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Countercyclical Pricing in Customer Markets

By Kyle Bagwell

Columbia University

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I present a dynamic model of price determination in customer markets that are subject to exogenous business cycle fluctuations. The business cycle is described in terms of a Markov process, in which market demand alternates stochastically between fast growth (boom) and slow growth (recession) phases. In the consumers' preferred equilibrium outcome, (1) prices are below the monopoly level, and (2) prices are countercyclical when demand growth rates are positively correlated through time. A firm faces a dynamic trade-off when making its current price selection. While a higher price may raise a firm's profit in the short term, it also may diminish the firm's reputation for low prices, leading to lower profits in the future.

Introduction

In many markets, firms set prices and consumers observe current price selections only after costly search. Price reputations naturally play a role in such markets, as consumers incorporate information about a firm's past pricing behaviour when forecasting its current prices. Consumer search behaviour of this form in turn confronts firms with a trade-off: while a higher price may raise profits in the short run, it also may diminish the firm's reputation for low prices, leading to lower profits in the future. A firm's pricing behaviour is thus sensitive to current and expected future market conditions, and as a consequence price choices may vary interestingly over the business cycle.

This dynamic perspective of firms' pricing decisions is described by Phelps and Winter (1970) and Okun (1975, 1980) in their classic discussions of 'customer markets'. Phelps, Winter and Okun emphasize that current prices affect profits in the short and long run when consumers' price information is dated, and they put forth two central hypotheses. First, they argue that this dynamic trade-off may induce firms to price below the monopoly level. The following quotes are illustrative:

> Our focus is on the problem of optimal price-making behavior for a firm which enjoys, at each instant of time, monopoly power with respect to its current consumers, yet which could not indefinitely maintain a price above the going market price without losing all its customers. (Phelps and Winter, 1970, p. 310)

> The price elasticity of demand for his product is relatively low in the short run, because of his established clientele (and that of his competitors), but much greater in the long run since information diffuses. Hence, the firm presumably 'underprices,' or sacrifices, for the near term if it maximizes the discounted future stream of profits (rather than current-period profits). (Okun 1980, p. 360)

Second, these authors argue that markups may be countercyclical in customer markets. For example, Okun writes that

> firms 'underprice' least in periods of weak demand and most (thus investing for future demand) in particularly good times. (Okun 1980, p. 361)

As is well known, the hypothesis of countercyclical markups is of particular importance, since it offers a possible reconciliation between the observation of (at least weakly) procyclical real wages and the Keynesian approach to business cycles.[1]

While the ideas put forth by Phelps, Winter and Okun are provocative and influential, their arguments have important limitations: Phelps and Winter do not endogenize consumer behaviour and the process of reputation formation; Okun's discussion is informal; and neither Phelps and Winter nor Okun explore formally the implications of business cycle fluctuations for equilibrium pricing in customer markets. With these limitations in mind, my purpose here is to construct and analyse an equilibrium model of the cyclical behaviour of markups in customer markets.

The model has two important ingredients. First, firms' price-setting and consumers' search behaviours are explicitly determined in an equilibrium analysis of reputation formation. I assume that consumers lack information concerning firms' current prices, so that the current-period search decisions of consumers are governed by the past prices that firms have selected. A firm thus has an incentive to choose a low current price, in order that it might acquire or maintain a reputation for low prices. Second, motivated by the description of US aggregate data put forth by Hamilton (1989), I assume that the business cycle is captured by a Markov process in which demand alternates stochastically between fast growth (i.e. boom) and slow growth (i.e. recession) phases. The Markov growth specification is attractive as it is both empirically plausible and analytically tractable.

While the dynamic model admits a large set of equilibria, one outcome that is of particular interest is the consumers' preferred equilibrium outcome. This outcome is shown to embody two main features:

1. prices are below the monopoly level;
2. prices are *countercyclical* when demand growth rates are positively correlated through time.

The nature of business cycle fluctuations varies across markets, but the assumption of positive correlation in demand growth rates seems sensible; for example, Hamilton's estimates indicate that US GNP growth rates are positively correlated through time. The analysis presented here thus provides an equilibrium foundation for the hypotheses of below-monopoly and countercyclical pricing in customer markets.

These results may be understood in the following way. When a firm operates in a dynamic search market, it must balance the short-term incentive to raise prices above the equilibrium level against the ramifications that such 'price gouging' would have for the firm's reputation and long-term profits. A firm is thus willing to price below the monopoly level in equilibrium if consumers would otherwise ascribe to the firm a 'bad' (i.e. high-price) reputation and purchase elsewhere in the future. Moreover, the long-term loss from a bad reputation is especially pronounced when the market is currently booming and growth rates are positively correlated through time, since a firm's sales are then expected to grow quickly in the near future if the firm maintains a 'good' (i.e. low-price) reputation. Consequently, firms are then willing to endure lower equilibrium prices in booms.

The findings presented here relate to two strands of recent research. First, the results contrast with those of models based on oligopolistic collusion, where prices are constrained by firms' incentives to maintain a collusive equilibrium. One finding from this literature is that persistent fluctuations in demand growth rates tend to generate procyclical markups. For example, Bagwell and Staiger (1997) model growth rate fluctuations as a Markov process and demonstrate that collusive markups are higher in fast growth (i.e. boom) phases.[2] In this paper I adopt the same Markov representation of the business cycle but focus instead on customer markets, and find that markups fluctuate countercyclically. The following broad conclusion is thus suggested: if the business cycle is well described by a Markov growth process, then customer markets may be a more likely source of countercyclical markups than collusive markets.[3]

A second related set of research concentrates on the cyclical behaviour of prices in customer markets that are characterized by switching costs. As Klemperer (1995) and Chevalier and Sharfstein (1996) discuss, when consumers have switching costs, firms have conflicting incentives.[4] A low price draws in more 'new' consumers, but a high price yields greater profit on 'old' consumers. A period of high demand can alter this trade-off in either way, depending on the manner in which the demand increase is distributed across the population of new and old consumers. For instance, if a boom corresponds to a rise in the fraction of new consumers, then firms may price countercyclically. Here, I offer a complementary theory that returns primary emphasis to price reputations. In contrast to the switching cost approach, this theory is easily developed in an infinite-horizon setting, and it delivers the prediction of countercyclical pricing provided only that demand growth rates are positively correlated though time.

The paper is organized as follows. In Section I, a simple stationary growth model is developed. This model illustrates the dynamic trade-off confronting firms and demonstrates that this trade-off can result in below-monopoly prices, confirming the first finding listed above. In Section II, a richer non-stationary environment is considered: the Markov growth business cycle is specified, and the equilibria of this Markov growth model are characterized. The consumers' preferred equilibrium for the Markov growth model is described in Section III, and it is here that the second main finding of the paper—that consumers' preferred equilibrium prices are countercyclical when growth rates are positively correlated though time—is presented. In Section IV I assume that a fraction of consumers is unresponsive to past pricing behaviour, perhaps because they are not informed as to firms' respective price reputations. I show that the main findings are robust to this possibility. Concluding remarks are offered in Section V.

I. The Stationary Growth Model

Basic assumptions

Consider a market in which a fixed set of $n \geqslant 2$ firms sell a homogeneous, nondurable good in each of an infinite number of periods, $t = 1, \ldots, \infty$. Consumers are assumed to be identical, and each consumer is described by an indirect utility function, $V(P)$, and a demand function, $D(P) = -V'(P)$, where P is the price per unit paid by the consumer. The assumptions placed on the demand function are:

$$(1) \qquad D(P) > 0 > D'(P) \text{ for } P \in (0, \overline{P}), \qquad \text{and} \qquad \lim_{P \to 0} D(P) = \infty,$$

where $\overline{P}$ is a choke price at and above which $D(\overline{P}) = 0$. These assumptions ensure that consumers prefer lower prices and that demand becomes large as price gets small.

Each firm has a constant unit cost of production, c, where $\overline{P} > c > 0$, and so a firm charging the price P earns profit per consumer in amount $\pi(P) \equiv (P - c)D(P)$. This profit function is assumed to satisfy the following properties:

$$(2) \qquad \begin{array}{l} \text{For } P > 0, \pi(P) \text{ has a unique maximizer}, P^*, \\ \quad \text{and } \pi'(P) > 0 \text{ when } P < P^*. \end{array}$$

Thus, P^* is the firm's monopoly price, and it follows directly that $\pi(P^*) > 0$.[5]

Before developing the dynamic games that follow, it is useful first to define the underlying stage game. In any period t, each consumer picks a firm $i \in \{1, \ldots, n\}$ from which to purchase, while each of the n firms picks a price $P \in (0, P^*]$ at which to sell its product. Furthermore, all decisions within period t are simultaneous: consumers pick firms at the same time that firms pick prices. The simultaneity of moves in turn implies that consumers choose firms before actually observing any firm's current-period price choice. In keeping with the emphasis of the earlier literature on customer markets, the stage game is thus structured in a manner that endows firms with considerable monopoly power over consumers in the short run. In a static or one-shot game of this form, for example, it is a (weakly) dominant strategy for each firm to pick its monopoly price, P^*.[6]

Firms may price below the monopoly level in a dynamic model, however, as the short-term incentive to raise price is then balanced against the long-term reduction in sales that such price gouging may induce. The resulting trade-off between profit in the short and long term is in turn affected by the discount factor, $\delta \in (0, 1)$, and by the state of the business cycle, which determines the relationship between the current and expected future levels of market demand. To model these ideas, I consider the infinite repetition of the stage game and allow the level of market demand to vary across periods. In particular, I assume that the period t level of market demand is summarized by the number of consumers active in period t, G_t, and I specify stationary as well as non-stationary processes that determine the evolution of these levels.

While the structure of the stage game precludes consumers from observing current prices prior to search, I assume that consumers are perfectly informed about firms' past pricing behaviour, so that firms may develop 'reputations' among consumers. One interpretation is that word-of-mouth communication among consumers is extensive and provides a channel through which information about past pricing behaviour can disseminate.[7] I also assume that consumers and firms are well informed about the nature of business cycle; namely, prior to making their decisions in the period-t stage game, consumers and firms know all past market demand levels as well the current market demand level. Finally, I assume that market participants are also familiar with the process through which market demand levels are generated, so that they may use this information in forming expectations as to future market conditions.

Payoffs in the dynamic game are given by the discounted payoff streams derived from the sequence of stage games. The set of subgame-perfect equilibria is large. Rather than characterizing the entire set, I focus upon equilibria in which consumers employ the *loyalty–boycott strategy*. To define this strategy, let R_t denote the set of reputable firms, where a firm is reputable if it has not deviated from its equilibrium strategy in any period preceding period t. A consumer in period t is assumed to be 'loyal' and to purchase from the firm from which he purchased in period $t-1$, so long as this firm is in the set R_t. If the firm is not in the set R_t, then the consumer 'boycotts' the non-reputable firm and picks randomly from the firms that are in the set R_t. Finally, if period t is the consumer's first period in the market, the consumer picks randomly from the set of reputable firms, R_t.[8]

The stationary growth game

I begin with a simple dynamic game, called the *stationary growth game*, in which the level of market demand grows according to a stationary growth rate. Specifically, the number of consumers active in period t, G_t, evolves as follows:

$$G_1 > 0 \qquad \text{and} \qquad G_t = gG_{t-1} \qquad \text{for } 0 < \delta g < 1.$$

Thus, the level of market demand expands (contracts) through time when $g > 1$ ($g < 1$), while the level of market demand is constant when $g = 1$. The assumption that $0 < \delta g < 1$ ensures that the discounted growth is finite. While this specification does not allow for non-stationary 'cycles' in the level of market demand, it does serve as a simple benchmark with which to fix ideas. The possibility of demand cycles is considered formally in subsequent sections.

Given that consumers employ the loyalty–boycott strategy, a firm now faces a trade-off between 'cheating' today with an unexpected price hike to the monopoly level P^*, and sacrificing future profits in a market that grows at rate g. Since the growth rate is stationary, this trade-off is the same at every date, and so it is natural to focus upon *single-price equilibria* (SPE), which are defined as subgame-perfect equilibria in which consumers use the loyalty–boycott strategy and where, along the equilibrium path, all firms choose a common price in every period. I now characterize the set of SPE, and identify the consumers' preferred SPE price, P^c, which is the lowest price that can be supported in an SPE of the stationary growth game.

To this end, let

$$\text{(3)} \qquad \Omega(P) = [\pi(P^*) - \pi(P)]/n$$

denote a firm's per-aggregate consumer incentive to cheat from a SPE specifying that all firms select the price P. Observe that Ω is positive and decreasing for $P < P^*$, while Ω is zero when $P = P^*$. Next, let

$$\text{(4)} \qquad \omega(P) = \pi(P)/n$$

give the per-period and per-aggregate-consumer cost of a future boycott. The function ω is increasing for $P < P^*$, and ω is zero when $P = c$. Implicit in these definitions is the loyalty–boycott strategy, whereby a firm sells to $(1/n)$th of the period t consumers in equilibrium and makes no sales in period t if it has previously deviated.

With these definitions in place, the basic incentive constraint that each firm faces in any period t of a SPE of the stationary growth game may be represented as

$$G_t\Omega(P) \leqslant G_t \sum_{\tau=t+1}^{\infty} (\delta g)^{\tau-t}\omega(P),$$

which may be written in the simpler form,

$$(5) \qquad \Omega(P) \leqslant [\delta g/(1-\delta g)]\omega(P).$$

Observe that the set of SPE is independent of the specific level of market demand and also the number of firms. This is an implication of the constant returns to scale technology.

The incentive constraint presented in (5) is illustrated graphically in Figure 1. Here, P^c is the price at which the incentive constraint binds. The incentive to cheat is even lower, and the long-term cost of a future boycott even higher, at prices above P^c, and so the set of SPE is described by $P \in [P^c, P^*]$.[9] The consumers' preferred equilibrium price, P^c, lies between the competitive price c and the monopoly price P^*, and is arbitrarily close to these values when the model's parameters are appropriately specified. To see this, suppose first that δg is close to zero, and observe then that P^c is near the monopoly price, P^*. Intuitively, if firms discount the future sufficiently, or if market demand growth is sufficiently low, then the prospect of a loss in future sales does not deter firms from seeking short-term profits, and so equilibrium can be maintained only if the equilibrium price is close to the monopoly level. Suppose next that δg is close to unity, and observe that P^c is then near the competitive price, c. In this case, the future is very important to firms, and equilibria can be supported in which the equilibrium price is close to the competitive level c, even though the short-term profits from a price hike are large.

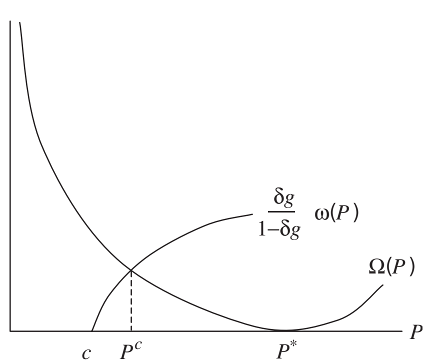

FIGURE 1. Incentive constraint.

This discussion may be summarized as follows.

Proposition 1. In the SPE of the stationary growth game, the consumers' preferred equilibrium price, P^c,

(i) is above the competitive price c but below the monopoly price P^*;
(ii) approaches the monopoly price as the discounted growth rate δg goes to zero;
(iii) approaches the competitive price as the discounted growth rate δg goes to unity.

The stationary growth game thus demonstrates that reputational considerations can induce firms to select below-monopoly prices, even when they have unlimited monopoly power in the short term. It also illustrates that the price-reducing influence of reputation formation is greater when firms are more patient and the market demand level grows at a faster rate.[10]

While the consumers' preferred equilibrium outcome exhibits many interesting features, the model also admits other equilibrium outcomes. In fact, as Figure 1 illustrates, there is an infinite number of SPE outcomes for the stationary growth game. The consumers' preferred equilibrium outcome, however, is of some special interest. First, given the basic premise that price reputations matter to firms, it is natural to analyse the extreme case, where reputational considerations lead firms to price at the lowest possible equilibrium level.[11] Second, the consumers' preferred equilibrium outcome becomes focal when coalition formation is considered. In SPE for which the equilibrium price exceeds P^c, a sub-coalition of two firms and consumers could all benefit under the alternative proposal to implement among themselves a SPE with a slightly lower price. The consumers' preferred equilibrium outcome cannot be broken in this fashion, however, since a price below P^c cannot satisfy the SPE incentive constraint (5) and thus is not a credible proposal.

II. The Markov Growth Model

I now relax the assumption of a stationary growth rate for the level of market demand and assume instead that the growth rate is stochastic and is determined by a Markov process. The purpose of this section is to formally define this Markov process, derive and interpret the corresponding incentive constraints for firms, and then characterize the set of equilibria of the Markov growth game.[12] A characterization of the consumers' preferred equilibrium prices is deferred until the next section.

Markov growth and the business cycle

The level of market demand is assumed to grow at one of two possible rates. Period t is said to be a *boom period* if $G_t = bG_{t-1}$, and period t is a *recession period* if $G_t = rG_{t-1}$, where $1 > \delta b > \delta r > 0$. In other words, if g_t denotes the period t growth rate, then period t is a boom (recession) period if $g_t = b (g_t = r)$. The transition between boom and recession periods is assumed to be governed by a Markov process, in which

$$
\begin{aligned}
&\rho \equiv \text{Prob}(g_t = r | g_{t-1} = b) \in (0,1) \\
(6) \qquad &\lambda \equiv \text{Prob}(g_t = b | g_{t-1} = r) \in (0,1) \\
&\mu \equiv \text{Prob}(g_1 = b) \in (0,1)
\end{aligned}
$$

The parameter ρ is thus the transition probability associated with moving from a boom to a recession, while λ is the transition probability corresponding to moves from recessions to booms. The parameter μ describes how the system begins. Assume further that $G_1 > 0$.

The parameters ρ and λ play important roles in two key measures associated with the business cycle. First, ρ and λ may be interpreted in terms of the expected duration of boom and recession phases, respectively. Suppose that $g_{t-1} = r$ and $g_t = b$, so that a switch to a boom period occurs at period t, and define $t^* \equiv \min\{\tau > t | g_\tau = r\}$. A *boom phase* may be now defined as a sequence of boom periods, $\{t, ..., t^* - 1\}$, and the *expected duration of a boom phase* is given by

$$\sum_{z=1}^{\infty} z\rho(1-\rho)^{z-1} = 1/\rho.$$

A *recession phase* may be defined analogously, and the *expected duration of a recession phase* is $1/\lambda$.

A second important measure for the business cycle concerns the correlation of growth rates through time. Observe that

$$
\begin{aligned}
&E(G_{t+1}|g_t = b) - E(G_{t+1}|g_t = r) \\
(7) \qquad &\quad = [E(g_{t+1}|g_t = b) - E(g_{t+1}|g_t = r)]G_t \\
&\quad = (1 - \lambda - \rho)(b - r)G_t,
\end{aligned}
$$

and so the expected growth rate is higher in period $t+1$ when period t is a boom period if and only if $1 - \lambda - \rho > 0$. I say therefore that business cycle growth rates are positively correlated through time when $1 - \lambda - \rho > 0$, and that they are negatively correlated through time when $1 - \lambda - \rho < 0$. Finally, growth rates are said to exhibit zero correlation when $1 - \lambda - \rho = 0$.

With the Markov growth business cycle now fully specified, I define the *Markov growth game* as the infinitely repeated stage game in which G_t evolves in the implied manner. Note that the stationary growth game is a limiting case of the Markov growth game (e.g. when $b = g$, $\mu \to 1$, $\rho \to 0$).

The incentive constraints

As in the stationary growth game, firms encounter a trade-off when making their respective price choices, as each must balance the short-term incentive to cheat against the value of maintaining a cooperative relationship with consumers. In other words, a price policy can be supported in equilibrium only if the incentive to cheat is no higher than the discounted expected cost of a boycott. Having already characterized the incentive to cheat in (3), I next derive a formal representation of the cost of a boycott so that the firms' incentive constraints for the Markov growth game may be presented.

The Markov structure is especially helpful here, since it implies that firms face the same incentives in any boom period regardless of the specific date, and similarly for any recession period. Therefore I now focus on *two-price equilibria* (TPE), which are equilibria for the Markov growth game in which consumers adopt the loyalty–boycott strategy and, along the equilibrium path, firms set a single price P_b in all boom periods and a single price P_r in all recession periods. An additional benefit of the Markov structure is that it admits a simple recursive structure, once the appropriate definitions are put forth. I now exploit these advantages and provide a simple representation for the incentive constraints facing firms in boom and recession periods, respectively.

To this end, I define $\overline{\omega}_b(P_b, P_r)$ as the expected discounted profit per aggregate consumer to a firm in period $t+1$ and all future periods, if period $t+1$ is a boom period and the prices P_b and P_r are charged in the future. In other words, $\overline{\omega}_b(P_b, P_r)$ is the expected discounted profit to a firm in period $t+1$ and thereafter, if $G_{t+1} = 1$, $g_{t+1} = b$ and P_b (P_r) is charged in boom (recession) periods. Analogously, the function $\overline{\omega}_r(P_b, P_r)$ can be defined when $g_{t+1} = r$. Both functions are evaluated in period $t+1$ dollars. Observe that $\overline{\omega}_b(P_b, P_r)$ and $\overline{\omega}_r(P_b, P_r)$ also provide a measure of the cost of a boycott, since a firm earns zero profit once such a boycott commences.

To fix ideas, consider the incentive constraint facing a firm in period t when period t is a boom period. This constraint appears as

$$G_t\Omega(P_b) \leqslant \delta\{\rho(rG_t)\,\overline{\omega}_r(P_b, P_r) + (1-\rho)(bG_t)\,\overline{\omega}_b(P_b, P_r)\},$$

since $g_{t+1} = r$ with probability ρ and $g_{t+1} = b$ with probability $1-\rho$, given that period t is a boom period. Observe now that the current-period level of market demand, G_t, cancels, resulting in a simpler expression for the incentive constraint:

$$\Omega(P_b) \leqslant \delta\{\rho r\,\overline{\omega}_r(P_b, P_r) + (1-\rho)b\,\overline{\omega}_b(P_b, P_r)\}.$$

The key idea here is that the future level of market demand is always proportional to the current level; consequently, the current demand level is simply a scaling factor that is irrelevant for firms' incentives.

Building on these insights, and simplifying notation slightly, I provide a complete system of incentive constraints with the following two inequalities:

$$\Omega(P_b) \leqslant \delta\{\rho r\,\overline{\omega}_r + (1-\rho)b\,\overline{\omega}_b\} \tag{8}$$

$$\Omega(P_r) \leqslant \delta\{\lambda b\,\overline{\omega}_b + (1-\lambda)r\,\overline{\omega}_r\}, \tag{9}$$

where

$$\overline{\omega}_b = \omega(P_b) + \delta\{\rho r\,\overline{\omega}_r + (1-\rho)b\,\overline{\omega}_b\} \tag{10}$$

$$\overline{\omega}_r = \omega(P_r) + \delta\{\lambda b\,\overline{\omega}_b + (1-\lambda)r\,\overline{\omega}_r\}. \tag{11}$$

Notice that (8) and (9) reflect the tension between the short-term incentive to cheat as defined by (3) and the expected discounted future profit that cheating would sacrifice, while through (10) and (11) the recursive nature of the model may be exploited so as explicitly to calculate the cost of a boycott (i.e. the value of a good reputation).

The next step is to solve (10) and (11) for $\overline{\omega}_b$ and $\overline{\omega}_r$, which yields

$$\overline{\omega}_b = \{\omega(P_b)[1-(1-\lambda)\delta r]/\delta + \omega(P_r)\rho r\}\Delta \tag{12}$$

$$\overline{\omega}_r = \{\omega(P_r)[1-(1-\rho)\delta b]/\delta + \omega(P_b)\lambda b\}\Delta, \tag{13}$$

where

$$\Delta = \frac{\delta}{[1-(1-\lambda)\delta r][1-(1-\rho)\delta b] - \delta^2\lambda b\rho r}. \tag{14}$$

Simple calculations indicate that $\Delta > 0$ and that Δ increases strictly in δ for $\delta \in (0, 1/b)$.[13] Substituting (12), (13) and (14) back into (8) and (9), I am now able to write the two incentive constraints in terms of the known functions, Ω and ω:

$$\Omega(P_b) \leqslant \{\omega(P_r)\rho r + \omega(P_b)b[1-\rho-\delta r(1-\lambda-\rho)]\}\Delta \tag{15}$$

$$\Omega(P_r) \leqslant \{\omega(P_b)\lambda b + \omega(P_r)r[1-\lambda-\delta b(1-\lambda-\rho)]\}\Delta. \tag{16}$$

Thus, as (15) indicates, the incentive to cheat in a boom period must be no greater than the expected discounted loss in future profit that would occur once the firm lost its good reputation and faced a boycott; moreover, this loss is a weighted average of the profit lost in future boom and recession periods, with the associated weights reflecting the expected duration in each of the respective types of period. A similar interpretation applies for inequality (16) in recession periods.

Equilibria of the Markov growth game

With the basic incentive constraints now in place, I next characterize the set of TPE for the Markov growth game. Let $P_b = B(P_r)$ be the function yielding combinations of boom- and recession-period prices that satisfy the boom-period incentive constraint (15) with equality. Similarly, the function $P_r = R(P_b)$ describes the prices that satisfy with equality the recession-period incentive constraint (16). I now characterize these functions and thereby determine the set of TPE for the Markov growth game.

The following lemma describes the basic features of the function B.

Lemma 1. In the Markov growth game,

(i) there exists $\underline{P}_r \in (0, P^*)$ such that $B(\underline{P}_r) = P^*$;
(ii) for $P_r \in [\underline{P}_r, P^*)$, $B(P_r) > 0 > B'(P_r)$.

Thus, the function B is negatively sloped.[14] Intuitively, as the price P_r rises, the cost of a boycott also rises, since greater profit is then earned in future recession periods by reputable firms; as a consequence, a lower boom-period price P_b can be charged without violating the boom-period incentive constraint, even though the lower price increases the associated incentive to cheat. The function R may be described similarly.

Lemma 2. In the Markov growth game,

(i) there exists $\underline{P}_b \in (0, P^*)$ such that $R(\underline{P}_b) = P^*$;
(ii) for $P_b \in [\underline{P}_b, P^*)$, $R(P_b) > 0 > R'(P_b)$.

Lemmas 1 and 2 are proved in the Appendix.

With functions B and R defined and described, I now examine the relationship between the two functions. To this end, it is useful to define price P_b^* as the price at which $P = B(P)$; in other words, P_b^* is the price that, if charged in both boom and recession periods, would satisfy the boom-period incentive constraint with equality. Similarly, price P_r^* satisfies $P = R(P)$ and is thus the price at which the recession-period incentive constraint binds, when a single price is charged in both boom and recession periods. The following lemma now may be stated.

Lemma 3. In the Markov growth game,

(i) $\underline{P}_r < R(P^*)$ and $\underline{P}_b < B(P^*)$;
(ii) at any (P_r, P_b) such that $P_b = B(P_r)$ and $P_r = R(P_b)$, $B'(P_r) > 1/R'(P_b)$;
(iii) $P_r^* \in (c, P^*)$, $P_b^* \in (c, P^*)$ and $\text{sign}\{P_r^* - P_b^*\} = \text{sign}\{1 - \lambda - \rho\}$.

Part (i) of the lemma guarantees that functions B and R intersect somewhere, while the 'single-crossing' condition (ii) ensures that they intersect at most once. To understand part (iii), recall from the stationary growth model that a faster growth rate raises the cost of a boycott, making firms willing to endure lower equilibrium prices. As derived in (7), when growth rates are positively correlated ($1 - \lambda - \rho > 0$), the expected future growth rate is faster when the current period is a boom period, and so the boom-period incentive constraint can be satisfied at a lower acyclic ($P_b = P_r$) price than can be the recession-period incentive constraint. The zero correlation ($1 - \lambda - \rho = 0$) and negative correlation ($1 - \lambda - \rho < 0$) cases may be understood similarly. Lemma 3 is proved in the Appendix.

Using Lemmas 1, 2 and 3, it is now possible to illustrate graphically functions B and R. This is done in Figures 2(a), 2(b) and 2(c), which correspond to the cases of positive correlation, negative correlation and zero correlation, respectively. In each case, the set of prices that satisfy both the boom-period incentive constraint (15) and the recession-period incentive constraint (16) is denoted by a shaded region. This region therefore also represents the set of prices that can be supported in TPE of the Markov growth game.[15] Clearly, a great many TPE outcomes are possible; moreover, given the negative slopes that the incentive constraints exhibit, it is not obvious which price pairing is preferred by the consumers. When a constraint binds, a lower recession-period price is possible only if the boom-period price is raised, and without an explicit representation of consumer preferences it is impossible to determine whether this trade-off benefits consumers.[16]

III. Consumers' Preferred Equilibria of the Markov Growth Game

In this section I derive a preference function for consumers and then characterize the consumers' preferred TPE of the Markov growth game. Consumer preferences are represented in terms of the discounted expected

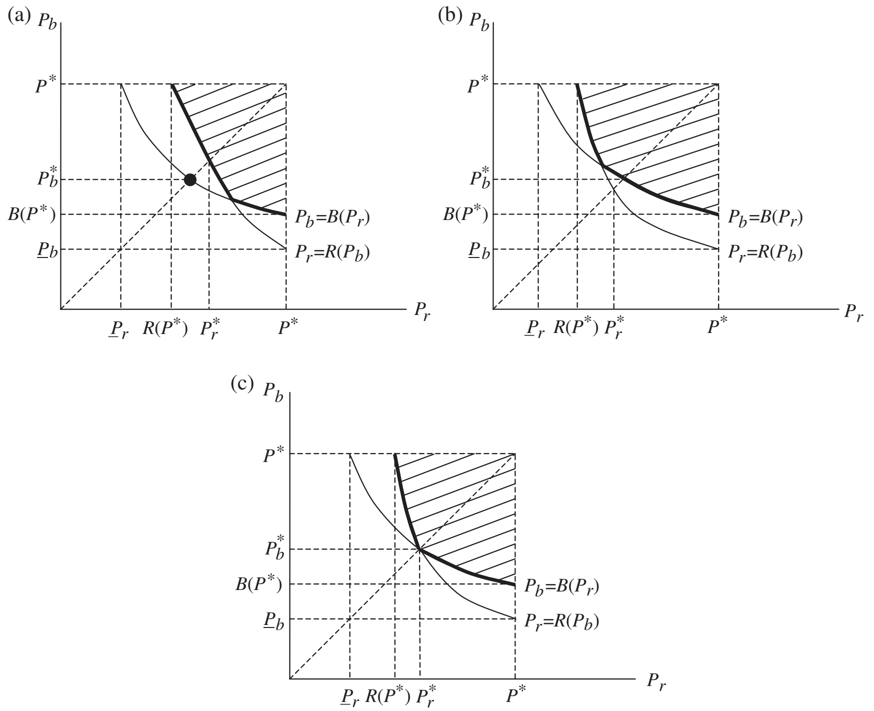

FIGURE 2. (a) Positive correlation, (b) Negative correlation, (c) Zero correlation.

indirect utility enjoyed by consumers in aggregate. After deriving this preference function, I characterize its properties and find that the consumers' preferred TPE has countercyclical prices when growth rates are positively correlated.

Consumers' preference function

I begin by deriving an expression for the discounted expected aggregate indirect utility that consumers enjoy. To this end, let $\overline{v}_b(P_b, P_r)$ denote the expected discounted aggregate indirect utility in period t and thereafter, if period t is a boom period ($g_t = b$), the measure of aggregate consumers in period t is unity ($G_t = 1$), and the prices P_b and P_r are charged by firms. Analogously, $\overline{v}_r(P_b, P_r)$ may be defined for recession periods. With some simplification in notation, these functions satisfy the following recursive relationships:

(17) $$\overline{v}_b = V(P_b) + \delta\{\rho r\,\overline{v}_r + (1-\rho)b\,\overline{v}_b\},$$

(18) $$\overline{v}_r = V(P_r) + \delta\{\lambda b\,\overline{v}_b + (1-\lambda)r\,\overline{v}_r\}.$$

Solving (17) and (18), I obtain

(19) $$\overline{v}_b = \{V(P_b)[1-(1-\lambda)\delta r]/\delta + V(P_r)\rho r\}\Delta,$$

(20) $$\overline{v}_r = \{V(P_r)[1-(1-\rho)\delta b]/\delta + V(P_b)\lambda b\}\Delta.$$

Since the probability that period 1 is a boom is given by μ, the expected discounted aggregate indirect utility for the whole game is given by

$$W(P_b, P_r) = \mu\,\bar{v}_b + (1-\mu)\,\bar{v}_r, \tag{21}$$

where I have set $G_1 = 1$ for convenience.

The value that probability μ takes has not yet been specified. Hamilton (1989) recommends $\mu = \lambda/(\lambda+\rho)$. In support of this recommendation, he considers the unconditional probability (i.e. the probability as viewed from period 1) that period t will be a boom period, and he deduces that this probability takes the asymptotic value of $\lambda/(\lambda+\rho)$. Hamilton also demonstrates that the unconditional probability of a boom is in fact $\lambda/(\lambda+\rho)$ for any period t, provided that the system is initiated with $\mu = \lambda/(\lambda+\rho)$. Absent any special information about period 1, therefore, it seems appropriate to follow Hamilton's recommendation and set $\mu = \lambda/(\lambda+\rho)$.

With this, the expected discounted aggregate indirect utility is given by

$$W(P_b, P_r) = [\lambda/(\lambda+\rho)]\,\bar{v}_b + [\rho/(\lambda+\rho)]\,\bar{v}_r, \tag{22}$$

which, using (19) and (20) takes the final form

$$\mathrm{W}(\mathrm{P}_b, \mathrm{P}_r) = \frac{\{[1-\delta r(1-\lambda)+\delta b\rho]\frac{\lambda}{\lambda+\rho}V(P_b) + [1-\delta b(1-\rho)+\delta r\lambda]\frac{\rho}{\lambda+\rho}V(Pr)\}\Delta}{\delta}. \tag{23}$$

Observe that function W is decreasing in both P_b and P_r, as would be expected. Using (23), it is convenient to define an indifference curve from the relationship $W(P_b, P_r) = k$. Letting this function be denoted by $P_b = I(P_r)$, it follows that $I'(P_r) < 0$.

Consumers' preferred equilibrium prices

The process of identifying the consumers' preferred TPE outcome can be understood with reference to Figures 2(a)–(c). The incentive constraints depicted there illustrate the TPE outcomes that are possible, and the preferences that consumers have with respect to these possibilities could be imposed on the same graph in terms of the downward-sloping indifference curves derived from the function W in (23). Simple calculations reveal that the indifference curves are also convex.

Let the consumers' preferred equilibrium prices be denoted as P_b^c and P_r^c. Graphically, these are the prices that place consumers on an indifference curve as close to the origin as possible, while still satisfying both incentive constraints. Given the convexity of the indifference curves and the possible convexity of the incentive constraints, corner solutions are a natural concern. As I discuss below, an interior solution is possible if the incentive constraints exhibit greater curvature than the indifference curve. As is demonstrated in the Appendix, a sufficient condition for an interior solution is found in the following assumption:

$$\psi'(P) < 0 \text{ for } P < P^*, \tag{24}$$

where the function $\psi(P)$ is defined by $\psi(P) = (P-c)D'(P)/D(P)$. This assumption holds, for example, if $P \geqslant c$ and $D''(P) \leqslant 0$. It also holds for all $P \geqslant 0$ if $D(P)$ is linear and the competitive output is positive, $D(c) > 0$.[17]

With this assumption in place, the following lemma may be stated.

Lemma 4. In the Markov growth game,

(i) for $P_r \geqslant P_b$, at any (P_r, P_b) such that $P_b = B(P_r)$ and $P_b = I(P_r)$, $I'(P_r) < B'(P_r)$;
(ii) for $P_b \geqslant P_r$, at any (P_r, P_b) such that $P_r = R(P_b)$ and $P_b = I(P_r)$, $I'(P_r) > 1/R'(P_b)$.

Thus, at any common point of intersection for which $P_r \geqslant P_b$, the indifference curve associated with the function W is steeper than the curve associated with the boom-period incentive constraint. Similarly, when $P_b \geqslant P_r$ and the recession-period incentive constraint intersects an indifference curve, the indifference curve is flatter. A proof of Lemma 4 is found in the Appendix.

The incentive constraints and indifference curves are featured together in Figures 3(a),(b) and (c), where the figures correspond to the cases of positive correlation, negative correlation and zero correlation, respectively. Consider first the case of positive correlation, as depicted in Figure 3(a). The incentive constraints cross at point X below the 45° line in this event, as Lemma 3 requires, and the featured indifference curve is steeper than the boom-period incentive constraint at point A and flatter than the recession-period incentive constraint at point B, as Lemma 4 requires. Clearly, the indifference curve passing through the points A and B does not correspond to the preferred TPE for consumers; i.e. the prices associated with point X yield higher consumer welfare. In fact, this argument establishes that, in the consumers' preferred

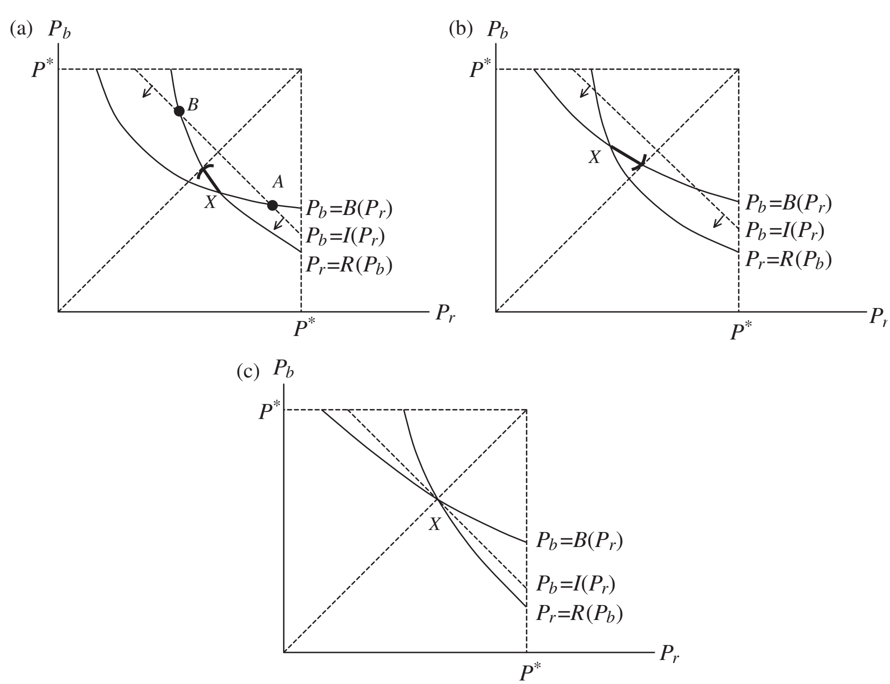

FIGURE 3. (a) Positive correlation, (b) Negative correlation, (c) Zero correlation.

TPE outcome, the recession-period incentive constraint binds, and the equilibrium prices are located at or above point X but below the 45° line. This set is marked in bold in Figure 3(a).

The case of negative correlation is handled similarly in Figure 3(b). Notice here that the point X is located above the 45° line, in keeping with the requirement of Lemma 3. The argument is otherwise the same as for the case of positive correlation. In the consumers' preferred TPE, when growth rates are negatively correlated, the equilibrium prices are located above the 45° line but below or on the point X. The set marked in bold in Figure 3(b) illustrates the set from which the consumers' preferred equilibrium prices must be drawn. Finally, in the case of zero correlation, point X occurs on the 45° line, and in fact the consumers' preferred equilibrium prices are found at point X. This case is illustrated in Figure 3(c).

Except in the case of zero correlation, I have not identified the precise location of the equilibrium prices that consumers prefer; nevertheless, the analysis conducted here does reveal the cyclical properties of these prices. The following proposition summarizes:

Proposition 2. In the TPE of the Markov growth game, the consumers' preferred equilibrium prices, P_b^c and P_r^c, are below the monopoly level and

(i) countercyclical ($P_b^c < P_r^c$) when growth rates are positively correlated;
(ii) procyclical ($P_b^c > P_r^c$) when growth rates are negatively correlated;
(iii) acyclic ($P_b^c = P_r^c$) when growth rates have zero correlation.

Of these various possibilities, the situation of positive correlation seems especially salient, suggesting that the possibility of countercyclical pricing is most relevant.[18]

To understand the main ideas, it is useful to consider the positive correlation case. When the business cycle is described by positive correlation in growth rates, the probability of fast growth in the future is larger if the current period is already a boom period. This implies that the benefit to a firm of a good price reputation is especially high during a boom period, and as a consequence the firm will tolerate a low equilibrium price in a boom period, even though the temptation to cheat with a higher price is then acute. Given the firm's asymmetric willingness to endure lower equilibrium prices during boom periods, consumers do not find acyclic prices most attractive. Since the firm will endure a lower equilibrium boom-period price without requiring much of an increase in the equilibrium recession-period price, the consumers' preferred equilibrium prices are countercyclical.

In the above discussion, the consumers' preferred equilibrium prices are defined as those that maximize the discounted expected indirect utility enjoyed by consumers in aggregate. This is a natural measure of consumer welfare, but other measures may be plausible candidates for analysis as well. Fortunately, as Figures 2(a)–2(c) suggest, the main findings are robust across a large range of consumer welfare measures. In particular, observe that the set of TPE is kinked, with the kink occurring in the countercyclical (procyclical) region when growth rates have positive (negative) correlation.

IV. Pricing and Heterogeneous Consumers

In the analysis presented above, consumers observe firms' past pricing decisions perfectly, and a firm makes no sales once it acquires a bad price reputation. The assumption that firms with bad reputations make no sales is convenient but strong; therefore it is important to demonstrate that the main results are robust to less extreme consumer behaviour. I develop in this section a model with heterogeneous consumers, in which high-price firms lose some but not all sales, and confirm that the main results developed above continue to hold in this alternative modelling framework.

There are many ways in which the possibility of heterogeneous consumer behaviour might be introduced. The essential feature that the modified model must include is that a firm experiences only a partial loss in future market share when it raises its current price above the 'going' (i.e. equilibrium) level. For example, it may be that in any period a fraction of consumers are uninformed 'tourists', or one-time buyers, who lack information as to firms' respective price reputations. Alternatively, it may be that non-price considerations, such as locational preferences or switching costs, are paramount for the visitation decisions of some consumers. In all of these situations, a firm that has priced opportunistically in the past nevertheless makes some sales in the present.

Consistent with these possibilities, I assume that in any period a fraction $\beta \in [0, 1)$ of consumers follows an exogenous search rule and divides up evenly over firms. The behaviour of the remaining fraction $1 - \beta$ of consumers is endogenous to the model. In particular, these consumers satisfy the informational assumptions developed above and adopt utility-maximizing strategies. I define this modified game as the *Markov growth game with heterogeneous consumers*. I focus on the TPE of this game, in which firms select a price P_b (P_r) in boom (recession) periods, and a fraction $1 - \beta$ of consumers in any period rationally employ the loyalty–boycott strategy. After characterizing the TPE, I return to the different interpretations available for the β 'exogenous' consumers and describe the assumptions under which their behaviour can be endogenized as part of an equilibrium strategy.

As before, in any period t of a TPE, a firm faces a dynamic trade-off. The firm can increase its short-term profit by deviating and increasing the period t price to the monopoly level, P^*. In doing so, however, it will suffer a loss in future period sales, and in fact the number of consumers to whom it sells in any future period will be a fraction β of those to whom it would have sold had it not pursued greater short-term profits. Consider now the price that the firm should charge in these future periods, given that it has already acquired a bad price reputation. Since the consumers to whom it sells in these periods are assumed to be 'captive' buyers, a firm with a bad reputation does best by continuing to select the monopoly price in all future periods.[19] The firm thus trades off the expected discounted equilibrium profits against the expected discounted profits available under the optimal deviation strategy, in which the firm prices at P^* in the current and all future periods.

To represent this trade-off formally, I first define

$$(25) \qquad \widetilde{\omega}(P) = [\pi(P) - \beta\pi(P^*)]/n = \omega(P) - \beta\omega(P^*)$$

as the per-period and per-aggregate-consumer cost of a future boycott. The function $\widetilde{\omega}(P)$ is increasing and concave for $P < P^*$, positive when $P = P^*$ and non-positive when $P = c$. With this definition in place, the derivation of incentive constraints follows that developed previously, but with function $\widetilde{\omega}(P)$ replacing function $\omega(P)$. Using (15) and (16), it thus follows that the incentive constraints now take the form

$$\Omega(P_b) \leqslant \{\widetilde{\omega}(P_r)\rho r + \widetilde{\omega}(P_b)b[1 - \rho - \delta r(1 - \lambda - \rho)]\}\Delta, \tag{26}$$

$$\Omega(P_r) \leqslant \{\widetilde{\omega}(P_b)\lambda b + \widetilde{\omega}(P_r)r[1 - \lambda - \delta b(1 - \lambda - \rho)]\}\Delta. \tag{27}$$

Of course, this gives back the original incentive constraints (15) and (16) when $\beta = 0$.

Inspecting (25), (26) and (27), it is apparent that the incentive constraints characterizing the TPE of the Markov growth game with heterogeneous consumers is the same as that derived previously in (15) and (16) for the original Markov growth game, except that the right-hand side of each constraint differs by a constant amount. Lemma 5 follows directly from this observation.

Lemma 5. In the Markov growth game with heterogeneous consumers, Lemmas 1–4 and Proposition 2 continue to hold.

In other words, all of the results derived above remain true when a fraction β of consumers continue to purchase from a firm with a bad price reputation. The predictions of the model are thus robust to the possibility that a firm loses some but not all future sales following an episode of price gouging. This lemma is proved in the Appendix.

I now return to the interpretation of the β exogenous consumers. Their behaviour can be endogenized in several interesting and distinct models. One possibility, for example, is that the market is described in any period by a fraction β of consumers who are ‘tourists’, i.e. who purchase only once and lack information as to firm’s respective price reputations. The remaining fraction $1 - \beta$ of consumers are then ‘locals’, who purchase in the market repeatedly and thus are well informed about existing price reputations. In this case, tourists are rational in picking a firm at random, since firms adopt symmetric equilibrium strategies and tourists are unable to observe historic deviations.

Other possibilities also exist. For instance, it may be that firms are differentiated in non-price dimensions, such as location, and that this dimension is paramount to a fraction β of consumers, while the remaining fraction $1 - \beta$ of consumers are ‘price-sensitive’ and uninterested in non-price considerations. The β ‘price-insensitive’ consumers are then rational in dividing themselves evenly among firms, if product differentiation is modelled in a symmetric way, so that each firm is attractive to $(1/n)$th of the β price-insensitive consumers. A final possibility is that a fraction β of the consumers have high ‘switching costs’ and find it prohibitively costly to switch from one firm to another. If in addition each such consumer lacks information as to firms’ price reputations when he or she enters the market, then the consumer is rational in initially picking a firm at random and then remaining with that firm for the duration, regardless of the firm’s subsequent price behaviour.

V. Conclusion

In many markets, consumers cannot costlessly observe all current-period prices, and their search decisions are guided by past pricing experiences and firms' price reputations. When consumers behave in this fashion, a firm's current-period sales are determined both by its past pricing performance and its current price level. An important implication is that firms face a dynamic trade-off in their pricing decisions: while a higher current-period price may raise profits in the short term, it will also result in fewer sales and thus lower profits in the long term. This suggests in turn that pricing decisions will be sensitive to current and expected future market conditions. Arguing in this fashion, it is then possible to forge a link between firms' prices and the state of the business cycle. In particular, I find that prices in the consumers' preferred equilibrium have two interesting properties: (1) they lie below the monopoly level, and (2) they are countercyclical if demand growth rates are positively correlated through time.

The finding of countercyclical pricing suggests that imperfect competition as represented by customer markets offers a possible reconciliation for the combination of procyclical real wages and demand-driven business cycle fluctuations. In this respect, the paper serves to provide an equilibrium foundation for arguments developed previously in the classic works of Phelps and Winter (1970) and Okun (1975, 1980). The findings developed here also reveal interesting distinctions between customer and collusive markets. In their analysis of collusion, Bagwell and Staiger (1997) also employ the Markov growth specification for the business cycle, but they argue that collusive prices are procyclical when demand growth rates are positively correlated through time. Assuming that the business cycle is well described by a Markov growth process with positively correlated demand growth rates, it follows that countercyclical markups may be more likely in markets in which price reputations play an important role than in markets in which firms collude.

The model presented in this paper is constructed to isolate and characterize the consequences of stochastic demand growth for pricing in customer markets. While a simple and focused model can reveal new insights, a number of additional influences may operate in actual markets. For example, the cyclical properties of markups may be affected by collusion, entry, capacity constraints and cyclical fluctuations in interest rates and marginal costs. In addition, actual markets may be characterized by price reputations that are not publicly held: a new consumer may have less information about a firm's past prices than an old consumer. If booms are associated with an influx of new consumers, a firm may have further incentive to reduce price in booms, so as to cultivate a low-price reputation among new consumers.[20] More generally, some of these influences may reinforce the predictions developed here, but others may yield countervailing effects. Interesting future research might examine the robustness of conclusions presented here to alternative modelling frameworks.

I emphasize here consumers' preferred equilibrium outcome. Given the basic premise that price reputations matter to firms, it is natural to explore the extreme case, wherein consumers use the loyalty–boycott strategy to enforce their preferred equilibrium outcome. The set of equilibria, however, is large. Fortunately, the analysis presented here (and in particular the location of the 'kink' in the set of TPE) suggests that the main findings are robust across a range of equilibrium selection criteria. Interesting future work might explore this suggestion in further detail.

APPENDIX

Proof of Lemma 1

If $P_b = P^*$, the boom-period incentive constraint (15) binds when

$$0 = \omega(P_r)\rho r + \omega(P^*)b[1 - \rho - \delta r(1 - \lambda - \rho)]. \tag{A1}$$

The solution to (A1) defines $\underline{P}_r$, and this solution is unique and satisfies $0 < \underline{P}_r < c$, since (1), (2) and (4) imply that $\omega(0) = -\infty < 0 = \omega(c)$ and that $\omega'(P) > 0$ for $P < P^*$. This proves part (i). Next, put $P_r = P^*$ so that (15) binds at $P_b = B(P^*)$, which must satisfy

$$\Omega(P_b) - \omega(P_b)b[1 - \rho - \delta r(1 - \lambda - \rho)]\Delta = \omega(P^*)\rho r\Delta. \tag{A2}$$

Let the LHS of (A2) be denoted as $G_b(P_b)$ and observe that $G_b(P^*) < 0 < \omega(P^*)\rho r\Delta < G_b(0) = \infty$ and that $G_b'(P) < 0$ for $P < P^*$. Thus, $B(P^*) \in (0, P^*)$ is uniquely defined. The proof of part (ii) is now complete if $B'(P_r) < 0$ for $P_r \in (\underline{P}_r, P^*)$. To see that this is true, differentiate the binding (15) to arrive at

$$B'(P_r) = \frac{\omega'(P_r)\rho r\Delta}{\Omega'(P_b) - \omega'(P_b)b[1 - \rho - \delta r(1 - \lambda - \rho)]\Delta}. \tag{A3}$$

The result then follows, since the numerator is positive over the given domain while the denominator is negative for $P_b < P^*$ and zero when $P_b = P^*$. □

Proof of Lemma 2

If $P_r = P^*$, the recession-period incentive constraint (16) binds when

$$0 = \omega(P_b)\lambda b + \omega(P^*)r[1 - \lambda - \delta b(1 - \lambda - \rho)]. \tag{A4}$$

The solution to (A4) defines $\underline{P}_b$, and this solution is unique and satisfies $0 < \underline{P}_b < c$ by an argument analogous to that given in the proof of Lemma 1. This proves part (i). Next, put $P_b = P^*$ so that (16) binds at $P_r = R(P^*)$, which must satisfy

$$\Omega(P_r) - \omega(P_r)r[1 - \lambda - \delta b(1 - \lambda - \rho)]\Delta = \omega(P^*)\lambda b\Delta. \tag{A5}$$

Let the LHS of (A5) be denoted as $G_r(P_r)$ and observe that $G_r(P^*) < 0 < \omega(P^*)\rho r\Delta < G_r(0) = \infty$ and that $G_r'(P) < 0$ for $P < P^*$. Thus, $R(P^*) \in (0, P^*)$ is uniquely defined. The remaining step in proving part (ii) is to show that $R'(P_b) < 0$ for $P_b \in (\underline{P}_b, P^*)$. Differentiation of the binding (16) reveals that

$$R'(P_b) = \frac{\omega'(P_b)\lambda b\Delta}{\Omega'(P_r) - \omega'(P_r)r[1 - \lambda - \delta b(1 - \lambda - \rho)]\Delta}. \tag{A6}$$

The numerator is positive over the given domain, the denominator is negative for $P_r < P^*$, and the denominator is zero for $P_r = P^*$. It follows that $R'(P_b) < 0$ for $P_b \in (\underline{P}_b, P^*)$. □

Proof of Lemma 3

To prove part (i), set $P_b = P^*$ and recall from (A1) and (A5) that $\underline{P}_r$ and $R(P^*)$ are respectively defined by

$$0 = \omega(\underline{P}_r)\rho r + \omega(P^*)b[1 - \rho - \delta r(1 - \lambda - \rho)], \tag{A7}$$

$$\Omega(R(P^*)) - \omega(R(P^*))r[1 - \lambda - \delta b(1 - \lambda - \rho)]\Delta = \omega(P^*)\lambda b\Delta. \tag{A8}$$

Using the definition of $\Omega(P)$ provided in (3), it follows that $\Omega(P) = \omega(P^*) - \omega(P)$, and so (A8) may be rewritten as

$$\text{(A9)} \qquad \omega(P^*)[1 - \lambda b\Delta] = \omega(R(P^*))\{r[1 - \lambda - \delta b(1 - \lambda - \rho)]\Delta + 1\}.$$

It thus follows that $\underline{P}_r$ and $R(P^*)$ satisfy

$$\text{(A10)} \qquad \omega(\underline{P}_r) = \frac{-\omega(P^*)b[1 - \rho - \delta r(1 - \lambda - \rho)]}{\rho r}$$

$$\text{(A11)} \qquad \omega(R(P^*)) = \frac{\omega(P^*)[1 - \lambda b\Delta]}{r[1 - \lambda - \delta b(1 - \lambda - \rho)]\Delta + 1}.$$

Hence, if $1 - \lambda b\Delta \geqslant 0$, then $R(P^*) \geqslant c > \underline{P}_r$, in conformity with Lemma 3(i). Suppose then that $1 - \lambda b\Delta < 0$. In this case, $R(P^*) > \underline{P}_r$ follows if

$$\text{(A12)} \qquad [1 - \lambda b\Delta]\rho r > -b[1 - \rho - \delta r(1 - \lambda - \rho)]\{r[1 - \lambda - \delta b(1 - \lambda - \rho)]\Delta + 1\}.$$

With further simplification, (A12) is equivalent to $0 > -b(1 - \rho) - \rho r$, which is true.

Next, set $P_r = P^*$ and recall from (A2) and (A4) that $B(P^*)$ and $\underline{P}_b$ are respectively defined by

$$\text{(A13)} \qquad \Omega(B(P^*)) - \omega(B(P^*))b[1 - \rho - \delta r(1 - \lambda - \rho)]\Delta = \omega(P^*)\rho r\Delta,$$

$$\text{(A14)} \qquad 0 = \omega(\underline{P}_b)\lambda b + \omega(P^*)r[1 - \lambda - \delta b(1 - \lambda - \rho)].$$

Using again that $\Omega(P) = \omega(P^*) - \omega(P)$, (A13) and (A14) may be rewritten as

$$\text{(A15)} \qquad \omega(B(P^*)) = \frac{\omega(P^*)[1 - \rho r\Delta]}{b[1 - \rho - \delta r(1 - \lambda - \rho)]\Delta + 1},$$

$$\text{(A16)} \qquad \omega(\underline{P}_b) = \frac{-\omega(P^*)r[1 - \lambda - \delta b(1 - \lambda - \rho)]}{\lambda b}.$$

Clearly, if $1 - \rho r\Delta \geqslant 0$, then $B(P^*) \geqslant c > \underline{P}_b$, which is consistent with Lemma 3(i). If instead $1 - \rho r\Delta < 0$, then $B(P^*) > \underline{P}_b$ follows if

$$\text{(A17)} \qquad [1 - \rho r\Delta]\lambda b > -r[1 - \lambda - \delta b(1 - \lambda - \rho)]\{b[1 - \rho - \delta r(1 - \lambda - \rho)]\Delta + 1\}.$$

But calculations reveal that (A17) holds if and only if $0 > -\lambda b - r(1 - \lambda)$, which is true. This completes the proof of part (i).

Turning to part (ii), calculations reveal that (A3) and (A6) may be rewritten as

$$\text{(A18)} \qquad B'(P_r) = -\left(\frac{\rho r\delta}{1 - (1 - \lambda)\delta r}\right)\left(\frac{\pi'(P_r)}{\pi'(P_b)}\right),$$

$$\text{(A19)} \qquad 1/R'(P_b) = -\left(\frac{1 - (1 - \rho)\delta b}{\lambda b\delta}\right)\left(\frac{\pi'(P_r)}{\pi'(P_b)}\right),$$

so that $B'(P_r) > 1/R'(P_b)$ at a point of intersection if and only if

$$\text{(A20)} \qquad \frac{\rho r\delta}{1 - (1 - \lambda)\delta r} < \frac{1 - (1 - \rho)\delta b}{\lambda b\delta},$$

which in turn holds if and only if $\Delta > 0$, which is true.

Finally, to prove part (iii), define

$$\text{(A21)} \qquad f_b(P) = \omega(P)\{\rho r + b[1 - \rho - \delta r(1 - \lambda - \rho)]\}\Delta - \Omega(P)$$

$$\text{(A22)} \qquad f_r(P) = \omega(P)\{\lambda b + r[1 - \lambda - \delta b(1 - \lambda - \rho)]\}\Delta - \Omega(P).$$

Observe from (A21) and (A22) that $f_b(P_b^*) = f_r(P_r^*) = 0$. Notice that $f_b(c) < 0 < f_b(P^*)$ and $f_b'(P) > 0$ for $P \in [c, P^*)$, and so $P_b^* \in (c, P^*)$ exists uniquely. A similar argument applies for P_r^*. Furthermore, calculations reveal that

$$f_b(P_r^*) = f_b(P_r^*) - f_r(P_r^*) = \omega(P_r^*)\Delta(b-r)(1-\lambda-\rho), \tag{A23}$$

and so $\operatorname{sign}\{P_r^* - P_b^*\} = \operatorname{sign}\{1-\lambda-\rho\}$, which proves part (iii). □

Proof of Lemma 4

Using (A7), calculations reveal that

$$I'(P_r) = -\frac{1-\delta b(1-\rho)+\lambda\delta r}{1-\delta r(1-\lambda)+\rho\delta b}\frac{\rho}{\lambda}\frac{D(P_r)}{D(P_b)}. \tag{A24}$$

Furthermore, using (A18), (A19) and $\pi'(P) = D(P)[1+\psi(P)]$, we set

$$B'(P_r) = -\frac{\rho r\delta}{1-(1-\lambda)\delta r}\frac{1+\psi(P_r)}{1+\psi(P_b)}\frac{D(P_r)}{D(P_b)}, \tag{A25}$$

$$1/R'(P_b) = -\frac{1-(1-\rho)\delta b}{\lambda b\delta}\frac{1+\psi(P_r)}{1+\psi(P_b)}\frac{D(P_r)}{D(P_b)}. \tag{A26}$$

Now, put $P_r \geqslant P_b$ and observe that $I'(P_r) < B'(P_r)$ at a point of intersection if and only if

$$\begin{aligned}[1-\delta b(1-\rho)+\lambda\delta r][1-(1-\lambda)\delta r][1+\psi(P_b)] > \lambda r\delta[1-\delta r(1-\lambda)\\ +\rho\delta b][1+\psi(P_r)].\end{aligned} \tag{A27}$$

Under (24), $1+\psi(P_b) \geqslant 1+\psi(P_r) \geqslant 0$, with the final inequality following from (1) and (2) since $[1+\psi(P)] = \pi'(P)/D(P)$. It thus suffices to show that

$$[1-\delta b(1-\rho)+\lambda\delta r][1-(1-\lambda)\delta r] > \lambda r\delta[1-\delta r(1-\lambda)+\rho\delta b]. \tag{A28}$$

But calculations reveal that (A28) is equivalent to $\Delta > 0$, which is true. This proves part (i).

Suppose next that $P_b \geqslant P_r$ and observe that $I'(P_r) > 1/R'(P_b)$ at a point of intersection if and only if

$$\begin{aligned}[1-\delta r(1-\lambda)+\rho\delta b][1-(1-\rho)\delta b][1+\psi(P_r)] > \rho b\delta[1-\delta b(1-\rho)\\ +\lambda\delta r][1+\psi(Pb)].\end{aligned} \tag{A29}$$

Arguing as above, in this case $1+\psi(P_r) \geqslant 1+\psi(P_b) \geqslant 0$, and so it suffices to find that

$$[1-\delta r(1-\lambda)+\rho\delta b][1-(1-\rho)\delta b] > \rho b\delta[1-\delta b(1-\rho)+\lambda\delta r], \tag{A30}$$

which is equivalent to $\Delta > 0$. This proves part (ii). □

Proof of Lemma 5

Recall that $\tilde{\omega}(P) = \omega(P) - \beta\omega(P^*)$, from which it follows that $\tilde{\omega}(0) = -\infty < 0 < \tilde{\omega}(P^*)$, and $\tilde{\omega}'(P) = \omega'(P) > 0$ for $P \in [0, P^*)$. It is now easy to show that Lemmas 1 and 2 hold, since the same arguments applied to (15) and (16) apply also to (26) and (27). (Given than $\tilde{\omega}(c) \leqslant 0$, it is no longer necessary that $\underline{P}_r < c$ and $\underline{P}_b < c$, however.) Next, the proof of Lemma 3 also carries over to (26) and (27), once $\tilde{\omega}(P_r^*) > 0$ is established. But this follows, since $P_r^* < P^*$ remains true, and so $\Omega(P_r^*) > 0$, whence (A21) (expressed with $\tilde{\omega}$ replacing ω) implies that $\tilde{\omega}(P_r^*) > 0$. Finally, Lemma 4 and Proposition 2 carry over as is, since $\tilde{\omega}'(P) = \omega'(P)$ (slopes are not affected by β). □

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NOTES

1. See e.g. Murphy *et al.* (1989); Rotemberg and Woodford (1991); Solon *et al.* (1994).
2. The pioneering paper in this literature is by Rotemberg and Saloner (1986), who model the business cycle in terms of i.i.d. shocks to the level of demand. Other extensions of this model include those by Haltiwanger and Harrington (1991), Kandori (1991) and Staiger and Wolak (1992).
3. This conclusion has tentative support in the empirical literature, if it is granted that collusion is more likely in concentrated industries. For example, Domowitz *et al.* (1986a,b) report evidence that markups are procyclical in concentrated industries and countercyclical in unconcentrated industries.
4. Other contributions to the literature on switching costs include Beggs and Klemperer (1992), Farrell and Shapiro (1988), Fishman and Rob (1995) and Rotemberg and Woodford (1991). Alternative models in which firms seek to develop low-price reputations are explored by Bagwell (1987), Bagwell and Ramey (1994) and Bagwell *et al.* (1997). The second paper is more related to the present effort. It is shown there that customer markets generate procyclical markups when demand fluctuations are modelled as i.i.d. shocks. Prices also may move cyclically when consumers lack information as to product quality, as Allen (1988), Bils (1989) and Stiglitz (1984) discuss. Finally, the classic paper by Klein and Leffler (1981) warrants mention: they show that reputational considerations may motivate firms to provide high-quality products. I emphasize price (rather than quality) reputations and introduce business cycle fluctuations.
5. The assumptions in (1) and (2) are all satisfied, say, if each consumer has a quasi-linear utility function of the form $U(X, Y) = \ln(X + k) + Y$, where Y denotes units of a numeraire good, X denotes units of the good studied here and k is a parameter satisfying $1/k > c$. With each consumer facing a budget constraint $PX + Y = 1$, the demand function is $D(P) = 1/P - k$. This function satisfies the assumptions in (1), with the choke price given as $1/k$. The assumptions in (2) are also satisfied, with $P^* = (c/k)^{1/2}$. The indirect utility function is then $V(P) = \ln(1/P) + kP$.
6. This is an instance of the well-known 'Diamond Paradox' (Diamond 1971).
7. This interpretation is consistent with that offered by Phelps and Winter for their model. They write: 'Over time, customers gradually shift from firms charging higher prices to those charging lower prices. The formulation of this dynamic process offered here is suggested by the thought that the process by which information about prices is transmitted is essentially one of "comparing notes" in the course of random encounters among consumers' (Phelps and Winter 1970, p. 311). The assumption of perfect historic price information is relaxed in Section IV.
8. Observe that the loyalty–boycott strategy can emerge as part of an equilibrium strategy only if firms adopt symmetric equilibrium pricing strategies. The loyalty–boycott strategy is appealing for two reasons: (1) it is simple, and (2) it inflicts upon non-reputable firms the maximal punishment (no future sales), thus making possible a large equilibrium set. With regard to the latter point, see Abreu (1988) for a discussion of 'optimal penal codes' in general games.
9. Consider, e.g. the following strategies. Consumers adopt the loyalty–boycott strategy, and each firm always chooses the equilibrium price P, no matter what history has unfolded. Consumers are then completely indifferent in each period as to which firm to visit, and they are therefore rational in employing the loyalty–boycott strategy. When consumers employ this strategy and a given firm has not previously deviated, that firm is also rational in selecting the price P, provided that $P \in [P^c, P^*]$. If the firm has previously deviated, then it gets no business in the future in any event, and so a policy of continuing to price at P is as good as any other. Alternatively, strategies supporting the given equilibrium outcome can be designed in which consumers strictly prefer not to return to a firm with a bad reputation. For example, a non-reputable firm may set the price P^* (perhaps hoping that some consumers will 'tremble in'). See Section IV for an extended model in which equilibrium strategies necessitate that non-reputable firms charge price P^*.
10. The analysis developed here builds from a stage game in which consumers and firms make simultaneous decisions. In effect, the assumption of simultaneous moves means that the cost to a consumer of sequential search within a pricing period is prohibitive. This is a simple means of capturing the idea that search costs generate short-run monopoly power. If the model were extended to allow for a lower search cost, then a firm's incentive to cheat

would be reduced, since consumers would refuse to purchase from the firm even in the current period if the price hike were too large. Consequently, when the search cost is lowered, consumers are able to enforce an even lower preferred equilibrium price. For further discussion, see Bagwell and Ramey (1994).

11. An analogous argument is often made for highlighting the most collusive equilibrium outcome in models of collusion. The firms' preferred equilibrium outcome is in fact easily described in the search model developed here, as an equilibrium always exists in which firms price at the monopoly level.
12. In deriving the incentive constraints, I employ methods developed by Bagwell and Staiger (1997) in their analysis of collusion.
13. Let $\Delta \equiv \delta/L(\delta)$, where L is the denominator of the expression in (14). Simple calculations reveal that $L(0) = 1 > (b-r)/b = L(1/b)$ and $L'(\delta) < 0$ for $\delta \in [0, 1/b]$. It follows that $L(\delta) > 0$ for $\delta \in [0, 1/b]$. These properties ensure that $\Delta > 0$ and that Δ increases strictly in δ for $\delta \in (0, 1/b)$.
14. While Lemma 1 states that $\underline{P}_r \in (0, P^*)$, the proof in the Appendix demonstrates the stronger result that $\underline{P}_r \in (0, c)$. Similar remarks apply as well to $\underline{P}_b$ in Lemma 2. Lemmas 1 and 2 are stated in this weaker form so that they will continue to hold as stated when the model is extended in Section IV.
15. As discussed in note 9, there are many ways to specify the actual equilibrium strategies. A simple strategy specification entails consumers following the loyalty–boycott rule and a non-reputable firm setting the price P^*. Notice that the assumption that players are well informed about the business cycle comes into play here. Firms need to observe the current state of the business cycle, so that they know whether to charge P_b or P_r. Similarly, consumers may need information about past business cycle conditions in order to determine whether a firm cheated and sacrificed its reputation.
16. By contrast, in models of collusion, the identification of the most collusive equilibrium is relatively simple, once the incentive constraints are characterized. Incentive constraints slope upward in the standard collusion model, since a higher price in any one state raises the cost of a price war, making possible a higher price in the other state as well. No trade-off arises.
17. The assumption also holds as stated for the convex demand function, $D(P) = 1/P - k$, where $1/k > c$, discussed in note 5. The solution method proposed here is similar to that employed by Bagwell and Ramey (1994) in their analysis of the consumers' preferred equilibrium when business cycles are modelled in terms of i.i.d. shocks to demand.
18. For example, as discussed in the Introduction, in his analysis of US GNP data, Hamilton (1989) argues that growth rates are positively correlated.
19. Thus, in TPE of the Markov growth game with heterogeneous consumers, a firm necessarily selects price P^* in all periods following its deviation. This may be contrasted with the TPE of the original Markov growth game (and the SPE of the stationary growth game), where, following a deviation, a firm is indifferent as to its price choice, since no sales are made in any case. See notes 9 and 15.
20. An extended model of this kind might thus have some predictions that are analogous to those in the switching-cost literature (as discussed in the Introduction). I thank a referee for suggesting this extension.

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