

No-Arbitrage Macroeconomic Determinants of the Yield Curve*

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First Draft: October 2004

This Revision: January 2, 2006

*We would like to thank Andrew Ang, Geert Bekaert, Jean Boivin, Larry Christiano, Pierre Collin-Dufresne, Greg Duffee, Silverio Foresi, Rene Garcia, Marc Giannoni, Mike Johannes, Stijn Van Nieuwerburgh, Andrea Roncoroni, Tano Santos, Suresh Sundaresan, Andrea Tambalotti and participants of Columbia's doctoral students, macro lunch and finance lunch workshops, CIREQ-CIRANO Financial Econometrics Conference in Montreal, the CEPR meetings at Gerzensee, Econometric World Congress in London, European Finance Association in Moscow, NYU Stern Five-Star Conference, and seminars at the Federal Reserve Board, the Federal Bank of New York, Goldman Sachs Asset Management, and NYU.

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Abstract

We determine which macroeconomic variables other than inflation and real activity drive the yield curve using a no-arbitrage affine term structure models. We construct a model-based dynamic projection of all the latent factors onto the observable macro factors, which are real activity and inflation. As a result, the factors are decomposed into a macro-component consisting of a linear function of inflation, real activity and their lags, and the truly novel part which is orthogonal to the entire history of the macro variables. The macro-component of a four-factor model can explain 80% of the variation in the short rate and 50% of the slope. Furthermore, we are able to explain the remaining part of the short rate and slope with such measures of monetary shocks as the AAA credit spread, the Money Zero Maturity measure of money supply, and public government debt growth as a measure of fiscal shocks. Finally, we decompose the term premia into the contributions of the identified macro sources of risk. Inflation and liquidity risk premia jointly explain 65% to 85% of the variation in the term premia across the yield curve. Inflation and fiscal shocks have the largest contributions to deviations from the expectation hypothesis.

1 Introduction

In this paper we propose a procedure which allows us to link the traditional latent variables to macroeconomic variables by maximizing the explanatory power of the latter in a term structure model. This procedure exploits the interaction between the macro and latent variables and extracts all macro-related information from the latent variables. As a result, we obtain an internally-consistent interest rate rule, which is a function of the macro variables, their lags, and truly “residual” latent variables that are orthogonal to the entire past history of macro shocks.

By doing this, we are able to address a number of questions left unanswered in the earlier literature. While real activity and inflation are clearly related to the short interest rate, it is less clear which aspects of the yield curve can be explained via these two variables. Because our procedure deliberately uses the latent variables as a last resort in building a term structure model, we can quantify precisely the impact of inflation and real activity and the corresponding risk premia on the yield curve. Finally, the latent variables which are constructed as part of our procedure have a clear interpretation of exogenous shocks orthogonal to inflation and real activity in a standard interest rate rule, and, thus, we can explore their impact on the short interest rate and the whole yield curve. We can also relate them to macro variables other than inflation and real activity.

Our exercise is important for at least four reasons. First, by taking the interaction between the macro and latent variables seriously, one can provide a prominent explanatory role to the macro factors. This is important if one wants to study how information about the macro economy feeds into bond prices.

Second, without proper identification of the role of various state variables, the impact of these variables on risk premia might be poorly identified, as an explanatory role attributed to a latent factor may not be truly orthogonal to observable macro factors. Our procedure is able to translate the information about macro variables into measures of macro risk premia. Therefore, we will be able to assess how risk premia vary with changes in the macro variables.

Third, it is not clear how to define exogenous shocks in the context of no-arbitrage models. The interest rate equation does not typically include an explicit error term as used in the macro literature. Therefore, some of the latent state variables could assume this role. The exogeneity of the latent variables could be enforced by imposing certain independence restrictions. However,

one faces a trade-off in the degree of independence and explanatory power of macro variables: the more parameters controlling correlation are switched off, the larger is the fraction of the yield curve variation that is explained by latent variables. In the traditional VAR literature, rich lag structures act similarly to the latent variables in terms of explanatory power and thus mitigate these issues. Such a solution is infeasible in the case of no-arbitrage models, as the resulting parameter proliferation leads to severe estimation difficulties. In contrast, the residual latent variables obtained from our methodology without any additional restrictions have a natural interpretation of the exogenous shocks because they are orthogonal, by construction, to the macro variables and their entire history.

Finally, despite their prominence in the interest rate rules, inflation and real activity cannot fully explain the term structure. It is desirable to study which other macro variables might impact the yield curve. The only systematic way to do so is to first extract maximum explanatory power from inflation and real activity and then try to relate the residuals to new sources of macro variation. This is precisely what we achieve by constructing the exogenous shocks.

We allow for a rich correlation structure between the macro and latent factors in our model. We explain this correlation via the macro variables by dynamically projecting the latent factors onto real activity and inflation in a fashion consistent with the model specification. As a result, the spot interest rate becomes a linear function of macro variables and their lags, which we interpret as the optimal backward-looking interest rate rule, and a set of new “projection residual” latent factors.¹ The new latent factors are exogenous to the information contained in the macro variables and their entire history, and, therefore, represent the part of the term structure unexplained by the pre-selected variables (real activity and inflation). Thus, our decomposition allows us to exert maximum pressure on the macro variables to explain the term structure.

Our setup is more flexible than the extant no-arbitrage papers because it allows us to study the impact of the lagged macro factors on the term structure, despite the single lag specification of the state variables.² Moreover, since the loadings are computed from the model parameters, we

¹Our lag structure is not arbitrary: recursive projection formulas imply the reliance on all lags and the loadings on these lags are optimal as they are selected to minimize the variance of the residuals.

²One of the specifications in Ang and Piazzesi (2003) allows for twelve lags in inflation and real activity simultaneously, with one lag in the latent factors. In contrast, in our setting, the lags of the macro variables effectively substitute out the latent ones.

avoid overparameterization, an issue reported by Ang and Piazzesi (2003) (AP henceforth). As a result, we are able to reconcile the conflicting findings based on both the VAR analysis of the policy shocks in the macro literature, which relies on observed macro variables and bond yields in a multiple-lag environment (e.g., Evans and Marshall, 1998), and the more recent no-arbitrage VAR specification based only on one lag of observed and latent factors. To date, the macro VAR models were much more successful in explaining the variability of the yield curve via the macro fundamentals because of the flexible lag specification. We uncover similar flexibility in the no-arbitrage models by implementing this projection.

We use the panel of eight yields, inflation and real activity observed at a monthly frequency from 1970 to 2002 in the empirical implementation. Our backward-looking interest rate rule can explain 80% of the short rate variation based exclusively on inflation and real activity and their lags. The quality of the fit deteriorates for slope (50%) and curvature (40%), indicating the need for additional variables in order to explain the whole yield curve. The exogenous residual factors explain the remaining 20% of the level and 50% of the slope, and improve the curvature fit by 10%. These results beg the question of whether these exogenous shocks could be related to other macro fundamentals not captured by inflation and real activity.

Only one of the two residual factors affects the short rate. This factor is correlated with such measures of liquidity as AAA credit spread and the growth rate of MZM measure of money supply. The impact of the other factor is most pronounced at long maturities. It is strongly correlated with the public government debt growth, which is a monthly counterpart of the quarterly budget deficit. We interpret these elements of the model as persistent monetary (AAA, MZM) and fiscal (debt) shocks.

The identified source of macro risk allows us to decompose the traditional affine stochastic discount factor into macro-related components. Specifically, we study the bond term premia and their determinants. Inflation and liquidity risk premia jointly explain 65% to 85% of the variation in the term premia, depending on a bond's maturity. The relative contributions of these two factors change over time with liquidity being more prominent on the short end of the curve. Finally, inflation and fiscal shock contribute most to the violation of the expectation hypothesis.

Our paper is related to the growing literature on the term structure models that incorporate

macro variables.³ Apart from the obvious link to the work of AP, our results are most closely related to four specific papers. Evans and Marshall (2002) pursue the similar goal of identifying the macro variables that drive the yield curve in the context of the traditional VAR models. Duffee (2005) focuses on the contribution of macro variables to the term structure as we do. He does it by avoiding specification and estimation of latent variables. This approach offers flexibility by allowing for a potentially large set of models that are consistent with the macro-side of the specification. The flexibility comes at the cost of partial term-structure implications. Rudebusch (2002) tries to distinguish the monetary-policy-inertia and serially-correlated-shock versions of the interest rate rule. He argues that it is impossible to distinguish the two without incorporating the information from the full term structure. We do this explicitly and find support for his conjecture of the policy inertia illusion. Finally, Dai and Philippon (2004) also argue, in the context of a no-arbitrage macro model, but in a different setup, that the budget deficit is an important ingredient of long-maturity bonds.

The paper is organized in five sections and one appendix. Section 2 introduces the theoretical model, describes the projection setup, and discusses the relationships to earlier approaches. In section 3 the estimation strategy is discussed and in section 4 the findings are presented. The final section concludes. Appendix contains technical details of the projection procedure.

2 The Model

We develop the theoretical underpinnings of our approach in this section. First, in section 2.1 we discuss the interest rate rule specification. Next, in section 2.2, we briefly review Gaussian term structure models. We then describe our projection approach in section 2.3. In section 2.4, we relate our backward-looking rule to the monetary policy inertia. We then discuss the relationship between traditional VAR models and our framework in section 2.5.

³This work includes Ang, Dong, and Piazzesi (2004), Bekaert, Cho, and Moreno (2003), Buraschi and Jiltsov (2005), Diebold, Rudebusch, and Arouba (2005), Gallmeyer, Hollifield, and Zin (2005), Hördahl, Tristani, and Vestin (2005), Law (2004), Rudebusch and Wu (2005), Wachter (2005), among others.

2.1 Interest Rate Rule

We build on the large literature regarding the Taylor rules, which relies on inflation and real activity as the only systematic response variables in interest rate rules. Under this interpretation, other macro variables that could potentially affect the short or long interest rates can come in through the channel of exogenous shocks only; i.e., they command an occasional response from the monetary authority.

Specifically, the state of the economy is captured by the vector $z_t = (m'_t, x'_t)'$. In particular, the vector of macroeconomic variables m_t is equal to $(g_t, \pi_t)'$, where g_t and π_t are monthly real activity and inflation rate, respectively. The remaining factors x_t are latent.

Similar to Clarida, Gali, and Gertler (2000), we assume that the market participants determine the spot interest rate based on their expectations of the average future values of both macro and latent variables over τ periods,

$$r_t = \gamma_0 + \gamma'_z E_t \left(\frac{1}{\tau} \sum_{i=1}^{\tau} z_{t+i} \right). \quad (2.1)$$

The basic form of this forward-looking rule can be justified as being optimal for an agent with a quadratic loss function under generic assumptions. We further assume that lags of the state variables embody all the information about the conditional expectations. Clarida, Gali, and Gertler (2000) point out that the rule (2.1) simplifies to a basic Taylor rule under this assumption. This implication allows us to be agnostic about the specifics of the forward-looking rule and view it simply as a background for our discussion. We now demonstrate the implications of our setup under specific assumptions about the dynamics of z_t .

2.2 The Affine No-Arbitrage Model

Our setup is similar to that of AP. The state vector z_t follows a VAR(1) process

$$\begin{aligned} z_t &= \mu + \Phi z_{t-1} + \Sigma \epsilon_t & (2.2) \\ &= \begin{bmatrix} \mu^m \\ \mu^x \end{bmatrix} + \begin{bmatrix} \Phi^{mm} & \Phi^{mx} \\ \Phi^{xm} & \Phi^{xx} \end{bmatrix} \begin{bmatrix} m_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \Sigma^{mm} & \Sigma^{mx} \\ \Sigma^{xm} & \Sigma^{xx} \end{bmatrix} \begin{bmatrix} \epsilon_t^m \\ \epsilon_t^x \end{bmatrix}, & (2.3) \end{aligned}$$

where $\epsilon_t \sim N(0, I)$. We denote the vector of parameters controlling the dynamics of state by Θ . The block representation will be useful for later discussions. In particular, the Markov structure of z_t is

consistent with our assumption that lags of the state variables incorporate all relevant information for the expectations. Therefore, consistent with Clarida, Gali, and Gertler (2000) and similar to Ang, Dong, and Piazzesi (2004) (ADP henceforth), (2.1) and (2.2) imply the following simple rule for the interest rate:

$$\begin{aligned} r_t &= \gamma_0 + \frac{\gamma'_z}{\tau} \left(\sum_{i=1}^{\tau} (I - \Phi)^{-1} (I - \Phi^i) \mu + (I - \Phi)^{-1} (I - \Phi^\tau) \Phi z_t \right) \\ &\equiv \delta_0 + \delta'_z z_t = \delta_0 + \delta'_m m_t + \delta'_x x_t. \end{aligned} \quad (2.4)$$

This representation of the interest rate allows us to complete the usual affine no-arbitrage framework by specifying the stochastic discount factor ξ_t

$$\log \xi_t = -r_{t-1} - \frac{1}{2} \Lambda'_{t-1} \Lambda_{t-1} - \Lambda_{t-1} \epsilon_t, \quad (2.5)$$

where the market prices of risk follow the essentially-affine specification (Duffee, 2002)

$$\Lambda_t = \Lambda_0 + \Lambda_z z_t. \quad (2.6)$$

Therefore, yields on zero-coupon bonds are linear in the state variables,

$$\begin{aligned} y_t(\tau) &= -\frac{1}{\tau} \log E_t \left(\prod_{s=t+1}^{t+\tau} \xi_s \right) = a^Q(\tau) + b^Q(\tau)' z_t \\ &\equiv \underbrace{a^P(\tau) + b^P(\tau)' z_t}_{\text{Short rate expectations}} + \underbrace{a^{TP}(\tau) + b^{TP}(\tau)' z_t}_{\text{Term premium}}, \end{aligned} \quad (2.7)$$

where τ is the respective maturity, and a^Q and b^Q solve recursive equations with boundary conditions $a^Q(1) = \delta_0$ and $b^Q(1) = \delta_z$ (see, e.g., Bekaert and Grenadier, 2001). In particular, one-month yield coincides with the short rate, $y_t(1) = r_t$. The last line decomposes the yields into the expectations of the future short rates and the term premium. The former component is equal to the usual factor loadings computed under the assumption of zero market prices of risk.

2.3 “Optimal” Interest Rate Rule

There is no clarity regarding the role of the latent variables x in a generic specification of the interest rate rule in (2.4). The key question is whether x represents macro variables which command systematic responses from the Fed or exogenous policy shocks. Typically, proposed resolutions of this ambiguity rely on additional assumptions regarding these factors. For example, the assigned

interpretations include the monetary shock (ADP and Law, 2004), and the budget deficit (Dai and Philippon, 2004).

We take a more agnostic view of this problem and assume that the latent variables are potentially correlated with macro variables, and can represent systematic or exogenous shock components, or both. This view allows us to decompose the latent factors into a macro-related component and an innovation component. We construct the decomposition by dynamically projecting the latent factors onto the macro factors. The projection residuals, which are orthogonal to the macro variables and their entire history, by construction, represent the true exogenous shocks.

To be more specific, we can rewrite the interest rate equation (2.4) as:

$$r_t = \delta_0 + \delta'_m m_t + \delta'_x x_t = \delta_0 + \delta'_m m_t + \delta'_x \hat{x}(M_t) + \delta'_x f_t, \quad (2.8)$$

where $M_t = \{m_t, m_{t-1}, \dots, m_0\}$ denotes the entire history of m up to time t ; $\hat{x}(M_t)$ denotes the linear projection of x_t onto M_t ; and f_t is the residual of x_t , which is orthogonal to M_t

$$f_t = x_t - \hat{x}(M_t). \quad (2.9)$$

The new latent factors f will be truly exogenous, given the set of macro variables m .

By definition, the linear projection \hat{x} has a simple functional form that fits nicely into the overall linear structure of the model and could be thought of as having a VAR structure

$$\hat{x}(M_t) = c(\Theta) + \sum_{j=0}^t c_{t-j}(\Theta) m_{t-j}, \quad (2.10)$$

where the matrices c are functions of parameters Θ that control the dynamics of the state variables. The dependence of the coefficients c on the model parameters allows us to avoid overparameterization, a problem in all multiple-lag studies. Appendix A provides the details of the procedure.

We can view this representation as the backward-looking interest rate rule considered both in the macro (e.g., Christiano, Eichenbaum, and Evans, 1996) and no-arbitrage (ADP) work. Duffee (2005) also advocates using a large number of lags of the macro variables in order to separate the contributions of the observable and latent variables.⁴ He selects the lag length based on Akaike and Bayesian Information Criteria in the framework of autoregressive processes estimated on the macro

⁴Duffee's purpose is to learn about joint dynamics of the macro variables and yields. He intentionally ignores their interactions with latent factors.

data. The distinguishing characteristic of our representation is the optimality of our functional form in the least-square sense. Therefore, the functional form of the coefficients and the lag structure are determined by the properties of the state variables in (2.2), rather than by assuming an arbitrary functional form.

Our approach simultaneously achieves two objectives. First, it eliminates the need for traditional latent variables, such as level, slope, or curvature, which are required for a successful fit in the absence of other variables. Our decomposition forces the macro variables to explain the term structure of the interest rates. Second, the residual variables have a clean interpretation of exogenous shocks.

We jointly incorporate the two features that are specific to either macro or no-arbitrage literature in a consistent fashion. As in the no-arbitrage literature, we construct a joint model of macro and term structure dynamics. Unlike this literature, we do not use arbitrary restrictions to identify exogenous factors, but rather, we rely on a well-defined projection methodology to separate systematic and exogenous shocks. Similar to macro work, we develop a flexible VAR model with a rich lag structure. However, instead of using arbitrary lag structure and lag weights determined by the best unconditional fit to the data, we use an optimal approach entirely consistent with the model dynamics. We elaborate on each of these two points in the following subsections.

2.4 Monetary Policy Inertia

Monetary policy inertia is a perfect ground for understanding our projection-based decomposition. ADP show that when a factor x_t is a scalar, the interest rate rule (2.4) can be rewritten in the equivalent form

$$r_t = \tilde{\delta}_0 + \delta'_m m_t + \tilde{\delta}'_m m_{t-1} + \tilde{\delta}_r r_{t-1} + \tilde{\epsilon}_t, \quad (2.11)$$

where tilde highlights parameters which are functions of the original parameters of the model. The case $\tilde{\delta}_m = 0$ corresponds to the traditional monetary policy inertia specification, which is empirically successful (for the details and references, see Rudebusch, 2002). Similarly, Piazzesi (2003), in a no-arbitrage model which explicitly accounts for the Fed decision-making process, finds that the implied interest rate rule incorporates a response to the two-year yield. Such rules imply an adjustment

of the interest rate target which suggests policy inertia, or interest rate smoothing behavior of the monetary authority. However, Rudebusch (2002) questions this interpretation because (2.11) implies counterfactually strong forecastability of the interest rates. He conjectures that “the illusion of monetary policy inertia” reflects persistent shocks. Moreover, it is hard to interpret $\tilde{\epsilon}_t$ as an exogenous shock because it is correlated with the lagged macro variables.

Our projection procedure decomposes a vector x of any dimension into a component associated with the macro variables and a new component f . On the one hand, the lagged macro component $\hat{x}(M_t)$ achieves the objective of interest rate smoothing. On the other hand, because f is orthogonal to the macro variables and their entire history by construction, f can be interpreted as exogenous shocks. This orthogonality also helps to disentangle the explanatory power of macro variables and that of the residual factors f . Our decomposition

$$r_t = \delta_0 + \delta'_x c(\Theta) + \delta'_m m_t + \delta'_x \sum_{j=0}^t c_{t-j}(\Theta) m_{t-j} + \delta'_x f_t, \quad (2.12)$$

implied by (2.8)-(2.10), is fundamentally different from a seemingly related one obtained via recursive substitution of r in (2.11),

$$r_t = \tilde{\delta}_0 \sum_{j=0}^{\infty} \tilde{\delta}_r^j + \delta'_m m_t + \left(\tilde{\delta}_r \delta'_m + \tilde{\delta}_m' \right) \sum_{j=1}^{\infty} \tilde{\delta}_r^{j-1} m_{t-j} + \sum_{j=0}^{\infty} \tilde{\delta}_r^j \tilde{\epsilon}_{t-j}. \quad (2.13)$$

In this MA(∞) representation, the error part is not constructed optimally. Moreover, while $\tilde{\epsilon}_t$ is orthogonal to the contemporaneous macro variables, the whole error term is correlated with the macro-component. As a result, it is difficult to assess the explanatory power of the macro variables versus the residual, as this will depend on the order of conditioning. Finally, our approach applies to any number of latent factors. Thus, the two decompositions have different properties and interpretations.

2.5 VAR and No-Arbitrage Models

It is instructive to understand the relation between macroeconomic VARs and no-arbitrage approaches. We will use the following VAR considered in Evans and Marshall (2002) as a reference in our discussion:

$$\begin{bmatrix} m_t \\ y_t \end{bmatrix} = \begin{bmatrix} \mu^m \\ \mu^y \end{bmatrix} + \begin{bmatrix} \Phi^{mm}(L) & 0 \\ \Phi^{ym}(L) & \Phi^{yy}(L) \end{bmatrix} \begin{bmatrix} m_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \Sigma^{mm} & 0 \\ \Sigma^{ym} & \Sigma^{yy} \end{bmatrix} \begin{bmatrix} \epsilon_t^m \\ \epsilon_t^y \end{bmatrix} \quad (2.14)$$

where y_t denotes a vector of yields $y_t(\tau)$ with various maturities τ . $\Phi(L)$ denotes matrix polynomials in the lag operator L . The number of lags is typically specified in an ad-hoc fashion related to the data frequency; i.e., 12 lags are used for monthly data. The blocks of zeroes imply exogeneity of macro variables with respect to the yields.

In this setting, ϵ_t^m is interpreted as a vector of structural macroeconomic shocks. The elements of ϵ_t^y are yield shocks. The identification scheme for Σ^{mm} , Σ^{ym} , and Σ^{yy} will determine the interpretation of the shocks' impact on the variables of interest (impulse response functions). The VAR specification in (2.14) makes no attempt to connect yields and macro variables in a structural way. Hence there is no particular relationship between the elements of Σ^{mm} and Σ^{ym} , or between ϵ_t^m and ϵ_t^y .

We can express the relationships between the yields and state variables in our no-arbitrage model as a VAR as well,

$$\begin{bmatrix} z_t \\ y_t \end{bmatrix} = \begin{bmatrix} \mu \\ a + b'\mu \end{bmatrix} + \begin{bmatrix} \Phi & 0 \\ b'\Phi & 0 \end{bmatrix} \begin{bmatrix} z_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \Sigma & 0 \\ b'\Sigma & 0 \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \omega_t \end{bmatrix}. \quad (2.15)$$

The no-arbitrage requirement imposes tight restrictions on the VAR specification. In particular, a researcher has to be concerned with an identifying scheme for matrix Σ only, which does not change with the number of yields considered. This is in sharp contrast to the macro VAR specification where the three matrices Σ^{mm} , Σ^{ym} , and Σ^{yy} have to be considered, and the latter two grow with the number of yields. This effect of the no-arbitrage models is well understood.

However, one of the criticisms of the no-arbitrage approach in the macro literature is that, despite well-motivated identification assumptions, these models explain much less of the variation in the yields via the macro factors than do the macro VARs (see, e.g., Evans and Marshall, 2002). We address this critique by showing that without our decomposition in (2.8)-(2.10), the macro variables are not given a fair chance to explain the term structure.

In addition to resolving this problem, we establish the optimal lag weights in the VAR (2.14) that are implied by the term structure model. Consequently, all optimal weights are computed from a small set of the parameters of our model which determine the dynamics of the state z . A macro VAR structure does not allow for this feature; the weights on lagged variables are determined by the best unconditional fit and can lead to parameter proliferation.

We can combine the no-arbitrage VAR (2.15) with the projection relationships (2.8)-(2.10) to obtain:

$$\begin{bmatrix} m_t \\ f_t \\ y_t \end{bmatrix} = \begin{bmatrix} \mu^m \\ 0 \\ \mu^y \end{bmatrix} + \begin{bmatrix} \Phi^{mm}(L) & \Phi^{mx} & 0 \\ 0 & \Phi^{ff} & 0 \\ \Phi^{ym}(L) & \Phi^{yf} & 0 \end{bmatrix} \begin{bmatrix} m_{t-1} \\ f_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \Sigma^{mm} & \Sigma^{mx} & 0 \\ \Sigma^{fm} & \Sigma^{ff} & 0 \\ \Sigma^{ym} & \Sigma^{yf} & 0 \end{bmatrix} \begin{bmatrix} \epsilon_t^m \\ \epsilon_t^x \\ \omega_t \end{bmatrix}. \quad (2.16)$$

This is a stylized representation; in order to focus on the most important issues, we do not reproduce full expressions for some matrices (e.g., $\Phi^{ym}(L)$).⁵ We can now develop a better appreciation of the no-arbitrage models by comparing this representation to (2.14).

At a first glance, the expression in (2.16) still appears to be more restrictive as compared to the macro VAR specification; many elements of the matrices are restricted to zero, and blocks associated with factor f , such as Φ^{yf} , do not have lags. A closer inspection of (2.16) reveals that if we restrict the block Φ^{ff} to zero, we will obtain a specification that is qualitatively similar to the macro VAR (2.14). In this case, f will play the role similar to that of the shocks ϵ^y . Hence, we do not lose much flexibility by imposing the no-arbitrage restrictions imposed on the matrices $\Phi^{mm}(L)$ and $\Phi^{ym}(L)$, in particular.

In summary, the no-arbitrage framework offers flexibility comparable to that of unrestricted VAR models. However, it is hard to establish the contributions of various factors without a proper normalization. Such a normalization is provided in our interest rate decomposition (2.8).

3 Empirical Setup

We start with a brief description of the dataset and then proceed with the estimation methodology. We conclude the section with the description of our parameter identification strategy.

⁵Expressions for most matrices are quite involved because they are simultaneously based on no-arbitrage pricing and filtering formulas. Appendix A provides the details.

3.1 Data

We use monthly time series of macro and bond data from 1970 to 2002. We use CPI and help wanted index taken from FRED to proxy for the price level and real activity, respectively.⁶ We compute annual log-changes in the CPI index to construct our measure of inflation. The differencing frequency is selected to mitigate the noise in month-to-month changes. Because the Help index ranges from 0 to 100, we divide it by 10 to obtain an order of magnitude similar to those of yields and inflation.

We use an unsmoothed Fama-Bliss approximation of the zero coupon bond prices of maturities three and six months and one, two, three, five, seven, and ten years.⁷ It is important to measure the full yield curve because its slope is correlated with the macro environment (Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998). Also, using rich yield data helps to identify the risk premia.

We provide preliminary descriptive analysis of the data in Table 1. We implement univariate regressions and unrestricted VARs in order to establish the explanatory power of inflation and real activity with respect to level, slope and curvature of the term structure. The omitted variables problem is always a potential issue in such an analysis, especially in the case of regressions. We provide the numbers as a simple benchmark for our analysis, which should be interpreted with care. The reported R^2 s indicate that the two macro variables can explain more than 50% of the level, especially when their lags are used as well. The contemporaneous macro variables seem to robustly explain about 40% of the slope. There is a lot of variation in the curvature depending on the particular methodology.

3.2 The Econometric Method

We estimate our term structure model via maximum likelihood with the Kalman filter. We place estimation errors on all yields so that the latent factors are not associated with pre-specified maturities. We assume that the macro variables are observed without error. There are a lot of compelling

⁶The index Help Wanted Advertising in Newspapers is used by AP and Dai and Philippon (2004) as well. It is thought of as a leading indicator of real activity. Its advantage is that it is stationary and, hence, can be used as is. We have also considered linearly detrended per capita employment as a proxy for real activity with largely similar results.

⁷We are grateful to Robert Bliss for providing us with the data.

arguments in favor of introducing macro measurement errors. However, since the literature is in the early stages of combining macro variables and the term structure, our model is likely to be misspecified. As a result, we may generate fitted macro variables in a model with measurement error that look different from the original series.

3.2.1 Observation Equations

The state equation in the state-space system is the Gaussian VAR(1) described in (2.2). The observation equations can be represented in the following way:

$$y_t = a + b'_m m_t + b'_x x_t + \xi, \quad (3.1)$$

where y represents the vector of eight yields of maturities from one month to ten years. The right-hand side of the equation is an expanded version of the no-arbitrage expression for the yield in (2.7). The measurement errors are denoted by ξ . We assume the simplest possible structure of the errors; that they are independent and normally distributed with zero mean and standard deviation σ_ξ (for each individual element of the vector ξ). We need not specify a more flexible error structure because these variables are introduced in addition to the VAR shocks that we considered earlier.

After estimating a model and filtering the state variables, we can use the projection (2.10) to construct our interest rate decomposition (2.8). We will denote the filtered state variables by $\hat{x}(M_t, Y_t)$, where, as before, the capital letters M and Y denote the entire history of m and y , respectively, up to time t .⁸ The estimated orthogonalized residual is $\hat{f}_t = \hat{x}(M_t, Y_t) - \hat{x}(M_t)$.

3.2.2 Number of Factors and Identification

We estimate a model with a total of four factors; i.e., $x_t = (x_{1t}, x_{2t})'$. First, the principal component analysis (available upon request) suggests at least four factors to explain the joint variation in the macro variables and the yield data. Second, we have estimated a three-factor model, and discovered that it had no hope of capturing the slope of the yield curve. Therefore, we must identify the maximally flexible four-factor model with two observable factors.

⁸Note the difference between the filter of the latent variable x based on all observables, $\hat{x}(M_t, Y_t)$, and the projection of the latent variable x onto the history of macro variables, $\hat{x}(M_t)$, discussed in the section 2.3.

Dai and Singleton (2000) show that if all factors are latent in the Gaussian system, the parameters of the model are identified if $\mu = 0$, Φ is lower triangular, Σ is diagonal, and $\delta = 1$. We also know from the macro literature that if all factors are observable, then μ , Φ , and δ are free, and Σ is lower triangular. As we have a mixture of observed and latent factors, we have to combine the insights from the two strands of the literature.

We set μ^m , Φ , δ_0 , and δ_m to be free. We restrict μ^x in such a way that the long-run mean of the factors x is equal to zero, i.e.:

$$e_i'(I - \Phi)^{-1}\mu = 0, \quad i = 1, 2, \quad (3.2)$$

where e_i 's are vectors of zeros with a one in positions corresponding to the factors x . Σ is⁹

$$\Sigma = \begin{bmatrix} \sigma_{gg} & 0 & 0 & 0 \\ \sigma_{\pi g} & \sigma_{\pi\pi} & 0 & 0 \\ \sigma_{1g} & \sigma_{1\pi} & \sigma_{11} & 0 \\ \sigma_{2g} & \sigma_{2\pi} & 0 & \sigma_{22} \end{bmatrix}. \quad (3.3)$$

These restrictions imply that we have to set $\delta_x = 1$. Finally, since all the risk premia parameters are identified in the case of the all-latent model as long as there are more yields than factors, these parameters will be identified when some of the factors are observable.

3.2.3 Estimation of the Risk Premia

Risk premia are hard to estimate in practice despite their theoretical identification. Typically, one encounters multiple local optima that have similar likelihood values, but imply dramatically different estimates of the risk premia. Additionally, a rich specification of market prices of risk might be a reason for concern, because they could be compensating for the misspecification of the factors dynamics instead of measuring the compensation for risk.

In order to mitigate these issues we augment the standard log-likelihood function, \mathcal{L} , with a penalization term which is proportional to the variation of the term premium in (2.7):

$$\mathcal{L}_p = \mathcal{L} - \frac{1}{2\sigma_p^2} \sum_{\tau} (a^{TP}(\tau))^2 + b^{TP}(\tau)' \cdot \text{Diag}(\text{var}(z_t)) \cdot b^{TP}(\tau), \quad (3.4)$$

⁹In particular, this form of Σ implies that $\Sigma^{mx} = 0$ in (2.2) and (2.16). This property simplifies many projection equations in Appendix A. One notable simplification is that $\Sigma^{ff} = \Sigma^{xx}$ in (2.16).

where σ_p controls the importance of the penalization term, and the Diag operator creates a diagonal matrix out of a regular one. If market prices of risk are equal to zero, the term premium will be equal to zero as well. Therefore, \mathcal{L}_p imposes an extra burden on the model to use the risk premia as a last resort in fitting the yields.

In practice we take $\sigma_p = 300$, which introduces a modest modification to the original log-likelihood. Nonetheless, it helps to stabilize the likelihood and simplifies the search for the global optimum. In particular, this setup helps us to avoid very large values of risk premia.

4 Results

We split the discussion of the results into two parts. Section 4.1 discusses the estimated latent variables, how well the model fits the yield curve, and contrasts different versions of the interest rate rule implied by the model. Section 4.2 identifies the latent variables with observable macro variables other than inflation and real activity and discusses their interactions.

4.1 The Model Properties

4.1.1 Parameter Estimates

Table 2 presents the estimated parameters. Because asymptotic standard errors are of a concern in the context of such persistent time series as interest rates, we compute the confidence bounds via the parametric bootstrap (Conley, Hansen, and Liu, 1997). Specifically, we simulate 1000 paths from the estimated model and re-estimate it along each path. This procedure yields a finite sample distribution of parameter estimates, which subsequently allows us to determine the confidence intervals.

While some parameters are individually insignificant, they appear to be jointly significant based on our parameter elimination routine. Following Dai and Singleton (2002) (DS2 henceforth), we restricted some of the insignificant parameters to zero, if such a restriction did not lead to a notable decline in the value of the log-likelihood function. The remaining parameters are therefore important for the model fit. The insignificance of the individual parameters stems from the fact that we are estimating a large model and the data might not be sufficiently informative about each

of the parameters, even if they are theoretically identified.¹⁰

Christiano, Eichenbaum, and Evans (1999) caution that the estimated parameters are difficult to interpret, as they represent a convolution of the parameters of the actual interest rate rule (e.g., the forward-looking rule in (2.1)), and the parameters of the projection of the missing data onto the econometrician’s dataset. Moreover, the values of parameters associated with the latent variables depend on the particular identification scheme. For example, the magnitude of the lower-right block of Σ depends on the restricted value of δ_x . Impulse response functions and related diagnostics represent the proper way of assessing the model implications. We discuss these in later sections.

4.1.2 The Model Fit

First, we highlight the value of σ_ξ , the standard deviation of the error in the yield observation equation (3.1), which is equal to 0.16. This implies that the model values the bonds within 32 basis points ($2\sigma_\xi$). Specifically, as indicated in panel (a) of Table 3, the average absolute pricing error ranges from 6.2 basis points for the one-year yield to 33 basis points for the ten-year yield. This is fairly reasonable, especially taking into account the noise in the approximated zero yields. The results are consistent with other macro studies (see ADP for a discussion).

We conduct a more thorough evaluation of the model performance by checking how well it fits certain moments. We again use the parametric bootstrap strategy and compute the finite sample distribution of the model-implied moments.¹¹ Panel (b) of Table 3 reports the results.

We see that the model successfully captures many important aspects of the data, such as means, standard deviations, kurtosis and autocorrelations. The difference between the fitted and actual means of the yields is quite large, at least relative to the extant studies. This happens for two reasons. First, we do not assume exactly observed yields. Second, we estimate the coefficient δ_0 in the interest rate rule, while most studies fix it to match unconditional mean of the short yield exactly.

¹⁰Note that the estimated process for the state vector z is covariance-stationary despite the fact that Φ_{44} is greater than one. The bootstrapped upper confidence bounds on the absolute value of each of the eigenvalues of Φ are below one.

¹¹In principle, we should have taken the parameter uncertainty and data sampling error into account when computing the confidence intervals. This would widen the intervals further.

The model primarily struggles with explaining the skewness of many of the variables and the curvature. The former is not surprising as a Gaussian model is incapable of generating non-normal skewness. In the case of curvature, the differences between the data and the model moments are significant. We intentionally selected a parsimonious model to investigate first-order effects in this paper and to minimize already formidable difficulties associated with a large set of parameters. The curvature fit can be improved by adding another latent factor. We do not pursue such extensions here, because the curvature, which explains at most one percent of the yield curve variation, does not have a first-order effect.

4.1.3 Orthogonalized Residuals, f_t

Table 4 reports the correlations of the filtered latent variables x , $\hat{x}(M_t, Y_t)$ and its estimated orthogonalized residual \hat{f}_t . We see that both f s are different from their x counterparts (the correlations are 0.22 and 0.31 for the first and second pair, respectively). The low correlations imply that inferring the impact of macro variables with x as the latent factors is very different from using f . Moreover, the correlation between the traditional latent factors level, slope, and curvature and either x s or f s are not as strong as in the latent factor models. The strongest relationship is between the slope and x_1 or f_1 (the correlation is above 0.60 in both cases). The fact that the correlations with the slope are nearly identical means that factor f_1 is more important for explaining the slope than is x_1 . We will highlight this effect later.

These results indicate that our orthogonalization procedure was worth pursuing, as it leads to new latent variables, that are substantively different from the ones typically studied in the literature. It appears that real activity and inflation variables have a bigger potential to explain the yield curve than would appear by considering the interest rate rule (2.4) directly. The next natural question is which fraction of the yield curve variability is explained by output, inflation, and their lags contained in the projection $\hat{x}(M_t)$.

4.1.4 Do Real Activity and Inflation Explain the Short Interest Rate?

In this section we evaluate how well different implementations of the interest rate rule explain the variation in the short end of the curve. We use “theoretical R^2 ,” or cumulative variance decom-

position, as an intuitive metric. In contrast to the regular R^2 , which is a side product of an OLS estimation, our measure is computed based on the parameter values obtained via an ML estimation. Because the factors f_t are orthogonal to the history of the macro variables M_t , there are no problems with attributing the explanatory power to latent versus macro factors. Table 5 reports the theoretical R^2 for the full model and a nested, macro-only specification.

If we consider the macro-component, the projection-based rule

$$r_t = \delta_0 + \delta'_m m_t + \hat{x}_{1t}(M_t) \quad (4.1)$$

explains the majority of the variation in the short rate: $R^2 = 80\%$. Panel (a) of Figure 1 complements our observations about the strong relationship. This result is in stark contrast to the previous no-arbitrage literature: AP and ADP report an $R^2 = 45\%$ based on contemporaneous macro variables only; AP report $R^2 = 53\%$ for an interest rate rule involving both macro variables and their lags and latent variables and Dai and Philippon (2004) report a high R^2 of 95% for their interest rate rule, however, it includes the contemporaneous Fed Funds rate.

Going back to Table 1, the result is stronger than the numbers implied by the unrestricted univariate regression and VAR. We would like to highlight three points regarding the comparison of the fit. First, because we use a fully specified model, biases in parameters and measures of fit due to omitted variables are not a concern. Second, because of our decomposition into the macro component and orthogonal latent factors, we are able to separate the contributions of the two. Hence, our R^2 of 80% reflects the explanatory power of the macro variables themselves, as opposed to their shocks in the usual VAR (i.e., the last column in Table 1). Third, the restrictions inherent in the no-arbitrage specification do not hurt the model performance, as the results are comparable overall. We will revisit these points when we discuss the overall term structure fit.

Panel (b) of Figure 1 illustrates the weights of the macro variables and their lags in the interest rate rule (4.1) implied by the projection procedure. The highest loading for real activity is at its contemporaneous value. The weights decline from there forward in an exponential fashion. The inflation loadings peak at the second lag and then decline similarly to real activity. We note that the weights die out quite slowly, which implies a need for lags well beyond the traditional twelve. Implementing such a lag structure in the VAR framework would be very difficult because of parameter proliferation.

Finally, after fully incorporating the latent variables in (2.8), the R^2 is close to one in Table 5. Despite a clear success of our strategy, there is still a room for improvement. We should be able to explain the remaining 20% of the variation in r . This is the purpose of the new latent factors f , which were omitted from our rule (4.1).

We note, from panel (a) of Figure 1, that there are clear periods when the actual interest rate is above (or below) the macro-based rule; i.e., the effective policy is more aggressive (or passive). We note that the latent factor f_2 accounts for the difference. Figure 2 complements this observation by showing that the factor loadings, or responses of the yield curve, to one standard deviation moves in the factors implied by the model.¹² We see that factor f_2 operates mostly on the short end of the curve. It might seem surprising that f_1 does not seem to affect the short rate. Indeed, our identification scheme imposes a unit loading on factor f_1 in the interest rate rule (2.8). In fact, the impact of a factor on the interest rate is controlled by its loading and by the variance of the factor's innovation. In our case, the volatility of the innovation in f_1 is so small that it has a negligible effect on the short interest rate.¹³

The deviations between the interest rate rule and the short yield seem to correspond to our intent of interpreting the latent residual factors as exogenous shocks. Reviewing some episodes would be helpful in developing intuition regarding this. Perhaps, the most striking deviation from the macro-based rule that we observe is during the period from November 1982 to Spring 1986. Goodfriend (1993) associates this period with the Fed establishing its credibility by handling the inflation scare of 1983 – 1984, when the Fed aggressively increased the funds rate by three percent during this period. A more recent, but similar, episode pertains to the “soft landing” of 1994, when the Fed hiked the interest rate by three quarters of a percentage point at once.

Blinder and Yellen (2001) cite the years 1990 through 1993 as a period of passive policy. Indeed, the Fed was accused of not cutting the interest rate in a sufficiently proactive fashion during and after the recession of 1990-91. The policy was characterized by long pauses in interest changes and by small cuts at a time. The deviations of the three-month yield from the macro-based

¹²The magnitudes of the changes appear to be larger than one would expect. This effect is driven by relatively large standard deviations of the factors, which is the result of the presence of the monetary experiment in the dataset.

¹³At longer horizons the loadings $b^Q(\tau)$ magnify the effect and make contribution of f_1 progressively more important.

rule in Figure 1 reflect this situation accurately.

4.1.5 Do Real Activity and Inflation Explain the Yield Curve?

In this section we establish which fraction of the variation in the yield curve is explained by various versions of the interest rate rule. We summarize the yield curve by its first three principle components: level, slope, and curvature. Note that evaluating the fit quality of the principle components is a more stringent exercise than evaluating the fit of certain yields. Because of interest rate persistence, success in explaining the short rate will translate into success in explaining the rest of the curve. Therefore, the key challenge to the model is to evaluate its ability to explain features of the curve that are not directly related to its level.¹⁴

Similar to the previous section, we consider two versions of the interest rate rule, which are based on different implementation of our specifications in (2.8). In the first case, we omit the orthogonalized residuals f . In the second case, we use the full state vector. The reported R^2 represent theoretical values based on the estimated model parameters, rather than the ones obtained from OLS. Indeed, because no-arbitrage theory allows for the computation of any bond yield based on each specification of r , we can compute the theoretical values of R^2 for all principal components, which are linear combinations of yields.

Table 5 reports these R^2 . As we observed in the previous section, our model is very successful in explaining the level based on the macro variables only. There is a large difference in how the two interest rate rules explain the slope of the term structure. First, our full model is successful in capturing the slope of the curve as it explains 97%. Second, the ability of real activity and inflation to capture the shape of the curve deteriorates. While the two macro variables could explain 80% of variation in the level, they explain only 52% of the slope.

Figure 3 complements this discussion by contrasting the macro-based and the observed slopes in panel (a) and by showing the weights of macro variables and their lags in the macro-based expression for the slope in panel (b). In contrast to the level, the weights die out almost immediately. This

¹⁴Moreover, since the likelihood was constructed based on yields, the errors will be small. However, if the model is misspecified, the errors will have complicated correlation structure. This will be revealed by looking at the linear combinations of yields, e.g., the principle components.

observation implies that the slope is mostly determined by the contemporaneous macro variables. This explains a “robust” explanatory power of macro variables with respect to the slope. In Table 1, the R^2 s range between 40% and 45% regardless of the implementation and our no-arbitrage model delivers a similar 52%.

As is the case with the interest rate rule in Figure 1, it is instructive to view the difference between the actual and macro-based slope not as model error, but as a response of the yield curve to the developments beyond those related to real activity and inflation. Indeed, factor f_1 accounts for almost the entire difference between the actual and macro-based slope in panel (a) of Figure 3. Figure 2 complements this observation; factor f_1 loadings indicate that the factor operates mostly on the long end of the curve.

It is interesting to contrast the plot of the slope with two U.S. government budget-related episodes highlighted in Blinder and Yellen (2001). First, President Clinton introduced a budget reduction package in February 1993, immediately after the recession of 1990-91. According to Blinder and Yellen, the unprecedented nature of this package awarded it instantaneous credibility. This perception was reflected in the 1.5% drop in the long interest rate from late 1992, when Clinton started advertising the package, to late 1993. Figure 1 shows that both macro-based and actual short interest rate remained flat during this time period. The macro-based slope in Figure 3 is flat as well. Therefore, the change in the actual slope may be attributed to the changes in the fiscal policy. Second, another Clinton budget agreement dated November 1999 “...shifted the norm for fiscal policy fundamentally by declaring the Social Security surplus off-budget..., the fiscal bar was thus raised enormously.” Similar to the first episode, the 2% drop in the slope over the period from November 1999 to October 2000 is attributable mostly to the fiscal policy or, in the context of our model, to factor f_1 .

While the explanatory power of the various variables follows the same pattern for the curvature, our model fails to generate the realistic pattern (the full model explains only 55%). This outcome is consistent with the model diagnostic results reported earlier. This problem could be remedied by introducing an extra latent variable. However, since curvature explains less than 1% of the whole term structure, we decided to leave out the model refinements.

4.2 What Are the Exogenous Shocks?

4.2.1 The Strategy

We want to relate the new latent factors f_1 and f_2 , which we interpret as exogenous shocks, to macro factors other than inflation and real activity that the literature deems to be important.¹⁵ The nature of the two shocks is different. The shock f_1 does not affect the short interest rate, but has an important contribution to the long rates. The shock f_2 affects the whole yield curve, but the major impact is on the short end. We identified three sources of economic activity which should influence the bond prices or could affect the exogenous monetary shocks.

The first two sources of activity are consistent with our initial identification of factor f_2 with exogenous monetary policy (see section 4.1.4). The first important factor is financial stability. Bernanke and Gertler (2001) argue that, given a strong stance on inflation targeting, the monetary authority should not react to such manifestations of financial instability as market booms and busts. However, Mishkin and White (2003) discuss that the Fed might react to other measures of financial instability, such as a large rise in interest rates for defaultable securities. Hence, our proxy for financial stability is the AAA credit spread taken from the FRED database.

The AAA credit spread represents a measure of another potential shock related to the monetary policy. Because of overall high credit quality of AAA companies, the spread will, to a large degree, measure the effect of liquidity – our second candidate for a missed component of the interest rate rule.¹⁶ Indeed, the analysis in Mishkin and White (2003) suggests that periods of financial instability are associated with liquidity crunches. Many analysts argue that monetary policy should react to such situations through the concept of the lender of last resort (see Freixas, Giannini, Hoggarth, and Soussa, 1999 for a review). We use month-to-month growth in MZM (money zero

¹⁵Rudebusch (2002) discusses shocks of a similar nature, however, he asserts that the “... rule deviations are not ‘exogenous policy shocks,’ that is, actions undertaken by central bankers that are independent of the economy ... Instead, these deviations are endogenous responses to a variety of influences that cannot be captured by some easily observable variable such as output or inflation.” Our interpretation is consistent with Rudebusch, and we use the word “exogenous” (without the qualifier “policy”) to emphasize the response to variables that are totally unrelated to inflation and real activity.

¹⁶The Treasury bonds are often perceived to be trading at “liquidity premium” – higher prices that reflect high demand for money-like instruments.

maturity) in order to directly measure the money supply.

The third source of shocks that we want to investigate is fiscal policy, which is motivated by our observations regarding factor f_1 in section 4.1.5. The impact of the fiscal policy on the term structure is very interesting in its own right. On the one hand, Engen and Hubbard (2004) review the macro literature, conduct their own analysis and conclude that a change in the government debt leads to minimal changes in the long-term real interest rate. On the other hand, Dai and Philippon (2004), in the context of their no-arbitrage model, find a large impact of the budget deficit (equivalent to the changes in debt) on the 10-year nominal yield. Apart from the direct impact debate, many researchers suggest tight link between the monetary and fiscal policy. In particular, Cochrane (2001) argues that inflation can be manipulated via the government issuing debt of various maturities. Moreover, Sargent and Wallace (1981) suggest that under certain circumstances, fiscal policy will limit the effectiveness of monetary policy. In order to understand the empirical relationship between monetary and fiscal policy and their interactions with the yield curve, we examine how the latent factors are related to the year-to-year growth of the public government debt.

The proposed macro variables are, most likely, correlated with our measures of real activity and inflation. These relationships might obscure the degree of association with the factors f . Therefore, we prewhiten the above-mentioned macro variables by regressing them on twelve lags of real activity and inflation. As a result we will be relating the factors f to the innovations in measures of liquidity or fiscal policy, which is consistent with our view of f as exogenous shocks.

In order to identify the factors f with the discussed variables clearly, we rotate them so that they are orthogonal to each other and the correlation between f_1 and the public debt growth is maximized. This rotation resolves the usual indeterminacy of latent factors. In the context of our model and our dataset, the proposed rotations lead to a mild change in the factors. The new versions are strongly correlated with the old ones. Factor f_1 became more volatile after the transformation, and factor f_2 is largely unaffected. Appendix B discusses the rotation details.

Finally, we compute impulse response functions in Figures 4 and 5 to aid our interpretations of factors. We compare the impulse responses from our model to the ones obtained from the regular VAR (2.14). We estimate a VAR(12) specification using three yields (three months, two and ten years) and four macro variables (real activity, g , inflation, π , growth in public debt, f_1 , and AAA

credit spread, f_2). The responses are based on the recursive identification scheme using the order (g, π, f_1, f_2) .

4.2.2 Impulse Responses of the State Variables

Figure 4 shows the impulse responses of the state variables. It appears, that after taking into account the statistical uncertainty the impulse responses implied by our model are not, in most cases, qualitatively different from the ones implied by VAR. This outcome gives us additional confidence in our model and the choice of macro variables related to f_1 and f_2 : it is restricted relative to VAR, but captures the same features of the state variables dynamics.

We conclude the following, based on the 95% confidence intervals and economic significance of the responses: real activity responds to itself (+0.20% immediately), inflation (-0.06% in three years), and factor f_1 (+0.15% in two years); inflation responds to real activity (+0.30% in three years) and itself (+0.30% immediately); and factor f_2 responds to real activity (+0.12% immediately), factor f_1 (-0.30% immediately), and itself (+0.50% immediately).

4.2.3 Fiscal Shock and Factor f_1

Figure 6 graphs factor f_1 with the public government debt annual growth. We find a very strong association between the two series as the correlation between the two is 59%. Thus, the deficit increases imply higher long-term yields. Indeed, a larger deficit leads to a large issuance of medium- to long-term debt; i.e., an increase in f_1 leads to an additional supply of bonds to the economy. The increased supply of bonds leads to a hike in the respective yields. The impulse response functions in Figure 5 complement our discussion. Factor f_1 has a big (40 basis points) impact on the slope, which dies out in about 2 years and it has a modest (15 basis points), but apparently permanent, impact on the ten-year yield. These observations are related to a number of studies of fiscal and monetary policies.

Dai and Philippon (2004) use the budget deficit, which is equivalent to the growth, as one of the factors in their model. They find an even stronger relationship between the fiscal policy and

the long-term debt.¹⁷ Engen and Hubbard (2004) argue that studying the relationship between the level of the long rate and the changes in debt is not appropriate because it is unrelated to the “crowding out” theory, which makes predictions about either levels of debt and interest rates, or changes in both debt and interest rates. These authors nonetheless estimate whether the changes in debt predict the level of the long interest rate and find that, after controlling for other sources of variation, there is no significant relationship between the two. We do not use as many macro variables as Engen and Hubbard do, yet our measures of fit indicate that our model describes the yield curve sufficiently well to take its implications seriously. Perhaps, it is different observation frequency (they use quarterly data), reliance on only one (long) yield and lack of no-arbitrage restrictions that account for the differences in our findings.

There is a strong association between the fiscal shock, inflation and the ten-year yield; both inflation and the long yield strongly respond to shocks in f_1 in Figures 4 and 5, respectively. Therefore, it is natural to ask whether it is the fiscal shock that really matters for the ten-year yield, or rather, its contribution via expected inflation. We compute the contribution of f_1 to the variation in expected average inflation over the ten-year horizon to address this question.¹⁸ It turns out that the fiscal shock explains only 0.45% of the variation in the expected inflation. We conclude that a strong reaction of the ten-year yield is driven by its direct response to f_1 .

4.2.4 Monetary Shock and Factor f_2

Figure 7, panel (a) shows the orthogonalized latent factor f_1 (with a minus sign) against the pre-whitened spread between AAA Moody’s corporate index and ten-year Treasury bond yield. The two series have many common spikes. The spikes could be interpreted as “flight to quality” events. When

¹⁷These authors find a different pattern in the ten-year yield response. We will revisit this issue when we discuss the risk premia. The magnitudes of the responses are different, perhaps, because of the differences in shock identification.

¹⁸Our model implies, see, e.g., (2.4):

$$E_t \left(\frac{1}{\tau} \sum_{i=1}^{\tau} \pi_i \right) = \frac{1}{\tau} e_2' \left(\sum_{i=1}^{\tau} (I - \Phi)^{-1} (I - \Phi^i) \mu + (I - \Phi)^{-1} (I - \Phi^{\tau}) \Phi z_t \right),$$

where e_2 is a vector of zeros with a one in the second position. In other words, in our affine model the expectation of the average inflation is a linear function of the state variables. Therefore, we can use our projection-based decomposition (2.8) into macro lags M and exogenous shocks f . As a result, it is easy to compute the contribution of the fiscal shock f_1 to the overall variability of the expected inflation, because it is orthogonal to all other factors.

bad news arrives, demand for Treasury bonds increases, which drives the yields down and credit spreads up. A correlation between f_2 and the credit spread of -38% indicates that the Fed might react to such events only occasionally, which qualifies them as shocks rather than the systematic response factors.

Alternatively, we could interpret this evidence as a Fed response to the variation of the demand in liquidity. Whenever there is a lack of liquidity, the Fed, as a lender of last resort, eases monetary policy in order to provide additional liquidity to the economy. This interpretation could be verified by correlating f_2 with the growth rate of MZM. Figure 7, panel (b) shows the two variables. While the correlation is weak at -12%, it is clear that there are periods of very strong association between the two (especially after the monetary experiment).

The negative correlation of both measures of liquidity shock with factor f_2 means that the short interest rate declines in response to illiquidity. In practice (outside of our model), the short interest rate declines in response to the bonds buyback by the Treasury; i.e., decline in f_2 leads to an additional supply of bonds to the economy. This operation brings money into the system and thereby mitigates liquidity problems.

The impulse response functions in Figure 5 show that increases in f_2 have a transient impact along the whole curve and are associated with the increases in the interest rates, with the short rate being the largest. Also, as indicated by Figures 4 and 5, all the yields move in opposite direction to inflation in response to the shock in f_2 . Thus, the response of the yield curve is driven by a liquidity effect rather than an expected inflation effect (this conclusion is consistent with Evans and Marshall, 1998). Taken together, the evidence and the observation that the fiscal factor f_1 has a minimal impact on the short end of the curve indicate that f_2 could be interpreted as a monetary policy shock.

4.3 Risk premia

Our analysis has direct implications for the role of risk premia because the state variables that we have identified through our projection-based decomposition affect the dynamics of the stochastic discount factor (2.5). There are at least three interesting questions that we can now explore. First, do shocks affect yields primarily through expectations about the future short yields, or they

directly affect the risk premium? Second, what is the contribution of the various macro risk factors to the term premia, and how do they change over time? Third, which particular macro factors are responsible for the deviations from the expectations hypothesis?

4.3.1 Impulse Responses of the Term Premia

To answer the first question, we decompose the yields into the expectations of the short rate and term premia parts. The expectations could be computed via the yield formula (2.7) by setting the risk parameters to zero. The difference between the yields and the expectations delivers the term premium part.

Figure 8 shows the impulse response functions for the expectations (depicted by circles) and term premia (depicted by asterisks) benchmarked against the impulse responses of the full yields (depicted by solid lines) in Figure 5. We see that, not surprisingly, the term premia have virtually no impact at the short end of the curve. The responses of the ten-year yield to inflation and liquidity shocks are primarily driven by the responses of the term premia. The responses of both expectations and term premia to real activity and fiscal shocks are large and of opposite directions.¹⁹

4.3.2 Decomposition of the Term Premia

To answer the second question, we further decompose the term premia into the contributions of the various macro variables using the projection-based representation. The yield equation (2.7) implies that the term premium of generic maturity τ can be expressed as:

$$\begin{aligned}
 TP(\tau) &= a^{TP}(\tau) + b^{TP}(\tau)'z_t & (4.2) \\
 &= a^{TP}(\tau) + \underbrace{b_g^{TP}(\tau)g_t + b_x^{TP}(\tau)'\hat{x}(M_t)|_{G_t}}_{\equiv B_g(\tau)G_t} + \underbrace{b_\pi^{TP}(\tau)\pi_t + b_x^{TP}(\tau)'\hat{x}(M_t)|_{\Pi_t}}_{\equiv B_\pi(\tau)\Pi_t} \\
 &+ b_{x_1}^{TP}(\tau)f_{1t} + b_{x_2}^{TP}(\tau)f_{2t}.
 \end{aligned}$$

The term $b_x^{TP}(\tau)'\hat{x}(M_t)|_{G_t}$ denotes the part of the projection of the latent factors x , which depends on G_t , the current and lagged values of only real activity. We use a similar notation for the inflation-

¹⁹Note that the response pattern of the short rate expectations over the ten-year horizon to the fiscal shock is similar to the one reported in Dai and Philippon (2004). Therefore, the differences in the yield responses highlighted earlier are driven by the differences in the risk premia. This is not surprising as we estimate different model specifications.

only component. The decomposition allows us to characterize how term premia vary together with changes in the macro variables and shocks.

Figure 9 shows the one-year and ten-year term premia as examples. The top panels provide the time series of the respective yields and the corresponding term premia and the short rate expectations parts. We note that the term premia are generally countercyclical and have a reasonable magnitude: at the one-year horizon the average absolute premium is 0.73% with a standard deviation of 0.63%; for the ten-year horizon the numbers are 2.29% and 1.59%, respectively.

The remainder of the panels show the decomposition of the term premium into the real activity, $B_g(\tau)G_t$, and inflation, $B_\pi(\tau)\Pi_t$, components of the premia in the panels of the intermediate row, and the fiscal, $b_{x_1}^{TP}(\tau)f_{1t}$, and liquidity, $b_{x_2}^{TP}(\tau)f_{2t}$, shock components of the premia in the panels of the bottom row. We see that, for the ten-year horizon, the real activity and inflation components of the premia are larger in magnitude than the fiscal and liquidity components, and tend to increase during recessions. The real activity component is less variable than the inflation component, and is smaller in magnitude except for the period beginning in December 1998. At that time, the Fed started the tightening streak leading to the eventual collapse of the stock market. The Figure indicates that concerns regarding the impact of these events on real activity dominated the inflation fears. The fiscal and liquidity components of the premia often move in opposite directions relative to each other. The liquidity component was the largest during the monetary experiment. The fiscal premium was large during the Bush presidency in 1989-1993 and went down as a result of the Clinton budget agreement of late 1993.

Subsequently, we compute the population variances of the various ingredients of the term premia and evaluate their relative contribution to the term premia variances. Because the factors f_1 and f_2 are orthogonal to the macro factors, the decomposition of the term premia variance for a generic maturity τ simplifies to:

$$\begin{aligned} \text{var}(TP(\tau)) &= \text{var}(B_g(\tau)G_t) + 2 \cdot \text{cov}(B_g(\tau)G_t, B_\pi(\tau)\Pi_t) + \text{var}(B_\pi(\tau)\Pi_t) \\ &+ \text{var}(b_{x_1}^{TP}(\tau)f_{1t}) + \text{var}(b_{x_2}^{TP}(\tau)f_{2t}). \end{aligned} \quad (4.3)$$

When computing the percentage contribution of the various factors to the term premia variance, we attribute half of the covariance term to the contribution of real activity and the other half to the

contribution of inflation. We compute the variance of the macro components by simulating a long path (500,000 observations) from the estimated model and computing the sample variance based on it.

Table 6 shows that our initial macro variables, real activity and inflation, contribute 53% to 68% to variation in the term premia. In independent work, Ludvigson and Ng (2005) explore the bond risk premia and their relation to the macro variables in the context of excess returns predictability regressions. These authors find, similar to us, that inflation and real activity are the most important factors in explaining variation in the risk premia. However, overall, inflation and liquidity risk premia have the largest and statistically significant effect; the combined impact of the two premia is about 65% to 85%, depending on a bond’s maturity. Naturally, the liquidity premium is more prominent at the short horizon. It explains 42% and 22% of the variation in the one-year and ten-year premiums, respectively. The contribution of real activity is the most pronounced at the intermediate maturities; it explains 28% of the five-year term premium.

4.3.3 The Expectation Hypothesis

As highlighted in DS2 and Duffee (2002), the essentially affine specifications of risk in (2.6) are important for replication of the expectation hypothesis failure observed in the data. We have verified that this claim holds in our macro-based model. Panel (a) of Figure 10 replicates the DS2 results by plotting the slope coefficients ϕ_τ from the regression,

$$y_{t+1}(\tau - 1) - y_t(\tau) = \text{constant} + \phi_\tau(y_t(\tau) - y_t(1))/(\tau - 1) + \text{residual} \quad (4.4)$$

implemented in our sample and implied by our model.

In the context of our model we can also characterize the contribution of the different macro factors to the explanation of the expectation hypothesis violation by relying on the risk premia decomposition. To this end we rely on DS2 who show that

$$y_{t+1}(\tau - 1) - y_t(\tau) = (y_t(\tau) - y_t(1) - D_{t+1}^*(\tau))/(\tau - 1) + \text{residual}, \quad (4.5)$$

where the “pure premium part” $D_{t+1}^*(\tau)$ is given in their equation (8). Therefore,

$$\phi_\tau = \frac{\text{cov}(y_{t+1}(\tau - 1) - y_t(\tau), (y_t(\tau) - y_t(1))/(\tau - 1))}{\text{var}((y_t(\tau) - y_t(1))/(\tau - 1))}$$

$$\begin{aligned}
&= \frac{\text{cov}((y_t(\tau) - y_t(1) - D_{t+1}^*(\tau))/(\tau - 1), (y_t(\tau) - y_t(1))/(\tau - 1))}{\text{var}((y_t(\tau) - y_t(1))/(\tau - 1))} \\
&= 1 + \frac{\text{cov}(-D_{t+1}^*(\tau), y_t(\tau) - y_t(1))}{\text{var}(y_t(\tau) - y_t(1))}.
\end{aligned} \tag{4.6}$$

Because D_{t+1}^* can be computed from our model and then decomposed into the contributions of the factors similar to the term premium in (4.2), the covariance part in the last expression could be split into four elements. We plot them in panel (b) of Figure 10.

We see that real activity and the monetary policy factor f_2 have an upward effect on the regressions at long horizons. Therefore, inflation and the fiscal factor f_2 contribute most to the violations of the expectations hypothesis. This outcome is intuitive as inflation and fiscal shocks are the major drivers of long yields.

5 Conclusion

We propose an approach that allows us to establish, in the no-arbitrage affine framework, how and which macroeconomic variables contribute to the evolution of the yield curve. We rely on two ingredients. First, we allow for a rich model specification involving both preselected macro variables, such as inflation, real activity, and latent factors. Second, in order to identify the novel information in the latent factors, we dynamically project them onto the macro variables, and study the projection residuals.

As a result, the linear relationship between the short interest rate and the factors (the interest rate rule) can be interpreted as a linear function of the macro variables and their lags (a backward-looking interest rate rule) and new orthogonal latent factors (the projection residuals). This new interpretation gives maximal flexibility to the measures of inflation and real activity to explain the yield curve. The residuals could be compared to other macro variables in order to identify additional macro factors and shocks affecting monetary policy.

In contrast to previous studies, we find that, in the context of a four-factor model, real activity, inflation and their lags weighted in the “optimal” projection-based way can explain 80% of the variation in the short interest rate.

In contrast to previous studies, in the context of a four-factor model we find that real activity,

inflation and their optimally weighted lags explain 80% of the variation in the short interest rate. We find that the unexplained part (the projection residual) is correlated with measures of the budget deficit and money supply (liquidity). These residuals, which we interpret as exogenous monetary and fiscal shocks, have a prominent impact on the short and long end of the yield curve, respectively. Jointly, they are as important as inflation and real activity in explaining the long part of the term structure. The residual factors explain 50% of the slope variation.

We can explore the impact of the macro variables on the term premia in our model. We find that the ten-year yield responses to inflation and liquidity shocks are primarily driven by the responses to the term premia. We also decompose the term premia into contributions of the four macro risk factors. Inflation and liquidity shock jointly provide the strongest explanatory power at any maturity (65% to 85%). Inflation and fiscal shock have the largest contributions to the violations of the expectations hypothesis.

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A Projection

In this appendix we first provide the projection formulas, and then use them to show how our model is related to the traditional VAR analysis.

A.1 Recursive Formulas

The model controlling the evolution of state z in (2.2) does not represent a state-space system. Nonetheless, Liptser (1997) and Liptser and Shiryaev (2001) derive the projection of one element of the VAR(1) on the other using the same ideas as in a standard Kalman filter. In particular, they derive the following expression for the conditional mean, $\hat{x}(M_t)$, often referred to as “forecast,” and variance, P_t , of the forecast error,

$$\begin{aligned} \hat{x}(M_t) &= \mu^x + \Phi^{xx} \hat{x}(M_{t-1}) + \Phi^{xm} m_{t-1} \\ &+ (\Sigma^{xx} \Sigma^{mx'} + \Sigma^{xm} \Sigma^{mm'} + \Phi^{xx} P_{t-1} \Phi^{mx'}) (\Sigma^{mx} \Sigma^{mx'} + \Sigma^{mm} \Sigma^{mm'} + \Phi^{mx} P_{t-1} \Phi^{mx'})^{-1} \\ &\times (m_t - \mu_m - \Phi^{mx} \hat{x}(M_{t-1}) - \Phi^{mm} m_{t-1}) \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} P_t &= \Phi^{xx} P_{t-1} \Phi^{xx'} + (\Sigma^{xx} \Sigma^{xx'} + \Sigma^{xm} \Sigma^{xm'}) \\ &- (\Sigma^{xx} \Sigma^{mx'} + \Sigma^{xm} \Sigma^{mm'} + \Phi^{xx} P_{t-1} \Phi^{mx'}) (\Sigma^{mx} \Sigma^{mx'} + \Sigma^{mm} \Sigma^{mm'} + \Phi^{mx} P_{t-1} \Phi^{mx'})^{-1} \\ &\times (\Sigma^{xx} \Sigma^{mx'} + \Sigma^{xm} \Sigma^{mm'} + \Phi^{xx} P_{t-1} \Phi^{mx'})' \end{aligned} \quad (\text{A.2})$$

We introduce additional notations to describe the projection initialization. The long run mean z is:

$$(I - \Phi)^{-1} \mu = \begin{bmatrix} \Theta^m \\ \Theta^x \end{bmatrix} \quad (\text{A.3})$$

$$(\text{A.4})$$

The steady-state matrix P satisfies

$$\begin{aligned} P &= \Phi^{xx} P \Phi^{xx'} + (\Sigma^{xx} \Sigma^{xx'} + \Sigma^{xm} \Sigma^{xm'}) \\ &- (\Sigma^{xx} \Sigma^{mx'} + \Sigma^{xm} \Sigma^{mm'} + \Phi^{xx} P \Phi^{mx'}) (\Sigma^{mx} \Sigma^{mx'} + \Sigma^{mm} \Sigma^{mm'} + \Phi^{mx} P \Phi^{mx'})^{-1} \\ &\times (\Sigma^{xx} \Sigma^{mx'} + \Sigma^{xm} \Sigma^{mm'} + \Phi^{xx} P \Phi^{mx'})' \end{aligned} \quad (\text{A.5})$$

Then the projection is initialized as follows:

$$\hat{x}(m_0) = \Theta^x + V^{xm}(V^{mm})^{-1}(m_0 - \Theta^m), \quad P_0 = P \quad (\text{A.6})$$

In this case $P_t = P$, and the projection is time-stationary. An alternative strategy is to initialize P_0 at the unconditional variance of z . In this case, the sequence P_t will converge to P . In our model it happens in twelve steps.

A.2 Relation to VAR

Recall from (2.9) that

$$f_t = x_t - \hat{x}(M_t). \quad (\text{A.7})$$

By construction, its variance is equal to P_t . Equations (2.2) and (A.1) imply that

$$\begin{aligned} f_t &= \Phi^{xx} f_{t-1} + \Sigma^{xx} \epsilon_t^x + \Sigma_{xm} \epsilon_t^m \\ &- \underbrace{(\Sigma^{xx} \Sigma^{mx'} + \Sigma^{xm} \Sigma^{mm'})}_{\mathcal{P}_{t-1}} (\Sigma^{mx} \Sigma^{mx'} + \Sigma^{mm} \Sigma^{mm'} + \Phi^{mx} P_{t-1} \Phi^{mx'})^{-1} \\ &\times (\Phi^{mx} f_{t-1} + \Sigma^{mx} \epsilon_t^x + \Sigma^{mm} \epsilon_t^m) \\ &= (\Phi^{xx} - \mathcal{P}_{t-1} \Phi^{mx}) f_{t-1} + (\Sigma^{xx} - \mathcal{P}_{t-1} \Sigma^{mx}) \epsilon_t^x + (\Sigma^{xm} - \mathcal{P}_{t-1} \Sigma^{mm}) \epsilon_t^m \end{aligned} \quad (\text{A.8})$$

Therefore, the ‘‘residual’’ factors f follow the VAR(1) process. However, in contrast to the dynamics of m and x the conditional mean and variance of f are state dependent. The steady-state Kalman filter theory implies that \mathcal{P}_t converges to a fixed matrix together with P_t .

Denote $\mathcal{P} = \lim_{t \rightarrow \infty} \mathcal{P}_t$. We introduce the following explicit notations for simplified referencing:

$$\Phi^{ff} \equiv \Phi^{xx} - \mathcal{P} \Phi^{mx} \quad (\text{A.9})$$

$$\Sigma^{ff} \equiv \Sigma^{xx} - \mathcal{P} \Sigma^{mx} \quad (\text{A.10})$$

$$\Sigma^{fm} \equiv \Sigma^{xm} - \mathcal{P} \Sigma^{mm} \quad (\text{A.11})$$

Therefore,

$$\begin{bmatrix} m_t \\ f_t \end{bmatrix} = \begin{bmatrix} \mu^m \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi^{mm} & \Phi^{mx} \\ 0 & \Phi^{ff} \end{bmatrix} \begin{bmatrix} m_{t-1} \\ f_{t-1} \end{bmatrix} + \begin{bmatrix} \Phi^{mx} \hat{x}(M_{t-1}) \\ 0 \end{bmatrix} + \begin{bmatrix} \Sigma^{mm} & \Sigma^{mx} \\ \Sigma^{fm} & \Sigma^{ff} \end{bmatrix} \begin{bmatrix} \epsilon_t^m \\ \epsilon_t^x \end{bmatrix}.$$

Recursive substitution of \hat{x} from (A.1) and bond valuation based on (2.4) yield the VAR system in (2.16).

B Latent Factor Indeterminacy

One of our objectives is to establish whether the new variables f_1 and f_2 are related to additional observables. The two latent factors could span the space generated by some macro variables, i.e., regressing f_1, f_2 on the proposed macro variables would generate high R^2 , but, in principle, we cannot easily interpret the latent factors. We can, however, exploit the indeterminant nature of the latent factors to our advantage.

While the previous section imposes identifying restrictions, Dai and Singleton (2000) point out that such restrictions are not necessarily unique. There are many sets of restrictions, or invariant transformations of the model, such that the yields are left unchanged. Naturally, when a parameter configuration changes, the respective latent variables change as well by “rotating.” It is sensible to rotate the factors to identify f with macro variables. We will use the invariant affine transformation, which scales factors by a matrix. Appendix A of Dai and Singleton (2000) describes how such a transformation affects model parameters.

We examine two types of rotations. The first rotation, \mathcal{O} , ensures that the two factors are orthogonal to each other; i.e., the variance-covariance matrix of f, P (see the expression in Appendix A), becomes diagonal. We define $\mathcal{O} = Rf_t$, where the matrix R is such that RPR' is diagonal.

The matrix R is not unique; i.e., the rotation of type \mathcal{O} can generate many pairs of orthogonal factors f . Our second proposed rotation, \mathcal{M} , can be applied after any of the rotations from the class \mathcal{O} , resolves this type of indeterminacy. Define $\mathcal{M} = Uf_t$, where the matrix U is the orthogonal matrix; i.e., $UU' = I$, that preserves the correlation structure between the factors. In our two-dimensional case, the matrix U is determined by a single parameter, which is established by maximizing the correlation between one of the latent factors and one of the observable macro factors, which we choose to be the public debt growth.

Table 1 : Descriptive Analysis

We provide the preliminary data analysis by reporting the adjusted R^2 from univariate regressions of level, slope and curvature on contemporaneous inflation and real activity in the first column, on 12 lags of inflation and real activity in the second column and from the VAR analysis of the vector comprised of level, slope, curvature, inflation, and real activity in the third column. The fourth column reports the combined effect of the macro shocks in this VAR implied from the unconditional variance decomposition based on the recursive identification scheme using the following order: real activity, inflation, level, slope, and curvature.

	m_t only	m_t +lags	VAR	m_t shocks
level	54.89	66.73	96.35	72.29
slope	40.83	43.90	88.21	45.33
curvature	9.57	14.23	70.20	41.36

Table 2 : Estimated Parameters

The table lists the parameter values for our model

$$\begin{aligned}
 z_t &= (g_t, \pi_t, x_{1t}, x_{2t}) \\
 r_t &= \delta_0 + \delta'_z z_t \\
 z_t &= \mu + \Phi z_{t-1} + \Sigma \epsilon_t \\
 \Lambda_t &= \Lambda_0 + \Lambda_z z_t \\
 \log \xi_t &= -r_{t-1} - \frac{1}{2} \Lambda'_{t-1} \Lambda_{t-1} - \Lambda'_{t-1} \epsilon_t
 \end{aligned}$$

The bootstrapped 95% confidence intervals are reported in parentheses. The parameters restricted by the identification requirements are highlighted by the letter combination ‘id’ in place of the confidence bounds. The dash ‘-’ indicates that a parameter was restricted in the course of estimation. Note that we report the long-run mean, $(I - \Phi)^{-1}\mu$, of the state variables.

Interest Rate Rule					
	δ_0	δ_z			
		g	π	x_1	x_2
	-13.03	1.62	1.45	1	1
	(-33.61, 10.61)	(-1.23, 4.60)	(-0.54, 4.60)	id	id

State equation												
	$(I - \Phi)^{-1}\mu$		Φ				Σ					
	g	π	x_1	x_2	g	π	x_1	x_2	g	π	x_1	x_2
g	7.41	0.96	0	-0.02	0.55	0.84	0	0	0	0	0	0
	(6.22, 8.62)	(0.86, 1.03)	-	(-0.04, -0.01)	(0.01, 5.86)	(0.77, 0.89)	id	id	id	id	id	id
π	4.19	0.07	0.95	0.01	-0.80	0.08	1.07	0	0	0	0	0
	(2.25, 5.96)	(0.02, 0.18)	(0.86, 0.99)	(0.00, 0.04)	(-8.83, -0.04)	(-0.02, 0.17)	(0.98, 1.13)	id	id	id	id	id
x_1	0	-0.10	0.04	0.92	0.98	-0.99	-1.40	1.96	0	0	0	0
	id	(-0.40, 0.27)	(-0.15, 0.27)	(0.80, 1.02)	(-3.93, 9.98)	(-3.46, 1.39)	(-4.57, 0.78)	(1.72, 2.10)	id	id	id	id
x_2	0	-0.01	0	0	1.02	0	-0.07	0	0	0	0	0.06
	id	(-0.04, 0.00)	-	-	(0.94, 1.09)	-	(-0.67, -0.01)	id	id	id	id	(0.01, 0.73)

Risk premia					
	Λ_0		Λ_z		
	g	π	x_1	x_2	
g	-93.24	10.62	6.31	1.68	0
	(-199.21, -31.83)	(3.51, 23.87)	(1.25, 16.56)	(-1.43, 6.77)	-
π	57.49	-5.64	-5.40	-0.83	-24.32
	(12.60, 135.34)	(-15.24, -0.68)	(-13.28, -1.84)	(-3.76, 1.09)	(-247.37, -1.92)
x_1	25.82	-2.91	-1.88	-0.83	0
	(2.44, 60.91)	(-7.71, -0.03)	(-5.95, 0.10)	(-2.50, -0.08)	-
x_2	0	0	0	0.10	-2.69
	-	-	-	(-0.09, 0.43)	(-47.05, -0.08)

Table 3 : Pricing Errors and Moments

We report average absolute pricing errors in yields by maturity in panel (a). Panel (b) reports various moments of the observables computed from the dataset (monthly observations from 1970 to 2002) and implied by the estimated model. The bootstrapped 95% confidence intervals are reported in parentheses. The boldfaced sample statistics are outside the confidence bounds.

Panel (a). Pricing Errors.										
Maturity, months	3	6	12	24	36	60	84	120		
Error, b.p.	19.88	12.05	6.20	27.15	28.01	8.93	13.12	33.64		

Panel (b). Moments.										
	Means, %		Std. Dev., %		Skewness		Kurtosis		Autocorr.	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
g	7.72	7.42	1.64	1.58	-0.27	-0.01	2.08	2.62	0.99	0.98
	(6.12, 8.72)		(0.99, 2.29)		(-0.89, 0.83)		(1.87, 3.92)		(0.97, 0.99)	
π	4.82	4.21	2.89	2.87	1.19	-0.01	3.64	2.50	0.99	0.99
	(2.34, 6.12)		(1.66, 4.21)		(-0.89, 0.89)		(1.74, 3.83)		(0.98, 1.00)	
$y(3)$, or level	6.49	5.53	2.78	2.95	1.05	0.00	4.44	2.56	0.98	0.97
	(2.33, 8.87)		(1.75, 4.38)		(-0.84, 0.79)		(1.78, 3.84)		(0.94, 0.99)	
$y(24)$	7.20	6.21	2.59	2.71	0.85	0.00	3.78	2.51	0.98	0.98
	(2.99, 9.56)		(1.52, 4.11)		(-0.90, 0.83)		(1.72, 3.86)		(0.95, 0.99)	
$y(120)$	7.85	7.01	2.21	2.27	0.97	-0.01	3.55	2.50	0.99	0.98
	(4.19, 9.78)		(1.27, 3.45)		(-0.95, 0.88)		(1.70, 3.87)		(0.96, 0.99)	
slope	1.36	1.48	1.43	1.44	-0.60	0.00	3.19	2.75	0.93	0.94
	(0.92, 2.01)		(1.04, 1.92)		(-0.65, 0.66)		(2.08, 3.82)		(0.90, 0.97)	
curvature	-0.04	0.13	0.76	0.38	0.04	0.01	4.03	2.52	0.81	0.97
	(-0.37, 0.63)		(0.21, 0.60)		(-0.79, 0.85)		(1.72, 3.75)		(0.94, 0.99)	

Table 4 : Correlations Between the Traditional Latent Factors and the Residual Latent Factors

We correlate the three latent factors (level, slope, and curvature) which jointly capture 99% of the yield curve variation with two sets of latent factors that feature in our model. The factors x enter our model directly, joint with macro factors. The factors f are residuals of projection of x on to the history of the macro variables. We also report correlations between the respective x s and f s. The low correlations imply that inferring the impact of macro variables with x as latent factors is very different from using f .

	x_1	x_2	f_1	f_2
level	-0.38	-0.26	-0.04	0.60
slope	0.64	0.38	0.62	-0.46
curvature	-0.02	0.08	-0.36	-0.35
x_1			0.22	
x_2				0.31

Table 5 : Variance Decomposition for Principle Components

We establish which fraction of the yield curve variation is explained by various versions of the interest rate rule. We represent the yield curve by its first three principle components: level, slope, and curvature. We consider three versions of the interest rule, which depend on the difference in how the factors are used and on how the rules is estimated. The different rules are derived from from the no-arbitrage model. Given the estimated coefficients, we compute the cumulative variance decomposition, i.e., fraction of the variance explained, for various nested models. The bootstrapped 95% confidence intervals are reported in parentheses. First, we evaluate the rule using the lagged macro variables

$$r_t = \delta_0 + \delta'_m m_t + \delta'_x \hat{x}(M_t).$$

Finally, we use the full set of state variables:

$$r_t = \delta_0 + \delta'_m m_t + \delta'_x x_t$$

PC	m_t +lags	full model
level	79.59 (43.86, 90.21)	99.75 (99.21, 99.86)
slope	52.36 (22.36, 75.77)	97.45 (95.17, 98.43)
curvature	42.74 (8.43, 64.04)	55.83 (24.87, 72.70)

Table 6 : The Variance Decomposition of Term Premia

We report the percentage contribution of macro risk factors to the overall unconditional variation in the term premia. The bootstrapped 95% confidence intervals are reported in parentheses. The contribution of the covariance between the inflation and real activity components is split equally. We consider three maturities: one, five, and ten years.

	g_t	π_t	f_{1t}	f_{2t}
1-year	11.43 (0.00, 39.91)	42.27 (12.00, 61.97)	4.44 (0.00, 15.76)	41.84 (21.86, 66.74)
5-year	28.47 (2.24, 49.18)	39.39 (17.23, 61.86)	6.39 (0.00, 34.26)	25.74 (10.01, 46.19)
10-year	17.24 (0.00, 39.40)	51.25 (14.57, 80.47)	8.84 (0.00, 44.34)	22.65 (7.17, 34.95)

Figure 1. The Interest Rate Rule.

We plot the time series of the three-month zero yield and the estimate of r_t based on the projection

$$\hat{r}_t = \delta_0 + \delta'_m m_t + \delta'_x \hat{x}(M_t)$$

in panel (a). Panel (b) shows the first 24 loadings on the macro variables and their lags that generate the interest rate rule.

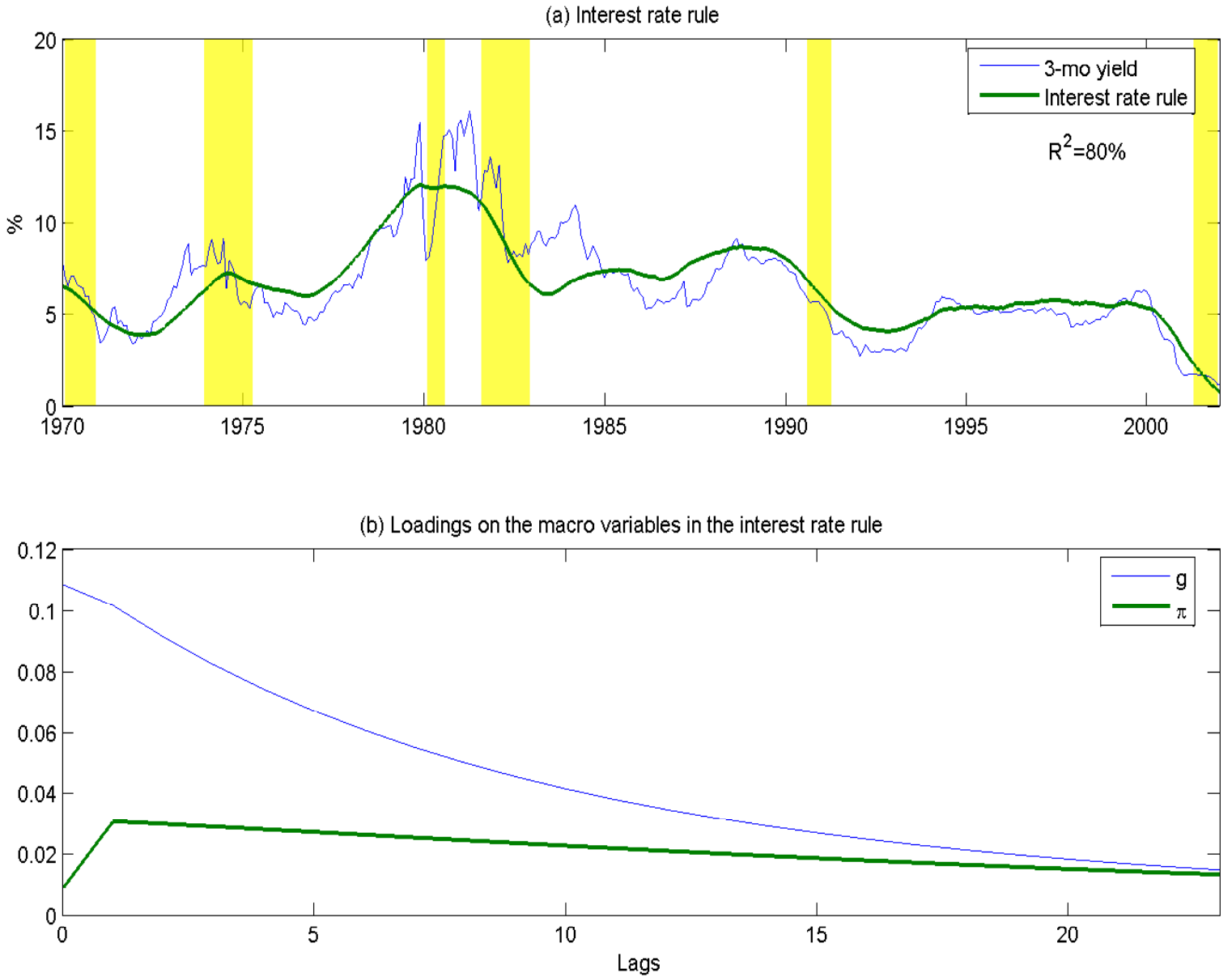


Figure 2. Term Structure Response to Shocks in the Latent State Variables.

We plot how term structure changes, in basis points, in response to one standard deviation change in one of the two residual factors f . The thin line corresponds to f_1 (left scale) and the thick line corresponds to f_2 (right scale).

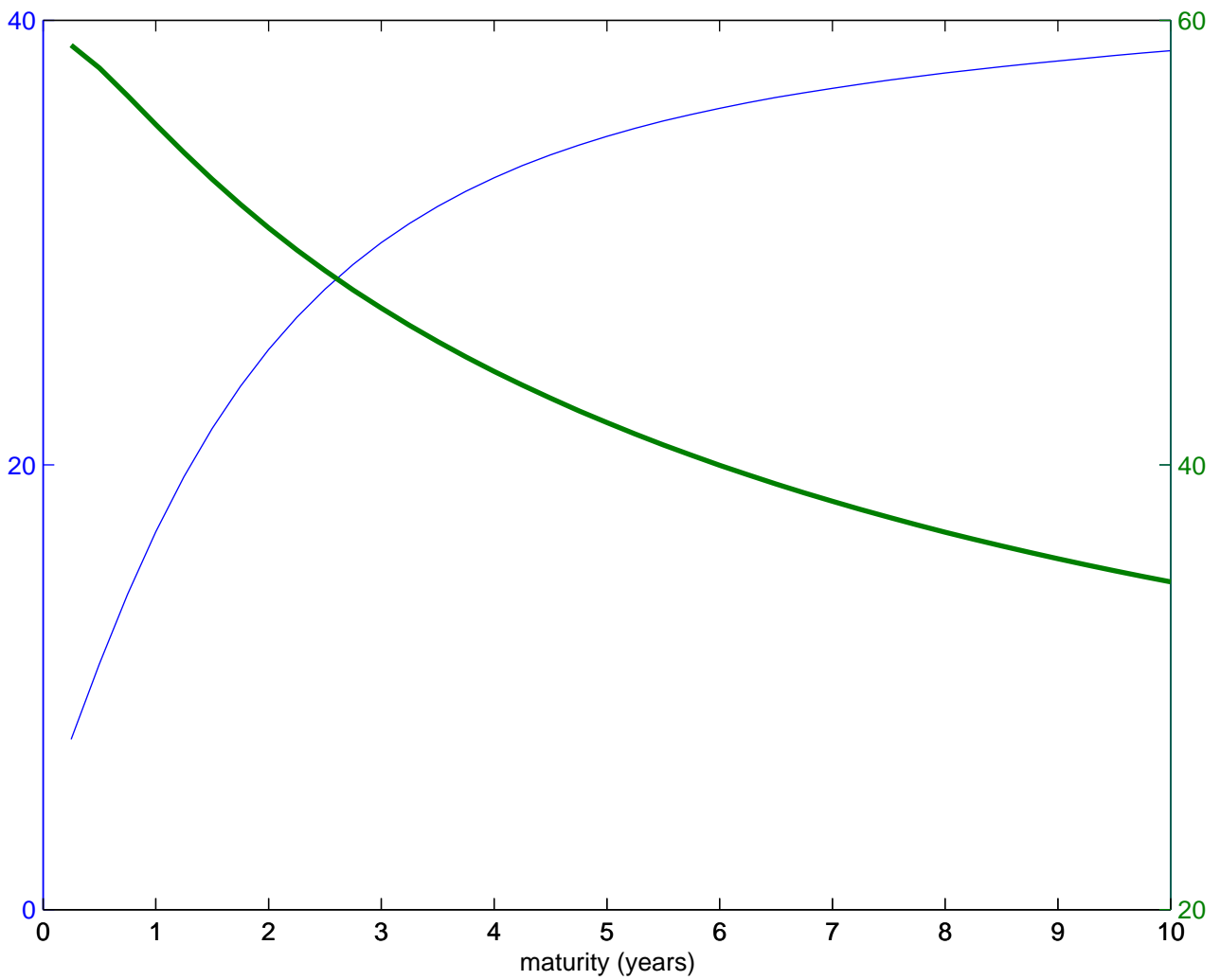


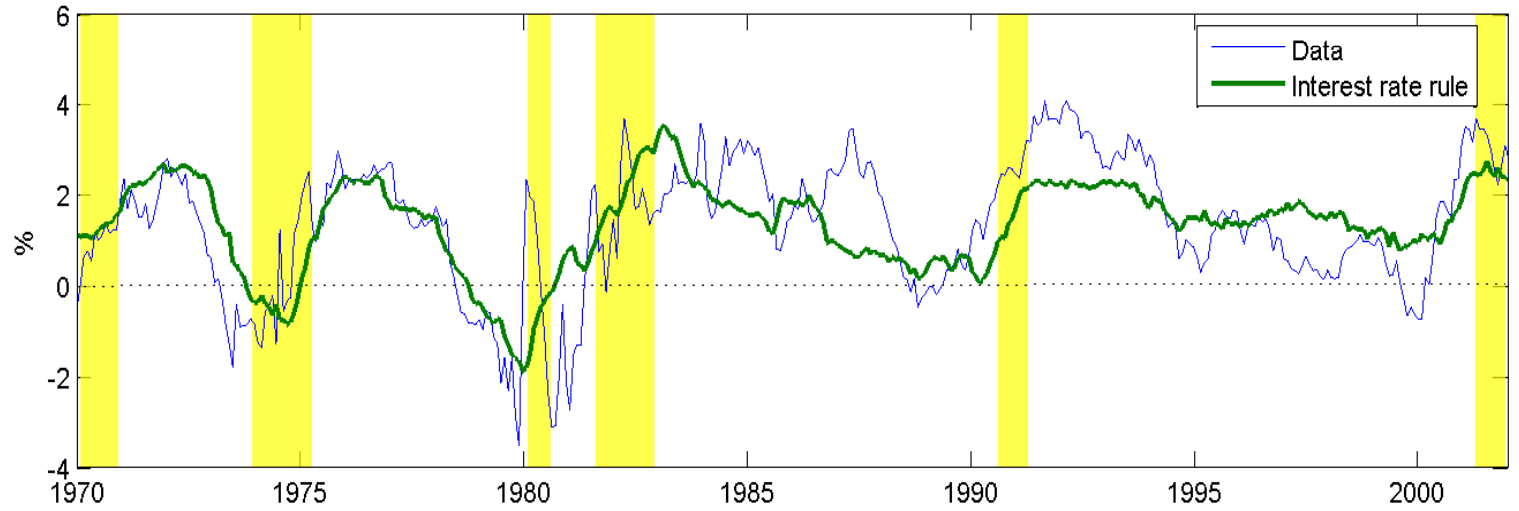
Figure 3. The Slope.

We plot the time series of the slope and its estimate computed using the projection-based interest rate rule

$$\hat{r}_t = \delta_0 + \delta'_m m_t + \delta'_x \hat{x}(M_t)$$

in panel (a). Panel (b) shows the first 24 loadings on the macro variables and their lags that generate the slope.

(a) Slope implied by the interest rate rule



(b) Loadings on the macro variables in the interest rate rule

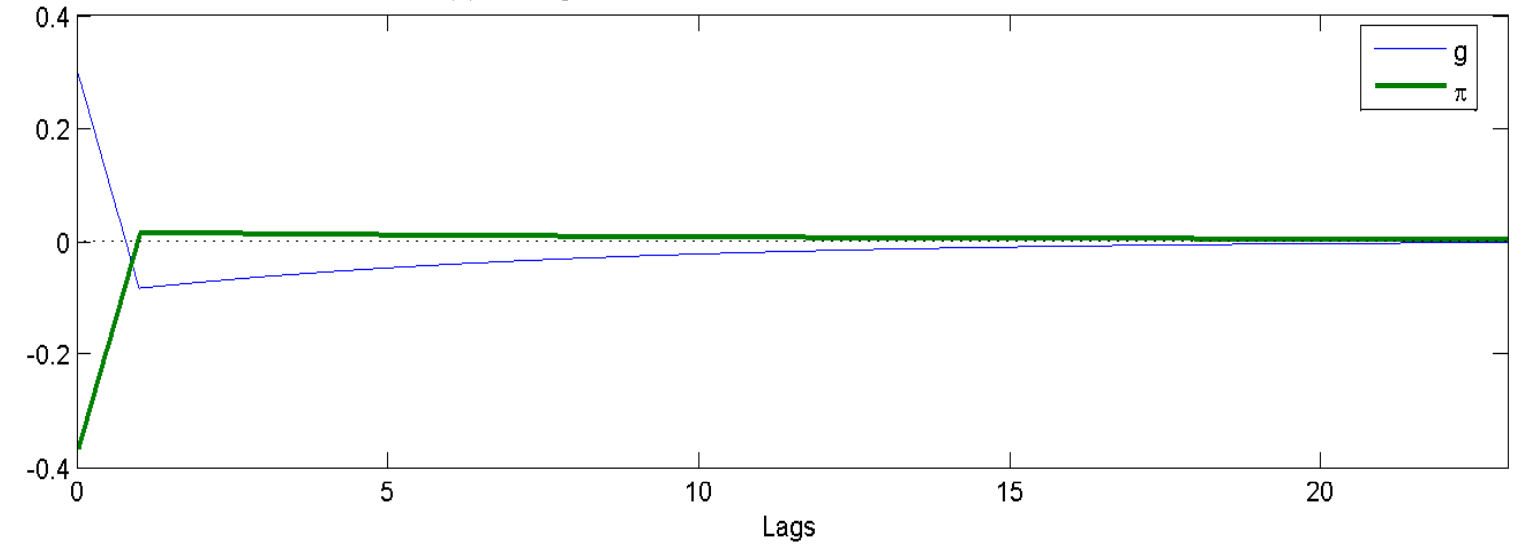


Figure 4. Impulse Response Functions: State Variables.

The figure shows the impulse responses to one standard deviation change of the factors in our model (thick line) and in a regular VAR (thin line). The bootstrapped 95% confidence intervals for the IR in our model are represented by dashed lines. We estimate a VAR(12) that is restricted as in (2.14) using three yields (3 months, 2 and 10 years) and four macro variables (real activity, g ; inflation, π ; growth in public debt, f_1 ; and AAA credit spread multiplied by negative one, f_2). We do not report confidence bounds for VAR to avoid clutter.

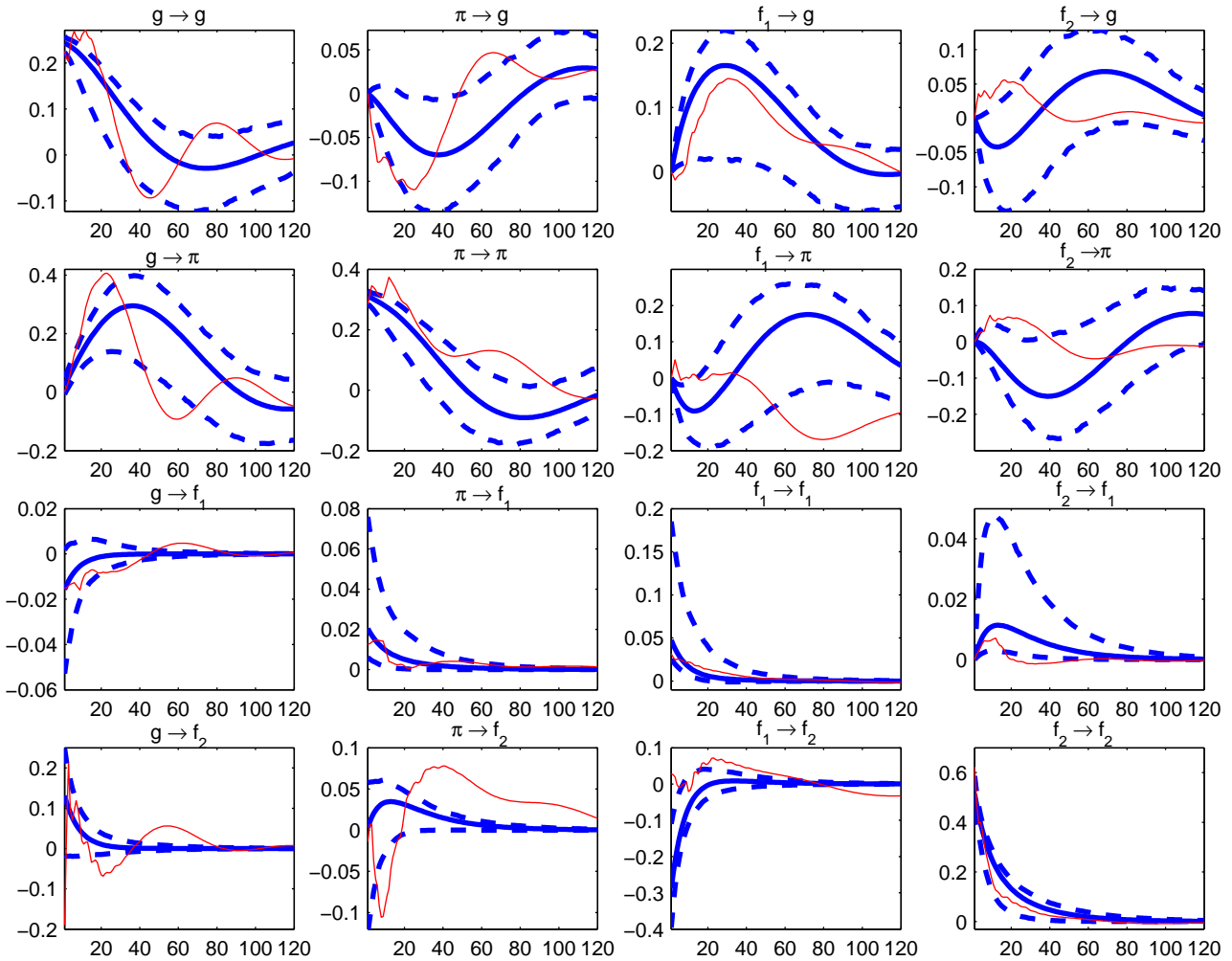


Figure 5. Impulse Response Functions: Yields and Slope.

The figure shows the impulse responses of the three-month and ten-year yields and the slope to one standard deviation shocks in the state variables in our model (thick line) and in a regular VAR (thin line). The bootstrapped 95% confidence intervals for the IR in our model are represented by dashed lines. We estimate a VAR(12) that is restricted as in (2.14) using three yields (3 months, 2 and 10 years) and four macro variables (real activity, g ; inflation, π ; growth in public debt, f_1 ; and AAA credit spread multiplied by negative one, f_2). We do not report confidence bounds for VAR to avoid clutter.

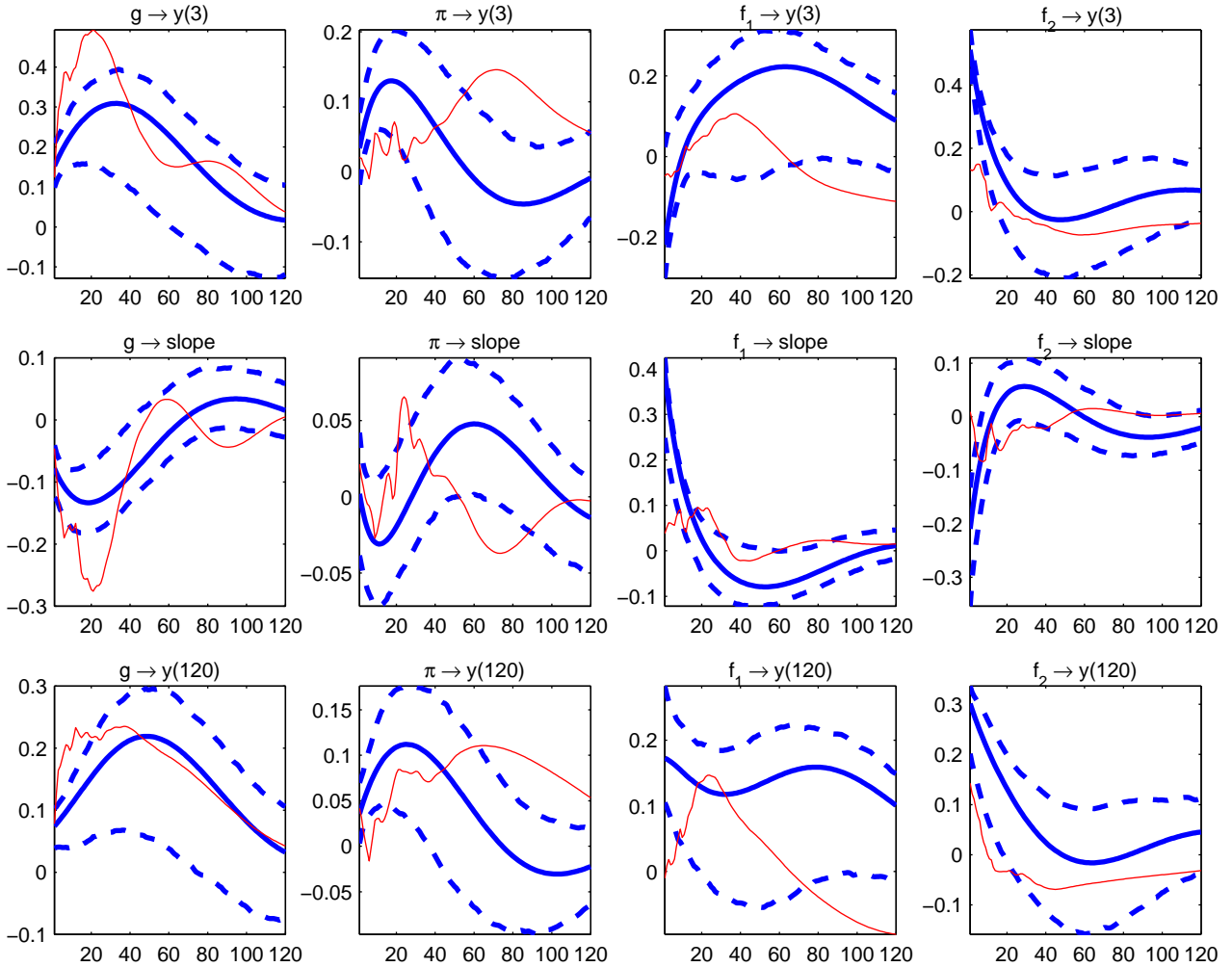


Figure 6. Orthogonalized Factor f_1 and the Annual Public Debt Growth Rate.

The plot shows the monthly series of the estimated factor f_1 (thin line, left scale) against the annual government public debt growth rate (thick line, right scale). The latter series are residuals from regressing the debt growth rate on twelve lags of inflation and real activity. Both series are standardized to facilitate comparison.

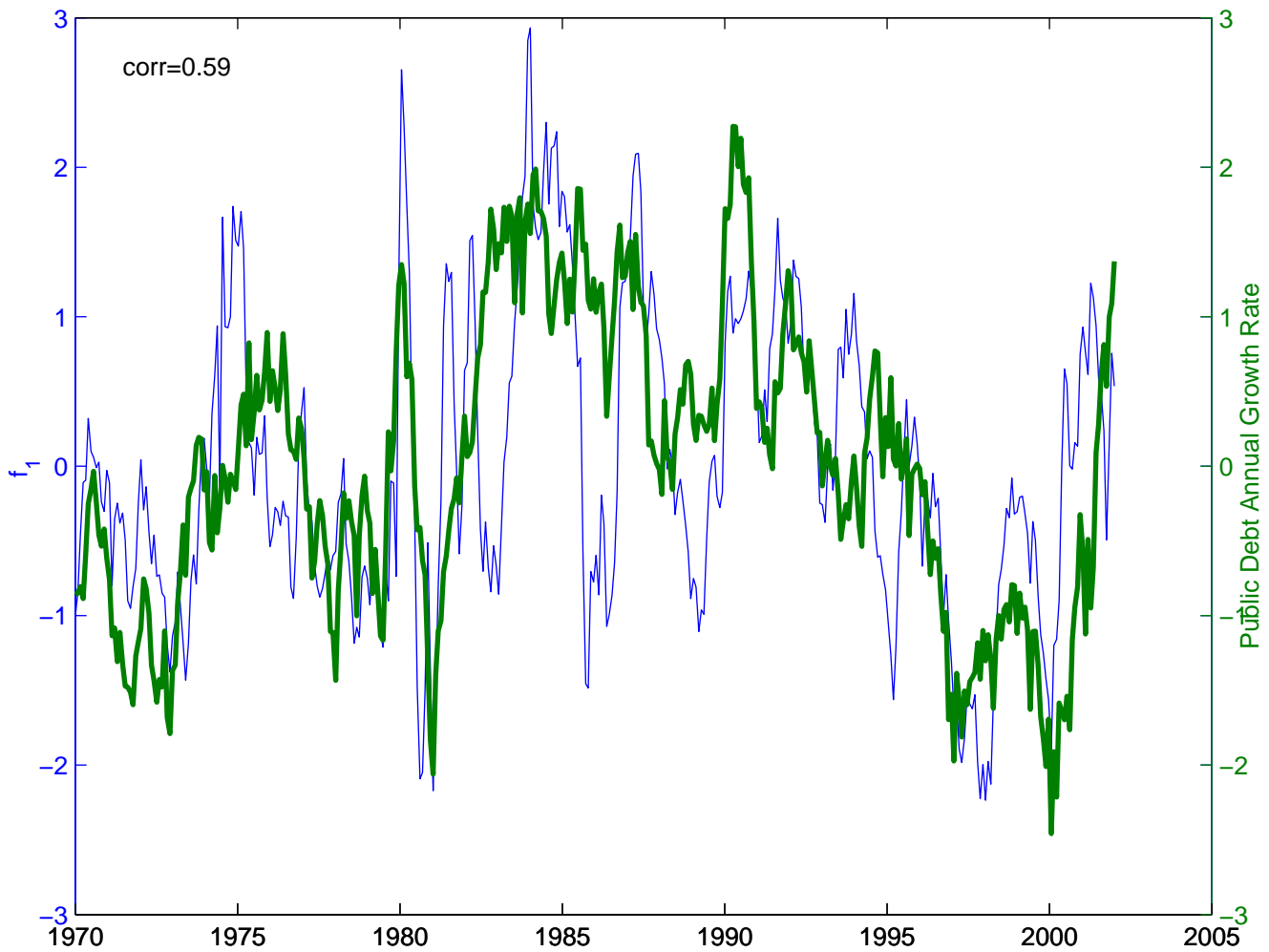
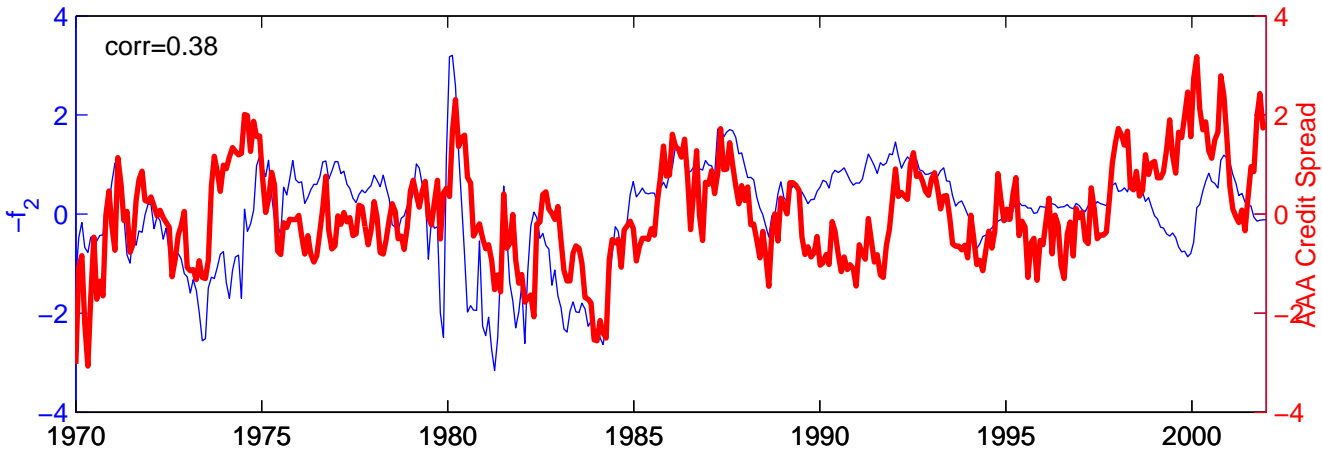


Figure 7. Orthogonalized Factor f_2 and the AAA Credit Spread and MZM growth rate

The plot shows the monthly series of the estimated latent factor f_2 (with minus sign) (thin line, left scale) against the spread between AAA Moody's corporate index and ten-year Treasury bond yield on panel (a) and MZM monthly growth rate on panel (b) (thick line, right scale). Both macro series are residuals from regressing the AAA spread (or MZM rate) on inflation and real activity. All three series are standardized to facilitate comparison. The MZM series are available from 1975.

Orthogonalized factor f_2 vs AAA Credit Spread



Orthogonalized factor f_2 vs MZM growth rate

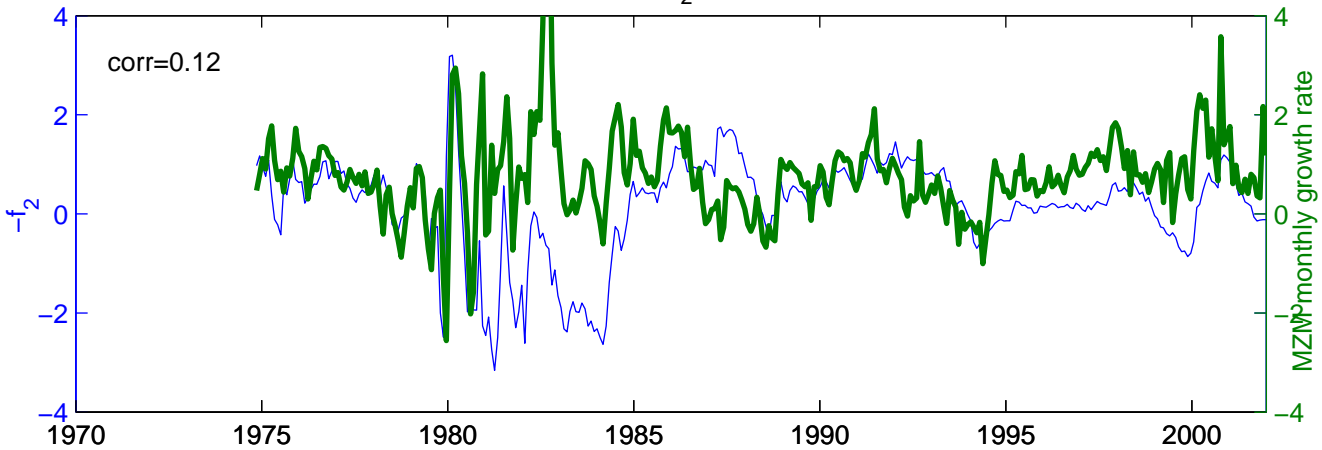


Figure 8. Impulse Response Functions: Expectations and Risk Premia.

The figure decomposes the impulse responses of the three-month and ten-year yields and the slope to one standard deviation shocks in the state variables into the expectations response and the term premia response. The solid line depicts the response of the yield (the sum of the two responses) – the same as in Figure 5.

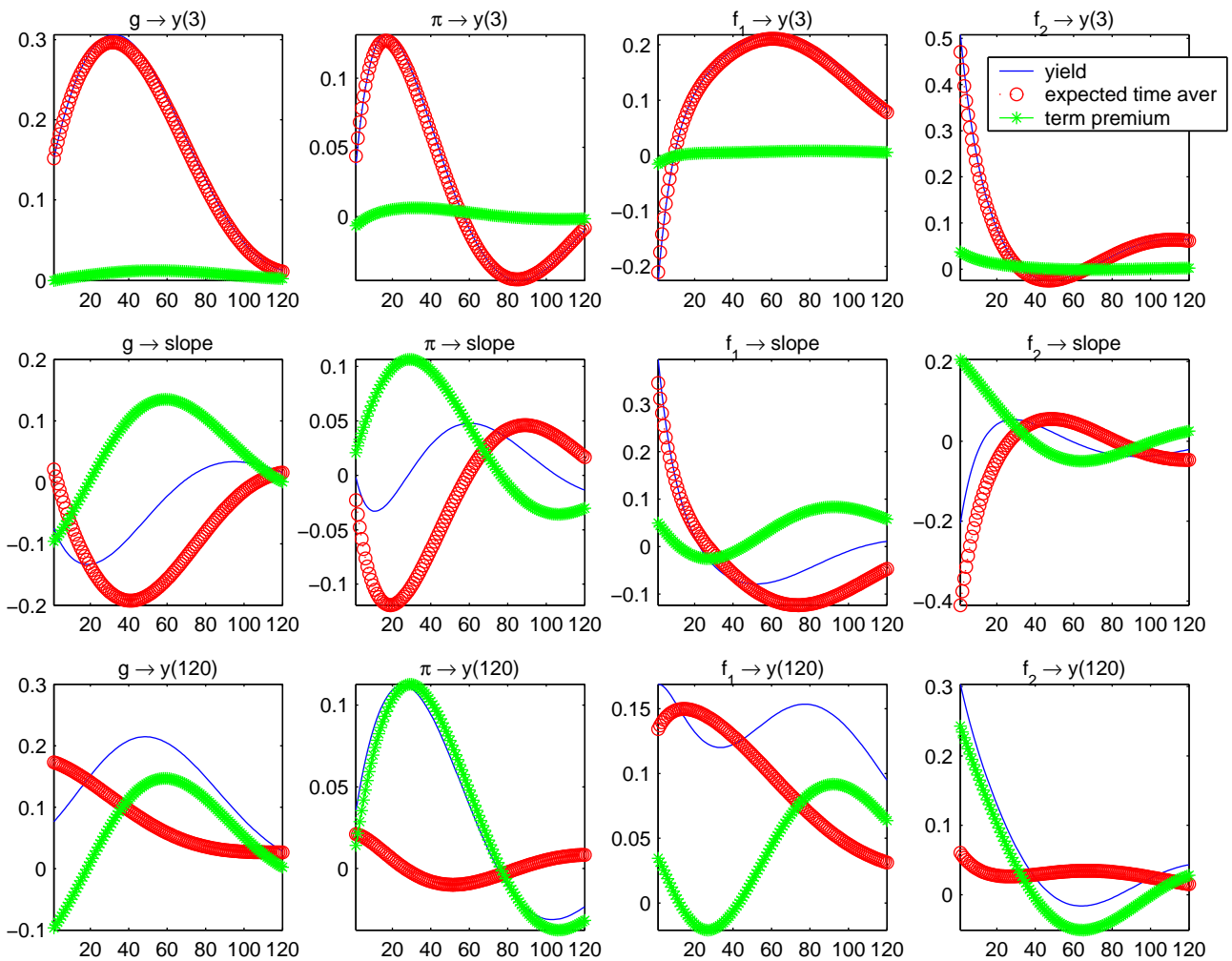


Figure 9. Term Premia Decompositions

The figure shows the time series of the one- and ten-year yields decomposed into the expectations and term premia. The term premia are subsequently decomposed into the contributions of the four determinants of the yield curve: real activity inflation, fiscal and liquidity shocks. The shaded regions show the NBER recessions.

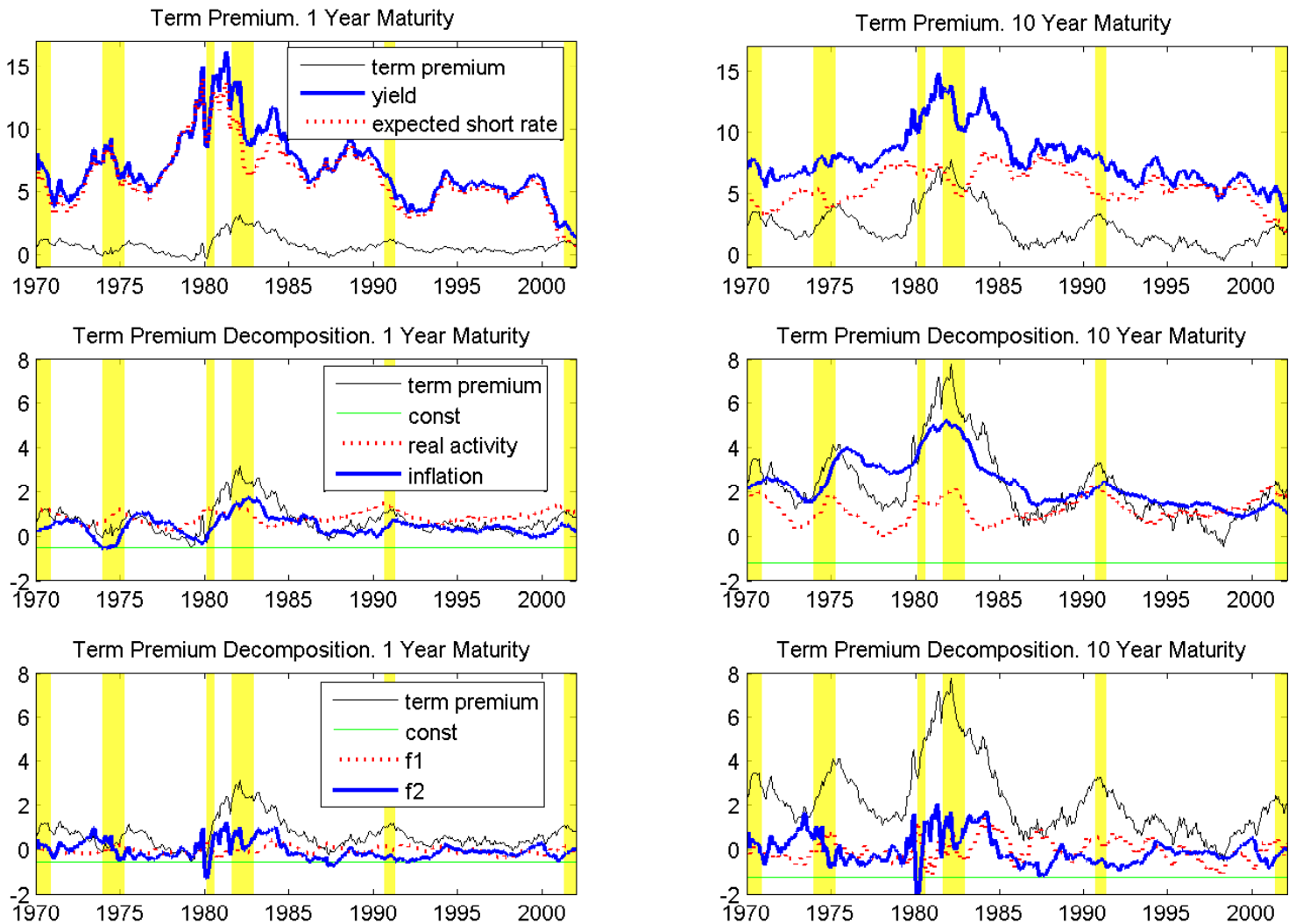


Figure 10. The Expectations Hypothesis

We show how well our model replicates the coefficients of the yield-predicting regressions ϕ_τ in panel (a) by plotting the data and model implied coefficients with the bootstrapped 95% confidence bounds. In panel (b) we decompose the model-implied coefficients according to the contributions of macro factors. These factor-based coefficients, together with the unity (zero risk premia) line add up to the model-implied ϕ_τ .

