Do Funds-of-Funds Deserve Their Fees-on-Fees?*

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Abstract

Since the after-fee returns of funds-of-funds are, on average, lower than hedge fund returns, it is easy to conclude that funds-of-funds do not add value compared to hedge funds. However, funds-of-funds should not be evaluated relative to hedge fund returns in publicly reported databases. Instead, the correct fund-of-funds benchmark is the set of direct hedge fund investments an investor could achieve on her own without recourse to funds-of-funds. We use asset allocation concepts to estimate characteristics of the fund-of-funds benchmark distribution. Since the benchmark characteristics are reasonable, we conclude that funds-of-funds, on average, deserve their fees-on-fees.

1 Introduction

A fund-of-funds is a hedge fund that invests in other hedge funds. Investors in funds-of-funds pay both the fees charged by the fund-of-funds, typically 1.5% and 10% in management and incentive fees, respectively, and the fees charged by the underlying hedge funds, often 1.5% and 20%. From the perspective of an endowment manager or chief investment officer considering the use of a fund-of-funds this "double" fee structure makes a fund-of-funds a costly alternative to direct hedge fund investment. How should an investor decide if funds-of-funds are worth their additional fees-on-fees?

The majority of the work on the hedge fund industry compares hedge funds with funds-of-funds and finds that, on average, funds-of-funds underperform hedge funds.[1] By running different factor regressions, many studies find little or no alpha from a fund-of-funds investment compared to a direct investment in hedge funds. Thus, many authors conclude that the extra fees charged by funds-of-funds outweigh any investment gains. Others, in particular, Brown, Goetzmann and Liang (2004), also claim that the extra fees do not provide an appropriate incentive for funds-of-funds managers. The general consensus is that the fund-of-funds industry offers poor value to investors.

The funds-of-funds industry is large and growing. In fact, a number of well-respected investors and endowment managers have chosen to use funds-of-funds even though they could have chosen to directly invest in hedge funds. Since 2000, funds-of-funds have received approximately 35 percent of the new inflows into hedge funds and now constitute about one quarter of the total assets under the direct management of hedge funds, according to the Tremont TASS database. In the early 1990's, they received just 11 percent of new inflows and accounted for just 20 percent of total hedge fund assets.[2]

The promoters of funds-of-funds offer an number of arguments in their favor. First, they allow investors to obtain exposure to hedge funds that are otherwise closed to new investments. Second, funds-of-funds generally have much lower required investment minimums than those required by hedge funds and they provide investors access to a diversified portfolio of managers. Only investors with very large amounts of capital could replicate this degree of diversification. Finally, they provide access to information and professional portfolio management that would otherwise be difficult and expensive to obtain. Brown, Fraser and Liang (2008) show that op-

[1] See, among others, Ackermann, McEnally and Ravenscraft (1999), Lhabitant and Learned (2002), Amin and Kat (2003), Kat and Amin (2003), Capocci and Hubner (2004), Fung and Hsieh (2004), and Fung et al. (2007).

[2] For funds in the TASS database, the total value of assets under management for funds of funds is $156 billion compared to $635 billion for hedge funds as of June 2006. We compute the inflows from the TASS data.

erational risk, as contrasted with financial risk, is a major source of hedge fund failure, and funds-of-funds' managers have significant economies of scale in conducting due diligence with resources not available to individual investors. All these reasons represent operational advantages of funds-of-funds that direct hedge fund investment may not provide. We address the issue that the relatively poor after-fee performance of funds-of-funds compared to hedge funds may not offset these advantages.

This paper has two goals. The first is to make it clear why observed hedge fund returns are not the correct benchmark against which to assess funds-of-funds. The second is to propose a method that investors can use to evaluate decision to choose a fund-of-funds versus investing directly in hedge funds.

The conventional analysis of directly comparing hedge fund and fund-of-funds returns does not adequately measure the true trade-off between a fund-of-funds and a hedge fund. To compare the decision to invest in funds-of-funds versus directly investing in hedge funds, an investor must compare her own skills, costs, and ability to find and monitor hedge funds with those of a fund-of-funds manager. Naturally, these costs would include any costs like employing consultants to evaluate or monitor hedge fund investments and represent any out-of-pocket expenses borne by the investor not using a fund-of-funds. Thus, an investor should not compare a fund-of-funds to the average hedge fund, but rather she should compare a fund-of-funds to the portfolio of hedge funds she could construct on her own. Our method allows an investor to ask what returns she could achieve in hedge fund investments by herself, without recourse to funds-of-funds, before investing in funds-of-funds becomes economically viable.[3]

We estimate the implied benchmark distribution for funds-of-funds for different types of investors. We find that it is not hard to justify the use of a fund-of-funds and the conditions under which investors choose a fund-of-funds over hedge funds are economically reasonable and plausible. This is particularly true for smaller and more risk-averse investors. Consider an investor holding a low-cost, benchmark portfolio of well-diversified domestic and international assets who cannot short more than -20% with a risk aversion of $\gamma = 6$ (a risk aversion level that implies that if she could invest in only the S&P 500 or the Lehman Aggregate Bond index, she would choose approximately a 50%-50% portfolio), and who only has enough capital to meet the minimum for one hedge fund. This investor finds that funds-of-funds add value if she believes that her own direct investment in a hedge fund would result in an average return just 0.3% per annum lower than the median return of funded hedge funds in data. Put another way,

[3] The analysis can also be used more broadly to examine the potential value of an asset class when markets are incomplete and to compute the economic value of access to a new asset class.

even if this investor had the skill to choose a typical hedge fund if her expenses associated with direct hedge fund investing were just 0.3% per annum on average, then she would still be better off with a fund-of-funds. It does not seem difficult to conclude that funds-of-funds can provide sensible investment vehicles to obtain exposure to hedge fund investment strategies.

We comment that our analysis focuses only on how to characterize the true benchmark for funds-of-funds and not on investigating the absolute performance of hedge funds or funds-of-funds relative to standard asset pricing models. Whether hedge funds have average returns in excess of their risk profiles is still an open question. Studies like Fung and Hsieh (2001) cannot reject that there is no average excess performance of hedge funds after factors with option-like payoffs are included. On the other hand, Bailey, Li and Zhang (2004) find evidence of the average out-performance of hedge funds under the null of no arbitrage, even when non-linear factor payoffs are considered. Jagannathan, Malakhov and Novikov (2006) also find statistically strong and economically large outperformance of hedge funds relative to their style benchmarks and evidence of strong persistence.[4] Our work is silent on the absolute investment performance of hedge funds and funds-of-funds as an asset class, and we focus only on the methodology an investor should employ in order to optimally choose to pay, or not to pay, the added fees of funds-of-funds.[5]

The rest of this paper is organized as follows. Section 2 shows why funds-of-funds should not be directly compared to hedge funds and discusses the true fund-of-funds benchmark. In Section 3, we formulate the asset allocation problem and describe our data. We lay out our empirical results evaluating the performance added of funds-of-funds relative to hedge funds in Section 4. Section 5 finds that our results are robust to alternative ways of computing expected returns. Finally, Section 6 concludes.

[4] Other authors computing alphas of funds-of-funds and hedge funds include Fung and Hsieh (1997, 2001), Ackermann, McEnally and Ravenscraft (1999), Liang (1999), Edwards and Caglayan (2001), Ben Dor, Jagannathan and Meier (2003), Agarwal and Naik (2000, 2004), Brown and Goetzmann (2004), and Fung et al. (2007), among many others.

[5] We also do not address the question of optimal fees for hedge funds or funds-of-funds. Recent studies focusing on the optimal fee structure of hedge funds or funds-of-funds include Anson (2001), Goetzmann, Ingersoll and Ross (2003), Bhansali and Wise (2005), Hodder and Jackwerth (2007), and Stavros and Westerfield (2007).

2 Benchmarking Funds-of-Funds

2.1 Hedge Fund and Fund-of-Funds Returns

Using the Tremont TASS database from January 1994 to June 2006, we briefly document some summary statistics of return performance of hedge fund and fund-of-funds returns in Table 1.[6] To be included in our sample, we require each fund to have at least 12 consecutive monthly net-of-fee returns. TASS divides the funds into two groups: live and graveyard. At the end of June 2006, the database contains 4,841 hedge funds, of which 2,460 are live funds and 1,671 are graveyard funds, and 1,576 funds-of-funds, of which 1,097 are live and 479 are no longer reporting. All the returns are after-fee returns. TASS only reports after-fee returns and generally not enough information to construct pre-fee returns for most funds.

Table 1 reports some details on the median fund incentive fee and management fee, along with the proportion of funds that have high watermark provisions. The median management fee for both hedge funds and funds-of-funds is 1.5%. Funds-of-funds have median incentive fees half the size of the median incentive fees for hedge funds, at 10% and 20% for funds-of-funds and hedge funds, respectively. Approximately half (56%) of funds-of-funds have high watermarks, whereas the proportion of hedge funds having high watermarks is 65%.

Consistent with a large literature on hedge fund and fund-of-funds performance, Table 1 reports that average returns and alphas for funds-of-funds are lower than the average returns and alphas of hedge funds. For example, the median monthly excess return for funds-of-funds is 0.35% per month, which is lower than 0.46% per month for hedge funds. As a simple illustration of hedge fund alphas, we compute monthly alphas for funds with respect to the aggregate TASS hedge fund index and with respect to the nine TASS hedge fund styles.[7] These alphas are also lower for funds-of-funds than for hedge funds. For example, the median fund-of-funds alpha with respect to the aggregate hedge fund index is -0.12% per month compared to 0.13% per month for hedge funds, so that the average fund-of-funds underperforms a simple hedge fund

[6] Prior to 1994, TASS backfills returns and does not include failed hedge funds (see comments by, among others, Agarwal and Naik, 2004). However, papers do use TASS data prior to 1994, like Fung and Hsieh (1997, 2001), Brown and Goetzmann (2003), and Brown Goetzmann and Liang (2004). Getmansky, Lo and Makarov (2004) and Gupta and Liang (2005) use the full TASS sample. We do not investigate the absolute level of hedge fund and fund-of-funds returns relative to performance benchmarks, where survivorship biases may create first-order effects.

[7] The TASS Tremont primary hedge fund strategy indices are convertible arbitrage; dedicated short bias; emerging markets; equity market neutral; event driven; fixed income arbitrage; global macro; long/short equity; managed futures; and multi-strategy.

benchmark. The median fund-of-funds alpha with respect to the TASS hedge funds styles is -0.17% per month compared to 0.03% per month for hedge funds. It is easy to conclude from these results that funds-of-funds do not add much value relative to hedge funds. We now discuss why this claim should not be made from comparing fund-of-fund returns to hedge fund returns.

2.2 The True Fund-of-Funds Benchmark

Comparing average risk-adjusted returns, or alphas, across two asset classes is valid if both assets are directly comparable. For example, mutual funds can be compared to index funds since investors can easily invest in either asset. However, investing in hedge funds is very different. First, the best hedge funds are closed (presumably filled with the capital of smart investors who recognized the superiority of these hedge funds at an early stage). Second, even if a wealthy investor meets the high minimum requirements for investing in hedge funds, there is no guarantee that a successful hedge fund will take that investor as a client. Third, and most importantly, unlike listed stocks that must provide timely disclosure notices and accounting reports, hedge funds are often secretive with little or no obvious market presence. Thus, it is plausible to assume that fund-of-funds managers and individual investors have different abilities in finding, evaluating and monitoring hedge funds, either because fund-of-funds managers have expertise in picking good hedge funds, or because they gather superior information at a cost.

Consider an investor deciding between investing in a fund-of-funds or directly making hedge fund investments. This investor is not comparing a fund-of-funds to the set of hedge funds in a published database. Rather, she is comparing the fund-of-funds to the set of hedge funds that she can locate, evaluate and invest in by herself – without using a fund-of-funds. To varying degrees, hedge funds are hard to find, evaluate and monitor; they have high minimums; and they are often closed to new investors. Thus, to choose between direct hedge fund investments and using funds-of-funds, an investor has to compare her own costs and skill at locating, evaluating and monitoring hedge funds she could enter to the costs and skill of a fund-of-funds manager.

For simplicity, we refer to an investor with sufficient expertise and skill to locate and evaluate hedge funds as an informed or skilled investor. We refer to an investor with little expertise in locating, evaluating and monitoring hedge funds as an uninformed or unskilled investor.[8] Natu-

[8] By unskilled investors, we do not mean investors with little capital or investors without any financial knowledge. By law, most hedge funds and funds-of-funds are organized under the qualified investor exemption and are limited to investors with a net worth of at least $5 million. By an unskilled investor, we mean an investor that does not have the same opportunity set to find hedge funds, or has inferior skills to evaluate and monitor hedge funds compared to a sophisticated, skilled investor. In our argument, we assume that fund-of-funds managers, on aver-

rally, an investor with a lot of expertise and skill in locating and monitoring hedge funds would prefer to invest directly in hedge funds, rather than pay a fund-of-funds. But, for an investor with little or no expertise, the probability of choosing a hedge fund that turns out to be a poor investment may be very high as there are large cross-sectional differences in the performance of individual hedge funds (see, for example, Li, Zhang and Zhao, 2005; Kosowski, Naik and Teo, 2007).

But wait a minute! Surely if the data show hedge funds do better on average than funds-of-funds, unskilled investors could just randomly select some hedge funds and would do better, on average, than being in a typical fund-of-funds. This is not true because the data are furnished by hedge funds and funds-of-funds which themselves are the result of an equilibrium where investors with different skills sort themselves into direct investors in hedge funds and indirect investors through funds-of-funds. The return differences from these hedge funds and funds-of-funds that have received capital from these two types of investors do not directly answer the question of whether funds-of-funds add value relative to hedge funds.

Remember that those people investing in hedge funds have *chosen* whether or not to use a fund-of-funds. Consider a world where the universe of hedge funds that we see in data are funded and monitored either by expert fund-of-funds managers or by investors with sufficient resources and skills that enable them to make high quality, direct hedge fund investments. (Obviously this does not mean that ex post all hedge funds and funds-of-funds have good performance because the decisions are made ex ante.) In this world, we do not observe the set of hedge funds that received little or no funding, but would have received funding if unskilled investors had to invest directly without recourse to funds-of-funds. In a world where unskilled investors do not invest directly in hedge funds, unskilled investors use funds-of-funds and thus manage to avoid the worst hedge funds.

This intuition extends to the real world. The hedge fund databases tend to contain returns only of hedge funds that are funded directly by skilled individuals and funded indirectly through unskilled investors using expert funds-of-funds. The data truncate the returns of most of the worst hedge funds that would have received funding from unskilled investors if they could not use funds-of-funds. Consequently, the observable set of hedge funds is biased upwards compared to the true set of hedge funds that would be available only to unsophisticated individuals

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age, have such skills. To prevent poor hedge fund allocations, sophisticated investors spend significant resources to evaluate the skill of the managers. William H. Donaldson, Chairman of the U.S. Securities and Exchange Commission, notes in his May 2003 testimony to Congress that sophisticated hedge fund investors "perform extensive due diligence prior to investing, often taking months to research a hedge fund before making an investment." See http://www.sec.gov/news/testimony/052203tswhd.htm forced to directly invest in hedge funds. The data do not directly tell whether funds-of-funds are adding value because the data do not necessarily contain returns of the hedge funds that unskilled investors would have invested in if funds-of-funds were not available.

Figure 1 illustrates this logic with a simple, but extreme, example. Assume that the investment returns of the true universe of hedge funds is normally distributed, as shown in the solid blue curve. Assume also that only skilled fund-of-funds managers, taking capital from unskilled individuals, and skilled individual investors are making direct hedge fund investments and they are able to avoid the worst 20% of hedge funds. For simplicity, we assume that the skill of these investors results in a truncation of the distribution so that the worst 20% of the true hedge funds never get funded. The mean of the truncated distribution of returns is higher than the mean of the original normal distribution, as skilled investors fund the best 80% of hedge funds.

The data available to researchers to evaluate funds-of-funds performance, includes only the set of hedge funds that receive funding by skilled investors. That data is too good relative to the true universe of hedge funds. We never observe the bad hedge funds that would have received funding if funds-of-funds did not exist. But, this true hedge fund distribution is the correct benchmark for funds-of-funds. In the Appendix, we present a formal model to illustrate these points.

We can push this example a little further. Suppose we generated observed hedge fund and fund-of-funds data according to the truncated distribution in Figure 1. We would find that funds-of-funds and direct hedge fund investments would roughly perform the same before fees. After fees, funds-of-funds would underperform hedge funds. Does this imply that funds-of-funds are not adding value? Funds-of-funds prevent unskilled investors from investing in the bottom 20% of funds. These funds do not show up in the data, but they are part of the true benchmark for funds-of-funds. Should an investor who is unskilled at choosing and monitoring hedge funds pay for this skill? Absolutely.

But why can't an investor just wait until the skilled investors have invested and then follow? First, it may not be clear who the top investors are or when they have chosen to invest. Unlike the stock market, there is no requirement to report large trades and so procuring this information may be very hard. Second, because many top hedge funds close to new investors, unskilled investors may not be able to invest in the top hedge funds and will likely receive larger allocations to worse funds. Third, because funds require ongoing monitoring, an unskilled investor may often still be in the now poor fund after the skilled investors have exited. Finally, and this is a point about operational rather than performance advantage, the best funds raise their minimums and no longer deal with smaller investors. Thus, even investors with hundreds of millions under management could not successfully follow this strategy. To state this another way, it is simply hard to believe that the least knowledgable investors can simply stand behind Yale Endowment and enter only once they know Yale is in.

We emphasize that this funding bias of existing hedge fund databases is very different from the survivorship or reporting bias discussed in the literature. The reporting bias deals with the many successful hedge funds which do not report to hedge fund databases (see Ackermann et al., 1999), making observed hedge fund returns biased downwards. On the other hand, Malkiel and Saha (2004) argue that many unsuccessful hedge funds, which ultimately fail, stop reporting to the hedge fund databases, which causes hedge fund returns to be biased upwards. Reporting biases cause the hedge fund databases to tend toward mediocrity. The funding bias is different from these survivorship biases because the funding bias involves whether hedge funds *that have received funding* report, or do not report, to a database. Our bias is that we never observe the hedge funds that do not receive funding, but would have received funding if funds-of-funds did not exist. It is this unobserved set of unfunded hedge funds, together with the observable set of funded hedge funds, that constitutes the true fund-of-funds benchmark.

2.3 Constructing the True Fund-of-Funds Benchmark

The key to evaluating a fund-of-funds manager is finding the true fund-of-funds benchmark. We have shown that the fund-of-funds benchmark is not the hedge fund universe in reported databases; rather it is the set of hedge funds that an investor could invest in on her own without recourse to funds-of-funds. The problem, of course, is that data on the true hedge fund universe are not directly observable and we should not directly benchmark funds-of-funds against hedge funds. In this section, we introduce a framework that uses revealed preference concepts which allows an investor to gauge whether a fund-of-funds investment is worth the additional fees. With this framework, and given an investor's attributes such as size and tolerance for risk, we estimate the implied fund-of-funds benchmark hurdle rate for funds-of-funds to add value.

We start with the observation that an investor selecting a fund-of-funds must at least be indifferent to directly selecting a hedge fund from her true hedge fund universe and a fund-of-funds. This revealed preference allows us to back out various characteristics of the true hedge fund distribution from observed fund-of-funds investments. In other words, we ask the indirect question: What would investors have to believe about their own ability to invest in hedge funds in order to make funds-of-funds a good idea? Answering this question also helps us to judge whether funds-of-funds add value. Since investors choose to invest in funds-of-funds, the true distribution of hedge funds these investors can access must be at least as bad as the distribution that makes them indifferent between the observed fund-of-funds investments and the true set of hedge funds to which the investors can access.

Formally, our methodology is as follows. We first determine the utility value of a typical fund-of-funds investment for an investor with a particular risk tolerance and a given portfolio composition and size. Note that the funds-of-funds have received funding and must represent optimal positions in investors' portfolios. We do not observe the true hedge fund distribution to which this typical fund-of-funds should be compared.

We construct the distribution of returns for the set of hedge funds in which the investor could invest directly on her own (the true fund-of-funds benchmark) that would provide the same utility as investing in a fund-of-funds. We compute the expected return (the hurdle rate) so that the investor is indifferent between going it alone in direct hedge fund investments and using a fund-of-funds. We denote this hurdle rate as μ_B, the mean of the benchmark fund-of-funds distribution. The hurdle rate is determined so that, at the margin, the ex-ante utility of adding a hedge fund drawn from the investor's hedge fund investment opportunity set is the same as the ex-ante utility of making the observable funds-of-funds investment.[9]

The hurdle rate can also be interpreted as the belief an investor has about their own hedge fund investments to think that funds-of-funds add value. If μ_B is relatively close to the observed returns of hedge funds reported in databases, then it is relatively easy to believe that an investor's direct investment returns minus her own costs would be less than the benchmark. Thus, for this investor it is easy to believe that a fund-of-funds would add value. Conversely, if we find that investors need to believe that their own direct hedge fund investments perform implausibly badly to justify a fund-of-funds, we would conclude that fund-of-funds are likely to be overcharging for their investment performance.

3 Methodology

3.1 The Portfolio Allocation Problem

In this section, we describe how a standard portfolio allocation framework can infer the characteristics of the benchmark fund-of-funds distribution. We assume that the investor has a standard

[9] Naturally, it is also possible to characterize a benchmark standard deviation given some assumptions on the benchmark return, which was considered in an earlier version. We focus on the benchmark return because it has a more direct practical application as an internal hurdle rate for gauging hedge fund investments.

CRRA utility function over final wealth:[10]

$$U(W) = \mathrm{E}\left(\frac{W^{1-\gamma}}{1-\gamma}\right), \tag{1}$$

where γ is the investor's coefficient of risk aversion. The investor's end-of-period wealth, W, is given by:

$$W = W_0 \left(\sum_{j=1}^{N} \exp(y^j)\alpha_j\right),$$

where y^j is the continuously compounded return of the jth asset, $j = 1 \ldots N$, for N assets, and α_j is the portfolio position in the jth asset, with $\sum \alpha_j = 1$. Since CRRA is homogenous in wealth we set $W_0 = 1$. The optimal asset allocation decision problem is:

$$\begin{aligned} &\max_{\alpha_j} \mathrm{E}\left(\frac{W^{1-\gamma}}{1-\gamma}\right) \\ &\text{subject to} \sum_{j=1}^{N} \alpha_j = 1 \end{aligned} \tag{2}$$

We choose CRRA utility because it is a standard utility function that is widely used in many asset allocation studies (see, for example, the review by Brandt, 2004). Naturally, our methodology can be extended to more complex utility functions. Since it is well known that unconstrained portfolio positions are sensitive to expected returns and can produce extreme portfolio positions, we consider only a small number of assets and impose the cases of no short-sale constraints as well as a shorting limit of -20%. To solve the asset allocation problem, we solve the first order conditions corresponding to equation (1) numerically using quadrature following Balduzzi and Lynch (1999) and Ang and Bekaert (2001), assuming asset returns are log-normally distributed.

To compute mean excess returns, we use the excess returns (raw returns less the one-month T-bill rate). In the asset allocation problem, we assume a risk-free rate of 0.04/12 = 0.33% per month and add this onto the mean excess return of each asset to produce total expected returns:

$$\mathrm{E}(y) = r_f + \mathrm{E}(y - r_f).$$

We present benchmark results for investors with risk aversion levels of $\gamma =$ 4, 6, 8, and 10. To intuitively grasp the effect of these risk aversion levels on a typical stock/bond allocation,

[10] In Ang, Rhodes-Kropf and Zhao (2006), we consider robustness of our results to using mean-variance utility and considering a much wider universe of assets including U.S. large and small stocks, U.S. value and growth stocks, U.S. bonds, commodities, foreign equity (U.K., Japan, Germany, France, and emerging markets), and foreign bonds (U.K., German, and Japan Eurobond returns).

consider an investor who only allocates capital between the S&P 500 and the Lehman Aggregate Bond index. Using data from January 1976 to June 2006, a standard position that is 60% stocks and 40% bonds corresponds to a risk aversion of $\gamma = 4$. The $\gamma = 6$ investor holds a 50%/50% portfolio. A more risk averse investor with $\gamma = 10$ holds a portfolio of 27% stocks and 73% bonds.

Benchmarking One Fund-of-Funds to One Hedge Fund

We first consider the problem of an investor deciding between adding a single hedge fund or a single fund-of-funds to a set of benchmark assets. This comparison is straightforward because the characteristics of a typical fund-of-funds can be easily estimated from observable fund-of-funds investments. After computing the utility of adding the fund-of-funds to the benchmark assets we can compute the hurdle rate μ_B as follows. Suppose that the distribution of hedge funds accessible by the investor has the same variance and correlations as the observed hedge fund returns in data. Of course, this is a conservative assumption because we would a priori expect the true set of hedge funds accessible by the investor to have at least as volatile returns as the reported hedge funds in data. With this assumption, we consider a second asset allocation problem of the investor adding a single hedge fund to the set of benchmark assets. We solve for the benchmark return μ_B that yields the same utility as the first asset allocation problem of adding the fund-of-funds investment. Other benchmark returns may be inferred by taking different assumptions on the variance and correlations of the true hedge fund returns.

Benchmarking a Fund-of-Funds to Many Hedge Funds

A small investor with $200M under management might allocate a small position, say $10M, to hedge funds. Given the large minimums required by many hedge funds this investor may only be able to invest in a single fund-of-funds or a hedge fund. In contrast, an investor with several billion dollars under management may be able to directly invest in a portfolio of hedge funds, and so should compare fund-of-funds to an investment in a number of hedge funds. The analysis in this section allows us to consider the funds-of-funds question for investors of different sizes.

To make this comparison we create artificial portfolios of randomly chosen hedge funds from the TASS database. At the end of each year, we randomly select 2, 5, 10, or 30 hedge funds from data to form an artificial fund-of-funds. We equally weight these randomly chosen hedge funds to form a portfolio and record the monthly returns of the portfolio over the next year. These portfolios are rebalanced annually.[11] This process is repeated 1,567 times to match

[11] If a hedge fund in the artificial fund-of-funds moves into the TASS graveyard file, we assume that the remaining

the number of funds-of-funds in our sample. We call portfolios of hedge funds created this way simple artificial funds-of-funds.

A large investor able to invest in many hedge funds may not choose hedge funds completely randomly but instead may optimally allocate to funds. To take this into account, we create artificial funds-of-funds that optimize over the different TASS hedge fund styles. This analysis uses hedge fund style data from January 1994 to June 2006, except for the multi-strategy category which starts in April 1994. We form one artificial funds-of-funds using an optimal weighting following equation (2) only over the TASS hedge fund styles, assuming no shorting of hedge funds. We choose a random fund in each category, but rather than equally-weighting the funds in the portfolio, we apply the optimal weighting scheme and repeat this process 1,567 times. Optimal artificial funds-of-funds created this way depend on the choice of risk aversion ($\gamma = 4 \ldots 10$) and are held mostly in market neutral, event driven, and global macro strategies. By comparing a fund-of-funds investment with these artificial hedge funds, we can gauge what an investor would have to believe about her own ability to select between many hedge funds on her own or to prefer a fund-of-funds.

3.2 Benchmark Assets and Moments

We specify a small investment universe consisting of:

1. U.S. Stocks – S&P 500 [Jan 1975 – Jun 2006]
2. U.S. Bonds – Lehman Aggregate Bond Index [Jan 1976 – Jun 2006]
3. Foreign Equities – MSCI world ex U.S. [Jan 1975 – Jun 2006]
4. Foreign Bonds – Lehman Global Aggregate Bond Index ex U.S. [Jan 1990 – Jun 2006]

together with a risk-free bond yielding 4%/12 = 0.33% per month. The small investment universe highlights the illustration of our methodology on a select set of assets that would be common to all institutional funds managers. The representative index returns are standard benchmarks and are provided by Ibbotson. Furthermore, this small set of assets also has relatively low correlations with each other. In Table 2, we report monthly summary statistics for these

capital is evenly allocated to the remaining hedge funds. In reality these artificial funds-of-funds also cannot be replicated because of lockup and redemption constraints. Thus, our artificial funds-of-funds are conservatively upward biased and represent the highest bound for the performance of an unskilled, institutional investor.

base assets. We use these historical moments as inputs for the asset allocation problem.[12] In Section 5 we consider other sources for obtaining expected returns.

To compute moments for hedge funds and funds-of-funds we use the cross-section of TASS data. This allows us to take estimates for a typical hedge fund, compared to an aggregated hedge fund or fund-of-funds index which smooths returns across a range of hedge funds and funds-of-funds. For example, to compute the representative volatility of fund-of-funds returns, we first compute the volatility of each individual fund-of-funds. Then, we take the median volatility of the cross section to use as the input volatility of fund-of-funds returns in the asset allocation exercise. The median not only represents a typical fund-of-funds, but the estimator is also robust to extremely low or high outliers.

Panel B of Table 2 lists the mean excess returns for hedge funds and funds-of-funds returns we use in the asset allocation problem. Since Asness, Krail and Liew (2001), among others, demonstrate that the returns of hedge funds are affected by nonsynchronous price movements, we also consider typical standard deviations to be greater than the numbers in Table 2. However, it is important to remember that the magnitude of the inputs is not very important to our study as we are interested in *comparing* funds-of-funds to direct hedge fund investments.[13]

Estimates of correlations drive diversification benefits in the optimal solution to any asset allocation problem. To produce estimates of correlations that are robust to non-synchronous reporting lags we introduce a methodology building on Dimson (1979), which is described in Appendix B. Not taking into account these non-synchronous lags overstates the benefits of investing in hedge funds or funds-of-funds. Panel B reports that the correlations of hedge funds and funds-of-funds with U.S. equities are 0.46 and 0.80, respectively, but are lower with U.S. bonds, at 0.08 and 0.23, respectively. These correlations are similar to those reported by Capocci and Hubner (2004), among others.

In Panel C of Table 2, we report the input moments to the asset allocation problem for the artificial funds-of-funds, which are created by randomly drawing hedge funds from the TASS database every year. The median excess return for the simple artificial funds-of-funds is slightly higher than the median hedge fund excess return. This is because we resample the database to

[12] The estimation of moments of hedge fund returns is sensitive to the choice of the sample period. In results available on request, we also compute hurdle rates when returns from 1998 are omitted. These results are very close to those reported in Table 3.

[13] Our evaluation technique does not suffer from the typical "error magnification" problems typical of optimal portfolio analysis. The central way in which the inputs matter is if the expectations are such that no investor wishes to invest in either hedge funds or funds-of-funds. Under this scenario, we cannot compare the relative benefit of one non-optimal asset class to another non-optimal asset class.

construct artificial funds-of-funds every year, and bad performing and liquidated hedge funds tend to have less impact on the artificial fund-of-funds portfolios.[14] As more funds are added to the artificial funds-of-funds, portfolio diversification reduces the median standard deviation from 3.74% per month for two funds to 1.84% per month for 30 funds. Funds-of-funds have a lower standard deviation at 1.55% per month, indicating that fund-of-funds managers obtain, on average, even better risk reductions than randomly selecting 30 hedge funds. The optimal artificial hedge funds have volatilities ranging from 2.76% per month for a risk aversion level of $\gamma = 6$, to 2.23% for a risk aversion of 10. These volatilities are also above the typical fund-of-funds standard deviation.

4 The Fund-of-Funds Benchmark

To estimate the hurdle rate, μ_B, we assume that the underlying benchmark volatility, σ_B, is either the observable median hedge fund volatility in Table 2 (Case 1) or 10% greater than the reported volatility (Case 2). We assume that the investor is adding the hedge fund or fund-of-funds to the four benchmark assets.

Table 3 reports the results for the fund-of-funds benchmark where the reported μ_B makes the investor indifferent between adding N hedge funds from the true, underlying hedge fund distribution and adding a typical fund-of-funds. Each row corresponds to a different comparison portfolio: $N = 1$ to 30 as well as the optimally chosen hedge fund portfolio. Entries in italics indicate that an investor already prefers a fund-of-funds, rather than a typical direct hedge fund investment. These investors would have to think they would do *better* than the typical hedge fund if they invested on their own in order for them not to use a fund-of-funds. This is because these investors are sufficiently risk-averse and favor the diversification benefits of funds-of-funds.

The entries in normal type in Table 3 report the highest possible μ_B that the investor could believe she would obtain on a direct hedge fund investment from the true hedge fund universe in order to choose a fund-of-funds. The median observable excess hedge fund return in TASS data is 0.460% per month. Let us examine a $\gamma = 6$ investor who starts with the four benchmark assets in her portfolio and who cannot short. Suppose this investor can add one hedge fund or one fund-

[14] This annual resampling also leads to the simple artificial fund-of-funds portfolio of two hedge funds to have a slightly larger median standard deviation than the median standard deviation of single hedge funds. This does not affect our analysis because we always compare the true hedge fund distribution with funds-of-funds, not funds-of-funds containing a certain number of underlying funds with other funds-of-funds containing a different number of underlying funds.

of-funds. We also conservatively assume that the underlying hedge fund volatility is the same as the reported hedge fund volatility in TASS in Case 1. Table 3 reports that this investor needs to believe that on her own she would obtain an excess return less than 0.405% per month for her to prefer to use a fund-of-funds. That is, she only needs to think her own hedge fund investments would result in a mean excess return after her own expenses $12 \times (0.460 - 0.405) = 0.66\%$ per annum worse on average before preferring a fund-of-funds. Imagine this investor had $5 million to put to work in hedge funds. Even if she could invest in the typical hedge fund, if she spent more than $33,000 finding, evaluating and monitoring her investment she would be better off in a fund-of-funds.

If the $\gamma = 6$ investor moves from being able to hold a single hedge fund or fund-of-fund and cannot short (where $\mu_B = 0.405\%$ per month) to being able to hold positions down to -20%, then μ_B increases to 0.434%. The ability to short increases the value of a fund-of-funds as the investor takes leveraged positions in equity and the fund-of-funds. This increases the investor's utility and the hurdle rate on the true hedge fund distribution correspondingly increases to match this higher utility.

Now suppose the $\gamma = 6$ investor is large enough to hold 10 hedge funds. This investor's μ_B is now 0.388% per month, or she would need to believe she must earn an expected excess return of $12 \times (0.460 - 0.388) = 0.86\%$ per annum worse than a typical hedge fund in TASS before preferring a fund-of-funds. Alternatively, if the investor can obtain at least an excess return of $\mu_B = 4.66\%$ per annum (compared to an excess return of 5.52% per annum for a typical hedge fund reported in data), then she prefers to directly enter hedge funds without a fund-of-funds. To meet the minimum investment requirement on 10 funds implies the inventors has on the order of $50 million to invest in hedge funds. Therefore, she can spend approximately $430,000 per annum finding, evaluating and monitoring her hedge fund portfolio before she prefers a fund-of-funds.

Finally, if the same investor can optimally allocate her money among the nine hedge fund styles, then the investor would have to believe she would earn an expected return of $12 \times (0.460 - 0.330) = 1.56\%$ per annum worse than a typical TASS hedge fund before choosing a fund-of-funds. However, this comparison is slightly off because it compares an optimally chosen benchmark against a typical fund. Actually the typical optimally chosen hedge fund for a $\gamma = 6$ investor earns an excess return of only 0.349% per month.[15] Therefore, compared to a typical optimal artificial hedge fund, the $\gamma = 6$ investor needs to believe that she does just

[15] This is less than the typical hedge fund because some high risk / high return fund styles are down-weighted in the optimal hedge fund style mix.

$12 \times (0.349 - 0.330) = 0.23\%$ per annum worse to prefer a fund-of-funds. So, if her own costs are greater than 23 basis points per annum, she would prefer a fund-of-funds. Overall, a $\gamma = 6$ investor does not have to believe that she would perform very poorly on her own in order to prefer a fund-of-funds.

Naturally, for Case 2 where the assumed benchmark volatility is 1.1 times the typical hedge fund volatility in TASS, the μ_B entries are larger than Case 1. Hence, investors have higher hurdle rates before preferring to use direct hedge fund investments or investors need to believe in fewer basis points of underperformance relative to a typical TASS hedge fund before they prefer a fund-of-funds. Note that these performance "reductions" also include the costs of finding, allocating and overseeing the hedge fund investments if a fund-of-funds is not employed. Thus, investors who have a higher cost structure would find funds-of-funds even more attractive.

5 Robustness

In our main analysis above we have used expectations that are the realized returns of the different assets. In this section, we take two alternative approaches to obtaining inputs of expected returns and volatilities. We consider an equilibrium approach to back out expected excess returns following Black and Litterman (1992) and we also take the expected returns and volatilities from a leading investment manager, Yale Endowment, that has many years of experience in investing in hedge funds and funds-of-funds.[16]

Equilibrium Expected Returns

We use a modified version of Black and Litterman (1992) to back out equilibrium expected excess returns using a representative mean-variance investor combined with market weights. A mean-variance agent with risk aversion γ holds a vector of weights w in N risky assets given by

$$w = \frac{1}{\gamma}\Sigma^{-1}\mu, \tag{3}$$

for the excess return vector μ and Σ the covariance matrix of asset returns. If this agent is the representative agent and w are the market capitalization weights of each asset class, then we

[16] We have also conducted robustness checks to determine the sensitivity of our results to different inputs for the fund-of-funds moments. These were done by taking different percentiles (25th and 75th percentiles) of the cross-sectional distribution of the expected returns, correlations, and volatilities across the fund-of-funds returns. We find that using realized moments for the fund-of-funds distribution in no way drives our results and that funds-of-funds are likely to add value using these alternative inputs. These results are available upon request.

can invert equation (3) to solve for the equilibrium expected excess returns using

$$\mu = \gamma \Sigma w. \tag{4}$$

We measure market weights of U.S. bonds, foreign bonds, U.S. equity, and foreign equity by taking the corresponding MSCI market capitalizations and proportions at June 30, 2006. These market values and proportions are:[17]

	Mkt Cap (Bil USD)	Proportion
U.S. Bonds	7,789	19.18%
Foreign Bonds	8,715	21.46%
U.S. Equities	11,930	29.37%
Foreign Equities	12,181	29.99%

Note that at this date the proportion of equity/bonds in terms of market values is very close to a 60%/40% mix.

While the MSCI indices are representative and measure accurately the total amount of capital invested in these assets, we do not have a similar measure of committed capital to hedge funds or funds-of-funds, particularly for a representative investor. We assume, arbitrarily, that the proportion of the portfolio committed to hedge funds or funds-of-funds is w_{FoF}. We then scale the vector of portfolio weights of U.S. bonds, international bonds, U.S. equities, international equities, and w_{FoF} to be:

$$w = \begin{pmatrix} (1 - w_{FoF})\ 0.19 \\ (1 - w_{FoF})\ 0.22 \\ (1 - w_{FoF})\ 0.29 \\ (1 - w_{FoF})\ 0.30 \\ w_{FoF} \end{pmatrix}.$$

We take w_{FoF} to be 5% or 10%, which represent typical holdings of an institutional fund invested in hedge funds or funds-of-funds.

Using these weights, we back out the expected excess returns of all using equation (4) by taking $\gamma = 4, 6, 8, 10$. We estimate the unconditional covariance matrix from data using the values in Table 2. We denote the equilibrium fund-of-funds excess return computed this way

[17] We use the market capitalizations of these MSCI indices as they represent a consistent measure across asset classes, but rely on the time-series of the indices in Section 3.2 to compute returns. This is because the historical data for the MSCI bond returns is very short. Moreover, while the Lehman bond indices are most widely used in industry, market capitalizations of the Lehman bond indices are not publicly available.

as μ_{FoF}. Then, we compute the benchmark hurdle rate μ_B to yield the same utility changing the w_{FoF} to a hedge fund position while making assumptions on the true hedge fund volatility and correlations. In particular, we conservatively assume that the hedge fund volatility is the median cross-sectional cross-sectional standard deviation of hedge fund excess returns in data (2.94% per month). We can also compare μ_B against an equilibrium hedge fund expected excess return, μ^e, which we compute in the same way as μ_{FoF} but now construct the covariance matrix using observed hedge fund volatilities and correlations. Note that in these calculations the asset weights are held fixed at their scaled market values together with the market weight assumption of the hedge fund or fund-of-funds proportion.

Table 4 reports the fund-of-funds hurdle rates computed using equilibrium expected returns. The columns labelled "Equilibrium Risk Premia μ^e" report the equilibrium expected excess returns in percentage terms per month. Table 4 shows that with equilibrium returns it is also likely that funds-of-funds add value. To more easily interpret the hurdle rate returns, we explicitly compare them with the equilibrium hedge fund returns in the last four columns. We report $12 \times (\mu^e - \mu_B)$, the annualized difference between the equilibrium hedge fund returns and the benchmark fund-of-fund return. These numbers are small, so investors do not have to believe that they would have to perform very poorly on their own in order to prefer a fund-of-funds. For example, if the equilibrium hedge fund or fund-of-funds weight is 5%, then a $\gamma = 6$ investor would only need to assume that she had a return 0.48% per annum worse than the equilibrium hedge fund return to choose a fund-of-funds. If the equilibrium hedge fund weight is 10%, then the difference is 0.60% per annum.

The return differences are very small at less than 30 basis points for simple artificial funds-of-funds created with 10 or 30 hedge funds, or for taking optimal artificial funds-of-funds. For a simple artificial hedge fund of 30 hedge funds, it would only take additional costs of 8-12 basis points of running an in-house portfolio of hedge funds rather than taking the after-fee returns of a fund-of-funds holding the same number of hedge funds for an institutional investor to prefer to invest in the fund-of-funds. These differences between μ^e and μ_B are even lower than looking at the difference between the historical hedge fund return and μ_B in Table 3. This is because the equilibrium expected returns for artificial funds-of-funds are much lower than the realized 0.46% per month return for hedge funds. Thus, using equilibrium expected returns makes the case that funds-of-funds add value even stronger.

Forecasted Expected Returns and Volatilities

Another alternative source of expected returns and volatilities is forecasts. The 2006 annual report of Yale Endowment provides the following forecasts of asset classes in our universe:[18]

	Excess Return	Volatility
Funds-of-Funds	6%	12.5%
U.S. Equities	7%	20%
U.S. Bonds	3%	10%
Foreign Equity	6%	20%

We conduct our exercise among these four asset classes, dropping foreign bonds, as opposed to the four main asset classes in the previous sections. We take the correlation structure from data, given in Table 2. To solve for the benchmark fund-of-funds return, μ_B, we assume that the hedge fund volatility is the same as that observed in the TASS database.

Table 5 lists the results. The hurdle rates derived using Yale Endowment's forecasts are very similar to the hurdle rates in Table 3 computed using historical returns. For example, for a $\gamma = 6$ investor who cannot short and is considering a single hedge fund versus a fund-of-funds has a hurdle rate of 0.332% per month using Yale's forecasts, versus 0.405% per month in Table 3 using historical inputs for means and variances. This is a difference of just 0.073% per month and well below 1% per annum. With an average hurdle rate of approximately 0.3% per month, Table 5 states that a typical investor only needs to think that she would do approximately $12 \times (0.46 - 0.3) \approx 2\%$ per annum worse than the reported median hedge fund return in TASS before preferring a fund-of-funds. In summary, the conclusion that funds-of-funds are likely to add value is robust to using Yale Endowment's forecasts of expected returns and volatilities.

6 Conclusion

Funds-of-funds charge comparatively large fees-on-fees that are paid in addition to the fees charged by the underlying hedge funds. The after-fee alphas and average returns of funds-of-funds are lower than the after-fee alphas and average returns of hedge funds. It is tempting to conclude from these results that funds-of-funds add little value relative to hedge funds. However, we argue that this comparison is not correct and funds-of-funds should not be evaluated relative to the set of hedge fund returns we observe in data.

The hedge funds that we observe in reported databases tend to receive capital either from sophisticated, skilled investors, or from skilled funds-of-funds. An investor with no skill in

[18] Available at http://www.yale.edu/investments/. The numbers for the funds-of-funds category is an average of event and value driven categories in the annual report. We translate 1% real return on cash, which is an assumption often used by practitioners (see, for example, Urwin, 2006).

locating and monitoring hedge funds would, on average, choose hedge funds that are worse than the typical hedge fund observed in data, if she were forced to directly invest in hedge funds without using a fund-of-funds intermediary. The existence of the fund-of-funds industry helps investors gain access to a better skill set of finding, evaluating, selecting, and monitoring hedge funds. For investors with relatively little skill or small investment amounts, funds-of-funds add value, even if their after-fee returns are lower than the returns of hedge funds. Thus, the correct benchmark for a fund-of-funds investor is the universe of hedge funds that she would face on her own minus her own costs, rather than the set of observable hedge fund returns in data.

This paper outlines a methodology investors can use to determine their hurdle rate for a fund-of-funds decision. We answer the question: “What return do I need to achieve on my own, given my capital, risk tolerances, and costs to decide that I should not use a fund-of-funds?” We find that for many types of investors the hurdle rate is close to the return earned by the typical hedge fund from the TASS database. Thus, if an investor thinks she would do just a bit worse on her own or if her costs were bigger than this difference, then she would prefer a fund-of-funds. Thus, it is not hard to conclude that funds-of-funds add value even for relatively large investors.

Recently, there has been some debate regarding the regulation of hedge funds. Much of this debate focuses on the trade-off between the potential benefits from allowing broader access to hedge funds and the potential for the abuse of unskilled investors (see Edwards, 2006, for a summary). Our analysis suggests that allowing broader access to funds-of-funds, who report more information to investors, may be an appropriate solution. Even though the average after-fee returns of funds-of-funds is lower than hedge fund returns in data, their ability to overcome the frictions of hedge fund investing provides value to less skilled investors.

Appendix

A A Simple Theoretical Model

This appendix provides a simple equilibrium model where unskilled investors pay extra fees relative to skilled investors but earn the same before-fee return. In equilibrium, unskilled investors use funds-of-funds and both funds-of-funds and hedge funds perform the same on a pre-fee basis, but funds-of-funds underperform hedge funds on an after-fee basis. Funds-of-funds still add value in this economy because the alternative for an unskilled investor is to directly invest in hedge funds, and consequently, earn lower returns than skilled investors.

Consider an economy with two types of hedge funds: good hedge funds (G) with per period after-fee returns $r_G \sim N(\mu_G, \sigma_G^2)$, and bad hedge funds ($B$) with per period after-fee returns $r_B \sim N(\mu_B, \sigma_B^2)$, where $\mu_G > \mu_B$ and $\sigma_G^2 < \sigma_B^2$. Thus, we assume that good hedge funds strictly dominate bad hedge funds. At each time period new hedge funds are born. A fraction φ of new funds are good. New funds either receive an investment of one unit of capital or they exit the market. Funded hedge funds produce a return for one period. At the end of that period their qualities are revealed. Investors would like to add money to funds revealed to be good, but these funds are closed to new investment. Investors withdraw money from bad funds which then exit the market. Good funds live one more period before retiring.

There are two types of investors in the economy. They are either skilled (S) or unskilled (U) at evaluating the quality of hedge funds. The probability that a skilled or unskilled investor evaluates the quality of a hedge fund correctly is θ_S and θ_U respectively, with $\theta_S > \theta_U \geq 0.5$. Thus, better skilled investors are more likely to know the true quality of the hedge funds. Let λ equal the fraction of new investors who are skilled at finding investments, and $(1-\lambda)$ equal the fraction of new investors who are unskilled. At each time period investors evaluate hedge funds until they find one that they think is a good fund and invest. Therefore, conditional on their level of skill, investors will invest in good funds with probability ρ_i, where ρ_i is given by:

$$\rho_i = \frac{\varphi\theta_i}{\varphi\theta_i + (1-\varphi)(1-\theta_i)}.$$

Unskilled investors can become better with time. We assume that after one period a fraction χ of unskilled investors become skilled. We solve the model for a steady state equilibrium that requires the assumption that $\chi = \lambda(\rho_S - \rho_U)/(1-\rho_U)$. All results hold without this assumption but all solutions would be time dependent.

Investors invest for two periods and then consume. For simplicity we assume hedge funds are the only asset with positive expected return and that investors have mean-variance utility over final wealth such that they maximize: $C = \mathrm{E}(r_p) - \frac{\gamma}{2}\,\mathrm{var}(r_p)$, where r_p is the two period return of the investor's portfolio of good and bad hedge funds, and γ is the investor's coefficient of risk aversion. There is also a riskless asset normalized to have a zero return.

In the steady-state equilibrium of the model, the universe of hedge funds includes good hedge funds that have survived, but are closed to new investment, and new good and bad hedge funds that have received funding. We are particularly interested in the expected return and variance of the average hedge fund in the market and the expected utility of the unskilled investors. We examine two cases: an economy in which investors choose not to use funds-of-funds and unskilled investors invest directly in hedge funds, and an economy in which unskilled investors use funds-of-funds to channel their hedge fund investments.

Since all unskilled investors are the same they will all make the same equilibrium choice with respect to funds-of-funds. So, we will first examine a world where unskilled investors choose not to use a fund-of-funds and compare it to a world in which they do use a fund-of-funds. We will also examine the decision of an individual unskilled investor.

If unskilled investors reject funds-of-funds, then both skilled and unskilled investors directly invest in hedge funds based on their own ability θ_S and θ_U to select good funds. It is easy to show that, in steady state, the average hedge fund ($\overline{h}$) in the economy has an expected return and variance of:

$$\begin{aligned} \mathrm{E}(r_{\overline{h}}) &= 2[\lambda\rho_S + (1-\lambda)\rho_U]\,\mu_G + [1 - \lambda\rho_S - (1-\lambda)\rho_U]\,\mu_B \\ \mathrm{var}(r_{\overline{h}}) &= 2[\lambda\rho_S + (1-\lambda)\rho_U]\,\sigma_G^2 + [1 - \lambda\rho_S - (1-\lambda)\rho_U]\,\sigma_B^2. \end{aligned} \tag{A-1}$$

Now we assume that unskilled investors choose to use funds-of-funds. We also assume that the funds-of-funds' managers have the same ability as the skilled investors, and thus evaluate fund quality correctly with probability θ_S. In the steady-state equilibrium of this economy, the average hedge fund ($\overline{h}$) has an expected return and variance of:

$$\begin{aligned} \mathrm{E}(r^*_{\overline{h}}) &= 2\rho_S\,\mu_G + (1-\rho_S)\,\mu_B \\ \mathrm{var}(r^*_{\overline{h}}) &= 2\rho_S\,\sigma_G^2 + (1-\rho_S)\,\sigma_B^2, \end{aligned} \tag{A-2}$$

where we use the asterisk to denote the economy with funds-of-funds. Since by assumption $\mu_G > \mu_B$ and $\sigma^2_G < \sigma^2_B$, it is straightforward to show that:

$$\mathrm{E}(r^*_{\overline{h}}) > \mathrm{E}(r_{\overline{h}}) \qquad \text{and} \qquad \mathrm{var}(r^*_{\overline{h}}) < \mathrm{var}(r_{\overline{h}}). \tag{A-3}$$

Thus, the existence of funds-of-funds alters the return distribution of funded hedge funds in the economy.

Regardless of what other investors choose to do, an individual unskilled investor is maximizing her own utility. With mean variance utility and 100% of her wealth in hedge funds, this is equivalent to maximizing $C = \mathrm{E}(r_p) - \frac{\gamma}{2}\,\mathrm{var}(r_p)$. For unskilled investors who choose not to use a fund-of-funds:

$$\begin{aligned} C_U &= \mathrm{E}(r^U_p) - \frac{\gamma}{2}\,\mathrm{var}(r^U_p) \\ &= 2\rho_U\left(\mu_G - \frac{\gamma}{2}\,\sigma^2_G\right) + (1-\rho_U)\left(\mu_B - \frac{\gamma}{2}\,\sigma^2_B\right), \end{aligned} \tag{A-4}$$

where r^U_p is the unskilled investor's return on direct hedge fund investments.

Suppose that for an unskilled investor who chooses to use a fund-of-funds, the fund-of-funds' manager charges a performance fee f as a percentage of capital gains or losses. Then, the utility of the unskilled investor in the economy with funds-of-funds is

$$\begin{aligned} C^*_U &= \mathrm{E}(r^{U^*}_p) - \frac{\gamma}{2}\,\mathrm{var}(r^{U^*}_p) \\ &= 2\rho_S\left[(1-f)\,\mu_G - \frac{\gamma}{2}\,(1-f)^2\sigma^2_G\right] \\ &\qquad +(1-\rho_S)\left[(1-f)\,\mu_B - \frac{\gamma}{2}\,(1-f)^2\sigma^2_B\right], \end{aligned} \tag{A-5}$$

where $r^{U^*}_p$ denotes the after-fee return of the unskilled investor that uses a fund-of-funds.

Discussion

While simple, this model illustrates four important points in comparing the returns of hedge funds and funds-of-funds.

Point 1 *The expected after-fee return of the average fund-of-funds investment is lower than the expected return of the average hedge fund, even though funds-of-funds' managers are skilled investors.*

In an economy where unskilled investors choose to use funds-of-funds, hedge fund investors tend to be either skilled direct investors or fund-of-funds managers. If they have a similar ability to evaluate hedge funds, then they should earn the same expected returns before fees. But, the returns after fee-on-fees of funds-of-funds are lower. Thus, to evaluate funds-of-funds, it is not meaningful to directly compare the average after-fee returns of funds-of-funds to hedge funds because by doing so we are comparing the returns of unskilled investors (through funds-of-funds) to the returns that skilled investors could achieve. Instead, we must compare the gains from funds-of-funds to the gains that the same investors would have achieved with direct hedge fund investments. That is, we need to compare the utility of the unskilled investors when they choose not to use a fund-of-funds with their utility when they choose to use funds-of-funds and pay their added fees. These are exactly the certainty equivalent comparisons we perform in Section 3.

Point 2 *Unskilled investors can potentially increase their utilities by investing in a fund-of-funds even though the average fund-of-funds does not outperform the average hedge fund.*

If unskilled investors directly invest in hedge funds without the benefit of a fund-of-funds intermediary, they might expect a lower return and higher variance than the skilled investors, and thus an inferior utility. If funds-of-funds exist, then the unskilled investors earn the same before-fee expected return and utility as the skilled investors. Thus, as long as the fees of funds-of-funds are low enough, unskilled investors increase their utility by using funds-of-funds, that is $C^*_U > C_U$. Hence, it is wrong to conclude that funds-of-funds are not adding value just by comparing the after-fee average returns of funds-of-funds and hedge funds.

Importantly, the presence of funds-of-funds also alters the distribution of funded hedge fund returns. In an economy with funds-of-funds, the distribution of all funded hedge funds returns is better (with higher mean and lower variance) than the distribution of hedge fund returns when no fund-of-funds exists.

Point 3 *Unskilled investors cannot just mimic skilled investors because good funds fill with the money from skilled investors.*

In an economy with funds-of-funds it might seem that an unskilled investor should just wait and enter funds after the skilled investors are in. However, in reality these top funds close, raise minimums, raise fees, or eventually earn lower returns as they become larger. Thus, investors that only followed would earn lower returns than the average hedge fund return. Furthermore, skilled investors monitor their investments and tend to withdraw when things go awry. Thus, passive following could easily generate lower utility than a fund-of-funds investment.

Point 4 *Theoretically, we could directly compare the utility of investors with and without the use of a funds-of-funds. However, in reality we only see the data from the economy that includes funds-of-funds.*

In data, we only observe funded hedge funds that receive investments either through expert funds-of-funds or by sophisticated, skilled, direct investors. If investors all invested directly in hedge funds without funds-of-funds, then the set of funded hedge funds would be much worse than what we observe in data. This causes the observable returns of hedge funds to not represent the full, true distribution of hedge fund returns. Thus, fund-of-funds returns should not be directly compared with hedge fund returns.

B Dimson (1979) Correlations

For each asset k and each fund i, we run a series of monthly excess returns on both contemporaneous and lagged asset returns (using up to $m = 1, \ldots, 6$ lags):

$$r_t^i = \alpha_k^i + \beta_{k,0}^i r_{k,t} + \beta_{k,1}^i r_{k,t-1} + \cdots + \beta_{k,m}^i r_{k,t-m}, \tag{B-1}$$

where r_t^i is the excess return of the ith fund and $r_{k,t}$ is the excess return of the kth benchmark asset. If non-synchronous trading exists, then the lagged betas $\beta_{k,m}^i$ are non-zero. The Dimson (1979) beta for the ith fund with respect to asset k, $\hat{\beta}_{k,m}^i$, is the sum of the $\beta_{k,m}$ betas across the lags:

$$\hat{\beta}_{k,m}^i = \beta_{k,0}^i + \beta_{k,1}^i + ... + \beta_{k,m}^i. \tag{B-2}$$

We compute the correlation implied by the Dimson beta, $\hat{\rho}_{k,m}^i$, as:

$$\hat{\rho}_{k,m}^i = \hat{\beta}_{k,m}^i \cdot \sigma_k / \sigma_i, \tag{B-3}$$

where σ_k is the standard deviation of asset k, and σ_i is the standard deviation of fund i. Since the beta estimates computed using the Dimson correction are closer to the true betas, the correlation estimates in equation (B-3) should provide more accurate estimates of the true correlations between fund returns and asset returns. We use the median cross-sectional correlation across all funds indexed by i in our portfolio allocation analysis.

The Dimson correlations lead to much higher correlations of hedge funds or fund-of-funds returns with other asset classes than correlations without any adjustments for non-synchronous trading. For example, the correlations with no Dimson adjustment for hedge funds and U.S. equities is 0.25, which increases to 0.45 with two Dimson lags and 0.46 for three Dimson lags. Similarly, the correlations between funds-of-funds and U.S. equities increase from 0.41 with zero lags to 0.77 (0.80) with two (three) Dimson lags. Thus, not taking into account the Dimson lags overstates the diversification benefits of hedge funds of funds-of-funds.

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Table 1: Hedge Fund and Fund-of-Funds Returns

		Hedge Fund	Fund-of-Funds
Number of Funds	Live	2666 (55%)	1097 (70%)
	Graveyard	2175 (45%)	479 (30%)
	Total	4841 (100%)	1576 (100%)
Fee Structure	Management Fee	1.5%	1.5%
(Median)	Incentive Fee	20%	10%
	Proportion with High Watermark	65%	56%
Monthly Excess Returns	Mean	0.41%	0.30%
	Median	0.46%	0.35%
	Std Dev	2.94%	1.55%
Alphas wrt Hedge Fund Indices	Mean	-0.02%	-0.19%
	Median	0.13%	-0.12%
	Std Dev	1.31%	0.59%
Alphas wrt Tremont Style Index	Mean	-0.07%	-0.29%
	Median	0.03%	-0.17%
	Std Dev	2.17%	0.90%

The table reports summary statistics of hedge fund and fund-of-funds returns from the TASS Tremont database from January 1994 to June 2006. For the fee structure statistics, the median management and incentive fees are reported, along with the proportion of all funds having a high watermark provision. The excess returns and alphas are all reported at a monthly frequency and excess returns are computed in excess of a one-month U.S. Treasury T-bill rate, all expressed in continuously compounded terms. For excess returns and alphas, we report the cross-sectional monthly mean, median, and standard deviation. To compute the alphas with respect to the hedge fund index, we run a regression of excess hedge fund returns onto the Tremont aggregate hedge fund index for each fund. The alphas with respect to the Tremont style indices are computed a similar way, except there are nine Tremont style factors representing the following styles: convertible arbitrage; dedicated short bias; emerging markets; equity market neutral; event driven; fixed income arbitrage; global macro; long/short equity; and managed futures.

Table 2: Input Variables for the Asset Allocation Problem

	Excess Return	Std Dev	Correlations with U.S. Equities	Correlations with U.S. Bonds	Correlations with Foreign Equities	Correlations with Foreign Bonds
Panel A: Base Assets						
U.S. Equities	0.55%	4.33%	1	0.25	0.60	0.04
U.S. Bonds	0.20%	1.67%	0.25	1	0.15	0.43
Foreign Equities	0.51%	4.73%	0.60	0.15	1	0.34
Foreign Bonds	0.23%	2.43%	0.04	0.43	0.34	1
Panel B: Hedge Funds and Funds-of-Funds						
Hedge Funds	0.46%	2.94%	0.46	0.08	0.35	0.04
Funds-of-Funds	0.35%	1.55%	0.80	0.23	0.69	0.15
Panel C: Artificial Funds-of-Funds						
Simple Artificial Funds-of-Funds						
2 Funds	0.45%	3.74%	0.39	0.06	0.37	0.01
5 Funds	0.49%	2.74%	0.53	0.08	0.49	0.01
10 Funds	0.51%	2.26%	0.65	0.10	0.59	0.01
30 Funds	0.52%	1.84%	0.80	0.12	0.73	0.01
Optimal Artificial Funds-of-Funds						
$\gamma = 4$	0.39%	2.39%	0.24	0.15	0.16	0.06
$\gamma = 6$	0.35%	2.76%	0.24	0.17	0.16	0.06
$\gamma = 8$	0.39%	2.46%	0.32	0.17	0.23	0.06
$\gamma = 12$	0.43%	2.23%	0.41	0.16	0.30	0.06

Panel A reports input moments for the base assets in the asset allocation problem. U.S. Equities represent returns on the S&P500 index from January 1975 to June 2006; U.S. Bonds are the Lehman Aggregate Bond index from January 1976 to June 2006; Foreign Equities are the MSCI World Index excluding the U.S. from January 1975 to June 2006; and Foreign Bonds are the Lehman Global Aggregate Bond Index excluding the U.S. from January 1990 to June 2006. Panel B lists moments for hedge funds and funds-of-funds, which are the median excess return and the median excess return volatility in the cross section. The correlations with other asset classes are computed using Dimson (1979) corrections for non-synchronous trading and are computed following the method outlined in Appendix B. Panel C reports medians and cross-sectional medians of volatility for artificial funds-of-funds (simple and optimal), which are described in more detail in Section 3.1. We use three Dimson lags to compute the correlations with the benchmark assets. In Panels A-C, all returns are expressed as monthly continuously compounded excess returns in percentages.

Table 3: Characterizing the Benchmark Fund-of-Funds Hurdle Rate

	No Short Sales				Short down to -20%			
Number of Funds	γ=4	γ=6	γ=8	γ=12	γ=4	γ=6	γ=8	γ=12
Median Hedge Fund Return in Data = 0.460% per month								
Case 1: $\sigma_B = \sigma_H$ = Median Hedge Fund Volatility in Data								
1	0.357	0.405	0.446	*0.481*	0.389	0.434	*0.475*	*0.516*
2	0.371	0.441	*0.501*	*0.554*	0.415	*0.483*	*0.545*	*0.608*
5	0.372	0.406	0.437	0.464	0.394	0.429	0.459	0.490
10	0.372	0.388	0.404	0.418	0.383	0.400	0.415	0.430
30	0.369	0.370	0.371	0.372	0.370	0.370	0.371	0.372
Optimal	0.277	0.330	0.371	0.388	0.297	0.361	0.396	0.411
Case 2: $\sigma_B = 1.1 \times \sigma_H$								
1	0.370	0.427	*0.476*	*0.518*	0.407	*0.461*	*0.511*	*0.560*
2	0.382	*0.465*	*0.535*	*0.597*	0.434	*0.513*	*0.586*	*0.659*
5	0.386	0.428	0.466	*0.499*	0.414	0.455	0.493	*0.531*
10	0.386	0.408	0.429	0.448	0.401	0.424	0.444	0.463
30	0.384	0.386	0.391	0.396	0.386	0.389	0.391	0.396
Optimal	0.283	0.346	0.394	0.416	0.307	0.382	0.424	0.444

The table reports the expected excess monthly return, μ_B, in monthly percentages which is constructed so that an investor is indifferent between adding a portfolio of hedge funds with this expected return, or adding a fund-of-funds to a set of four benchmark assets (U.S. equities, U.S. bonds, foreign equity, and foreign bonds). In Case 1, we assume that the fund-of-funds benchmark standard deviation, σ_B, is equal to the median cross-sectional standard deviation of hedge fund excess returns in data, σ_H. (see Table 2). In Case 2, we assume that $\sigma_B = 1.1 \times \sigma_H$. Entries in italics indicate that the mean is greater than the observed median hedge fund excess return in data and consequently, in these cases, investors already prefer a fund-of-funds even if they can obtain somewhat higher returns on their own.

Table 4: The Fund-of-Funds Hurdle Rate Computed Using Equilibrium Expected Returns

	Equilibrium Risk Premia μ^e				Fund-of-Fund Benchmark μ_B				Annualized Difference $12 \times (\mu^e - \mu_B)$			
Asset	γ=4	γ=6	γ=8	γ=12	γ=4	γ=6	γ=8	γ=12	γ=4	γ=6	γ=8	γ=12
HF/FoF Weight = 5%												
Fund-of-Funds μ_{FoF}	0.131	0.197	0.262	0.328								
1 Hedge Fund	0.144	0.216	0.288	0.359	0.107	0.175	0.244	0.313	0.45	0.48	0.52	0.56
2 Hedge Funds	0.176	0.264	0.353	0.441	0.107	0.189	0.271	0.354	0.83	0.90	0.97	1.04
5 Hedge Funds	0.160	0.240	0.319	0.399	0.129	0.207	0.284	0.361	0.36	0.40	0.43	0.46
10 Hedge Funds	0.155	0.233	0.311	0.388	0.139	0.216	0.292	0.368	0.19	0.21	0.22	0.24
30 Hedge Funds	0.153	0.229	0.306	0.382	0.147	0.223	0.299	0.375	0.07	0.08	0.08	0.09
Optimal Hedge Fund	0.067	0.120	0.180	0.242	0.047	0.086	0.154	0.223	0.24	0.40	0.31	0.23
HF/FoF Weight = 10%												
Fund-of-Funds μ_{FoF}	0.129	0.194	0.259	0.323								
1 Hedge Fund	0.154	0.232	0.309	0.386	0.111	0.182	0.253	0.324	0.52	0.60	0.67	0.75
2 Hedge Funds	0.197	0.295	0.393	0.491	0.115	0.202	0.289	0.375	0.97	1.11	1.25	1.39
5 Hedge Funds	0.167	0.251	0.334	0.418	0.131	0.210	0.288	0.367	0.43	0.49	0.55	0.61
10 Hedge Funds	0.158	0.237	0.316	0.395	0.139	0.215	0.292	0.368	0.23	0.26	0.29	0.32
30 Hedge Funds	0.152	0.228	0.304	0.380	0.145	0.220	0.295	0.370	0.08	0.10	0.11	0.12
Optimal Hedge Fund	0.075	0.137	0.196	0.255	0.052	0.096	0.163	0.230	0.28	0.50	0.40	0.31

Note to Table 4

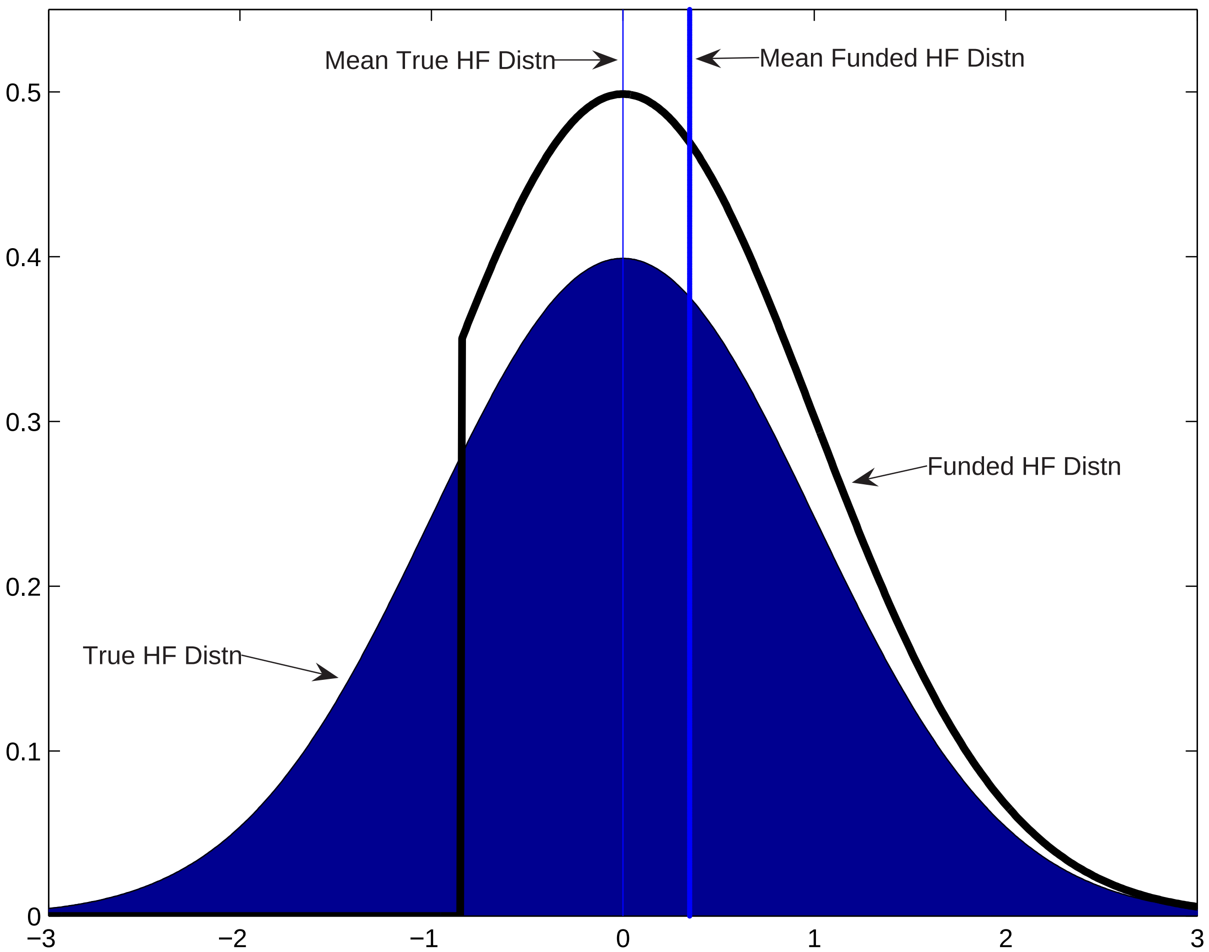
The first four columns titled "Equilibrium Risk Premia μ^e" report the equilibrium expected excess returns computed using equation (4), which backs out expected excess returns from market weight capitalizations as of 30 June 2006. We take the market weight proportions of U.S. equity, U.S. bonds, international equity, and international bonds and scale these weights to sum to 95% or 90%. The remaining 5% or 10% weight we assume is invested in hedge funds or funds-of-funds. The covariance matrix is estimated from data over the full sample using the inputs in Table 2. The columns titled "Fund-of-Fund Benchmark μ_B" report the hurdle rate μ_B which makes an investor indifferent between holding a fund-of-fund in addition to the four benchmark assets with the Black-Litterman (1992) equilibrium expected returns reported in the first four columns and hedge funds drawn from the investor's true hedge fund distribution with expected excess return μ_B. We assume that the benchmark volatility σ_B is equal to the observed median cross-sectional standard deviation of hedge fund excess returns in data. Both μ^e and μ_B are expressed in monthly percentage terms. The last four columns titled "Annualized Difference $12 \times (\mu^e - \mu_B)$" report the annualized difference between μ^e and μ_B expressed in percentages.

Table 5: The Fund-of-Funds Hurdle Rate Computed Using Forecasted Expected Returns

	No Short Sales				Short down to -20%			
Number of Funds	γ=4	γ=6	γ=8	γ=12	γ=4	γ=6	γ=8	γ=12
Median Hedge Fund Return in Data = 0.460% per month								
1	0.359	0.332	0.323	0.323	0.348	0.323	0.323	0.323
2	0.387	0.380	0.378	0.378	0.384	0.378	0.378	0.378
5	0.370	0.338	0.327	0.327	0.358	0.327	0.327	0.327
10	0.361	0.318	0.296	0.296	0.344	0.299	0.296	0.296
30	0.352	0.300	0.265	0.257	0.331	0.274	0.258	0.257
Optimal	0.282	0.269	0.259	0.254	0.266	0.261	0.259	0.254

The table reports the expected excess monthly return, μ_B, in monthly percentages which is constructed so that an investor is indifferent between adding a portfolio of hedge funds with this expected return, or adding a fund-of-funds to a set of three benchmark assets (U.S. equities, U.S. bonds, and foreign equity). Forecast returns and volatilities for these asset classes are provided by Yale Endowment in its 2006 annual report. We compute the correlation matrix using the inputs from data in Table 2. We conservatively assume that the fund-of-funds benchmark standard deviation, σ_B, is equal to the median cross-sectional standard deviation of hedge fund excess returns in data (see Table 2).

Figure 1: Simple Example of Truncation Bias



Assume that the true hedge fund (HF) distribution is normal, which is shown in the solid blue curve. If only skilled fund-of-funds managers and skilled individual investors are making direct hedge fund investments and they are able to avoid the worst 20% of hedge funds, then the distribution of funded hedge funds becomes a truncated normal distribution, shown in the solid black line. The mean of the truncated distribution is shifted to the right of the mean of the true HF distribution.