

Risk Disclosures with Asymmetric Information and Costly Investment in Information Technology^{*}

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^{*} We would like to thank Hakan Orbay, Timothy Van Zandt, and seminar participants at Universidad of Carlos III and SED 2000 for helpful comments.

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Abstract

Risk disclosures, including quantitative disclosures about variances, have increased as a result of recent regulations. However mandating such disclosure may require additional investments to obtain the required information about variances. Our research first characterizes the equilibrium strategies for voluntary risk disclosures with asymmetric information and then analyzes the impact of mandatory risk disclosures when this requires additional investment in information technology.

We initially consider a capital market with a safe bond and a single risky firm, where the risk-averse investors are uncertain about the variance of future cash flows and about whether the manager possesses private information about this variance. If the manager knows this variance, he may truthfully disclose the variance prior to selling the firm to the investors. We establish the existence of an equilibrium characterized by a disclosure threshold in which the managers, if informed, voluntarily discloses variances below a threshold (favorable news) but not variances above the threshold (unfavorable news). The market-clearing stock prices correctly incorporate the investors' beliefs that, for some exogenous reason, the manager may be uninformed and hence cannot disclose. We show that an increase in the investors' assessed likelihood of the manager being uninformed leads to a higher disclosure threshold and we provide conditions under which an increase in the expected variance causes a higher disclosure threshold.

We extend these results to a capital market with multiple firms whose future cash flows are affected by a common market-wide risk factor with known variance and an idiosyncratic risk factor with unknown variance. We assume each manager can disclose the variance of his firm's idiosyncratic risk. We find that the betas depend on the market-wide disclosures, even though the disclosure strategy of each manager may not.

Finally, we consider the setting where managers can acquire risk information through a costly investment in information technology. We show that in this case, cutoff levels of both the variance and the cost characterize the partial disclosure equilibrium, so that disclosure occurs if the variances and costs are both sufficiently low, but not otherwise. We also show that mandating disclosure may not increase the level of disclosure even for positive investment costs.

1 INTRODUCTION AND BACKGROUND

The accounting literature has a long-standing tradition for economics-based analyses of discretionary disclosures of non-proprietary information.¹ The “unraveling principle” argues that a privately informed seller may voluntarily disclose all non-proprietary information about his product (see Grossman, 1981 and Milgrom, 1981). The unraveling principle assumes that the seller and buyers interpret the information in the same way and that buyers know the seller possesses information that can be communicated costlessly and credibly. The seller in these models was naturally interpreted as a manager choosing a financial disclosure strategy in order to maximize firm value.

Subsequent models relax the underlying assumptions to analyze situations where the unraveling principle fails. For example, Dye (1985) introduces a probability that the manager is uninformed about the future value of a firm. He establishes that managers disclose favorable information and withhold unfavorable information. Further, any equilibrium is characterized by a disclosure threshold in future firm value such that the firm manager discloses if and only if future firm value exceeds this threshold. Jung and Kwon (1988) derive comparative static results on the disclosure threshold. Their analyses help us assess the effect of managers being able to credibly disclose future firm value.² Recent papers, including Newman and Sansing (1993), Gigler (1994), and Stocken (2000), avoid the unraveling principle by introducing multiple audiences and non-truthful disclosures. These models address disclosures about the realized or expected value of the firm; the relation to risk or variance was tangential at best. While Verrecchia (1983)

¹ The accounting literature distinguishes between proprietary versus non-proprietary disclosures, where the proprietary disclosure decision may affect the operations of the firm, while non-proprietary disclosures do not. Proprietary disclosures may arise when firms face imperfect product market competition or potential entry (see, among others, Darrough (1993), Darrough and Stoughton (1990), and Wagenhofer (1990)).

² Alternatively, Verrecchia (1983, 1990) introduces a pecuniary cost of discretionary disclosures of non-proprietary information about future firm profitability. He finds that risk-neutral investors *ex ante* prefer to commit to non-discretionary non-disclosure. In contrast, risk-neutral investors *ex post* prefer partial disclosure equilibrium in which managers voluntarily disclose favorable information while withholding unfavorable information.

considers risk-averse investors, most models assume investors in the capital market are risk-neutral.

Theoretical studies regarding disclosure of financial accounting information focus on disclosures relating to financial statement variables, such as asset values, earnings, or operating cash flow. This focus was natural, as discretionary disclosures are often made to complement mandatory disclosures, and until recently, virtually all the mandated disclosures related to financial statement variables. With the issuance of new disclosure standards, however, in particular the US Securities and Exchange Commission's Financial Reporting Release No. 48, disclosures concerning market risk have drawn the focus of management, regulators and researchers. In this paper we investigate the effect of discretionary disclosures about risk when the variance of firm value is uncertain. We consider a setting in which a risk-neutral entrepreneur can disclose the variance of the firm's future cash flows prior to offering the firm for sale in a competitive market of risk-averse investors. If investors knew with certainty that the manager possessed this information, full disclosure would be the unique equilibrium disclosure strategy. However, we allow for the possibility that the manager is uninformed. In this case we show partial disclosure is the unique equilibrium strategy.

Uncertainty concerning the precision of amounts disclosed in financial statements, such as earnings disclosures, has been addressed in a number of recent studies. Subramanyam (1996) considers the relation between prices and earnings when the precision of the earnings' reports is uncertain, while Kirschenheiter and Melumad (2002) study earnings' smoothing in a two period model. Penno (1996, 1997) analyzes the effect of a manager's unobservable choice of precision. In Penno (1996), a manager chooses the level of precision, and investors make inferences about this choice based on the disclosures about firm value. Penno (1997) extends the model of Dye (1985) to analyze inferences made by investors about the unobservable precision of the reports concerning firm value being made by managers. In all these studies, inferences about the precision

are made from the earnings reports themselves, and direct disclosure of the level of precision is not considered.

Studies of the interdependence of disclosures across firms, including Dye (1990) and Dye and Sridhar (1996) also relate to the current paper. Dye (1990) compares the optimal voluntary disclosures that a firm value maximizing manager would make to optimal mandated disclosures in a multi-firm setting where firms' future cash flows are correlated. He shows how manager's disclosures depend on nature of the externality associated with the disclosures and correlation among future cash flows. While Dye (1990) considers *ex ante* disclosures, the current paper considers *ex post* disclosures. We also find that the optimal disclosure strategy depends on the covariance structure assumed to hold. Dye and Sridhar (1995) extend the disclosure strategies of Dye (1985) to a setting with multiple firms where each firm manager may receive private information. They show that whether a firm manager discloses depends on that firm's information about other firms' information. This differs from our analysis in which the disclosure strategy of one firm may depend on the competing firm's disclosure, not on the information that the manager of the competing firm may possess.

Our paper differs in another aspect from the prior research summarized above; we derive the asset prices under partial disclosure from the investors' portfolio choice problem. First, we investigate a setting with a single risky asset. We show that the unique partial disclosure equilibrium has an upper-tailed non-disclosure region: an informed manager voluntarily discloses low variances (favorable news) and withholds high variances (unfavorable news). We establish that the higher the likelihood that the manager is informed, the more disclosures. We also provide comparative static results when the variance is uniformly distributed and exponentially distributed on the average level and the variation of the variance.

Next, we investigate a setting with multiple risky assets where the future cash flows from each asset are based on a market-wide risk factor plus an idiosyncratic risk factor.

We consider the simplest case, where the manager's reported variance does not inform investors about the cash flow factor loading for each firm. We find that the disclosure decision for each manager affects the beta of each firm, estimated using the CAPM, even though the manager is disclosing information about idiosyncratic cash flow risk. This result follows in an economy with a finite number of assets since market-clearing prices invariably reflect idiosyncratic cash flow risk through the market portfolio.

Finally, we extend the model by investigating the manager's strategy when he can obtain information about risk by investing in information technology. This approach is analogous to Pae (1999), who investigates the role played by the acquisition of information about future firm value. Similar to Verrecchia (1990), Pae finds that ex ante welfare is maximized when the signal is completely uninformative, or when precommitting to no investments in such technology is possible. Unlike our paper, Pae (1999) allows the manager to exert effort that increases the value of the firm. Further, as mentioned above, we consider risk disclosures, not disclosures of a signal about the future liquidating value of the firm. The focus on risk disclosures makes this modeling approach arguably more appealing, both from the standpoint that managers may be uninformed about risk information and that this information may be acquired at a cost.

We derive the market-clearing prices and the equilibrium disclosures when managers decide endogenously whether to expend resources on becoming informed about risks. In contrast to when the receipt of information is exogenous and cannot be acquired, managers may employ full disclosure or no disclosure strategy when costly investment in information technology is endogenous. Further, we show that even with positive costs of acquiring risk information, managers may prefer full disclosure. This indicates that mandating full disclosure of risk may not increase the level of disclosure.

The paper proceeds as follows. In Section 2 we introduce the basic model and discuss its assumptions. Section 3 presents the results assuming managers have an exogenous probability of being uninformed, first with a single asset and second with

multiple assets. In Section 4 we analyze the situation when firm managers must also decide whether to invest in costly information acquisition. We conclude in Section 5.

2 Model

We present the model through four assumptions regarding firms (A1), risk (A2), managers (A3), and investors (A4).

Assumption A1: Firm j has a risky investment project in place that pays X_j units of consumption good at time 2. These payoffs are described by a single factor model, that is, $X_j = \mathbf{m}_j + \mathbf{g}_j F + \mathbf{e}_j$, where \mathbf{m}_j represents the expected payoff, F denotes the common factor, \mathbf{g}_j is the factor loading, and the idiosyncratic risk is \mathbf{e}_j for $j=1, \dots, J$. The risky j 'th asset is offered in total supply of one share that costs P_j . The riskless bond is in zero net supply and offers a certain return normalized to zero.

Assumption A2: The common factor F is distributed $N(0, \mathbf{s}^2)$, and idiosyncratic risk is $\mathbf{e}_j | V_j \sim N(0, V_j)$. This implies that the conditional distribution of the payoff is $X_j | V_j \sim N(\mathbf{m}_j, \mathbf{g}_j^2 \mathbf{s}^2 + V_j)$. The variance V_j is drawn from a probability distribution, $f(V_j)$ with support $[\underline{V}_j, \bar{V}_j]$ where $0 \leq \underline{V}_j \leq \bar{V}_j \leq \infty$.

Assumption A3: At time 0, the manager of firm j privately observes the realized variance of V_j with probability $(1 - q_j)$, where $0 < q_j < 1$. If the manager is informed he then chooses whether to truthfully disclose V_j . A manager who is uninformed cannot credibly disclose that he is uninformed. The disclosure strategy of an informed manager j is characterized by $N_j \subseteq \mathfrak{R}_+$, the set of values of the variance that the informed manager j will not disclose. Let \underline{N} denote the vector of disclosure strategies for the $j=1, \dots, J$ managers. Since the event that one manager is informed is

independent of the event that other managers are informed, each manager holds the same beliefs towards the disclosure strategy of other managers as the investors.

Assumption A4: The capital market consists of I individual investors indexed by $i = 1, \dots, I$. Each investor has initial wealth of W_i^0 and constant absolute risk aversion, $a_i > 0$, and chooses to buy S_{ij} shares of the j 'th stock and B_i bonds taking stock and bond prices as given. Let $\hat{N}_j \subseteq \mathfrak{R}_+$ denote the set of values of the variance for which the investors believe that an informed manager j would not disclosure, where the "hat" denotes an inferred disclosure strategy to distinguish investors' beliefs about the manager's disclosure strategy from the actual disclosure strategy. Consistent with A3, let $\hat{\underline{N}}$ denote the vector of these inferred disclosure strategies. Finally, consistent with A1 and A3 let $P_j(v_j / \underline{N}, \hat{\underline{N}})$ denote the price of firm j 's share under strategies \underline{N} and inferred strategies $\hat{\underline{N}}$, and denote the vector of these prices as \underline{P} .

Assumption A1 summarizes the information about the profitability of each firm's project. Firms' expected profitability, \underline{m} , and their factor loadings, \underline{g}_j , are common knowledge. Assumption A2 provides the information concerning the distributions of the factor, the idiosyncratic risks and their variance, and these distributions are also assumed to be common knowledge. Assumption A3 specifies the information which managers may possess and the probability of the event that each manger becomes informed. Further, A3 clarifies that managers who are informed of their firm's idiosyncratic risk choose a disclosure strategy, which involves either truthfully disclosing this risk or withholding this information. Assumption A4 states that risk-averse investors are initially endowed with wealth which they trade for shares of riskless bonds and shares in the risky firms. Investors trade after observing all public information, including which firm managers choose to disclose and what variance they disclose. At the end of the model, investors consume based on the cash flows received from the bonds and shares in firms.

The managers will choose their disclosure strategies in order to maximize the price of their firm. The investors trade in order to maximize their expected utility, and prices are determined by investors' demand to clear the market for the shares of the firm. Following Jung and Kwon (1988), we let A denote the event that the manager is uninformed and does not disclose, let B denote the event that the manager is informed but does not disclose, and let C denote the event that the manager is informed and does disclose. Further, let N denote the set of variance realizations for which an informed manager does not disclose. Clearly,

$$\Pr\{NoDisclosure\} = \Pr\{A\} + \Pr\{B\} = q + (1-q)\Pr\{V \in N\}$$

$$\Pr\{Disclosure\} = \Pr\{C\} = (1-q)\Pr\{V \notin N\}.$$

Investors update their beliefs towards firm value after observing no voluntary disclosures.

To characterize this revision of investors' beliefs, we denote their assessment of the probability that a manager is uninformed given no disclosure by

$$\mathbf{x}(\hat{N}) = \frac{q}{q + (1-q)\Pr\{V \in \hat{N}\}}.$$

We now define an equilibrium for our model.

Definition of a Credible Equilibrium: The pair $\{\underline{N}, \hat{N}\}$ is an equilibrium if, for each j , each $v_j \geq 0$ and for all $N_j' \subseteq \mathfrak{R}^+$, the following inequality holds.

$$P_j(v_j | N_j, \underline{N}_{-j}, \hat{N}) \geq P_j(v_j | N_j', \underline{N}_{-j}, \hat{N}).$$

Further this equilibrium is a credible equilibrium when $\underline{N} = \hat{N}$.

The concept of a credible equilibrium is standard in the disclosure literature, see Kirschenheiter and Melumad (2002). Assumptions A1 through A4 are assumed to hold for the analysis of the equilibrium disclosures, the results of which are presented next.

3 Results Without Investment

In this section, we characterize the equilibrium voluntary disclosure strategies assuming that managers cannot invest in additional informational technology. The

resulting prices and disclosure strategies are complex, yet they can be understood intuitively. To ameliorate this intuitive understanding, we divide the discussion in this section into two parts. We begin, in section 3.1, by considering a setting with a single risky firm and suppress the firm index j to introduce the intuitive structure of the equilibrium. We extend the analysis to a setting with multiple firms in section 3.2 to communicate how risk premium is reflected in stock returns through CAPM betas.

3.1 Results for Single Firm Setting

A crucial step in analyzing the equilibrium strategies is to characterize the market clearing prices as a function of the disclosures made by the managers. This is done for a single firm setting in Lemma 1.

Lemma 1: For any beliefs, the market clearing prices are as follows.

(a) If v is disclosed ($v \notin N$) then the price of the firm is $P(v | v \notin N, \hat{N}) = \mathbf{m} - av$

where $a = \left(\sum_{i=1}^I a_i^{-1} \right)^{-1}$ as in Wilson (1968).

(b) If v is not disclosed then the price of the firm is $P(v | v \in N, \hat{N}) = \mathbf{m} - aH(\hat{N})$,

where $H(\hat{N}) = \frac{\mathbf{x}(\hat{N})E[Ve^{a^2V/2}] + (1 - \mathbf{x}(\hat{N}))E[Ve^{a^2V/2} | V \in \hat{N}]}{\mathbf{x}(\hat{N})E[e^{a^2V/2}] + (1 - \mathbf{x}(\hat{N}))E[e^{a^2V/2} | V \in \hat{N}]}$.

Part (a) of lemma 1 confirms that the standard mean-variance prices arise when the manager discloses the variance. In this case, the risk is priced as the product of the global risk aversion, a , times the disclosed variance. Part (b) provides the main contribution of the lemma, so we discuss this result in more detail.

Comparing the price equations in part (a) to part (b), we see that they appear quite similar, with one difference. In the risk premium term, we replace the known variance, v , with the measure $H(\hat{N})$ which depends on the investors' inferences about the manager's

disclosure strategy. Being risk-averse, investors require an additional risk premium implicit in the price for the uncertainty about the variance of the future cash flows. When the manager does not disclose, the expected variance given no disclosure is

$$E[V | No Disclosure] = \mathbf{x}(\hat{N})E[V] + (1 - \mathbf{x}(\hat{N}))E[V | V \in \hat{N}].$$

While this expected variance measures a portion of the risk premium, it does not capture all of it as it differs from the term $H(\hat{N})$. To see this, define the factor $\mathbf{q}(\hat{N})$ so that $H(\hat{N}) = E[V | No Disclosure] \times \mathbf{q}(\hat{N})$. We prove that $\mathbf{q}(\hat{N}) \geq 1$ (see the appendix), so it follows that investors require an additional risk premium, beyond the expected variance, due to the uncertainty about the level of the variance. In the limit as the risk aversion coefficient goes to zero, investors become risk neutral and $\mathbf{q}(\hat{N})$ goes to one.

Lemma 1 does not impose any structure on the investors' beliefs regarding the no disclosure sets. For our subsequent analysis, Lemma 2 characterizes the structure of the disclosure strategies that support credible equilibria.

Lemma 2: For any credible equilibrium $\{N, \hat{N}\}$, $\exists \hat{y} : \hat{N} = \{v | v \geq \hat{y}\}$.

Lemma 2 establishes that the no disclosure set, N , is connected and upper-tailed. In any credible equilibrium, the manager's disclosure strategy has the familiar structure from earlier disclosure studies. Using Lemma 2 allows us to eliminate two natural disclosure strategies in Theorem 1.

Theorem 1: Assume that $0 < q < 1$ then

- (a) always disclosing ($y = \underline{V}$) is not a credible equilibrium, and
- (b) never disclosing ($y = \bar{V}$) is not a credible equilibrium.

The proof of Theorem 1 follows by contradiction. If investors were to hold the belief that all informed firm managers disclose then they would also believe no disclosure

means that the manager is uninformed and hence of average type. These beliefs are not confirmed in equilibrium because managers who observe high variances would then maximize firm value by not disclosing. Similarly, if investors hold the belief that none of the informed managers disclose then the price for a firm that does not disclose would be below the price of a firm with known average variance. Hence firms with low variance have an incentive to provide voluntary disclosures to maximize firm value.

Next we provide three examples that convey intuition concerning the credible disclosure equilibrium. We start with a simple example that characterizes the set of equilibrium disclosure strategies.

Example 1: Consider the special case in which the variance can attain one of two values: High or Low. We can then enumerate the four possible no disclosure sets: $\hat{N} \in \{\{\emptyset\}, \{Low, High\}, \{Low\}, \{High\}\}$

Theorem 1 rules out the first two no disclosure sets. Further, Lemma 2 rules out the third no disclosure set. Hence, in the unique partial disclosure equilibrium a firm manager observing Low variance discloses while a manager observing High variance withholds. In this case $\hat{N} = \{High\}$ since the no disclosure set of an informed manager contains only a single variance, High.

Having established in Theorem 1 that full disclosure and no disclosure do not support equilibria, we now characterize the disclosure strategy that supports a credible equilibrium.

Theorem 2: When $0 < q < 1$, there exists a partial disclosure equilibrium characterized by a threshold level of variance, y , where $y \geq E[V]$.

Theorem 2 demonstrates the existence of a credible equilibrium where the disclosure threshold exceeds the unconditional expected level of variances. While this equilibrium threshold is unique for our conjectured market-clearing prices, we cannot rule out that other price functions might support a credible equilibrium.

Corollary 1: Consider the credible disclosure strategy determined by the threshold of Theorem 2. If \tilde{v} has continuous support with a differentiable probability density function, $f(v)$, then $\frac{\partial y}{\partial q} \leq 0$

Corollary 1 demonstrates that, consistent with Jung and Kwon (1988), an increase in the ex ante information asymmetry between managers and investors induce more disclosures by managers in equilibrium. To provide additional comparative static results we make parametric assumptions about the distribution of the variance. Next, we provide the results for two additional specific examples.

Example 2: Let the variance V be uniformly distributed on $[m-s, m+s]$, where $m > s > 0$. Then an increase in the mean, the standard deviation, or the risk aversion increases the disclosure threshold, that is, $\frac{\partial y}{\partial m} = 1$, $0 \leq \frac{\partial y}{\partial s} \leq 1$, and $\frac{\partial y}{\partial a} \geq 0$.

Example 3: Let the variance be exponentially distributed with parameter $I > 0$. Then the disclosure threshold will be higher the more risk averse the investors ($\partial y / \partial a > 0$) and the higher the prior mean and variance of the unknown variance ($\partial y / \partial I < 0$).

The comparative static results in examples 2 and 3 are consistent with Jung and Kwon (1988), showing that the intuition of these earlier results extend to the case of risk disclosures. The results of examples 2 and 3 reveal that the amount of disclosure, as

measured by the level of the disclosure threshold, increases as the prior mean and variance of the distribution of the cash flow variance increase. Also, they show that the amount of disclosure increases with the level of risk aversion in the market.

Next we turn to an analysis of the multiple stock setting.

3.2 Results for Multiple Stock Setting

We now consider the case with multiple stocks, that is, $J > 1$. We can show (a detailed proof is available from the authors) that Lemma 1 extends naturally to this setting. In this case, separation results similar to those found in Jorgensen and Kirschenheiter 2003 emerge.

More specifically, we find that the market-clearing price of each stock depends on the disclosure decision by its manager, but does not depend on the disclosure decisions of other managers. Further, the market-clearing demand for each stock j is $S_{ij} = a_i^{-1} a$, that is, two fund separation holds. In the Capital Asset Pricing Model, the excess return on each stock depends on its beta, the covariance between the return on stock j with the return on the market portfolio divided by the variance of the return on the market portfolio, that is, $\mathbf{b}_j = \frac{COV[r_j, r_m]}{VAR[r_m]}$. Given two fund separation holds in our credible equilibrium, the excess returns on stock j are fully characterized by its beta. The difference, however, is that we allow for discretionary disclosures to an extent determined by manager's disclosure incentives in equilibrium. The next theorem formalizes this result.

Theorem 3: The beta of stock j , \mathbf{b}_j , is affected by the disclosure threshold chosen by manager j , y_j .

The feature that the beta of firm j is affected by the manager's discretionary disclosure decision may initially surprise given that the managers' disclosures, if any, concern the variance of idiosyncratic future cash flows. With a finite number of assets, or firms, market-clearing prices reflect idiosyncratic cash flow risk through the market portfolio. In the limit as the number of firms goes to infinity, this feature of the model disappears because the market portfolio is unaffected by individual stock. Nonetheless, we consider these results robust. For example, Jorgensen and Kirschenheiter (2003) show that they hold when, instead of investors being unsure about whether the manager is informed, pecuniary disclosure costs break the "unraveling principle".

4 Results with Investment

In this section we analyze the role played by the opportunity to acquire information about risk measures through the investment in costly improvements in information technology. In particular, we characterize the impact of imposing mandatory risk disclosures in such a setting.

Assumption A3*: At time 0, the manager of firm j does not know the variance of his cash flows. However, the manager can obtain this information by investing in additional information technology at a cost of $c_j \geq 0$. This cost is assumed to be a random variable, the realization of which the manager privately observes at date zero. We assume that the costs are independently distributed and that the distribution of the costs is common knowledge. As before, if the manager is informed he then chooses whether to truthfully disclose V_j . A manager who is uninformed cannot credibly disclose that he is uninformed. In addition, the manager may, or may not, be able to disclose incurred investment costs, c_j . However, we maintain the assumption that the manager cannot credibly disclose investment costs that were not incurred. Based on the results of section 3, we know that a non-disclosure strategy can be characterized

by a variance, so we continue to denote the strategy in this manner, using the notation \hat{y}_j for the inferred threshold and y_j for the threshold actually chosen by the manager. As before, each manager holds the same beliefs towards the disclosure strategy of other managers as the investors.

Assumption A3* relaxes the assumption that an exogenous random event determines whether each managers is informed. Clearly, the previous assumption A3 arises as the special case when the investment cost is either 0 with probability $(1 - q_j)$ or ∞ with probability q_j where $1 > q_j > 0$. Based on the results from section 3 and the assumption that the costs of acquiring the information are independent, we can, without loss of generalization, again focus on the prices and disclosure strategy of a manager in a single firm economy. Hence, we again suppress the subscript in the remaining results.

Using the more general assumption A3*, we proceed, in parallel to Lemma 1 above, by determining the prices when the variance information is available through investment in additional information technology. We provide this characterization in the following lemma.

Lemma 3: Let $a = \left(\sum_{i=1}^I a_i^{-1} \right)^{-1}$ as in Wilson (1968), and let $\hat{C} > 0$ denote an inferred

level of disclosure cost. Then for any beliefs, the market clearing prices are as follows.

- (a) If v is disclosed ($v \leq \hat{y}$) and the actual cost of information acquisition, c , is observable then the price of the firm is $P(v, c) = \frac{\mathbf{m} - av - c}{R_f}$.
- (b) If v is disclosed ($v \leq \hat{y}$) and the actual costs of information acquisition is not observable then $P^*(v, c | c \leq \hat{C}) = \frac{\mathbf{m} - av - J(a | c \leq \hat{C})}{R_f}$,

$$\text{where } J(a|c \leq \hat{C}) = \frac{E\left[C e^{-a1_{\{c \leq \hat{C}\}}^C} \mid c \leq \hat{C} \right]}{E\left[e^{-a1_{\{c \leq \hat{C}\}}^C} \mid c \leq \hat{C} \right]}.$$

(c) If v is not disclosed, then the price of the firm is given as follows

$$P^*(v, c | c \geq \hat{C} \text{ or } v \geq \hat{y}) = \frac{m - K(a | \hat{C}, \hat{y})}{R_f}$$

$$\text{where } K(a | \hat{C}, \hat{y}) = \frac{E^{g(v, c | \hat{y}, \hat{C})} \left[\mathbb{1}_{\{c \leq \hat{C}\}} C + aV \right] e^{a1_{\{c \leq \hat{C}\}}^C} e^{a^2V/2}}{E^{g(v, c | \hat{y}, \hat{C})} \left[e^{a1_{\{c \leq \hat{C}\}}^C} e^{a^2V/2} \right]}$$

and

$$g(v, c | \hat{y}, \hat{C}) = 0 \quad v \leq \hat{y} \text{ and } c \leq \hat{C}$$

$$g(v, c | \hat{y}, \hat{C}) = \frac{f(v, c)}{q(\hat{C}) + (1 - q(\hat{C})) \Pr\{v \geq \hat{y}\}} \quad \text{otherwise.}$$

In all three cases, two fund separation holds, that is, $S_i^*(P^*) = a_i^{-1} a$.

Lemma 3 provides market-clearing prices under the various disclosure strategies, for a given cutoff level of investment costs. Each manager will voluntarily incur the investment cost if he believes that the investment raises the expected price of the firm. We use $\hat{C} > 0$ to denote the level of costs above which a manager will voluntarily refrain from investing in the information technology and, being uninformed, he is incapable of disclosing the variance. As in Lemma 1, we provide the prices when managers disclose the variance and when investors, prior to trading shares, observe the investment costs in (a) or investors do not observe the investment costs in (b). Part (a) is straight forward; the price is the disclosure price of lemma 1 reduced by the known investment cost. Part (b) shows that when the investment cost is unknown, the price of the firm is reduced by an amount based on the inferred investment cost. Of course, since the manager only invests for costs below \hat{C} , the expectation reflects this.

The price under non-disclosure is more complex. Non-disclosure may occur either because the information technology costs are high, or because the actual variance is high.

Hence the price under non-disclosure includes a risk premium, represented by $K(a|\hat{C}, \hat{y})$, which depends on investors' uncertainty concerning both the variance and the cost. While the expression for $K(a|\hat{C}, \hat{y})$ is complicated to evaluate, it seems likely to be increasing in the cutoff level of costs, \hat{C} . Given this assumption, we find that a mandatory disclosure requirement may not increase disclosure, even if investment costs are positive, as the following theorem illustrates.

Theorem 4: Assume a partial disclosure equilibrium exists in the voluntary disclosure regime in which the investment costs, c , are observable to investors. Then mandating disclosure increases the amount of disclosure if and only if $c > K(a|c = \hat{C}, \hat{y} = \underline{V}) - aE[V]$.

Clearly if the investment costs of generating the necessary information to meet the mandated disclosure requirements are sufficiently high, mandated disclosure will generate additional disclosure versus that achieved in a voluntary disclosure regime. However, Theorem 4 shows that these costs need not be zero in order to negate the impact of mandated disclosure on the level of disclosure in equilibrium. The reason is that the managers may voluntarily disclose to save on the inferred risk premium under non-disclosure as captured by $K(a|\hat{C}, \hat{y})$. If the savings from the reduced risk premium are sufficiently large, full disclosure may occur even under positive investment costs.

Finally, we complete the analysis of disclosure with investment by returning to the question of what equilibria are possible. Recall that in the basic model analyzed in section 3, neither full disclosure nor no disclosure arises in equilibrium. This is no longer guaranteed when managers' investments in information technology arise endogenously. Instead, an equilibrium may be supported by either full disclosure or no disclosure, as the final theorem shows.

Theorem 5: There exist parameter values for costs and variances where either full disclosure or no disclosure supports an equilibrium.

For no disclosure to support an equilibrium, the minimum cost must be sufficiently high to make disclosure unattractive to the manager. Perhaps more surprising is that full disclosure may also support an equilibrium. Full disclosure may occur if the maximum cost is sufficiently low to justify the investment, but the intuition derives from the benefits obtained. On an ex ante basis, the initial shareholders (as represented by the manager) benefit from disclosure by receiving a higher price. They receive this higher price, even though it is reduced by the investment costs, because this reduction is more than offset by the increase in utility of the future shareholders from the reduction in uncertainty concerning the variance. In addition to illustrating the additional subtleties in the analysis, this result also shows that introducing investment in information technology broadens the potential equilibrium outcomes.

5 Summary and Discussion

Dye (1985) and Jung and Kwon (1988) study a manager's discretionary disclosures about future firm value, when investors are unsure of whether the manager is informed. They establish that when investors are risk neutral, there exists a unique partial disclosure equilibrium. This paper studies discretionary disclosures about variances when investors are unsure of whether the manager is informed.

We find that disclosures about variances lead to a partial disclosure equilibrium when investors are risk averse. Due to investors' risk aversion, however, market-clearing prices take into account managers' discretionary disclosures, that is, investors' demand a risk premium for the additional uncertainty about unknown risk in the absence of disclosure. These results can be viewed as characterizing the impact of managers who are

able to make credible disclosures about risks relative to a setting in which such credible disclosures are impossible.

We next extend our basic model to consider a setting with multiple stocks. We characterize how, in a small stock market, a manager's discretionary disclosures about risk affect the equilibrium systematic risk premium of each firm, characterized by the beta. We then consider the case where a manager may, for a cost, invest in information acquisition technology that allows him to obtain risk information. We find that this expands the possible equilibrium outcomes, such that either full disclosure or no disclosure strategies may support a credible equilibrium. This indicates that even if the cost of investing in information acquisition is positive, full disclosure may be preferred, so that mandating disclosure need not increase the amount of disclosure in equilibrium.

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7 Appendix: Proofs

Proof of Lemma 1 (a): The price of the stock after a manager has disclosed the variance is standard (the proof follows from, among others, Huang and Litzenberger, 1988, section 4.15, p. 100-102).

Proof of Lemma 1 (b): To derive the market-clearing price of stock after a manager has not disclosed the variance, recall that, conditional on non-disclosure, either the manager is uninformed (event A) or the manager is informed (event B). If A then the variance is drawn from its prior distribution, whereas if B then $v \in N$. Further recall that in the event of no disclosure, investors hold common beliefs about a no disclosure region, \hat{N} . That is, investors believe that if the manager is informed then the variance is in the no disclosure region, $v \in \hat{N}$.

After observing that the manager did not disclose the variance, each investor solves the following problem:

$$\begin{aligned} \max_{S_i, B_i} \quad & E[-a_i^{-1}e^{-a_i W_i} | \text{No disclosure}] = \mathbf{x}(\hat{N})E[-a_i^{-1}e^{-a_i W_i}] + (1 - \mathbf{x}(\hat{N}))E[-a_i^{-1}e^{-a_i W_i} | V \in \hat{N}] \\ \text{s.t.} \quad & W_i = S_i X + B_i R_f \\ & W_i^0 = S_i P + B_i \end{aligned}$$

where $\Pr\{A\} = q$ and $\mathbf{x}(N) = \frac{q}{q + (1-q)\Pr\{V \in N\}}$. We can substitute out the number of

bonds through $B_i = (W_i^0 - S_i P)$ and hence

$$W_i = S_i X + (W_i^0 - S_i P)R_f = S_i(X - PR_f) + W_i^0 R_f$$

or

$$\begin{aligned} \min_{S_i} \quad & \mathbf{x}(\hat{N})E\left[e^{-a_i(S_i(X - PR_f) + W_i^0 R_f)}\right] + (1 - \mathbf{x}(\hat{N}))E\left[e^{-a_i(S_i(X - PR_f) + W_i^0 R_f)} | V \in \hat{N}\right] \\ = & e^{-a_i W_i^0 R_f} e^{-a_i S_i(m - PR_f)} \left\{ \mathbf{x}(\hat{N})E\left[e^{-a_i S_i e}\right] + (1 - \mathbf{x}(\hat{N}))E\left[e^{-a_i S_i e} | V \in \hat{N}\right] \right\} \\ = & e^{-a_i W_i^0 R_f} e^{-a_i S_i(m - PR_f)} \left\{ \mathbf{x}(\hat{N})E\left[e^{(a_i S_i)^2 V / 2}\right] + (1 - \mathbf{x}(\hat{N}))E\left[e^{(a_i S_i)^2 V / 2} | V \in \hat{N}\right] \right\} \end{aligned}$$

which is a convex optimization problem. Define the probability measure

$$g(v) = \mathbf{x}(\hat{N})f(v) = \frac{q}{q + (1-q)\Pr\{v \in \hat{N}\}} f(v) \quad \text{for } v \notin \hat{N}$$

and

$$\begin{aligned} g(v) &= \frac{q}{q + (1-q)\Pr\{v \in \hat{N}\}} f(v) + \frac{(1-q)\Pr\{v \in \hat{N}\}}{q + (1-q)\Pr\{v \in \hat{N}\}} f(v | v \in \hat{N}) \\ &= \frac{f(v)}{q + (1-q)\Pr\{v \in \hat{N}\}} \quad \text{for } v \in \hat{N} \end{aligned}$$

Then the optimization problem can be written as

$$\min_{S_i} \quad e^{-a_i S_i(m - PR_f)} E^{g(v|\hat{N})} \left[e^{(a_i S_i)^2 V / 2} \right]$$

where the superscript indicates that the expectation is taken under the probability measure, g . Taking natural logarithm, this optimization problem can be rewritten as

$$\max_{S_i} -a_i S_i (\mathbf{m} - PR_f) + \ln \left(E^{g(v|\hat{N})} \left[e^{(a_i S_i)^2 V / 2} \right] \right)$$

The first order condition for an interior solution is

$$(\mathbf{m} - PR_f) = a_i S_i h_i(\hat{N}, S_i)$$

where

$$h_i(\hat{N}, S_i) = \frac{E^{g(v|\hat{N})} \left[V e^{(a_i S_i)^2 V / 2} \right]}{E^{g(v|\hat{N})} \left[e^{(a_i S_i)^2 V / 2} \right]}.$$

Hence,

$$S_i = a_i^{-1} \frac{(\mathbf{m} - PR_f)}{h_i(\hat{N}, S_i)}.$$

We conjecture (and subsequently verify) that there exists an equilibrium in which $h_i(S_i^*, \hat{N}) = H(\hat{N})$ when evaluated at the market-clearing demand for stock, S_i^* .

$$S_i = a_i^{-1} \frac{(\mathbf{m} - PR_f)}{H(\hat{N})}$$

which leads to the investor's demand function for stock

$$S_i = a_i^{-1} \frac{(\mathbf{m} - PR_f)}{H(\hat{N})}.$$

Since this holds for any investor, we can sum across all investors on both sides

$$\sum_{i=1}^I S_i = \sum_{i=1}^I a_i^{-1} \frac{(\mathbf{m} - PR_f)}{H(\hat{N})}.$$

Using the market clearing condition that $\sum_{i=1}^I S_i = 1$, we find that

$$aH(\hat{N}) = (\mathbf{m} - PR_f)$$

where

$$a = \left(\sum_{i=1}^I a_i^{-1} \right)^{-1}$$

denotes the aggregate global risk aversion and the market-clearing price of stock given no disclosure of variance, is

$$P(V \in \hat{N}) = \frac{\mathbf{m} - aH(\hat{N})}{R_f}.$$

To verify the conjecture that $h_i(\hat{N}, S_i^*) = H(\hat{N})$, substitute the market-clearing price into the individual investor's demand for stock

$$S_i^* = a_i^{-1} \frac{\left(\mathbf{m} - \frac{\mathbf{m} - ah(\hat{N})}{R_f} \right) R_f}{H(\hat{N})} = a_i^{-1} a$$

and note that

$$h_i(\hat{N}, S_i^*) = \frac{E^{g(v|\hat{N})} \left[V e^{a^2 v / 2} \right]}{E^{g(v|\hat{N})} \left[e^{a^2 v / 2} \right]} = H(\hat{N})$$

which is independent of i . This completes the proof of lemma 1 (b).

Proof of Lemma 2: Proof by contradiction. Suppose a credible equilibrium in which \hat{N} is assumed not to be upper-tailed. Then there exist variances $v_1 < v_2$ where $v_1 \in N$ and $v_2 \notin N$, that is,

$$P(V = v_1) = \frac{\mathbf{m} - av_1}{R_f} \leq \frac{\mathbf{m} - aH(\hat{N})}{R_f} = P(\text{No Disclosure})$$

$$P(V = v_2) = \frac{\mathbf{m} - av_2}{R_f} \geq \frac{\mathbf{m} - aH(\hat{N})}{R_f} = P(\text{No Disclosure})$$

However, the contradiction now arises since

$$P(\text{No Disclosure}) \geq P(V = v_1) = \frac{\mathbf{m} - av_1}{R_f} > \frac{\mathbf{m} - av_2}{R_f} = P(V = v_2) \geq P(\text{No Disclosure}).$$

Proof of Theorem 1 (a)

Proof by contradiction. Assume that full disclosure is an equilibrium, then the price given no disclosure would be.

$$P(\bar{V}) = \frac{\mathbf{m} - aH(\bar{V})}{R_f}$$

where

$$H(\bar{V}) = \frac{E[V e^{kV}]}{E[e^{kV}]}$$

From Gron, Jorgensen, and Polson (2002)

$$H(\bar{V}) = \frac{E[V e^{kV}]}{E[e^{kV}]} \geq E[V]$$

But this implies that an informed manager who observes firm variance above the mean would prefer to not disclose. Hence the contradiction.

Proof of Theorem 1 (b)

Proof by contradiction. Assume that no disclosure is an equilibrium, then the price given no disclosure would be.

$$P(\underline{V}) = \frac{\mathbf{m} - aH(\underline{V})}{R_f}$$

where

$$H(\underline{V}) = \frac{qE[Ve^{kV}] + (1-q)\Pr\{V \geq \underline{V}\}E[Ve^{kV} | V \geq \underline{V}]}{qE[e^{kV}] + (1-q)\Pr\{V \geq \underline{V}\}E[e^{kV} | V \geq \underline{V}]} = \frac{E[Ve^{kV}]}{E[e^{kV}]}$$

From Gron et al. (2002)

$$H(\underline{V}) = \frac{E[Ve^{kV}]}{E[e^{kV}]} \geq E[V]$$

But this implies that an informed manager who observes firm variance below the mean would prefer to disclose. Hence the contradiction.

Proof of Theorem 2

We first establish the existence of a unique partial disclosure equilibrium. We then establish that the disclosure threshold exceeds the unconditional mean of the variance.

If a partial disclosure equilibrium exists then the disclosure threshold, y , is characterized by $H(y) = y$ or

$$\frac{\left(q \int_{\underline{V}}^{\bar{V}} v e^{kv} f(v) dv + (1-q) \int_y^{\bar{V}} v e^{kv} f(v) dv \right)}{\left(q \int_{\underline{V}}^{\bar{V}} e^{kv} f(v) dv + (1-q) \int_y^{\bar{V}} e^{kv} f(v) dv \right)} = y$$

or

$$\left(q \int_{\underline{V}}^{\bar{V}} v e^{kv} f(v) dv + (1-q) \int_y^{\bar{V}} v e^{kv} f(v) dv \right) = y \left(q \int_{\underline{V}}^{\bar{V}} e^{kv} f(v) dv + (1-q) \int_y^{\bar{V}} e^{kv} f(v) dv \right)$$

or

$$(1-q) \int_y^{\bar{V}} (v-y) e^{kv} f(v) dv = q \int_{\underline{V}}^{\bar{V}} (y-v) e^{kv} f(v) dv \quad (*)$$

The left hand side is always non-negative. Further, the right hand side is continuous and monotonically increasing in y . Evaluated at $y = \underline{V}$, (*) becomes

$$(1-q) \int_y^{\bar{V}} v e^{kv} f(v) dv > 0 > -q \int_{\underline{V}}^{\bar{V}} v e^{kv} f(v) dv$$

and evaluated at the upper limit

$$\lim_{y \rightarrow \bar{V}} (1-q) \int_y^{\bar{V}} (v-y) e^{kv} f(v) dv = 0$$

$$\lim_{y \rightarrow \bar{V}} q \int_0^{\bar{V}} (y-v) e^{kv} f(v) dv > 0$$

Hence, there exists a unique solution to (*).

Finally we establish that the disclosure threshold exceeds the unconditional mean of the variance using proof by contradiction. Assume that $y < E[V]$. From Gron et al. (2002) it follows that

$$\begin{aligned}
H(y) &\geq E[V \mid \text{No Disclosure}] \\
&= \mathbf{x}(y)E[V] + (1 - \mathbf{x}(y))E[V \mid V \geq y] \\
&> \mathbf{x}(y)E[V] + (1 - \mathbf{x}(y))E[V \mid V \geq E[V]] = E[V]
\end{aligned}$$

The contradiction now follows from the equilibrium condition, $y = H(y) > E[V]$.

Proof of Corollary 1:

Since

$$H(y) = \frac{\left(\int_{\underline{v}}^{\bar{v}} v e^{kv} f(v) dv + (1-q) \int_y^{\bar{v}} v e^{kv} f(v) dv \right)}{\left(\int_{\underline{v}}^{\bar{v}} e^{kv} f(v) dv + (1-q) \int_y^{\bar{v}} e^{kv} f(v) dv \right)} = \frac{\left(\int_{\underline{v}}^{\bar{v}} v e^{kv} f(v) dv + \int_y^{\bar{v}} v e^{kv} f(v) dv \right)}{\left(\int_{\underline{v}}^{\bar{v}} e^{kv} f(v) dv + \int_y^{\bar{v}} e^{kv} f(v) dv \right)}$$

It follows that

$$\frac{\partial}{\partial q} H(y) = \frac{\left\{ \int_{\underline{v}}^{\bar{v}} v e^{kv} f(v) dv \right\} \left(\int_{\underline{v}}^{\bar{v}} e^{kv} f(v) dv + \int_y^{\bar{v}} e^{kv} f(v) dv \right) - \left\{ \int_{\underline{v}}^{\bar{v}} e^{kv} f(v) dv \right\} \left(\int_{\underline{v}}^{\bar{v}} v e^{kv} f(v) dv + \int_y^{\bar{v}} v e^{kv} f(v) dv \right)}{\left(\int_{\underline{v}}^{\bar{v}} e^{kv} f(v) dv + \int_y^{\bar{v}} e^{kv} f(v) dv \right)^2}$$

or

$$\begin{aligned}
&\frac{\partial}{\partial q} H(y) \left(\int_{\underline{v}}^{\bar{v}} e^{kv} f(v) dv + \int_y^{\bar{v}} e^{kv} f(v) dv \right)^2 \\
&= \left\{ \int_{\underline{v}}^{\bar{v}} v e^{kv} f(v) dv \right\} \left\{ \int_y^{\bar{v}} e^{kv} f(v) dv \right\} - \left\{ \int_{\underline{v}}^{\bar{v}} e^{kv} f(v) dv \right\} \left\{ \int_y^{\bar{v}} v e^{kv} f(v) dv \right\} \\
&= \left\{ \int_{\underline{v}}^y v e^{kv} f(v) dv + \int_y^{\bar{v}} v e^{kv} f(v) dv \right\} \left\{ \int_y^{\bar{v}} e^{kv} f(v) dv \right\} - \left\{ \int_{\underline{v}}^y e^{kv} f(v) dv + \int_y^{\bar{v}} e^{kv} f(v) dv \right\} \left\{ \int_y^{\bar{v}} v e^{kv} f(v) dv \right\} \\
&= \left\{ \int_{\underline{v}}^y v e^{kv} f(v) dv \right\} \left\{ \int_y^{\bar{v}} e^{kv} f(v) dv \right\} - \left\{ \int_{\underline{v}}^y e^{kv} f(v) dv \right\} \left\{ \int_y^{\bar{v}} v e^{kv} f(v) dv \right\}
\end{aligned}$$

or

$$\frac{\partial}{\partial q} H(y) \frac{\left(\int_{\underline{v}}^{\bar{v}} e^{kv} f(v) dv + \int_y^{\bar{v}} e^{kv} f(v) dv \right)^2}{\left(\int_{\underline{v}}^y e^{kv} f(v) dv \right) \left(\int_y^{\bar{v}} e^{kv} f(v) dv \right)} = \frac{\left\{ \int_{\underline{v}}^y v e^{kv} f(v) dv \right\}}{\left(\int_{\underline{v}}^y e^{kv} f(v) dv \right)} - \frac{\left\{ \int_y^{\bar{v}} e^{kv} f(v) dv \right\}}{\left(\int_y^{\bar{v}} e^{kv} f(v) dv \right)}$$

For $0 < a < b$, define the probability distribution function

$$h(v \mid a, b) = \frac{e^{kv} f(v)}{\int_a^b e^{kv} f(v) dv}$$

on the interval $[a, b]$, and zero elsewhere. Then

$$\frac{\partial}{\partial q} H(y) = \frac{\left(q \int_{\underline{v}}^{\bar{v}} e^{kv} f(v) dv + \int_y^{\bar{v}} e^{kv} f(v) dv \right)^2}{\left(\int_{\underline{v}}^y e^{kv} f(v) dv \right) \left(\int_y^{\bar{v}} e^{kv} f(v) dv \right)} = \int_{\underline{v}}^y v h(v | \underline{v}, y) dv - \int_y^{\bar{v}} v h(v | y, \bar{v}) dv \leq 0$$

Proof of Example 2:

Let the variance V be uniformly distributed on $[m-s, m+s]$, where $0 < s < m$, then

$$\begin{aligned} H(y) &= \frac{\left(q \int_{m-s}^{m+s} v e^{kv} dv + (1-q) \int_y^{m+s} v e^{kv} dv \right)}{\left(q \int_{m-s}^{m+s} e^{kv} f dv + (1-q) \int_y^{m+s} e^{kv} dv \right)} \\ &= \frac{\left(q[(m+s)e^{k(m+s)} - (m-s)e^{k(m-s)}] + (1-q)[(m+s)e^{k(m+s)} - ye^{ky}] \right)}{\left(q[e^{k(m+s)} - e^{k(m-s)}] + (1-q)[e^{k(m+s)} - e^{ky}] \right)} - k^{-1} \\ &= \frac{q[(m+s)e^{k2s} - (m-s)] + (1-q)[(m+s)e^{k2s} - ye^{k[y-(m-s)]}]}{q[e^{k2s} - 1] + (1-q)[e^{k2s} - e^{k[y-(m-s)]}]} - k^{-1} \\ &= \frac{(m+s)e^{k2s} - q(m-s) - (1-q)ye^{k[y-(m-s)]}}{e^{k2s} - q - (1-q)e^{k[y-(m-s)]}} - k^{-1} \end{aligned}$$

Using the indifference condition, we find that

$$(y + k^{-1})(e^{k2s} - q - (1-q)e^{k[y-(m-s)]}) = (m+s)e^{k2s} - q(m-s) - (1-q)ye^{k[y-(m-s)]}$$

or

$$(y + k^{-1})(e^{k2s} - q) - k^{-1}(1-q)e^{k[y-(m-s)]} = (m-s)e^{k2s} + 2se^{k2s} - q(m-s)$$

or

$$[y - (m-s)](e^{k2s} - q) + k^{-1}(e^{k2s} - q) - k^{-1}(1-q)e^{k[y-(m-s)]} = 2se^{k2s}$$

Substitution by $z=y-m$ reveals that

$$[z+s](e^{k2s} - q) + k^{-1}(e^{k2s} - q) - k^{-1}(1-q)e^{k[z+s]} = 2se^{k2s}$$

or

$$k[z+s](e^{k2s} - q) + (e^{k2s} - q) - (1-q)e^{k[z+s]} - k2se^{k2s} = 0$$

Since this equation does not depend on m , it must be true that

$$\frac{\partial y}{\partial m} = 1$$

Further define

$$M(z | k, s, q) = k[z+s](e^{k2s} - q) + (e^{k2s} - q) - (1-q)e^{k[z+s]} - k2se^{k2s}$$

Note that

$$\begin{aligned} \frac{\partial}{\partial z} M(z | k, s, q) &= k[(e^{k2s} - q) - (1-q)e^{k[z+s]}] = k[e^{k2s} - \{q + (1-q)e^{k[z+s]}\}] \\ &\geq k[e^{k2s} - \{q + (1-q)e^{k2s}\}] \geq 0 \end{aligned}$$

Further note that

$$\begin{aligned}
\frac{\partial}{\partial s} M(z | k, s, q) &= k(e^{k2s} - q) + [z + s]2k^2 e^{k2s} + 2ke^{k2s} - (1 - q)ke^{k[z+s]} - 2ke^{k2s} - 4k^2 se^{k2s} \\
&= k(e^{k2s} - q) + 2kM(z | k, s, q) + 2kq + (1 - q)ke^{k[z+s]} - 2ke^{k2s} \\
&= k(e^{k2s} - q) + 2kq + (1 - q)ke^{k[z+s]} - 2ke^{k2s} \\
&= -k[e^{k2s} - q - (1 - q)e^{k[z+s]}] \leq 0
\end{aligned}$$

Therefore, from implicit function theorem we get that

$$\frac{\partial y}{\partial s} = \frac{\partial z}{\partial s} = -\frac{\frac{\partial M}{\partial s}}{\frac{\partial M}{\partial z}} \geq 0.$$

Further,

$$\frac{\partial y}{\partial s} \leq \frac{[e^{k2s} - q - (1 - q)e^{k[z+s]}]}{[e^{k2s} - q - (1 - q)e^{k2s}]} \leq 1$$

where the latter inequality follows from the fact that $x < s$. Finally, since

$$\begin{aligned}
\frac{\partial}{\partial k} M(z | k, s, q) &= [z + s](e^{k2s} - q) + k[z + s]2se^{k2s} + 2se^{k2s} - (1 - q)[z + s]e^{k[z+s]} - 2se^{k2s} - k4s^2 e^{k2s} \\
&= [z + s](e^{k2s} - q) + 2sM(z | k, s, q) - (1 - q)[z - s]e^{k[z+s]} - 2se^{k2s} \\
&= [z + s](e^{k2s} - q) - (1 - q)[z - s]e^{k[z+s]} - 2se^{k2s} \\
&= [z - s]e^{k2s} - [z + s]q - (1 - q)[z - s]e^{k[z+s]} \\
&= [z - s]qe^{k2s} - [z + s]q \leq 0
\end{aligned}$$

it follows by the implicit function theorem

$$\frac{\partial y}{\partial a} = \frac{\partial y}{\partial z} \frac{\partial z}{\partial k} \frac{\partial k}{\partial a} = \frac{\partial z}{\partial k} 2a = \frac{\frac{\partial M}{\partial k}}{\frac{\partial M}{\partial z}} 2a \geq 0.$$

Proof for Example 3: Assume that V is exponentially distributed with parameter $I > 0$, that is, $f(V) = Ie^{-IV}$ for $V > 0$ and 0 otherwise. Defining $k \equiv a^2/2$ we note that for all $y > 0$, the following five facts, F.1 through F.4, hold.

$$\text{F.1: } E[e^{kV}] = \int_0^{\infty} e^{kV} f(V) dV = \frac{I}{(I - k)}$$

$$\text{F.2: } E[Ve^{kV}] = \int_0^{\infty} Ve^{kV} f(V) dV = \frac{I}{(I - k)^2}$$

$$\text{F.3: } E[e^{kV} | V \geq y] = \int_0^{\infty} e^{kV} f(V | V \geq y) dV = \frac{Ie^{ky}}{(I - k)}$$

$$\text{F.4: } E[Ve^{kV} | V \geq y] = \int_0^{\infty} Ve^{kV} f(V | V \geq y) dV = \frac{Ie^{ky}}{(I - k)} \left\{ \frac{1}{(I - k)} + y \right\}$$

F.1 through F.2 are shown below. First, for $k = a^2 / 2 < I$, F.1 holds because

$$\int_0^{\infty} e^{kV} f(V) dV = \int_0^{\infty} I e^{-(I-k)V} dV = \frac{I}{(I-k)} \int_0^{\infty} (I-k) e^{-(I-k)V} dV = \frac{I}{(I-k)}$$

and F.2 holds because

$$\int_0^{\infty} V e^{kV} f(V) dV = \frac{I}{(I-k)} \int_0^{\infty} V (I-k) e^{-(I-k)V} dV = \frac{I}{(I-k)^2}.$$

Further, F.3 holds since

$$\begin{aligned} \int_y^{\infty} e^{kV} f(V | V \geq y) dV &= \int_y^{\infty} e^{kV} I e^{-I(V-y)} dV = e^{ky} \int_y^{\infty} e^{k(V-y)} I e^{-I(V-y)} dV \\ &= e^{ky} \int_0^{\infty} e^{kz} I e^{-Iz} dz = e^{ky} \frac{I}{(I-k)} \int_0^{\infty} (I-k) e^{-(I-k)z} dz = \frac{I e^{ky}}{(I-k)} \end{aligned}$$

and

$$\begin{aligned} \int_y^{\infty} V e^{kV} f(V | V \geq y) dV &= \int_y^{\infty} (V-y+y) e^{kV} I e^{-I(V-y)} dV = e^{ky} \left\{ \int_y^{\infty} (V-y) e^{k(V-y)} I e^{-I(V-y)} dV + y \int_y^{\infty} e^{k(V-y)} I e^{-I(V-y)} dV \right\} \\ e^{ky} \left\{ \int_0^{\infty} Z I e^{-(I-k)Z} dZ + y \int_y^{\infty} I e^{-(I-k)Z} dZ \right\} &= \frac{I e^{ky}}{(I-k)} \left\{ \int_0^{\infty} Z (I-k) e^{-(I-k)Z} dZ + y \right\} = \frac{I e^{ky}}{(I-k)} \left\{ \frac{1}{(I-k)} + y \right\}. \end{aligned}$$

Using facts F.1 through F.4, we can write $H(y)$ as follows

$$\begin{aligned} H(y) &= \frac{qE[Ve^{kV}] + (1-q)E[Ve^{kV} | V \geq y]}{qE[e^{kV}] + (1-q)E[e^{kV} | V \geq y]} = \frac{q \frac{I}{(I-k)^2} + (1-q) \frac{I e^{ky}}{(I-k)} \left\{ \frac{1}{(I-k)} + y \right\}}{q \frac{I}{(I-k)} + (1-q) \frac{I e^{ky}}{(I-k)}} \\ &= \frac{q \frac{e^{-ky}}{(I-k)} + (1-q) \left\{ \frac{1}{(I-k)} + y \right\}}{q e^{-ky} + (1-q)} = \frac{1}{(I-k)} + y \left\{ \frac{(1-q)}{q e^{-ky} + (1-q)} \right\} = \frac{1}{(I-k)} + y \left\{ 1 - \frac{q e^{-ky}}{q e^{-ky} + (1-q)} \right\}. \end{aligned}$$

This in turn means we can write the equilibrium indifference condition, $H(y) - y = 0$, as follows:

$$H(y) - y = \frac{1}{(I-k)} - \frac{y q e^{-ky}}{q e^{-ky} + (1-q)} = \frac{1}{(I-k)} - \frac{y}{1 + e^{ky}(1-q)/q} = 0.$$

Such a y can be shown to exist. First, this implies that at $y=0$ we have

$$H(0) = (I-k)^{-1} > 0. \text{ Further, we have}$$

$$\frac{\partial(H(y) - y)}{\partial y} = \frac{(1 + e^{ky}(1-q)/q) - k y e^{ky}(1-q)/q}{(q e^{-ky} + (1-q))^2} = \frac{(1 - ky)(e^{ky}(1-q)/q) + 1}{(q e^{-ky} + (1-q))^2}.$$

The denominator is positive and bounded. The numerator is initially positive, but becomes negative and stays negative for y sufficiently large, going to negative infinity as y goes to infinity. Hence, a unique partial disclosure equilibrium exists, as shown in theorem 2. Further, the equilibrium threshold solves the equilibrium condition given above.

For the comparative statics, first note that the ratio $(1-q)/q$ is decreasing in q . Hence the second term in the equation for $H(y) - y = 0$ is increasing in q . This means the threshold must decrease in order for this condition to hold, so that $\partial y / \partial q < 0$ holds proving part a of example 3.

Next, consider increasing $k \equiv a^2/2$. This will increase the first term and decrease the second term in the indifference condition, so that the threshold must increase in order for the indifference condition to hold, proving part b of example 3.

Last, consider increasing I , which means lowering the prior mean and variance on the distribution of the unknown variance. This will decrease the first term, requiring y to decrease in order for the indifference condition to hold, proving part c of example 3 and completing the proof of example 3.

Proof of Theorem 3:

Consider without loss of generality a setting with $J = 2$ stocks. To simplify notation let $\Delta_j = H(y_j)$ if no disclosure and $\Delta_j = v_j$ if disclosure. The initial wealth of the economy can be written as $W_m^0 = \sum_{i=1}^I W_i^0$ and the terminal wealth of the economy is

$$W_m = \sum_{i=1}^I W_i = W_m^0 R_f + (X_1 - P_1 R_f) + (X_2 - P_2 R_f). \text{ Substituting in the liquidating cash}$$

flows from assumption (A1) and the market-clearing price from Lemma 1, we find that

$$\begin{aligned} W_m &= W_m^0 R_f + (\mathbf{g}_1 F + \mathbf{e}_1 + a \mathbf{g}_1 \mathbf{s}^2 (\mathbf{g}_1 + \mathbf{g}_2) + a \Delta_1) + (\mathbf{g}_2 F + \mathbf{e}_2 + a \mathbf{g}_2 \mathbf{s}^2 (\mathbf{g}_1 + \mathbf{g}_2) + a \Delta_2) \\ &= W_m^0 R_f + (\mathbf{g}_1 + \mathbf{g}_2) F + a \mathbf{s}^2 (\mathbf{g}_1 + \mathbf{g}_2)^2 + (\mathbf{e}_1 + a \Delta_1) + (\mathbf{e}_2 + a \Delta_2) \end{aligned}$$

Hence, the excess return on the market portfolio is

$$r_m - R_f = \frac{W_m}{W_m^0} - R_f = (W_m^0)^{-1} \{ (\mathbf{g}_1 + \mathbf{g}_2) F + a \mathbf{s}^2 (\mathbf{g}_1 + \mathbf{g}_2)^2 + (\mathbf{e}_1 + a \Delta_1) + (\mathbf{e}_2 + a \Delta_2) \}$$

and the expected excess return on the market is

$$E[r_m] - R_f = (W_m^0)^{-1} a \{ \mathbf{s}^2 (\mathbf{g}_1 + \mathbf{g}_2)^2 + \Delta_1 + \Delta_2 \}$$

and the variance on the return on the market is

$$\text{VAR}[r_m] = \text{VAR}[r_m - R_f] = (W_m^0)^{-2} \{ (\mathbf{g}_1 + \mathbf{g}_2)^2 \mathbf{s}^2 + \text{VAR}[\mathbf{e}_1] + \text{VAR}[\mathbf{e}_2] \}$$

where the variance of the idiosyncratic cash flow risk, $\text{VAR}[\mathbf{e}_j]$, depends on whether the variance was disclosed. The return on stock j , $r_j = X_j / P_j(v_j | \Delta_j)$, can be calculated from assumption (A1) and lemma 1. Therefore,

$$\begin{aligned} \text{COV}[r_m, r_j] &= (P_j(v_j | \Delta_j))^{-1} \text{COV}[r_m - R_f, X_j] \\ &= (W_m^0 P_j(v_j | \Delta_j))^{-1} \text{COV}[(\mathbf{g}_1 + \mathbf{g}_2) F + \mathbf{e}_1 + \mathbf{e}_2, \mathbf{g}_j F + \mathbf{e}_j] \\ &= (W_m^0 P_j(v_j | \Delta_j))^{-1} \{ \mathbf{g}_j (\mathbf{g}_1 + \mathbf{g}_2) \mathbf{s}^2 + \text{VAR}[\mathbf{e}_j] \} \end{aligned}$$

Therefore, the beta of stock j is

$$\mathbf{b}_j = \frac{COV[r_m, r_j]}{VAR[r_m]} = \frac{W_m^0 \{ \mathbf{g}_j (\mathbf{g}_1 + \mathbf{g}_2) \mathbf{s}^2 + VAR[\mathbf{e}_j] \}}{[P_j(v_j | \Delta_j)] \{ (\mathbf{g}_1 + \mathbf{g}_2)^2 \mathbf{s}^2 + VAR[\mathbf{e}_1] + VAR[\mathbf{e}_2] \}}$$

which depends on the disclosure decisions by both firm managers.

Detailed Proof of Lemma 3 (b):

The proof of part 3 (a) is a special case of the proof of part 3 (b), so we begin with the latter. After observing the manager's disclosure of the variance, each investor solves the following problem:

$$\begin{aligned} \max_{S_i, B_i} \quad & E \left[-a_i^{-1} e^{-a_i W_i} \mid C \leq \hat{C} \right] \\ \text{s.t.} \quad & W_i = S_i X + B_i R_f \\ & W_i^0 = S_i P + B_i \end{aligned}$$

We can substitute out the number of bonds through

$$B_i = W_i^0 - S_i P$$

and hence

$$W_i = S_i (X - 1_{\{C \leq \hat{C}\}} C) + (W_i^0 - S_i P) R_f = S_i (X - 1_{\{C \leq \hat{C}\}} C - PR_f) + W_i^0 R_f$$

Consequently, the investor's problem reduces to

$$\max_{S_i} E \left[-e^{-a_i (S_i (X - 1_{\{C \leq \hat{C}\}} C - PR_f) + W_i^0 R_f)} \mid C \leq \hat{C} \right] = -e^{-a_i W_i^0 R_f} E \left[e^{-a_i S_i (X - 1_{\{C \leq \hat{C}\}} C - PR_f)} \mid C \leq \hat{C} \right]$$

It is easily seen that initial wealth does not matter, hence investors solve:

$$\min_{S_i} E \left[e^{-a_i S_i (X - 1_{\{C \leq \hat{C}\}} C - PR_f)} \right],$$

Assume that $V = v$ has been disclosed truthfully by the manager. Then the objective function reduces to

$$\min_{S_i} e^{-a_i S_i (\mathbf{m} - PR_f) + \frac{1}{2} (a_i S_i)^2 v} E \left[e^{-a_i S_i 1_{\{C \leq \hat{C}\}} C} \mid C \leq \hat{C} \right]$$

or

$$\min_{S_i} -a_i S_i (\mathbf{m} - PR_f) + \frac{(a_i S_i)^2}{2} v + \ln \left(E \left[e^{-a_i S_i 1_{\{C \leq \hat{C}\}} C} \mid C \leq \hat{C} \right] \right)$$

The first order condition for an interior optimum is

$$0 = -(\mathbf{m} - PR_f) + a_i S_i v + \frac{E \left[C e^{-a_i S_i 1_{\{C \leq \hat{C}\}} C} \mid C \leq \hat{C} \right]}{E \left[e^{-a_i S_i 1_{\{C \leq \hat{C}\}} C} \mid C \leq \hat{C} \right]}$$

and the second order condition for a global maximum is fulfilled since

$$v + \frac{E \left[C^2 e^{-a_i S_i 1_{\{C \leq \hat{C}\}} C} \mid C \leq \hat{C} \right]}{E \left[e^{-a_i S_i 1_{\{C \leq \hat{C}\}} C} \mid C \leq \hat{C} \right]} > 0.$$

An equilibrium consists of (I) a set of I individual investors' market-clearing demand, $S_i^*(P^*)$, that fulfill their respective first order condition and (II) an market-clearing price, P^* , such that demand equals supply, that is

$$1 = \sum_{i=1}^I S_i^*(P^*).$$

It is easily verified that the following ensures market clearing

$$P^*(V = v) = \frac{\mathbf{m} - av - J(a|C \leq \hat{C})}{R_f}$$

$$S_i^*(P^*) = a_i^{-1} a$$

where

$$J(a|C \leq \hat{C}) = \frac{E\left[C e^{-a1_{\{c \leq \hat{C}\}}C} \mid C \leq \hat{C}\right]}{E\left[e^{-a1_{\{c \leq \hat{C}\}}C} \mid C \leq \hat{C}\right]} \text{ and } a = \left(\sum_{i=1}^I a_i^{-1}\right)^{-1} \text{ as claimed in the Lemma.}$$

Proof of Lemma 3 (c): To derive the market-clearing price of stock after a manager has not disclosed the variance, recall that, conditional on non-disclosure, either the manager is uninformed (event A) or the manager is informed (event B). If A then the variance is drawn from its prior distribution, whereas if B then $v \in N$. Further recall that in the event of no disclosure, investors hold common beliefs about a no disclosure region, $\hat{N} = \{V \mid V \geq \hat{y}\}$. That is, investors believe that if the manager is informed then the variance is in the no disclosure region, $v \in \hat{N}$. We use the notation $v \geq \hat{y}$ interchangeably with $v \in \hat{N}$.

After observing that the manager did not disclose the variance, each investor solves the following problem:

$$\max_{S_i, B_i} E\left[-a_i^{-1} e^{-a_i W_i} \mid \text{No disclosure}\right] = \mathbf{x}(\hat{y}) E\left[-a_i^{-1} e^{-a_i W_i}\right] + (1 - \mathbf{x}(\hat{y})) E\left[-a_i^{-1} e^{-a_i W_i} \mid V \in \hat{N}\right]$$

$$\text{s.t. } W_i = S_i \left(X - 1_{\{C \leq \hat{C}\}} C\right) + B_i R_f$$

$$W_i^0 = S_i P + B_i$$

where $q(\hat{C}) = \Pr\{c > \hat{C}\}$ and $\mathbf{x}(\hat{C}, \hat{y}) = \frac{q(\hat{C})}{q(\hat{C}) + (1 - q(\hat{C})) \Pr\{V \geq \hat{y}\}}$. We can substitute out the

number of bonds through $B_i = (W_i^0 - S_i P)$ and hence

$$W_i = S_i \left(X - 1_{\{C \leq \hat{C}\}} C\right) + (W_i^0 - S_i P) R_f = S_i \left(X - 1_{\{C \leq \hat{C}\}} C - PR_f\right) + W_i^0 R_f$$

or

$$\begin{aligned} \min_{S_i} & \mathbf{x}(\hat{C}, \hat{y}) E\left[e^{-a_i (S_i (X - PR_f) + W_i^0 R_f)}\right] + (1 - \mathbf{x}(\hat{C}, \hat{y})) E\left[e^{-a_i (S_i (X - 1_{\{C \leq \hat{C}\}} C - PR_f) + W_i^0 R_f)} \mid c \leq \hat{C}, v \geq \hat{y}\right] \\ & = e^{-a_i W_i^0 R_f} e^{-a_i S_i (\mathbf{m} - PR_f)} \left\{ \mathbf{x}(\hat{C}, \hat{y}) E\left[e^{-a_i S_i e}\right] + (1 - \mathbf{x}(\hat{C}, \hat{y})) E\left[e^{a_i S_i C - a_i S_i e} \mid c \leq \hat{C}, v \geq \hat{y}\right] \right\} \\ & = e^{-a_i W_i^0 R_f} e^{-a_i S_i (\mathbf{m} - PR_f)} \left\{ \mathbf{x}(\hat{C}, \hat{y}) E\left[e^{(a_i S_i)^2 V / 2}\right] + (1 - \mathbf{x}(\hat{C}, \hat{y})) E\left[e^{a_i S_i C} e^{(a_i S_i)^2 V / 2} \mid c \leq \hat{C}, v \geq \hat{y}\right] \right\} \end{aligned}$$

Taking natural logarithm, this optimization problem can be rewritten as

$$\min_{S_i} -a_i S_i (\mathbf{m} - PR_f) + \ln(L(S_i | \hat{C}, \hat{y}))$$

where

$$L(S_i | \hat{C}, \hat{y}) = \mathbf{x}(\hat{C}, \hat{y}) E \left[e^{(a_i S_i)^2 V / 2} \right] + (1 - \mathbf{x}(\hat{C}, \hat{y})) E \left[e^{a_i S_i 1_{\{c \leq \hat{c}\}}^C} e^{(a_i S_i)^2 V / 2} \mid c \leq \hat{C}, v \geq \hat{y} \right]$$

which is a convex optimization problem.

Let f denote the bivariate distribution of variance, v , and information costs, c . For notational convenience we define the probability measure, $g(v, c | \hat{y}, \hat{C})$, as follows.

First, let $g(v, c | \hat{y}, \hat{C}) = 0$ if $v \leq \hat{y}$ and $c \leq \hat{C}$. Next, for $c \leq \hat{C}$, let

$$g(v, c | \hat{y}, \hat{C}) = \frac{f(v, c)}{q(\hat{C}) + (1 - q(\hat{C})) \Pr\{v \geq \hat{y}\}}$$

such that

$$q(\hat{C}) f(v) = \int_{\hat{C}}^{\infty} f(v, c) dc$$

$$q(\hat{C}) = \int_0^{\infty} \left(\int_0^{\hat{C}} f(v, c) dc \right) dv$$

Last, for $v > \hat{y}$ and $c < \hat{C}$, let

$$g(v, c) = \frac{f(v, c)}{q(\hat{C}) + (1 - q(\hat{C})) \Pr\{v \geq \hat{y}\}}.$$

Hence,

$$(1 - q(\hat{C})) f(v) = \int_0^{\hat{C}} f(v, c) dc$$

$$(1 - q(\hat{C})) \Pr\{v \geq \hat{y}\} = \int_{\hat{y}}^{\infty} \left(\int_0^{\hat{C}} f(v, c) dc \right) dv$$

Then the optimization problem can be simplified because

$$L(S_i | \hat{C}, \hat{y}) = E^{g(v, c | \hat{y}, \hat{C})} \left[e^{a_i S_i 1_{\{c \leq \hat{c}\}}^C} e^{(a_i S_i)^2 V / 2} \right]$$

where the superscript indicates that the expectation is taken under the probability measure, g . Since

$$\begin{aligned} \frac{\partial L(S_i | \hat{C}, \hat{y})}{\partial S_i} &= E^{g(v, c | \hat{y}, \hat{C})} \left[\left(a_i 1_{\{c \leq \hat{c}\}}^C + a_i^2 S_i V \right) e^{a_i S_i 1_{\{c \leq \hat{c}\}}^C} e^{(a_i S_i)^2 V / 2} \right] \\ &= a_i E^{g(v, c | \hat{y}, \hat{C})} \left[\left(1_{\{c \leq \hat{c}\}}^C + a_i S_i V \right) e^{a_i S_i 1_{\{c \leq \hat{c}\}}^C} e^{(a_i S_i)^2 V / 2} \right] \end{aligned}$$

The first order condition for an interior solution is

$$a_i (\mathbf{m} - PR_f) = l(S_i | \hat{C}, \hat{y})$$

where

$$l(S_i | \hat{C}, \hat{y}) = \frac{\frac{\partial L(S_i | \hat{C}, \hat{y})}{\partial S_i}}{L(S_i | \hat{C}, \hat{y})}$$

An equilibrium consists of (I) a set of I individual investors' market-clearing demand, $S_i^*(P^*)$, that fulfill their respective first order condition and (II) an market-clearing price, P^* , such that demand equals supply, that is

$$1 = \sum_{i=1}^I S_i^*(P^*).$$

The proof is complete if we verify that the following ensures market clearing

$$P^*(V=v) = \frac{m - l(a_i^{-1}a | c \leq \hat{C})}{R_f}$$

$$S_i^*(P^*) = a_i^{-1}a$$

where

$$L(a_i^{-1}a | \hat{C}, \hat{y}) = E^{g(v, c | \hat{N})} \left[e^{a1_{\{c \leq \hat{C}\}} C} e^{a^2 V / 2} \right]$$

Hence, the proof is complete.

Proof of Theorem 4:

Assume that the cost of the information acquisition is observable to investors. The existence of a partial disclosure equilibrium means the manager is indifferent between investing or not investing at the cutoff level of investment costs. This means the expected market value of the firm (prior to learning the variance, V) is the same whether he invests or not. In the latter case, he always gets the non disclosure price, while in the former, he gets the disclosure price if both variance and cost are sufficiently low. Hence, for cost above the cutoff, the prices are identical, while for cost below the cutoff, the following equation holds.

$$P^*(v, c | c \geq \hat{C} \text{ or } v \geq \hat{y}) = \Pr\{v \geq \hat{y}\} P^*(v, c | c \geq \hat{C} \text{ or } v \geq \hat{y}) + \Pr\{v \leq \hat{y}\} E[P(v, c) | v \leq \hat{y}]$$

or

$$\frac{m - K(a | \hat{C}, \hat{y})}{R_f} = E \left[\frac{m - av - \hat{C}}{R_f} \Big| v \leq \hat{y} \right]$$

or

$$\hat{C} = K(a | \hat{C}, \hat{y}) - aE[v | v \leq \hat{y}] = \frac{E^{g(v, c | \hat{y}, \hat{C})} \left[\left(1_{\{c \leq \hat{C}\}} C + aV \right) e^{a1_{\{c \leq \hat{C}\}} C} e^{a^2 V / 2} \right]}{E^{g(v, c | \hat{y}, \hat{C})} \left[e^{a1_{\{c \leq \hat{C}\}} C} e^{a^2 V / 2} \right]} - aE[v | v \leq \hat{y}].$$

Mandatory disclosure increases the amount of disclosure only if full disclosure is not a possible equilibrium under voluntary disclosure. This is true only if the cost on the left hand side exceeds the benefits on the right hand side, evaluated at $\hat{y} = \underline{v}$. This is the condition given in theorem 4, completing the proof.

Proof of Theorem 5:

Assume that the costs, variances and distributions are given by the binary distributions.

Let cost equal $c = \bar{C} > 0$ with probability p_C and let cost equal $c = \underline{C} \geq 0$ with probability $(1 - p_C)$. Let the variance equal $v = \bar{V} > 0$ with probability p_V and let the variance equal

$v = \underline{v} \geq 0$ with probability $(1 - p_v)$. Recall that for any cutoff variance $\underline{v} < \hat{y} < \bar{v}$ and any cutoff costs, $\underline{c} < \hat{c} < \bar{c}$, the probability measure, $g(v, c | \hat{y}, \hat{c})$ from the proof of lemma 4 (c) equals zero for realizations below both of the cutoffs and is given as

$$g(v, c | \hat{y}, \hat{c}) = \frac{f(v, c)}{q(\hat{c}) + (1 - q(\hat{c})) \Pr\{v \geq \hat{y}\}}$$

for realizations above either cutoff. Hence $g(v, c | \hat{y}, \hat{c})$ is given as follows for the three realizations having a non-zero probability.

$$g(v = \bar{v}, c = \bar{c} | \hat{y}, \hat{c}) = (p_v p_c) / D; \quad g(v = \underline{v}, c = \bar{c} | \hat{y}, \hat{c}) = ((1 - p_v) p_c) / D; \quad \text{and} \\ g(v = \bar{v}, c = \underline{c} | \hat{y}, \hat{c}) = (p_v (1 - p_c)) / D, \quad \text{where } D = (p_c + p_v (1 - p_c)).$$

Using these probabilities, the risk premium under non-disclosure, $K(a | \hat{y}, \hat{c})$, can be written explicitly as follows:

$$K(a | \hat{y}, \hat{c}) = \frac{\frac{p_v p_c}{D} (\underline{c} + a \bar{v}) e^{a(\underline{c} + a \bar{v} / 2)} + \frac{(1 - p_v) p_c}{D} (\underline{c} + a \underline{v}) e^{a(\underline{c} + a \underline{v} / 2)} + \frac{(1 - p_c) p_v}{D} (\underline{c} + a \bar{v}) e^{a(\underline{c} + a \bar{v} / 2)}}{\frac{p_v p_c}{D} e^{a(\underline{c} + a \bar{v} / 2)} + \frac{(1 - p_v) p_c}{D} e^{a(\underline{c} + a \underline{v} / 2)} + \frac{(1 - p_c) p_v}{D} e^{a(\underline{c} + a \bar{v} / 2)}} \\ = \frac{p_v (\underline{c} + a \bar{v}) e^{a(\underline{c} + a \bar{v} / 2)} + (1 - p_v) p_c (\underline{c} + a \underline{v}) e^{a(\underline{c} + a \underline{v} / 2)}}{p_v e^{a(\underline{c} + a \bar{v} / 2)} + (1 - p_v) p_c e^{a(\underline{c} + a \underline{v} / 2)}}.$$

Next, recall from the proof of theorem 4 that a partial disclosure equilibrium is characterized by the following condition on the cost cutoff:

$$\hat{c} = K(a | \hat{y}, \hat{c}) - aE[v | v \leq \hat{y}].$$

Using the above computation of the non-disclosure risk premium, $K(a | \hat{y}, \hat{c})$, and recognizing that for this example, $E[v | v \leq \hat{y}] = \underline{v}$, we can write this condition as follows:

$$\hat{c} = K(a | \hat{y}, \hat{c}) - \underline{v} = \frac{p_v (\underline{c} + a \bar{v}) e^{a(\underline{c} + a \bar{v} / 2)} + (1 - p_v) p_c (\underline{c} + a \underline{v}) e^{a(\underline{c} + a \underline{v} / 2)}}{p_v e^{a(\underline{c} + a \bar{v} / 2)} + (1 - p_v) p_c e^{a(\underline{c} + a \underline{v} / 2)}} - \underline{v} \\ = \frac{p_v (\underline{c} + a (\bar{v} - \underline{v})) e^{a(\underline{c} + a \bar{v} / 2)} + (1 - p_v) p_c \underline{c} e^{a(\underline{c} + a \underline{v} / 2)}}{p_v e^{a(\underline{c} + a \bar{v} / 2)} + (1 - p_v) p_c e^{a(\underline{c} + a \underline{v} / 2)}}.$$

Cross multiplying and rearranging terms means we can rewrite this equation as follows:

$$(\hat{c} - \underline{c}) (p_v e^{a(\underline{c} + a \bar{v} / 2)} + (1 - p_v) p_c e^{a(\underline{c} + a \underline{v} / 2)}) = p_v a (\bar{v} - \underline{v}) e^{a(\underline{c} + a \bar{v} / 2)}.$$

Full disclosure holds as long as the left hand side is always less the right hand side, while no disclosure holds if the reverse inequality holds strictly. Since the choice of the bounds on the cost and variances was never restricted, it is clear that we can choose them so that either of these conditions holds. Hence both no disclosure and full disclosure may support an equilibrium, completing the proof of the theorem.

FOR REFEREE ONLY

Appendix B: More Details on Proofs

Detailed Proof of Lemma 1 (a):

After observing the manager's disclosure of the variance, each investor solves the following problem:

$$\begin{aligned} \max_{S_i, B_i} \quad & E\left[-a_i^{-1} e^{-a_i W_i} \mid C \leq \hat{C}\right] \\ \text{s.t.} \quad & W_i = S_i X + B_i R_f \\ & W_i^0 = S_i P + B_i \end{aligned}$$

We can substitute out the number of bonds through

$$B_i = W_i^0 - S_i P$$

and hence

$$W_i = S_i \left(X - 1_{\{C \leq \hat{C}\}} C \right) + (W_i^0 - S_i P) R_f = S_i \left(X - 1_{\{C \leq \hat{C}\}} C - PR_f \right) + W_i^0 R_f$$

Consequently, the investor's problem reduces to

$$\max_{S_i} E\left[-e^{-a_i (S_i (X - 1_{\{C \leq \hat{C}\}} C - PR_f) + W_i^0 R_f)} \mid C \leq \hat{C}\right] = -e^{-a_i W_i^0 R_f} E\left[e^{-a_i S_i (X - 1_{\{C \leq \hat{C}\}} C - PR_f)} \mid C \leq \hat{C}\right]$$

It is easily seen that initial wealth does not matter, hence investors solve:

$$\min_{S_i} E\left[e^{-a_i S_i (X - 1_{\{C \leq \hat{C}\}} C - PR_f)}\right],$$

Assume that $V = v$ has been disclosed truthfully by the manager. Then the objective function reduces to

$$\min_{S_i} e^{-a_i S_i (m - PR_f) + \frac{1}{2} (a_i S_i)^2 v} E\left[e^{-a_i S_i 1_{\{C \leq \hat{C}\}} C} \mid C \leq \hat{C}\right]$$

or

$$\min_{S_i} -a_i S_i (m - PR_f) + \frac{(a_i S_i)^2}{2} v + \ln \left(E\left[e^{-a_i S_i 1_{\{C \leq \hat{C}\}} C} \mid C \leq \hat{C}\right] \right)$$

The first order condition for an interior optimum is

$$0 = -(m - PR_f) + a_i S_i v + \frac{E\left[C e^{-a_i S_i 1_{\{C \leq \hat{C}\}} C} \mid C \leq \hat{C}\right]}{E\left[e^{-a_i S_i 1_{\{C \leq \hat{C}\}} C} \mid C \leq \hat{C}\right]}$$

and the second order condition for a global maximum is fulfilled since

$$v + \frac{E\left[C^2 e^{-a_i S_i 1_{\{C \leq \hat{C}\}} C} \mid C \leq \hat{C}\right]}{E\left[e^{-a_i S_i 1_{\{C \leq \hat{C}\}} C} \mid C \leq \hat{C}\right]} > 0.$$

An equilibrium consists of (I) a set of I individual investors' market-clearing demand, $S_i^*(P^*)$, that fulfill their respective first order condition and (II) an market-clearing price, P^* , such that demand equals supply, that is

$$1 = \sum_{i=1}^I S_i^*(P^*).$$

It is easily verified that market clearing results if

$$P^*(V = v) = \frac{\mathbf{m} - av - J(a|C \leq \hat{C})}{R_f}$$

$$S_i^*(P^*) = a_i^{-1}a$$

where

$$J(a|C \leq \hat{C}) = \frac{E\left[Ce^{-a1_{\{C \leq \hat{C}\}}}\right]}{E\left[e^{-a1_{\{C \leq \hat{C}\}}}\right]} \text{ and } a = \left(\sum_{i=1}^I a_i^{-1}\right)^{-1} \text{ as claimed in the Lemma.}$$

Proof of Lemma 1 for 2 stocks:

A detailed proof for the general case of J stocks is available from the authors.

When there are two stocks, the investor's portfolio choice problem can be written as

$$\begin{aligned} \max_{S_{i1}, S_{i2}} \quad & S_{i1}(\mathbf{m}_1 - P_1 R_f) + S_{i2}(\mathbf{m}_2 - P_2 R_f) - \frac{a_i}{2}(S_{i1}\mathbf{g}_1 + S_{i2}\mathbf{g}_2)^2 \mathbf{s}^2 \\ & - 1_{\{v_1 \notin N_1\}} \frac{a_i}{2}(S_{i1})^2 - a_i^{-1} 1_{\{v_1 \in N_1\}} \ln \left(E^{g(v_1|\hat{N}_1)} \left[e^{-\frac{(a_i S_{i1})^2}{2} V_1} \right] \right) \\ & - 1_{\{v_2 \notin N_2\}} \frac{a_i}{2}(S_{i2})^2 - a_i^{-1} 1_{\{v_2 \in N_2\}} \ln \left(E^{g(v_2|\hat{N}_2)} \left[e^{-\frac{(a_i S_{i2})^2}{2} V_2} \right] \right) \end{aligned}$$

where the expectations in term in the second line is taken conditional on whether or not disclosure occurred. Two associated first order conditions characterize the optimal demand for shares:

$$\begin{aligned} (\mathbf{m}_1 - P_1 R_f) &= a_i (S_{i1}\mathbf{g}_1 + S_{i2}\mathbf{g}_2)\mathbf{g}_1 \mathbf{s}^2 + a_i S_{i1} \left\{ 1_{\{v_1 \notin N_1\}} v_1 + 1_{\{v_1 \in N_1\}} h_1(\hat{N}_j, S_{i2}) \right\} \\ (\mathbf{m}_2 - P_2 R_f) &= a_i (S_{i1}\mathbf{g}_1 + S_{i2}\mathbf{g}_2)\mathbf{g}_2 \mathbf{s}^2 + a_i S_{i2} \left\{ 1_{\{v_2 \notin N_2\}} v_2 + 1_{\{v_2 \in N_2\}} h_2(\hat{N}_2, S_{i2}) \right\} \end{aligned}$$

where

$$h_j(\hat{N}_j, S_{ij}) = \frac{E^{g(v_j|\hat{N}_j)} \left[V_j e^{-\frac{(a_i S_{ij})^2}{2} V_j} \right]}{E^{g(v_j|\hat{N}_j)} \left[e^{-\frac{(a_i S_{ij})^2}{2} V_j} \right]}$$

Under the conjecture that $h_j(\hat{N}_j, S_{ij}) = H_j(\hat{N}_j)$, we find each investor's demand function for stocks 1 and 2 by solving two linear equations with two unknowns:

$$\begin{aligned} a_i^{-1}(\mathbf{m}_1 - P_1 R_f) &= S_{i1}(\mathbf{g}_1^2 \mathbf{s}^2 + \Delta_1) + S_{i2}\mathbf{g}_1\mathbf{g}_2 \mathbf{s}^2 \\ a_i^{-1}(\mathbf{m}_2 - P_2 R_f) &= S_{i1}(\mathbf{g}_1\mathbf{g}_2 \mathbf{s}^2) + S_{i2}(\mathbf{g}_2^2 \mathbf{s}^2 + \Delta_2) \end{aligned}$$

where

$$\Delta_j = 1_{\{v_j \in N_j\}} H_j(\hat{N}_j) + 1_{\{v_j \notin N_j\}} v_j$$

Hence the demand for shares by each individual investor is

$$S_{i1} = a_i^{-1} D^{-1} \left[(\mathbf{g}_2^2 \mathbf{s}^2 + \Delta_2) (\mathbf{m}_1 - P_1 R_f) - (\mathbf{g}_1 \mathbf{g}_2 \mathbf{s}^2) (\mathbf{m}_2 - P_2 R_f) \right]$$

$$S_{i2} = a_i^{-1} D^{-1} \left[(\mathbf{g}_1^2 \mathbf{s}^2 + \Delta_1) (\mathbf{m}_2 - P_2 R_f) - (\mathbf{g}_1 \mathbf{g}_2 \mathbf{s}^2) (\mathbf{m}_1 - P_1 R_f) \right]$$

where

$$D = (\mathbf{g}_1^2 \mathbf{s}^2 + \Delta_1) (\mathbf{g}_2^2 \mathbf{s}^2 + \Delta_2) - (\mathbf{g}_1 \mathbf{g}_2 \mathbf{s}^2)^2.$$

The sum of all individual investors' demand functions must equal the aggregate supply of shares, which we have normalized to one. Hence,

$$\sum_{i=1}^I S_{i1} = \sum_{i=1}^I a_i^{-1} D^{-1} \left[(\mathbf{g}_2^2 \mathbf{s}^2 + \Delta_2) (\mathbf{m}_1 - P_1 R_f) - (\mathbf{g}_1 \mathbf{g}_2 \mathbf{s}^2) (\mathbf{m}_2 - P_2 R_f) \right]$$

$$\sum_{i=1}^I S_{i2} = \sum_{i=1}^I a_i^{-1} D^{-1} \left[(\mathbf{g}_1^2 \mathbf{s}^2 + \Delta_1) (\mathbf{m}_2 - P_2 R_f) - (\mathbf{g}_1 \mathbf{g}_2 \mathbf{s}^2) (\mathbf{m}_1 - P_1 R_f) \right]$$

implies that

$$1 = a^{-1} D^{-1} \left[(\mathbf{g}_2^2 \mathbf{s}^2 + \Delta_2) (\mathbf{m}_1 - P_1 R_f) - (\mathbf{g}_1 \mathbf{g}_2 \mathbf{s}^2) (\mathbf{m}_2 - P_2 R_f) \right]$$

$$1 = a^{-1} D^{-1} \left[(\mathbf{g}_1^2 \mathbf{s}^2 + \Delta_1) (\mathbf{m}_2 - P_2 R_f) - (\mathbf{g}_1 \mathbf{g}_2 \mathbf{s}^2) (\mathbf{m}_1 - P_1 R_f) \right]$$

or

$$aD - \mathbf{m}_1 (\mathbf{g}_2^2 \mathbf{s}^2 + \Delta_2) + \mathbf{m}_2 (\mathbf{g}_1 \mathbf{g}_2 \mathbf{s}^2) = -P_1 R_f (\mathbf{g}_2^2 \mathbf{s}^2 + \Delta_2) + P_2 R_f (\mathbf{g}_1 \mathbf{g}_2 \mathbf{s}^2)$$

$$aD + \mathbf{m}_2 (\mathbf{g}_1 \mathbf{g}_2 \mathbf{s}^2) - \mathbf{m}_2 (\mathbf{g}_1^2 \mathbf{s}^2 + \Delta_1) = P_1 R_f (\mathbf{g}_1 \mathbf{g}_2 \mathbf{s}^2) - P_2 R_f (\mathbf{g}_1^2 \mathbf{s}^2 + \Delta_1)$$

we can now solve for the market-clearing prices of stocks 1 and 2. The solution is

$$P_1(v_1 | \Delta_1) = \left[\mathbf{m}_1 - a \mathbf{g}_1 \mathbf{s}^2 (\mathbf{g}_1 + \mathbf{g}_2) - a \Delta_1 \right] / R_f$$

$$P_2(v_2 | \Delta_2) = \left[\mathbf{m}_2 - a \mathbf{g}_2 \mathbf{s}^2 (\mathbf{g}_1 + \mathbf{g}_2) - a \Delta_2 \right] / R_f$$

Substitution back into each investors demand function, we get find each investor's market-clearing demand for stock j as $S_{ij}^* = a_i^{-1} a$. This shows that there is two-fund separation: Investors hold the same relative weight of all stocks but vary in their relative demand for the bonds according to their risk preferences. Further, we confirm the conjecture by substitution of both market-clearing prices and market-clearing demand for stock into

$$h_j(\hat{N}_j, S_{ij}^*) = \frac{E^{g(v_j | \hat{N}_j)} \left[V_j e^{-\frac{a^2}{2} V_j} \right]}{E^{g(v_j | \hat{N}_j)} \left[e^{-\frac{a^2}{2} V_j} \right]} = H_j(\hat{N}_j).$$

This completes the proof.

Proof of Lemma 1 for general case with J stocks:

Below, we follow Huang and Litzenberger (1988), chapter 4.15. Each investor solves the following problem

$$\max_{\{S_{ij}\}_{j=1}^J, B_i} E \left[-a_i^{-1} e^{-a_i W_i} \right]$$

$$s.t. \quad W_i = B_i R_f + \sum_{j=1}^J S_{ij} X_j$$

$$W_i^0 = B_i + \sum_{j=1}^J S_{ij} P_j$$

where expectations are taken conditional on whether the idiosyncratic variance for each stock is known by investors. As in the proof of lemma 1, the investor's problem can be written as

$$\max_{\{S_{ij}\}_{j=1}^J} E \left[-e^{-a_i \left(\sum_{j=1}^J S_{ij} (X_j - P_j R_f) + W_i^0 \right)} \right] = -e^{-a_i W_i^0} E \left[e^{-a_i \sum_{j=1}^J S_{ij} (X_j - P_j R_f)} \right]$$

$$= -e^{-a_i W_i^0 R_f} e^{-a_i \sum_{j=1}^J S_{ij} (\mathbf{m}_j - P_j R_f)} E \left[e^{-a_i \sum_{j=1}^J S_{ij} (\mathbf{g}_j F + \mathbf{e}_j)} \right]$$

$$= -e^{-a_i W_i^0 R_f} e^{-a_i \sum_{j=1}^J S_{ij} (\mathbf{m}_j - P_j R_f)} E \left[e^{-a_i \left(\sum_{j=1}^J S_{ij} \mathbf{g}_j \right) F} \prod_{j=1}^J E \left[e^{-a_i S_{ij} \mathbf{e}_j} \right] \right]$$

$$= -e^{-a_i W_i^0 R_f} e^{-a_i \sum_{j=1}^J S_{ij} (\mathbf{m}_j - P_j R_f)} e^{\frac{a_i^2}{2} \left(\sum_{j=1}^J S_{ij} \mathbf{g}_j \right)^2 \mathbf{s}^2} \prod_{j=1}^J E \left[e^{-\frac{(a_i S_{ij})^2}{2} V_j} \right]$$

where expectations are taken either (a) conditional on disclosure of the variance or (b) under the g measure conditional on no disclosure. Changing sign, taking logs, and dividing through by $-a$, the investor's problem reduces to

$$\max_{\{S_{ij}\}_{j=1}^J} \sum_{j=1}^J S_{ij} (\mathbf{m}_j - P_j R_f) - \frac{a_i}{2} \left(\sum_{j=1}^J S_{ij} \mathbf{g}_j \right)^2 \mathbf{s}^2 - a_i^{-1} \sum_{j=1}^J \ln \left(E \left[e^{-\frac{(a_i S_{ij})^2}{2} V_j} \right] \right)$$

As in the single stock case of lemma 1, initial wealth does not matter. The first order conditions for an interior maximum are

$$0 = (\mathbf{m}_j - P_j R_f) - a_i \mathbf{g}_j \left(\sum_{n=1}^J S_{in} \mathbf{g}_n \right) \mathbf{s}^2 - a_i^{-1} \frac{E \left[a_i^2 S_{ij} V_j e^{-\frac{(a_i S_{ij})^2}{2} V_j} \right]}{E \left[e^{-\frac{(a_i S_{ij})^2}{2} V_j} \right]}$$

or

$$\mathbf{g}_j \left(\sum_{n=1}^J S_{in} \mathbf{g}_n \right) \mathbf{s}^2 + S_{ij} \frac{E \left[V_j e^{-\frac{(a_i S_{ij})^2}{2} V_j} \right]}{E \left[e^{-\frac{(a_i S_{ij})^2}{2} V_j} \right]} = a_i^{-1} (\mathbf{m}_j - P_j R_f)$$

where expectations are taken conditional on whether the idiosyncratic variance for each stock is known by investors. Clearly, if $V_j = v_j$ has been disclosed by manager j , then

$$\mathbf{g}_j \left(\sum_{n=1}^J S_{in} \mathbf{g}_n \right) \mathbf{s}^2 + S_{ij} v_j = a_i^{-1} (\mathbf{m}_j - P_j R_f).$$

If instead V_j has not been disclosed by manager j , then

$$\mathbf{g}_j \left(\sum_{n=1}^J S_{in} \mathbf{g}_n \right) \mathbf{s}^2 + S_{ij} h_j(\hat{N}_j, S_{ij}) = a_i^{-1} (\mathbf{m}_j - P_j R_f)$$

Where

$$h_j(\hat{N}_j, S_{ij}) = \frac{E^{g(v_j|\hat{N}_j)} \left[V_j e^{-\frac{(a_i S_{ij})^2}{2} V_j} \right]}{E^{g(v_j|\hat{N}_j)} \left[e^{-\frac{(a_i S_{ij})^2}{2} V_j} \right]}$$

To consolidate notation we define the indicator function for the event that disclosure occurs, $1_{\{v_j \notin N_j\}}$, and define

$$\Delta_j = 1_{\{v_j \in N_j\}} v_j h_j(\hat{N}_j, S_{ij}) + 1_{\{v_j \notin N_j\}} v_j$$

Such that

$$\mathbf{g}_j \left(\sum_{l=1}^J S_{il} \mathbf{g}_l \right) \mathbf{s}^2 + S_{ij} \Delta_j = a_i^{-1} (\mathbf{m}_j - P_j R_f)$$

As in the proof of lemma 2, we conjecture (and then verify) that there exists an equilibrium in which $h_j(\hat{N}_j, S_{ij}^*) = H(\hat{N}_j)$ when evaluated at the market-clearing demand for stock, S_{ij}^* . The first order condition for investor i 's demand for stock j can be written as

$$\mathbf{g}_j \left(\sum_{l=1}^J S_{il} \mathbf{g}_l \right) \mathbf{s}^2 + S_{ij} \Delta_j = a_i^{-1} (\mathbf{m}_j - P_j R_f)$$

All J first order conditions for investor i 's demand for stocks can be written on vector form as

$$\underline{\underline{M}} S_i = a_i^{-1} (\underline{\underline{m}} - \underline{\underline{P}} R_f)$$

where

$$\underline{S}_i = (S_{i1}, \dots, S_{iJ})$$

$$\underline{m} = (\mathbf{m}_1, \dots, \mathbf{m}_j)$$

$$\underline{P} = (P_1, \dots, P_J)$$

$$\underline{g} = (\mathbf{g}_1, \dots, \mathbf{g}_J)$$

and

$$\underline{M} = \underline{s}^2 \underline{\Gamma} + \underline{D}$$

$$\underline{\Gamma} = \underline{g} \otimes \underline{g}$$

$$\{\mathbf{g}_i \mathbf{g}_j\}$$

$$\underline{D} = \text{diag}(\Delta_1, \dots, \Delta_J)$$

where the Cronecker product means that each

$$\underline{\Gamma}_{ij} = \{\mathbf{g}_i \mathbf{g}_j\}.$$

Since $D_j > 0$, the matrix M is invertible. Hence,

$$\underline{S}_i = \underline{M}^{-1} a_i^{-1} (\underline{m} - \underline{P} R_f)$$

Summing over i 's on both sides, we find the aggregate demand for stock j

$$\sum_{i=1}^I \underline{S}_i = \sum_{i=1}^I \underline{M}^{-1} a_i^{-1} (\underline{m} - \underline{P} R_f)$$

Since each stock is normalized to be in net supply of one,

$$\underline{1} = \underline{M}^{-1} \left(\sum_{i=1}^I a_i^{-1} \right) (\underline{m} - \underline{P} R_f)$$

where the left hand side is a vector of ones.

$$\underline{M} \underline{1} = \underline{M} \underline{M}^{-1} \left(\sum_{i=1}^I a_i^{-1} \right) (\underline{m} - \underline{P} R_f)$$

which reduces to

$$\underline{M} \underline{1} = a^{-1} (\underline{m} - \underline{P} R_f)$$

where $a^{-1} = \left(\sum_{i=1}^I a_i^{-1} \right)$. Rearranging terms, the market-clearing stock prices follow

$$\underline{P} = [\underline{m} - a \underline{M} \underline{1}] / R_f.$$

To verify the conjecture, substitute the market-clearing stock price into the individual investor's demand function:

$$\underline{S}_i^* = \underline{M}^{-1} a_i^{-1} (\underline{m} - [\underline{m} - a \underline{M} \underline{1}]) = \underline{M}^{-1} a_i^{-1} a \underline{M} \underline{1} = a_i^{-1} a \underline{1}$$

Since $a_i a_{ij}^* = a$ is the same for all investors and all firms, the conjecture that

$$h_j(\hat{N}_j, S_{ij}^*) = H(\hat{N}_j) \text{ follows.}$$