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Optimal debt contracts and moral hazard along the business cycle*

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Summary. We analyze the Pareto optimal contracts between lenders and borrowers in a model with asymmetric information. The model generalizes the Rothschild-Stiglitz pure adverse selection problem by including moral hazard. Entrepreneurs with unequal "abilities" borrow to finance alternative investment projects which differ in degree of risk and productivity. We determine the endogenous distribution of projects as functions of the amount of loanable funds, when lenders have no information about borrowers' ability and technological choices. Then, we embed these results in a dynamic competitive economy and show that the average quality of the selected projects in equilibrium may be high in recessions and low in booms. This phenomenon may generate (a) multiple steady states, (b) a smaller impact of exogenous shocks on output relative to the full information case, (c) endogenous fluctuations.

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1 Introduction

It is commonly argued that the business cycle is likely to be amplified in economies with imperfect financial markets. Different versions of this proposition are in [13, 1, 2, 12, 17]. When lenders are not well informed about borrowers' investment projects they may devise "second-best" contracts inducing the borrowers to take desirable

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actions or reveal some information. Since these contracts are often entailing collateral requirements and credit rationing, investment and consumption turn out to be highly dependent on the borrowers' balance sheet position.

Most models predict that the agency costs associated to informational problems in financial markets (monitoring, moral hazard, adverse selection, etc.) are more important in recessions rather than booms. In fact, these costs are a decreasing function of firms' liquidity or collateralized assets and most models predict that these variables are highly procyclical when endogenized in a general equilibrium framework.

One problem with this literature is that it cannot explain why booms may sometime revert to recessions. The idea that agency costs and distortions are negatively affected by firms' net worth is good for explaining the amplification of business cycles (e.g., the fact that recessions may last longer than predicted by the real business cycle models), much less for understanding endogenous reversion mechanisms.

This paper analyzes the relation between informational asymmetries in financial markets and business cycle fluctuations from a rather different perspective. We explore the effects of asymmetric information in financial markets by focusing on the joint effects of adverse selection and moral hazard in a world with heterogeneous entrepreneurs and investment projects. Our goal is to generate the distribution of entrepreneurs and projects in equilibrium and to see how this distribution evolves along the business cycle.

Projects chosen by a given pool of heterogeneous borrowers-entrepreneurs (differentiated by ability or location) can be ranked in terms of risk and productivity. Some of them have a higher probability of default and a lower expected output for given investment. Entrepreneurs have no endowment and they can only invest by applying for loan contracts. The type of investment projects that they choose is a function of the set of loan contracts offered by lenders with no information about borrowers' types and investment choices. We endogenize the distribution of projects by assuming that loan contracts are Pareto optimal and decentralizable and by imposing that the opportunity cost of lending is determined by a market clearing condition in the market for loans. Due to asymmetric information and limited liability, the social costs generated by the risky projects are not fully internalized and some entrepreneurs may choose to adopt them. The higher the proportion λ of entrepreneurs undertaking these projects, the higher is the loss of efficiency and real resources characterizing competitive equilibria.

The key step in the paper is to relate the optimal contracts and the distribution of bad projects λ to the opportunity cost of lending (the risk free interest rate) and the amount of loanable funds. We show that λ is an increasing function of interest payments (loan rate times loan size) on good projects, i.e., higher interest payments and limited liability make the moral hazard and adverse selection problems more important. A rise in the interest rate induces a price and a quantity effect on interest payments. The price effect increases the cost of borrowing (higher loan rate), while the quantity effect reduces the loan size. If the quantity effect dominates over the price effect, interest payments go down along with the proportion of bad investment projects. As the interest rate is determined by a market-clearing condition in the market for loans, a rise in loanable funds may have a negative effect on the loan rates and a positive effect on the average loan size. Hence, when the quantity effect dominates over the price effect, the proportion of bad projects λ is increasing in the size of loanable funds.

In the final section we embed this framework in an overlapping generations model where the amount of loanable funds is equal to the competitive wage rate arising from a neoclassical production function. This feature allows for a characterization of the joint evolution of projects distribution and output and the evaluation of the impact of exogenous shocks. We find that the model may produce multiple steady states and that a rise in productivity increases the steady state values of the wage rate along with the proportion of bad projects. In addition, when the production function is not "too concave", a productivity shock has a smaller impact on steady state real wages and output in the asymmetric information than in the full information model. Finally, we show that the evolution of equilibrium contracts along the cycle may produce discontinuities in the relation between the amount of loanable funds and the opportunity cost of lending that may be responsible for endogenous cycles. These cycles may exist because the high set-up costs generated during the upswing (when more borrowers choose high risk and wasteful projects) may eventually grow large enough to decrease output and wages.

Discontinuities are possible in our model because optimal contracts are subject to a no cross subsidizing (NCS) condition. The latter allows for optimal contracts to be decentralizable in a competitive environment (In Reichlin and Siconolfi [14] we show how we can decentralize optimal contracts as the Nash equilibrium outcome of the three stage game proposed by Hellwig [9]). NCS may generate "regime switches": an increase of the opportunity cost of lending may cause a switch of equilibrium contracts from "pooling" (borrowers get the same contract irrespective of their project selection) to "separating" (borrowers "self-select" by choosing contracts designed for their own project) or vice versa. This is the source for the existence of endogenous cycles.

The "cleansing effect of recessions", a phenomenon documented by Caballero and Hammour [6] in a different context, may be a way of describing concisely what may happen in our economy. During the cyclical upswing (when there is a large amount of loanable funds) competitive lenders devise contracts attracting a high proportion of bad projects; the opposite occurs during the downswing. In other words, contrary to other papers in this field, our model predicts that the adverse selection and moral hazard problem in financial markets may be more severe during booms than recessions. Clearly, the reason for these diverging results is that we focus on the equilibrium selection of different investment projects instead of focusing on the interaction between credit limits and asset prices along the cycle. The two approaches are not competing with each other.

One may argue that having entrepreneurs with no endowment is a strong assumption. In fact, the degree of moral hazard and adverse selection could be a function of the cyclical behavior of entrepreneurs' net worth so as to alter the dynamics of our model. In Appendix 2 we show that, when Pareto optimal contracts are obtained by maximizing the borrowers' surplus, the existence of positive endowments does not significantly alter the results of the model as long as the endowment is small enough. In particular, if the down payment earns the opportunity cost r

and if there is no cost of liquidation, borrowers' capital does not affect the nature of pooling contracts and it may reduce the set of economies for which equilibrium contracts are separating. Since our results hold under much weaker conditions when contracts are pooling, assuming that the borrower has some endowment may actually help in making our point.

Our model parallels Mankiw [13] by assuming a standard adverse selection model where liquidity and collateral have no role. However, we endogenize the opportunity cost of lending as an equilibrium interest rate on deposit, allow for separating contracts and add a specific form of moral hazard in the lenders-borrowers relations so as to endogenize the distribution of borrowers in terms of risk and productivity. Since we show that, in some cases, the proportion of bad projects may be procyclical, our results are at variance with Mankiw [13].

Suarez and Sussman [17] explore the possibility that the costs associated to asymmetric information may be more important in booms than in recessions. Because of moral hazard and limited liability, the probability of success of any given investment project is positively correlated with the value of first period output. Since entrepreneurs face a downward sloping demand curve, this value is anti cyclical in equilibrium. It follows that a boom generates a low price of output and a low liquidity thereby increasing external finance. Due to moral hazard, this leads to excessive risk taking and a high rate of failure. However, the prediction that borrowers' net worth is counter-cyclical is at variance with empirical regularities.

Carlstrom and Fuerst [7] study a computable general equilibrium model based on agency costs and find that the output response to exogenous shocks is characterized by more propagation and less amplification than in the standard Real Business Cycle model. The agency problem derives from the costly state verification model of Townsend [18] and contractual relations between lenders and borrowers are assumed to be anonymous. Since in their model the response of agents' net worth to exogenous shocks is sluggish, aggregate fluctuations may loose amplitude while gaining persistence.

2 The model

Agents and technologies

We consider an economy with overlapping generations of two sets of agents, lenders and entrepreneurs (or borrowers). Each set has a continuum of two-period-lived identical individuals in an interval of size one.

The cumulative distribution function of entrepreneurs is G (with density $G' = g$), where G is assumed to be twice continuously differentiable with support in $[0, 1]$ and such that

Assumption 1. $sg'(s)/g(s) > -2$, *for all* $s \in [0, 1]$.

The precise role of the above assumption will be apparent in Section 4, where we derive the loan contracts allowing entrepreneurs to finance their projects. Essentially, the assumption simplifies the exposition, by ruling out a non convexity of the programming problem from which contract choices are derived.

Lenders are endowed with one unit of labor when young, which they supply inelastically to a large set of competitive firms engaged in the production of a final good y. Entrepreneurs are endowed with the ability to run investment projects, whose output is an intermediate good z. All agents save and produce while young and consume in old age only. Given this assumption, it is inessential to specify the utility function of the "old", assumed to be strictly monotonic. Good y can be consumed or used as a capital good for the production of z, which is an input (materials) for the production of the final good y. Capital and materials fully depreciate in the production process. Production of y is instantaneous and described by a production function $y = AF(z, L)$, where $A > 0$ is a productivity parameter and L is a labor input.

Assumption 2. *$F(z, L)$ is homogeneous of degree 1 and strictly concave.*

Under full employment and by the linear homogeneity of $F(.)$ we can write:

$$y = AF(z, 1) \equiv Af(z)$$

where $f'(z) > 0$ and $f''(z) < 0$.

Prices are defined in units of the final good and we let w and q be the relative prices of labor and materials respectively. Then, perfect competition and profit maximization imply:

$$w = A\left[f(z) - zf'(z)\right] \equiv AW(z), \qquad q = Af'(z) \tag{1}$$

An entrepreneur $s \in [0, 1]$ can undertake, in the first period, one of two possible projects, L and H. In particular, if entrepreneur s invest k units of good y at time $t-1$ in the j-project ($j = H, L$), he obtains a random output

$$\tilde{z}_s^j(k) = \max\{0, \tilde{\alpha}^j(k - e_s^j)\}$$

in period t, where e_s^j is a set-up cost. This technology satisfies the following assumptions.

Assumption 3. *$\tilde{\alpha}^j$ is iid across entrepreneurs and it takes two values, $\alpha^j > 0$ with probability p^j and zero with probability $1-p^j$, with $p^H < p^L$, $p^L\alpha^L = p^H\alpha^H = \bar{\alpha}$.*

Assumption 4. *$\tilde{z}_s^j(k) = \tilde{z}_s^j(1)$ for all $k \geq 1$.*

Assumption 5. *$e_s^L = 0$, $e_s^H = es$, with $e > 0$.*

Both project types are affected by two parameter values: the marginal products α^j and the set-up cost $e^j s$. Since the latter is a function of j (the riskiness of the project) and s (the entrepreneur's ability), the model is characterized by both moral hazard and adverse selection.

Assumption 3 is almost a replica of a key assumption in Stiglitz and Weiss [16], i.e., projects differ according to a mean preserving spread of the marginal rates of return. The alternative assumption $p^L\alpha^L > p^H\alpha^H$ would not change the basic predictions of our model, as long as the two expected marginal products are not too far apart.

Assumption 4 guarantees the existence of solutions to the entrepreneurs' profit maximization problem for a wide range of relative prices.

Assumption 5, together with 3, imply that any investment k yields a higher expected output with L-projects than with H-projects. For this reason we will occasionally refer to the H-project as the "bad" project. Notice that no qualitative property of our model depends on the fact that the set-up cost is proportional to the marginal product of capital $\tilde{\alpha}^j$.

Assumptions 4, 3 and 5 generalize the standard adverse selection model (e.g., Stiglitz and Weiss [16], Bester [3], Boyd and Smith [5]) introducing a potential moral hazard problem in the relation between entrepreneurs and outside investors. In a pure adverse selection model projects are not choice variables, the output of a project is independent of s and, in general, of k, since investment is usually assumed to be indivisible.

We consider the above assumptions to be the natural generalization of the standard assumptions employed in the literature on adverse selection to a setting including moral hazard (however, see Bester [4] for a different approach).

The assumptions about the production of materials is where we depart from more standard models. Here we choose a simple condition to capture the idea that only the "best" entrepreneurs (low values of s) find the risky project more profitable than the safe project. In Stiglitz and Weiss [16], at equilibrium, the risky project yields, by the mean preserving spread assumption, a higher expected profit, while, in our model, this is true only for the best entrepreneurs. This creates an incentive for some borrowers to select the safe project. The reference to a set-up cost is just a way, computationally convenient, to obtain this property.

Loan contracts and information structure

The type of an entrepreneur and his investment choice are private information, both before and after the realization of the random variable $\tilde{\alpha}^j$. However, it is publicly known whether an investment project is successful or not, i.e., whether $\tilde{\alpha}^j = 0$ or $\tilde{\alpha}^j > 0$.

Since entrepreneurs have no physical endowment and projects yield no output in the bad state, the entire production of the intermediate good is financed externally with limited liability loan contracts. Namely, the borrower repays the loan when the investment project is successful, while no payment occurs otherwise.

As in many other applications of incentive problems to the analysis of business cycles (e.g., Bernanke and Gertler [1,2], Suarez and Sussman [17]), we assume that loan contracts are constrained Pareto Optimal, i.e., they are the best arrangement under which borrowing can take place, given the information structure. More precisely, contracts are obtained by maximizing the borrowers' profit subject to the lenders' participation constraint. Since lenders are only consuming when old, their reservation utility at time t is $r_t q_t w_{t-1}$, where r is a deterministic "opportunity cost of lending" in units of the intermediate good. When deriving contracts, we take r as given. However, this is an endogenous variable, which is determined, at equilibrium, by the equality between demand and supply of loanable funds.

A contract is a pair $c = (B, R)$ specifying the size of the loan and the amount of repayment per unit of loan in the good state. For convenience, the loan size, B, is defined in units of the final good and the amount of repayment per unit of loan, R, is defined in units of the intermediate good. This specification of c allows us to define the expected profit of an entrepreneur in units of the intermediate good independently of q. In particular, the expected profit at time t of an entrepreneur s investing at time $t-1$ in the j-project with a contract $c_{t-1} = (B_{t-1}, R_t)$ signed at time $t-1$ is:

$$\pi^j_{s,t}(c_{t-1}) = (\bar{\alpha} - p^j R_t)B_{t-1} - \bar{\alpha}e^j s, \qquad j = H, L,$$

where $e^H = e$ and $e^L = 0$.

The presence of asymmetric information rules out contracts contingent on the specific realizations of $\tilde{\alpha}^j$, the borrowers' type and their project choice. In principle, lenders may devise a continuum of contracts, each of them inducing self selection of s-types. However, in our model, for a given project choice, the borrower's ranking of the available contracts is independent of the type s. More precisely, if c and $c^{'}$ are two distinct contracts, $\pi^j_s(c) - \pi^j_s(c^{'})$ is s- independent for $j = L, H$. Therefore, there is no loss of generality in restricting our attention to at most two contracts (designed for each project). We call these *separating* if they are distinct, and *pooling* if they are equal.

Let $C(r)$ be the family of optimal contracts for a given (next period) opportunity cost of lending r. An element $c \in C(r)$ is a pair $c^s = (c^H, c^L)$ (if contracts are separating) or it is a single contract c^p (if contracts are pooling), with $c^j = (R^j, B^j)$ and $j = H, L, p$. Separating contracts induce self selection of borrowers. Thus, a borrower adopting the risky (safe) project (weakly) prefers c^H to c^L (c^L to c^H). Equivalently, separating contracts satisfy the incentive compatibility constraints, i.e., $\pi^L_s(c^L) \geq \pi^L_s(c^H)$ and $\pi^H_s(c^H) \geq \pi^H_s(c^L)$.

Whether c is separating or pooling depends on which of these alternative arrangements provides the largest surplus. Whenever pooling and separating contracts yield the same surplus, we allow for randomization. Namely, a random contract is an array $c = (c^s, c^p, \theta)$ with $\theta \in [0, 1]$ denoting the probability for the lender to get the separating contract. With random contracts, the lender selects the investment project after having observed the realization of the lottery over the pair (c^s, c^p). If the separating contract realizes, the lender is free to choose the c^H or c^L component. By construction, randomization does not alter the optimal surplus. However, it is essential for the existence of an equilibrium in the market of loanable funds (see next section). Thus, the optimal contract $c(r)$ is separating (pooling) if $\theta(r) = 1$ ($\theta(r) = 0$). The precise definition of the Planner's problem and the derivation of the contracts is described in Appendix 1.

Intertemporal equilibria

To simplify the exposition, we now define intertemporal equilibria when contracts are deterministic. However, as already mentioned, existence of an equilibrium may require a randomization. Later on in this section, we explain how to generalize the definition and the analysis to random contracts.

For given opportunity cost of lending, r_t, set of optimal contracts, $C(r_t)$, and proportion of entrepreneurs selecting the H-project, $\lambda(r_t)$, are determined at time $t-1$. The aggregate demand for loans at $t-1$ is denoted as $D(r_t)$, where

$$D(r) = \left(1 - G(\lambda^{\kappa(r)}(r))\right) B^{L(\kappa(r))}(r) + G(\lambda^{\kappa(r)}(r)) B^{H(\kappa(r))}(r),$$

for $\kappa(r) = p$ if $c^p(r) = C(r)$, $\kappa(r) = s$ if $c^s(r) = C(r)$, $L(p) = H(p) = p$, $L(s) = L$ and $H(s) = H$. Hence,

$$D(r) = \begin{cases} B^p(r) & \text{if } \kappa(r) = p, \\ (1 - G(\lambda^s(r))\, B^L(r) + G(\lambda^s(r)) B^H(r) & \text{if } \kappa(r) = s. \end{cases}$$

Since lenders work in young age and consume when old, the (per capita) supply of funds at $t-1$ coincides with the wage rate w_{t-1} and the market clearing condition in the loan market at time $t-1$ is:

$$w_{t-1} = D(r_t) \tag{2}$$

The set $r(w_{t-1})$ of values of r solving Equation 2 is defined as the set of *temporary equilibrium interest rates*. By the law of large numbers (with the usual caveat applying to a continuum of random variables (see Judd [11]) and by the assumption that borrowers get all the surplus generated by the optimal contracts, lenders earn the deterministic gross return r on any unit of loaned funds. Hence, their (per capita) consumption at t is simply $r_t q_t w_{t-1}$. Since $p^H \alpha^H = p^L \alpha^L = \bar{\alpha}$, the supply of the intermediate good is defined by:

$$Z(r) = \bar{\alpha} \left(\int_0^{\lambda^\kappa(r)} (B^{H(\kappa)}(r) - es) dG(s) + \int_{\lambda^\kappa(r)}^1 B^{L(\kappa)}(r) dG(s) \right).$$

Hence, $Z(r) = \bar{\alpha} D(r) - M_e(r)$, where:

$$M_e(r) = \bar{\alpha} \int_0^{\lambda^\kappa(r)} es dG(s)$$

is the aggregate set-up cost.

Equations 1 (i.e., $w_t = AW(z_t)$ and $q_t = Af'(z_t)$) determine the real wage and the relative price of the intermediate good for given demand of the intermediate good, z_t, while the market clearing condition of the intermediate good market at time t is:

$$z_t = Z(r_t) = \bar{\alpha} D(r_t) - M_e A(r_t). \tag{3}$$

At a general equilibrium, the markets for loanable funds, labor, final and intermediate goods clear. The equilibrium conditions determine three relative prices: the interest rates r (in units of the intermediate good), the wage rate w and the price of the intermediate good q. Exploiting Walras' Law, we eliminate the market clearing condition for the final good.

Definition 1 (Intertemporal equilibrium). *For given initial value w_0, an intertemporal equilibrium is a trajectory $\{(z_t, w_{t-1}, r_t, q_t, C(r_t)) : t \geq 1\}$, such that, for all t, $C(r_t)$ is a family of optimal contracts and Equations 1, 2 and 3 are satisfied.*

Definition 2 (Stationary state). *A stationary state is an array (z_e^*, w_e^*, r_e^*) such that*

$$w_e^* = D(r_e^*) = AW\left(\bar{\alpha}w_e^* - M_e(r_e^*)\right), \qquad z_e^* = Z(r_e^*), \qquad q_e^* = Af^{'}(z_e^*).$$

By trivial manipulations, we can more directly define an intertemporal equilibrium as a sequence $\{(w_t, r_t) : t \geq 1\}$ satisfying Equation 2 and:

$$w_t = AW\left(\bar{\alpha}w_{t-1} - M_e(r_t)\right), \tag{4}$$

where the remaining variables, z_t and q_t are determined by the conditions $z_t = Z^j(r_t)$ and $q_t = Af^{'}(z_t)$.

Evidently, for $e = 0$ (i.e., when all entrepreneurs are equal), the dynamic behavior of the real wage (and therefore, of materials and final output) is independent of the loan market. Intertemporal equilibria can be derived from:

$$w_t = AW(\bar{\alpha}w_{t-1}). \tag{5}$$

In the next section we show that the above also characterizes intertemporal equilibria of the full information model.

Full information

When projects are observable, optimal contracts are contingent on project types. By limited liability, these contracts are a pair (c^L, c^H) such that $p^L R^L = p^H R^H = r$ and $B^L = B^H = 1$ if $R^j < \alpha^j$, $B^j \in [0, 1]$ if $R^j = \alpha^j$ $(j = H, L)$.

For $w < 1$, equilibrium in the market for loans implies $B^j = w$, $R^j = \alpha^j$, $j = H, L$. Then, all borrowers choose the L-project and the opportunity cost of lending is $r_t = \bar{\alpha}$ for all t. An intertemporal equilibrium of the full information model is, therefore, the unique monotonic sequence $\{w_t : t \geq 1\}$ solving Equation 5 with initial condition $w_0 < 1$.

To single out the role of informational asymmetries in the dynamics of the model, we only consider economies that, under full information, have stationary states $w_0^* \in (0, 1]$. The following assumption guarantees this:

Assumption 6. $\lim_{z\to 0} -zAf''(z) > 1/\bar{\alpha}$, $Af(\bar{\alpha}) - \bar{\alpha}Af'(\bar{\alpha}) \leq 1$.

Competitive allocations of the benchmark model are Pareto optimal if and only if they are *dynamically efficient*. As usual, we define a stationary intertemporal equilibrium as dynamically efficient if there is no way to increase agents' total consumption by destroying the existing stock of capital. This type of inefficiency (which typically arises in overlapping generation models) is ruled out when the marginal product of capital is sufficiently high. The relevant condition in this model is $\bar{\alpha}Af'(\bar{\alpha}w_0^*) \geq 1$.

3 Intertemporal equilibria with asymmetric information

In this section we partially characterize the dynamic properties of competitive equilibria with asymmetric information. We first provide an informal discussion of the contractual problem and we state the main properties of the optimal contracts. Then, we examine the consequences of these properties on intertemporal equilibria, under a set of simplifying assumptions. These are the minimum set of assumptions guaranteeing the uniqueness of a temporary equilibrium interest rate r (and optimal random contract) for any given wage w. It will be explained in a moment that uniqueness can be insured whenever the set-up cost e is not "too small" (the critical size of e depends on the shape of the distribution function $G(s)$). Furthermore, we will show that, when e is big enough so as to guarantee the uniqueness of an equilibrium interest rate, $r(w)$, the latter is a decreasing function and the associated proportion of bad projects, $\lambda(r(w))$, is an increasing function of w. We regard this as the "regular case" when the inefficiency arising from asymmetric information is quantitatively significant.

The existence of a unique temporary equilibrium interest rate, $r(w)$, has important implications for the dynamics of intertemporal equilibria. Indeed, when $r(w)$ is decreasing and $\lambda(r(w))$ is increasing in w, the aggregate set-up cost

$$M_e(r(w)) = \bar{\alpha} \int_0^{\lambda(r(w))} esdG(s) \equiv m_e(w)$$

is an increasing function of w. Then, Equation 4 becomes:

$$w_t = AW\left(\bar{\alpha}w_{t-1} - m_e(w_{t-1})\right).$$

A detailed analysis of the dynamics of the above map will be derived later on in this section. However, it is immediate to see that the positive effect of an increase in past wages w_{t-1} on current wage w_t is tempered by the rise of the aggregate set-up cost. Hence, adverse selection and moral hazard imply a slower wage growth and a smaller impact of exogenous shocks as compared to the full information case.

When these assumptions are not satisfied, optimal contracts may not be unique and intertemporal equilibria may display a variety of different dynamic patterns. In the final section of the paper we concentrate on a specific type of multiplicity related to the coexistence of different type of optimal contracts (pooling and separating) for a given wage. This multiplicity calls for a selection criterion and it may be responsible for the existence of endogenous fluctuations. We postpone the discussion of these cases to the last section of the paper. An example shows that, for e below the value above which uniqueness is guaranteed, the model does not behave (not even approximately) like the full information model. The reason is that optimal contracts may "switch" from separating (pooling) to pooling (separating) as w changes along the cycle and "switching regimes" are associated to discontinuities of the map defining the equilibrium dynamics.

Main properties of loan contracts

Contracts are obtained by maximizing the borrowers' profit subject to the lenders' participation constraint. The presence of a continuum of borrowers' types allows for many possible and alternative choices of the Planner's objective function. The propositions, in this section, state some of the most relevant properties of the optimal contracts. The analysis of the contractual problem together with the proof of all the propositions in this section are in Appendix 1. Although we provide some intuition for the results, the propositions that follow can be read, at this stage, as assumptions satisfied by the optimal contracts.

As it is customary in environments with asymmetric information, we design the contractual problem by imposing a no cross subsidizing (NCS) condition (see Henriet and Rochet [10] for a discussion of the role of the NCS condition in competitive markets). More specifically, we require that (separating) contracts (c^L, c^H) cannot produce a negative surplus for the lender when offered separately. The (NCS) condition guarantees that optimal contracts are decentralizable in a competitive environment. In Reichlin and Siconolfi [14] we provide a specific game form, with lenders (banks) and borrowers, which delivers (decentralizes) as a (subgame perfect) Nash equilibrium the efficient contracts used in the paper. As usual, the decentralization of efficient contracts requires an exclusivity assumption, i.e., each borrower can apply at most for one contract.

By the NCS condition, optimal contracts may be either pooling or separating (or both), depending on the value of r. A value $r^* \in (0, \bar{\alpha})$ is called a *switching point* if c^p and c^s are both in $C(r^*)$ (are both optimal). r^* is called a switching point because, as shown in Appendix 1 (Lemma 4), for e in a generic set of $\mathbb{R}_{++}$, r^* is locally isolated and optimal contracts "switch" from pooling (separating) to separating (pooling) as r crosses r^*. The (generically finite and possibly empty) set of switching points in $[0, \bar{\alpha}]$ is denoted with S^*. From now on we will implicitly assume that e is either zero or it belongs to the generic subset of $\mathbb{R}_{++}$ for which S^* is finite.

Proposition 1. *For all $r \in [0, \bar{\alpha}]$, the set of optimal contracts $C(r)$ is non empty. Moreover,*

- *if $S^* = \emptyset$, $\theta(r) = 0$ (i.e., the optimal contract is pooling) for all $r \in [0, \bar{\alpha}]$ and if $S^* \neq \emptyset$, $\theta(r) = 0$ for all $r \in [0, r_1^*)$ and some $r_1^* \in S^*$;*
- *the optimal proportion of H-projects, $\lambda(r)$, is strictly positive for all $r \in (0, \bar{\alpha})$ and $\lambda(0) = \lambda(\bar{\alpha}) = 0$.*

Proposition 1 guarantees that the set $\{r : \theta(r) = 0\}$ is non empty, i.e., a deterministic pooling contract is optimal for r small enough. The proposition, however, does not rule out that $\theta(r') = 1$ for some $r' \in (0, \bar{\alpha})$, i.e., the possibility for optimal contracts to be separating. Clearly, if $\theta(r') = 1$ for some $r' \in (0, \bar{\alpha})$, Proposition 1 implies that $S^* \neq \emptyset$.

In the pure adverse selection model, if cross subsidization is allowed, Pareto optimal contracts are, typically, separating. The loan contract offered to high risk borrowers is associated with a higher loan rate and it generates a surplus which can be used to subsidize the contract (generating a negative surplus) offered to low risk borrowers. The NCS condition imposes a lower bound on the loan rate of the (safe) L-contract. Thus, the benefit from cream skimming may not be fully exploitable and a pooling contract may dominate. This is a known possibility in pure adverse selection environments with exogenously given values of risky projects λ (e.g., Rothschild and Stiglitz [15]). In our economy, instead, the presence of moral hazard and adverse selection makes λ endogenous and thus, separating and pooling contracts may be optimal for different values of r (or for the same r). Proposition 1 shows that "cream skimming" is not beneficial when the opportunity cost of lending is relatively low.

The following two propositions provide two alternative sufficient conditions under which switching points are either absent or unique.

Proposition 2. *Let $S^* \neq \emptyset$. Then, r_1^* is increasing in e. Moreover, there is a value e^o large enough and such that, for $e \geq e^o$, $S^* = \emptyset$.*

Proposition 3. *If $sg(s)/G(s) \leq 1$, S^* is either empty or it contains a unique switching point $r^* \in (0, \bar{\alpha})$.*

Proposition 3 shows that, for a large class of distributions functions, either optimal contracts are pooling for all r, or they are pooling for $r < r_1^* \in (0, \bar{\alpha})$, while they are separating for $r \in (r_1^*, \bar{\alpha})$. A specific case is the uniform distribution, i.e., $G(s) = s$, for which the unique switching point is:

$$r_1^* = \frac{\bar{\alpha}}{\mu + 4(1-\mu)^2/e}.$$

In this case, optimal contracts are always pooling if $e \geq 4(1-\mu)$ and they are separating for $r \in [r_1^*, \bar{\alpha}]$ otherwise.

The general equilibrium properties of the model are critically related to the behavior of the optimal contracts as a function of r. The next propositions characterize the map $C(r)$ when $\theta(r) = 0$ or $\theta(r) = 1$ (deterministic contracts).

Proposition 4. *Consider a deterministic contract $c^\kappa(r) \in C(r)$ ($\kappa = p, s$). Then, $B^\kappa(r)$ is non increasing and $R^\kappa(r)$ is strictly increasing for all $r \in [0, \bar{\alpha}]$. Moreover, $B^H(r) = 1 > B^L(r)$ and $B^p(r) < 1$ for all $r \in (r^o, \bar{\alpha}]$, for some $r^o \in (0, \bar{\alpha})$.*

Proposition 4 is in line with known results in the adverse selection literature. Asymmetric information may generate credit rationing of all borrowers when contracts are pooling and credit rationing of the safe borrowers (those selecting project L) when contracts are separating. Since contracts guarantee the lenders a reservation utility defined by the opportunity cost r, a higher value of r is compensated by a higher repayment R^κ. Since high repayments induce borrowers to take more risk, the Planner reduces the loan size to compensate for the loss generated by the increase in risk.

Separating contracts (c^H, c^L) satisfy two conditions. The first is the no cross subsidizing (NCS) condition $p^L R^L = p^H R^H = r$. The second states, as in the pure adverse selection model (e.g., Rothschild and Stiglitz [15]), that the risky borrowers are not rationed, i.e., $B^H = 1$, and that they are indifferent between the c^H and the c^L contract, i.e., $\pi_s^H(c^H) = \pi_s^H(c^L)$. When the NCS condition is taken into account, the latter implies that:

$$B^L = \left(\frac{\bar{\alpha} - p^L R^L}{\bar{\alpha} - p^H R^L}\right).$$

Rationing of safe borrowers is the cost paid in order to make a separating contract incentive compatible (cream skimming). By optimality, B^L must be the largest loan size satisfying the incentive compatibility constraint of the risky borrower.

The proportion of risky (bad) projects λ plays a fundamental role in a general equilibrium. When $r \notin S^*$, this variable will be denoted with $\lambda^\kappa(r)$, where $\kappa = p$ if $C(r) = c^p(r)$ and $\kappa = s$ if $C(r) = c^s(r)$.

Proposition 5. *The functions $\lambda^j(r)$ ($j = s, p$) are continuous and unimodal in $[0, \bar{\alpha}]$ (with $\lambda^j(0) = \lambda^j(\bar{\alpha}) = 0$). Furthermore, $\lambda^p(r)$ is strictly decreasing in r, for $r \in (r^o, \bar{\alpha}]$. Finally, if $\lambda \in (0, 1)$, it is:*

$$\lambda^\kappa = \frac{(p^L - p^H)}{\bar{\alpha} e} R^{L(\kappa)}(r) B^{L(\kappa)}(r),$$

for $\kappa = p, s$ and $L(p) = p$, $L(s) = L$.

The particular functional form of λ^κ in Proposition 5 is obtained by solving for λ the equation $\pi_\lambda^H(c^p) = \pi^L(c^p)$, when contracts are pooling, and the equation $\pi_\lambda^H(c^H) = \pi^L(c^L)$, when they are separating.

For $r \in S^*$, optimal contracts are a correspondence and the expression of λ depends on the particular contracts $C(r)$ selected. Hence, for $r \in S^*$ and $\theta \in (0, 1)$, the law of large numbers implies that:

$$\lambda = \lambda^\theta(r) = (1 - \theta)\lambda^p(r) + \theta \lambda^s(r).$$

The basic content of Proposition 5 is in the characterization of the proportion of risky projects as a function of r. The statement is quite intuitive. Limited liability debt contracts induce borrowers to undertake risky projects because of the lower expected interest payments associated with these choices. Hence, an increase in r has an ambiguous effect on the proportion of risky projects. This effect can be decomposed into a quantity (fall in loan size) and a price (rise in repayments per unit of loan) effect. This is the reason why, as stated in Proposition 5, $\lambda^\kappa(r)$ is increasing for low values of r (dominating price effect) and decreasing for high values of r (dominating quantity effect).

Dynamics of equilibrium allocations

Two distinct problems arise in the model with asymmetric information. First, the set of equilibrium interest rates $r(w)$ defined by Equation 2 may be empty, i.e., an equilibrium with deterministic contracts may fail to exists. Second, $r(w)$ may contain several elements, i.e., the pairs (w_t, r_t) solving Equations 2 and 4, for some given value of w_{t-1}, may be indeterminate.

Recall that, by Proposition 4, $B^H(r) = 1$ and the aggregate demands of loans are,

$$\begin{array}{ll} (1 - G(\lambda^s(r))\, B^L(r) + G(\lambda^s(r)) \equiv B^s(r) & \text{if } \theta(r) = 1, \\ B^p(r) & \text{if } \theta(r) = 0. \end{array}$$

Then, given $w \in [0, 1]$, a credit market equilibrium with random contracts is a pair (r, ϑ), such that:

$$\vartheta B^s(r) + (1 - \vartheta) B^p(r) = w; \qquad \vartheta \in \theta(r) \tag{6}$$

and $\vartheta = \theta(r)$ if $r \in [0, \bar{\alpha}] \backslash S^*$, while $\vartheta \in [0, 1]$ otherwise. With some abuse of notation, we keep denoting with $r(w)$ the non empty set of values of r solving Equation 6.

When $S^* \neq \emptyset$, a temporary equilibrium with deterministic contracts may not exist for some values of w, i.e., $r(w)$ may be empty. The reason is simple. Consider an arbitrarily small interval around r^*, $I(r^*)$. By definition of switching point, for $r \in I(r^*)$, $r < r^*$, optimal contracts are pooling (separating), while for $r \in I(r^*)$, $r > r^*$, optimal contracts are separating (pooling). Thus, at r^*, the demand of loanable funds changes discontinuously from $B^p(r^*)$ to $B^s(r^*)$.

When market clearing requires random contracts, i.e., when $r(w) \in S^*$, $\vartheta(w)$ will denote the *market clearing mixing parameter*. Moreover, for $\kappa = p, s$, we denote with $r^\kappa(w)$ the set of solutions to $B^\kappa(r) = w$ and, for $r_i^* \in S^*$, $w_i^\kappa = B^\kappa(r_i^*)$.

Proposition 6. *For all $w \in [0, 1]$, there exists a credit market equilibrium $(\vartheta(w), r(w)) \geq 0$, with $r(0) = \bar{\alpha}$ and $r(1) = 0$. Moreover, if $S^* \neq \emptyset$, $w_i^p > w_i^s$ for all $r_i^* \in S^*$ and, if $\vartheta(w) \in (0, 1)$,*

$$\vartheta(w) = \frac{w_i^p - w}{w_i^p - w_i^s} \in (0, 1).$$

Since, by Proposition 4, $B^p(r)$ is non increasing, $r^p(w)$ is either a singleton or empty. Thus, $S^* = \emptyset$ is a sufficient condition for the uniqueness of the equilibrium interest rate $r(w) = r^p(w)$.

However, $B^s(r)$ may be a non monotone function. In fact, notice that

$$\frac{\partial B^s}{\partial r} = (1 - G(\lambda^s)) \frac{\partial B^L}{\partial r} + (1 - B^L) G'(\lambda^s) \frac{\partial \lambda^s}{\partial r}.$$

By Proposition 4, $\partial B^L / \partial r \leq 0$, for $j = H, L$, and $B^L < 1$. However, by Claim 5, $\partial \lambda^s / \partial r$ cannot be signed and, hence, $B^s(r)$ may not be monotonic. Hence, $r^s(w)$ may be a non trivial correspondence. This problem can only arise when $\lambda^s(r)$ is increasing. In this case, a rise in the opportunity cost of lending has two effects on the aggregate demand for loans. On the one hand, $B^s(r)$ falls for given λ because borrowers get a smaller loan size. On the other hand, $B^s(r)$ rises because the proportion of risky borrowers increases and these are the borrowers who get a larger loan size when contracts are separating. Hence, when $S^* \neq \emptyset$, $r(w)$ may be a correspondence.

Avoiding multiplicity

Proposition 6 leaves open the possibility of multiple credit market equilibria and clarifies that multiplicity may occur under the following two distinct possibilities:

(a) $r^s(w)$ contains at least two distinct elements, r and r', such that $\theta(r) = \theta(r') = 1$;
(b) $\theta(r^p(w)) = 0$ and $\theta(r^s(w)) = 1$ (and/or, $\theta(r(w)) \in (0, 1)$).

Case (a) never arises when the set-up cost e is large enough. In particular, let $A^s(e) = \{r : \theta(r) = 1\}$. Then, we can state the following.

Proposition 7. *There is a large enough value $\hat{e}$ such that, for all $e \geq \hat{e}$, $\lambda^s(r)$ is a decreasing function in $A^s(e)$ and $r^s(w) \cap A^s(e)$ is decreasing in w.*

In turn, case (b) never arises when the number of switching points is not greater than 1 and $r^s(w) \cap A^s(e)$ is decreasing in w. In particular, we can state the following proposition.

Proposition 8. *Assume that $r^s(w) \cap A^s(e)$ is a decreasing function and S^* contains at most one switching point. Then, $r(w)$ is unique for all $w \in [0, 1]$.*

Recall that there exists a value $e^o > 0$ such that $S^* = \emptyset$ for all $e \geq e^o$ (cf. Proposition 2) and that, if $sg(s)/G(s) \leq 1$, S^* contains a single switching point r^* (cf. Proposition 3). In either one of these cases (by Proposition 8) we can rule out any multiplicity arising from the coexistence of separating and pooling contracts for a given w. Hence, multiplicity may only arise from the non monotonicity of $B^s(r)$. By Proposition 7, we can state the following proposition, almost as a corollary of Propositions 6, 7 and 8.

Proposition 9. *If $sg(s)/G(s) \leq 1$ for all $s \in [0, 1]$, there exists a value $\bar{e} > \min\{e^o, \hat{e}\} > 0$ such that $r(w)$ is a decreasing differentiable function and $\lambda^{\vartheta(w)}(r(w))$ an increasing differentiable function of w.*

Intertemporal equilibria when $r(w)$ is unique.

The analysis of the dynamic behavior of the economy is carried out by imposing uniqueness of credit market equilibria. At the end of the paper, we will discuss the consequences generated by the existence of discontinuous regime switches (i.e., discontinuities in the relation between w and the equilibrium opportunity cost $r(w)$).

From now on in this section we study economies that satisfy the assumption of Proposition 9, i.e.,

Assumption 7. $e \geq \bar{e} > 0$, $sg(s)/G(s) \leq 1$ *for all* $s \in [0, 1]$.

By Proposition 9, we can express the aggregate set-up cost, $M_e(r(w))$ as an increasing function of $w \in [0, 1]$ denoted by $m_e(w)$. Thus, substituting Equation

2 into Equation 4, the equilibrium dynamics of the wage rate w_t is defined by the following equation:

$$w_t = \Phi_e(w_{t-1}) \equiv AW\left(\bar{\alpha} w_{t-1} - \bar{\alpha} m_e(w_{t-1})\right), \tag{7}$$

The equilibrium sequence of wage rates is generated by Equation 7 provided that $w_t \leq 1$, for all t (otherwise, the capacity constraint of the intermediate good technology is binding and Φ_e no longer describes the equilibrium of the system). This is an immediate consequence of the next proposition.

Proposition 10. *Under Assumption 7:*

- $\Phi_e(w)$ *is increasing in* $[0, 1]$*;*
- $\Phi_e(w)$ *has at least one fixed point* w_e^* *in* $[0, 1)$ *such that* $\Phi_e^{'}(w_e^*) < 1$*;*
- $w_e^* < w_0^*$.

By Proposition 10, all the stationary states of the asymmetric information economy are smaller than w_0^*, the stationary state of the benchmark model. Furthermore, $\Phi_e(w)$ is increasing in $[0, 1]$, it has at least one fixed point $w_e^* \in [0, 1)$ with $\Phi_e'(w_e^*) < 1$, and since $\Phi_0(w) > \Phi_e(w)$, by Assumption 6, $1 > \Phi_e(1)$. Hence, for any given initial condition $\bar{w} \in (0, 1)$, the equilibrium sequence of wage rates converges monotonically to some steady state $w_e^* < w_0^*$. Hence, for any initial condition in $(0, 1)$, the equilibrium sequence of wage rates $\{w_t\}$ is bounded above by 1.

Evidently, since there are no more restrictions on the shape of the map $\Phi_e(w)$ other than the ones specified in Proposition 10, we cannot rule out the existence of multiple fixed point. In this case, the equilibrium sequence may converge to different steady states for different initial conditions and asymmetric information may be responsible for the existence of "poverty traps" that would otherwise be absent in the model.

Since λ is an increasing function of w, the monotonicity of the sequence of equilibrium wages translates into monotonicity of the equilibrium proportion of bad projects. Figure 1 shows a diagrammatic representation of the equilibrium dynamics of the model with full and asymmetric information.

By inspection of the map Φ_e, a rise in w generates two opposite effects. On one hand, higher wages increase the amount of loanable funds, reinforcing the initial increase in wages. However, higher wages produce a fall in the opportunity cost of lending, an increase in the proportion of bad projects and an increase in the amount of resources lost because of the set-up cost. If the effect of w on λ is positive and strong, this mechanism damps business fluctuations (relatively to the full information benchmark), thereby introducing a role for asymmetric information somewhat in contrast with the standard view in this literature. Most importantly, this phenomenon may occur (locally around a steady state) irrespectively of whether the equilibrium contracts are separating or pooling. Although we do not investigate this issue, we conjecture that the phenomenon holds independently of the no-cross subsidizing condition. In the absence of this condition, optimal contracts would always be separating, but a positive relation between w and λ may still be there.

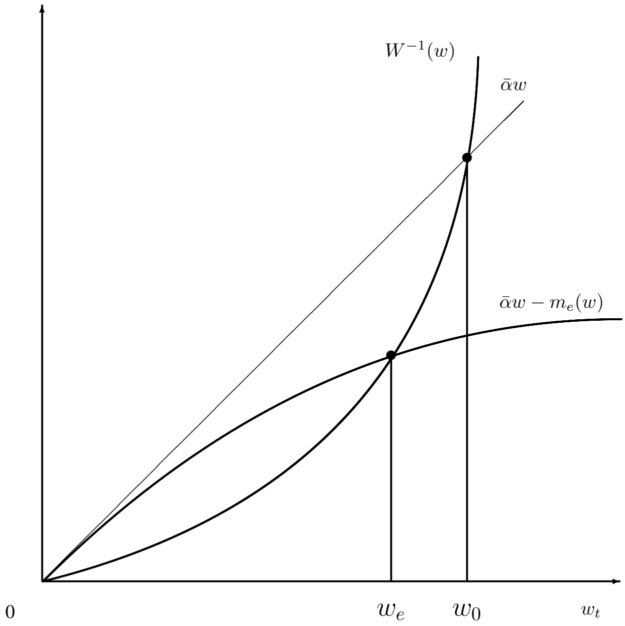

Figure 1

Finally, the same mechanism is responsible for the (possible) existence of multiple steady states.

To evaluate the long-run effect of a productivity shock with asymmetric information, consider a stable steady state w_e^* and define the elasticity of a change in A as:

$$\epsilon(e) = \frac{\partial w_e^* / \partial A}{w_e^* / A} = \frac{1}{1 - \Phi_e'(w_e^*)}$$

where

$$\Phi_e^{'}(w_e^*) = AW'(.)\bar{\alpha}\left[1 - e\lambda g(\lambda)\frac{\partial \lambda}{\partial w}\right]$$

and $\epsilon(e)$ is well defined and positive by the stability of w_e^*. Hence, a stable steady state λ_e^* is increasing in the productivity parameter A. Furthermore, since $\Phi_e'(w_e^*)$ defines the approximate "speed of adjustment" to the steady state wage rate, $\epsilon(e)$ is positively correlated with the speed of adjustment of real wages to their steady state value.

We can compare the steady state effects of a productivity shock in the asymmetric information model relative to the full information case by looking at the values of $\epsilon(e)$ and $\epsilon(0)$. The results are summarized by the following proposition.

Proposition 11. $\epsilon(e) < \epsilon(0)$ *for* $e > 0$ *iff:*

$$e\lambda g(\lambda)\frac{\partial \lambda}{\partial w} > 1 - \frac{W'\left(\bar{\alpha} w_0^*\right)}{W'\left(\bar{\alpha} w_e^* - \bar{\alpha} m_e(w_e^*)\right)}. \tag{8}$$

where λ *and* $\partial\lambda/\partial w$ *are computed at* $r(w_e^*)$.

Proposition 11 states that, if inequality 8 is satisfied, a productivity shock has a relatively smaller impact on steady state real wages and output in the asymmetric information than in the full information model. How should we interpret these results? The right hand side of the inequality in Proposition 11 is non positive for all convex wage functions $W(.)$. In these cases the inequality is always verified. When the wage function is strictly concave, $\epsilon(e) < \epsilon(0)$ requires a sufficiently high value of $\partial\lambda/\partial w$ at w_e^*. In the latter case, the speed of adjustment with $e > 0$ tends to be smaller because of adverse selection (i.e., the adverse effect of λ on output), but it tends to be higher because of concavity (the steady state capital stock with asymmetric information is smaller than the capital stock with full information).

4 Endogenous cycles

We now give conditions under which separating and pooling contracts may coexist for the same w in a credit market equilibrium and we point out a general property of any selection criterium that could resolve this multiplicity problem.

A selection criterium $\mathcal{E}$ is a rule assigning to $w \in [0, 1]$ a unique credit market equilibrium $(\vartheta_{\mathcal{E}}, r_{\mathcal{E}})$, (remember that, by definition of equilibrium, $\vartheta_{\mathcal{E}} \in \theta(r_{\mathcal{E}})$). We define this rule as a map

$$\mathcal{E}(w) = (r_{\mathcal{E}}(w), \vartheta_{\mathcal{E}}(w)).$$

The existence of multiple market clearing contracts may generate, at some w, a “sudden" discontinuity in the behavior of the equilibrium interest rate $r(w)$ (and, hence, of the proportion of risky contracts λ and the production of materials z as functions of w). The discontinuity may be induced, for instance, by a change of regime, i.e., a switch from a deterministic pooling to a deterministic separating contract (or vice versa), or by a discontinuous choice among the multiple separating equilibria. Let $\lambda_{\mathcal{E}}(w)$ be the credit market equilibrium proportion of H-projects generated by the selection criterium $\mathcal{E}$. More precisely, we give the following definition.

Definition 3. *Given a selection $\mathcal{E}$, we say that there is a discontinuous switch at w', if $(r_{\mathcal{E}}(w), \lambda_{\mathcal{E}}(w))$ is discontinuous at w'.*

Now recall that n denotes the cardinality of the set of switching points $r_i^* \in (0, \bar{\alpha})$. The following proposition states that discontinuous switches always occur when $n \geq 2$.

Proposition 12. *Let $n > 1$. Then, (i) there exists a multiplicity of market clearing contracts, (ii) for each selection $\mathcal{E}(w)$ there exists a w at which there is a discontinuous switch.*

The last proposition implies that the lack of continuous selections is independent of the existence of multiple separating contracts. However, if $r^s(w) \cap A^s$ is a continuous (and decreasing) function and $n \leq 1$, $r(w)$ is, by Proposition 9, a non increasing function and $\vartheta(w)$ is trivially continuous. Furthermore, Proposition 3 provides a sufficient condition for $n \leq 1$, while Proposition 7 shows that $e \geq \hat{e}$ suffices to make $r^s(w) \cap A^s$ a (decreasing) function.

To illustrate the consequences on the equilibrium dynamics of regime switching, we consider a specific example. Suppose that $e \geq \hat{e}$, so that, by Proposition 7, $r^s(w)$ is a decreasing function. It will be apparent that this assumption plays no role. Most importantly, suppose that $S^* = \{r_1^*, r_2^*\}$, with $\bar{\alpha} > r_2^* > r_1^* > 0$ and $w_1^p = 1$, $w_1^s > w_2^p$. Remember that, in this situation, optimal contracts must assign $\theta(r) = 0$ for $r \in [0, r_1^*) \cup (r_2^*, \bar{\alpha}]$ and $\theta(r) = 1$, for $r \in (r_1^*, r_2^*)$. Since $n > 1$, by Proposition 12 there is no selection criterium $\mathcal{E}$ making $r_{\mathcal{E}}(w)$ a continuous function.

It will be sufficient, for our purposes, to define a selection criterium $\mathcal{E}$ in the interval $[w_2^s, w_2^p]$. Setting $\bar{w} = (w_2^p + w_2^s)/2$, $\mathcal{E}$ is implicitly defined in $[w_2^s, w_2^p]$, by the following conditions:

$$w \in [w_2^s, \bar{w}] \Rightarrow r_{\mathcal{E}}(w) = r^s(w), \qquad \vartheta_{\mathcal{E}}(w) = 1;$$
$$w \in (\bar{w}, w_2^p] \Rightarrow r_{\mathcal{E}}(w) = r^p(w), \qquad \vartheta_{\mathcal{E}}(w) = 0;$$

$\mathcal{E}$ selects the separating contract in the interval $[w_2^s, \bar{w}]$ and the pooling contract in $(\bar{w}, w_2^p]$. In particular, $r_{\mathcal{E}}(\bar{w}) = r^s(\bar{w})$, while

$$\lim_{w \downarrow w_2^p} r_{\mathcal{E}}(w) = r^p(\bar{w}).$$

Most importantly, the adopted selection criterium generates a discontinuity in the proportion of H-projects: as w crosses $\bar{w}$ from the right, $\lambda(r_{\mathcal{E}}(w))$ jumps upward discontinuously. This is shown in the next proposition:

Proposition 13. $\lambda(r_{\mathcal{E}}(\bar{w})) = \lambda^s(r^s(\bar{w})) < \lim_{w \downarrow w_2^p} \lambda(r_{\mathcal{E}}(w)) = \lambda^p(r^p(\bar{w}))$.

This proposition points out a general problem with selection criteria for economies with $n > 1$. Every time the selection criterium implies a regime switch from separating to pooling contracts as w increases, the proportion of H-projects goes up abruptly. Although for $n = 2$ this type of regime switch is avoidable (i.e., it is always possible to construct selections where the switches are from pooling to separating), the phenomenon seems somewhat structural for high values of n.

Evidently, the discontinuity in $\lambda(.)$ translates into a discontinuity of $\Phi_e(w)$, as defined in Equation 7. Since a sudden increase in λ generates a sudden decrease in the production of the material, the following inequality holds true

$$\Phi_e(\bar{w}) > \lim_{w \downarrow \bar{w}} \Phi_e(w).$$

This discontinuity may be a source of endogenous dynamic fluctuations. In particular, the map $\Phi_e(w)$ may not have a steady state w_e^*. If, given our adopted selection criterium, there is no steady state w_e^*, we have the following situation:

$$\Phi_e(w) > w \text{ for } w \in [0, \bar{w}],$$
$$\Phi_e(w) < w \text{ for } w \in (\bar{w}, 1].$$

This implies that the equilibrium trajectory $\{w_t; t \geq 0\}$ permanently fluctuates in the interval $(0, 1]$ for all initial conditions $w_0 \in (0, 1]$. Figure 2 shows a diagrammatic representation of a cycle of period 2. The example proves that the model

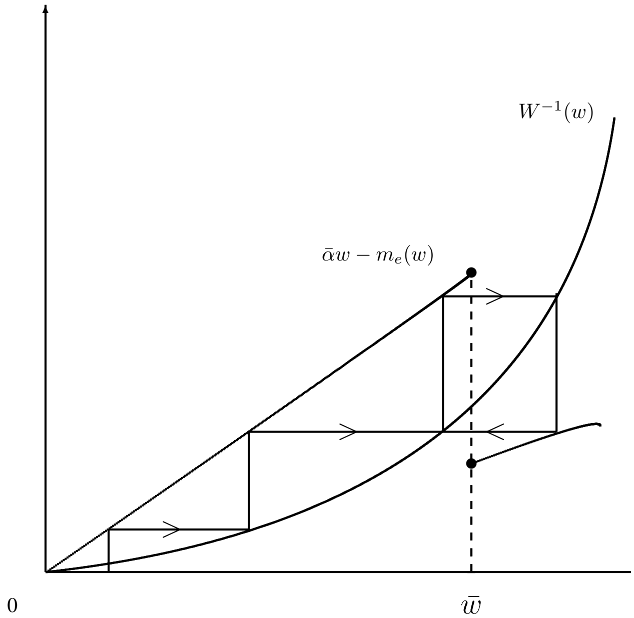

Figure 2

can explain an endogenous reversion mechanism. Low wages induce borrowers to select good projects so as to produce higher wages in next periods. In turn, high wages induce agents to choose bad projects setting up the conditions for a downturn. It should be stressed, however, that discontinuous regime switches may as well generate the opposite type of discontinuity for the map $\Phi_e(w)$. Namely, the curve representing this map may jump upward as w crosses a given threshold from the left. In this case we would expect multiple steady states.

5 Conclusions and extensions

Using a version of a celebrated model by Stiglitz and Weiss [16], this paper shows that the interplay of moral hazard and adverse selection in the market for loans implies that risky and socially costly actions made by profit maximizing entrepreneurs may be more pervasive in booms other than recessions. Hence, by embedding the Stiglitz-Weiss model in a general equilibrium framework, we may argue that credit market imperfections do not necessarily amplify the effects of exogenous shocks. Multiple steady states and endogenous fluctuations are a more likely phenomena. This is in contrast with what comes out from most of the literature in the field (for example, [1,2] and [12]). Our results show that business cycle theory may be very sensitive to the way information and market frictions in credit markets are modeled.

Our model is admittedly very simple, mainly because we assume that contracts are single-period and borrowers have no endowment. In particular, a natural objection to this model is that we are overlooking the importance of the borrowers' balance sheet and the role of collateral. As a partial answer to this type of criticism, in Appendix 2 we study the case in which entrepreneurs are endowed with some positive amount $E < 1$ of the consumption good in the first period of their life. In this case, contracts may include the following additional components: some amount $W \in [0, E]$ to be invested by the borrower in a "secure deposit" (yielding the return r) and some payment $V \geq 0$ to be delivered by the borrower to the lender in the case in which the project fails. We find that:

- W and V play no role in defining the optimal contracts in the sense that their value is either zero or left undefined in the optimal contract;
- credit rationing still occurs when $E + w < 1$;
- pooling contracts are independent of E when credit is rationed, $i.e.$, $B^p < 1-E$;
- the range of r for which optimal contracts are pooling is increasing in E.

The key result is that pooling contracts are more likely to be optimal when borrowers have a (relatively) large endowment. The intuition is that a larger E reduces the lenders' exposure with high risk borrowers and, hence, their need to separate risky from safe types. To give a sense of the effects of introducing endowments on the type of contracts prevailing in the model, consider that, with the uniform distribution $G(s) = s$, optimal contracts are pooling for all r (and w) when $E \geq 1 - e/4(1 - \mu)$. If we assume, as usual in business cycle models, that E is procyclical, we should expect pooling contracts to be more likely in booms, both because a high w implies a low r (as shown in Section 4, optimal contracts are always pooling when r is small) and because E is larger. Hence, the effect of introducing the endowment E is ambiguous. On the one hand, a large E may eliminate rationing along with any interesting dynamics of intertemporal equilibria. On the other hand, when E is not too big, the set of wage rates for which optimal contracts are pooling is larger and λ is more likely to be procyclical.

Appendix 1: Optimal contracts and proofs

In this appendix we derive the family of optimal (random) contracts $C(r)$ for a given opportunity cost of lending r (in units of materials).

As anticipated in Section 2, contracts are obtained by maximizing the borrowers' profit subject to the lenders' participation constraint. The presence of a continuum of borrowers' types allows for many possible and alternative choices of the Planner's objective function. We make this function equal to the profit of the most inefficient borrower, i.e., the borrower of type $s = 1$. When $\lambda < 1$ the profit of the least efficient borrower is $\pi^L(.)$, while it is $\pi_1^H(.)$ when $\lambda = 1$. Since every borrower is free to select projects, our criterium is equivalent to the maximization of a "reservation" profit for all types of entrepreneurs.

Hence, for all given $r \in [0, \bar{\alpha}]$, we seek triples $C = (c^s, c^p, \theta)$, with $(c^s, c^p) \geq 0$ and $y \in [0, 1]$, maximizing:

$$\mathcal{W}(C) = \theta\pi^L(c^L) + (1 - \theta)\max\{\pi^L(c^p), \pi_1^H(c^p)\},$$

subject to :

$$B^{\kappa} \leq 1, \ (\kappa = H, L, p) \qquad \text{(CC)}$$

$$(\bar{\alpha} - p^H R^H) B^H \geq (\bar{\alpha} - p^H R^L) B^L, \ \text{(IC1)}$$

$$(\bar{\alpha} - p^L R^L) B^L \geq (\bar{\alpha} - p^L R^H) B^H, \ \text{(IC2)}$$

$$p^j R^j \geq r, \ (j = H, L), \qquad \text{(ZPS)}$$

$$R^p \left[p^H G(\lambda) + p^L (1 - G(\lambda)) \right] \geq r. \quad \text{(ZPP)}$$

The form of the objective function follows from the following consideration. If, at a separating contract $c'^s = (c'^L, c'^H)$, every borrower applies for the contract c'^H, then $\lambda = 1$. Hence, we can identify, without loss of generality, c'^s with the pooling contract $c^p = c'^H$. By adopting this convention, we rule out the possibility that, at a separating contract, $\lambda = 1$.

Condition (CC) is the capacity constraint, (IC1) and (IC2) are the incentive compatibility constraints and (ZPS) is a NCS condition. By (ZPS) and (ZPP), contracts satisfy the standard participation constraint, i.e., the cost of lending cannot exceed the expected return of each contract in the optimal offer. Furthermore, given the adopted objective function, we need not write down explicitly the borrowers' individual rationality constraints. Since $B^{\kappa} = 0$ $(\kappa = H, L, p)$ is always feasible, it must be $\pi^L(c^{\kappa}) \geq 0$ (and hence $\pi^H(c^{\kappa}) \geq 0$) at any solution of the programming problem. These inequalities, together with the other constraints, imply $B^{\kappa} \in [0, 1]$ for $\kappa = p, s$, $R^j \in [0, \alpha^j]$ for $j = H, L$ and $R^p \in [0, \alpha^H]$. Hence, the (relevant part of) the feasible set is compact.

The separability in c^s and c^p, the linearity in θ of the objective function and the NCS condition imply that the optimality problem separates in two programming problems defining, respectively, optimal separating and optimal pooling contracts. More specifically, for given r, an optimal pooling contract c^p is a solution to

$$(P.p) \ \max_{c^p \geq 0} \{\max\{\pi^L(c^p), \pi_1^H(c^p)\}\} \ \text{s.t.: (CC), (ZPP),}$$

while, an optimal separating contracts $c = (c^L, c^H)$ is a solution to

$$(P.s) \ \max_{(c^H, c^L) \geq 0} \pi^L(c^L) \ \text{s.t.: (CC), (IC1), (IC2), (ZPS).}$$

In the sequel, $c^j(r)$ and $\pi^j(r)$, $j = p, s$, will denote the set of optimal solutions to and the value functions of the programs $(P.\kappa)$, $\kappa = s, p$. If, at r, $\pi^p(r) > (<) \pi^s(r)$, the optimal random contract implies $\theta(r) = 0$ $(\theta(r) = 1)$ and it degenerates in the pooling (separating) contract. Optimal random contracts are non trivial lotteries over the two deterministic contracts only when $\pi^p(r) = \pi^s(r)$. In such a situation, any $\theta \in [0, 1]$ is optimal, i.e., $\theta(r) = [0, 1]$. It is evident that if, for instance, $\theta(r) = 0$, the actual specification of the separating component is immaterial. However, it will be convenient and without loss of generality to specify the set of optimal contracts as the array $C(r) = (c^s(r), c^p(r), \theta(r))$. Since $\theta \in (0, 1)$ only when $\pi^p(r) = \pi^s(r)$, the introduction of random offers does not increase the value of the objective function and it is inessential for optimality.

Proof of Propositions 1, 4, 5

The proof of Proposition 1 is divided in three steps. In the first we characterize $c^s(r)$, in the second $c^p(r)$ and in the last $\theta(r)$. From now on we simplify the notation by setting $\mu = p^H/p^L$ and $\sigma = p^L(1-\mu)/\bar{\alpha}e$.

Step 1: optimal separating contracts. As we have already observed, $\pi^L(c^L) \geq 0$ (and hence $\pi^H(c^H) \geq 0$) at any optimal solution, since $B^L = B^H = 0$, $R^j = \alpha^j$ is feasible. Furthermore, all additional constraints in the programming problem $(P.s)$ are weak inequalities. Hence, the search for optimal separating contracts is in a compact region contained in the compact set $\{(c^H, c^L):\ B^j \in [0,1],\ R^j \in [0, \alpha^j],\ j = L, H\}$. The following lemma characterizes the properties of the unique optimal separating contract $c^s(r) = (c^H(r), c^L(r))$. The argument is standard and it is therefore omitted.

Lemma 1. *A contract $c^s(r)$ solving problem (P.s) is such that:*

$$p^H R^H(r) = p^L R^L(r) = r, \tag{9}$$

$$B^H(r) = 1, \tag{10}$$

$$B^L(r) = \frac{\bar{\alpha} - r}{\bar{\alpha} - \mu r}. \tag{11}$$

By the properties of the separating contract, it is readily verified that the proportion of H-projects is equal to:

$$\lambda^s(r) = \sigma R^L(r) B^L(r) = \sigma \frac{(\bar{\alpha} - r)r}{\bar{\alpha} - \mu r}.$$

The function $\lambda^s(r)$ is hump shaped, equal to zero at $r = 0$ and $r = \bar{\alpha}$ and possibly greater than or equal to 1 for some r. Then, there is a closed subinterval $I^s = [r_1^s, r_2^s] \subset (0, \bar{\alpha})$, possibly empty, such that $\lambda^s(r) = 1$, for $r \in I^s$, i.e., no optimal contract can be separating for $r \in I^s$ (or, equivalently, separating and pooling contracts are, for all practical matters, indistinguishable).

Now let $\pi^s(r) = \max\{\pi^L(c^s), \pi_1^H(c^s)\}$ for c^s in the constrained set defined by (CC), (IC1), (IC2), (ZPS). The above discussion implies that:

$$\pi^s(r) = \begin{cases} (\bar{\alpha} - r) - \bar{\alpha}e & for\ r \in I^s; \\ (\bar{\alpha} - r)^2/(\bar{\alpha} - \mu r) & for\ r \in [0, \bar{\alpha}] \setminus I^s, \end{cases}$$

Step 2: optimal pooling contracts. At a pooling contract, $\lambda^p(r) = \min\{1, \sigma B^p R^p\}$. When $\lambda^p(r) = 1$ the contract is defined by $R^p = r/p^H$, $B^p = 1$. Otherwise, it is a solution to:

$$(P.p')\ \max_c (\bar{\alpha} - p^L R)B \text{ s.t.: (CC), (ZPP), } \sigma B^p R^p \leq 1.$$

Once again, $B = 0$ is always feasible and all constraints in $(P.p')$ are weak inequalities. Hence, the search for optimal pooling contracts is in a compact region contained in the compact set $\{c^p:\ B^p \in [0,1],\ R^p \in [0, \alpha^H]\}$.

Lemma 2. *Let $I^p = \{r : \lambda^p(r) = 1\}$. Under Assumption 1, there is a unique optimal pooling contract $c^p(r)$ with the following characterization. If $I^p \neq \emptyset$, $I^p = [r_1^p, r_2^p] \subset (0, \bar{\alpha})$ and there is a value $\hat{r}$ and a continuous function $\lambda^p(r)$ such that:*

- *$\hat{r} = r_1^p$, if $I^p \neq \emptyset$, $\hat{r} \in (0, \bar{\alpha})$, otherwise;*
- *$\lambda^p(r) = 1$ for $r \in I^p$, $\lambda^p(r)$ strictly increasing in $[0, \hat{r})$, strictly decreasing in $(\hat{r}, \bar{\alpha}] \setminus I^p$ and $\lambda^p(0) = \lambda^p(\bar{\alpha}) = 0$;*
- *$B^p(r) = 1$, for $r \in [0, \hat{r}] \cup I^p$, $B^p(\bar{\alpha}) = 0$, $B^p(r)$ strictly decreasing in $[\hat{r}, \bar{\alpha}] \setminus I^p$ and $R^p(r) = (r/p^L)[1 - (1-\mu)G(\lambda^p(r))]^{-1}$ for $r \in [0, \bar{\alpha}]$.*

Proof. Since both the objective function and λ^p are monotonic in B, solutions in $\mathcal{C}^p(r)$ satisfy (ZPP) with equality. Hence:

$$p^L R^p(r)[1 - (1-\mu)G(\lambda^p(r))] = r. \tag{12}$$

Now define $T(\lambda) = \bar{\alpha}\,(1 - (1-\mu)G(\lambda))\,\lambda$. By Equation 12 and the definition of λ, problem $(P.p')$ can be restated as

$$(P.p^o)\ \max\nolimits_{\lambda \in [0,1]} Q(\lambda, r) \text{ s.t.: } T(\lambda) \leq \sigma r \bar{\alpha}/p^L,$$

where $Q(\lambda, r) = T(\lambda) - r\lambda$.

Let $\Lambda^p(r)$ be the set of solutions to problem $(P.p^o)$. Notice that $Q(0, r) = 0$, $T(0) < \sigma r$ for $r > 0$ and $\partial Q(0, r)/\partial\lambda = \bar{\alpha} - r$, $\partial T(0)/\partial\lambda = p^L\bar{\alpha} > 0$. Then, $\lambda > 0$ for all $\lambda \in \Lambda^p(r)$ with $r \in (0, \bar{\alpha})$ and $\Lambda^p(\bar{\alpha}) = \Lambda^p(0) = \{0\}$. Furthermore, by the maximum theorem, the continuity of $T(.)$ and $Q(.)$ and the compactness of the constrained set imply that $\Lambda^p(r)$ is an upper hemi continuous correspondence.

Now we show that $\Lambda^p(r) = \{\lambda^p(r)\}$, where $\lambda^p(r)$ is a function. Since all $\lambda \in \Lambda^p(r)$ are positive, $T'(\lambda) \geq r$, i.e., $T'(\lambda) > 0$ for $\lambda \in \Lambda^p(r)$. Assumption 1 implies that $Q(.)$ and $T(.)$ are strictly concave in λ. Hence, the constraint set in $(P.p^o)$ is the union of two disjoint intervals, $I_1 = [0, \xi_1(r)]$, $I_2 = [\xi_2(r), 1]$, where I_2 is possibly empty, $\xi_1(r) \in (0, 1]$ and $\xi_1(r) < \xi_2(r)$. Since $T'(0) > 0$, $T'(\xi_1(r)) > 0$, $T'(\xi_2(r)) < 0$ and $T'(\lambda) > 0$ for all $\lambda \in \Lambda^p(r)$, must be $\Lambda^p(r) \subset I_1$. Hence, the strict concavity of $Q(.)$ and the convexity of I_1 imply that $\Lambda^p(r) = \{\lambda^p(r)\}$ is a continuous function.

By the implicit function theorem, $\xi_1(r)$ is an increasing function of r. By the strict concavity of the objective function in $(P.p^o)$, there exists a unique $\eta(r)$ such that $\partial Q(\eta(r), r)/\partial\lambda = 0$. Then, $\lambda^p(r) = \min\{\eta(r), \xi_1(r)\}$. Since $\xi_1(0) = 0$, $\eta(0) > 0$, $\eta(\bar{\alpha}) = 0$, $\xi_1(r)$ is increasing, while $\eta(r)$ is decreasing. Then, there is a subinterval $I^p = [\rho_1^p, \rho_2^p] \subset (0, \bar{\alpha})$, possibly empty, and a value $\hat{r} \in (0, \bar{\alpha})$, with $\hat{r} = r_1^p$, when $I^p \neq \emptyset$, such that:

$$\lambda^p(r) = \begin{cases} \xi_1(r) & \text{for } r \in [0, \hat{r}] \setminus I^p, \\ 1 & \text{for } r \in I^p, \\ \eta(r) & \text{for } r \in [\hat{r}, \bar{\alpha}] \setminus I^p. \end{cases}$$

Hence, the unique contract $\bar{c}(r)$ solving problem $(P.p^o)$ satisfies:

$$\sigma\bar{B}(r)\bar{R}(r) = \lambda^p(r), \qquad \bar{R}(r) = \frac{r}{p^L[1 - (1-\mu)G(\lambda^p(r))]}.$$

Since the Planner's objective is to maximize $\max\{\pi^L(c), \pi_1^H(c)\}$, we know that an optimal contract is characterized by $R = r/p^H$ and $B = 1$ when $\lambda = 1$. Hence, the unique optimal pooling contract $c^p(r)$ satisfies $c^p(r) = \bar{c}(r)$ for $r \in [0, \bar{\alpha}] \setminus I^p$, $c^p(r) = (1, r/p^H)$ for $r \in I^p$. □

Lemma 3. $\pi^p(r) = \max\{\pi^L(c^p(r)), \pi_1^H(c^p(r))\}$ *is strictly decreasing in* $[0, \bar{\alpha}]$ *and such that* $\partial^2\pi^p(r)/\partial r^2 < 0$*, for* $r \in (0, \hat{r})$ *and, if* $I^p \neq \emptyset$*,* $\partial^2\pi^p(r)/\partial r^2 = 0$ *for* $r \in (r_1^p, r_2^p)$*.*

Proof. Consider $r \in [\hat{r}, \bar{\alpha}) \setminus I^p$. By the envelope theorem:

$$\frac{\partial \pi^p(r)}{\partial r} = \frac{\partial Q(\lambda^p(r), r)}{\partial r} = -\frac{p^L\bar{\alpha}}{r^2}\left(1 - (1-\mu)G(\lambda^p(r))\right)\lambda^p(r) < 0.$$

For $r \in (0, \hat{r})$, $T(\lambda^p(r)) = \bar{\alpha}\sigma r/p^L$ and $\pi^p(r) = 1 - (p^L\lambda^p(r)/\sigma)$. Then:

$$\partial\pi^p/\partial r = -(p^L/\sigma)\partial\lambda^p(r)/\partial r, \qquad \partial^2\pi^p/\partial r^2 = -(p^L/\sigma)\partial^2\lambda^p(r)/\partial r^2,$$

where the first and second derivatives of $\lambda^p(r)$ are obtained by total differentiation of the equation $T(\lambda^p(r)) = \sigma r$, i.e.:

$$\begin{aligned}\partial\lambda^p(r)/\partial r &= (\sigma/p^L)[1 - (1-\mu)H(\lambda^p(r)],\\ \partial^2\lambda^p(r)/\partial r^2 &= \frac{\sigma(1-\mu)H'(\lambda^p(r))}{p^L[1-(1-\mu)H(\lambda^p(r))]^2}\partial\lambda^p(r)/\partial r,\end{aligned}$$

where $H(\lambda) = G(\lambda) + \lambda g(\lambda)$.

Lemma 2 implies that $\partial\lambda^p(r)/\partial r > 0$, and, therefore, $\partial\pi^p/\partial r < 0$, for $r \in (0, \hat{r})$. Furthermore, for $r \in [r_1^p, r_2^p]$, the constraint $\lambda(r) \leq 1$ is binding and, hence,

$$Q(\lambda^p(r), r) = (\bar{\alpha}[1 - (1-\mu)G(\lambda^p(r))] - r)\,\lambda = \bar{\alpha}\mu - r. \qquad \square$$

Step 3: optimal contracts. We are now ready to characterize $\theta(r)$, thereby completing the study of the optimal contract $(c^s, c^p, \theta)(r)$. If $\pi^s(r) > \pi^p(r)$ ($\pi^s(r) < \pi^p(r)$), the optimal contract is separating (pooling), i.e., $\theta(r) = 1$ ($\theta(r) = 0$). If r is such that $\pi^s(r) = \pi^p(r)$, the contract $(c^s(r), c^p(r), \theta)$ is optimal for all $\theta \in [0, 1]$. By direct computations and by the previous lemma, $\pi^s(0) = \pi^p(0) = \bar{\alpha}$, $\pi^s(\bar{\alpha}) = \pi^p(\bar{\alpha}) = 0$ and, at $r = 0$, $\partial\pi^s/\partial r = \mu - 2 < -1 = \partial\pi^p/\partial r$. Hence, there exists $r^* \in (0, \bar{\alpha}]$ such that $\pi^p(r) > \pi^s(r)$ for $r \in (0, r^*)$. This completes the proof of Proposition 1. □

We have now all the elements needed to characterize the properties of the set of switching points, S^*. We call a subset $A \subset \mathbb{R}_{++}$ generic if it is both open and of full Lebesgue measure. The proof of the next Lemma is a trivial application of the joint trasversality theorem.

Lemma 4. *For* e *in a generic subset of* $\mathbb{R}_{++}$*, the set* S^* *of switching point is either empty or finite. For each* $r_i^* \in S^*$*,* $i = 0, 1, ..., n$*, either* $\pi^p(r) > \pi^s(r)$ *for all* r *in a left neighborhood of* r_i^* *and* $\pi^p(r) < \pi^s(r)$ *for all* r *in a right neighborhood of* r^**, or vice versa.*

Proof. First we show that the lemma is true for e in a full Lebesgue measure set of $\mathbb{R}_{++}$. Then, we prove that this set is open as well.

Consider the function $f(r,e) = \pi^s(r,e) - \pi^p(r,e)$, for $e > 0$. For each $e > 0$, $f(\bar{\alpha},e) = f(0,e) = 0$. From Lemma 3 , $\partial f(0,e)/\partial r < 0$, and, hence, $r = 0$ is a locally isolated zero of f, for all $e > 0$. Furthermore, from the proof of Proposition 2, $\partial f(\bar{\alpha},e)/\partial r = 0$, $\partial^2 \pi^s(\bar{\alpha},e)/\partial r^2 = -2/\bar{\alpha}(1-\mu)$ and $\partial^2 \pi^p(\bar{\alpha},e)/\partial r^2 = -\sigma/\bar{\alpha}$. Thus, for $e \in K = \{e > 0 : \sigma \neq 2/(1-\mu)\}$, $\partial^2 f(\bar{\alpha},e)/\partial r \neq 0$. Hence, for each $e \in K$, $r = \bar{\alpha}$ is a locally isolated zero of $\partial f/\partial r$, and, since $f(\bar{\alpha},e) = 0$, $\bar{\alpha}$ a locally isolated zero of f (just take a second order Taylor expansion in a neighborhood of $\bar{\alpha}$). Now, consider the map $f : (0,1) \times K \to \mathbb{R}$. Bear in mind that we are restricting the domain of the map f to not include the points $r = \bar{\alpha}$ and $r = 0$. Since, $\partial \pi^s/\partial e = 0$, while $\partial \pi^p/\partial e < 0$, the map f is transversal to zero. Let $f_e(r)$ denote the map f for given value of e. By the joint trasversality theorem, there exists a full Lebesgue measure subset of K, K^*, such that either $f_e^{-1}(0) = \emptyset$ or, for $r \in f_e^{-1}(0)$, $\partial f(r,e)/\partial r \neq 0$, for $e \in K^*$. Thus, by the implicit function theorem, the solutions r_i^* are locally isolated. If they are not finite, they must have either $r = 0$ or $r = \bar{\alpha}$ as accumulation points. However, this is excluded by the fact that both of them are locally isolated solutions of $f(r,e) = 0$. Hence, $S^*(e)$ is a finite set for all $e \in K^*$. Furthermore, since, if $r \in f_e^{-1}(0) \neq 0$, $\partial f(r,e)/\partial r \neq 0$, for $e \in K^*$, the second part of the lemma holds true.

In order to show that K^* is open, pick a point $\hat{e} \in K^*$. If $f_{\hat{e}}^{-1}(0) = \emptyset$, by the continuity of f and by the fact that both 0 and $\bar{\alpha}$ are locally isolated zeros, $f_e^{-1}(0) = \emptyset$, for e in an open neighborhood of $\hat{e}$. If, on the other hand, $f_{\hat{e}}^{-1}(0) \neq 0$, the implicit function theorem immediately implies that f_e is transversal to $zero$, for e in some open neighborhood of $\hat{e}$. Hence, K^* is open and of full Lebesgue measure. □

In the analysis that follows, the set up cost is assumed to be in the generic set of Lemma 4. Thus, S^* is a finite collection of points, $\{r_1^*, ..., r_n^*\}$ and, by convention, $r_i^* < r_{i+1}^*$.

Proof of Proposition 2

From the proof of Proposition 1, the Planner's objective function with $\theta = 1$ is:

$$\pi^s(r) = (\bar{\alpha} - r)^2/(\bar{\alpha} - \mu r),$$

whereas the Planner's objective function with $\theta = 0$ is:

$$\begin{aligned} \pi^p(r) &= \bar{\alpha} - (p^L/\sigma)\lambda^p(r) && \text{if } r \leq r^o \in (0,\bar{\alpha}) \\ \pi^p(r) &= e(\bar{\alpha})^2 \lambda^p(r)^2 g(\lambda^p(r))/r && \text{if } r > r^o, \end{aligned}$$

where $r^o \in (0,\bar{\alpha})$ is such that $B^p(r) = 1$, for $r \in [0,r^o]$, while $B^p(r) < 1$, for $r \in (r^o,\bar{\alpha}]$. Equivalently, the capacity constraint $T(\lambda) \leq \bar{\alpha}(\sigma/q)r/p^L$ is binding for $r \in [0,r^o)$, while it is not binding, for $r \in [r^o,\bar{\alpha}]$. Hence, $\lambda^p(r)$ is decreasing in e for $r < r^o$ and independent of e otherwise. Then, $\pi^p(r)$ is increasing in e and

$\pi^s(r)$ is independent of e. Since at r_1^* we have $\pi^p(r_1^*) = \pi^s(r_1^*)$, $\pi^p(r) > \pi^s(r)$ for $r < r_1^*$, r_1^* is an increasing function of e.

Now assume that $r_1^* \to r^* < \bar{\alpha}$ as $e \to \infty$. Notice that, for all r, there is a big enough value $e(r)$ such that (CC) becomes a binding constraint. Hence, for all $e \geq e(r^*)$, must be $(\bar{\alpha} - r)^2/(\bar{\alpha} - \mu r) > \bar{\alpha} - (p^L/\sigma)\lambda^p(r)$ for all $r \in (r^*, \bar{\alpha})$. However, $e \to \infty$ implies $\lambda^p(r) \to 0$. Hence, for e big enough and $r \in (r^*, \bar{\alpha})$, $\bar{\alpha} > (\bar{\alpha} - r)^2/(\bar{\alpha} - \mu r) > \bar{\alpha}$, which is a contradiction. □

Proof of Proposition 3

Recall that in the proof of Proposition 1 I^p and I^s have been defined as the subintervals of $[0, \bar{\alpha}]$ such that $\lambda^p(r) = \lambda^s(r) = 1$. For $r \in I^s$, the optimal contract is pooling and, for $r \notin I^s$, $\pi^s(r) = (\bar{\alpha} - r)/(\bar{\alpha} - \mu r)$. Hence, we just need to compare this expression with $\pi^p(r)$. Notice that $\pi^p(r)$ is concave in $[0, \hat{r}] \cup I^p$, while $\pi^s(r)$ is strictly convex in $[0, \bar{\alpha}]$. Furthermore, $\pi^s(0) = \pi^p(0) = \bar{\alpha}$ and $\partial\pi^s/\partial r < \partial\pi^p/\partial r$ at $r = 0$. Hence, there is at most one switching point in $r^* \in [0, \hat{\rho}] \cup I^p$. Finally, recall from the proof of Proposition 1 that $B^p(r) = 1$, for $r \in [0, \hat{r}] \cup I^p$ and, for $r \in (\hat{r}, \bar{\alpha}) \setminus I^p$, $B^p(r) < 1$ and:

$$G(\lambda^p(r)) + \lambda^p(r)g(\lambda^p(r)) = (\bar{\alpha} - r)/(1 - \mu)\bar{\alpha}.$$

Now let

$$A(r) = \frac{p^L\lambda[\bar{\alpha} - r - \bar{\alpha}(1 - \mu)G(\lambda)]}{(\sigma)(\bar{\alpha} - r)^2}, \qquad N(r) = \frac{r}{\bar{\alpha} - \mu r},$$

and observe that, for $r \in (\hat{r}, \bar{\alpha}) \setminus \{I^p \cup I^s\}$, $N(r) \geq A(r) \Leftrightarrow \pi^s(r) \geq \pi^p(r)$. It is $N'(r) > 0$ and $A'(r) \leq 0 \Leftrightarrow sg(s)/G(s) \leq 1$. Hence, if there is a switching point $r_2^* \in (\hat{r}, \bar{\alpha}) \setminus I^p$, $A(r_2^*) = N(r_2^*)$ and, then, $\pi^s(r) > \pi^p(r)$, for $r \in (r_2^*, \bar{\alpha})$, while $\pi^s(r) < \pi^p(r)$ for $r \in \{(\hat{r}, \bar{\alpha}) \setminus I^p\} \cap (0, r_2^*)$. However, $\pi^s(r) < \pi^p(r)$ for $r \in (\hat{r}, \bar{\alpha}) \setminus I^p \cap (0, r_2^*)$ if and only if it does not exists a switching point in $(0, \hat{r}] \cup I^p$, otherwise the inequality is reversed. Hence, the thesis. □

Proof of Proposition 6

By Proposition 1, $B^p(r) = 1$, for $r \in [0, \hat{r}] \cup I^p$, while $\partial B^p(r)/\partial r < 0$, for $r \in (\hat{r}, \bar{\alpha}] \setminus I^p$. Hence, $r^p : [0, 1) \to [0, \bar{\alpha}]$ is a decreasing function. $B^s(r)$ is continuous with $B^s(0) = 1$, $B^s(\bar{\alpha}) = 0$. Hence, for any given $w \in [0, 1]$, there $r^s(w)$ is non empty. However, $B^s(r)$ is not monotonic and, hence, $r^s(w)$ may be non unique.

If $S^* = \emptyset$, the optimal contracts are pooling for any $r \in [0, \bar{\alpha}]$. Hence, since $B^p([0, \bar{\alpha}]) = [0, 1]$, if $S^* = \emptyset$, there exists a market clearing equilibrium for each $r \in [0, \bar{\alpha}]$. Now suppose that there are n switching points and let $r_0^* = 0$, $r_{n+1}^* = \bar{\alpha}$. The closed interval $\mathcal{I}_i = [r_i^*, r_{i+1}^*]$ is called pooling (separating) if the pooling (separating) contract is optimal for $r \in \mathcal{I}_i$. By Proposition 1, $\mathcal{I}_0$ is pooling and, by the definition of switching points, $\mathcal{I}_i$ is pooling, if i is even, and separating, if i is odd. Let w_i^j $(j = p, s)$ be defined by $r_i^* = r^p(w_i^p)$ and $r_i^* \in r^s(w_i^s)$. The following lemma shows that the relative magnitude of these points can be evaluated.

Lemma 5. $w_i^p > w_i^s$.

Proof. By construction, at a separating contract $\lambda^s(r) < 1$. Therefore, since $r_i^* \in (0,1)$, $w_i^s < 1$. Suppose that $w_i^p < 1$ (otherwise, there is nothing to prove). By the definitions of π^s and π^p given in Section 4 and since we are assuming $w_i^j < 1$, ($j = s, p$), we are only considering switching points r_i^* for which $\lambda^s(r_i^*)$ and $\lambda^p(r_i^*)$ are both less than 1. Then, omitting the index i, $\pi^s(r^*) = B^L(r^*)(\bar{\alpha} - r^*)$, $\pi^p(r^*) = \bar{\alpha}w^p - p^L\lambda^p(r^*)/\sigma$. It follows that $w^p = B^L(r^*)(1 - r^*/\bar{\alpha}) + p^L\lambda^p(r^*)/\sigma\bar{\alpha}$ and:

$$w^s = B^L(r^*) + G(\lambda^s(r^*))(1 - B^L(r^*)).$$

Using the above expressions along with the definitions of $\lambda^s(r)$ and $\lambda^p(r)$ we get:

$$w^p - w^s = \frac{e\bar{\alpha}}{\bar{\alpha} - r^*}[\lambda^p(r^*)G(\lambda^p(r^*)) - \lambda^s(r^*)G(\lambda^s(r^*))].$$

Since $\lambda^p(r^*) > \lambda^s(r^*)$, we have $w^p - w^s > 0$. □

By the continuity of $B^p(r)$ and $B^s(r)$, both $B^p(\mathcal{I}_i)$, i odd, and $B^s(\mathcal{I}_i)$, i even, are intervals contained in $[0,1]$. However, by the last claim, $w_i^p > w_i^s$, and, hence, there might not exists deterministic contracts that clear the market for $w \in (w_i^s, w_i^p)$. If, for some i, this is the case, consider an optimal random contract offer $(c^s, c^p, \theta)(r_i^*)$. For this contract offer, the market clearing condition is:

$$\vartheta B^s(r_i^*) + (1 - \vartheta)B^p(r_i^*) = w,$$

which can be rewritten as $\vartheta w_i^s + (1 - \vartheta)w_i^p = w$. Hence, for $w \in (w_i^s, w_i^p)$ and for:

$$\vartheta(w) = \frac{w_i^p - w}{w_i^p - w_i^s} \in [0,1],$$

$(c^s, c^p, \vartheta(w))(r_i^*)$ clears the loan market. Since $B^p(0) = 1$ and $B^p(\bar{\alpha}) = B^s(\bar{\alpha}) = 0$, the proof is complete. □

Proof of Proposition 7

In the previous section we have shown that

$$\lambda^s(r) = \frac{(p^L - p^H)}{\bar{\alpha}e}R^L B^L, \qquad B^L = \frac{\bar{\alpha} - p^L R^L}{\bar{\alpha} - p^H R^L}, \qquad R^L = r/p^L.$$

By direct computations, $\partial\lambda^s(r')/\partial r < 0$ for $r > \tilde{r}(\mu) = \frac{\bar{\alpha}}{\mu}\left(1 - (1 - \mu)^{1/2}\right)$. Moreover, by Proposition 2, the first switching point r_1^* is a strictly increasing function of e. Hence, there exists $\hat{e}$ such that, for $e \geq \hat{e}$, $r_1^*(e) \geq \tilde{r}(\mu)$. By the definition of the map $B^s(.)$ and by the implicit function theorem, $\partial\lambda^s(r')/\partial r < 0$ implies $\partial B^s(r')/\partial r < 0$. Then, since $A^s(e) \subset [r_1^*(e), \bar{\alpha}]$, $\partial B^s(r')/\partial r < 0$ for $r \in A^s(e)$, $e \geq \hat{e}$. The latter and the definition of market clearing imply the thesis. □

Proof of Proposition 8

If $S^* = \emptyset$, the proposition is trivial. Hence, suppose that $S^* = \{r^*\}$. By Proposition 1, optimal contracts are pooling for $r \in [0, r^*)$ and separating for $r \in (r^*, \bar{\alpha})$. By Proposition 6, $w^p > w^s$, for $w^\kappa = B^\kappa(r^*)$ and $\kappa = s, p$. Hence, for $w \in [0, w^s) \cup (w^p, 1]$, the optimal contracts are deterministic and $r(w) = r^p(w)$ for $w \in (B^p(r^*), 1]$, $r(w) = r^s(w)$ for $w \in [0, B^s(r^*))$, while $r(w) = r^*$ for $w \in [B^s(r^*), B^p(r^*)]$

Consider $w \in (w^s, w^p)$. Contract $c_{\theta(w)} = (c^s(r^*), c^p(r^*), \theta_w)$ is optimal for all $\theta_w \in [0, 1]$ and it clears the credit market for $\vartheta_w = (w - w_1^p)/(w_1^p - w_1^s)$. Thus, it remains to show that there does not exists any other deterministic optimal contract clearing the market for $w \in (w^s, w^p)$. This follows immediately from the fact that $r^s(w) \cap A^s(e)$ is, under the stated assumption, unique and decreasing. Hence, $r^s(w) < r^*$ and $r^p(w) > r^*$ for $w \in (w^s, w^p)$. However, for $r < r^*$, optimal contracts are pooling, while, for $r > r^*$, they are separating. □

Proof of Proposition 9

By the assumptions, S^* is either empty or it contains a unique point r^*. If $\vartheta(w) = 0$, $\lambda^{\vartheta(w)}(r(w)) = \lambda^p(r^p(w))$ and, hence, it is an increasing function of w (just recall that $\lambda^p(r)$ is decreasing and $r^p(w)$ is decreasing in w). If $\vartheta(w) = 1$, $\lambda^{\vartheta(w)}(r(w)) = \lambda^s(r^s(w))$ where, by the assumption $e \geq \hat{e}$ and Proposition 7, $\lambda^s(r)$ is decreasing and $r^s(w)$ is decreasing. It follows that $\lambda^{\vartheta(w)}(r(w))$ is again an increasing function of w. Now let $\vartheta(w) \in (0, 1)$, $r(w) = r^*$, then, by the law of large number and Proposition 6, we have

$$\lambda = \lambda^{\vartheta(w)}(r(w)) = \lambda^p(r^*) - \vartheta(w)\left(\lambda^p(r^*) - \lambda^s(r^*)\right), \tag{13}$$

where $\vartheta(w)$ is the linearly decreasing function defined in Proposition 6.

Now we can show that, if $S^* \neq \emptyset$, $\lambda^p(r^*) > \lambda^s(r^*)$. In fact, by the definition of switching point, $r^* = p^L R^p(r^*)[1 - (1 - \mu)G(\lambda^p(r^*))] = p^L R^L(r^*)$. Then, $R^p(r^*) > R^L(r^*)$. Using again the definition of a switching point

$$\pi^L(c^L(r^*)) = (\bar{\alpha} - r^*)B^L(r^*) = \pi^L(c^p(r^*) = (\bar{\alpha} - p^L R^p(r^*))B^p(r^*).$$

Hence, $R^p(r^*) > R^L(r^*)$ implies $B^p(r^*) > B^L(r^*)$ and the proposition follows. □

Proof of Proposition 10

First we show that the map Φ_e is increasing in $[0, 1]$. By direct computations, $m_e'(w) = e\lambda g(\lambda)(\partial\lambda/\partial w) < 1$. Hence, by Proposition 9, $\Phi_e(w)$ is increasing in the intervals $[0, B^s(r^*))$ and $(B^p(r^*), 1]$. To conclude this part of the argument we have to show that, at the switching point r^*,

$$\Phi_e(w^p) - \Phi_e(w^s) = w^p - w^s - (m_e(w^p) - m_e(w^s)) > 0.$$

As we have shown in the proof of Proposition 6, at the switching point, it is

$$w^p - w^s = \frac{e\bar{\alpha}}{\bar{\alpha} - r^*}[\lambda^p(r^*)G(\lambda^p(r^*)) - \lambda^s(r^*)G(\lambda^s(r^*))] > 0.$$

Furthermore, integrating by parts (and taking into account that $\lambda^p(r^*) > \lambda^s(r^*)$,

$$m_e(w^p) - m_e(w^s) = \int_{\lambda^s(r_1^*)}^{\lambda^p(r_1^*)} esdG(s) =$$

$$= e(\lambda^p(r^*)G(\lambda^p(r^*)) - \lambda^s(r^*)G(\lambda^s(r^*)) - \int_{\lambda^s(r_1^*)}^{\lambda^p(r_1^*)} eGds.$$

Hence,

$$\Phi_e(w^p) - \Phi_e(w^s) \geq \frac{er^*}{\bar{\alpha} - r^*}[\lambda^p(r^*)G(\lambda^p(r^*)) - \lambda^s(r^*)G(\lambda^s(r^*))] > 0.$$

Since at both $w = 0$ and $w = 1$, $m_e(w) = m_0(w)$, the Assumption 6 guarantees the existence of a fixed point of Φ_e, for $e \geq 0$. This assumption also shows that $\Phi_e^{'}(w_e^*) < 1$ for at least one fixed point w_e^*. Finally, $w_e^* < w_0^*$ for $e > 0$ follows from $\Phi_0(w) > \Phi_e(w)$ for $e > 0$. □

Proof of Proposition 12

It is sufficient to analyze the case $n = 2$. It is immediate from the argument, that this is without loss of generality. Since the case in which $r^s(w)$ is not a singleton for $r \in A^s$ can only make multiple equilibria and discontinuous selections more likely, we will also assume $e \geq \hat{e}$. By Proposition 7, this assumption guarantees that $r^s(w) \cap A^s$ is a decreasing function. Finally, to save notation, we just use $r^s(w)$ to denote $r^s(w) \cap A^s$.

For $n = 2$, $[0, 1]$ is partitioned in three intervals, $\mathcal{I}_j = [r_j^*, r_{j+1}^*]$, $j = 0, 1, 2$, with $r_0^* = 0$ and $r_3^* = \bar{\alpha}$. Furthermore, $A = \mathcal{I}_0 \cup \mathcal{I}_2$ and $A^s = \mathcal{I}_1$. By Lemma 5 and Proposition 7, $w_i^p > w_i^s$ for all i and both w_i^p and w_i^s are decreasing in i. Hence, either (a) $w_1^s \in (w_2^p, w_1^p)$ or (b) $w_1^s \leq w_2^p$.

To prove (i), observe that, if (a) holds, for $w \in (w_2^s, w_2^p)$ the contracts

$$\begin{aligned} &C(r^s(w)) \text{ with } \theta(r^s(w)) = 1, \\ &C(r^p(w)) \text{ with } \theta(r^p(w)) = 0, \\ &C(r_2^*) \qquad \text{with } \theta(r_2^*) = \vartheta_i^*(w), \end{aligned}$$

are all optimal and they all satisfy the market clearing requirement.

To prove (ii) we argue by contradiction. Evidently, if there exists a continuous selection for all $w \in [0, 1]$, it must be

$$C(r) = \begin{cases} (c^p(r^p(w)), c^s(r^p(w)), 0) & \text{for } w \in [w_1^p, 1], \\ (c^s(r_1^*), c^p(r_1^*), \vartheta_1^*(w)) & \text{for } w \in (w_1^s, w_1^p), \\ (c^s(r^s(w)), c^p(r^s(w)), 1) & \text{for } w \in [w_2^s, w_1^s]. \end{cases}$$

However, since $\mathcal{I}_2$ is pooling, the only possible contract that we can select for $w \in [0, w_2^s)$ is $C(r) = (c^s(r^p(w)), c^p(\rho^p(w)), 1)$. Hence, for such a selection, there is a discontinuous regime switch at w_2^s. □

Proof of Proposition 13

By the market clearing conditions:

$$B^p(r^p(\bar{w})) = (1 - G(\lambda^s(r^s(\bar{w}))) B^L(r^s(\bar{w}))) + G(\lambda^s(r^s(\bar{w})) = \bar{w}.$$

Hence, $B^p(r^p(\bar{w}))$ is a convex combination of $B^L(r^s(\bar{w}))$ and 1. Therefore,

$$B^p(r^p(\bar{w})) < 1 \qquad \Rightarrow \qquad B^p(r^p(\bar{w})) > B^L(r^s(\bar{w})).$$

Furthermore, since $\bar{w} \in [0, w_2^p) \cap (w_2^s, w_1^s)$ and $r^p(w)$ is strictly decreasing in $(0,1)$, it follows that $r^p(\bar{w}) > r_2^*, r^s(\bar{w}) \in [r_1^*, r_2^*]$. Hence, $r^p(\bar{w}) > r^s(\bar{w})$. By the definition of $R^p(.)$ and by the inequalities $p^L > p^H G(\lambda^p(.)) + p^L(1 - G(\lambda^p(.))$ and $r^p(\bar{w}) > r^s(\bar{w})$, we have $R^p(r(\bar{w})) > r^p(\bar{w})/p^L > r^s(\bar{w})/p^L = R^L(r(\bar{w}))$. Then, the definitions of λ^p and λ^s imply the thesis. □

Appendix 2: Borrowers' endowment

In this appendix we show that the optimal contracts derived in our model would not change under the assumption that entrepreneurs have some endowment and that lenders were allowed to use this endowment to secure loans.

Suppose that each borrower $s \in [0,1]$ is endowed with a positive amount E of the final good. Also, for logical consistency with our model, E can only be used as an input in the production of materials. All other assumptions defining technologies and borrower types are maintained. We now characterize contracts allowing for part of the borrowers' endowment to be used to "secure" the loan. More specifically, we consider a class of contracts defined by the array (B, W, T, V). Each of the components of the contract is defined as follows:

- $W \in [0, E]$ is the amount of the borrowers' endowment invested in a secure deposit yielding the opportunity cost of borrowing r.
- $B \in [0, 1 - E - W]$ is the loan size.
- T and V are, respectively, the payment by the borrower to the lender when the project succeeds and the payment by the lender to the borrower when the project fails. Since failure makes the borrower penniless, limited liability implies $V \geq 0$.

For convenience, we define $D = B - W$ and, from now on, a contract is characterized with an array $c = (D, W, T, V)$. A borrower with contract c invest $E + D$ in one of the two projects. When the project succeeds, the borrower receives $\alpha^j(E+D) + rW - T$ and the lender receives T. When the project fails, the borrower receives V and the lender receives $rW - V$. Evidently, by setting $W = 0, T = RB$ and $V = 0$, we are back to the loan contracts defined in Section 4. We start the analysis by studying a separating contract $c^s = \{(B^j, W^j, T^j, V^j),\ j = H, L\}$. The borrowers' expected profits from project $j = H, L$ and contract $i = H, L$ is:

$$\pi_s^j(c^i) = \bar{\alpha}(E + B^i) + p^j(rW^i - T^i) + (1 - p^j)V^i - \bar{\alpha}e^j s,$$

and the lenders' non negative profit conditions now read:

$$p^j T^j + (1 - p^j)(rW^j - V^j) \geq rB^j \qquad (j = H, L),$$

Then, the optimal separating contract maximizes $\pi^L(c^L)$ subject to the following constraints:

$$\begin{aligned}
&E + B^j \leq 1, && j = H, L, \text{ (CC)}\\
&\pi_s^H(c^H) \geq \pi_s^H(c^L), && \text{(IC1)}\\
&\pi^L(c^L) \geq \pi^L(c^H), && \text{(IC2)}\\
&V^j \geq 0, && j = H, L, \text{ (LL)}\\
&p^j T^j \geq rB^j + p^j rW^j + (1 - p^j)V^j, && j = H, L, \text{ (ZPS)}
\end{aligned}$$

where constraints (IC1) and (IC2) are s-invariant and take the following form:

$$\bar{\alpha}B^j + p^j(rW^j - T^j) + (1 - p^j)V^j \geq \bar{\alpha}B^i + p^j(rW^i - T^i) + (1 - p^j)V^i,$$

for $j \neq i$ and $i, j = H, L$.

It can be easily show that, at optimality, the lenders' participation constraint (ZPS) is binding. Hence, an optimal separating contract must solve the following programming problem:

$$\begin{aligned}
&\max\ \bar{\alpha}E + (\bar{\alpha} - r)B^L \text{ s.t.:}\\
&E + B^j \quad \leq \quad 1, && \text{(CC)}\\
&(\bar{\alpha} - r)B^H \quad \geq \quad (\bar{\alpha} - \mu r)B^L + (1 - \mu)V^L && \text{(IC1)}\\
&(\bar{\alpha} - r)B^L \quad \geq \quad (\bar{\alpha} - (1/\mu)r)B^H - (1/\mu - 1)V^H, && \text{(IC2)}
\end{aligned}$$

where $j = H, L$. Using the standard arguments, it can be shown that (IC1) must be binding, $B^H = 1 - E$ and $V^L = 0$. Hence,

$$B^L(r) = \frac{\bar{\alpha} - r}{\bar{\alpha} - \mu r}(1 - E), \qquad \lambda^s(r) = \frac{(1 - \mu)r(\bar{\alpha} - r)}{\bar{\alpha} - \mu r}(1 - E).$$

Since the value of W (secured deposit) is unspecified, we can set $W = 0$ and claim that the loan sizes (B^L, B^H) and the proportion of risky projects λ are the same as with $E = 0$, except for the multiplicative factor $(1-E)$. In particular, letting $\lambda^s(r, E)$ be the proportion of risky projects with an optimal separating contract, we get:

$$\lambda^s(r, E) = \lambda^s(r)(1 - E).$$

Consider now the pooling case, $c^p = (D, W, T, V)$. The profits to the borrowers are

$$\pi_s^j(c^p) = \bar{\alpha}(E + D) + p^j(rW - T) + (1 - p^j)V - \bar{\alpha}e^j s$$

The lenders' participation constraint now is:

$$(T{+}V - rW)\left(p^L(1{-}G(\lambda)){+}p^H G(\lambda)\right) + p^L(1 - G(\lambda)){+}p^H G(\lambda) \geq rD{+}V.$$

Hence, $\lambda = \min\{1, \sigma(T + V - rW)\}$.

When $\sigma(T{+}V{-}rW) \geq 1$, the contractual problem reduces to the maximization of $\pi_1^H(c^p)$ subject to the capacity and the lenders' participation constraint. Since the latter must be binding and $\lambda = 1$, we get: $p^H(rW - T) + (1 - p^H)V = -rD$. By plugging this equation into $\pi_1^H(c^p)$, we get $\pi_1^H(c^p) = \bar{\alpha}E + (\bar{\alpha} - r)D - \bar{\alpha}e$. In this case, we have to maximize π_1^H subject to the capacity constraint $E + D \leq 1$ and the limited liability constraint $V \geq 0$. Evidently, the optimal pooling contract has $D = 1 - E$, whereas the values of the remaining components T, V and W are irrelevant. By setting $V = W = 0$, the lenders' participation constraint becomes $p^H T = r(1 - E) = rD = rB$, i.e., we are back to the optimal pooling contract described in the text, except that the loan size is now equal to $1 - E$.

Now consider the case $\sigma(T{+}V{-}rW) < 1$. In this case, $\lambda = \sigma(T{+}V{-}rW)$. Manipulating the objective function and the relevant constraints, the search for an optimal pooling contract can be reduced to the search of a solution to the following problem:

$$\max_{\{\lambda,V\}} T(\lambda){-}r\lambda{-}(\sigma/p^L)(\bar{\alpha}{-}r)V \text{ s.to: } T(\lambda) \leq (\sigma\bar{\alpha})/p^L)[r(1{-}E){+}V],$$
$$T(\lambda) \leq (\sigma\bar{\alpha})/p^L)[r(1{-}E){+}V], \qquad \text{(CC")}$$

where $T(\lambda) = \bar{\alpha}[1{-}(1{-}\mu)G(\lambda)]\lambda$ (as defined in Appendix 1). The key observation is that any solution of the above problem has $V = 0$. This is trivial when (CC") is not binding, since the objective function is non increasing in V, for $r \leq \bar{\alpha}$. Now assume that (CC") is binding and define λ_V as the value of λ satisfying (CC") with equality. Then,

$$\pi^L(c^p) = (\sigma\bar{\alpha}/p^L)r(1 - E) + (\sigma r/p^L)V - r\lambda_V,$$
$$\partial\lambda_V/\partial V = (\sigma/p^L)\left(1 - (1 - \mu)H(\lambda_V)\right)^{-1},$$

where $H(\lambda) = G(\lambda) + \lambda g(\lambda)$. Notice that $(1 - \mu)H(\lambda_V) < 1$ because of the second order conditions. It follows that

$$\partial\pi^L(c^p)/\partial V = -\frac{\sigma r}{p^L}\frac{(1 - \mu)H(\lambda_V)}{1 - (1 - \mu)H(\lambda_V)} < 0.$$

Hence, we can set $V = 0$ and restate the optimality problem as:

$$\max_{\lambda} T(\lambda) - r\lambda \qquad \text{s.t.:} \qquad T(\lambda) \leq (\sigma r\bar{\alpha}/p^L)(1 - E).$$

The programming problem above is exactly problem $(P.p')$ (cf. Appendix 1) other than for the multiplicative factor $(1 - E)$ on the right hand side of equation (CC").

Here again, the value of W is irrelevant for the specification of the contract and we can set $W = 0$ with no loss of generality.

Thus, we can mimic the analysis already performed for the case $E = 0$. As before, we denote by $\xi_1(r, E)$ the smallest value of λ which makes the lender participation constraint binding. Evidently, $\xi_1(r, E) < \xi_1(r, 0) = \xi_1(r)$ and there exists a value $\hat{r}_E \in (0, \bar{\alpha})$ such that $B^p(r, E) = 1 - E$, and $\lambda^p(r, E) = \xi_1(r, E)$, for $r \leq \hat{r}(E)$, while

$$(\lambda^p(r, E), B^p(r, E)) = (\lambda^p(r, 0), B^p(r, 0)) = (\lambda^p(r), B^p(r),$$

for $r \geq \hat{r}(E)$. Moreover, $\hat{r}(1) = \bar{\alpha}$, $r(0) = \hat{r}$ (as defined in Appendix 1), $\hat{r}(E) \in (\hat{r}(0), \hat{r}(1))$ for $E \in (0, 1)$ and $\hat{r}(E)$ is increasing in E. The next proposition clarifies the relation between the profit of the pooling contracts with and without entrepreneurs endowments.

Proposition 14. $\pi^P(r, E) > \bar{\alpha}E + (1 - E)\pi^P(r)$, *for* $E \in (0, 1)$ *and* $r \in (0, \bar{\alpha})$

Proof. For $r \geq r(E)$, profits are

$$\pi^p(r, E) = \bar{\alpha}E + \pi^p(r, 0) = \bar{\alpha}E + \pi^p(r).$$

For $r \leq r(E)$, $B^p(r, E) = 1 - E$ and, hence,

$$\pi^p(r, E) = \bar{\alpha}E + \bar{\alpha}(1 - E) - (p^L/\sigma)\xi_1(r, E).$$

Consider the interval $(0, \hat{r}] \subset (0, \hat{r}(E)]$. Then, since $\lambda^p(r) = \xi_1(r)$, $r \in (0, \hat{r})$, it suffices to show that $\xi_1(r)(1-E) > \xi_1(r, E)$, which is verified if $T(\lambda)(1-E) < T((1-E)\lambda)$. But this is equivalent to the condition $(1-E)(1-\mu)\lambda[G((1-E)\lambda) - G(\lambda)] < 0$, which is surely verified.

For $r \in [\hat{r}, \hat{r}(E)]$, it suffices to observe that $T(\eta(r)) > T(\xi_1(r, E)) = (1 - E)(\sigma r\bar{\alpha}/p^L)$. If $\eta(r)(1 - E) \leq \xi_1(r, E)$, $\bar{\lambda} = \eta(r)(1 - E)$ is feasible and

$$T(\bar{\lambda}) - r\bar{\lambda} > (1 - E)[T(\eta(r)) - \eta(r)r].$$

Otherwise, it follows from $B^p(r, E) = (1-E) > (1-E)B^p(r)$ and $\eta(r)(1-E) > \xi_1(r, E)$. □

For separating contracts, $\pi^s(r, E) = \bar{\alpha}E + (1 - E)\pi^s(r)$, for $r \in [0, \bar{\alpha}]$. By the last proposition, $(\pi^p(r, E) - \pi^s(r, E))/(1 - E) > (\pi^p(r) - \pi^s(r))$. Hence, we can state the following

Proposition 15. *Let* $U(E)$ *be the collection of sub-intervals of* $[0, \bar{\alpha}]$ *for which optimal contracts are pooling. Then,* $U(E) \neq \emptyset$ *for all* $E \in [0, 1]$, $U(1) = [0, \bar{\alpha}]$ *and* $U(E) \supset U(E')$ *whenever* $E > E'$.

For the uniform distribution $G(s) = s$, optimal contracts are always pooling for $E \geq 1 - e/4(1 - \mu)$.

The analysis in this appendix shows that the introduction of borrowers' endowment $E \in (0, 1)$ does not change the basic properties of the contracts studied for the case $E = 0$. Chan and Thakor [8] study the optimal contracts between borrowers and lenders in a model with both moral hazard and adverse selection, risk neutrality (of both borrowers and lenders) and unconstrained access to a collateral good. The authors prove that, when lenders maximize the borrowers' rent, the optimal contract involves no rationing of either type of borrowers and fully collateralized loans. Chan and Thakor study a partial equilibrium model where collateral is a good that cannot be used for investment. Applying this framework in our setting would require an additional storable good in the economy. In any case, for credit rationing to be absent in an optimal contract, it is crucial to assume that the borrowers' access to the collateral good is unconstrained. When firms have a limited amount of collateral and this amount is sufficiently low, credit rationing can still arise. This shows that the result obtained by Chan and Takor is not in contrast with our findings. In fact, we have rationing only if $E < 1$.

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