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LACK-OF-RECALL AND CENTRALIZED MONETARY TRADE*

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We introduce *lack-of-recall* of past transactions as an alternative assumption to anonymity in a model where trade is centralized. In environments where there is an intertemporal lack-of-double-coincidence of wants problem and lack-of-commitment, lack-of-recall can give rise to monetary equilibria that dominate nonmonetary outcomes in terms of welfare.

1. INTRODUCTION

Several monetary models assume *anonymity* in order to rule out credit arrangements. Indeed, overlapping generations, turnpike, or random matching assumptions can be thought of as giving rise to a lack-of-double-coincidence problem in conjunction with anonymous trades.[2] In this article, we introduce *lack-of-recall* of past transactions as an alternative to anonymity and demonstrate that this can lead to money being useful in achieving desirable allocations.

In a related paper, Kocherlakota (1998) identifies "lack of public memory" as a *necessary* friction for money to be useful in a variety of models.[3] One difference between our model and the ones considered in Kocherlakota (1998) is that lack-of-commitment will be the only other friction in our model. Thus, the restriction on memory that we need to employ is stronger than lack of public memory. The reason for this is the following. In a model with centralized trade and a finite number of agents, each individual's *own* history may contain sufficient information about whether someone else has deviated from the equilibrium trade pattern in the past, since the resulting consumption basket will be slightly different even

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[2] References on these models include Wallace (1980, 1997), Townsend (1980), Kiyotaki and Wright (1989, 1993), Corbae et al. (2003), and Rocheteau and Wright (2003). Levine (1991) assumes anonymity directly.

[3] It is important to note that this does not readily imply that "lack of public memory" is *sufficient* for money to be useful. In general, stronger restrictions on memory, such as the one employed here, will be needed. Kocherlakota and Wallace (1998) bridge the extremes of no public memory and full public memory by introducing a parameter that determines the degree of public memory.

if one other agent deviates in a given period. Thus, under perfect recall, for high enough discount factors, one can easily demonstrate the existence of nonmonetary mechanisms that use this information to support desirable allocations by triggering a collective punishment. Unlike anonymity, lack-of-recall limits the applicability of such punishments.

Our analysis is based on an environment in which intertemporal trade is necessary for desirable allocations to be supported. Intrinsically useless fiat money may be valued playing, in a sense, the role of receipts, indicating that certain transfers of goods took place in the past. We present two examples showing that such objects may lead to improved allocations over the best allocations that can be supported in the absence of money.

Our first example is a trading game similar in spirit to the pure coordination mechanism in Kocherlakota and Wallace (1998). Under lack-of-recall, equilibrium behavior implies that intertemporal trades are excluded. We introduce money and demonstrate that there is a monetary equilibrium that supports a superior allocation through intertemporal trade. Our second example suggests how the lack-of-commitment/lack-of-recall friction can be incorporated within a Walrasian model. That lack-of-commitment is not inconsistent with markets has been used, for example, by Kehoe and Levine (1993), who consider a model in which individual rationality (IR) constraints ensure that agents are at no time better off reverting to autarky. We impose related IR constraints and demonstrate the existence of a monetary equilibrium, which supports an efficient allocation through intertemporal trade.

One way to think about our setup is as follows. At the beginning of time, agents agree on a system of economic interaction, having full knowledge of the economic environment, including the fact that there will be lack-of-recall of any past transactions. Agents might agree on a prescription for all future trades, say because the trades will lead to an efficient allocation. They *do* recall the prescription, but in any future period they will not recall whether the prescription was followed in the past; that is, there is no record of past transactions. We are looking for conditions that make it individually desirable for agents to follow the prescription at any given time. Money is one instrument that makes this possible.

2. TWO EXAMPLES

2.1. *A Trading Game.* In this section, we discuss how lack-of-recall of past transactions makes money useful within a centralized trading game. Time is discrete and the horizon is infinite. There is one perfectly divisible and nonstorable physical commodity in every period. There is a large but finite and equal number of each of two (types of) agents, $\mathbb{I} = \{1, 2\}$. To generate a need for intertemporal trade, we assume that the agents' endowment sequences are $e^1 = \{e_h, e_l, e_h, e_l, \ldots\}$ and $e^2 = \{e_l, e_h, e_l, e_h, \ldots\}$, with $e_h > e_l > 0$. Agents have a common utility function from consumption, denoted by $u(c_t)$, where u is smooth, monotonically increasing, and strictly concave. Agents discount the future at a common discount rate $\beta \in (0, 1)$. There is no uncertainty. Throughout the article, we impose lack-of-recall by assuming that, for all t, agents do not recall trades or utilities from periods $0, \ldots, t-1$.[4]

We consider a game and solution concept that are similar in spirit to the study of the pure coordination mechanism in Kocherlakota and Wallace (1998). The difference is that here everyone participates in the game (trade is centralized), whereas they study an environment in which agents are matched bilaterally and at random. Clearly, under lack-of-recall the only perfect equilibrium outcome is autarky, where $u(c_t^i) \geq u(e_t^i)$ is satisfied for all i and t.

Next, we introduce money and study a monetary trading mechanism. We assume that money consists of intrinsically useless, indivisible, perfectly storable fiat objects. Furthermore, we assume that each type 2 agent is endowed with one unit of money in period 1. Each player's information set includes his current money holdings, m_t. The mechanism works as follows. In each period, player i makes a trade proposal $s_t^i(m_t^i) = (l_t^i(m_t^i), b_t^i(m_t^i))$, where $l_t^i(m_t^i)$ is a monetary proposal, and $b_t^i(m_t^i)$ is consumption good proposal. A positive sign denotes a request and a negative sign denotes an offer.[5] Let $s = \{s_t^i(\cdot)\}_{i\in\mathbb{I}, t\in\mathbb{N}}$ denote a trade proposal profile. We impose the constraint that $\text{sign}[l_t^i(m_t^i)] = -\text{sign}[b_t^i(m_t^i)]$. Thus, in any given period, an agent has three options regarding trade. He can offer an amount of money and request an amount of good, offer an amount of good and request an amount of money, or he can stay in autarky for the period. The proposals are implemented iff $\sum_i l_t^i(m_t^i) = \sum_i b_t^i(m_t^i) = 0$. Autarky prevails otherwise. Let $v_t^i(m_t^i, s)$ be the discounted utility of player i with m_t^i units of money if he follows s_t^i. The following proposition guarantees the existence of an efficient monetary equilibrium. This equilibrium is more likely to exist if agents are sufficiently patient, so that they are willing to trade goods for money, or if the utility function is sufficiently concave, so that agents are willing to engage in trade in order to smooth consumption overtime.[6]

PROPOSITION 1. *Provided that β is sufficiently high, or u is sufficiently concave, there exists a monetary equilibrium with trade that Pareto dominates autarky. In any monetary equilibrium, $v_t^i(m_t^i, s) \geq u(e_t^i) + \beta v_{t+1}^i(m_t^i, s)$ is satisfied for all i and t.*

PROOF. We will argue that the s_t satisfying the following condition constitutes an equilibrium strategy profile. For all i and t, let

$$s_t^i(m_t^i) = \begin{cases} (-1, \frac{e_h - e_l}{2}) & \text{if } m_t^i = 1 \text{ and } e_t^i = e_l \\ (1, -\frac{e_h - e_l}{2}) & \text{if } m_t^i = 0 \text{ and } e_t^i = e_h \\ (0, 0) & \text{otherwise} \end{cases} \tag{1}$$

This proposal implies that an agent with high current endowment offers $\frac{e_h - e_l}{2}$ units of consumption good in exchange for one unit of money, which he uses

[4] See Kuhn (1953) for a discussion on bounded recall.

[5] We restrict $l_t^i(m_t^i) \geq -m_t^i$ and $b_{gt}^i(m_t^i) \geq -e_t^i$ to ensure feasibility. This is without loss of generality since we are implicitly assuming that any individually infeasible offer leads to autarky.

[6] As in Kocherlakota and Wallace (1998), our coordination mechanism has many equilibria. One corresponds to a nonmonetary arrangement (autarky) in which money is not valued. Any equilibrium with trade will be one where money is valued and will Pareto dominate autarky.

in the next low-endowment period. If, during a given period, an agent deviates from the above strategy, then $\sum_i l_t^i(m_t^i) = \sum_i b_t^i(m_t^i) = 0$ is violated, and autarky prevails. The incentive condition that binds is the one for the buyer. Hence, any such deviation will not be part of a best response as long as

$$u\left(\frac{e_h + e_l}{2}\right) + \beta u\left(\frac{e_h + e_l}{2}\right) \geq u(e_h) + \beta u(e_l) \tag{2}$$

The above inequality requires that a player holding e_h is better off following s than staying in autarky for the current period. The corresponding constraint for the case where $m_t^i = 1$ and $e_t^i = e_l$ is not binding. Provided that β is sufficiently high, or u is sufficiently concave, the inequality always holds. Clearly, $v_t^i(m_t^i, s) \geq u(e_t^i) + \beta v_{t+1}^i(m_t^i, s)$ holds in any monetary equilibrium. ■

Under s, each agent consumes $\frac{e_h + e_l}{2}$ in each period, which clearly dominates autarky. In fact, this equilibrium achieves an efficient allocation.

2.2. *A Walrasian Model.* Here, we study an example of an intertemporal pure exchange economy that is subject to the lack-of-commitment/recall frictions. We again study the case with two (types of) agents, $\mathbb{I} = \{1, 2\}$, and one consumption good per period. As in the previous example, let the agents' endowment sequences be $e^1 = \{e_h, e_l, e_h, e_l, \ldots\}$ and $e^2 = \{e_l, e_h, e_l, e_h, \ldots\}$, with $e_h > e_l > 0$. As a result of our lack-of-recall assumption, in the absence of money, the allocation in any given period cannot depend on past transactions. The individual's problem under Walrasian markets is standard, but with the additional IR constraint that no individual will give up current consumption in exchange for higher future consumption. This constraint captures the lack-of-recall/lack-of-commitment assumption in a model with Walrasian prices. Once again, without money, the only equilibrium allocation corresponds to autarky. We introduce money next.

As in the game studied earlier, money consists of indivisible, perfectly storable fiat objects. Let m_t^i stand for agent i's money holdings in period t, and let $v_t^i(m_t^i)$ denote i's discounted sum of future utility. Assume that each type 2 agent is endowed with one unit of fiat money in period 1. Although money holdings may act as a state variable, there is still no commitment. Thus, any candidate equilibrium allocation has to offer each agent, and at each point in time, a higher continuation utility than the utility that would be gained from staying in autarky for the period. We, therefore, impose the IR constraint that $v_t^i(m_t^i) \geq u^i(e_t^i) + \beta v_{t+1}^i(m_t^i)$, $\forall i \in \mathbb{I}$, $t \in \mathbb{N}$. Let q_t denote the price of money at date t in units of the consumption good. Agent i's problem becomes

$$\left.\begin{array}{l} \sup\limits_{\{c_t^i, m_{t+1}^i\}_{t\in\mathbb{N}}} \sum\limits_{t\in\mathbb{N}} \beta^{t-1} u\big(c_t^i\big) \\ \text{s.t. } c_t^i + q_t m_{t+1}^i \leq e_t^i + q_t m_t^i, \quad \forall t \\ v_t^i\big(m_t^i\big) \geq u\big(e_t^i\big) + \beta v_t^i\big(m_{t+1}^i\big), \quad \forall t \\ c_t^i, m_t^i \geq 0, \quad \forall t \\ \{q_t\}_{t\in\mathbb{N}} \text{ and } m_1^i \text{ given.} \end{array}\right\} \tag{3}$$

A *stationary monetary equilibrium* (SME) consists of an allocation together with a constant price of money, q, such that the allocation solves each consumer's problem, and the markets for money and the consumption good clear in each period. The following proposition establishes the existence of an SME.

PROPOSITION 2. *Assume that* $\frac{u'(e_h)}{u'(e_l)} < \beta < 1$. *There exists an SME in which any agent who holds money exchanges it for q units of the consumption good in each period.*

The proof of this proposition involves ensuring that there exists a q such that it is optimal for an agent with high endowment to obtain exactly one unit of money, which he uses to improve his consumption when his endowment is low. We remark that a nonmonetary equilibrium in which agents do not accept money anticipating that the price of money in the future will be zero always exists in our setup.

PROOF. First, we assume that any individually optimal plan has the property that an agent will accumulate one unit of money when his endowment is e_h, and he will offer one unit of money in exchange for additional consumption when his endowment is e_l. Such a plan consists of an infinite repetition of a two-period "save-and-spend" cycle, which can be sustained as long as the price q ensures that each agent holds exactly zero or one unit of money, respectively. This, in turn, follows if

$$u(e_h) + \beta u(e_l) < u(e_h - q) + \beta u(e_l + q) \tag{4}$$

and

$$u(e_h - 2q) + \beta u(e_l + 2q) < u(e_h - q) + \beta u(e_l + q) \tag{5}$$

Clearly, such an equilibrium price q exists provided that $\beta > \frac{u'(e_h)}{u'(e_l)}$. In the remainder of the proof, we argue that an individual's trading plan satisfies the assumed property. Because the marginal utility from consumption is at its highest, the agent with endowment e_l in period 1 will not accumulate money but will in fact spend his money during period 1. Consider the agent starting period 1 with endowment e_h. If it is optimal for him to accumulate exactly one unit of money, then we have the claimed property. Let us rule out all other cases. It is not optimal for the agent endowed with e_h to stay in autarky in period 1 since (4) implies that

$$u(e_h) + \beta u(e_l) + \beta^2 v_3^1(0) < u(e_h - q) + \beta u(e_l + q) + \beta^2 v_3^1(0) \tag{6}$$

Thus, the agent is better off accumulating one unit of money. We now demonstrate that it is also not optimal to accumulate more than one unit of money in period 1.

Suppose, instead, that $m_2^1 \geq 2$. In that case, the additional money will not be spent in period 2 because, if $m_2^1 - m_3^1 > 1$, expression (5) implies that

$$
\begin{aligned}
(7)\qquad & u(e_h - m_2^1 q) + \beta u\big(e_l + (m_2^1 - m_3^1)q\big) + \beta^2 v_3^1(m_3^1) \\
& < u\big(e_h - (m_3^1 + 1)q\big) + \beta u(e_l + q) + \beta^2 v_3^1(m_3^1)
\end{aligned}
$$

Therefore, the agent is better off accumulating less in period 1 and offering less money in period 2. That is, any additional period 1 accumulation is just carried over to period 3. But if the period 3 total money accumulation is no greater than one unit, the agent could have delayed accumulation until period 3 and be better off due to discounting. Finally, accumulating more than one unit of money during period 3 is clearly suboptimal because, by the previous argument, the agent will be accumulating unused money holdings. The same argument applies for any other period. We conclude that agents will accumulate exactly one unit of money during each e_h period. The existence of an SME follows. ■

3. CONCLUSION

We proposed a setup that incorporates lack-of-recall/lack-of-commitment and centralized monetary trade. Lack-of-recall can be thought of as an alternative to anonymity, a common assumption in monetary models. Our work is related to several existing models. Unlike the prototypical random matching model, here there is an *intertemporal* lack-of-double-coincidence problem. In our environment, this problem cannot be overcome without money even though agents meet in every period and trade in centralized markets. One advantage of dealing with a Walrasian model is that the determination of prices is straightforward because the "institution-free" Walrasian model abstracts from the details of how transactions take place.[7] We believe that the study of these details is not essential in order to understand why money has value.[8]

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[7] For an eloquent discussion on this issue, see Gale (1986).

[8] We emphasize that this does not mean that these specifics are not relevant in general. For example, the details of how agents trade might be essential in the evaluation of monetary policy since transaction patterns might change as a result of the policy itself.

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