P}^{H},\hat{A}^{H})$.

It is useful to define the following two sets:

$$\underline{\mathrm{H}}\equiv\{(P,A)\,|\,\Pi^{H}(P,A)\geqq\Pi^{H}(P^{H},A^{H})+\delta[\tilde{\Pi}^{H}(0)-\tilde{\Pi}^{H}(1)]\equiv\underline{\Pi}^{H}\},$$

$$\bar{\mathrm{L}}\equiv\{P,A)\,|\,\Pi^{L}(P,A)\leqq\Pi^{L}(P^{L},A^{L})+\delta[\tilde{\Pi}^{L}(0)-\tilde{\Pi}^{L}(1)]\equiv\bar{\Pi}^{L}\}.$$

Arguing as before, it is straightforward to establish that $(\hat{P}^{H},\hat{A}^{H})\in\underline{\mathrm{H}}\cap\bar{\mathrm{L}}$ in any separating equilibrium. Furthermore, using the belief $\hat{\rho}(P,A)=0$ for all

[6]Graphically, the assumption $\tilde{\Pi}^{L}(0)-\tilde{\Pi}^{L}(1)\geqq\tilde{\Pi}^{H}(0)-\tilde{\Pi}^{H}(1)$ entails the level curve $\Pi^{L}(P,A)=\underline{\Pi}^{L}$ completely surrounding the level curve $\Pi^{H}(P,A)=\bar{\Pi}^{H}$. That is, under this assumption, if $\bar{\Pi}^{H}(P,A)=\bar{\Pi}^{H}$, then $(P,A)\in\underline{\mathrm{L}}$. The set of separating equilibria is thus immense. Yet there exists but one undominated separating equilibrium.

$(P,A)\neq(\hat{P}^{H},\hat{A}^{H})$, the existence of a separating equilibrium with $(\hat{P}^{L},\hat{A}^{L})=(P^{L},A^{L})$ and $(\hat{P}^{H},\hat{A}^{H})\in \underline{H}\cap\bar{L}$ is easily established.

Once more, an embarrassingly large set of separating equilibria generally exists. These equilibria are supported by excessively unfavorable disequilibrium beliefs. In particular, the entrant is allowed to believe that a price-advertising pair comes from a low-demand incumbent, even if the low-demand incumbent would *never* have incentive to select such a pair.

Accordingly, we proceed as above and eliminate dominated strategies. A pair (P,A) is now said to be *dominated when demand is* α^{i} if

$$\Pi^{i}(P,A)+\delta\tilde{\Pi}^{i}(1)<\Pi^{i}(P^{i},A^{i})+\delta\tilde{\Pi}^{i}(0).$$

Thus, (P,A) is dominated when demand is α^{i} if it can never generate more profit than can be guaranteed by setting $(\hat{P}^{i},\hat{A}^{i})=(P^{i},A^{i})$. An equilibrium will now be said to be *undominated* if $\hat{\rho}(P,A)=1$ whenever (P,A) is dominated when demand is α^{L} but not when demand is α^{H}.

The set $\underline{H}\cap\bar{L}$ is now seen to be the set of (P,A) that are weakly dominated when demand is low and not dominated when demand is high. The definition of undominated equilibrium gives the high-demand incumbent freedom in choosing over (P,A) strictly in $\bar{L}$. Consequently, the high-demand incumbent chooses that $(P,A)\in\bar{L}$ that maximizes his period-one profit. The interesting case has $(P^{H},A^{H})\notin\bar{L}$.

In order to ensure that $(\hat{P}^{H},\hat{A}^{H})$ is bounded, we make the technical assumption that $\alpha>\alpha^{L}$ exists such that $\Pi(\psi(\alpha)\mid\alpha^{L})<\bar{\Pi}^{L}$.[7] We now have the following theorem, proved in the appendix:

Theorem 2. Suppose $\tilde{\Pi}^{i}(\rho)$ is strictly increasing. If $(P^{H},A^{H})\notin\bar{L}$, then there exists at most one undominated separating equilibrium, in which $(\hat{P}^{H},\hat{A}^{H})=(P(\bar{\alpha}),A(\bar{\alpha}))$, for some $\bar{\alpha}>\alpha^{H}$. Moreover, the equilibrium exists if $\tilde{\Pi}^{H}(1)-\tilde{\Pi}^{H}(0)\geqq\tilde{\Pi}^{L}(1)-\tilde{\Pi}^{L}(0)$.

As fig. 2 illustrates, $\Pi^{H}(P,A)$ is maximized on $\bar{L}$ at the point of intersection between $\psi(\alpha)$ and $\bar{\Pi}^{L}$, represented as $(P(\bar{\alpha}),A(\bar{\alpha}))$. Thus, in the unique undominated separating equilibrium, a demand-increasing distortion occurs in that the high-demand incumbent acts as if demand were commonly known to be even higher than it truly is. Notice that this equilibrium exists if the high-demand incumbent gains at least as much as the low-demand incumbent from inducing the most favorable belief.

We can now easily assess the distortion occurring in pre-entry variables when $\tilde{\Pi}^{i}(\rho)$ is strictly increasing. To wit, pre-entry price is distorted upward, demand-enhancing advertising is also distorted upward, and dissipative

[7]This assumption would be met, for example, if there were a reservation price associated with the function $X(P,A)$.

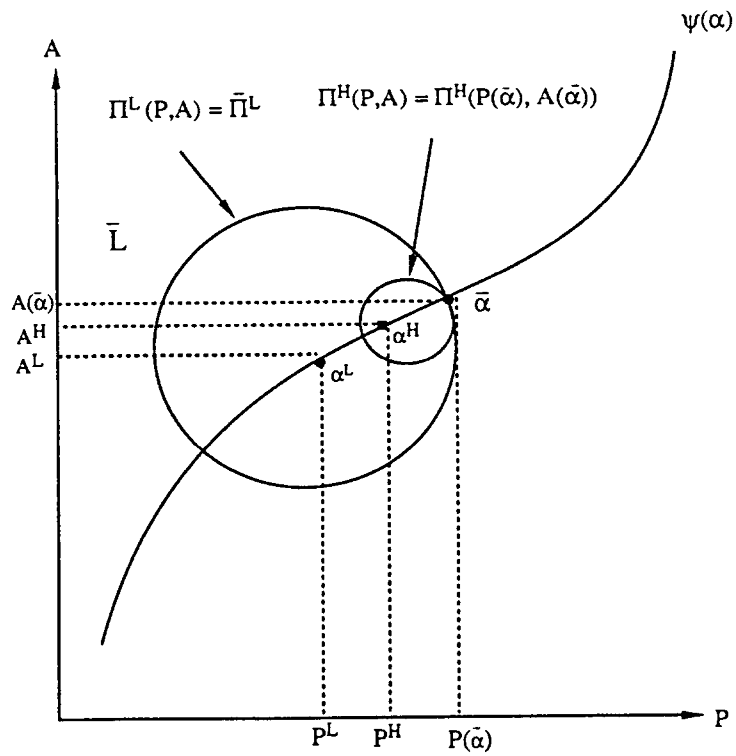

Fig. 2

advertising is not employed. High prices and high advertising precede entry when it is advantageous to suggest high demand.

4. Pooling equilibria

We have yet to consider the possibility of pooling, where pre-entry behavior is not informative to the entrant. In this section, we characterize and refine the set of pooling equilibria, when $\tilde{\Pi}^i(\rho)$ is strictly decreasing and when $\tilde{\Pi}^i(\rho)$ is strictly increasing.

$\tilde{\Pi}^i(\rho)$ strictly decreasing

We assume now that entry deterrence and accommodation effects are such that the incumbent prefers the entrant to believe that demand is low. The set of pooling strategies is easily established as the set of (P, A) satisfying

$$\Pi^H(P, A) \geqq \Pi^H(P^H, A^H) + \delta[\tilde{\Pi}^H(1) - \tilde{\Pi}^H(\rho^0)], \tag{3}$$

$$\Pi^{L}(P,A) \geqq \Pi^{L}(P^{L},A^{L}) + \delta[\tilde{\Pi}^{L}(1) - \tilde{\Pi}^{L}(\rho^{0})]. \tag{4}$$

In words, the set of (P, A) charged in pooling equilibria is exactly the set of (P, A) such that both the high-and-low-demand incumbent prefers pooling at (P, A) to deviating to the respective monopoly price-advertising pair and being believed to be in a high-demand market. Certainly, this set will typically be very large. The arbitrariness with which disequilibrium beliefs can be specified removes predictive content from the pooling equilibrium concept.

Elimination of dominated strategies is also insufficient to give predictive power to the pooling possibility. This is easily understood. When dominated strategies are removed, the low-demand incumbent can deviate to (P, A) in $\bar{\mathrm{H}}$ and be correctly perceived. Yet there is typically a large range of pooling possibilities not in $\bar{\mathrm{H}}$ which are unaffected by belief changes for, perhaps, 'far away' points in $\bar{\mathrm{H}}$. Thus, while elimination of dominated strategies generally removes many pooling equilibria, the remaining set of pooling equilibria is unlikely to be characterized by distortions in any particular direction.

We therefore refine the equilibrium concept still further. Following Cho and Kreps (1987), we say that equilibrium beliefs are *unintuitive* if there exists $(\tilde{P}, \tilde{A}) \neq (\hat{P}^{L}, \hat{A}^{L}), (\hat{P}^{H}, \hat{A}^{H})$ such that

$$\Pi^{L}(\tilde{P},\tilde{A}) + \delta\tilde{\Pi}^{L}(0) > \Pi^{L}(\hat{P}^{L},\hat{A}^{L}) + \delta\tilde{\Pi}^{L}(\hat{\rho}(\hat{P}^{L},\hat{A}^{L})), \tag{5}$$

$$\Pi^{H}(\tilde{P},\tilde{A}) + \delta\tilde{\Pi}^{H}(0) < \Pi^{H}(\hat{P}^{H},\hat{A}^{H}) + \delta\tilde{\Pi}^{H}(\hat{\rho}(\hat{P}^{H},\hat{A}^{H})). \tag{6}$$

To understand this 'intuitive criterion', suppose an equilibrium exists with the low-and-high demand incumbent's profit given by the right hand sides of (5) and (6), respectively. Consider now the possibility of a deviant selection $(\tilde{P}, \tilde{A})$ satisfying (5) and (6). By (5), this selection could increase the low-demand incumbent's profit, *if* the selection were correctly perceived as being his. By (6), the $(\tilde{P}, \tilde{A})$ selection could never increase the high-demand incumbent's profit, as he makes less with $(\tilde{P}, \tilde{A})$ than in equilibrium *even* when $\hat{\rho}(\tilde{P}, \tilde{A}) = 0$. The 'intuitive' inference is then that the deviant selection was made by the low-demand incumbent. But, by (5), the equilibrium would then fail. Accordingly, we say that an equilibrium is *intuitive* if there does not exist a disequilibrium $(\tilde{P}, \tilde{A})$ satisfying (5) and (6). Thus, an equilibrium is intuitive if it can be supported by beliefs that are not unintuitive.

Before stating our next theorem, we note that pooling cannot occur at (P, A) giving profits near $\tilde{\Pi}^{H}$, since a high-demand incumbent will not venture as far away from (P^{H}, A^{H}) to pool as he will to induce the belief $\hat{\rho}(P, A) = 0$. In fact, pooling cannot occur at any (P, A) such that $\Pi^{H}(P, A) < \Pi^{H}(P^{H}, A^{H}) + \delta[\tilde{\Pi}^{H}(1) - \tilde{\Pi}^{H}(\rho^{0})]$. As shown in the appendix, there exists at most one $\underline{\alpha}^P < \alpha^L$ such that $\Pi^H(P(\underline{\alpha}^P), A(\underline{\alpha}^P)) \equiv \Pi^H(P^H, A^H) + \delta[\tilde{\Pi}^H(1) - \tilde{\Pi}^H(\rho^0)]$. We now have:

Theorem 3. Suppose $\tilde{\Pi}^i(\rho)$ is strictly decreasing. If $\tilde{\Pi}^L(0) - \tilde{\Pi}^L(\rho^0) \geqq \tilde{\Pi}^H(0) - \tilde{\Pi}^H(\rho^0)$, then in any intuitive pooling equilibrium with strategies $(\hat{P}, \hat{A})$, $(\hat{P}, \hat{A}) = \psi(\hat{\alpha})$ for some $\hat{\alpha} \in [\underline{\alpha}^P, \alpha^L]$.

Theorem 3 is proved in the appendix. It establishes that, when $\tilde{\Pi}^i(\rho)$ is strictly decreasing, a demand-decreasing distirtion also occurs in intuitive pooling equilibria. To understand the sufficient condition $\tilde{\Pi}^L(0) - \tilde{\Pi}^L(\rho^0) \geqq \tilde{\Pi}^H(0) - \tilde{\Pi}^H(\rho^0)$, imagine a pooling equilibrium with $(\hat{P}, \hat{A}) \notin \psi(\alpha)$. It is easy to locate $(\tilde{P}, \tilde{A}) \in \psi(\alpha)$, such that the low-demand incumbent finds the deviation more profitable in the first period than does his high-demand counterpart. For the equilibrium to be broken, it is thus sufficient to assume that the low-demand incumbent gains no less in the second period from inducing favorable beliefs than does his high-demand counterpart. $\tilde{\Pi}^L(0) - \tilde{\Pi}^L(\rho^0) \geqq \tilde{\Pi}^H(0) - \tilde{\Pi}^H(\rho^0)$ therefore guarantees that all intuitive pooling equilibria fall on $\psi(\alpha)$. A related argument establishes $\hat{\alpha} \leqq \alpha^L$.

Intuitive pooling equilibria need not be unique. Indeed, if $\tilde{\Pi}^L(0) - \tilde{\Pi}^L(\rho^0) = \tilde{\Pi}^H(0) - \tilde{\Pi}^H(\rho^0)$, then every $\hat{\alpha} \in [\underline{\alpha}^P, \alpha^L]$ corresponds to an intuitive pooling equilibrium with strategies $(\hat{P}, \hat{A}) = \psi(\hat{\alpha})$.[8] Fig. 3 represents the set of possible intuitive pooling equilibria for the general case.

To summarize, when $\tilde{\Pi}^i(\rho)$ is strictly decreasing, intuitive pooling equilibria are characterized by demand-decreasing distortions. That is, the incumbent acts as if demand were lower than it truly is and then chooses the corresponding monopoly price-advertising selection. Pre-entry pricing and pre-entry demand-enhancing advertising are thus distorted downward, as the incumbent attempts to suggest a low demand. Dissipative advertising is not employed.

$\tilde{\Pi}^i(\rho)$ strictly increasing

We now return to the possibility that $\tilde{\Pi}^i(\rho)$ is strictly increasing. Recall that this assumption would be appropriate, for example, if entry were certain and followed by a price game with incomplete information.

The set of pooling equilibria is again large and easily decribed. Specifically, the set of pooling equilibria is simply the set of (P, A) satisfying

$$\Pi^H(P, A) \geqq \Pi^H(P^H, A^H) + \delta[\tilde{\Pi}^H(0) - \tilde{\Pi}^H(\rho^0)], \tag{7}$$

[8] When $\tilde{\Pi}^L(0) - \tilde{\Pi}^L(\rho^0) > \tilde{\Pi}^H(0) - \tilde{\Pi}^H(\rho^0)$, some points on $\psi(\alpha)$ between $\psi(\underline{\alpha}^P)$ and $\psi(\alpha^L)$ may fail the intuitive criterion, as the low-demand incumbent may be willing to move 'down' the $\psi(\alpha)$ curve, which is costly in the first period, in order to get a relatively large post-entry gain from the corresponding change in beliefs.

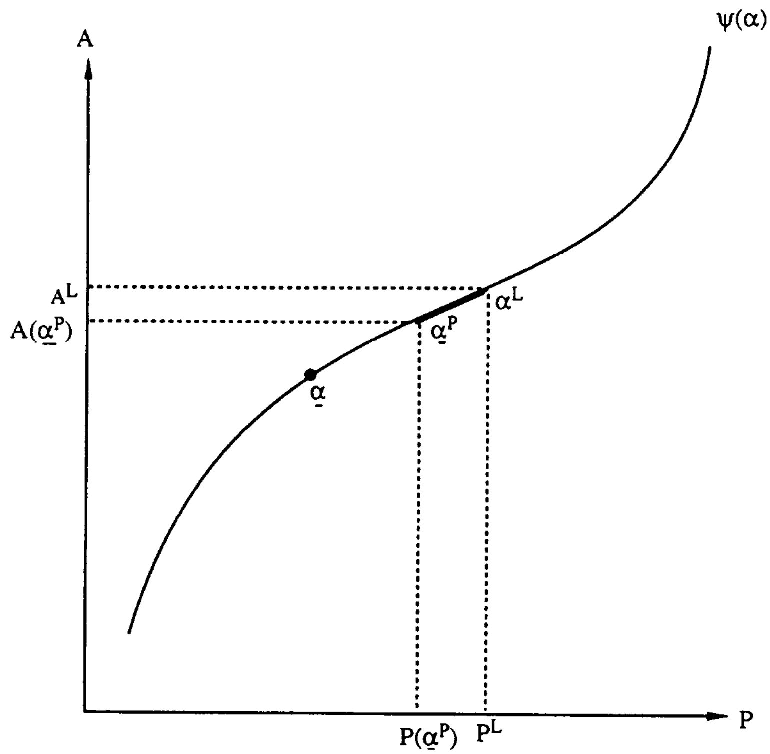

Fig. 3. Set of possible pooling strategies ≡ shaded set.

$$\Pi^{L}(P, A) \geqq \Pi^{L}(P^{L}, A^{L}) + \delta[\tilde{\Pi}^{L}(0) - \tilde{\Pi}^{L}(\rho^{0})]. \tag{8}$$

These equilibria are supported by the unfavorable belief that disequilibrium choices are made only by the low-demand incumbent. Since such beliefs may well be implausible, we define the Cho and Kreps intuitive criterion for the game when $\tilde{\Pi}^{i}(\rho)$ is strictly increasing. To wit, we say that equilibrium beliefs are *unintuitive* if there exists $(\tilde{P}, \tilde{A}) \neq (\hat{P}^{L}, \hat{A}^{L}), (\hat{P}^{H}, \hat{A}^{H})$ such that

$$\Pi^{L}(\tilde{P}, \tilde{A}) + \delta\tilde{\Pi}^{L}(1) < \Pi^{L}(\hat{P}^{L}, \hat{A}^{L}) + \delta\tilde{\Pi}^{L}(\hat{\rho}(\hat{P}^{L}, \hat{A}^{L})), \tag{9}$$

$$\Pi^{H}(\tilde{P}, \tilde{A}) + \delta\tilde{\Pi}^{H}(1) > \Pi^{H}(\hat{P}^{H}, \hat{A}^{H}) + \delta\tilde{\Pi}^{H}(\hat{\rho}(\hat{P}^{H}, \hat{A}^{H})). \tag{10}$$

Under these conditions, only the high-demand incumbent has incentive to deviate to $(\tilde{P}, \tilde{A})$, so the intuitive belief is $\hat{\rho}(\tilde{P}, \tilde{A}) = 1$. But, by (10), the pooling equilibrium would then fail. We now say that an equilibrium is *intuitive* if a disequilibrium $(\tilde{P}, \tilde{A})$ satisfying (9) and (10) does not exist.

Note that pooling cannot occur at (P,A) giving profits near $\bar{\Pi}^{L}$, as the low-demand incumbent will go further from (P^{L},A^{L}) to induce the belief $\hat{\rho}(P,A)=1$ then the belief $\hat{\rho}(P,A)=\rho^{0}$. In fact, pooling will not occur at any (P,A) such that $\Pi^{L}(P,A)<\Pi^{L}(P^{L},A^{L})+\delta[\tilde{\Pi}^{L}(0)-\tilde{\Pi}^{L}(\rho^{0})]$. In the appendix, we show that at most one $\bar{\alpha}^{P}>\alpha^{H}$ can exist such that $\Pi^{L}(P(\bar{\alpha}^{P}),A(\bar{\alpha}^{P}))\equiv \Pi^{L}(P^{L},A^{L}))+\delta[\tilde{\Pi}^{L}(0)-\tilde{\Pi}^{L}(\rho^{0})]$. With this definition, we have:

Theorem 4. Suppose $\tilde{\Pi}^{i}(\rho)$ is strictly increasing. If $\tilde{\Pi}^{H}(1)-\tilde{\Pi}^{H}(\rho^{0})\geqq \tilde{\Pi}^{L}(1)-\tilde{\Pi}^{L}(\rho^{0})$, then in any intuitive pooling equilibrium with strategies $(\hat{P},\hat{A})$, $(\hat{P},\hat{A})=\psi(\hat{\alpha})$ for some $\hat{\alpha}\in[\alpha^{H},\bar{\alpha}^{P}]$.

Theorem 4, which is proved in the appendix, can be interpreted analogously to Theorem 3. Note in particular that a demand-increasing distortion occurs in intuitive pooling equilibria when $\tilde{\Pi}^{i}(\rho)$ is strictly increasing and that intuitive pooling equilibria are not unique.[9]

Thus, if $\tilde{\Pi}^{i}(\rho)$ is strictly increasing, then in any intuitive pooling equilibrium, the incumbent acts as if demand is commonly known to be higher than it truly is and then chooses the corresponding monopoly price-advertising selection. Pre-entry prices and pre-entry advertising are each distorted upward, as shown in fig. 4. Dissipative advertising does not occur.

5. Conclusion

When an incumbent possesses private information about the size of demand, he realizes that his pre-entry behavior may affect the probability of entry as well as the nature of post-entry play. Our general thesis is that the entrant's effort to infer the incumbent's information leads the incumbent to behave as if there were complete information but he were 'stronger' than he actually is. Thus, if the prospects of a low demand deters some entry and/or a post-entry quantity game follows entry, the incumbent is strongest when the entrant believes demand to be low. The incumbent then engages in a demand-decreasing distortion, reflected in lower pre-entry prices and lower pre-entry demand-enhancing advertising. Alternatively, if entry is certain (or at least very likely) to occur and a post-entry price game follows entry, then the incumbent is in the strongest position when demand is believed to be high. A demand-increasing distortion then occurs, with pre-entry price and pre-entry demand-enhancing advertising distorted upward. The general thesis of strength-increasing distortions occurs in both separating and pooling

[9]Again, if $\tilde{\Pi}^{H}(1)-\tilde{\Pi}^{H}(\rho^{0})=\tilde{\Pi}^{L}(1)-\tilde{\Pi}^{L}(\rho^{0})$, then every $\hat{\alpha}\in[\alpha^{H},\bar{\alpha}^{P}]$ corresponds to an intuitive pooling equilibrium. However, if $\tilde{\Pi}^{H}(1)-\tilde{\Pi}^{H}(\rho^{0})>\tilde{\Pi}^{L}(1)-\tilde{\Pi}^{L}(\rho^{0})$, some of these equilibria may fail to be intuitive, as the high-demand incumbent moves 'up' the $\psi(\alpha)$ curve, in order to receive a relatively large post-entry gain from the associated alteration in beliefs. See note 8 also.

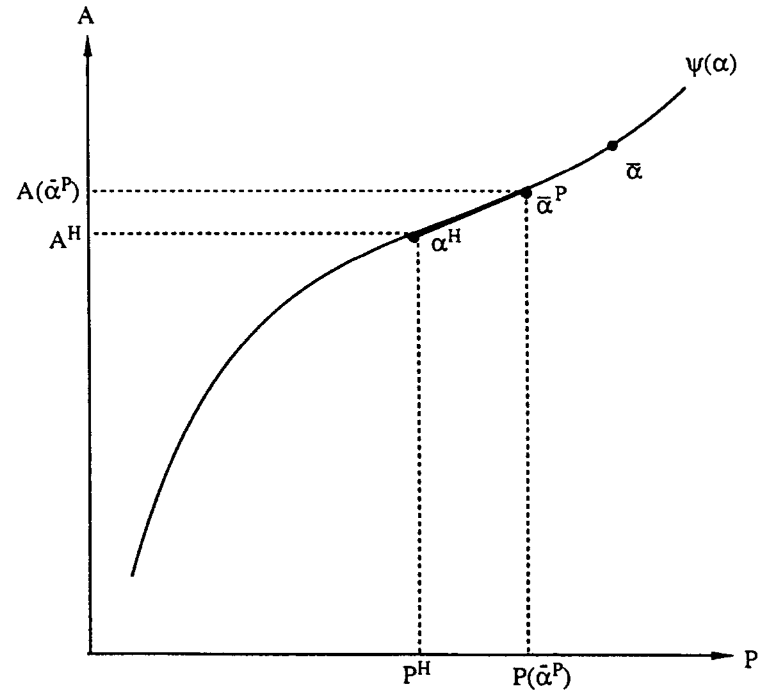

Fig. 4. Set of possible pooling strategies ≡ shaded set.

equilibria, once the equilibrium set is refined.[10] An immediate corollary is that dissipative advertising is never undertaken.

Appendix

Proof of Theorem 1. First, we establish that a unique $\underline{\alpha}<\alpha^{H}$ exists at which $\Pi(\psi(\underline{\alpha})\,|\,\alpha^{H})=\bar{\Pi}^{H}$, with $P(\underline{\alpha})>0$ and $A(\underline{\alpha})\geqq 0$. Since $\Pi(\psi(\alpha^{H})\,|\,\alpha^{H})>\bar{\Pi}^{H}>0$, $\Pi(\psi(\alpha)\,|\,\alpha^{H})$ is continuous in α, and by assumption $\alpha<\alpha^{H}$ exists at which $\psi(\alpha)\geqq 0$ and $\Pi(\psi(\alpha)\,|\,\alpha^{H})<\bar{\Pi}^{H}$, it is sufficient to show that $\Pi(\psi(\alpha)\,|\,\alpha^{H})$ is strictly increasing in α for $\alpha<\alpha^{H}$.

To this end, pick $\alpha_1<\alpha_2<\alpha^{H}$ and let $(P_1,A_1)\equiv(P(\alpha_1),A(\alpha_1))$ and $(P_2,A_2)\equiv(P(\alpha_2),A(\alpha_2))$. Then

$$(P_1-c)(\alpha_1+X(P_1,A_1))-A_1-(P_2-c)(\alpha_1+X(P_2,A_2))+A_2>0,$$

$$(P_2-c)(\alpha_2+X(P_2,A_2))-A_2-(P_1-c)(\alpha_2+X(P_1,A_1))+A_1>0. \quad \text{(A.1)}$$

[10] Pooling equilibria exist over a small range, or even fail to exist, when ρ^0 is low (high) and $\bar{\Pi}^i(\rho)$ is increasing (decreasing), since each type of incumbent prefers then to deviate to its single-period optimum. In general, however, both pooling and separating equilibria exist satisfying the refinements we study.

Adding inequalities gives $(P_1 - P_2)(\alpha_1 - \alpha_2) > 0$, so that $P_1 < P_2$ follows necessarily. Using (A.1):

$$(P_2 - c)(\alpha^H + X(P_2, A_2)) - A_2 - (P_1 - c)(\alpha^H + X(P_1, A_1)) + A_1 > 0,$$

which proves monotonicity and establishes the existence of the unique intersection point $(P(\underline{\alpha}), A(\underline{\alpha}))$. Clearly, a related argument shows that $\Pi(\psi(\alpha) \mid \alpha^H)$ is strictly decreasing in α for $\alpha > \alpha^H$. Also, analogous monotonicity properties hold for $\Pi(\psi(\alpha) \mid \alpha^L)$. Note that $\underline{\alpha} < \alpha^L$ since $\Pi(\psi(\alpha^L) \mid \alpha^H) > \bar{\Pi}^H$.

Second, we argue that $\Pi^H(\hat{P}^L, \hat{A}^L) = \bar{\Pi}^H$. Suppose not. Let $(\underline{P}, \underline{A}) \equiv (P(\underline{\alpha}), A(\underline{\alpha}))$. Then

$$(\underline{P} - c)(\alpha^H + X(\underline{P}, \underline{A})) - \underline{A} - (\hat{P}^L - c)(\alpha^H + X(\hat{P}^L, \hat{A}^L)) + \hat{A}^L > 0,$$

$$(\hat{P}^L - c)(\alpha^L + X(\hat{P}^L, \hat{A}^L)) - \hat{A}^L - (\underline{P} - c)(\alpha^L + X(\underline{P}, \underline{A})) + \underline{A} \geqq 0, \tag{A.2}$$

since $(\hat{P}^L, \hat{A}^L)$ maximizes Π^L on $\bar{\mathrm{H}}$. Adding gives $(\underline{P} - \hat{P}^L)(\alpha^H - \alpha^L) > 0$, whence $\underline{P} > \hat{P}^L$. But, using (A.2) and $\underline{\alpha} < \alpha^L$,

$$(\hat{P}^L - c)(\underline{\alpha} + X(\hat{P}^L, \hat{A}^L)) - \hat{A}^L - (\underline{P} - c)(\underline{\alpha} + X(\underline{P}, \underline{A})) + \underline{A} > 0,$$

which is a contradiction.

The necessary portion of the proof can now be completed by showing that $(\underline{P}, \underline{A})$ uniquely maximizes $\Pi^L(P, A)$ over (P, A) such that $\Pi^H(P, A) = \bar{\Pi}^H$. Pick any $(P, A) \neq (\underline{P}, \underline{A})$ such that $\Pi^H(P, A) = \bar{\Pi}^H$. Then

$$(\underline{P} - c)(\underline{\alpha} + X(\underline{P}, \underline{A})) - \underline{A} - (P - c)(\underline{\alpha} + X(P, A)) + A > 0,$$

$$(P - c)(\alpha^H + X(P, A)) - A - (\underline{P} - c)(\alpha^H + X(\underline{P}, \underline{A})) + \underline{A} = 0. \tag{A.3}$$

Adding gives $(\underline{P} - P)(\underline{\alpha} - \alpha^H) > 0$, whence $\underline{P} < P$. Using (A.3)

$$(\underline{P} - c)(\alpha^L + X(\underline{P}, \underline{A})) - \underline{A} - (P - c)(\alpha^L + X(P, A)) + A$$
$$= (P - \underline{P})(\alpha^H - \alpha^L) > 0.$$

Note that this also proves $\underline{P} < P$ for all $(P, A) \notin \bar{\mathrm{H}}$.

We now establish existence of the equilibrium when $\tilde{\Pi}^L(0) - \tilde{\Pi}^L(1) \geqq \tilde{\Pi}^H(0) - \tilde{\Pi}^H(1)$. From above, $(\underline{P}, \underline{A}) \in \bar{\mathrm{H}}$. We next show $(\underline{P}, \underline{A}) \in \underline{\mathrm{L}}$. Observe

$$\Pi^L(\underline{P}, \underline{A}) + \delta\tilde{\Pi}^L(0) - \Pi^L(P^L, A^L) - \delta\tilde{\Pi}^L(1)$$
$$= \Pi^L(\underline{P}, \underline{A}) + \delta\tilde{\Pi}^L(0) - \Pi^L(P^L, A^L) - \delta\tilde{\Pi}^L(1)$$

$$-\Pi^{H}(\underline{P},\underline{A})-\delta\tilde{\Pi}^{H}(0)+\Pi^{H}(P^{H},A^{H})+\delta\tilde{\Pi}^{H}(1)$$

$$=[\Pi^{L}(\underline{P},\underline{A})-\Pi^{H}(\underline{P},\underline{A})]-[\Pi^{L}(P^{L},A^{L})-\Pi^{H}(P^{H},A^{H})]$$

$$+\delta[\tilde{\Pi}^{L}(0)-\tilde{\Pi}^{L}(1)-\tilde{\Pi}^{H}(0)+\tilde{\Pi}^{H}(1)]$$

$$\geqq[\Pi^{L}(\underline{P},\underline{A})-\Pi^{H}(\underline{P},\underline{A})]-[\Pi^{L}(P^{L},A^{L})-\Pi^{H}(P^{H},A^{H})]$$

$$>[\Pi^{L}(\underline{P},\underline{A})-\Pi^{H}(\underline{P},\underline{A})]-[\Pi^{L}(P^{L},A^{L})-\Pi^{H}(P^{L},A^{L})]$$

$$=(\underline{P}-P^{L})(\underline{\alpha}-\alpha^{L}).$$

Since $(P^{L},A^{L})\notin\bar{\text{H}}$, we know by monotonicity that $\underline{P}<P^{L}$. Thus, the above expression is positive and $(\underline{P},\underline{A})\in\underline{\text{L}}$. We can therefore support $\{(\hat{P}^{L},\hat{A}^{L})=(\underline{P},\underline{A}),\ (\hat{P}^{H},\hat{A}^{H})=(P^{H},A^{H})\}$ as a separating equilibrium. Since $(\underline{P},\underline{A})$ maximizes $\Pi^{L}(P,A)$ over $\bar{\text{H}}$, the separating equilibrium is undominated.

Proof of Theorem 2. Using the assumptions that $(P^{H},A^{H})\notin\bar{\text{L}}$ and $\alpha>\alpha^{L}$ exists such that $\Pi(\psi(\alpha)\,|\,\alpha^{L})<\bar{\Pi}^{L}$, familiar arguments establish a unique $\bar{\alpha}>\alpha^{H}$ such that $\Pi(\psi(\bar{\alpha})\,|\,\alpha^{L})=\bar{\Pi}^{L}$. The theorem can then be proved with arguments similar to those above.

Proof of Theorem 3. Suppose to the contrary that $(\hat{P},\hat{A})\notin\psi(\mathbb{R})$ gives pooling strategies. For the incumbent to participate when demand is high, it must be that $\Pi^{H}(\hat{P},\hat{A})\geqq\Pi^{H}(P^{H},A^{H})+\delta[\tilde{\Pi}^{H}(1)-\tilde{\Pi}^{H}(\rho^{0})]$. Now since $\Pi(\psi(\alpha^{H})\,|\,\alpha^{H})>\Pi^{H}(\hat{P},\hat{A})+\delta[\tilde{\Pi}^{H}(\rho^{0})-\tilde{\Pi}^{H}(0)]\geqq\bar{\Pi}^{H}$, under our assumptions there must exist $\hat{\alpha}<\alpha^{H}$ such that $P(\hat{\alpha})>0$, $A(\hat{\alpha})\geqq0$, and $\Pi(\psi(\hat{\alpha})\,|\,\alpha^{H})=\Pi^{H}(\hat{P},\hat{A})+\delta[\tilde{\Pi}(\rho^{0})-\tilde{\Pi}^{H}(0)]$. We thus have

$$(P(\hat{\alpha})-c)(\alpha^{H}+X(P(\hat{\alpha}),A(\hat{\alpha})))-A(\hat{\alpha})-(\hat{P}-c)(\alpha^{H}+X(\hat{P},\hat{A})) \tag{A.4}$$

$$+\hat{A}-\delta[\tilde{\Pi}^{H}(\rho^{0})-\tilde{\Pi}^{H}(0)]=0,$$

$$(\hat{P}-c)(\hat{\alpha}+X(\hat{P},\hat{A}))-\hat{A}-(P(\hat{\alpha})-c)(\hat{\alpha}+X(P(\hat{\alpha}),A(\hat{\alpha}))+A(\hat{\alpha})<0$$

Adding

$$(P(\hat{\alpha})-\hat{P})(\alpha^{H}-\hat{\alpha})-\delta[\tilde{\Pi}^{H}(\rho^{0})-\tilde{\Pi}^{H}(0)]<0,$$

implying $P(\hat{\alpha})<\hat{P}$. From (A.4),

$$(P(\hat{\alpha})-c)(\alpha^{L}+X(P(\hat{\alpha}),A(\hat{\alpha})))-A(\hat{\alpha})-(\hat{P}-c)(\alpha^{L}+X(\hat{P},\hat{A}))$$

$$+\hat{A}-\delta[\tilde{\Pi}^{H}(\rho^{0})-\tilde{\Pi}^{H}(0)]>0.$$

Since $\delta[\tilde{\Pi}^{\mathrm{L}}(0)-\tilde{\Pi}^{\mathrm{L}}(\rho^0)]-\delta[\tilde{\Pi}^{\mathrm{H}}(0)-\tilde{\Pi}^{\mathrm{H}}(\rho^0)]\geqq 0$, by assumption, we can add this quantity in to get

$$(P(\hat{\alpha})-c)(\alpha^{\mathrm{L}}+X(P(\hat{\alpha}),A(\hat{\alpha})))-A(\hat{\alpha})+\delta\tilde{\Pi}^{\mathrm{L}}(0)$$
$$>(\hat{P}-c)(\alpha^{\mathrm{L}}+X(\hat{P},\hat{A}))-\hat{A}+\delta\tilde{\Pi}^{\mathrm{L}}(\rho^0).$$

Using continuity and monotonicity of $\Pi^i(P,A)$ on $\psi(\alpha)$, there exists $(\tilde{P},\tilde{A})$ near $(P(\hat{\alpha}),A(\hat{\alpha}))$ satisfying (5) and (6). Hence, $(\hat{P},\hat{A})\in\psi(\mathbb{R})$.

Now suppose $(\hat{P},\hat{A})=(P(\hat{\alpha}),A(\hat{\alpha}))$ for $\hat{\alpha}>\alpha^{\mathrm{L}}$. Since $\Pi^{\mathrm{L}}(\hat{P},\hat{A})+\delta[\tilde{\Pi}^{\mathrm{L}}(\rho^0)-\tilde{\Pi}^{\mathrm{L}}(0)]\leqq\Pi^{\mathrm{L}}(P^{\mathrm{L}},A^{\mathrm{L}})$ the fact that $P(\alpha)=0$ for sufficiently small α, together with the continuity of the functions, guarantees the existence of $\alpha'<\alpha^{\mathrm{L}}$ such that $P(\alpha')>0$, $A(\alpha')\geqq 0$, and $\Pi^{\mathrm{L}}(P(\alpha'),A(\alpha'))=\Pi^{\mathrm{L}}(\hat{P},\hat{A})+\delta[\tilde{\Pi}^{\mathrm{L}}(\rho^0)-\tilde{\Pi}^{\mathrm{L}}(0)]$. We thus have

$$(\hat{P}-c)(\alpha^{\mathrm{L}}+X(\hat{P},\hat{A}))-\hat{A}+\delta\tilde{\Pi}^{\mathrm{L}}(\rho^0)-(P(\alpha')-c)(\alpha^{\mathrm{L}}+X(P(\alpha'),A(\alpha')))$$
$$+A(\alpha')-\delta\tilde{\Pi}^{\mathrm{L}}(0)=0, \tag{A.5}$$

$$(P(\alpha')-c)(\alpha'+X(P(\alpha'),A(\alpha')))-A(\alpha')-(\hat{P}-c)(\alpha'+X(\hat{P},\hat{A}))+\hat{A}>0.$$

Adding gives

$$(\hat{P}-P(\alpha'))(\alpha^{\mathrm{L}}-\alpha')+\delta[\tilde{\Pi}^{\mathrm{L}}(\rho^0)-\tilde{\Pi}^{\mathrm{L}}(0)]>0,$$

whence $\hat{P}>P(\alpha')$. By (A.5)

$$(\hat{P}-c)(\alpha^{\mathrm{H}}+X(\hat{P},\hat{A}))-\hat{A}+\delta\tilde{\Pi}^{\mathrm{L}}(\rho^0)-(P(\alpha')-c)(\alpha^{\mathrm{H}}+X(P(\alpha'),A(\alpha')))$$
$$+A(\alpha')-\delta\tilde{\Pi}^{\mathrm{L}}(0)>0.$$

Adding $\delta[\tilde{\Pi}^{\mathrm{L}}(0)-\tilde{\Pi}^{\mathrm{L}}(\rho^0)]-\delta[\tilde{\Pi}^{\mathrm{H}}(0)-\tilde{\Pi}^{\mathrm{H}}(\rho^0)]>0$ in gives

$$(\hat{P}-c)(\alpha^{\mathrm{H}}+X(\hat{P},\hat{A}))-\hat{A}+\delta\tilde{\Pi}^{\mathrm{H}}(\rho^0)>(P(\alpha')-c)(\alpha^{\mathrm{H}}+X(P(\alpha'),A(\alpha')))$$
$$-A(\alpha')+\delta\tilde{\Pi}^{\mathrm{H}}(0).$$

Arguing as before, we have that $(P(\hat{\alpha}),A(\hat{\alpha}))$, with $\hat{\alpha}>\alpha^{\mathrm{L}}$, cannot be intuitive pooling equilibrium strategies. It follows that $(\hat{P},\hat{A})=\psi(\hat{\alpha})$, for some $\hat{\alpha}\in[\underline{\alpha}^{\mathrm{P}},\alpha^{\mathrm{L}}]$ in any intuitive pooling equilibrium.

Proof of Theorem 4. $(\hat{P},\hat{A})\notin\psi(\mathbb{R})$ is impossible, as one can argue as above to find $\hat{\alpha}>\alpha^{\mathrm{L}}$ with $\Pi(\psi(\hat{\alpha})\,|\,\alpha^{\mathrm{L}})=\Pi^{\mathrm{L}}(\hat{P},\hat{A})+\delta[\tilde{\Pi}^{\mathrm{L}}(\rho^0)-\tilde{\Pi}^{\mathrm{L}}(1)]$, leading to a violation of (9) and (10). Next, $(\hat{P}, \hat{A}) = (P(\hat{\alpha}), A(\hat{\alpha}))$ with $\hat{\alpha} < \alpha^H$ is impossible by a related argument.

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