

# The sensitivity of strategic and corrective R&D policy in oligopolistic industries

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We evaluate the case for R&D subsidies in export sectors when the outcome of R&D is uncertain and the product market is oligopolistic. When R&D reduces expected costs in the particular sense of first-order stochastic dominance, a national strategic basis for R&D subsidies exists, whether firms choose prices or quantities. This must be balanced against a corrective incentive to tax R&D whenever the number of domestic firms exceeds one. However, when R&D increases the riskiness of the cost distribution as well, the results may switch: there emerges a strategic incentive to tax and a corrective incentive to subsidize R&D.

## 1. Introduction

Recent work in international trade theory has established a strategic role for trade policy in oligopolistic industries. Brander and Spencer (1985) and Spencer and Brander (1983) have shown that domestic export and R&D subsidies can enable domestic firms to commit to 'aggressive' output and investment strategies, inducing a 'soft' response from foreign firms and thereby shifting profits from the foreign to the domestic country.

A number of well-known criticisms have been leveled against the position

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that exports should be subsidized, among them that the case for such subsidies is sensitive both to whether firms choose prices or quantities and to the number of domestic firms in the industry.<sup>1</sup> However, there has been comparatively little analysis of the possible limitations of an R&D subsidy policy.<sup>2</sup> Certainly, an understanding of the robustness of the R&D subsidy logic is important, as there are many countries that do subsidize the R&D of domestic firms, especially those involved in international markets.<sup>3</sup> Moreover, with the exception of certain primary products, developed countries are explicitly forbidden to subsidize exports under the Subsidies Code of the General Agreement on Tariffs and Trade (GATT), a ban that does not extend to R&D subsidies. For these reasons, we analyze here the extent to which the desirability of an R&D subsidy depends upon the nature of product market competition, the number of domestic and foreign firms, and the existence and form of uncertainty associated with R&D investments.

The R&D model developed by Spencer and Brander (1983) has two exporting countries, each with one firm, and a single importing country. Product market competition in quantities occurs after R&D leads in a deterministic fashion to lower production costs. In this setting, the appropriate strategic policy is an R&D subsidy.

A potentially crucial element not captured by the Spencer and Brander (1983) analysis is the inherent uncertainty associated with R&D. This uncertainty could concern the potential results of the R&D project at the time of investment, with different levels of investment yielding different distributions over R&D outcomes. Alternatively, if the production process is itself *ex ante* uncertain, and is associated *ex ante* with a distribution of costs, then investment might simply affect this distribution directly. The former interpretation would reflect most closely R&D investments to develop a new cost technology. The latter interpretation would reflect investments that alter the distributional properties of an existing technology. In either case, the stochastic properties of the investment could take on a number of plausible forms.

We begin with the natural stochastic analogue to the Spencer and Brander (1983) deterministic model, and assume that R&D lowers the mean of a firm's cost distribution in the particular sense of a first-order stochastic shift. The nature of optimal R&D policy then centers around the resolution of two questions: (1) Does greater R&D activity by one firm diminish the expected profits of other firms; that is, is there a negative externality associated with R&D? (2) Are investment reaction curves negatively or positively sloped?

<sup>1</sup>See, in particular, Dixit (1984) and Eaton and Grossman (1986).

<sup>2</sup>Exceptions include our companion paper, Bagwell and Staiger (1992), as well as Cheng (1987) and Dixit (1988).

<sup>3</sup>See, for example, the discussion in Hufbauer and Erb (1984, esp. pp. 103–104), and also in Komiya et al. (1988, esp. chs. 7 and 8).

A key conclusion of the paper is that the resolution of these questions – and thus the nature of appropriate R&D policy – is independent of the form of product market competition. In fact, employing standard models of product market competition we find that a negative externality is associated with investment and that investment reaction curves are negatively sloped, whether firms compete in prices or quantities. Moreover, while the models of product market competition that we consider exhibit linear demands, we argue that a presumption in favor of a negative investment externality and a negatively-sloped investment reaction curve extends well beyond the linear demand setting that typifies the standard models.

With these resolutions in place, it is straightforward to demonstrate that the optimal strategic policy involves an R&D subsidy. Specifically, with a single firm in each country, a strategic domestic R&D subsidy raises domestic R&D, which then lowers foreign R&D (due to the negatively-sloped investment reaction curve), and this response is in turn beneficial to the domestic country (due to the negative externality received from foreign investment). When there is more than one domestic firm engaged in R&D, however, a negative externality arises also among these domestic firms, and so a corrective incentive emerges for an R&D tax. Our results here thus parallel those developed for the case of an export subsidy: the appropriate R&D policy balances the strategic incentive to subsidize with the corrective incentive to tax, and a subsidy is relatively more attractive the smaller the number of domestic firms. In contrast to the export subsidy literature, however, our conclusions are not sensitive to the form of product market competition.

As noted above, though, the effect of R&D on the distribution of costs could take on any of several plausible forms, of which a first-order stochastic shift is just one. In particular, the assumption of a first-order stochastic shift limits the degree to which the riskiness of the cost distribution can change as a result of R&D investments. Thus, to explore the generality of our results with regard to the riskiness of R&D, we next consider more general distributional shifts, whereby R&D continues to lower a firm's mean cost, but now without imposing any additional restrictions on changes in higher moments of the cost distribution. In this way, we can examine cases where R&D investments reduce expected costs and also involve greater 'risk'.

Regardless of the risk properties of R&D, we continue to find that investment reaction curves are negatively sloped, as long as R&D reduces mean cost. When mean reduction is associated with increasing risk, however, it is no longer true that investment always imparts a negative externality, as the 'risk effect' can overwhelm the 'mean effect'. This can be seen most clearly by considering the limiting case of a 'mean-preserving spread', in which R&D preserves the mean of the cost distribution and simply increases its riskiness. In the standard models of product market competition that we consider, a

firm's profit is convex in its rival's costs, and consequently investment which increases a firm's cost risk imparts a positive externality to rival firms. Extending this logic to more general cases, in which R&D lowers the mean cost and increases the riskiness of the cost distribution, it is direct to identify two opposing effects. The risk effect noted above suggests a positive externality from investment, but this effect must be balanced against the mean effect which, as in the case of the first-order shift, corresponds to a negative externality.

The optimal R&D policy thus depends on the balance of these two effects. When the mean effect dominates and the externality is negative, then the appropriate policy continues to be characterized by strategic subsidies and corrective taxes. However, if the risk effect dominates and the externality is positive, then strategic taxes, which slow domestic R&D and thereby spur foreign R&D, and corrective subsidies are optimal. The mean and risk effects therefore have exactly opposite implications for appropriate R&D policy. We thus conclude that statements as to optimal R&D policy will require a convincing treatment of the role that uncertainty plays in the R&D process.

The remainder of the paper proceeds as follows. Our approach is to determine the nature of optimal R&D intervention as a function of the properties of general firm-level profit functions in the product market stage of competition, and then to assess the sensitivity of R&D intervention by evaluating the properties of these profit functions under specific models of product market competition. Section 2 presents the basic model and discusses at a general level strategic and corrective issues. Section 3 considers appropriate R&D policy when investment in R&D leads to a first-order stochastic shift in the cost distribution, while section 4 considers the case of general mean shifts. Section 5 concludes.

## **2. The basic model**

### *2.1. Basic assumptions*

We consider two exporting countries and a third importing country. For now, we assume each exporting country has a single firm. The exporting countries are referred to as home and foreign countries, respectively, and asterisks will be used to denote foreign country variables.

We examine subgame perfect equilibria of a three-stage game. First, the exporting governments simultaneously choose the (positive) unit costs of investment,  $r$  and  $r^*$ , for their respective firms. Let  $\tilde{r}$  represent the social cost of investment, assumed for simplicity to be constant across countries. A home country subsidy (tax) on investment then occurs if  $r < \tilde{r}$  ( $r > \tilde{r}$ ). Since all consumption takes place in the third country, each exporting country chooses its cost of investment in order to maximize its firm's expected profit less subsidy costs.

Next, having observed the policy choices of both exporting governments, both firms then simultaneously choose non-negative investment levels,  $I$  and  $I^*$ , in stage two. We assume that production costs are randomly determined as a function of investment. Thus,  $f(c|I)$  is the density of possible constant costs  $c$ , given the investment level  $I$ .  $f^*(c^*|I^*)$  is defined analogously. We assume throughout that the two investment technologies are symmetric and well-behaved:  $f(c|I) \equiv f^*(c|I)$ ,  $f(c|I) > 0$ , and  $f(c|I)$  is continuously differentiable in  $c$  and  $I$ , for every  $c \in [\underline{c}, \bar{c}]$ .<sup>4</sup> At this stage, each firm seeks to maximize its expected profit, given its cost of investment.

In the third stage, each firm observes its own realized production cost and that of its rivals, and the firms then compete in the product market (in prices or quantities) with each firm maximizing its profit given the realized pair of production costs.

## 2.2. Strategic issues

We now define national welfare and discuss the general determinants of appropriate R&D policy in a strategic international setting. Home welfare is simply expected profits less subsidy costs, or

$$W(r, r^*) \equiv E\pi(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r) - (\bar{r} - r) \cdot \hat{I}(r, r^*). \quad (1)$$

Here  $E\pi(\cdot)$  gives the expected profits of the home firm (net of investment costs), and  $\hat{I}(\cdot)$  and  $\hat{I}^*(\cdot)$  denote the respective equilibrium domestic and foreign investment levels.  $W^*(r, r^*)$  is defined analogously for the symmetric foreign profit function  $E\pi^*(\cdot)$ . Differentiating (1) with respect to  $r$ , using the envelope theorem, and imposing  $E\pi_r(\cdot) = -\hat{I}(\cdot)$ , we obtain

$$W_r(r, r^*) = E\pi_{r^*}(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r) \cdot \hat{I}^*_r(r, r^*) - (\bar{r} - r) \cdot \hat{I}_r(r, r^*), \quad (2)$$

where subscripts denote partial derivatives. Thus, the effect of a change in  $r$  on domestic welfare is captured by two terms. First, a change in  $r$  will affect foreign investment and thereby domestic profits, and second, a change in  $r$  will also alter domestic investment and thus domestic subsidy payments.

The sign of  $W_r(r, r^*)$  therefore depends upon the signs of  $E\pi_{r^*}(\cdot)$ ,  $\hat{I}^*_r(\cdot)$ , and  $\hat{I}_r(\cdot)$ . The effect of greater foreign investment on home profits is captured by  $E\pi_{r^*}(\cdot)$ , while  $\hat{I}^*_r(\cdot)$  and  $\hat{I}_r(\cdot)$  determine the equilibrium response of foreign and domestic investment, respectively, to a change in  $r$ . To characterize this

<sup>4</sup>If R&D acts to generate a new cost technology, then we may interpret the model as applying to a situation in which  $\underline{c} > 0$  and (i) the firm is developing a new product, for which no previous technology exists, (ii) a previous technology exists but  $\bar{c}$  lies below the unit cost of production under the old technology, or (iii) there are switching costs associated with returning to the previous technology once investment in the new technology has begun. Alternatively, if R&D affects the distributional properties of a given random production technology, then the model has a direct interpretation (provided  $\underline{c} > 0$ ).

response we begin with the first-order conditions,  $E\pi_I(I, I^*, r) = 0$  and  $E\pi_{I^*}^*(I, I^*, r^*) = 0$ , which respectively define investment reaction functions,  $I(I^*, r)$  and  $I^*(I, r^*)$ . We assume the determinant of the Jacobian,  $J$ , associated with the first-order conditions is globally positive:

$$|J| \equiv E\pi_{II}(I, I^*, r) \cdot E\pi_{I^*I^*}^*(I, I^*, r^*) - E\pi_{I^*I}(I, I^*, r) \cdot E\pi_{II^*}^*(I, I^*, r^*) > 0. \quad (3)$$

The reaction curves then have at most one intersection, which we assume exists.  $\hat{I}(r, r^*)$  and  $\hat{I}^*(r, r^*)$  are the solution. Total differentiation of the first-order conditions gives

$$\hat{I}_r(r, r^*) = \frac{E\pi_{I^*I^*}^*(I, I^*, r^*)}{|J|}; \quad \hat{I}^*_r(r, r^*) = \frac{-E\pi_{II^*}^*(I, I^*, r^*)}{|J|}. \quad (4)$$

If second-order conditions hold, a home subsidy increases home investment. A home subsidy decreases (increases) foreign investment if  $E\pi_{I^*I^*}^*(I, I^*, r^*) < 0$  ( $E\pi_{II^*}^*(I, I^*, r) > 0$ ), i.e. if investment reaction curves are negatively (positively) sloped.

Finally, we maintain the assumption that  $W(r, r^*)$  is concave in  $r$  and set  $W_r(r, r^*)$  in (2) equal to zero. Using (4) then yields an expression for the optimal domestic strategic R&D subsidy (if  $\tilde{r} - r > 0$ ) or tax (if  $\tilde{r} - r < 0$ ), for a fixed  $r^*$ :

$$\tilde{r} - r = \frac{-E\pi_{I^*}(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r) \cdot E\pi_{II^*}^*(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r^*)}{E\pi_{I^*I^*}^*(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r^*)}. \quad (5)$$

Thus, provided second-order conditions hold [ $E\pi_{I^*I^*}^*(I, I^*, r^*) < 0$ ], the decision to subsidize or tax R&D in a given environment can be deduced from the knowledge of whether the signs of  $E\pi_{II^*}^*(I, I^*, r)$  and  $E\pi_{I^*I^*}^*(I, I^*, r^*)$  agree or disagree, respectively. Moreover, (5) can be used to characterize the Nash equilibrium between governments, defined by two policy choices,  $\hat{r}$  and  $\hat{r}^*$ , satisfying  $W_r(\hat{r}, \hat{r}^*) = 0$  and  $W_{r^*}^*(\hat{r}, \hat{r}^*) = 0$ . Assuming a Nash equilibrium exists and that second-order conditions hold, (5) implies that both countries subsidize (tax) in the Nash equilibrium if the signs of  $E\pi_{II^*}^*(I, I^*, r)$  and  $E\pi_{I^*I^*}^*(I, I^*, r)$  agree (disagree).

### 2.3. Corrective issues

It is also of interest to know if too little or too much R&D is undertaken relative to some socially optimal level. To this end, let  $r_c = r_c^*$  be the corrective R&D policy that maximizes  $W(r, r^*) + W^*(r, r^*)$  over the set of symmetric policies. Thus,  $r_c$  maximizes the combined welfare of exporters; equivalently,  $r_c$  is the policy that would be optimal if both exporting firms

were domestic. Maintaining the assumption of an interior maximum, the solution must satisfy  $W_r(r_c, r_c) + W_r^*(r_c, r_c) = 0$ .

Direct calculation yields

$$\tilde{r} - r_c = E\pi_{I^*}(\hat{I}(r_c, r_c), \hat{I}^*(r_c, r_c), r_c). \quad (6)$$

Hence, the sign of the symmetric corrective R&D policy will reflect the sign of  $E\pi_{I^*}(\cdot)$ . Free trade is not optimal if  $E\pi_{I^*}(\cdot) \neq 0$ , since then each country's investment imposes an externality on the other's expected profits. R&D policies are then set to counteract the excessive or deficient levels of non-cooperative investment that result.

Having identified the critical role played by the signs of  $E\pi_{I^*}^*(\cdot)$ ,  $E\pi_{I^*}(\cdot)$ , and  $E\pi_{I^*}^*(\cdot)$  in determining the appropriate strategic or corrective R&D policy, we now proceed to analyze the sign of these terms in a variety of settings.

### 3. R&D and first-order shifts

In this section we explore the case where investment in R&D leads to a reduction in the mean of the firm's cost distribution as captured by the notion of first-order stochastic dominance. For the present we limit our discussion to the case of one domestic and one foreign firm. Thus, we first consider the national incentives for strategic R&D policy, and postpone consideration of the interaction between strategic and corrective issues at the national level until later.

#### 3.1. Strategic issues

As discussed in the previous section, the sign of the appropriate strategic R&D policy intervention is determined by the signs of  $E\pi_{I^*}^*(\cdot)$ ,  $E\pi_{I^*}(\cdot)$ , and  $E\pi_{I^*}^*(\cdot)$ , or equivalently, given the symmetry of the expected profit functions,  $E\pi_{II}(\cdot)$ ,  $E\pi_{I^*}(\cdot)$ , and  $E\pi_{I^*}(\cdot)$ . Our approach is to define general foreign and domestic profit functions,  $\pi^*(c, c^*)$  and  $\pi(c, c^*)$ , respectively, and to characterize the link between the properties of these functions and the signs of  $E\pi_{II}(\cdot)$ ,  $E\pi_{I^*}(\cdot)$ , and  $E\pi_{I^*}(\cdot)$  through a series of lemmas. We then evaluate the properties of these general profit functions under specific market conditions, and assess the implications for strategic R&D policy.

##### 3.1.1. Basic assumptions

We let  $\pi(c, c^*)$  represent the home firm's profits (gross of investment costs) in the third stage if home costs are  $c$  and foreign costs are  $c^*$ , and define  $\pi^*(c, c^*)$  as the symmetric function for the foreign firm. We assume that these functions are continuously differentiable and positive functions of  $c$  and  $c^*$ .

We also maintain the assumption that profits are decreasing in own costs, i.e.  $\pi_c(c, c^*) < 0$  and  $\pi_{c^*}(c, c^*) < 0$ , for all  $c \in [\underline{c}, \bar{c}]$  and  $c^* \in [\underline{c}, \bar{c}]$ .<sup>5</sup> In particular, we have in mind that they correspond to either Cournot competition in quantities or Hotelling competition in prices with differentiated products. We must also make a distributional assumption to convey the mean-cost-reducing nature of R&D investment. Defining  $F(c|I) = \int_{\underline{c}}^c f(s|I) ds$ , and with  $F_I(c|I)$  denoting the partial derivative of  $F(c|I)$  with respect to  $I$ , we assume:

*Assumption A.* For every  $I$ ,  $F_I(c|I) > 0$  for all  $c \in (\underline{c}, \bar{c})$ .

Assumption A depicts the usual first-order stochastic dominance condition [Hadar and Russell (1969)]. Thus, an increase in investment shifts the density to lower costs. We also assume that this shifting process occurs at a decreasing rate as investment increases.

*Assumption B.* For every  $I$  and  $c \in [\underline{c}, \bar{c}]$ ,  $F_{II}(c|I) \leq 0$ , with a strict inequality holding over some positive measure of costs.

Finally, it is useful to note that, from our assumption that  $\underline{c}$  and  $\bar{c}$  are independent of  $I$ :

$$\frac{d}{dI} F(\bar{c}(I)|I) \equiv F_I(\bar{c}|I) \equiv 0 \quad \text{and} \quad \frac{d}{dI} F(\underline{c}(I)|I) \equiv F_I(\underline{c}|I) \equiv 0.<sup>6</sup>$$

### 3.1.2. The investment stage

Fixing  $r$  and  $r^*$ , we now consider the choice of investment levels. The home firm chooses its investment level to maximize its expected profit:

$$E\pi(I, I^*, r) \equiv \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} f(c|I) f^*(c^*|I^*) \pi(c, c^*) dc^* dc - rI. \quad (7)$$

The first- and second-order conditions, respectively, are

$$E\pi_I(I, I^*, r) \equiv \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} f_I(c|I) f^*(c^*|I^*) \pi(c, c^*) dc^* dc - r = 0 \quad (8)$$

<sup>5</sup>This assumption ensures that firms find mean-cost-reducing investments potentially attractive, and it also plays a role in establishing the satisfaction of second-order conditions (Lemma 3.1). We verify in subsection 3.3 that this assumption holds in the standard models of product market competition that we consider.

<sup>6</sup>A class of examples that satisfies our assumptions is  $F(c|I) = h(c)f(I)^{g(c)}$ , where  $h(c) = g(\bar{c}) = 0 < 1 = h(\bar{c}) = g(\underline{c})$ ,  $h_c(c) > 0 > g_c(c)$ , and  $f(I) \in (0, 1)$  with  $f_I(I) > 0$  and  $f_{II}(I) \leq 0$  over the relevant investment domain. A particular example is

$$F(c|I) = \left[ \frac{c - \underline{c}}{\bar{c} - \underline{c}} \right] \cdot I^{(c - \underline{c})/(\bar{c} - \underline{c})} \quad \text{for } I \in (0, 1).$$

and

$$E\pi_{II}(I, I^*, r) \equiv \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} f_{II}(c | I) f^*(c^* | I^*) \pi(c, c^*) dc^* dc < 0. \quad (9)$$

If the second-order condition holds, then under our assumptions a solution to (8) with positive investment will exist provided  $r$  is not too large, and this solution corresponds to a reaction curve,  $I = I(I^*, r)$ . Exactly symmetric arguments apply for the foreign firm. We begin with the following lemma:

*Lemma 3.1. For all  $I, I^*$ , and  $r$ ,  $E\pi_{II}(I, I^*, r) < 0$ .*

*Proof.* Observe that

$$E\pi_{II}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} f_{II}(c | I) K(c | I^*) dc,$$

where

$$K(c | I^*) \equiv \int_{\underline{c}}^{\bar{c}} f^*(c^* | I^*) \pi(c, c^*) dc^* > 0 \quad \text{for all } c \in [\underline{c}, \bar{c}]. \quad (10)$$

We also have that

$$K_c(c | I^*) = \int_{\underline{c}}^{\bar{c}} f^*(c^* | I^*) \pi_c(c, c^*) dc^* < 0 \quad \text{for all } c \in [\underline{c}, \bar{c}]. \quad (11)$$

Thus, integrating by parts and noting that  $F_{II}(c | I) \equiv 0$  at  $\underline{c}$  and  $\bar{c}$  we obtain

$$E\pi_{II}(I, I^*, r) = - \int_{\underline{c}}^{\bar{c}} F_{II}(c | I) K_c(c | I^*) dc < 0. \quad \text{Q.E.D.} \quad (12)$$

The reaction functions  $I = I(I^*, r)$  and  $I^* = I^*(I, r)$  are thus well-defined. Using Lemma 3.1, (3), and (4), we also have that  $\hat{I}_r(r, r^*) < 0$ , indicating that a domestic R&D subsidy will raise domestic investment. We next determine the external effect of greater investment by one firm on the expected profit of the other.

*Lemma 3.2. For all  $I, I^*$  and  $r$ ,  $\text{sign}[E\pi_{I^*}(I, I^*, r)] = -\text{sign}[\pi_{c^*}(c, c^*)]$  provided that  $\text{sign}[\pi_{c^*}(c, c^*)]$  is the same for all  $c \in [\underline{c}, \bar{c}]$  and  $c^* \in [\underline{c}, \bar{c}]$ .*

*Proof.* Observe that

$$E\pi_{I^*}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} f(c|I) f_{I^*}^*(c^*|I^*) \pi(c, c^*) dc^* dc \quad (13)$$

which, using (10), can be rewritten as

$$E\pi_{I^*}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} f(c|I) K_{I^*}(c|I^*) dc, \quad (14)$$

where

$$K_{I^*}(c|I^*) = \int_{\underline{c}}^{\bar{c}} f_{I^*}^*(c^*|I^*) \pi(c, c^*) dc^*. \quad (15)$$

But integrating by parts yields

$$K_{I^*}(c|I^*) = - \int_{\underline{c}}^{\bar{c}} F_{I^*}^*(c^*|I^*) \pi_{c^*}(c, c^*) dc^*. \quad (16)$$

Using Assumption A, (14), and (16), the lemma is thus proved. Q.E.D.

Intuitively, as  $I^*$  increases, the density on low  $c^*$ 's also increases. Hence, expected home profits decline (rise) if  $\pi_{c^*}(c, c^*)$  is positive (negative). Finally, the next lemma examines the slope of the investment reaction curves.

*Lemma 3.3. For all  $I, I^*$ , and  $r$ ,  $\text{sign}[E\pi_{I^*}(I, I^*, r)] = \text{sign}[\pi_{cc^*}(c, c^*)]$  provided that  $\text{sign}[\pi_{cc^*}(c, c^*)]$  is the same for all  $c \in [\underline{c}, \bar{c}]$  and  $c^* \in [\underline{c}, \bar{c}]$ .*

*Proof.* Observe that

$$E\pi_{I^*}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} f_I(c|I) f_{I^*}^*(c|I^*) \pi(c, c^*) dc^* dc \quad (17)$$

which, with (15), simplifies to

$$E\pi_{I^*}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} f_I(c|I) K_{I^*}(c|I^*) dc. \quad (18)$$

Integrating by parts yields

$$E\pi_{I^*}(I, I^*, r) = - \int_{\underline{c}}^{\bar{c}} F_I(c|I) K_{I^*}(c|I^*) dc. \quad (19)$$

But using (16):

$$E\pi_{I^*I}(I, I^*, r) = \int_{\underline{c}}^{\bar{c}} F_I(c|I) \int_{\underline{c}}^{\bar{c}} F_{I^*}(c^*|I^*) \pi_{cc^*}(c, c^*) dc^* dc$$

which, by Assumption A, yields the statement of the lemma. Q.E.D.

To gain some intuition for this result, suppose  $\pi_{cc^*}(c, c^*) < 0$ . Then the gain in profit from a reduction in own cost diminishes as rival cost drops. It follows that as the rival firm increases its investments, thus lowering its expected cost, the incentive for the domestic firm to lower its cost with investment is reduced, i.e.  $E\pi_{I^*I}(I, I^*, r) < 0$ .

Thus, investment reaction curves are downward sloping if  $\pi_{cc^*}(c, c^*) < 0$  for all  $c$  and  $c^*$ , and upward sloping if  $\pi_{cc^*}(c, c^*) > 0$  for all  $c$  and  $c^*$ . We now turn to the implications of the properties of  $\pi(c, c^*)$  for the resulting R&D policies.

### 3.1.3. The policy stage

Lemmas 3.1–3.3, together with (5), yield the following proposition.

*Proposition 3.1.* (a) Facing a fixed foreign policy  $r^*$ , the home government's optimal strategic R&D policy as characterized in (5) will take the form of an R&D subsidy if (i)  $\text{sign}[\pi_{c^*}(c, c^*)] = -\text{sign}[\pi_{cc^*}(c, c^*)]$  for all  $c \in [\underline{c}, \bar{c}]$  and  $c^* \in [\underline{c}, \bar{c}]$  and will take the form of an R&D tax if (ii)  $\text{sign}[\pi_{c^*}(c, c^*)] = \text{sign}[\pi_{cc^*}(c, c^*)]$  for all  $c \in [\underline{c}, \bar{c}]$  and  $c^* \in [\underline{c}, \bar{c}]$ .

(b) Any Nash equilibrium between the two governments will be characterized by strategic subsidies to R&D if (i) holds and by strategic taxes on R&D if (ii) holds.

### 3.2. Corrective issues

We now consider whether 'excessive' R&D is undertaken. Our results are summarized in the next proposition.

*Proposition 3.2.* The symmetric joint welfare maximizing corrective R&D policy as characterized in (6) involves an R&D tax (subsidy) if  $\pi_{c^*}(c, c^*) > 0$  ( $\pi_{c^*}(c, c^*) < 0$ ) for all  $c \in [\underline{c}, \bar{c}]$  and  $c^* \in [\underline{c}, \bar{c}]$ . Moreover, any Nash equilibrium between the two governments will be characterized by strategic R&D policies which conflict (agree) in sign with the symmetric joint welfare maximizing corrective R&D policy whenever  $\pi_{cc^*}(c, c^*) < 0$  ( $\pi_{cc^*}(c, c^*) > 0$ ) for all  $c \in [\underline{c}, \bar{c}]$  and  $c^* \in [\underline{c}, \bar{c}]$ .

We now extend this line of inquiry to consider the potential national

conflict between corrective and strategic incentives that arises when there are many domestic firms. To this end, we suppose that there are  $H$  home firms and  $F$  foreign firms, and we define domestic welfare as

$$W(r, r^*) \equiv \sum_{i=1}^H E\pi^i(\hat{I}(r, r^*), \hat{I}^*(r, r^*), r) - (\tilde{r} - r) \sum_{i=1}^H \hat{I}^i(r, r^*),$$

where  $\hat{I}(r, r^*) \equiv \hat{I}^1(r, r^*) \dots \hat{I}^H(r, r^*)$  and  $\hat{I}^*(r, r^*) \equiv \hat{I}^{*1}(r, r^*) \dots \hat{I}^{*F}(r, r^*)$ . Maintaining the assumption that  $W(r, r^*)$  is concave in  $r$  and imposing symmetry among domestic and foreign firms, we may set  $W_r(r, r^*) = 0$ , and then derive explicit expressions for  $\hat{I}_r^j(r, r^*)$  and  $\hat{I}_r^i(r, r^*)$ , in order to solve for the optimal domestic R&D policy for a fixed  $r^*$ :

$$\tilde{r} - r = \frac{-HFE\pi_{I^*j}^i(\cdot)E\pi_{I^*j}^{*j}(\cdot)}{E\pi_{I^*j}^{*j}(\cdot) + (F-1)E\pi_{I^*j}^{*j}(\cdot)} + H(H-1)E\pi_{Ij}^i(\cdot). \quad (20)$$

The first term on the right-hand side of (20) captures the national strategic R&D policy incentive. Provided second-order and stability conditions are met, there will be a strategic incentive to subsidize (tax) R&D if the signs of  $E\pi_{I^*j}^i(\cdot)$  and  $E\pi_{I^*j}^{*j}(\cdot)$  agree (disagree).<sup>7</sup> But when the number of home firms  $H$  exceeds one, there is a second term on the right-hand side of (20), and this term captures the national corrective incentive to intervene. This term has the same sign as  $E\pi_{Ij}^i(\cdot)$ , and reflects the fact that domestic firm  $j$ 's investment imposes a negative (positive) externality on domestic firm  $i$ 's profits if  $E\pi_{Ij}^i(\cdot) < 0$  ( $E\pi_{Ij}^i(\cdot) > 0$ ) and should be taxed (subsidized) accordingly.

Since  $E\pi_{I^*j}^i(\cdot)$  and  $E\pi_{Ij}^i(\cdot)$  share the same sign, we conclude from (20) that whether or not the signs of the strategic and corrective incentives for a country to pursue R&D policy differ is determined by the sign of  $E\pi_{I^*j}^{*j}(\cdot)$ . If  $E\pi_{I^*j}^{*j}(\cdot) < 0$ , so that investment reaction curves slope downward, the two policy incentives will conflict in sign, and the sign of the optimal national R&D policy will depend on the relative numbers of foreign and domestic firms. In contrast, if  $E\pi_{I^*j}^{*j}(\cdot) > 0$ , so that investment reaction curves slope upward, the two policy incentives will agree in sign, and the sign of the optimal national R&D policy will correspond to the sign of  $E\pi_{Ij}^i(\cdot)$ .

### 3.3. Specific market conditions

Having established the general relationship between the properties of  $\pi(c, c^*)$  and both strategic and corrective incentives for R&D intervention, we

<sup>7</sup>The denominator of the first term in (20) corresponds to a diagonal term in the relevant Jacobian, and thus takes a negative sign. Note that  $I^{*j}$  indicates the investment of foreign firm  $j$ , while  $I^i$  denotes that of domestic firm  $i$ .

now turn to a consideration of specific models. We make the assumption of a single firm in each country, and then note what our results imply for the many-firm case.

We begin with the Hotelling model of price competition. In the standard model, each firm faces a linear demand, as a consequence of individual consumers possessing unitary demands and being uniformly distributed over the  $[0, 1]$  interval, with a total mass of one. The two firms are located at opposite endpoints and simultaneously choose prices.<sup>8</sup> Letting  $t$  denote the consumers' transportation cost, if the good is valued sufficiently that all consumers buy in equilibrium and if each firm has positive equilibrium sales, with  $3t > \bar{c} - \underline{c}$  being sufficient for the latter, then it is straightforward to show that the equilibrium profit function is  $\pi(c, c^*) = (t + (c^* - c)/3)^2/2t$ . The foreign profit function is exactly symmetric. It follows that

$$\pi_c(c, c^*) < 0; \quad \pi_{c^*}(c, c^*) > 0; \quad \pi_{cc^*}(c, c^*) < 0 \quad \text{for all } c \text{ and } c^*. \quad (21)$$

Thus, a higher home unit cost lowers home profit. As for the other inequalities, as  $c^*$  rises so too will the foreign price, and this serves to improve home profit. This improvement diminishes, however, as  $c$  rises, because the home demand increase generated by the rise in foreign price then becomes less valuable.<sup>9</sup>

With the relevant properties of  $\pi(c, c^*)$  established in (21), Proposition 3.1 can now be employed to characterize the incentives for R&D intervention in this setting. In particular, with  $\pi_c(c, c^*) < 0$  and  $\text{sign}(\pi_{c^*}(c, c^*)) = -\text{sign}(\pi_{cc^*}(c, c^*))$ , condition (i) of Proposition 3.1 applies. Thus, despite the price-setting nature of product market competition between firms, each government has a strategic incentive to subsidize its firm's R&D investment, and any Nash equilibrium between the two governments will involve R&D subsidies. The fact that  $\pi_{c^*}(c, c^*) > 0$  indicates a negative externality to investment, so that Proposition 3.2 implies that non-cooperative firms 'overinvest' for given  $r$  and  $r^*$ . Moreover, with  $\pi_{cc^*}(c, c^*) < 0$ , investment reaction curves are downward sloping. Hence, while the strategic incentive is

<sup>8</sup>The model as developed below follows Tirole (1988), who also notes a negatively-sloped investment reaction curve, though in a deterministic setting.

<sup>9</sup>The conditions listed in (21) also can be supported in more general non-linear settings. In particular, for substitute goods,  $\pi_{c^*}(c, c^*) > 0$  is a direct consequence of the normal assumptions of positively-sloped price reaction functions and reaction function stability, while  $\pi_c(c, c^*) < 0$  follows similarly but with a slight strengthening of the stability condition. Sufficient conditions for  $\pi_{cc^*}(c, c^*) < 0$  are more elusive; however, we have verified this inequality [and the others in (21)] for certain tractable non-linear demand structures, such as when the home demand is given by  $p^{-\beta} p^{*\alpha}$  or by  $f(p^*) - bp$ , where  $\beta > 1$ ,  $\alpha > 0$ , and  $(b, f, f', b - f') > 0 \geq f''$ . We also emphasize that the conditions in (21) hold for any linear-demand model of product differentiation, provided only that own effects are not overwhelmed by cross effects. Specifically, if the demand for the home product is  $a - bP + dP^*$ , with  $(a, b, d) > 0$  and  $a$  large, and symmetrically for the foreign product, then (21) is satisfied if  $2b^2 > d^2$ . The additional properties attributed to the Hotelling model in the next section also hold under this condition.

for each government to subsidize R&D, the symmetric corrective R&D policy for both governments acting together would be to tax R&D. The analogous result implied by  $\pi_{c,c^*}(c, c^*) < 0$  when there are many domestic and foreign firms is that the national strategic incentive to subsidize R&D will run counter to the national corrective incentive to tax: the national R&D policy will thus depend on the balance of these two effects, and hence on the relative number of domestic and foreign firms.<sup>10</sup>

We now consider quantity competition. The conditions listed in (21) are typically assumed to hold in quantity choice models, and hold in particular when demand is linear and costs are constant.<sup>11</sup> To see this, let  $P(q+q^*) = 1 - q - q^*$  be the market price when domestic (foreign) output is  $q$  ( $q^*$ ). Assuming that  $1 - 2\bar{c} + \bar{c} > 0$  so that each firm earns positive equilibrium profit, equilibrium Cournot profit is given by the function  $\pi(c, c^*) = (1 - 2c + c^*)^2/9$ .  $\pi^*(c, c^*)$  is defined analogously. It is now straightforward to verify that the conditions listed in (21) hold. Our results for quantity competition are thus analogous to those established by Spencer and Brander (1983), although we do generalize their analysis to allow for uncertainty.

#### 4. R&D and general mean shifts

In the previous section we assumed that R&D leads to a reduction in the firm's mean cost in the particular sense of first-order stochastic dominance. As noted in the introduction, this assumption limits the degree to which the 'riskiness' of the cost distribution can change as a result of R&D investments. We now extend the model to allow that R&D lowers mean cost without imposing additional restrictions on the way higher moments of the cost distribution change, and explore the generality of our results with regard to the riskiness of R&D investments. Specifically, we now replace Assumption A with:

*Assumption A'*. For every  $I$ ,  $\int_c^{\bar{c}} F_I(c | I) dc > 0$ .

To interpret this assumption, observe that integration by parts implies

<sup>10</sup>Analogous properties for the equilibrium profit function can be derived in the many-firm case when firms are located around a circle, as in Salop (1979).

<sup>11</sup>As Brander and Spencer (1985, Proposition 1) establish, under the normal Cournot assumptions,  $\pi_{c,c^*}(c, c^*) > 0 > \pi_c(c, c^*)$ . The requirement that  $\pi_{c,c^*}(c, c^*) < 0$  is more difficult to establish under general conditions; however, Besley and Suzumura (1992) and Reinganum (1983) argue that this latter condition also holds for some popular non-linear demand structures. Thus, there is a strong presumption that (21) holds for Cournot competition. It should also be emphasized that (21) holds for Cournot competition in differentiated products, too, as may be verified by inverting the linear demand system given in footnote 9 above.

$$\frac{d}{dI} \int_{\underline{c}}^{\bar{c}} f(c|I)c \, dc = - \int_{\underline{c}}^{\bar{c}} F_I(c|I) \, dc,$$

so that Assumption A' holds exactly when investment always reduces mean cost. Of course, Assumption A' is necessary but not sufficient for first-order stochastic dominance, as defined by Assumption A.

The next step is to examine the signs of  $E\pi_{II}(\cdot)$ ,  $E\pi_{I^*}(\cdot)$ , and  $E\pi_{I^*I}(\cdot)$  when Assumption A' is in place. In order to save space and direct attention to the most important issues, we omit a formal analysis of second-order conditions in this section, choosing instead to simply suppose that they hold. This investigation of the remaining expressions is considerably simplified when (21) is strengthened to also require that

$$\pi_{c^*c^*}(c, c^*) > 0 \quad \text{and} \quad \pi_{cc^*c^*}(c, c^*) \equiv 0 \equiv \pi_{cc^*}(c, c^*) \quad \text{for all } c \text{ and } c^*. \quad (22)$$

These assumptions are all met by our earlier Hotelling and Cournot examples.

We consider first the slope of the investment reaction curves. Repeated integration by parts then reveals the following inequality, which holds for all  $I$  and  $I^*$ :

$$E\pi_{I^*I}(I, I^*, r) = \left[ \int_{\underline{c}}^{\bar{c}} F_I(c|I) \, dc \right] \left[ \int_{\underline{c}}^{\bar{c}} F_{I^*}^*(c^*|I^*) \, dc^* \right] \pi_{cc^*}(\bar{c}, \bar{c}) < 0. \quad (23)$$

Thus, investment reaction curves slope down provided that investment reduces the mean cost, whether or not it does so in the particular sense of a first-order stochastic shift.

Consider next the effect of greater foreign investment on domestic profits. Successive integration by parts reveals that

$$\begin{aligned} E\pi_{I^*}(I, I^*, r) = & - \left[ \int_{\underline{c}}^{\bar{c}} F_{I^*}^*(c^*|I^*) \, dc^* \right] \left[ \int_{\underline{c}}^{\bar{c}} f(c|I) \pi_{c^*}(c, \bar{c}) \, dc \right] \\ & + \int_{\underline{c}}^{\bar{c}} \left[ \int_{\underline{c}}^{c^*} F_{I^*}^*(s|I^*) \, ds \right] \pi_{c^*c^*}(c, c^*) \, dc^*. \end{aligned} \quad (24)$$

We see here that  $E\pi_{I^*}(\cdot)$  is comprised of two terms. The first term captures the 'mean effect', and will be negative under Assumption A', since it indicates that greater foreign investment lowers the mean foreign cost, which in turn lowers domestic profit. Given that investment reaction curves are negatively sloped, and using (5) and (6), it follows that the mean effect favors a strategic R&D subsidy and a corrective R&D tax.

There is, however, a second term which can be either positive or negative under Assumption A'. For instance, this term will be positive if investment leads to a second-order stochastic shift [Hadar and Russell (1969)] in the cost distribution, so that for all  $I^*$ ,

$$\int_{\underline{c}}^{c^*} F_{I^*}^*(c^* | I^*) dc^* \geq 0 \quad \text{for all } c \in [\underline{c}, \bar{c}]. \quad (25)$$

If the second term in (24) is positive and large enough to outweigh the mean effect embodied in the first term, then  $E\pi_{I^*}(\cdot)$  will be positive and R&D investment will impart a positive externality on rival firms.

To illustrate this possibility, we focus on the case in which (25) holds. In addition, let us momentarily abandon Assumption A' and suppose instead that, for all  $I^*$ ,

$$\int_{\underline{c}}^{\bar{c}} F_{I^*}^*(c^* | I^*) dc^* = 0. \quad (26)$$

Conditions (25) and (26) correspond to the case of a 'mean-preserving spread' [Rothschild and Stiglitz (1970)], in which investment preserves the mean but increases the riskiness of the cost distribution.<sup>12</sup>

When R&D increases risk without altering the mean, it is evident from (23) and (24) that investment reaction curves are flat and that investment imparts a positive externality, with the latter result arising since under (22) domestic profits are also convex in foreign costs. It then follows from (5) and (6) that the optimal strategic policy is simply laissez faire while the appropriate corrective policy is an R&D subsidy.<sup>13</sup>

Let us now relax (26) and reimpose Assumption A', while still maintaining the possibility that R&D increases riskiness in the sense of (25). Specifically, consider next the possibility that R&D adds 'risk' [i.e. (25) holds] but that R&D only slightly lowers the mean (i.e. the integral in Assumption A' is slightly positive).<sup>14</sup> In this event, (23) again ensures that investment reaction curves are negatively sloped. Furthermore, it is straightforward to show that the risk effect, associated with the second term in (24), then dominates the mean effect, in that investment imparts a positive externality. But with

<sup>12</sup>While our primary interest is in R&D investments that also reduce mean cost, analyzing the case of a mean-preserving spread has some direct as well as interpretive value, especially given that a Hotelling or Cournot firm's profit function is convex in its own cost, which makes risk-increasing investments potentially attractive.

<sup>13</sup>A complete analysis of the mean-preserving-spread case is found in our working paper [Bagwell and Staiger (1990)].

<sup>14</sup>Formally (25) can be interpreted as adding risk only when (26) holds. However, for purposes of exposition, we prefer to continue to call (25) a 'risk' effect even when the mean changes and (26) is violated.

$E\pi_{j,s}(\cdot) > 0 > E\pi_{j,r}(\cdot)$ , (5) and (6) imply that in this case the appropriate strategic policy is an R&D tax, while the appropriate corrective policy is an R&D subsidy.

We thus conclude that the optimal R&D policy is highly sensitive to the respective mean and risk effects embodied in the R&D process itself: when the mean effect dominates, strategic subsidies and corrective taxes are called for, but when the risk effect is strongest, strategic taxes and corrective subsidies are desired. On this basis we argue that the pivotal issue for determining appropriate strategic and corrective policy appears to be the way in which uncertainty in R&D outcomes is modeled.

## 5. Conclusion

We have found that the strategic role played by an R&D subsidy remains attractive for a variety of forms of product market competition when R&D leads to a first-order reduction in a firm's expected costs. In this setting, a country's strategic incentive is to subsidize R&D, although the corrective incentive is to tax R&D. However, we have also shown that the optimal R&D policy is highly sensitive to the exact way in which greater R&D acts to lower a firm's mean cost. Strategic taxes and corrective subsidies may be desired if R&D greatly increases the riskiness of the distribution of costs and only modestly lowers the mean cost. We conclude that the crucial determinant of appropriate R&D policy is not the form of product market competition, but rather the nature of uncertainty in the R&D process itself.

Finally, an assumption we have maintained throughout is the non-negativity of profits for each firm when all firms enter the final (production) stage of the market. This suggests that our model is most relevant in markets characterized by a limited degree of scale economies. In a companion paper [Bagwell and Staiger (1992)] we consider the alternative case in which firms battle for a monopoly position and show that the sensitivity of appropriate R&D policy to the nature of uncertainty is mitigated: in 'battles for monopoly', strategic subsidies and corrective taxes are optimal for both first-order and mean-preserving (risk-increasing) stochastic shifts. Whether such battles for monopoly or the oligopolistic setting of the present paper is more representative of 'typical' international competition is not an issue we attempt to resolve here. Rather, we view these results simply as pointing to the key structural characteristics on which appropriate R&D policy depends.

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