Exposure to Currency Risk: Definition and Measurement

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Introduction

U.S. corporations, including those with no foreign operations and no foreign currency assets, liabilities, or transactions, are generally exposed to foreign currency risk. Regional electrical utilities are a somewhat extreme case in point. With no foreign currency accounts on their books, they have no accounting exposure to exchange risk. From an economic perspective, however, the story is different. If their customer base is dominated by either importing or exporting firms whose activities and demand for electricity are affected by exchange rate changes, the electrical utilities' operations and stock prices themselves will also in principle be exposed to exchange risk.[1]

Accounting rules for foreign currency translation are largely powerless in this situation. Economic exposure to foreign currency risk must be defined and measured quite apart from the choice between current or historical exchange rates for translating particular foreign currency positions. What is needed, in short, is to replace the accounting approach with a technique for measuring the economic exposures of the market prices of financial and physical assets. This is done in the pages that follow which draw out the implications for practice of recent innovations on the theoretical front. The objective, it should be noted, is limited to definition and measurement; it is not the analysis of the determinants of exchange risk or exposure thereto. It is also not to analyze whether individuals or firms should hedge. These last are important problems which are best addressed once the exposure phenomenon itself has been precisely identified.

Section I distinguishes between currency risk and exposure. The latter is defined in Section II and measured in Section III. The practical implications of the approach are discussed in Sections IV and V.

[1]This point is not new. It has its origins in traditional trade theory and the notion that exchange rate changes can cause, or coincide with, fluctuations in domestic aggregate demand, employment and output and, therefore, also with the activities of non-trading domestic firms. See Laursen and Metzler [11].

I. Risk vs. Exposure

A currency is not risky because devaluation is likely. If the devaluation were certain as to magnitude and timing, there would be no risk at all. A weak currency can be less risky than a strong currency. A strong currency does not become risky because it has been used to denominate an individual's or a firm's debt. In short, expected exchange changes can be anticipated. Risk or uncertainty is a question of randomness, *i.e.*, unexpected exchange rate variations.[2]

Currency risk is not the exposure. Currency risk is to be identified with statistical quantities which summarize the probability that the actual domestic purchasing power of home or foreign currency on a given future date will differ from its originally anticipated value. Exposure, in contrast, should be defined in terms of what one has at risk.[3] The question at hand is how to measure it.

II. Currency-Risk Exposure

For purposes of financial analysis, a reasonable measure of exposure to currency-risk should meet the following three criteria:

(1) Its dimension should be an amount of currency, domestic for domestic currency risk and foreign for foreign currency risk.

(2) It should be a characteristic of any asset or liability, physical or financial, that a given investor might own or owe, defined from that investor's viewpoint.

(3) It should be implementable in the dual sense that, first, measurement can be accomplished with available techniques; and, second, exposure so measured can be hedged or covered with available financial instruments, such as forward exchange contracts.[4]

Assume a world with only two currencies, U.S. dollars and French francs, and that U.S. investors, who are all alike, perhaps improbably expect U.S. inflation, and therefore their domestic currency risk, to be zero. They expect, however, that the French inflation rate and therefore the exchange rate will be random. Forward contracts are available.

Consider next a representative U.S. investor who expects *with certainty* to receive exactly FF 1000 in three months from today. What will be his exposure on the target date three months away? The intuitive (and correct) answer is: precisely FF 1000. Why? The answer is apparent from Exhibit 1. FF 1000 represents exactly the sensitivity of the future *dollar* value of the franc balance to variations in the exchange rate. Furthermore, a hedge in the form of a forward contract to sell 1000 francs for three months at the forward rate F, which will pay off \$1000(F-S) at maturity, is exactly what is required to insulate the *dollar* value of the balance from unanticipated variations in the exchange rate. The dollar value of the hedged position remains constant, at \$1000F, across states of nature.

We can summarize these considerations in the following linked definitions of exposure and hedging, based on the theory set forth in the Appendix.

Exposures = The amounts of foreign currencies which represent the sensitivity of the future, real domestic-currency (market) value of any physical or financial asset to random variations in the future domestic purchasing powers of these foreign currencies, at some specific future date.

Hedges = The amounts of the foreign-currency financial transactions (*i.e.*, forward contracting or its equivalent) required to render the future, real, domestic-currency market value of an exposed position statistically independent of unanticipated, random variations in the future domestic purchasing powers of these foreign currencies.

There are several points to observe about these definitions. First, the concept of exposure has a definite target date. This is partly because the financial instruments used for hedging in practice have contractually fixed maturity dates. They can, therefore, protect exposures only within the period covered by the contract. In addition, exposures themselves may vary over time. They are future quantities which may be imagined as the equivalent of foreign currency deposits or debts

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[2]The identification of exchange risk as randomness, distinct from expected currency strength or weakness, is a by-product of studies of exchange rates as random walks, beginning with Poole [13]. There is no need for present purposes to distinguish between nominal exchange risk, which may not matter under some assumptions, and relative price risk, which may. A critical review of the arcane debate on this point appears in Adler and Dumas ([4], section 6).

[3]Dumas [7] seems to have been the first to make this distinction in print. Though obvious, it bears repeating.

[4]The hedging scenarios to be considered are extremely simple by design. Useful insights can nonetheless be achieved. Futures contracts are excluded mainly because most large firms still prefer the forward to the futures markets. We also elide multi-period or dynamic hedging strategies, restricting attention to a one-period setting. On both accounts, interest rate risk or reinvestment rate uncertainty remains outside our present purview. These exclusions indicate the direction that future extensions and modifications of the ideas should take.

Exhibit 1. Exposure of FF 1000, To Be Received in the Future, When There Are Three States of Nature

Future Quantities	State 1	State 2	State 3
FF Balance: P*	1000	1000	1000
Exchange Rate: S = $/FF	0.250	0.225	0.200
Dollar Value: SP*	$250	$225	$200
Proceeds of Forward Sale: $1000(F-S)	$1000(F-0.25)	$1000(F-0.225)	$1000(F-0.20)
Dollar Value of Hedged Position	$1000F	$1000F	$1000F

which will exist or mature on some future date.[5] In most practical circumstances, exposure in any currency will not be a certain accounting quantity but will rather be a magnitude which must be projected and estimated by statistical means. Seen from the present, exposures six months ahead may be greater or smaller than exposures a year ahead.

A second point to be emphasized, and one that we shall investigate in detail below, is that the notion of exposure is not restricted to non-traded financial assets or liabilities with fixed, nominal, foreign-currency payoffs on the maturity date of the hedge. The exposure of such assets is easy to determine. They are 100% exposed in the specific sense that their exposure is exactly equal to their foreign-currency face value on that date, as the example in Exhibit 1 demonstrates. However, the (real) domestic-currency price of *any* asset or liability, physical or financial, whose future foreign currency value is uncertain, may also be sensitive to, or correlated with, exchange rate fluctuations. To the extent of this sensitivity or correlation, such assets should indeed be considered exposed. This leads to the notion of exposure as a statistical property.

In the Appendix and in the next section we show that exposure is best measured with statistical regression techniques, as the coefficient(s) of the purchasing power variables(s), (*i.e.*, the exchange rates when domestic inflation is zero), in a multiple linear regression of an asset's future domestic-currency market price on (the set of) the contemporaneous foreign exchange rate(s).[6] Each such regression coefficient has the dimension of units of foreign currency. The idea behind defining exposure in this fashion is that it decomposes the probability distribution of a risky asset's domestic-currency price at a future instant into two parts: one that is correlated with the (set of) exchange rate(s) and a second that is independent of them. The first component may be considered exposed as its variability may be removed by financial hedging. It is the equivalent of the French franc deposit in the previous example. Residual variability remains, but it is not exposed as it cannot be further reduced by hedging. It should be clear that in this approach exposures can be positive, zero or even negative depending on the direction of the correlations between an asset's price and the exchange rates. If, furthermore, one thinks of long-term foreign bonds with nominally fixed redemption values but with maturities beyond the target date, the exposures of their *market* prices, translated into dollars, may be more or less than one-hundred percent.[7]

III. Exposure Is a Regression Coefficient

An asset may be simultaneously exposed to more than one currency. The reader should keep this in mind

[5]This emphasis, that exposure depends on the covariance of *future* prices with *future* exchange rates, marks a radical departure from the current textbook wisdom. For example, Cornell and Shapiro [6] indicate that exposure should be measured in terms of the impact of unexpected exchange rate changes on the *present* value of the firm's future cash flows without reference to a target date. This apparently represents an attempt to collapse the dynamic evolution of exposure over time into a single measure. In our opinion, it is unsuccessful. When one thinks in terms of market prices, as one should, present-values are currently known. They are certain. The very idea of a covariance between the non-random current market price and random exchange rate variations over an indefinite horizon is, therefore, vacuous. The possibility of devising any other single-valued measure of association between a present value and exchange rates at every future date seems equally remote. In finance, notional changes in present values are used to evaluate the impact of current decisions that will affect future outcomes: but no financial decision is required for measuring exposure. The approach taken here enables financial decision analysis and exposure measurement to be kept apart: Exposure is a matter of the future, instantaneous covariation of contemporaneous prices and exchange rates either at discrete points of, or continuously over, time. It is the *value* of hedging that requires present value comparisons.

[6]The intellectual pedigree of this approach appears partly in the Appendix. It originated in Dumas [7] which followed several of his working papers beginning in 1976.

[7]See Adler and Dumas [2] for a two-period analysis of the determinants of exposure in this case. In the one-period framework of the present paper, a financial or physical asset is risky if its end-of-period, dollar market price is random. Financial assets include stocks and short and long-term bonds; physical assets are (manufactured) goods like plant and equipment and inventory. Financial and physical assets need not further be distinguished, as they must in multiperiod or continuous time models, according to the source of uncertainty: interest-rate risk in the case of long-term bonds; and cash flow or replacement-value risk, combined with interest-rate risk, in that of stocks and physical assets. Nominally riskless bonds and deposits are free of default risk in their respective currencies. Deposits are not traded prior to maturity and therefore are also free of interest rate risk. Nominally riskless foreign currency deposits are risky in dollar terms.

throughout what follows. It is only for simplicity of exposition that we construct the examples of this section using a single exchange rate. The intention of these examples is to demonstrate how exposure may be measured using well-known linear regression techniques. That it should be so measured is proved in the Appendix. When one wants to determine exposure to many currencies, the regressions in question simply become multiple linear regressions. This extension of the methodology will be mentioned again in Section IV.

For the purpose of the exercises that follow, we need no more than a simple setting with three possible future states of nature and two currencies, dollars and francs. We continue to assume that dollar inflation is non-random. The setting is fully described by Exhibit 2. Looking ahead from now, time 0, to a future target date, the dollar value of a nominally riskless or risky French asset is uncertain. Uncertainty is captured by the fact that one out of three possible states of nature, subscripted by s, will occur. In each state the French asset will have a franc price denoted by P^*_s. P^*_s will have the same value in each state if the asset is nominally riskless, such as a bank deposit or a T-bill, and if it matures on the target date; and it will have different values across states if it is risky like a stock or a long-term bond which matures after the target date. Seen from now, the exchange rate, S_s, is also uncertain: it will take on different values within its possible range of variation in each future state of nature. Consequently, whether or not P^*_s itself is constant, the dollar value of the French asset, $S_sP^*_s = P_s$, will be random.

In the Appendix we show that the exposure of the French asset, from the viewpoint of a U.S. investor who assumes that U.S. inflation is non-random and who has access to forward contracts, is properly measured by the coefficient, b, in a linear regression of P on S across the states of nature:

$$P = a + b \cdot S + e$$

where

a = regression constant;
e = random error term such that $E(e) = 0 = \text{cov}(e,S)$.

Note that the dimension of b is units of French francs: it therefore meets the first of our criteria for an exposure measure in the previous section.

We now demonstrate the application of this exposure measurement technique through two numerical examples. In each, the state probabilities are 1/3, that is, the states are equi-probable, a condition often associated with ignorance. The first, in Exhibit 3, which calculates the exposure of a French franc deposit, is included for comfort. It should be consoling to discover that the computed exposure is in fact equal to the deposit's face value as intuition demands. The second deals with a slightly more complex situation in which the French franc asset-price is also random. We defer discussion of the results till later.

Exhibit 3 provides the requisite consolation. In the situation where we know that the exposure is FF 1000, the regression technique produces exactly that result. This is also exactly the amount that can be hedged. Following the procedures of Exhibit 1 above, the outcome is that a hedge in the amount of FF 1000 will make the dollar value of the franc deposit *constant* across states of nature.

This constancy of the dollar value of the hedged position is possible *only* when the franc price of the French asset is itself constant across states of nature, that is, when the French asset is nominally riskless in franc terms and its maturity coincides with that of the forward contract. It is only such assets that are 100% exposed in the dual sense that their exposures equal their face values and they can be perfectly hedged.

When the underlying French franc price at the hedging horizon is *random*, the risky asset cannot be perfectly hedged. The Appendix teaches us that in such

Exhibit 2. The Dollar Price of a Risky Foreign Asset in Three States of Nature

	French Franc Price: P^*_s	Exchange Rate: S_s	U.S. Dollar Price: $S_sP^*_s = P_s$
	P^*_1	S_1	$S_1P^*_1 = P_1$
Time 0	P^*_2	S_2	$S_2P^*_2 = P_2$
	P^*_3	S_3	$S_3P^*_3 = P_3$

Exhibit 3. Exposure and Hedging When There are Three States of Nature and a Fixed Amount, FF 1000, Due in the Future

(1) To Show: Exposure is the regression coefficient of the future dollar value of the FF position in a regression of that position on the exchange rate. In this case, exposure must also necessarily be FF 1000.

(2) Definitions:
P^*_s = FF 1000 in each state
P_s = Dollar value of FF 1000 in state s at time-t
= $\$S_sP^*_s$ = \$1000 S_s in this case.

(3) Regresion:
$P = a + bS + e$, where here, $a = \bar{P} - b\bar{S} = 0$;
$b = \text{cov}(P,S)/\text{Var}(S)$;
p_s = Probability of state s;
$\bar{P} = \Sigma_s p_s S_s P^*_s = \$1000\ \bar{S} = \$225$ below, since
$\bar{S} = \Sigma_s p_s S_s = 0.225$ \$/FF, from the data,
$\text{cov}(P,S) = \Sigma_s p_s(P_s - \bar{P})(S_s - \bar{S})$
$\text{var}(S) = \Sigma_s p_s(S_s - \bar{S})^2$

(4) Data:

States	p_s	S_s	P_s	$S_s - \bar{S}$	$P_s - \bar{P}$	$(S_s - \bar{S})^2$	$(P_s - \bar{P})(S_s - \bar{S})$
1	1/3	0.25	\$250	0.025	\$25	0.000625	0.625
2	1/3	0.225	\$225	0	0	0	0
3	1/3	0.20	\$200	−0.025	−\$25	0.000625	0.625

(5) Calculated Regression Coefficient:

$$b = \frac{\text{cov}(P,S)}{\text{var}(S)} = \frac{(2/3)(0.625)}{(2/3)(0.000625)} = \text{FF } 1000$$

cases the best one can do is to hedge in the amount of the exposure. This will minimize the variance of the hedged asset and render the residual variability of the asset price in dollars, after hedging, *independent* of the exchange rate. The effect of determining exposure via regression, in other words, is to decompose the randomness of $P = SP^*$ into two components: one which is correlated with the exchange rate and is removed by hedging; and a second, that remains after hedging, whose variance is minimal and which is statistically unrelated to the exchange rate. This is demonstrated in Exhibits 4 and 5.

The top panel of Exhibit 4 provides the data for the dollar value of a risky French asset in each of three states of nature. In the middle panel we compute the regression coefficient, b, in the regression of P on S. This coefficient represents the exposure: it is FF 2000 in this example. The bottom panel reveals the results of a hedge in the amount of the exposure. Quite clearly, the dollar value of the hedged asset is not constant across states of nature. The hedge in this situation is obviously not perfect: the hedged asset is still risky.

It is, however, the best that can be done in a very specific sense. As shown in Exhibit 5, the residual variability in the dollar value of the hedged asset, denoted by R, is statistically independent of the exchange rate: that is, as shown in the third panel, cov(R,S) = 0. What then has been accomplished by hedging? This is shown in the fourth panel of Exhibit 5. The variance of the unhedged position, computed in the rightmost column of panel 3 in Exhibit 4, is 1867 $(\$)^2$. The variance of the hedged but still risky asset, calculated in panel 2 of Exhibit 5, is 200 $(\$)^2$. Hedging has removed the part of the variance due to exchange rate variability, in the amount of 1667 $(\$)^2$. The remaining variability has been minimized and cannot further be reduced by forward exchange contracts or their equivalent. Other hedging instruments or methods such as diversification must be sought if it is to be avoided.

IV. Summary Observations on the Meaning of Exposure

It is perhaps worth reconsidering the concept of exposure in light of the measurement procedures described above. From the viewpoint of U.S. investors exposure, to be sure, is the regression coefficient or, if there are many currencies, the vector of partial regression coefficients, when an asset's dollar price is regressed on exchange rates. Seen in this way, it is clear that any asset, *regardless of its location,* may be exposed. U.S. stocks and real estate are exposed, for example, if their (future) market prices are correlated

Exhibit 4. Exposure Measurement When the French Franc Price is Random

1. Regression Equation $\tilde{P} = a + b\tilde{S} + \tilde{e}$, where $\tilde{P} = \tilde{S}\tilde{P}^*$

2. Data

States	P_s	P_s^*	S_s	P_s
1	1/3	1200	0.25	\$300
2	1/3	978	0.225	\$220
3	1/3	1000	0.200	\$200
Means			0.225	\$240

3. Calculation of the Parameters of the Regression of P on S.

States	$S_s - \bar{S}$	$(S_s - \bar{S})^2$	$(P_s - \bar{P})$	$(P_s - \bar{P})(S_s - \bar{S})$	$(P_s - \bar{P})^2$
1	0.025	0.000625	\$60	1.5	3600
2	0	0	−\$20	0	400
3	−0.025	0.000625	−\$40	1	1600
Means	0	(2/3)(0.000625)	0	(1/3)(1.5) + (1/3)(1)	1867

$$b = \frac{\text{cov}(P,S)}{\text{var}(S)} = \frac{[(1/3)(1.5) + (1/3)(1)]}{(2/3)(0.000625)} = \text{FF } 2000$$

$$a = \bar{P} - b\bar{S} = \$240 - \$(2000)(0.225) = -\$210$$

4. Hedge the Exposure: Sell FF 2000 Forward @ F: Proceeds = \$2000(F − S)

Results	State 1	State 2	State 3
Unhedged Dollar Value	\$300	\$220	\$200
Proceeds of Contract	\$2000(F-0.25)	\$2000(F-0.225)	\$2000(F-0.20)
Value of Hedged Asset	\$(2000F-200)	\$(2000F-230)	\$(2000F-200)

with one or more foreign exchange rates.[8] Nominally riskless U.S. deposits that are held to maturity, on the other hand, are not exposed to foreign currency risk on the maturity date. Their nominal dollar payoffs are constant, regardless of exchange rate changes.[9] They are, however, exposed to domestic currency risk if U.S. investors perceive the purchasing power of the dollar to be nonrandom. Nominally riskless foreign currency deposits whose maturities coincide with the hedging horizon are, in contrast, always exposed fully (*i.e.*, 100%) to their own exchange rate and to no other. Risky foreign assets whose market prices at the hedging horizon are uncertain are, in principle, no different from risky U.S. assets. Their exposures to one or more currency risks are an empirical matter, to be determined by the procedures already described.

Measuring an asset's exposure by its regression coefficient or coefficients essentially means splitting the random future dollar price into two components. One is the exposure. It is the equivalent of a foreign currency deposit (or a portfolio of foreign currency deposits if the regression is run over many exchange rates).

[8]This point enables us to comment on the faulty claim, now enshrined in FAS 52, according to which local debt hedges fixed assets in foreign subsidiaries. In our approach, of course, this would not generally be true since the measured exposures of physical assets and foreign currency liabilities would not necessarily be equal or even offsetting for any given planning horizon. The exposures of physical assets like plant, equipment and inventories, wherever located, are simply the regression coefficients of their future dollar market values on exchange rates. These exposure coefficients, were one to analyze them further, would be seen to depend on the impact of exchange rates, via macro-economic and trade linkages, on the productivity of, and dollar cash flows from, the assets. Equally, the exposures of a firm's nominal short and long-term foreign currency liabilities are the coefficients from regressing their dollar market values on exchange rates. Following Adler and Dumas [2], these latter exposures will depend on the serial correlation of exchange rates, and their links, via their impact on inflationary expectations and default risks, with local borrowing rates. The determinants and, therefore, any potentially reasonable estimates of the exposures of physical assets and nominal liabilities are completely different. The latter may hedge the former to a greater or lesser extent, but this is an empirical matter. Finally, though parenthetically, physical asset exposures are independent of deviations from commodity or purchasing power parity; and the exposures of foreign bonds or deposits are independent of violations of interest-rate parity. For the reasons, see Adler and Dumas ([4], Sec. 8).

[9]Long-term U.S. bonds, on the other hand, are of course generally exposed at any hedging target date short of maturity. The main determinant of this exposure, which can be analyzed only in two-or-more-period models, is the correlation between interest rate risk and exchange risk. This, in turn, depends on the serial correlation properties of exchange rate and inflation rate expectations as these are embodied in the interest rate.

Exhibit 5. Demonstration That the Hedged French Asset Is Independent of the Exchange Rate

1. Dollar Value of Hedged Asset $= R_s = P_s + b(F - S_s)$

2. From Exhibit 4 calculate the following data:

States	P_s	R_s	$R_s - \bar{R}$	$S_s - \bar{S}$	$(R_s - \bar{R})(S_s - \bar{S})$	$(R_s - \bar{R})^2$
1	1/3	\$(2000F-200)	+\$10	0.025	0.25	100
2	1/3	\$(2000F-230)	−\$20	0	0	400
3	1/3	\$(2000F-200)	+\$10	−0.025	−0.25	100
Means		\$(2000F-210)	0	0	(1/3)(0.25) − (1/3)(0.25)	200

3. $\text{cov}(R,S) = E[(R_s - \bar{R})(S_s - \bar{S})] = (1/3)(0.25) - (1/3)(0.25) = 0$

4. Summary of Results of Hedging

Variance of Unhedged Asset	$= \text{Var}(P) = 1867\ (\$)^2$
Variance of Hedged Asset	$= \text{Var}(P - bS) = \text{Var}(R) = 200\ (\$)^2$
Variance Removed by Hedging	$= b^2\text{Var}(S) = (2000)^2(2/3)(0.000625) = 1667\ (\$)^2$

This component can be perfectly hedged by an offsetting forward exchange transaction, just as foreign currency deposits themselves could be hedged. This equivalence of exposure to a (portfolio of) foreign currency deposit(s) is an idea to which we shall subsequently return. The second component is not exposed to exchange risk, in the specific sense that its randomness is not correlated with any exchange rate. It may, of course, be exposed to other identifiable risks: but these cannot be hedged with forward currency transactions.

Risky assets other than domestic and foreign currency deposits and long-term bonds, which promise payment of a specific amount of currency at maturity, have no intrinsic denomination in any currency. In other words, they are generally risky regardless of the choice of reference or measurement currency. An asset can be said to be denominated in a currency to the extent that its future dollar value, or some fraction thereof, behaves as if it were a riskless deposit in that currency.[10] Another way of looking at our concept of exposure is that it measures the extent to which any asset can be said to be denominated in one or more currencies. Common stocks, for example, regardless of where they are traded, and physical assets, regardless of where they are located, can therefore equally well be said to have no intrinsic currency denomination or to be denominated, to the extent of their exposures, in one or more currencies. In these cases, denomination like exposure cannot be identified in advance but instead is an empirical matter.

Finally, it is perhaps worthwhile emphasizing what hedging with forward exchange contracts can do and what it cannot. Hedging with forward exchange transactions does not in general remove currency risk.[11] It merely substitutes exposure to domestic purchasing power risk for exposure to foreign currency risk. When domestic purchasing power uncertainty is negligible, forward exchange transactions can remove the impact of exchange risk variability. They do so in effect by redenominating the exposed fraction of an asset's future domestic value, from foreign currency into domestic currency. When foreign asset prices are uncertain and/or when domestic inflation risk is present, the (real) dollar values of these assets cannot be fixed in advance merely with forward contracts. At most, one can hedge away that part of the nominal variation which is linearly correlated with exchange-rate randomness. The dollar prices of these assets will remain uncertain after hedging. The remaining uncertainty will be independent of the exchange rate. Only to this extent can the asset's total risk be said to have been hedged.

[10]The question of denomination, it should be clear, is quite different from the issue of the correct deflator for a firm's results discussed by Eaker [8]. His suggested numeraire is, in any event, problematic: one has few choices between the owners' CPI and separate indices for each of the firm's output and input goods. The latter, furthermore, will generally have different exposures. Exchange rate changes can indeed affect a firm's competitiveness even if CPP holds for every good and PPP holds at the level of the CPI, if the relative prices of inputs and outputs vary.

[11]Different limitations of hedging with forward contracts have been discussed by other authors. One key issue, which is largely outside the province of this paper, is whether hedging foreign exchange risk can add value to the firm. In informationally symmetric, quasi-complete markets, it cannot: see Adler and Dumas [1], Baron [5], Logue and Oldfield [12], and Dumas [7].

V. Conclusion

Until now, corporate exposure to foreign exchange risk has largely been investigated in the literature from the viewpoint of the firm and its managers. Here we have looked at it in a way that conforms to the interests of stockholders and analysts. The notion that exposure can be measured as a regression coefficient should have immediate appeal to this group.

Stock analysts have long been accustomed to measuring the riskiness of a security or a portfolio by its beta in a regression of the portfolio's returns on the market index. Viewed slightly differently, beta represents a transformation of the *exposure* of the portfolio to the market. To hedge a portfolio with initial value V_t, one should short a quantity of index futures equal to (beta) (V_t).[12] Such a hedge, if interest rates remain constant, will render the future value of the portfolio largely independent of the index for the life of the contract.

Clearly, the idea of exposure to exchange risk is not intrinsically different from that of exposure to market risk. It is not, therefore, surprising that the two can be estimated in similar ways. Thus, for example, a portfolio's average exposure to exchange risk in the past can, in principle, be obtained by regressing its total dollar value on a vector of exchange rates: the partial regression coefficients will represent its exposure to each currency. If the future is like the past, portfolio managers could conceivably use the procedure usefully to strip the exchange risk component out of portfolio returns by hedging these exposures. The promise is there. Before it can be implemented, however, one must confront the usual but intricate problems of stationarity and multicollinearity. As exposures vary over time, it will be necessary further to attempt to derive multiperiod hedging rules. These are matters for future research.

In addition, several implications for corporate practice emerge from redefining exposure properly in terms of market rather than book values and measuring it as a regression coefficient. For one thing it may change or refine the way some treasurers think about exposure: that indeed is the paper's main aim. Exposure is a statistical quantity rather than a (projected) accounting number. Foreign currency liabilities and forward contracts have their own measurable exposures. They cannot therefore be expected to perfectly hedge fixed assets or foreign operating results, whose exposures are generally different, and in some circumstances may not do so at all. The firm as a whole is exposed. Its global exposure is not necessarily the sum of the exposures of the individual foreign operations or of specific foreign currency accounts, for this ignores the exposure of domestic operations. Nor is its global exposure some sum of its so-called "accounting" and "economic" exposures.[13] *Ex ante*, the two cannot be measured or hedged separately, in as much as future financial reports will reflect the impact of the economic environment and the firm's responses via current and future managerial decisions.

What the regression coefficient concept of exposure can provide is a single comprehensive measure that summarizes the sensitivity of the whole firm, as of a given future date, to all the various ways in which exchange rate changes can affect it. While problems of implementing this approach must be left to be treated elsewhere, it offers a further insight.[14] Once exposures are so measured they can, for purposes of managerial control, be decomposed into components, one for each kind of influence that exchange rates can have. If necessary, the decomposition can produce the counterparts of translation and transaction exposures as part of the total. Neither should necessarily be ignored, nor should they be hedged alone. When and if hedging is a desirable corporate decision, it is exposure as a whole, not some part of it, that should be protected. All this is at least conceptual progress in the direction of clarity.

We cannot end without a warning. Adapting firms' planning systems to the requirements of the proposed approach to exposure measurement may be expensive. Furthermore, financial theory suggests that hedging any exposure, however measured, might be useless to stockholders if they had the information to hedge for themselves. There is no empirical evidence to show that exchange risk exposure reduces, or that compa-

[12]More precisely, since the regression is run on rates of return, beta is the elasticity of the stock price with respect to the market. The elasticity is a dimensionless number while exposure is a dollar amount in this case. It is apparent from the Appendix that in this case: $\text{Exposure}_t = (\text{beta})(V_t)$, where V_t is the initial price of the stock. Naturally, if beta is stationary, exposure will change as V_t varies over time. This raises the issue of the specification of dynamic hedging strategies which is the object of much current research in connection with the use of index futures.

[13]The notion that economic exposure and accounting exposure are different is not new: the origin of the idea is obscure and the references too many to list. Most textbooks, *e.g.*, Eiteman and Stonehill [10] and Shapiro [16], rightly emphasize the distinction. However, they leave the impression that economic exposure is something which one somehow adds on to accounting exposure. Nor do they offer a summary measure of economic exposure.

[14]An extended example which illustrates the requisite simulation analysis in a simple one-period setting with two states of nature appears in Adler and Dumas [2]. Adding states presents no particular difficulties. The multi-period extension is more problematic. It could conceivably be structured as a (complex) dynamic program. Shapiro and Rutenberg [15] reveals the hazards of such efforts.

nies' hedging activities improve, stock values. The issues addressed here could conceivably be irrelevant from this viewpoint. There seem, however, to be two ways to justify costly exchange risk management in the name of the stockholders' interests. Once they are preliminarily identified, exposures can often be wholly or partly closed by shifting funds across borders through channels, other than forward contracts, that also reduce taxes. Identifying and hedging large exposures, furthermore, can lead to reduced perceptions of default risk, improved bond ratings and lower borrowing rates. When such savings are available, managers may balance the benefits against the costs.

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Appendix

This Appendix offers a general and intuitively appealing definition of exposure and links it with hedging. Under specific assumptions exposure boils down to a (partial) regression coefficient. Hedging in the amount of the exposure minimizes the variance of the hedged position and leaves a residual randomness that is independent of exchange rates.

Consider the random dollar price, P, of a risky asset on a given future date. The number of states of nature, K, is finite with known probabilities. In a given state, k, the outcome P_k is associated with a vector of state variables, $S_k = \{S_1, \ldots, S_n\}_k$, which may include exchange rates if these are called for by some underlying theory. Let S_i represent the ith state variable. Following Adler and Dumas [2], we may then define:

$$\text{Exposure of P to } S_i = E(\partial P/\partial S_i)$$

This defines exposure as the current expectation, across future states of nature, of the partial sensitivity of P to S_i, the effects of all other variables held constant. The definition is intuitive and quite general. For the exposure to be hedgeable, side bets such as forward contracts on S_i must be possible and available.

When P and S are jointly normal, exposure as defined by (3) becomes the partial regression coefficient of S_i in a linear regression of P on S. This rests on two previous results. Rubinstein [14] considered the pricing of a contingent claim on P, g(P), in the presence of a single state variable S. When P and S are jointly normal, he obtained that:

$$\text{cov}[g(P), S] = E[g'(P)]\ \text{cov}(P,S)$$

Utilizing the same setting, Adler and Dumas [2] extended this result and showed that:

$$E\{\frac{\partial E[g(P)|S]}{\partial S}\} = \frac{\text{cov}[g(P), S]}{\text{var}(S)} = E[g'(P)]b_{p|s}$$

where $b_{p|s}$ is the regression coefficient of P on S, in the bivariate case.

The application of these relationships to present needs is immediate. First, note that in connection with exposure measurement, $g(P) = P$ so that $g'(P) = 1$ in all states of nature. Second, the definition of exposure can therefore be rewritten in the multivariate case as:

$$\begin{aligned} E(\partial P/\partial S_i) &= E\{\frac{\partial E[(P)|\mathcal{S}]}{\partial S_i}\} \\ &= \mathrm{cov}[S_i, P|\mathcal{S}]/\mathrm{var}(S_i) \\ &= b_{p \cdot s_i|\underline{s}} \end{aligned}$$

where $b_{p \cdot s_i|\underline{s}}$ is the partial regression coefficient on S_i in a regression of P on $\mathcal{S}$. The extension to jointly lognormal random variables is equally immediate, with similar results.

The link between exposure and hedging can easily be inferred from the futures literature. The requisite insight emerges from a modification of Ederington [9].

Let P, as before, be the future price of a risky asset with current price P_0. Assume that there exists a costless forward sale contract on state variable S: the current forward price is F_0. At maturity the random gain or loss on the contract is $(F_0 - S)$. Define the expected return and variance of a portfolio consisting of w_p units of P and w_s units of the forward contract:

$$\begin{aligned} E(R) &= w_p E(P - P_0) + w_s E(F_0 - S) \\ \mathrm{var}(R) &= w_p^2 \mathrm{var}(P) + w_s^2 \mathrm{var}(S) - 2 w_p w_s \mathrm{cov}(P,S) \end{aligned}$$

Let: $w_s = aw_p$

In the futures literature, $a = w_s/w_p$ is called the hedge ratio or the proportion of the spot position that is hedged. In our terminology, the optimal value of a is the exposure. Substituting, we rewrite

$$\mathrm{var}(R) = w_p^2 \mathrm{var}(P) + a^2 w_p^2 \mathrm{var}(S) - 2 w_p^2 a \mathrm{cov}(P,S)$$

The regression coefficient definition emerges from computing the minimum variance hedge, a*:

$$\begin{aligned} \partial \mathrm{var}(R)/\partial a &= 2w_p^2[a \mathrm{var}(S) - \mathrm{cov}(P,S)] = 0 \\ \Rightarrow a^* &= \mathrm{cov}(P,S)/\mathrm{var}(S) = b_{p|s} \end{aligned}$$

In short, when hedging so as to minimize the variance of the hedged position, one should hedge in the amount of the exposure.

We next show that, after hedging the exposure, the residual randomness of the hedged position is independent of S. First, we specify the regression to measure the exposure:

$$P = a + bS + e$$

where:
- a = regression constant;
- b = regression coefficient, the exposure;
- e = regression residual; where by construction,
- E(e) = 0 = cov(e,S); and
- E = expectation, denoted by a superbar: $E(X) = \bar{X}$

The linear regression technique produces the following definitions:

$$\begin{aligned} a &= \bar{P} - b\bar{S} \\ b &= \mathrm{cov}(P,S)/\mathrm{var}(S), \text{ so that} \\ E(P|S) &= a + b\bar{S} \\ \mathrm{var}(P|S) &= b^2 \mathrm{var}(S) + \mathrm{var}(e), \text{ since } \mathrm{cov}(e,S) = 0. \end{aligned}$$

The random dollar value, R, of a position consisting of one dollar's worth of P hedged by the forward sale of b units of S at the forward rate F_0 is:

$$R = P + b(F_0 - S)$$

The variance of the hedged position is given by:

$$\begin{aligned} \mathrm{var}(R) &= \mathrm{var}(P - bS) \\ &= E(P - bS - \bar{P} + b\bar{S})^2 \\ &= E[(P - \bar{P}) - b(S - \bar{S})]^2 \\ &= E[(P - \bar{P})^2 - 2b(P - \bar{P})(S - \bar{S}) + b^2(S - \bar{S})^2] \\ &= \mathrm{var}(P) - 2b\,\mathrm{cov}(P,S) + b^2 \mathrm{var}(S). \end{aligned}$$

Recall from the definition of the regression parameters that:

$$\begin{aligned} \mathrm{cov}(P,S) &= b \mathrm{var}(S) \\ b^2 \mathrm{var}(S) &= \mathrm{var}(P) - \mathrm{var}(e). \end{aligned}$$

Hence, after substitution and rearrangement:

$$\mathrm{var}(R) = \mathrm{var}(e).$$

What we have shown is that the variance of the hedged position is equal to the variance of the regression error. Since this residual is *by construction* independent of S, so therefore is R. In other words,

$$\begin{aligned} \mathrm{cov}(R,S) &= E[(P - bS - \bar{P} + b\bar{S})(S - \bar{S})] \\ &= \mathrm{cov}(P,S) - b \mathrm{var}(S) \\ &= 0, \text{ by virtue of the definition of b.} \end{aligned}$$