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Price Improvement in Dealership Markets*

Price improvement refers to the practice whereby dealers offer executions that improve on quoted prices. Why are these improvements given? Standard thinking is that competition causes dealers to give better prices to customers with less information. This paper contrasts this with a novel theory in which customers negotiate improvements and differential pricing arises from differences in customers' market power. Each theory affects the formation of bid/ask spreads in empirically distinguishable ways. Understanding price improvement and its impact on market participants is critical the regulation of markets, particularly since equal execution is such an important stated goal of the SEC.

I. Introduction

To understand price formation we must understand price improvements and why improvements are given. Price improvements may be given to customers with less information. This is the standard theory in market microstructure initially formalized by Seppi (1990). However, it may be that some customers have different amounts of market power due to their size, knowledge, technology, discount rate, and the like, regardless of their information. Those customers with more market power negotiate better prices. The introduction of this second theory is important both to understand price improvement and to regulate markets but also because it recognizes that financial markets may not be perfectly competitive and therefore the market power of customers relative to dealers may play a significant role in the formation of prices.

The simultaneous examination of both models demonstrates the effects of price improvement that are general and those that depend on the specific reason improvements are given (adverse selection or market power). This comparison results in predictions that can be empirically tested to

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determine the dominant explanation for price improvements. Since equal execution is an important goal of the regulatory process an understanding of why execution may be unequal is critical. Furthermore, price improvements and market power may be important unexamined components of the bid/ask spread.

Price improvement is a pervasive feature of many financial markets, especially dealer markets such as the NASDAQ and London equity markets and the foreign exchange market. Furthermore, almost all new financial innovations are traded through dealer markets. In London, evidence provided by Reiss and Werner (1996) and Hansch, Naik, and Viswanathan (1999) shows that mid-size and large trades receive a price improvement over small trades. This practice is controversial, as it implies that customers that are able to negotiate may be able to obtain better executions than other customers. On NASDAQ, the ability to negotiate posted quotes has been suggested by Kleidon and Willing (1995) and Grossman et al. (1997) as an important reason spreads on the NASDAQ market are so wide, as documented in Christie and Schultz (1994). Even auction-type markets like the New York Stock Exchange (NYSE) feature price improvement on the best quotes through hidden limit orders or stopped orders, as documented in Petersen and Fialkowski (1994) and Ready (1999). However, none of these papers model how quotes are determined in the presence of price improvement.

In the canonical models of market microstructure laid out in Kyle (1985, 1989), Glosten and Milgrom (1985), and Ho and Stoll (1983), price improvement is not considered and market makers expect zero profit from each trade. However, in this paper, as in Dennert (1993), dealers expect profits on individual trades (though, with fixed costs, the expected market-making profit may still be zero). Thus, dealers have the market power to set the spread wider than the zero-expected-profit-per-trade prices.[1] This gives them room to give price improvements.

Less-than-perfect competition among dealers is modeled by allowing heterogeneous dealers to choose bids in a first price auction.[2] While bidding for order flow due to dealer heterogeneity is allowed in Ho and Stoll (1983)[3] and Biais (1993), the best quotes in these models are the

1. On SEAQ, the average number of dealers per stock ranges from 12.6 for the FTSE-100 to 6.2 and 4.7 for the medium and small equities (Reiss and Werner 1996). Furthermore, for the FTSE-100, seven dealers execute 90% of public orders (Hansch, Naik, and Viswanathan 1998). On NASDAQ, the average number of dealers is 10, although some stock have only 1 and others have over 60. Therefore, less- than-perfect competition seems a reasonable characterization of these markets. Q11

2. The use of the auction model is not meant to suggest that dealer markets are exactly like auctions, in the same way that Kyle (1989) is not meant to suggest that orders are batched. Rather, an auction model is used because dealer markets have similarities to an auction. What an auction model sacrifices in reality it makes up for in tractability and with insightful results.

3. Ho and Stoll (1983) is effectively a form of English auction, as the spread is defined by the reservation value of the second-highest dealer. However, under particular conditions the reservation value of every dealer is the same in equilibrium.

prices at which trades are executed. However, dealership markets are characterized by the posting of wide quotes followed by price improvement to some customers. In this paper, unlike a standard first price auction, where no negotiation of bids is allowed, a fraction of customers receive a price better than the first-stage dealer quotes.

The first model considered here develops information-based price improvement and originates from the theories proposed by Seppi (1990), Barclay and Warner (1993), and Hansch et al. (1999). Seppi (1990) noted that, with negotiated improvements, the transaction is not anonymous; thus, implicit contracts can bind repeat customers.[4] A penalty technology could force repeat customers to acknowledge when they are informed by trading at the quotes and ask for an improvement only when they are uniformed.[5] Competition then forces dealers to give better prices to less-informed customers. Barclay and Warner (1993) and Hansch et al. (1999) suggest that dealers can examine customers to assess their information. So informed customers remain anonymous and trade at the quotes, while the uninformed submit to examination and receive an improvement. The result of either theory is the same: less-informed customers receive price improvements.

The second model considered is novel. In this theory, price improvements are negotiated and determined by the relative market power of customers and dealers. This model relaxes the assumption implicit in any information model, that either the dealers expect zero profit and therefore could not negotiate or that dealers have all the market power and thus post take-it-or-leave-it prices. Instead, we assume that some customers can make counteroffers to the posted quotes. Since competition between dealers is less than perfect, dealers bid below their true value, leaving the surplus up for negotiation. Those customers who can negotiate may be larger, own a negotiation technology, have lower discount rates, posses greater skill, and so forth, but they do not necessarily have less or more information than a customer who cannot negotiate.

Note that, with information-based improvements, the uninformed customers receive improvements because competition forces the dealers to raise their prices to obtain these more profitable customers. However, in the bargaining model, the market power of the customer requires the dealer to give up some surplus even though all customers are equally profitable.

Either theory affects price formation. Dealers quote prices with the knowledge that either competition or negotiation may result in an

4. Recent work by Dan Bernhardt and Werner (2002) and Desgranges and Foucault (2002) models this repeated interaction. Q12

5. See Seppi (1990), proposition 2. Seppi also demonstrates a partial-pooling equilibrium, where some informed customers ask for price improvements. However, even in this equilibrium customers can credibly signal (although imperfectly) that they are uninformed. For simplicity, this paper focuses on the separating equilibrium.

improvement. Focusing first on the similarities between the theories clarifies the relevant differences: in both models, an increase in the fraction of informed customers widens the bid/ask spread as in a standard model. An increase in the probability that a customer negotiates or the market power of a negotiator (in the second model) also widens the bid/ask spread. However, with either form of price improvement, the dealers' profits are unaffected by the probability that a customer will Q1 negotiate. The notion that dealers are not affected by price improvements seems counterintuitive. Intuition might suggest that, when the dealers are forced to give up some surplus to customers with market power, their profits should go down. Or, when the dealers are able to discriminate based on the customer's information, their profits should go up. This is not the case. A more accurate intuition is that, if prices may be improved, the dealers post wider quotes but competition among dealers causes their overall expected profit to remain unchanged.

The stability of dealer profits follows from the celebrated revenue equivalence theorem of Myerson (1981), Harris and Raviv (1981), and Riley and Samuelson (1981). Since the true value of a trade does not change with the probability that a customer can negotiate and the competition is not changing, dealers simply adjust their quotes, in expectation, to earn the same amount. This is important because it tells us that dealers are affected only by changes that alter the size of the available pie, such as the level of adverse selection, taxes, the quantities negotiated, competition, or deadweight costs like negotiation effort. However, the dealers are not affected by the type or level of improvements.

The bid/ask spread, however, is wider as a result of price improvements. Therefore, the width of the spread is determined in part by the negotiator's market power. Most research on the components of the spread has examined order-processing costs, inventory-holding costs, and adverse selection costs[6] (see Huang and Stoll 1997 for a summary Q2 of the literature). This paper suggests that a fraction of the market spread is due to price improvements.

Since the dealer is not affected by improvements, the first-order effect of price improvement is simply a transfer between customers. This tells us that the public policy issue is whether one group of traders should gain relative to another group of traders. Price improvement based on information allows negotiating uninformed customers and informed customers to both receive prices commiserate with their information. Also, if improvements are banned, then informed customers extract even more from the uninformed customers, who cannot negotiate. If improvements are due to customers with market power, then they allow institutional players to extract surplus from the customers who

6. Such as Glosten and Harris (1988) and Foster and Viswanathan (1993).

cannot negotiate improvements. Thus, this work demonstrates how the costs and benefits of price improvement depend on the type of price improvement.

The initial results provide a broad intuition that the type or existence of price improvement does not affect the dealer. However, the different models imply different correlations between the quoted and negotiated spreads. This paper shows how it may be possible to empirically determine which form of price improvement is dominant in a particular market.

If the fraction of informed customers in the market increases and the information model is correct, then the quoted spread should increase but the improved spread should not. In contrast, if the market power-based model is correct, then both the quoted and negotiated spreads should increase. If a proxy could be found for an increase in the probability of negotiation, then a similar test could be run on a change in this probability.

If we examine how competition affects the price improvements, we can then find both another test of the models and an interesting explanation for a seeming anomaly. In a market with many dealers, the standard prediction is that greater competition forces dealers to increase the price improvements, see for example Harris (1994). However, Reiss and Werner (1996) find the opposite result. They find that the price improvement on securities for which many dealers make a market is less than the price improvement on securities with little competition.

Smaller price improvement is the natural outcome of greater dealer competition if improvements are based on the market power of customers. In the market-power model, greater competition decreases the magnitude of the price improvements, because each dealer earns less, so less is available for negotiators to acquire. However, in the information-based model, greater competition forces dealers to give larger improvements, as expected by Harris (1994). Thus, in the market-power model, improvements decrease with increased competition, but in the information model, improvements increase with competition. Consequently, Reiss and Werner's 1996 result lends support to the theory that customer market power plays a significant role in price formation and suggests that information may not be as important in price improvements.

This paper examines two different explanations why dealers give price improvements. Thus, it provides a better understanding of how prices are formed. Although the focus is on microstructure, the results apply more generally to any situation where an item is exchanged through a dealer. Why are improvements given? Is it because some customers are uninformed or because some customers have market power? We conclude that the market power of customers is an important and unexplored aspect of market microstructure.

Section II of this paper presents both models of price improvement. Section III analyzes the general affects of price improvement. Section IV focuses on the differences between the models and the tests to determine which theory is correct, and Section V examines the effect of competition. Section VI concludes.

II. The Model

The securities market is modeled as a three-stage game with private information. In the first stage N risk-neutral dealers post quotes to purchase (bid) and sell (ask) a security.[7] In the second stage, a customer arrives. A fraction α of the customers have the potential to receive a price improvement through one of two interactions, discussed in detail later. Those customers that receive an improvement trade at the improved price and those customers that do not receive an improvement trade at the posted quotes. In the third stage, the value of the security is revealed.

The dealers have common beliefs about the value of the security. Dealers also have an independent private characteristic, such as inventory positions or variable transaction costs.[8] The variable costs ensure that each dealer's willingness to pay for the security, v_i^b, is below his willingness to sell the security, v_i^a, where the subscript represents the different dealers and the superscript indicates the bid side (b) or the ask side (a) of the market. On either side of the market, the individual dealer's dollar values accounting for his private characteristics are distributed independently and identically and drawn from $F^b(v)$ with $F^b(\underline{v}^b) = 0, \mathrm{F}^{\mathrm{b}}(\overline{v}^b) = 1$, or $F^a(v)$ with $F^a(\underline{v}^a) = 0, \mathrm{F}^{\mathrm{a}}(\overline{v}^a) = 1$; although the dealers' values are independent, clearly an individual dealer's willingness to buy and sell are not independent. Here, $F^a(v)$ and $F^b(v)$ Q3 are strictly increasing and differentiable over their respective intervals $[\underline{v}^b, \overline{v}^b]$ or $[\underline{v}^a, \overline{v}^a]$, with $\overline{v}^b < \overline{v}^a$. The assumption that $\overline{v}^b < \overline{v}^a$ is equivalent to the assumption that all interdealer trades have already occurred or that trading costs exceed the benefits of interdealer trades.[9] Without loss of generality the primary focus is on the bid side of the market, therefore, the superscript b or a is suppressed.

In the tradition of Glosten and Milgrom (1985) and (Kyle 1985, 1989), customers are asymmetrically informed. With probability $\theta, 0 \leq \theta \leq 1$, a customer has private information about the value of the security.

7. With $N = \{1, \ldots, n\}$ representing the set of n dealers.

8. The assumption of diverse values is justified by following Amihud and Mendelson (1980), Ho and Stoll (1983), and Biais (1993) and assuming that risk-averse dealers have different inventories and hence different expected values. Hansch et al. (1998) provide evidence that a dealer's desire to sell is related to his relative inventory position. Although the risk aversion also affects the auction, these affects are ignored, following Biais (1993), as they are of second-order importance. The variable transaction costs may be order handling costs and settlement and delivery costs. Fixed costs do not affect the value of a particular trade.

9. See Wang and Viswanathan (2001) for a paper that focuses on interdealer trading.

In the third stage, the revealed value of the security is either H or L. The informed customers know the true value in the first stage. Rational uninformed customer participation is motivated by a liquidity shock, risk aversion, or inventories different from dealers. Therefore, a dealer is willing to trade because the customer may be uninformed, but if the customer is informed, the dealer loses the difference between his bid and L or his ask and H.[10] Uninformed customers have a quantity demand q. For simplicity, we assume uninformed customers demand the quoted depth, which is normalized to unity. Informed customers have infinite quantity demands, because they have perfect information.[11] Therefore, they trade as much as they can with any dealer who has a quote better than the true value. While the assumption of infinite informed demand overemphasizes the effect of adverse selection, it does not drive any results; the effects are the same if dealers who post worse quotes have a lower chance of trading with the informed customer.

The dealers' quote decisions are modeled as a first-price auction. This basic structure captures the salient aspects of the dealer market. Quotes are formed by competing dealers, who take into account heterogeneous dealer inventories and costs, the level of competition, adverse selection from informed customers, and as we will see, potential price improvements. The use of a first-price auction model results in quotes that are not conditioned on the simultaneous bids of other dealers. In a fragmented market, such as a dealer market, price formation is characterized less by repeated price increases that drive prices to the Bertrand equilibrium and more by repeated quotes and trades. At each moment in time, dealers post quotes. A trade then either occurs or does not occur. Direct price competition is a characteristic of centralized or open outcry markets (such as the Chicago options pit), where each agent knows that a trade is about to occur and can call out a better price. In dealership markets, dealers are not immediately aware of the trades of other dealers. This is particularly true in a fast-moving market.[12] This logic drives our decision to use a first-price auction.[13] However, none of the normative predictions of the paper would change under an English auction model. Thus, the paper's results apply more generally to other types of markets.

10. For simplicity, we assume $\underline{v}^b > L$ and $\bar{v}^a < H$. This is logical, since L and H are common knowledge; therefore, dealers with values below L or above H could not except to trade and do not participate in the market.

11. This is a simplification of the idea formalized by Easley and O'Hara (1987), that informed traders prefer larger quantities.

12. For a more compete discussion on the difference between a fragmented dealer market and a centralized market, see Biais (1993).

13. In a more dynamic setting, the market would be characterized through time as a repeated first-price (sealed-bid) auction, where in each auction, the "values" of the other dealers are unknown because recent trades are not in the other dealers information set. Furthermore, the "values" of every dealer change through time, due to information and the consummation of trades in the market.

In the standard market-microstructure auction model with bidder heterogeneity, such as Biais (1993), different dealers post different take-it-or-leave-it quotes, and customers then accept these quotes as fixed and decide to trade. In contrast to these models, quotes in a dealership market are often improved. Thus, some prices are set by competitive bids and some are set by secondary interaction. In this way, this paper brings together some of the theories of Wolinsky (1990) and Biais (1993). It is an open question as to who receives the improved prices and why they receive them.

This paper considers two mechanisms through which some customers receive improvements. In the first model, price improvements depend on the customers' information; those customers who can demonstrate or commit to a lack of information receive improvements. In the second model, improvements are based on the market power of the customer; some customers have the market power to negotiate an improvement. The consideration of a combination of a fragmented market, competition, and different types of price improvement allows greater insights into the price formation process.

A. Price Improvement Based on Customer Information

The most common theory of price improvement, first formalized by Seppi (1990) in the context of NYSE block trades, is that improvements are given on the basis of customer information. With improvements, the transaction is no longer anonymous; therefore, implicit contracts can bind repeat customers. The theory is straightforward: some customers repeatedly interact with the dealers. When these customers are informed they anonymously accept the quoted price, and when they are uninformed they ask for an improvement.[14] This equilibrium is enforced through a penalty technology employed by the dealer if he is hurt by the trade after giving an improvement.

Barclay and Warner (1993) form and test a similar hypothesis. They suggest that interaction allows the dealers to better assess the customer's information. Uninformed traders, therefore, like to interact to convince the dealer that they are uninformed. If a customer cannot certify that he is uninformed, he does not receive an improvement and may even face a price reduction. Therefore, informed customers trade anonymously at the quotes.

We take the theories of Seppi (1990) and Barclay and Warner (1993) as the basis for the model of improvements based on customer information.[15] We assume that only α fraction of the customers have the

14. This idea is proposition 2 in Seppi (1990). More generally, customers can only imperfectly signal that they are uninformed. The intuition of this paper does not depend on the perfect signal, although for tractability the signal is assumed to be perfect.

15. Both hypotheses are further supported by evidence from Hansch et al. (1999), who find evidence on the London exchange that is consistent with the hypothesis "that customers have trading relationships with dealers." And, they show that dealers make money on the small and large trades but lose money on medium-sized trades (trades that use all of the depth at the quote).

relationship that allows them to interact with the dealer and commit to or reveal a lack of information. Furthermore, while only α fraction of the customers could receive an improvement, only the subset $(1-\theta)$ of that group who are actually uninformed do receive an improvement.

The summation of model 1 is as follows. First, the dealers post quotes. Second, a customer arrives. If the customer cannot interact with dealers and is uninformed, he trades 1 unit with the dealer posting the best quote. If the customer is informed, he trades 1 unit with every dealer. If the customer is uninformed and can interact with dealers then each dealer makes him another offer.[16] This second offer is also modeled as a first-price auction.[17] In the final stage, the true value is revealed.

The dealer's goal is to choose his quoted price and the amount of price improvement to maximize profits. To trade with an uninformed customer who cannot receive an improvement, the dealer must offer the best quote. To trade with a customer who can negotiate, the dealer must offer the best price-improved quote. Let b_{1i} represent dealer i's quoted bid, and b_{2i} represent dealer i's price-improved bid. In expectation, the dealer earns $(v_i - b_{1i})[\text{Prob}(b_{1i} > b_{1j} \text{ for all } j \neq i)]$ if an uninformed customer who cannot interact with dealers arrives at the market. The dealer loses $(b_{1i} - L)$ if an informed customer arrives at the market.[18] And the dealer expects to earn $(v_i - b_{2i})[\text{Prob}(b_{2i} > b_{2j} \text{ for all } j \neq i)]$ if an uninformed customer who can interact with dealers arrives at the market. Therefore, dealer i's expected profit is

$$\begin{aligned}\Pi_i = {} & (1-\theta)\alpha(v_i - b_{2i})[\text{Prob}(b_{2i} > b_{2j} \quad \text{for all } j \neq i)] \\ & + (1-\theta)(1-\alpha)(v_i - b_{1i})[\text{Prob}(b_{1i} > b_{1j} \quad \text{for all } j \neq i)] - \theta(b_{1i} - L).\end{aligned} \tag{1}$$

To determine the probabilities of winning, assume that all bidders except bidder i use the conjectured invertible equilibrium bid functions, $b_1(v_j)$ and $b_2(v_j)$ (invertibility will be verified in equilibrium). Therefore, dealer i beats dealer j if $b_{1i} > b_1(v_j)$ or $b_{2i} > b_2(v_j)$, depending on the type of customer. Since the equilibrium bid functions are assumed to be invertible, these inequalities can be rewritten as $b_1^{-1}(b_{1i}) > v_j$ or $b_2^{-1}(b_{2i}) > v_j$. Given the distribution of values, $F(\cdot)$, the probability of

16. Since only uninformed customers ask for price improvements, the dealer faces no adverse selection from these customers. The dealers can therefore give better prices. Whether they choose to give price improvements depends on the competition in the market.

17. We assume that the information from the first posting of quotes is still unknown to the other dealers. However, this is not necessary for the results. The reader may prefer to think that the quoted prices reveal the dealers' information. In this case, the equilibrium best offer in the second round is the true value of the second-highest bidder. Thus, the release of information would change the model of competition for the uninformed customer to an English (ascending-bid) auction but would not change any of the results in the paper.

18. No probability is associated with the loss, because the informed customers trade with every dealer who quotes a price above the true value, L.

i beating j is $F[b_1^{-1}(b_{1i})]$ or $F[b_2^{-1}(b_{2i})]$, and the probability that dealer i beats all other dealers is $F^{n-1}[b_1^{-1}(b_{1i})]$ or $F^{n-1}[b_2^{-1}(b_{2i})]$, since $F^{n-1}(\cdot)$ is the distribution function of the highest of $n-1$ draws from $F(\cdot)$.

Therefore, dealer i chooses b_{1i} and b_{2i} to maximize expected profits:

$$\max_{b_{1i},b_{2i}} \Pi_i = (1-\theta)\alpha(v_i - b_{2i})F^{n-1}[b_2^{-1}(b_{2i})] + (1-\theta)(1-\alpha)(v_i - b_{1i})F^{n-1}[b_1^{-1}(b_{1i})] - \theta(b_{1i} - L). \quad (2)$$

Result 1. The unique symmetric equilibrium bid is

$$b_1(v) = \left[v - \int_{\underline{v}}^{v} \frac{F^{n-1}(x)}{F^{n-1}(v)}dx\right]\frac{(1-\alpha)(1-\theta)F^{n-1}(v)}{(1-\alpha)(1-\theta)F^{n-1}(v)+\theta} + \frac{\theta L}{(1-\alpha)(1-\theta)F^{n-1}(v)+\theta}, \quad (3)$$

with equilibrium price improvement equal to $b_2(v) - b_1(v) =$

$$\left[v - \int_{\underline{v}}^{v} \frac{F^{n-1}(x)dx}{F^{n-1}(v)}\right]\left[\frac{\theta}{(1-\alpha)(1-\theta)F^{n-1}(v)+\theta}\right] - \frac{\theta L}{(1-\alpha)(1-\theta)F^{n-1}(v)+\theta} \geq 0, \quad (4)$$

and the price improved bid is equal to

$$b_2(v) = \left[v - \int_{\underline{v}}^{v} \frac{F^{n-1}(x)dx}{F^{n-1}(v)}\right] \quad (5)$$

Proof. See the appendix.

The dealer's market bid is simply the weighted average of the dealer's optimal bid to the uninformed customers and the optimal bid to the informed customers. More specifically, the dealers bid their value, v, minus a term because the competition is not perfect, $\int_{\underline{v}}^{v}[F^{n-1}(x)]/F^{n-1}(v)]dx$. This is the standard bid in a first price auction because $v_i - \int_{\underline{v}}^{v_i}[F^{n-1}(x)/F^{n-1}(v)]$ is the expected value of the next-highest dealer given that i has the highest value; the dealers bid to just beat the competition in expectation. However, v is the dealer's value only if the customer is uninformed. Thus, the standard bid must be multiplied by the probability that the customer is uninformed given that the customer is not negotiating, $[(1-\alpha)(1-\theta)F^{n-1}(v)]/[(1-\alpha)(1-\theta)F^{n-1}(v)+\theta]$. If the customer is informed, then the dealer's value is L. In this case, the customer trades with every dealer, so the level of competition is irrelevant. Since L is the dealer's value only if the customer is informed, it must be multiplied by the probability the customer is informed, given that the customer is not negotiating, $\theta/[(1-\alpha)(1-\theta)F^{n-1}(v)+\theta]$. And, $b_2(v)$ can be interpreted similarly; since improvements are given only to uninformed customers, the bid is just the optimal bid to an uninformed customer (multiplied by the probability that the customer is uninformed, given that the customer is uninformed, which is just 1). So the bids are just the weighted averages of the optimal bids in each situation.[19]

The equilibrium asks, $a_1(v)$, can be found and interpreted similarly. The only differences are that the probability of winning is $[1-F(v)]^{n-1}$, and the informed customer's value is H, and some signs are changed. Thus, a dealer's quoted spread is $a_1(v)-b_1(v)$.

Note that, throughout this model, as in any standard information based model, all of the market power is implicitly assumed to reside with the dealer, as each dealer posts take-it-or-leave-it offers. Dealers improve the price only if competition forces them to do so. The model that follows allows for the possibility that customers can respond.

Before analyzing the effects of price improvement, it is important to first develop an alternative model of price improvement based on customers with the market power to negotiate improvements. This allows us to consider general effects of price improvement and determine what distinguishes the theories.

B. Price Improvement Driven by Customers with Market Power

An alternative theory of price improvement relaxes the assumption, implicit in the information model, that the dealer has all the market power and therefore posts take-it-or-leave-it offers. The alternative theory assumes that customers can make counteroffers and negotiate with the dealers. This form of price improvement does not depend on the information of the customers. The theory simply assumes that customers have different amounts of market power and those customers with more market power negotiate better prices. Clearly, this market poweraffects price formation.

The market power of customers arises from their large size, their knowledge of markets, their technology, their low discount rates, and the like. The negotiations literature has enumerated many potential reasons why customers could differ in their market power relative to the dealers.

For simplicity we assume that the market power differences between customers are such that only α fraction of customers can extract an improvement. Furthermore, all negotiating customers have the same market power relative to dealers. These negotiating customers are neither more nor less informed than customers who accept the quote.[20] Since dealers

19. It is easily confirmed that, as long as $\theta < 1$, both bids are increasing in *v*, which was assumed to start the problem. As long as some customers are not informed, $\theta < 1$, then the dealers can profit with a small enough quote, and those with higher values bid higher.

20. For simplicity, we assume the probability of being informed, θ, and the ability to negotiate, α, are independent. As long as the correlation is the same across the models, it does not alter any normative conclusion.

earn a profit from the customers that negotiate, we assume that dealers cannot commit ex ante not to negotiate. Let $Q(\hat{b}_1)$ represent the outcome of negotiations between a customer and a dealer who quotes a bid of $\hat{b}_1$ in the market. For simplicity, we assume that the negotiation depend only on the quoted bid, which represents the dealers signal to the market about his desire to trade. Thus, $Q(\hat{b}_1)$ represents the price-improved quote. The hat signifies the market power model; a distinction that is useful later. We now elaborate on $Q(\cdot)$.

The summation of model 2 is as follows. First, the dealers post quotes. Second, a customer arrives. If the customer is uninformed and cannot negotiate, he trades with the dealer who posts the highest quote. If the customer is uninformed and can negotiate, he trades with the dealer who provides the highest improved quote, $Q(\hat{b}_1)$. If the customer is informed, he trades with every dealer at a price of $\hat{b}_1$ or negotiates with every dealer.[21]

A dealer's goal is to choose his quoted price to maximize profits, accounting for the possibility of future negotiations. Since the negotiations depend on the marketwide quote and even the probability of negotiating with an uninformed customer depends on the marketwide quote, the dealer must account for the fact that his quote is influencing negotiations and the probability of negotiating with an uninformed customer. Let $\hat{b}_{1i}$ represent dealer i's quoted bid. In expectation the dealer earns $(v_i - \hat{b}_{1i})[\text{Prob}(\hat{b}_{1i} > \hat{b}_{1j}, \text{for all } j \neq i)]$ if an uninformed customer who cannot negotiate arrives at the market. The dealer loses $(\hat{b}_{1i} - L)$ or $[Q(\hat{b}_{1i}) - L]$ if an informed customer arrives at the market, depending on whether or not the customer can negotiate. And the dealer expects to earn $[v_i - Q(\hat{b}_{1i})][\text{Prob}(Q(\hat{b}_{1i}) > Q(\hat{b}_{1j}), \text{for all } j \neq i)]$ if an uninformed customer who can negotiate arrives at the market. Therefore, dealer i's expected profit is

$$\begin{aligned}\max_{\hat{b}_{1i}} \hat{\Pi}_i = {} & (1-\theta)\alpha[v_i - Q(\hat{b}_{1i})][\text{Prob}(Q(\hat{b}_{1i}) > Q(\hat{b}_{1j}), \quad \text{for all } j \neq i)] \\ & - \theta\alpha[Q(\hat{b}_{1i}) - L] + (1-\theta)(1-\alpha)(v_i - \hat{b}_{1i}) \\ & \times [\text{Prob}(\hat{b}_{1i} > \hat{b}_{1j}, \quad \text{for all } j \neq i)] - \theta(1-\alpha)(\hat{b}_{1i} - L). \qquad (6)\end{aligned}$$

To determine the probabilities of winning, assume that all bidders except bidder i use the conjectured invertible equilibrium bid function, $\hat{b}_1(v_j)$, and negotiations result in the conjectured invertible equilibrium, $Q(\cdot)$(invertibility is verified in equilibrium). Therefore, dealer i beats dealer j if $b_{1i} > b_1(v_j)$ or $Q(\hat{b}_{1i}) > Q[\hat{b}_1(v_j)]$, depending on the type of customer. Since the equilibrium bid functions are assumed to be

21. This overemphasizes the effect of information but is not important for the normative conclusions.

invertible, these inequalities can both be rewritten as $b_1^{-1}(b_{1i}) > v_j$. Given the distribution of values, $F(\cdot)$, the probability of i beating j is $F[b_1^{-1}(b_{1i})]$, and the probability that dealer i beats all other dealers is $F^{n-1}[b_1^{-1}(b_{1i})]$, since $F^{n-1}(\cdot)$ is the distribution function of the highest of $n-1$ draws from $F(\cdot)$.

Therefore, dealer i chooses b_{1i} to maximize expected profits:

$$\max_{\hat{b}_{1i}} \hat{\Pi}_i = (1-\theta)\alpha[v_i - Q(\hat{b}_{1i})]F^{n-1}[b_1^{-1}(b_{1i})] - \theta\alpha[Q(\hat{b}_{1i}) - L] + (1-\theta)(1-\alpha)(v_i - \hat{b}_{1i})F^{n-1}[b_1^{-1}(b_{1i})] - \theta(1-\alpha)(\hat{b}_{1i} - L). \tag{7}$$

To determine a closed-form solution, we must impose more structure on the negotiation. The dealer and the customer bargainover money or, essentially, transferable utility. We assume that, if no trade occurs, the customer can get another dealer to match the BBO (best bid or offer) so he can trade at $\hat{b}_{1i}$. Thus, $\hat{b}_{1i}$ is the disagreement outcome for the customer.

The dealer must account for the fact that he could negotiate with an informed customer. To negotiate with an uninformed customer, the dealer must have the highest $Q(\cdot)$. Thus, the dealer's expected value for a negotiated trade (and thus his highest willingness to pay) is a weighted average of v_i and L:

$$\frac{v_i(1-\theta)F^{n-1}[b_1^{-1}(b_{1i})] + \theta L}{(1-\theta)F^{n-1}[b_1^{-1}(b_{1i})] + \theta} \equiv V(v_i, \hat{b}_{1i}). \tag{8}$$

Therefore, if the dealer does not trade, he gets the same utility he would achieve from a trade at $V(v_i, \hat{b}_{1i})$. This defines his maximum willingness to pay.

We assume that the customer's beliefs about the dealer's willingness to pay defines the upper bound on the negotiation. This is the assumption that the dealer is signaling his interest in trade through the aggressiveness of his quote. Given $\hat{b}_{1i}$ customers believe that the dealer is willing to pay $V[\hat{b}_1^{-1}(\hat{b}_{1i}), (\hat{b}_{1i})]$. In equilibrium, this is the dealer's true willingness to pay, but we assume that dealers believe that, out of equilibrium, negotiations still depend on the customer's beliefs.[22] Therefore, any negotiation results in the dealer paying $\hat{b}_{1i}$ plus a fraction of $V[\hat{b}_1^{-1}(\hat{b}_{1i}),\ \hat{b}_{1i}] - \hat{b}_{1i}$.[23]

22. This simplification eliminates the possibility that alternate off-equilibrium-path beliefs about the negotiation alter the equilibrium quotes. However, the normative conclusions about bargaining vs. information-based improvements still hold under many other reasonable off-equilibrium-path beliefs.

23. Imposing the restriction that $\hat{b}_{1i}$ plus a fraction of $V[\hat{b}_1^{-1}(\hat{b}_{1i}), \hat{b}_{1i}] - \hat{b}_{1i}$ is less than $V(v, \hat{b}_{1i})$ is logical, but this restriction never binds and does not affect the equilibrium.

For simplicity, we assume a reduced form for the negotiations. Negotiation results in the customer gaining a fraction β of the difference between $V[\hat{b}_1^{-1}(\hat{b}_{1i}), \hat{b}_{1i}]$ and $\hat{b}_{1i}$.[24] Thus,

$$Q(\hat{b}_{1i}) = \hat{b}_{1i} + \beta\{V[\hat{b}_1^{-1}(\hat{b}_{1i}), \hat{b}_{1i}] - \hat{b}_{1i}\}. \tag{9}$$

This assumption covers a broad class of negotiation models and allows a closed-form solution while providing excellent intuition about how customers with market power affect prices. In equilibrium, for example, this functional form encompasses the Nash (1950) bargaining solution. The Nash bargaining solution in this context provides each bargainer with a disagreement utility and then the bargainers split the remaining utility equally. The quote $Q(\cdot)$could also represent some form of repeated game such as a Rubienstein alternating-offers model.[25] In either case, β, represents the relative bargaining power of the customers and dealers. If dealers are more patient, more skillful, have lower negotiating costs or lower search costs, and so forth, then β is smaller and vice versa for customers.

The dealer's problem can now be rewritten as

$$\begin{aligned}\max_{\hat{b}_{1i}} \hat{\Pi}_i = {} & \alpha\big\{(1-\theta)vF^{n-1}][\hat{b}_1^{-1}(\hat{b}_{1i})] + \theta L - (1-\beta)\hat{b}_1 F^{n-1}[\hat{b}_1^{-1}(\hat{b}_{1i})](1-\theta)\big\} \\ & - \alpha\big((1-\beta)\hat{b}_1\theta + \beta\big\{(1-\theta)\hat{b}_1^{-1}(\hat{b}_{1i})F^{n-1}[\hat{b}_1^{-1}(\hat{b}_{1i})] + \theta L\big\}\big) \\ & + (1-\alpha)\big\{(1-\theta)\big(v - \hat{b}_1\big)F^{n-1}[\hat{b}_1^{-1}(\hat{b}_{1i})] - \theta\hat{b}_1 + \theta L\big\}. \end{aligned} \tag{10}$$

Results 2. If condition (A22) holds, then the unique symmetric equilibrium bid is

$$\begin{aligned}\hat{b}_1(v) = {} & \left[v - \int_{\underline{v}}^{v} \frac{F^{n-1}(x)}{F^{n-1}(v)(1-\alpha\beta)}dx\right] \\ & \times \frac{(1-\theta)F^{n-1}(v)}{(1-\theta)F^{n-1}(v)+\theta} + \frac{\theta L}{(1-\theta)F^{n-1}(v)+\theta}, \end{aligned} \tag{11}$$

24. Engelbrecht-Wiggins and Kahn (1991), RothKopf, Tiesberg, and Kahn (1991), and Waehrer (1999) all show that, if the information revealed by the bids in an auction is used against the bidder, then a separating equilibrium in the auction may not exist. Waehrer (1999) seems most applicable, because he follows an auction by an alternating-offers bargaining model and shows that this eliminates separability. In each of these papers, the incentive compatibility is violated because both the information is sure to be used and too much is taken from the bidder. In Katzman and Rhodes-Kropf (2001) information is revealed by the bids and thus bidders reduce their bids. However, if the impact of the information revelation on the bidder is small, then a separating equilibrium is shown to exist. Therefore, in the model chosen here, the bargaining power of the customer must be low relative to the dealer or very few customers must be able to negotiate. The formal condition for this restriction is shown in eq. (A22).

25. In a working paper by Dan Bernhardt, Hughson, and Werner (2002), the dealer weighs the value of maximizing his one-time revenue from the customer by not offering a price improvement against the future value of trades from offering present and future price improvements. In equilibrium, the dealer offers just enough price improvement to keep the trader from switching dealers. In the equilibrium of a dynamic model, β would be the amount that just keeps the customer from switching dealers.

with equilibrium price improvement equal to $\beta\{V[v, \hat{b}_1(v)] - \hat{b}_1(v)\} =$

$$Q(\hat{b}_1(v)) - \hat{b}_1(v) = \left[\frac{\int_{\underline{v}}^{v} F^{n-1}(x)dx(1-\theta)}{(1-\alpha\beta)((1-\theta)F^{n-1}(v)+\theta}\right]\beta \geq 0, \quad (12)$$

and the price improved bid is equal to $\hat{b}_1(v) + \beta\{V[v, \hat{b}_1(v)] - \hat{b}_1(v)\} =$

$$Q[\hat{b}_1(v)] = \frac{v(1-\theta)F^{n-1}(v)+\theta L}{(1-\theta)F^{n-1}(v)+\theta} - \frac{(1-\beta)\int_{\underline{v}}^{v} F^{n-1}(x)dx(1-\theta)}{(1-\alpha\beta)[(1-\theta)F^{n-1}(v)+\theta]}. \quad (13)$$

Proof. See the appendix.

This bid can be interpreted similarly to result 1, as a weighted average dealer's optimal bid to the uninformed customers and the optimal bid to the informed customers. However, the dealer must also account for improvements when setting his bid. If no customer were informed and no customer could negotiate, then the optimal bid would be the standard first-price auction bid, $v - \int_{\underline{v}}^{v}[F^{n-1}(x)/F^{n-1}(v)]dx$. If the customer is uninformed and dealer wins the auction, $(1-\theta)F^{n-1}(v)$, then the dealer pays the customer's bid; and if the customer can negotiate, then the dealer pays the customer's bid plus the improvement. If these were the only two possibilities, then the optimal bid would be the standard bid reduced by a term $(1-\alpha\beta)$ relating to the probability of negotiating, α, and the negotiation power of the customer, β. However, the customer may be informed. In this case the dealer should bid L.

The equilibrium ask, $\hat{a}_1(v)$, can be found and interpreted similarly. Thus, a dealer's quoted spread is $\hat{a}_1(v) - \hat{b}_1(v)$.

There are two key differences between the information-based model of price improvement and the model based on customers with market power. The central difference, of course, is the information level of the customers who receive improvements. Customers who receive price improvements in the information model have less information than the average customer who accepts the quoted price. However, in the market-power model, customers who negotiate are as informed on average as the customers who accept the quoted price.

This fundamental difference leads to the other key distinction in the models: in the information model, the improved price does not depend on the quoted price, whereas in the market-power model we assume that the quoted price influences the negotiations. In the information model, a customer cannot negotiate a better price simply because he is less informed; he receives a better price only because the dealers compete for his uninformed trade. Therefore, the dealer's quoted price does not influence the improved price. In contrast, a customer with market power negotiates directly with dealers. Since these customers are as informed as the average market participant, competition plays the same role that it did in the original posting of quotes. Therefore, competition does not improve prices. However, negotiating power allows the customer to make a counteroffer and negotiate an improvement in spite of the lack of competition. Since the initial quote is the basis for negotiations, the initial quote influences the improved price.

Thus, in the information model, the price to the uninformed customer is higher because they are more valuable and there is competition among dealers, but this price does not depend on the quoted price. In the market-power model, the negotiated price depends on the original quote and the market power of the negotiator.

This completes the basic outline of both models of price improvement. The next section considers those aspects of price improvement that are fundamental and do not depend on how the improvements are determined.

III. Effects of Price Improvements

The two distinct notions of price improvements have different effects on the trading participants. However, before examining their differences, it is important to establish those aspects of price improvement that are universal. This provides a framework to focus attention on the relevant differences between the two rationales for price improvements.

In this section, the first theorem demonstrates the basic interaction between information, the ability to negotiate, and the quoted prices in both models. Then, the second theorem examines how the dealers' expected revenue is affected by the price improvements.

Results 1 and 2 developed the dealers' quotes and the quoted spread. The prices most often examined are the best bid and ask, which are the same prices quoted in the newspaper. The best bid or ask is the expected highest of n bids or the expected lowest of n asks. It is easy to see that the expected highest bid (the first-order statistic) when improvements are based on information is

$$\overline{b_1(v)} = n \int_{\underline{v}}^{\bar{v}} \frac{\left[v - \int_{\underline{v}}^{v} \frac{F^{n-1}(x)}{F^{n-1}(v)} dx\right](1-\alpha)(1-\theta)F^{n-1}(v) + \theta L}{(1-\alpha)(1-\theta)F^{n-1}(v) + \theta} \times F'(v)F^{n-1}(v)dv, \tag{14}$$

and the expected lowest ask, $a_1(v)$, is similar. The expected highest bid when improvements are based on market power is

$$\overline{\hat{b}_1(v)} = n \int_{\underline{v}}^{\bar{v}} \left\{ \frac{\left[v(1-\alpha\beta) - \int_{\underline{v}}^{v} \frac{F^{n-1}(x)}{F^{n-1}(v)} dx\right](1-\theta)F^{n-1}(v) + \theta(1-\alpha\beta)L}{(1-\alpha\beta)[(1-\theta)F^{n-1}(v) + \theta]} \right\} \times F'(v)F^{n-1}(v)dv, \tag{15}$$

and the expected lowest ask, $\overline{\hat{a}_1(v)}$, is similar. The inside market spread is the distance between the best ask and the best bid or, $a_1(v) - b_1(v)$ and $\overline{\hat{a}_1(v)} - \overline{\hat{b}_1(v)}$.

Theorem 1. As the percentage of customers who can negotiate, α, their ability to negotiate, β, or the percentage of informed customers, θ, increase, then the quoted spread of the individual dealers and the inside market spread also increase.

Proof. See the appendix.

The increased spread is the intuitive result of price improvements. If a dealer knows that a customer may ask for a price improvement, then the dealer has an incentive to bid lower. The more likely it is that dealers are asked to improve the price, the greater the dealer's incentive to lower their bid. If the customers' ability to negotiate is greater, then the dealer expects to pay negotiators more, thus dealers widen their quotes to reduce the effect of the greater negotiating ability. If instead, the dealer has the ability to determine the information content of some customers' trades, then as more customers are able to show they are uninformed, the pool of customers who cannot (or choose not) to reveal their information is increasingly dominated by informed customers. Thus, the value of trading with these customers decreases and the bid decreases in response.

It is interesting to note that increased quoted and inside market spreads require no agreements among colluding dealers or between dealers and customers. Instead, it is the natural consequence of the dealers' ability to determine the customers' information or the customers' ability to negotiate. This is consistent with the historical reality that dealers posted narrower quotes on the Electronic Communication Networks (ECNs), where no customers can negotiate, than on the NASDAQ market.[26]

As expected, the quoted spread and the inside market spread are both increased by information asymmetries and the possibility of price improvements. But how are dealers' expected profits affected by these market attributes?

Theorem 2. With either form of price improvement, expected dealer profits are unaffected by the percentage of customers who can negotiate, α, or their ability to negotiate, β. Furthermore, the dealer's expected profit under either form of price improvement is the same.

Proof. See the appendix.

This surprising result shows that the dealer's expected profit is the same with either type of price improvement. Furthermore, neither the probability that the customer can negotiate nor the ability of the customer to bargain affects the dealers' profits. Although the preciseness of

26. ECNs, such as Instinet, are accessible only to institutional firms. A recent SEC amendment to the "Quote Rule," rule 11Ac1-1, requires a dealer who enters a proprietary order into an ECN that is priced better than its published quote on NASDAQ to display that order's price in his quote.

this result is likely to deteriorate with the addition of more complex frictions, the intuition still holds. The first-order impact of price improvements is that they do not affect the dealers.[27]

Theorem 2 is a consequence of the celebrated revenue equivalence theorem of Harris and Raviv (1981), Myerson (1981), and Riley and Samuelson (1981). The intuition is that an auction is essentially Bertrand competition in expectation. Each bidder bids the highest amount he expects the bidder just below him to be able to pay, given he is the highest bidder. Thus, as negotiating customers take more or less or as the number of customers who can negotiate changes, the dealers adjust their bid to pay, in expectation, the true value of the bidder below them, which is unchanging. Intuition might suggest that, when the dealers are forced to give up some surplus by customers with market power, their profits should go down. Or, when the dealer's are able to discriminate based on the customer's information, their profits should go up. However, this is not the case. Dealers post prices that are weighted averages of the proper bids to each type of customer. Thus, as the relevant variables change, competitive pressures change the weights and bids to keep the expected profits the same.[28]

The understanding provided by theorem 2 points to what does matter to the dealer: adverse selection.

Corollary 1. Dealers' expected profit decreases as the percentage of informed customers, θ, increases.

Proof. The derivative with respect to θ of the expected dealer profits (eqq. [A30] and [A33]) are less than zero.

In this model or market making, the dealers make positive profits because the competition is less than perfect. However, the dealers cannot make a profit on an informed customer. Therefore, as the percentage of informed customers goes up the dealer's profits go to zero.

In the seminal models of quote formation, dealers' except zero profits; therefore, the level of adverse selection does not affect the dealers' profit. However, it seems more reasonable that a dealership market is characterized by less than perfect competition for each trade (although fixed costs may still result in zero overall profits). If dealers make positive profits, intuition suggests that their profits are altered by the level and type of price improvements given by the dealers. However, theorem 2 and corollary 1 together demonstrate that only the level of adverse selection affects the dealers' profits.

27. Empirically, the profits of dealers are difficult to measure because the actual dealer inventories and costs are unknown. Instead, researchers examine the effective spread, since this distance is the average dealer revenue. The effective spread is also unchanged by an increase in the percentage of customers who can negotiate or their ability to negotiate.

28. This is similar to the weighted average cost of capital in the no-tax, no-friction case. Since no money leaves the system, the dealer simply rearranges the quotes and receives the same amount in total.

Combining the results so far tells us the circumstances under which price improvement affect the dealers. The dealers are affected if any aspect of price improvements reduce the size of the available profits, such as taxes, deadweight costs, or the quantity traded. For example, if improving prices requires costly effort, then we expect the dealers' profits to decrease if the probability of price improvement increases. I suspect that these types of effects are small, but this is fundamentally an empirical question.

It is interesting to note the results so far provide a possible understanding of why Petersen and Fialkowski (1994) find that the simple correlation between the inside and the effective market spread for data from the Boston, New York, Midwest, and Pacific stock exchanges is less than 0.1 (although these are not dealer markets, they feature price improvements). Numerous things may adjust quoted spreads, such as the probability of negotiation or the ability of the negotiating customer. However, the only variables that influence the effective spread are those that change the size of the pie available to market makers. Therefore, changes in the quoted spread are far more common than changes in the effective spread.

This section provided the basic intuition about how the dealer's quotes and profits change and how the inside market spread changes with the relevant variables. However, so far, all the effects have been the same for each type of price improvement. The next section considers the differences and how an empiricist might determine the correct theory from the data.

IV. How Can We Tell the Difference between the Two Models?

The initial results provide the broad intuition that the type or existence of price improvement does not matter. More specific, it does not matter to the dealer. However, this section demonstrates that the type of price improvement affects the price formation process and, therefore, the customers. Understanding how the prices are affected demonstrates how we may be able to determine which form of price improvement is more dominant in a particular market and whether it is detrimental to market quality.

To determine the differences between the models, we must examine the negotiated spread. The negotiated spread is the distance between the price-improved bid and the price-improved ask. Sections II.A and II.B demonstrate that the dealer with the highest bid also gives the best price-improved bid. Therefore, the expected highest price-improved bid when improvements are based on information is

$$\overline{b_2(v)} = n\int_{\underline{v}}^{\overline{v}} \left[v - \int_{\underline{v}}^{v} \frac{F^{n-1}(x)dx}{F^{n-1}(v)} \right] F'(v)F^{n-1}(v)dv, \qquad (16)$$

and the expected lowest ask, $\overline{a_2(v)}$, can be found similarly. The expected highest price-improved bid when improvements are based on market power is

$$\overline{Q[\hat{b}_1(v)]} = n\int_{\underline{v}}^{\overline{v}}\left\{\frac{v(1-\theta)F^{n-1}(v)+\theta L}{(1-\theta)F^{n-1}(v)+\theta} - \frac{(1-\beta)\int_{\underline{v}}^{v}F^{n-1}(x)dx(1-\theta)}{(1-\alpha\beta)[(1-\theta)F^{n-1}(v)+\theta]}\right\} \times F'(v)F^{n-1}(v)dv, \qquad (17)$$

and the expected lowest price-improved ask, $\overline{Q[\hat{a}_1(v)]}$, can be found similarly. The negotiated spread is the distance between the expected highest price-improved quotes, or $\overline{a_2(v)} - \overline{b_2(v)}$ and $\overline{Q[\hat{a}_1(v)]} - \overline{Q[\hat{b}_1(v)]}$.

Examining the negotiated spread demonstrates a central difference between the two models of price improvement.

THEREOM 3. Neither the percentage of customers who can negotiate, α, nor the percentage of informed customers, θ, affects the negotiated spread if price improvements are based on customer information. However, if price improvements are negotiated by customers with market power, then the negotiated spread widens if the percentage of customers who can negotiate, α, or the percentage of informed customers, θ, increases.[29]

Proof. See the appendix.

Therefore, if price improvements are based on information, then neither the probability that a customer can negotiate nor even the fraction of informed customers affects the negotiated spread. However, if price improvements are negotiated by customers with market power, then the negotiated spread is a function of the adverse selection, the probability of negotiation, and the ability to negotiate.

A. Testing the Models

Combining the results of theorem 3 with theorems 1 and 2 and corollary 1 demonstrates how the models may be distinguished empirically. Although the effective spread and therefore the dealers profits are unaffected by the differences in these models, the quoted spread and the negotiated spread are different. Therefore, the different models imply different correlations between the effective, quoted. and negotiated spreads.

29. An obvious corollary is that, if the customer's ability to negotiate, β, increases, then the negotiated spread narrows. The derivative of the negotiated bid with respect to β is

$$\frac{\partial\overline{Q[\hat{b}_1(v)]}}{\delta\beta} = n\int_{\underline{v}}^{\overline{v}}\left\{\frac{(1-\alpha)\int_{\underline{v}}^{v}F^{n-1}(x)dx(1-\theta)}{(1-\alpha\beta)^2[(1-\theta)F^{n-1}(v)=\theta]}\right\}F'(v)F^{n-1}(v)dv > 0.$$

Q.E.D.

Test 1. If the percentage of customers who can negotiate, α, increases and price improvements are based on information, then the quoted and the inside spread increase and the negotiated spread does not change. However, if price improvements are given to customers with market power, then the quoted, the inside, and the negotiated spread all increase.

If a proxy could be found for the probability that a customer will negotiate, then this test could be used to determine the type of price improvements. However, finding a proxy may be difficult. It may be easier to find a proxy for the adverse selection in the market, θ, in which case, the following test could be used.

Test 2. If the percentage of informed customers, θ, increases and price improvements are based on information, then the quoted and the inside spreads increase and the negotiated spread does not change. However, if price improvements are given to customers with market power, then the quoted, the inside, and the negotiated spreads all increase.

The intuition is simply that, with information-based price improvements, the negotiated spread does not depend on either α or θ. This is because the price-improved bid is only for the uninformed. Therefore, given that a customer is uninformed and the dealer knows that the customer is uninformed, competition prevents that customer's price from depending on ex ante probabilities.

The distinctions between the two models should allow a clever empiricist the opportunity to determine which theory is dominant.

B. Why Is It Important to Know the Difference?

Theorem 2 shows that dealers are unaffected by the type or probability of price improvements, so on average, dealers pay and customers receive the same amount regardless of the type or probability of price improvement. However, some customers must accept the quoted bid and some receive an improved bid. Theorem 1 shows that the quoted spread is affected by the probability of price improvements. And theorem 3 shows that the negotiated spread is affected by the type of price improvement. Therefore, particular customers are affected by the type and level of price improvements, although the effect is simply a transfer.

Corollary 2. In either model, any benefit received by the customers who trade at improved quotes is extracted from those customers who must trade at the inside quotes.

Proof. See the appendix.

As the number of negotiators (or their negotiating power) increases, the negotiating group's total profit increases by exactly the amount that those who cannot negotiate lose. Therefore, in general, wide quotes are not necessarily indicative of excess dealer profits. Instead, wide quotes may indicate a wealth transfer between the heterogeneous groups of customers.

If the first-order effect of price improvements is simply a transfer between groups, then understanding why price improvements are given will shed light on whether it is a beneficial or costly feature of the market. If price improvements are given based on information, then improvements help some customers receive prices more commiserate with their level of information, although those uninformed customers who cannot negotiate receive worse prices. However, if improvements are banned, then informed customers will extract even more from the uninformed customers as a group. If improvements are negotiated by customers with market power, then the group that interacts repeatedly with the market has an advantage over the less-frequent traders, that is, institutional customers extract surplus from households. Since execution equality is a stated goal of the Securities and Exchange Commission (SEC), we must understand which form of price improvement is correct to understand the trade-offs involved in preventing or encouraging improvements. If improvements are based on information, then it would seem that they should be encouraged, since the informed will extract less from the uninformed on average. While, if improvements are based on market power, then it would seem that they should be discouraged.

V. Competition among Dealers

The level of competition plays an important role in the formation of both prices and price improvements. Section III noted that the dealer is affected only by changes that alter the expected available profit. With an increase in the number of dealers, the profit available for a particular dealer decreases. Therefore, the dealer and the quotes are affected by the level of competition.

The following theorem describes the relationship between the number of dealers making a market in a particular security and the expected price improvement in each model.

Theorem 4. As the number of dealers increases, the price improvements increase if improvements are based on information and decrease if improvements are due to customers' market power.

Proof. See the appendix.

In the information model, increasing competition among dealers has two effects. The first is that dealers increase their quotes as competition increases. However, because of adverse selection, a second effect counters the first effect. This occurs because every dealer is less likely to trade with an uninformed customer (i.e., win the auction) but equally likely to trade with an informed customer because the informed customers trade with every dealer.[30] The improved bid in the information model is not affected

30. The idea that the informed trader trades with every dealer overemphasize the magnitude of the adverse selection effect, but the logic and direction of the effect are correct. The implications of the adverse selection effect are examined more thoroughly in Dennert (1993).

by the adverse selection. Thus, the improved bid is certain to increase as the number of dealers increases.

In the market-power model, increasing competition affects the quote in the same two ways as just discussed. However, unlike information-based improvements, the improvements negotiated by customers with market power are affected by both the standard competition effect and the adverse selection effect.

Therefore, in the information model, the distance between the quote and the improved quote increases with the number of dealers, but that distance decreases with the number of dealers in the market-power model.

As a side note, in both models, as the number of dealers increases, the increased competition narrows the spread. A tighter spread with no change in α or β ensures that all customers are better off. Thus, increasing n transfers wealth from dealers to customers.

Theorem 4 provides another method to test the models.

Test 3. As the number of dealers increases, then, all else constant, if the price improvements increase, it is more likely that the improvements are based on information. However, if they decrease, then it is more likely that customers have market power.

This result provides an interesting explanation for a seeming anomaly. In a market with many dealers, the standard prediction is that the competition increases the price improvements. If price improvements are given because the minimum tick size holds quotes wider than the zero profit point, then more competition would cause dealers to give more of the surplus. Also, under this theory, price improvements would be less than one tick. However, Reiss and Werner (1996) find empirically that the price improvement on securities for which many dealers make a market is less than the price improvement on securities with few market makers and average improvements are greater than one tick. In the market-power-based model, when many dealers participate, every dealer bids closer to their value. Less surplus is available after the auction, so there is less for negotiators to acquire. Thus, the magnitude of price improvement is reduced. This provides empirical support for the theory that customer market power plays a significant role in price formation.

Furthermore, this argument is not predicated on the tick; therefore, improvements could be greater than one tick. This corresponds with numbers from NASDAQ, Huang and Stoll (1996), that the quoted half-spread averages 24.6 cents and the effective half spread averages 18.7 cents. So, the average price improvement is 6 cents. Since only 26.7% of trades occur inside the quotes, the average price improvement on those trades that receive an improvement is 0.06/0.267 = $0.22, greater than one tick.

If n is endogenous, then with a fixed cost to making a market, dealers enter the market until the next entrant expects to raise the bids to the point where he could not expect to cover his costs. Quotes at this point, however, may still be wide, since customers still negotiate price improvements. Wide quotes indicate dealers make more from some trades than others, but with endogenous entry, in expectation, dealers only cover costs and make zero profits, as is consistent with the empirical evidence of Hansch et al. (1999).

Finally, it is important to note that the dealer profit's in either model, equations (A30) and (A33), decrease when n increases. Thus, overall the level of competition affects the profits of the dealers, while price improvements are transfers from those customers who cannot negotiate to those who can.

VI. Conclusion

Although price improvement is a pervasive feature of many financial markets, especially dealer markets, it is not considered in the canonical models of market microstructure. This paper adds to the literature by exploring and contrasting the standard theory with a novel theory as to why dealers give price improvements. The standard model is an theory initially formalized by Seppi (1990) in the context of the NYSE, that improvements are given to customers with less information. The novel theory is that some customers have the market power to negotiate improvements on the dealers' posted quotes. The examination of both models is important, because it demonstrates the general effects of price improvement and those effects that depend on the specific reason improvements are given (adverse selection or market power). Furthermore, if equal execution is an important goal of a market, then understanding why execution may be unequal is critical.

Either theory affects the formation of the quoted and negotiated spreads but not the effective spread. Therefore, the dealers' expected revenue is unaffected by the type or the probability of price improvement. Although it seems counterintuitive at first, Section III shows that the size of the available profit and the level of competition determine the dealers' revenue, Therefore, only deadweight costs affect the dealers and prices can change in numerous ways without affecting the dealer. Thus, we should expect a low correlation between the quoted and negotiated spread. Interestingly, price improvement is a virtually unexplored component of the bid/ask spread.

Although the dealer is unaffected by price improvements, those customers that negotiate extract surplus from those customers who cannot. The important public policy issue is whether one group of traders should gain relative to another group of traders. Price improvement based on information may be deemed beneficial, since uninformed customers who negotiate and informed customers both receive prices commiserate with their information. Also, if improvements are banned, then informed customers extract even more from the uninformed customers. If improvements are due to customers with market power, then they allow institutional players to extract surplus from the customers who cannot negotiate improvements. Thus, this work demonstrates how the costs and benefits of price improvement depend on the type of price improvement.

To determine which type of price improvement is dominant, their differences must be highlighted. In both models, an increase in the level of adverse selection increases the quoted spreads, as in a standard model. Increasing the probability that a customer negotiates or the market power of a negotiator also increases the quoted spread in both models. However, the probability of negotiation and the level of adverse selection asymmetrically affect the negotiated spreads. The negotiated spread in the information model does not change with adverse selection or the probability of negotiation, because the improved bid is used only by the uninformed. Therefore, competition drives the bid to the same price regardless of the level of adverse selection or negotiation. However, the negotiated spread in the market-power-based model is a function of the quoted bid. Therefore, the dealer must set his quoted spread to account for the effect it will have on the negotiated spread. Thus, the negotiated spread in the market-power-based model increases with the level of adverse selection and the probability of negotiation. These differences should be empirically valuable.

Examining the effects of competition yields another test of the models and an explanation for a seeming anomaly. Previous work suggests that greater competition should force dealers to give larger price improvements. Reiss and Werner (1996), however, find the opposite result. In either model of price improvement, increased competition decreases the quoted spread, but with information-driven improvements, the negotiated spread decreases even more. Therefore, if competition increases, then only the market-power model predicts a decrease in price improvements.

The overall examination of price improvement helps elucidate the price formation process and the strategic behavior of dealers. Establishing the correct model of price improvement and its impact on the participants helps determine whether improvements are beneficial or detrimental to market quality and provides a better understanding of our markets. Furthermore, it is important to recognize that the market power of customers, as well as adverse selection, may play a significant role in price formation.

Appendix

This appendix first presents the general calculation of the dealer's bid or ask, then proves results 1 and 2, the theorems 1–4, and corollary 2. Any of the profit functions written in this paper are a function of the dealer's private value v, the dealer's bid b, and the beliefs of the customer about the dealer's value given the bid, $v(b)$. The dealer's problem is then to choose a bid to maximize their profit:

$$b(v) \in \arg \max_{b \in \{\Re^+\}} \Pi[b, v(b), v]. \tag{A1}$$

Myerson's (1981) revelation principle ensures the solution(s) to this maximization are subset of the solutions to the direct revelation problem:

$$v \in \arg \max_{x \in \{\underline{v}, \overline{v}\}} \Pi[b(x), x, v]. \tag{A2}$$

This yields the first-order condition

$$\Pi_1[b(v), v, v]b'(v) + \Pi_2[b(v), v, v] = 0 \tag{A3}$$

and the envelope condition

$$\frac{d\Pi[b(v), v, v]}{dv} = \Pi_3[b(v), v, v]. \tag{A4}$$

Lemma 1. If $v > x \Rightarrow \Pi_3[b(v), v, s] > \Pi_3[b(x), x, s]$, for all s, then the first-order conditions are sufficient to ensure global incentive compatibility.

Proof. For the first-order conditions to be sufficient, the inequality

$$\Pi[b(v), v, v]\Pi[b(x), x, v], \quad \text{for all } x \tag{A5}$$

must be satisfied for all v. We can use the envelope condition to rewrite this inequality as

$$v > x \Rightarrow \int_x^v \Pi_3[b(s), s, s]ds > \int_x^v \Pi_3[b(x), x, s]ds. \tag{A6}$$

If $x = v$, then this inequality is an equality. Therefore, a related necessary condition is

$$\Pi_{31}[b(v), v, v]b'(v) + \Pi_{32}[b(v), v, v] \geq 0. \tag{A7}$$

Therefore, a sufficient condition that implies equation (A5) is

$$v > x \Rightarrow \Pi_3[b(v), v, s] > \Pi_3[b(x), x, s], \quad \text{for all } s. \tag{A8}$$

Thus, as each bid function is calculated, equation (A8) can be checked to ensure global incentive compatibility.

Result 1. Note that b_{1i} is only in the second part of the maximization and that b_{2i} is only in the first part. Therefore, the dealer essentially faces two different maximizations. To set the quote in the market, the dealer chooses b_{1i} to solve

$$b_1(v) \in \arg \max_{b \in \{\Re^+\}} \{(1-\theta)(1-\alpha)(v - b_{1i})q_1 F^{n-1}[b_1^{-1}(b_{1i})] - \theta(b_{1i} - L)\}. \tag{A9}$$

The revelation principle and the assumption that we consider only invertible equilibrium (invertibility is verified in equilibrium) allows the dealer's problem to be rewritten as

$$v \in \arg\max_{x \in \{\underline{v}, \overline{v}\}} \{(1-\theta)(1-\alpha)[v - b_1(x)]F^{n-1}(x) - \theta[b_1(x) - L]\}. \qquad (A10)$$

The first-order condition is

$$(1-\theta)(1-\alpha)v\frac{dF^{n-1}(x)}{dx} = \frac{d}{dx}\{(1-\theta)(1-\alpha)b_1(x)F^{n-1}(x) - \theta[b_1(x) - L]\}, \qquad (A11)$$

where $F^{n-1}(x)$ is an increasing function; therefore, lemma 1, equation (A8), is satisfied and global incentive compatibility is ensured. An equilibrium requires this first-order condition to be true when $x = v$, for all v. Therefore, the equality still holds if both sides of the equation are integrated from v to v:

$$(1-\theta)(1-\alpha)\left[vF^{n-1}(v) - \int_{\underline{v}}^{v} F^{n-1}(x)dx\right] = (1-\theta)(1-\alpha)b_1(v)F^{n-1}(v) + \theta[b_1(v) - b_1(\underline{v})]. \qquad (A12)$$

The lowest bidder stands no chance of winning and, therefore, faces only the informed customer. Thus, the lowest bidder cannot bid above L, as this guarantees losses. If the lowest bid is below L, then the lowest bidder, who stands to make no money with his current bid, has a chance to make money with a slightly higher bid. Therefore, he raises his bid. Thus, $b_1(v) = L$. Hence,

$$b_1(v) = \left[v - \int_{\underline{v}}^{v_i} \frac{F^{n-1}(x)}{F^{n-1}(v)}dx\right]\frac{(1-\theta)(1-\alpha)F^{n-1}(v)}{(1-\theta)(1-\alpha)F^{n-1}(v) + \theta} + \frac{\theta L}{(1-\theta)(1-\alpha)F^{n-1}(v) + \theta}. \qquad (A13)$$

Rearranging reveals that the optimal quote is equation (3).

To set the price improved quote the dealer chooses b_{2i} to maximize

$$(1-\theta)\alpha(v - b_{2i})F^{n-1}[b_2^{-1}(b_{2i})]. \qquad (A14)$$

Identical steps to those just completed for b_1 demonstrate that the optimal price improvement is equation (5).

The price improvement is obviously positive, since the quoted bid is just a weighted average of the price improved bid and L, and the quoted bid is always greater than or equal to L.

Result 2. With improvements negotiated by the customers, the dealer has to solve only his quoted bid because the assumed bargaining process is such that the improved bid is a deterministic function of the quoted bid and not a choice of the dealer. However, the dealers must take this into account in choosing their quote. The revelation principle and the assumption that we consider only invertible equilibrium (invertibility is verified in equilibrium) allows the dealer's problem, equation (10), to be rewritten as

$$v \in \arg\max_{x \in \{\underline{v},\overline{v}\}} \Big(\alpha\big[(1-\theta)vF^{n-1}(x) + \theta L - (1-\beta)\hat{b}_1(x)F^{n-1}(x)(1-\theta)\big] - \alpha\big\{(1-\beta)\hat{b}_1(x)\theta + \beta[(1-\theta)xF^{n-1}(x) + \theta L]\big\} + (1-\alpha)\big\{(1-\theta)[v - \hat{b}_1(x)]F^{n-1}(x) - \theta\hat{b}_1(x) + \theta L\big\}\Big). \quad \text{(A15)}$$

This can be reduced to

$$v \in \arg\max_{x \in \{\underline{v},\overline{v}\}} \big\{\big[v - \hat{b}_1(x) + \alpha\beta\hat{b}_1(x) - \alpha\beta x\big](1-\theta)F^{n-1}(x) + \theta(1-\alpha\beta)\big[L - \hat{b}_1(x)\big]\big\}. \quad \text{(A16)}$$

The first-order condition is

$$(1-\theta)v\frac{dF^{n-1}(x)}{dx} = \frac{d}{dx}\big\{\big[\hat{b}_1(x) - \alpha\beta\hat{b}_1(x) + \alpha\beta x\big](1-\theta)F^{n-1}(x) + \theta(1-\alpha\beta)\big[\hat{b}_1(x)\big] - L\big\}, \quad \text{(A17)}$$

where $F^{n-1}(x)$ is an increasing function; therefore, lemma 1, equation (A8), is satisfied and global incentive compatibility is ensured. An equilibrium requires this first-order condition to be true when $x = v$ for all v. Therefore, the equality still holds if both sides of the equation are integrated from v to v. Under

$$(1-\theta)\left[vF^{n-1}(v) - \int^{v} F^{n-1}(x)dx\right] = \big[\hat{b}_1(v) - \alpha\beta\hat{b}_1(v) + \alpha\beta v\big](1-E)F^{n-1}(v) + \theta(1-\alpha\beta)\big[\hat{b}_1(v) - \hat{b}_1(\underline{v})\big], \quad \text{(A18)}$$

the dealer will face only the informed customer. Thus, the lowest bidder cannot bid above L, as this will guarantee losses. If the lowest bid is below L, then the lowest bidder, who stands to make no money with his current bid, has a chance to make money with a slightly higher bid, so he raises his bid. Thus, $b_1(v) = L$. Therefore, Q4

$$\hat{b}_1 = \left[v - \int_{\underline{v}}^{v} \frac{F^{n-1}(x)}{F^{n-1}(v)}dx\right] \frac{(1-\theta)F^{n-1}(v)}{(1-\alpha\beta)[(1-\theta)F^{n-1}(v) + E]} + \frac{\theta(1-\alpha\beta)L - \alpha\beta v(1-\theta)F^{n-1}(v)}{(1-\alpha\beta)[(1-\theta)F^{n-1}(v) + \theta]}. \quad \text{(A19)}$$

In this case, the price improvement is positive as long as

$$\beta\frac{(1-\theta)vF^{n-1}(v) + \theta L}{(1-\theta)F^{n-1}(v) + \theta} > \beta\hat{b}_1(x). \quad \text{(A20)}$$

where $[(1-\theta)vF^{n-1}(v)+\theta L]/[(1-\theta)F^{n-1}(v)+\theta]$ is the dealer's expected value given that he is negotiating with the customer.[31]

This equilibrium is predicated on the assumption that the bid is invertible or increasing in v and positive. As Waehrer (1999) points out, negotiation following an auction may eliminate an invertible equilibrium. The problem arises if, in expectation, the negotiation is able to extract too large a portion of the dealer's surplus. If the extraction is too large, then the dealers are better off pooling and hiding their value. This alters the customer's ability to extract the dealer's surplus. To consider equilibria that are invertible, we must assume that expected extraction is not too large. Specifically, $(\partial/\partial v)\hat{b}_1(v) > 0$ requires

$$0 < \frac{\partial}{\partial v}\frac{\left[v - \int_{\underline{v}}^{v}\frac{F^{n-1}(x)}{F^{n-1}(v)}dx\right](1-\theta)F^{n-1}(v) + \theta(1-\alpha\beta)L - \alpha\beta v(1-\theta)F^{n-1}(v)}{(1-\alpha\beta)[(1-E)F^{n-1}(v)+\theta]}, \tag{A21}$$

$$0 < (v-L)(1-\alpha\beta)\theta - \alpha\beta\frac{F^{n-1}(v)}{\frac{\partial}{\partial v}F^{n-1}(v)}(1-\theta)[F^{n-1}(v)+\theta] + \int_{\underline{v}}^{v}F^{n-1}(x)dx(1-\theta). \tag{A22}$$

While there is no simple interpretation of this constraint, it essentially says that the parameters relating to extraction by the negotiator or the informed customer must be small enough to ensure participation. For example, if α or β is smaller, then the constraint is more likely to hold. That is, the fraction of customers who can negotiate, α, and their ability to negotiate, β, both reduce the benefit from participating in the auction. Therefore, the ability to extract surplus because the customer can negotiate must be low or the lowest customers are unwilling to bid. Also, $[F^{n-1}(v)]/[(\partial/\partial v)F^{n-1}(v)]$ must never become too large. That is, each bidder must believe that there is a reasonable probability that there is another bidder with a value just above him who he might beat if he raise his bid. Condition (A22), itself simple, formalizes how these parameters can be traded off and still ensure an equilibrium.

31. Therefore, price improvement is positive as long as

$$[(1-\theta)vF^{n-1}(v)+\theta L] > \frac{\left[v - \int_{\underline{v}}^{v}\frac{F^{n-1}(x)}{F^{n-1}(v)}dx\right](1-E)F^{n-1}(v) + E(1-\alpha\beta)L - \alpha\beta v(1-E)F^{n-1}(v)}{(1-\alpha\beta)}$$

A sufficient condition is that

$$[(1-\theta)vF^{n-1}(v)+\theta L] \geq \frac{v(1-\theta)F^{n-1}(v)+\theta(1-\alpha\beta)L - \alpha\beta v(1-\theta)F^{n-1}(v)}{(1-\alpha\beta)}$$

This reduces to

$$(1-\theta)vF^{n-1}(v) + EL \geq v(1-E)F^{n-1}(v) + \theta L.$$

Therefore, the price improvement is positive.

If condition (A22) holds, then the bids are invertible and the dealer's true value can be determined by the negotiating customer. Furthermore, it is straightforward to show that the negotiated outcome $Q(\cdot)$ is an increasing function of the bid and, therefore, also an increasing function of v. Throughout the paper, we assume this condition holds.

Proof of Theorem 1

The derivatives of the bid and the expected best bid with respect to the relevant variable are

$$\frac{\partial \overline{b_1(v)}}{\partial \alpha} = n \int_{\underline{v}}^{\bar{v}} \frac{\left\{L - \left[v - \int_{\underline{v}}^{v} \frac{F^{n-1}(x)}{F^{n-1}(v)} dx\right]\right\}(1-\theta)F^{n-1}(v)\theta}{[(1-E)(1-\theta)F^{n-1}(v)+\theta]^2} F'(v)F^{n-1}(v)dv, \tag{A23}$$

$$\frac{\partial \overline{b_1(v)}}{\partial \theta} = n \int_{\underline{v}}^{\bar{v}} \frac{\left\{L - \left[v - \int_{\underline{v}}^{v} \frac{F^{n-1}(x)}{F^{n-1}(v)} dx\right]\right\}(1-\alpha)F^{n-1}(v)}{[(1-\alpha)(1-\theta)F^{n-1}(v)+\theta]^2} F'(v)F^{n-1}(v)dv. \tag{A24}$$

Both derivatives are less than zero because $v - \int_{\underline{v}}^{v} [F^{n-1}(x)]/[F^{n-1}(v)]dx > L$. Then,

$$\frac{\partial \overline{\hat{b}_1(v)}}{\partial \alpha} = n \int_{\underline{v}}^{\bar{v}} \left(- \frac{\beta \int_{\underline{v}}^{v} F^{n-1}(x)dx(1-\theta)}{\{(1-\alpha\beta)^2[(1-\theta)F^{n-1}(v)+\theta]\}} \right) F'(v)F^{n-1}(v)dv, \tag{A25}$$

which is less than zero. The derivative with respect to β, of course, also is less than zero. Finally,

$$\frac{\partial \overline{\hat{b}_1(v)}}{\partial \theta} = n \int_{\underline{v}}^{\bar{v}} \left\{ - \frac{v(1-\alpha\beta)F^{n-1}(v) - \int_{\underline{v}}^{v} F^{n-1}(x)dx}{[(1-\theta)F^{n-1}(v)+\theta]^2(1-\alpha\beta)} \right\} F'(v)F^{n-1}(v)dv, \tag{A26}$$

which is negative, since condition (A22) holds, the bid is positive when $\theta = 0$, therefore, $v(1-\alpha\beta)F^{n-1}(v) - \int_{\underline{v}}^{v} F^{n-1}(x)dx > 0$. The derivatives of the asks are not shown but are similar. Q.E.D.

Proof of Theorem 2

In equilibrium, a dealer's expected profit, when improvements are based on information, is

$$\Pi(v) = (1-\theta)\alpha[v - b_2(v)]F^{n-1}(v) + (1-\theta)(1-\alpha)[v - b_1(v)]F^{n-1}(v) - \theta[b_1(v) - L]. \tag{A27}$$

Rearranging yields

$$\Pi(v) = (1-\theta)vF^{n-1}(v) - b_2(v)(1-\theta)\alpha F^{n-1}(v) - b_1(v)[(1-\theta)(1-\alpha)F^{n-1}(v)+\theta] + \theta L. \tag{A28}$$

Substituting in the equilibrium bid function for $b_1(v)$, equation (3), yields

$$\Pi(v) = (1-\theta)vF^{n-1}(v) - b_2(v)(1-\theta)\alpha F^{n-1}(v) - \left\{\left[v - \int_{\underline{v}}^{v}\frac{F^{n-1}(x)}{F^{n-1}(v)}dx\right](1-\alpha)(1-\theta)F^{n-1}(v) + \theta L\right\} + \theta L. \quad \text{(A29)}$$

Substituting in the equilibrium bid function for $b_2(v)$, equation (5), yields

$$\Pi(v) = \int_{\underline{v}}^{v} F^{n-1}(x)dx(1-\theta). \quad \text{(A30)}$$

In equilibrium, a dealer's expected profit when improvements are given because customers have market power is

$$\hat{\Pi}(v) = [v - \hat{b}_1(v) + \alpha\beta\hat{b}_1(v) - \alpha\beta v](1-\theta)F^{n-1}(v) + \theta(1-\alpha\beta)[L - \hat{b}_1(v)]. \quad \text{(A31)}$$

Rearranging yields

$$\hat{\Pi}(v) = (1-\theta)(1-\alpha\beta)vF^{n-1}(v) - \hat{b}_1(v)(1-\alpha\beta)[(1-\theta)F^{n-1}(v) - \theta] + \theta(1-\alpha\beta)L. \quad \text{(A32)}$$

Substituting in the equilibrium bid function for $\hat{b}_1(v)$, equation (11), yields

$$\hat{\Pi}(v) = \int_{\underline{v}}^{v} F^{n-1}(x)dx(1-\theta). \quad \text{(A33)}$$

Note that equation (A30) equals equation (A33) and neither equation depends on α or β. Q.E.D.

Proof of Theorem 3

Examining the expected highest price-improved bid with information based improvements, $b_2(v)$, equation (16), it is immediately obvious that this price is not affected by either α or θ. However, the derivative of the expected highest price-improved bid when customers have market power, $\overline{Q[\hat{b}_1(v)]}$, equation (17), with respect to α is

$$\frac{\partial\overline{Q[\hat{b}_1(v)]}}{\partial\alpha} = n\int_{\underline{v}}^{\overline{v}} -\frac{(1-\beta)\beta\int_{\underline{v}}^{v} F^{n-1}(x)dx(1-\theta)}{\{(1-\alpha\beta)^2[(1-\theta)F^{n-1}(v)+\theta]\}}F'(v)F^{n-1}(v)dv, \quad \text{(A34)}$$

which is less than zero. The derivative of $\overline{Q[\hat{b}_1(v)]}$ with respect to θ is

$$\frac{\partial\overline{Q[\hat{b}_1(v)]}}{\partial\theta} = n\int_{\underline{v}}^{\overline{v}} -\frac{v(1-\alpha\beta)F^{n-1}(v) - (1-E)\int_{\underline{v}}^{v} F^{n-1}(x)dx}{[(1-\theta)F^{n-1}(v)+\theta](1-\alpha\beta)[(1-\theta)F^{n-1}(v)+\theta]} \times F'(v)F^{n-1}(v)dv, \quad \text{(A35)}$$

which is negative, since condition (A22) holds. The bid is positive when $\theta = 0$; therefore, $v(1-\alpha\beta)F^{n-1}(v) - \int_{\underline{v}}^{v} F^{n-1}(x)dx > 0$, hence, $v(1-\alpha\beta)F^{n-1}(v) - (1-\beta)\int_{\underline{v}}^{v} F^{n-1}(x)dx > 0$. Q.E.D

Proof of Corollary 2

To see the transfer from the nonnegotiators to the negotiators, we need to look at the expected payments to the groups. A customer who can negotiate receives $b_2(v)$, from the dealer if the customer is uninformed and the price improvements are based on information, and $Q[\hat{b}_1(v)]$, if the improvements are due to market power. As a group, those customers who negotiate expect to receive from a dealer the improved bids times the probability that the customer negotiates times the probability that dealer wins (which is $\alpha(1-\theta)F^{n-1}(v)$ if improvements are information based, since the negotiator must be uninformed, and just $\alpha[(1-\theta)F^{n-1}(v)+\theta]$ if the improvements are due to market power). Therefore, the change in the negotiating group's profit with a change in α is

$$\frac{\partial}{\partial\alpha} nE[\alpha(1-\theta)F^{n-1}(v)b_2(v)] = n\int_{\underline{v}}^{\bar{v}} (1-\theta)F^{n-1}(v)\left[v - \int_{\underline{v}}^{v}\frac{F^{n-1}(x)dx}{F^{n-1}(v)}\right]F'(v)F^{n-1}(v)dv, \quad \text{(A36)}$$

if improvements are based on information. And the change in the negotiating group's profit with a change in α is

$$\frac{\partial}{\partial\alpha} nE\left\{\alpha Q\overline{[\hat{b}_1(v)]}[(1-\theta)F^{n-1}(v)+\theta]\right\} = n\int_{\underline{v}}^{\bar{v}}\left[v(1-\theta)F^{n-1}(v)+\theta L - \frac{(1-\beta)\int_{\underline{v}}^{v}F^{n-1}(x)dx(1-\theta)}{(1-\alpha\beta)^2}\right]F'(v)F^{n-1}(v)dv, \quad \text{(A37)}$$

pay $(1-\alpha)(1-\theta)F^{n-1}(v)b_1(v)+\theta b_1(v)$, given their v, to those customers who must accept the inside spread. Since there are n dealers, the total expected payment to the group that cannot negotiate is

$$nE[(1-\alpha)(1-\theta)F^{n-1}(v)b_1(v)+\theta b_1(v)] \text{ or } nE[(1-\alpha)\overline{\hat{b}_1(v)}[(1-\theta)F^{n-1}(v)+\theta]\}, \quad \text{(A38)}$$

depending on the type of improvements in the market. The derivatives of each group's profit with respect to α is

$$\frac{\partial}{\partial\alpha} nE\{[(1-\alpha)(1-\theta)F^{n-1}(v)+\theta]b_1(v)\} = n\int_{\underline{v}}^{\bar{v}} -(1-\theta)F^{n-1}(v)\left[v - \int_{\underline{v}}^{v}\frac{F^{n-1}(x)}{F^{n-1}(v)}dx\right]F'(v)F^{n-1}(v)dv, \quad \text{(A39)}$$

$$\frac{\partial}{\partial\alpha}\left\{(1-\alpha)\overline{\hat{b}_1(v)}[(1-\theta]F^{n-1}(v)+\theta\right\} = n\int_{\underline{v}}^{\bar{v}}\left[-v(1-\theta)F^{n-1}(v)-\theta L + \frac{(1-\beta)\int_{\underline{v}}^{v}F^{n-1}(x)dx(1-\theta)}{(1-\alpha\beta)^2}\right]F'(v)F^{n-1}(v)dv. \quad \text{(A40)}$$

Since

$$\frac{\partial}{\partial \alpha} nE[\alpha(1-\theta)F^{n-1}(v)b_2(v)] = -\frac{\partial}{\partial \alpha} nE\{[(1-\alpha)(1-\theta)F^{n-1}(v)+\theta]b_1(v)\} \tag{A41}$$

and

$$\frac{\partial}{\partial \alpha}\left\{\alpha\overline{Q[\hat{b}_1(v)]}[(1-\theta]F^{n-1}(v)+\theta\right\} = -\frac{\partial}{\partial \alpha}\left\{(1-\alpha)\overline{\hat{b}_1(v)}[(1-\theta)F^{n-1}(v)+\theta]\right\}, \tag{A42}$$

then as the probability of negotiation increases from zero, the benefit to the negotiating group is exactly offset by the reduction to the nonnegotiators.[32] Q.E.D.

Proof of Theorem 4

The price improvement with information-based improvements is

$$b_2(v) - b_1(v) = \frac{\left[v - \int_{\underline{v}}^{v} \frac{F^{n-1}(x)}{F^{n-1}(v)} dx\right]\theta - \theta L}{(1-\alpha)(1-\theta)F^{n-1}(v)+\theta}. \tag{A43}$$

This is clearly increasing, since the numerator is increasing and the denominator is decreasing with n.The price improvement with market-power-based improvements is

$$Q[\hat{b}_1(v)] - \hat{b}_1(v) = \left\{\frac{\int_{\underline{v}}^{v} F^{n-1}(x)dx(1-\theta)}{(1-\alpha\beta)[(1-\theta)F^{n-1}(v)+\theta]}\right\}\beta. \tag{A44}$$

The derivative of the numerator with respect to n is

$$\frac{\int_{\underline{v}}^{v} F^{n-1}(x)\ln F^{n-1}(x)dx(1-\theta)}{(1-\alpha\beta)[(1-\theta)F^{n-1}(v)+\theta]} - \frac{\int_{\underline{v}}^{v} F^{n-1}(x)dx(1-\theta)\ \ln F^{n-1}(v)}{\{(1-\alpha\beta)[(1-\theta)F^{n-1}(v)+\theta]\}^2}(1-\alpha\beta)(1-\theta)F^{n-1}(v). \tag{A45}$$

Further,

$$\int_{\underline{v}}^{v} F^{n-1}(x)\ln F^{n-1}(x)dx < \int_{\underline{v}}^{v} F^{n-1}(x)dx \ \ln F^{n-1}(v)\frac{(1-\theta)F^{n-1}(v)}{(1-\theta)F^{n-1}(v)+\theta} \tag{A46}$$

is true since $\ln F^{n-1}(v)$ is a negative number and

$$\int_{\underline{v}}^{v} F^{n-1}(x)\ln F^{n-1}(v)dx > \int_{\underline{v}}^{v} F^{n-1}(x)\ln F^{n-1}(x)dx. \tag{A47}$$

Therefore, the improvement decreases when n increases. Q.E.D.

32. The same is true for the market power of negotiators.

If β Depends on the Bid

The bid in the market-power game is a function of v and the parameters θ, α, β, L and $n : \hat{b}_1 = h(v, \theta, \alpha, \beta, L, n)$. It is easy to demonstrate that, if β is a function of the bid, then the bid can be defined by the implicit function $\hat{b}_1 = h[v, \theta, \alpha, \beta(\hat{b}_1), L, n]$, as long as condition (A22) with $\beta(\hat{b}_1)$is still assumed to hold. Using this implicit function and the preceding proofs, we can show that the theorems are unchanged, as long as $[\partial\beta(\hat{b}_1)]/\partial\hat{b}_1$ is not too small.

For example, the derivative of the bid function with respect to α is

$$\frac{\partial\hat{b}_1}{\partial\alpha} = h_3[v, \theta, \alpha, \beta(\hat{b}_1), L, n] + h_4[v, \theta, \alpha, \beta(\hat{b}_1), L, n]\frac{\partial\beta(\hat{b}_1)}{\partial\hat{b}_1}\frac{\partial(\hat{b}_1)}{\partial\alpha}, \qquad \text{(A48)}$$

where the subscript represents the derivative with respect to the nth argument:

$$\frac{\partial\hat{b}_1}{\partial\alpha} = \frac{h_3[v, \theta, \alpha, \beta(\hat{b}_1), L, n]}{1 - h_4[v, \theta, \alpha, \beta(\hat{b}_1), L, n]\frac{\partial\beta(\hat{b}_1)}{\partial(\hat{b}_1)}}. \qquad \text{(A49)}$$

Thus, the effect of a change in α has the same sign as for a fixed β as long as

$$1 - h_4[v, \theta, \alpha, \beta(\hat{b}_1), L, n]\frac{\partial\beta(\hat{b}_1)}{\partial\hat{b}_1} > 0. \qquad \text{(A50)}$$

Note 29 demonstrates that the derivative of the old bid function with respect to β is negative $[h_4(v, \theta, \alpha, \beta(\hat{b}_1), L)] < 0$. Therefore, as long as $[\partial\beta(\hat{b}_1)]/\partial\hat{b}_1$ is positive or not too negative,

$$\frac{\partial\beta(\hat{b}_1)}{\partial\hat{b}_1} > \frac{1}{h_4[v, \theta, \alpha, \beta(\hat{b}_1), L]}, \qquad \text{(A51)}$$

Then the effect of a change in α (or a change in θ) has the same sign as before, so theorems 1 and 3 hold. The same logic can be used on equation (A44) to demonstrate theorem 4 is true as long as condition (A51) holds. Theorem 2 is unchanged.

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