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## Research Note

# The Role of Production Lead Time and Demand Uncertainty in Marketing Durable Goods

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Firms often have to make their production decisions under conditions of demand uncertainty. This is especially true for product categories such as automobiles and technology goods where the lead time needed for manufacturing forces firms to make production decisions well in advance of the selling season. Once the firm has produced the goods, the available production volume affects the firm's subsequent marketing decisions. In this paper, we study the relationship between the firm's production and marketing decisions for a durable goods manufacturer. We develop a dynamic model of a durable product market in which the demand functions are developed from a micromodeling of consumer utility functions and an equilibrium analysis of consumer strategies. After taking into account the demand uncertainty as well as the potential for cannibalization of future sales, the manufacturer makes its production and sales decisions. We find that the firm's optimal inventory level is U-shaped in the durability of the product and that the firm suffers a larger loss due to uncertainty when it is leases rather than sells its products. Furthermore, unlike the case for nondurables, for durable goods we find that the effect of uncertainty persists even after the uncertainty has been resolved.

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## 1. Introduction

Consider a firm planning to introduce a new product in the market. Given the lead time needed for manufacturing, the production decision may be based on the firm's expectations about the market demand for the new product. Thus, subsequent marketing decisions, such as price or advertising, are conditional not only on the realized demand, but also on the production decision made earlier, when the firm did not have complete information about market demand. This is precisely the problem faced by publishing houses as they play a guessing game about which books will become best sellers, or by toy companies guessing what the hot toy will be during the Christmas selling season. The fundamental question that this raises is, How does the separation of the production and the sales decisions for durable goods in the presence of demand uncertainty affect a firm's marketing strategy? The goal of this paper is to get a better understanding of this problem.

Even though there may be many reasons why realized demand may end up being different from the firm's prediction, one of the short-term consequence of this uncertainty is that the firm may have excess supply or face a shortage. Recent examples of excess supply include DreamWorks Animation, which grossly overpredicted its sales of the *Shrek 2* DVD, with the result that thousands of unsold DVDs were returned by retailers (Merissa 2005); Chinese cell phone makers licensed and produced huge quantities of low-cost phones only to discover that consumers wanted phones with new features such as cameras and MP3 players; and with rising gas prices, dealers of SUVs are facing bloated inventories and manufactures are resorting to new incentives to move these vehicles off dealer lots (Shirouzu 2005). On the shortage side, similar examples abound: The huge popularity of iPods led to shortages in the United States and led Apple to delay its European launch (Gibson 2004); there is a worldwide shortage of production facilities to manufacturer large LCDs for televisions leading to alliances between Sony and Samsung, and Philips and LG (Dvorak 2003); and consumers have to wait several months to get a Toyota Prius after placing an order (Eldridge 2004). A common theme running through these examples is that they are all durable goods and that their production decisions were made before their subsequent marketing decisions. Marketing researchers have not devoted much effort to this separation between production and marketing in the face of demand uncertainty. Yet, as the examples above suggest, this is a very real problem that has effects that persist even after the uncertainty is resolved.

One consequence of a mismatch between consumer demand and a firm's production quantity is that the firm can end up with varying levels of inventory. Yet, how this uncertainty affects inventory levels or firm profitability is not clear. As a motivating example, consider the automobile industry and its use of the days-of-supply metric to capture a car model's inventory level. Such a measure accounts for each model's base sales rate and allows a comparison of inventory levels across both high-volume and low-volume models. In 2000, General Motors had 68 days of supply for its Buick Century, 94 days of supply for its Chevrolet Prizm, and 114 days of supply for its Chevrolet Camaro. At first blush, one could argue that such differences in inventory levels are driven solely by demand uncertainty and errors in forecasts. On the other hand, in this paper we show that these differences in days of supply can be driven not only by demand uncertainty, but also by the inherent durability of the product.

The lead time preceding a firm's marketing decisions is crucial for a number of reasons. For example, before firms can market any new products, they first need to create manufacturing capacities, prepare the production infrastructure, procure parts and components, and manage the delivery logistics. For products such as automobiles that require a lot of outsourced parts and components, the production processes of suppliers further adds to production lead times. Traditionally, marketing researchers have focused on the retail price or selling quantity decisions and not paid much attention to the effects of production lead time and demand uncertainty; a typical assumption in their models is that firms can instantaneously produce the required number of goods and deliver them to the market. However, uncertainty about the length of the lead time or about the level of market demand can significantly affect the firm's pricing decision. In this paper, we focus on demand uncertainty and production lead time such that the firm has to commit to a specific production quantity before knowing the demand for its goods. As a result, the subsequent market price chosen by the firm may be affected by the available supply of units.

The other focus of this paper is on durable goods. Marketing durables such as automobiles or refrigerators is significantly different from the marketing and manufacturing of nondurables such as cereals or soft drinks. From a strategic perspective, the main differences arise because durables can also be sold by consumers in secondhand markets, thus leading to competition between used and new goods. In addition, there is the classic durable goods problem: A monopolist firm's inability to commit to maintaining high prices over time forces it to lower prices immediately, and thus lowers firm profits (Coase 1972).[1] Although there is a vast literature on durable goods, most of these papers do not consider demand uncertainty and production lead time. The few papers that do consider demand uncertainty (e.g., Bhatt 1989, Goering 1993), assume instantaneous production and delivery of the goods to the market. In addition, there are two papers that find over- or underproduction in durable goods models. In Wolinsky (1991), the durable goods manufacturer maintains an inventory, and in Denicolo and Garella (1999), the durable goods manufacturer produces less than the demand. However, both of these papers are concerned with addressing issues related to time consistency and neither has any demand-side uncertainty. In contrast, in our paper over- or underproduction arises as a result of demand-side uncertainty and production lead time.

Outside the durable goods area, a large number of papers, starting with Mills (1954), deal with demand uncertainty. In the operations area, research that addresses lead time and optimal inventory levels typically does not address the effect of production decisions on marketing decisions and demand typically is not a function of price (see Zipkin 2000 for a review). There are notable exceptions that allow for endogenous prices, e.g., Van Mieghem and Dada (1999), Petruzzi and Dada (1999), and Ferguson and Koenigsberg (2007). Our work is different from these papers in that we address specific issues related to durable goods such as the existence of resale markets, the effects of product durability, and the ability of the firm to lease or sell the good to consumers. In the spirit of the new research on the marketing-manufacturing interface (see, for example, Kekre and Srinivasan 1990, Kulp et al. 2004), we develop a more integrative view of the firm. Thus, we model both marketing and manufacturing decisions in an environment of uncertain demand.

In order to address the issues that arise from demand uncertainty and production lead time for durable goods, we develop a two-period model in which a

[1] Further research in this area has formalized Coase's conjecture (e.g., Bulow 1982, Stokey 1981) and pointed out conditions under which it does not hold, e.g., if there is a constant inflow of new customers (Conlisk et al. 1984), if the good depreciates (Bond and Samuelson 1984), or if the firm has increasing marginal production costs (Kahn 1986). See Tirole (1988) for a good review.

single manufacturer makes production and sales decisions for a durable product in each period. At the beginning of Period 1 when there is uncertainty about the level of demand, the manufacturer chooses the production quantity. After this quantity is produced, the demand state is revealed and the firm chooses the quantity to market in Period 1. In Period 2, the firm knows the demand level and makes its next set of production and marketing decisions. We begin with the analysis of the firm's decisions when it sells its products to consumers, and we subsequently analyze the case where it leases its product. In both cases, we derive the firm's optimal decisions and characterize the effects of lead time and uncertainty on the firm's profits.

The remainder of this paper is organized as follows. In the next section, we lay out the model and our assumptions. We solve for the equilibria and present our basic analysis in §3. In §4, we investigate the effect of lead time and demand uncertainty on inventory levels and profits. We conclude in §5.

## 2. Model

We develop a two-period model in which a manufacturer produces a durable product. The product can potentially provide two periods of service—a unit sold in Period 1 provides service in Periods 1 and 2, while a unit sold in Period 2 provides service only for that period. The product deteriorates with use and the extent of this deterioration is related to the inherent durability of the product. We capture durability through the parameter $\delta$, $0 \le \delta \le 1$, which represents how well a unit sold in Period 1 holds up in Period 2 (see Desai and Purohit 1998). If $\delta = 1$, the product does not deteriorate and new units are identical to used units. If $\delta = 0$, then the product is a nondurable and it deteriorates fully after one period of use. Note that deterioration $(1-\delta)$ should not be confused with depreciation, which incorporates deterioration as well as supply and demand conditions that may affect market price (see Desai and Purohit 1999).

An implication of product durability is that once the product has been sold by the manufacturer in Period 1, it can be resold by the subsequent owner in the secondhand market in Period 2. Because of a potentially large number of individual sellers, the secondhand or used market is competitive and there exists a market price for the used good. It is important to note that this price is determined by the total number of used units that potentially are available—whether or not the used good changes hands does not influence the market price. Furthermore, in the case of selling, because the manufacturer has no direct control over the secondhand market, used units compete against new units that the manufacturer sells; in the case of leasing, the old and new products still compete but the firm has control over both prices. Finally, note that in order for the product to deteriorate, it has to be used. Thus, a product stored in the manufacturer's inventory is not used and is identical to new production.

Consumers value the durable product for the stream of services that it provides over time. If a product lasts for $n$ periods, then consumers' total valuation of the product at the beginning of Period 1 is the discounted sum of the value provided in each period over its lifetime of $n$ periods. In our model, we assume that $n = 2$. We use the parameter $\phi \in [0, \alpha]$ to represent a consumer's valuation of the per period service provided by a new product. In this vertical differentiation model, a consumer with a higher $\phi$ values the product more than a consumer with a lower $\phi$. We assume that $\phi$ is distributed uniformly in the interval $[0, \alpha]$ and, in any period, each consumer uses at most one product. Finally, because the product deteriorates with use, the consumers' valuation of the per period services from a used product are $\delta\phi$. Thus, the net utility from using a product for a single period is $U = \delta^m \phi - r$, where $m$ is an indicator variable such that $m = 0$ if the product is new and $m = 1$ if the product is used, and $r$ is the one-period price. This underlying utility model leads to the following inverse demand system:

$$r_{1n} = \alpha - q_{1n}, \tag{1}$$

$$r_{2n} = \alpha - q_{2n} - \delta q_{2u}, \tag{2}$$

$$r_{2u} = \delta(\alpha - q_{2n} - q_{2u}), \tag{3}$$

where $r_{im}$ is the one-period price of the product in period $i$ $(i = 1, 2)$ of product $m$, where $m$ can be either a new $(n)$ or used $(u)$ good. Similarly, $q_{im}$ represents the quantities sold in period $i$ of product $m$. Note that a product sold by the manufacturer in Period 1 becomes a used product in Period 2, i.e., $q_{2u} = q_{1n}$. Because consumers value the product for the services that it renders, the selling price in Period 1, $p_{1n}$, is the discounted sum $p_{1n} = r_{1n} + \rho r_{2u}$, where $\rho \in [0, 1]$ is the discount factor common to consumers and the firm. Because the product lasts for only two periods, the selling price reflects only two one-period prices. Note that this selling price is also derived from our underlying utility model.

There is a single firm that manufactures a durable product with a constant marginal cost of production, $c > 0$ per unit. In addition, the firm incurs a production setup cost of $k > 0$ for every production run and a per unit holding cost of $h > 0$ if it carries any inventory for one period. Unused units in inventory are indistinguishable from new production in Period 2. It is straightforward to see that if holding cost $(h)$ is sufficiently low, and the production setup cost ($k$) is sufficiently high, it is possible that the firm may produce only once—at the beginning of Period 1—and produce no new units in Period 2. For our analysis, we only consider those parameter values for which the firm produces in both periods.

We model uncertainty by assuming that demand can either be high or low. In terms of the demand system in Equations (1)–(3), if demand is high, $\alpha = \alpha^H$, and if demand is low, $\alpha = \alpha^L$. The high-demand state is assumed to occur with probability $\theta$ and the low-demand state with probability $(1-\theta)$. The manufacturer is uncertain about the demand for its product prior to Period 1. However, as Period 1 begins, the firm observes the true demand state and this state persists for both periods. For example, prior to a new product launch, a firm may be uncertain about how the market may respond to its new product. However, once the product is launched and the firm observes how the market responds to the product, it does not have any uncertainty as long as the product and the consumers remain unchanged. Thus, uncertainty in our model merely reflects the firm's uncertainty about the market's reaction to the new product.

## 3. Analysis and Results

The sequence of events in our analysis are as follows.

*Stage* 1. The firm chooses the production quantity for the first period, $Q_1$. After this decision is made, the firm observes the demand state (high or low).

*Stage* 2. The firm chooses the first-period quantity to market, $q_{1n}$.[2] If sales are less than production, $q_{1n} < Q_1$, the firm carries over $I = Q_1 - q_{1n}$ units of inventory to Period 2.

*Stage* 3. The firm chooses the second period production level, $Q_2$ followed by the quantity to market for the second period, $q_{2n}$. Note that $q_{2n} = Q_2 + I$.

An important assumption of our model is the lead time between the production and marketing decisions above. The main point is that there is a separation between when the firm chooses its production level, when the nature of demand is revealed, and when the firm decides the quantity to market. Lead time in this framework can also be interpreted as any decision that cannot be changed in the short run. For example, capacity allocation to various product lines can only be changed in the long run. Thus, if a firm underestimates demand for its new product, there will be some length of time before it can make the necessary adjustments to its production decisions. If there were no lead time and we had instantaneous production, then the firm would make a single production-marketing decision. As a result, the optimal level of sales would depend not only on the expected demand, but also on the marginal cost of the product. On the other hand, when there is lead time and a separation between production and marketing decisions, then the production decision depends on the expected demand and the marginal cost, but the subsequent decision on the quantity to market is conditional only on the production decision made earlier. In this case, the firm's marketing decision is made after the uncertainty is resolved and the marginal cost is sunk (the firm has already produced a given quantity).

We begin our analysis with the case in which the firm sells the product to consumers. Subsequently, we consider the case when the firm leases its product to consumers.

### 3.1. Selling

In order to focus on the results, we have delegated most of the details of the analysis to an online technical appendix (provided in the e-companion).[3] Below, we provide only an overview of our approach. In order to derive a subgame-perfect equilibrium, we solve the game backward, starting from Stage 3 in Period 2. At the beginning of Stage 3, the firm does not face any uncertainty and knows whether $\alpha = \alpha^H$ or $\alpha = \alpha^L$. In addition, the firm knows the level of inventory, $I$, that has been carried from Period 1. Thus, the firm's problem is to choose its optimal production quantity $Q_2$, and its optimal sales quantity, $q_{2n}$. Recall that we are considering only those values of parameters for which the optimal second-period production $Q_2$ is positive. Thus, the firm maximizes profit in Stage 3,

$$\begin{aligned}\Pi_2 &= r_{2n}q_{2n} - cQ_2 - k = (r_{2n} - c)Q_2 + r_{2n}(I) - k \\ &= (r_{2n} - c)Q_2 + r_{2n}(Q_1 - q_{1n}) - k,\end{aligned}$$

by choosing an optimal quantity to produce in Period 2, $Q_2^*$.[4]

Given the optimal $Q_2^*$ in Stage 2, the firm chooses how much of its available volume to sell in Period 1. The firm's Stage 2 profits are given by

$$\begin{aligned}\Pi_1 &= (r_{1n} + \rho r_{2u})q_{1n} - hI \\ &= (\alpha - q_{1n} + \rho\delta(\alpha - Q_{2n}^* - I - q_{1n}))q_{1n} - hI.\end{aligned}$$

The firm maximizes the discounted sum of its Stage 2 and Stage 3 profits, $\Pi_1 + \rho\Pi_2^*$, by choosing a sales quantity, $q_{1n}$, subject to $q_{1n} \le Q_1$. This constraint captures the fact that the firm cannot sell more than it has produced. We denote the constrained and

[2] We have also analyzed a variant of our model in which the firm chooses prices rather than selling quantities. Our results remain unchanged.

[3] The e-companion to this paper, which is part of the online version, is available at http://mansci.pubs.informs.org/.

[4] In the absence of uncertainty, allowing the firm to choose $Q_2$ followed by $q_{2n}$ is identical to letting the firm choose $Q_2$.

unconstrained solutions to the above program by $q_{1n}^{j*}$ and $q_{1n}^{j\circ}$, respectively.

At the beginning of Stage 1, the firm does not know the state of demand and has to choose the first-period production quantity, $Q_1$, before the uncertainty is resolved. At this stage, the firm maximizes profits,

$$\Pi = \theta\left[\Pi_1^*(\alpha^H) + \rho\Pi_2^*(\alpha^H)\right] + (1-\theta) \cdot \left[\Pi_1^*(\alpha^L) + \rho\Pi_2^*(\alpha^L)\right] - Q_1 c - k, \tag{4}$$

by choosing an optimal quantity, $Q_1^*$ to produce, where $\Pi_1^*(\alpha^j)$ and $\Pi_2^*(\alpha^j)$ are the optimal profits under the low- (L) and high- (H) demand realizations. Table 1 provides the optimal quantities chosen by the firm.

This leads to the following proposition:

Proposition 1. *When $\theta(\alpha^H - \alpha^L) - c(1-\rho) - h > 0$, the firm carries inventory only in the low-demand state, $q_{1n}^{L\circ} < Q_1^* < q_{1n}^{H\circ}$.*

The optimal production quantity depends on $\theta(\alpha^H - \alpha^L) - c(1-\rho) - h$ being positive (we refer to this expression as the *inventory condition*). Essentially, this condition holds when the expected gain from the high state materializing $\theta(\alpha^H - \alpha^L)$ outweighs the relative cost of producing in the current and carrying the unit in inventory. A strategy of producing anything less than $q_{1n}^{L\circ}$ has the disadvantage that it requires selling suboptimal quantities in both the low- and the high-demand states. Producing a quantity greater than $q_{1n}^{L\circ}$ allows the firm to sell its optimal quantity in the low-demand state and it brings it closer to its optimal quantity in the high-demand state. A strategy of producing anything more than $q_{1n}^{H\circ}$ results in costly inventories in both demand states, and higher production costs without any demand-side benefit. These results seems intuitive in that the firm chooses a production quantity that strikes the middle ground between carrying an inventory and facing a shortage. However, the overall strategy of the firm is more complex as evidenced by the following result:

Proposition 2. *When $\theta(\alpha^H - \alpha^L) - c(1-\rho) - h < 0$, then $Q_1^* < q_{1n}^{L\circ} < q_{1n}^{H\circ}$.*

When the difference between the two states is sufficiently small or the high-demand state is less likely, the inventory condition is negative. In this case, a strategy of producing a quantity between $q_{1n}^{L\circ}$ and $q_{1n}^{H\circ}$

**Table 1 Equilibrium Values**

| | Selling | Leasing |
|---|---|---|
| Panel A. For $\theta(\alpha^H - \alpha^L) - c(1-\rho) - h > 0$ ($q_{1n}^{L*} = q_{1n}^{L\circ}$ and $q_{1n}^{H*} = Q_1^*$) | | |
| $q_{1n}^{L\circ}$ | $\frac{2[\alpha^L + h - c(1-\delta)\rho]}{4 + 4\delta\rho - 3\delta^2\rho}$ | $\frac{\alpha^L + h - \rho c(1-\delta)}{2[1 + \delta\rho(1-\delta)]}$ |
| $q_{1n}^{H\circ}$ | $\frac{2[\alpha^H + h - c(1-\delta)\rho]}{4 + 4\delta\rho - 3\delta^2\rho}$ | $\frac{\alpha^H + h - \rho c(1-\delta)}{2[1 + \delta\rho(1-\delta)]}$ |
| $Q_1^*$ | $\frac{2[\alpha^H\theta - c(1-\rho+\theta\rho-\delta\theta\rho) - h(1-\theta)]}{\theta(4 + 4\delta\rho - 3\delta^2\rho)}$ | $\frac{\alpha^H\theta - c(1-\rho+\theta\rho-\delta\theta\rho) - h(1-\theta)}{2\theta(1 + \delta\rho - \delta^2\rho)}$ |
| $Q_2^{L*}$ | $\frac{\theta[\alpha^L[8 - \delta(2-4\rho+3\delta\rho)] - 2h\delta - 4\alpha^H] + c[4(1-\rho) - \theta[4 + \delta\rho(2-\delta)]] + 4h}{2\theta(4 + 4\delta\rho - 3\delta^2\rho)}$ | $\frac{\theta[\alpha^L[2 - \delta(1+\rho-\delta\rho)] - h\delta - \alpha^H] + c(1-\theta-\rho) + h}{2\theta(1 + \delta\rho - \delta^2\rho)}$ |
| $Q_2^{H*}$ | $\frac{\theta\alpha^H[4 - \delta(2-4\rho+3\delta\rho)] - c[4\theta - \delta[2 - \rho(2+2\theta-\delta\rho)]] + 2h\delta(1-\theta)}{\theta(4 + 4\delta\rho - 3\delta^2\rho)}$ | $\frac{\theta[\alpha^L[1 - \delta(1+\rho-\delta\rho)] - \delta\alpha^H] - c[\theta - \delta(1-\rho)] + h\delta(1-\theta)}{2\theta(1 + \delta\rho - \delta^2\rho)}$ |
| Panel B. For $\theta(\alpha^H - \alpha^L) - c(1-\rho) - h < 0$ ($q_{1n}^{L*} = q_{1n}^{H*} = Q_1^*$) | | |
| $q_{1n}^{L\circ}$ | $\frac{2[\alpha^L + h - c(1-\delta)\rho]}{4 + 4\delta\rho - 3\delta^2\rho}$ | $\frac{\alpha^L + h - \rho c(1-\delta)}{2[1 + \delta\rho(1-\delta)]}$ |
| $q_{1n}^{H\circ}$ | $\frac{2[\alpha^H + h - c(1-\delta)\rho]}{4 + 4\delta\rho - 3\delta^2\rho}$ | $\frac{\alpha^H + h - \rho c(1-\delta)}{2[1 + \delta\rho(1-\delta)]}$ |
| $Q_1^*$ | $\frac{2[\alpha^H\theta + \alpha^L(1-\theta) - c(1-\delta\rho)]}{4 + 4\delta\rho - 3\delta^2\rho}$ | $\frac{\alpha^H\theta + \alpha^L(1-\theta) - c(1-\delta\rho)}{2(1 + \delta\rho - \delta^2\rho)}$ |
| $Q_2^{L*}$ | $\frac{\alpha^L[4 - \delta[2(1-\theta) - \rho(4-3\delta)]] - 2\theta\delta\alpha^H - c[4 - \delta[2 - \rho(4-\delta)]]}{2(4 + 4\delta\rho - 3\delta^2\rho)}$ | $\frac{\alpha^L[1 - \delta(1-\theta) + \delta\rho(1-\delta)] - \delta\theta\alpha^H - c(1-\delta+\delta\rho)}{2(1 + \delta\rho - \delta^2\rho)}$ |
| $Q_2^{H*}$ | $\frac{(4 + 4\delta\rho - 3\delta^2\rho)(\alpha^H - c) - 2\delta[\alpha^L(1-\theta) + \theta\alpha^H - c(1-\delta\rho)]}{2(4 + 4\delta\rho - 3\delta^2\rho)}$ | $\frac{\alpha^H - \delta[\alpha^L(1-\theta) + \alpha^H[\theta - \rho(1-\delta)]] - c(1-\delta+\delta\rho)}{2(1 + \delta\rho - \delta^2\rho)}$ |

is not feasible and the firm produces a quantity that will be guaranteed to be sold in the subsequent stage. One might argue that the production quantity dictated by the low-demand state should be the lower bound for the selling quantity, i.e., there is no point producing a quantity below what we would expect to sell in the low-demand case. However, this line of thinking is not accurate. In particular, as shown in Proposition 2, we find that the firm is better off producing a quantity that is strictly lower than the selling quantity of the low-demand state, $Q_1^* < q_{1n}^{L\circ}$. This occurs principally because of the lead time and the fact that production and sales decisions are separate. Specifically, the production decision is made before the uncertainty is resolved and the firm incorporates its marginal cost of production in choosing the optimal production quantity. After the uncertainty is resolved and the firm gets to the sales decision, the marginal cost of production is sunk and does not affect the optimal sales quantity. As a result, the firm has a tendency to "oversell," compared to a situation where the demand was known to be low and there was no lead time. Anticipating this overselling behavior down the road, the firm lowers its production quantity and chooses a lower level. Note that this result holds even in the absence of uncertainty.

Finally, note that lead time and uncertainty might imply that the firm may want to base its sales decision on its production decision. We find that when $Q_1 > q_{1n}^{L\circ}$, the firm's optimal selling quantity, $q_{1n}^{j*}$, is not a function of the firm's first-period production quantity, $Q_1$. Thus, it is not optimal for the firm to increase its sales simply because it has a greater-than-necessary production quantity available. In particular, the firm's optimal sales quantity is affected by the available production only in that the latter acts as an upper bound on the former. If the production constraint is not binding, i.e., the firm has overproduced, the firm does not find it optimal to sell any more than what the demand condition dictates. For the intuition behind this result, note that there are three adverse effects of trying to sell more simply because there are more units available. The first is that as the firm sells more units, $r_{1n}$ declines, which also reduces the first-period selling price, $p_{1n}$. Furthermore, due to the linkages between the two periods, any increase in $q_{1n}$ increases the size of the future used market, thus reducing the second-period price of used goods, $r_{2u}$, and as a consequence, the first-period price, $p_{1n}$. Finally, any increase in $q_{1n}$ also reduces the second-period price of new goods, $r_{2n}$. The net effect is that increasing $q_{1n}$ results in lower prices for the firm's products in both periods. The firm, therefore, prefers to carry $(Q_1 - q_{1n})$ units in inventory and sell them in Period 2. In this manner, the firm can reduce the second-period production quantity to offset the additional inventory carried over from Period 1, and it can do this without moving away from its optimal sales choices in either period.

### 3.2. Leasing

We now consider the case where the firm leases the product to consumers in each period. As a result, the used goods are returned to the firm at the end of Period 1, and they are subsequently leased again along with any new production in Period 2 (Desai et al. 2004). Because the steps to solving the model are similar to those outlined earlier, we delegate all the intermediate steps to the technical appendix and display the optimal quantities in Table 1. Under a leasing strategy, our results in the case are summarized by the following proposition:[5]

Proposition 3. *If* $\theta(\alpha^H - \alpha^L) - c(1-\rho) - h > 0$, *the firm carries inventory in the low-demand state and* $q_{1n}^{L\circ} < Q_1^* < q_{1n}^{H\circ}$. *If* $\theta(\alpha^H - \alpha^L) - c(1-\rho) - h < 0$, *then* $Q_1^* < q_{1n}^{L\circ} < q_{1n}^{H\circ}$.

These results are similar to those in the case of selling. As with the first-period optimal selling quantity, as long as there is adequate production available, the first-period optimal leasing quantity does not depend on the production quantity. That is, even when the firm discovers that it has overproduced, the firm does not lease a greater number of units. In addition, when the inventory condition is not satisfied, the firm chooses a production quantity that is lower than the leasing levels in both the high- and low-demand states.

## 4. Impact of Lead Time and Demand Uncertainty

As discussed earlier, demand uncertainty in the presence of production lead time compels the firm to make its first-period production decisions without fully knowing the market conditions, resulting in over- or underproduction in the first period. Furthermore, by explicitly incorporating durability, we find that the effects of demand uncertainty and production lead time persist in Period 2 even after the uncertainty is resolved and the firm is able to adjust its production and selling or leasing quantities. To understand this effect, consider the case of a nondurable product, i.e., $\delta = 0$. In this case, if the low-demand state arises, the firm has excess supply that it carries as inventory

[5] As noted earlier, an assumption of our analysis is that parameters are such that, regardless of the demand state, the firm chooses a positive production level in both periods.

to Period 2. Regardless of these outcomes, the firm's optimal selling or leasing quantity in Period 2 is driven solely by the amount dictated by the demand state: $q_2^* = (\alpha - c)/2$. Now consider the case where the product is a durable, $\delta > 0$. In Period 2, even though the demand uncertainty has been resolved, the firm's optimal choices depend on the decisions in Period 1: $q_2^* = (\alpha - c - \delta q_{1n})/2$. Thus, any change in $q_1$ due to uncertainty affects not only the first-period prices and profits, but also the second-period prices, quantities, and profits. For example, an increase in the first-period selling quantity, $q_{1n}$, also increases the number of used units in Period 2, resulting in a greater competition for the firm's new units from the used units in Period 2, which affects the second-period quantities, prices, and profits of the firm. This suggests that the effect of uncertainty tends to persist for durable goods, well after the uncertainty has been resolved.

Uncertainty combined with production lead time leads to our results regarding inventory in both the leasing and selling cases. In particular, given the optimal production and sales quantities discussed earlier, it is clear that only when the demand is low will the firm produce more than $q_{1n}^{L\circ}$ and carry inventory to Period 2. In particular, if the demand turns out to be low and $\theta(\alpha^H - \alpha^L) - c(1-\rho) - h > 0$, the firm carries an inventory. The optimal inventory levels in the selling and leasing states are given by

$$I^{s*} = \frac{2[\theta(\alpha^H - \alpha^L) - c(1-\rho) - h]}{\theta(4 + 4\rho\delta - 3\rho\delta^2)},$$
$$I^{l*} = \frac{(\alpha^H - \alpha^L)\theta - c(1-\rho) - h}{2\theta[1 + \delta\rho(1-\delta)]}. \quad (5)$$

It is easy to see that as the high-demand state becomes more likely ($\theta$ increases) or as the high-demand state becomes more attractive relative to the low-demand state ($\alpha^H - \alpha^L$ increases), the firm is likely to carry more inventory. If the firm carries an inventory, the next question to address is how this level changes with durability, $\delta$? We answer as follows:

Proposition 4. *The optimal inventory level, $I^*$, for a durable product is U-shaped in the level of durability, $\delta$, and $I^*$ is highest when the product is a nondurable, i.e., $\delta = 0$.*

Proposition 4 shows that the optimal inventory level exhibits a U-shaped curve with durability. Recall that we began this paper by noting the widely different inventory levels of cars produced by the same manufacturer. In particular, General Motors had 68 days of supply for its Buick Century, 94 days of supply for its Chevrolet Prizm, and 114 days of supply for its Chevrolet Camaro. A further analysis of these three car models reveals that they vary in terms of their reliability ratings as assessed by *Consumer Reports*. On a 1 (low reliability)–5 (high reliability) scale, the Century has a rating of 3, the Prizm has a rating of 5, and the Camaro has a rating of 1. Within the context of our model, reliability ratings can be considered a proxy for the durability of the product. Thus, Proposition 4 is consistent with the example of GM's days of supply for the three vehicles mentioned. As we discuss below, the relationship between the optimal inventory level and product durability depends on two effects: One creates incentives for a reduction in inventory as durability increases, and the other creates incentives for an increase in inventory as durability increases.

Consider a situation in which the low-demand state is realized. The firm's inventory level represents the trade-off between selling one more unit in Period 1 versus keeping it in inventory and selling it in Period 2. As the product becomes more durable, the price that the firm can get in Period 2 from sales of new units declines. This occurs because changes in $\delta$ affect the extent of competition between the old and the new units in Period 2. In particular, as products become more durable, the old units in the secondhand market are closer substitutes for new units. Therefore, holding all else constant, higher levels of $\delta$ decrease the price of new units in Period 2 (see Equation (2)). However, because a higher level of $\delta$ implies that used units are more similar to new ones, this increases the price of the used units, $r_{2u}$. This, in turn, increases the selling price of new units in Period 1, $p_{1n} = r_{1n} + \rho r_{2u}$. As $\delta$ increases, this favors selling a unit in Period 1 rather than in Period 2, thus creating an incentive for the firm to lower the level of inventory.

On the other hand, the firm's decisions are also based on what happens if demand turns out to be high. Because the firm sells its entire production in the high-demand state, the production quantity is the same as the sales quantity. Furthermore, as $\delta$ increases, the firm can charge a higher price in Period 1, thus increasing its incentives to sell additional units in Period 1. This effect induces the firm to produce more as $\delta$ increases. But if demand turns out to be low, the firm ends up with a larger inventory for higher values of $\delta$. These two effects together result in the U-shaped relationship between the optimal inventory level and $\delta$. In addition, the optimal inventory level increases with the difference between the two demand states ($\alpha^H - \alpha^L$) and with the probability of the high-demand state ($\theta$). However, in both these cases, the optimal inventory curves maintain the U-shaped relationship between $I^*$ and $\delta$.

The final question we address relates to the sizes of the inventory levels under leasing and selling. This leads to the following:

Proposition 5. *Holding all else constant, the firm carries a higher level of inventory when it leases than when it sells.*

Proposition 5 presents an interesting result on the difference between inventory levels under leasing and selling. As one would expect in an environment of uncertain demand, the inventory level is affected by the trade-off between producing and holding an additional unit and the opportunity cost of supplying less than the optimal quantity to the market. Since the firm internalizes the competition between new and used goods in the second period in leasing but not in selling, the firm's optimal prices and quantities are different in selling and leasing. Specifically, for a given state of demand ($\alpha^L$ or $\alpha^H$), the optimal prices and quantities in the first period are higher in leasing than in selling. Moreover, this difference between leasing and selling quantities is higher in the high-demand state than in the low-demand state. Stated differently, the variability in the firm's optimal quantities ($q_{1n}^{H\circ}$ and $q_{1n}^{L\circ}$) is greater in leasing than in selling. Therefore, the firm bears the cost of higher inventory levels when it leases than when it sells.

### 4.1. Profits

The durable goods literature has shown that leasing, unlike selling, does not suffer from problems of time consistency. Hence, profits are higher when the firm leases rather than sells its output (e.g., Bulow 1982). However, demand uncertainty imposes costs on the firm in that uncertainty moves the firm away from the optimal decisions under certainty. Although inventories help manage the uncertainty, inventory levels can also be measures of the inefficiency arising due to demand uncertainty. This raises the possibility that due to higher inventory levels, the firm suffers more inefficiency when it leases than when it sells. In this section, we focus on the differential impact of uncertainty on the profits associated with leasing and selling.

We first analyze the firm's profit under demand certainty in the case of selling. Define the firm's profit in the selling certainty case as $\Pi^{sc} = \theta\pi^{sc}(\alpha^H) + (1-\theta)\cdot\pi^{sc}(\alpha^L)$ where $\pi^{sc}(\alpha^j)$ is the firm's profit when it knows with certainty that $\alpha = \alpha^j$ $(j = H, L)$. Similarly, the firm's comparable leasing profits in the case of demand certainty is $\Pi^{lc} = \theta\pi^{lc}(\alpha^H) + (1-\theta)\pi^{lc}(\alpha^L)$ where $\pi^{lc}(\alpha^j)$ is the firm's profit when it knows with certainty that $\alpha = \alpha^j$ $(j = H, L)$. The following proposition characterizes the effect of uncertainty on the firm's profits:

Proposition 6. *The loss in the firm's profit due to demand uncertainty is greater when the firm is leasing than when it is selling*: $\Pi^{lc} - \Pi^{l*} > \Pi^{sc} - \Pi^{s*}$. *This effect increases as the product durability increases*:

$$\frac{\partial}{\partial\delta}\big((\Pi^{lc} - \Pi^{l*}) - (\Pi^{sc} - \Pi^{s*})\big) > 0.$$

The standard result in the durable goods literature is that because leasing does not suffer from the problem of time inconsistency, leasing is more profitable than selling. In other words, a leasing strategy affords the firm a greater degree of flexibility; hence, one might intuitively expect that compared to a selling strategy, demand uncertainty will have less of an impact on a leasing strategy. However, Proposition 6 suggests that demand uncertainty entails greater inefficiency for the firm when it leases than when it sells, i.e., $\Pi^{lc} - \Pi^{l*} > \Pi^{sc} - \Pi^{s*}$. This is because the opportunity cost of not having adequate production is higher when the firm is leasing. Clearly, this also means that the superiority of leasing over selling is diminished in the presence of uncertainty; $\Pi^{lc} - \Pi^{sc} > \Pi^{l*} - \Pi^{s*}$. As the level of durability increases, the firm suffers relatively more inefficiency in leasing, and as a result the difference between leasing and selling profits continues to shrink. Proposition 6 extends the durable goods results by highlighting that even though uncertainty causes leasing to lose some of its advantages over selling, it does not becomes less profitable than selling.

## 5. Conclusion

This paper addresses the problems created by the interaction between demand uncertainty and production lead time for the manufacturer of a durable good. We find that if inventorying costs are sufficiently low, the firm carries inventory when the demand state turns out to be low. Interestingly, for higher holding costs we find that the firm chooses a production quantity that is lower than the level dictated by both the low- and the high-demand states in a world with no uncertainty. This is a surprising result because one could easily argue that the lowest production level should be dictated by the number of units the firm would choose to market if it expected demand to be low. Our result that the firm constrains itself with a low production quantity in both demand states is driven solely by the separation between the production and the marketing decisions. In particular, after production has occurred, the firm views the marginal cost as sunk; as a result, it has a tendency to "oversell" its product. In order to curb its incentive to overmarket when the demand turns out to be low, the firm chooses a lower production level. Note that a low production level in Period 1 is a commitment to keep sales at a low level only in Period 1—in Period 2, there is no demand uncertainty and the firm increases its production to make up for its restrained production in Period 1. We further emphasize that constrained production with lead time and demand uncertainty in our model is not a solution to the Coase problem, i.e., the restrained production in Period 1 is not a commitment by the firm to sell a lower quantity in Period 2. Importantly, in our model, restrained production in Period 1 is as likely to occur under selling as under leasing (when the Coase problem does not exist).

A central theme that emerges from our analysis is that, in the face of demand uncertainty, the presence of an inventory allows the firm to be more aggressive with its production strategy. Being more aggressive simply means that the firm chooses higher production levels to capitalize on the upside of demand turning out to be high. However, counterintuitively, we also find that an aggressive production policy does not translate to an aggressive sales strategy; in particular, the firm's sales strategy is not affected by its inventory level. This suggests that the firm is better off taking bets on the demand state through its inventory level rather than through its sales strategy. However, if the firm cannot inventory units for some reason, such as high holding costs, then the optimal policy is to choose a lower production quantity for all cases; effectively, this leads to a less aggressive sales strategy. Given that the role of an inventory is to capitalize on the chance that demand may turn out to be high, an interesting issue to consider is how the inventory levels vary with the durability of the product. In particular, we find that inventory is U-shaped in product durability, $\delta$. This is interesting because it suggests that "medium" durability products should have lower inventory levels than high durability products. Although this result is consistent with anecdotal evidence from the automobile industry, we caution the reader that durability is not endogenous in our model. Further exploring the links between optimal durability and inventory levels would be an important direction in which to extend our work. Finally, we find that the firm chooses higher inventory levels when it leases than when it sells its product.

Our work also has interesting implications for future research. For example, we have not modeled marketing mix variables such as advertising. An interesting avenue for future research would be to examine how advertising decisions interact with demand uncertainty: Is it optimal for firms to advertise more when they find that they have overproduced? Another interesting topic for future research would be to further study the adverse impact of uncertainty under leasing strategy to further characterize how selling can be more profitable than leasing. Finally, additional research on how product durability affects optimal inventory policies strikes us as an important area for future work.

An e-companion to this paper is available as part of the online version that can be found at http://mansci.pubs.informs.org/.

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