

# Simple Relational Contracts to Motivate Capacity Investment: Price Only vs. Price and Quantity

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Because of long lead times associated with product development and building capacity, a supplier must initiate investment in capacity when the product development effort is ongoing. Because the product is ill defined at this point in time, the buyer is unable to commit to the future terms of trade through a court-enforceable contract. Instead, to provide incentives for capacity investment, the buyer informally *promises* future terms of trade. The prospect of future interaction creates an incentive for the buyer to pay the supplier as promised. We characterize optimal price-only and price-and-quantity promises and compare their performance. If the production cost is low and either the capacity cost is low or the discount factor is high, then the buyer should promise to purchase a specific quantity rather than simply promise to pay a per unit price; otherwise, the buyer should simply promise to pay a specified unit price.

*Key words:* relational contracts; supply chain management; capacity investment; price-only contracts

*History:* Received: January 28, 2006; accepted: August 20, 2006.

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## 1. Introduction

Partners in a supply chain often rely on informal agreements in addition to contracts. In a typical scenario, a supplier must build capacity contemporaneously with the buyer's product-development effort because of the long lead times associated with each process. While the product development is ongoing, the product specification is evolving, which makes the buyer unable to commit to the future terms of trade through a court-enforceable contract (because the firms cannot specify the product to be delivered, they cannot specify the quantity to be delivered or the unit price).

When the product development effort is finished and the product's specifications are completely determined, the firms can write a court-enforceable contract that specifies the terms of trade. However, if the supplier were to delay making capacity investments until this point, the resulting delay in getting the product to market would be unacceptable. Consequently, prior to initiating capacity investment, the supplier must rely on informal assurances from the buyer as to what unit price the buyer intends to pay

and perhaps the quantity the buyer intends to purchase. The prospect of repeated interaction and the associated future value of a trusting, cooperative relationship provide an incentive for the buyer to pay the supplier as promised. In effect, the firms cannot adopt a formal contract before the supplier invests in capacity, so they adopt an informal agreement to create incentives for capacity investment.

These dynamics arise frequently in innovative, high-tech industries such as electronics and semiconductor equipment manufacturing. For example, Toshiba informally promises the *price* per unit it will pay for electronic components while the design is still evolving, in order to give its supplier enough time to prepare for production. Because the design is changing significantly until large-scale production begins, Toshiba and its supplier do not commit to a formal contract until just before large-scale production begins (Sako 1992).

In the semiconductor equipment industry, buyers informally commit to order *quantities*. Typically, in advance of placing a binding order, the buyer shares a demand forecast with its supplier. The forecast serves

as an informal or “soft” order, intended to guide the supplier’s production decisions (Cohen et al. 2003, Johnson 2003).

In both the electronics and semiconductor equipment industries, the extent to which a supplier commits production resources on the basis of a buyer’s informal promise to purchase depends on the extent to which the supplier views the promise as being credible. In the electronics example, long-term trading relationships generally ensure that Toshiba will adhere to its promised price, so its suppliers prepare for production on the basis of this promise. In the semiconductor equipment industry, if a buyer either purchases in line with the forecast or compensates her supplier on canceling soft orders, the supplier will tend to commit production resources on the basis of the buyer’s forecast. However, to the extent that the buyer cancels soft orders without compensating the supplier, the supplier will tend not to commit production resources before receiving a firm order.

This paper describes how a buyer should make promises to purchase, describing whether the buyer should merely make a unit-price promise or if she should also make a purchase-quantity promise. In deciding what type of promise to make and how large it should be, the buyer must balance two concerns. If the promise is stingy, the supplier may invest little in capacity, constraining the buyer’s ability to satisfy market demand. If the promise is highly generous, the supplier may find it “too good to be true” and conclude that the buyer will not adhere to her promise.

We show how to optimally balance these two concerns. To do so we analyze a stylized repeated game of new product introduction and capacity investment. For each new product, the supplier must invest in capacity when the product development effort is ongoing, well in advance of the selling season. At this point, because the product is ill defined, the production cost, retail price, and demand are uncertain, and the buyer is unable to commit to the future terms of trade through a court-enforceable contract and so instead promises to purchase.

The buyer’s promised terms are an important component of a relational contract, which describes the firms’ informal agreement about how they will behave. Roughly speaking, a relational contract specifies for every period the buyer’s promised terms,

the supplier’s capacity investment, and whether the firms will adhere to the buyer’s promised terms. The relational contract must be self-enforcing: in each period the parties adhere to their informal agreement because failure to do so will destroy trust and hence the value of future cooperation.

Taylor and Plambeck (2003) characterize a self-enforcing relational contract that maximizes total expected discounted profit and allows the buyer to capture any fraction of the gain from cooperation.<sup>1</sup> This optimal relational contract is stationary (payment terms and capacity investment are the same in every period, on the equilibrium path). Nevertheless, even in the simplest setting, where the production cost is zero, this optimal relational contract can be very complex and thus difficult to implement. The buyer’s promised order quantity and payment to the supplier are piecewise linear, increasing functions of her realized demand; in some cases, the order quantity function exhibits multiple discontinuities.

Another complex feature is that the firms make a fixed transfer payment at the beginning of each period that is not contingent on any action by either firm. This feature could be criticized on the grounds that a manager might find it undesirable to make a payment with no strings attached. Consequently, to provide guidance to managers involved in procurement, it is important to examine relational contracts that are simple enough to be implemented and that do not have noncontingent transfer payments.

In this paper, we consider two simple, plausible kinds of promises to purchase that a buyer may make. In a price-only ( $P$ ) relational contract, the buyer promises to pay a specified price per unit but does not commit to purchase a specific quantity. In a price-and-quantity ( $PQ$ ) relational contract, the buyer promises to purchase a specified number of units for a specified price.

We focus on these two forms of relational contracts for three reasons. First, in designing a contract or relational contract, firms trade off complexity

<sup>1</sup> Taylor and Plambeck (2003) employ the same basic modeling framework as in this paper, although they allow for certain extensions (e.g., nonlinear capacity cost) that this paper does not, and this paper allows for extensions (e.g., uncertain retail price) that they do not. Both papers’ results apply to the core setting of fixed retail price and linear capacity cost; the differences in the extensions do not drive the differences in the results.

versus performance. The simplest form of a procurement agreement is simply to specify a unit price ( $P$ ); arguably the next simplest form is to specify both a price and a quantity ( $PQ$ ). Second, the supply chain contracting literature has extensively studied  $P$  contracts owing to their simplicity and has compared the formal contracts of these two forms. Third, the  $PQ$  relational contract is motivated by the use of “soft orders,” which are promises, but not legally binding commitments, to purchase a specified quantity.

In designing a relational contract, we take the perspective of the buyer: We identify the buyer’s promise to purchase that maximizes her expected discounted profit, subject to the constraints that the relational contract is self-enforcing and takes the simple  $P$  (or  $PQ$ ) form. In addition to characterizing the buyer’s optimal  $P$  and  $PQ$  relational contracts, we characterize when the buyer should offer which type of relational contract: If the production cost is low and either the capacity cost is low or the discount factor is high, then the buyer should promise to purchase a specified quantity (i.e., employ the  $PQ$  relational contract). Otherwise, the buyer should simply promise to pay a specified unit price. Finally, we characterize conditions under which these two simple promises perform nearly as well as the complex optimal relational contract.

### 1.1. Literature Review

The supply chain contracting literature examines the impact of court-enforceable contracts on capacity and inventory decisions. If both price and capacity are contractible, then by properly specifying the terms of the contract, the buyer can maximize the total supply chain profit and appropriate it entirely (Cachon and Lariviere 2001). When capacity is not contractible, firms may employ contracts that only specify the unit price. A number of papers (e.g., Van Mieghem 1999, Cachon and Lariviere 2001, Lariviere and Porteus 2001, Özer and Wei 2006) consider  $P$  contracts in newsvendor settings. Cachon and Lariviere (2001) consider asymmetric demand forecast information and examine how a buyer can signal her private information through the contract she offers. Özer and Wei (2006) extend Cachon and Lariviere (2001) and further examine how a supplier can screen buyers with private demand forecast information by offering a menu

of contracts. Corbett (2001) and Ha (2001) allow for asymmetric cost information.

Van Mieghem (1999) observes that firms may be unable to commit to a per unit price before the supplier invests in capacity. Furthermore, even if firms can write  $P$  contracts, they may be better off not doing so and instead agreeing to negotiate after demand is observed.

Other research considers formal contracts that expand the terms of trade beyond the unit price in newsvendor settings, examining returns (e.g., Pasternack 1985), quantity-flexibility contracts (e.g., Tsay 1999), revenue sharing (e.g., Cachon and Lariviere 2005), quantity discounts (e.g., Tomlin 2003), and channel rebates (e.g., Taylor 2002). Cachon (2003) and Chen (2003) provide comprehensive reviews of the supply chain contracting literature.

In contrast to the aforementioned papers, which employ formal, court-enforceable contracts to coordinate capacity or inventory decisions, we focus on informal agreements that are sustained by repeated interaction (relational contracts). In this vein, Debo and Sun (2004) consider a repeated game in which the supplier sets the wholesale price and the buyer must order before realizing demand. Debo and Sun show that the firms can increase their per period expected profit by adopting an informal agreement in which the supplier offers a price that is lower than his optimal stage-game price, and the buyer responds by ordering a quantity that is larger than her myopically optimal order quantity. The informal agreement is sustained by the threat of punishing deviation by noncooperative behavior in all subsequent periods (i.e., the firms employ trigger strategies).

Ren et al. (2005) consider a repeated game in which the buyer shares forecast information and then the supplier builds capacity. Their relational contract specifies the price per unit and punishment (noncooperation for a specified number of periods) when the buyer reports a “high” forecast but realized demand is “low.” When the discount factor is sufficiently large, the buyer reports the forecast truthfully, but the system is not perfectly coordinated because the punishment occurs with positive probability.

Tunca and Zenios (2006) model the interplay between relational contracts and supply auctions, with multiple suppliers that differ in quality. For a review

of the economics and sociology literature on relational contracts, see Plambeck and Taylor 2006.

The paper is organized as follows. Section 2 introduces a single-period game. Sections 3 and 4 characterize optimal  $P$  and  $PQ$  relational contracts. Section 5 characterizes when the buyer prefers each type of relational contract; §6 discusses the implications of relaxing our assumptions, and §7 provides concluding remarks. Proofs of all our analytic results are in Taylor and Plambeck (2007).

## 2. The Single-Period Game

Consider a simple two-firm supply chain for an innovative product. The downstream firm, denoted the buyer, sells the product to a market in which demand is uncertain. The buyer (she) purchases the product from an upstream supplier (he). The supplier must invest in production capacity before the demand is revealed and before the firms can contract for production.

This section describes the physical model of capacity investment and examines the case where the firms interact only once. Subsequent sections consider the implications of repeated interaction. Demand  $\xi$  is a random variable with distribution function  $\Phi(\xi)$  and density  $\phi(\xi)$ , where  $\phi(\xi) > 0$  for  $\xi \in [0, m)$ , where  $m \leq \infty$ , and  $\phi(\xi) = 0$  otherwise. Let  $r$  denote the retail price,  $c$  the per unit cost of capacity, and  $p$  the per unit cost of production. The firms have common information about the distribution of demand  $\xi$ , and the salvage value of excess product or capacity is assumed to be negligible.

The sequence of events is as follows:

1. The supplier invests in production capacity  $K$  and incurs cost  $cK$  (unobserved by the buyer).
2. The buyer observes demand  $\xi$  (privately).
3. The firms contract on the wholesale price per unit  $w$ . The buyer orders, and the supplier produces and delivers.

Because the product is ill defined when the supplier initiates his capacity investment (Step 1), at that time the firms are unable to write a court-enforceable procurement contract. However, close to the selling season (Step 3), the product is well defined and the firms can contractually specify the price and quantity.

The assumption that the buyer does not observe the supplier's capacity is motivated by the observation that typically the buyer is unable to determine

how many physical and human assets the supplier has dedicated to the buyer's product. The assumption that only the buyer observes the realized demand is motivated by the observation that a supplier often lacks visibility into the specifics of the buyer's end market (e.g., individual customers, orders); this assumption is inessential, as all the results extend when it is relaxed. Similarly, the assumption that both firms know the supplier's capacity cost  $c$  is inessential. However, the assumption that the production cost and retail price are commonly observed is essential for our analysis and results (see §6). The retail price assumption is reasonable when the buyer targets a price point and the supplier has a general understanding of the product's positioning in the end market. The assumption that both firms observe the production cost is reasonable if the bulk of the cost is from commodity material or labor inputs or if the buyer is otherwise able to become familiar with the production technology and associated costs.

In §6, we describe how the results extend when the retail price, production cost, and realized capacity are stochastic, and when the supplier has private information about the cost of capacity. These generalizations are important because our focus is on innovative products, that is, products that are ill-defined at the time when the supplier initiates capacity investment. At this time, it is natural that the retail price, production cost, and effectiveness of a dollar investment in capacity would be uncertain. We postpone the extensions to §6 to convey the main insights in the simplest framework and make it clear that the relaxations do not drive the main results.

We assume that in this single-period game, the firms split the ex post gain from trade  $(r - p) \min(K, \xi)$  according to the generalized Nash bargaining solution, with share  $\sigma \in (0, 1)$  for the supplier. Specifically, the buyer contracts to pay  $w = \sigma r + (1 - \sigma)p$  per unit and purchases the efficient quantity  $q = \min(K, \xi)$ .<sup>2</sup> The profit for the supplier (excluding the sunk cost of

<sup>2</sup> Suppose that the supplier and buyer bargain noncooperatively by making alternating offers of the per unit price, as in Rubinstein (1982). Then in the unique subgame perfect equilibrium, by the theorem on page 106 of Rubinstein (1982), the price  $\sigma r + (1 - \sigma)p$  is immediately offered and accepted. The parameter  $\sigma \in (0, 1)$  depends on which firm makes the first offer, the time between offers, and the discount factor. The trick in extending Rubinstein's

capacity) is  $\sigma(r - p) \min(K, \xi)$ , and the profit for the buyer is  $(1 - \sigma)(r - p) \min(K, \xi)$ . The economics literature on incomplete contracts and on relational contracts adopts this generalized Nash bargaining solution (e.g., Grossman and Hart 1986, Baker et al. 2002). The Nash bargaining solution is the unique outcome that satisfies a set of axiomatic properties, including Pareto optimality and independence of irrelevant alternatives (Nash 1950).

Anticipating the per unit price  $\sigma r + (1 - \sigma)p$ , the supplier's expected profit when he builds  $K$  units of capacity is

$$\sigma(r - p)E \min(K, \xi) - cK. \quad (1)$$

The supplier faces a newsvendor problem, and his optimal capacity is  $\underline{K} \equiv \Phi^{-1}([1 - c/\sigma(r - p)]^+)$ . The supplier's and buyer's expected profit are

$$\begin{aligned} \underline{\Pi}_S &= \sigma(r - p)E \min(\underline{K}, \xi) - c\underline{K} \\ \underline{\Pi}_B &= (1 - \sigma)(r - p)E \min(\underline{K}, \xi). \end{aligned}$$

In contrast, the expected profit of the total supply chain  $(r - p)E \min(K, \xi) - cK$  is maximized at capacity  $\bar{K} \equiv \Phi^{-1}([1 - c/(r - p)]^+)$ . If the supplier captures all the gain from trade so that all system revenue accrues to him ( $\sigma = 1$ ), then he will build the first best level of capacity  $\bar{K}$ . If the buyer captures a portion of the gain from trade ( $\sigma < 1$ ), then the supplier will build a level of capacity that is smaller than the first best  $\bar{K}$ . This is a classic hold-up problem: The supplier invests too little because he will capture only a fraction of the return on investment. If the capacity cost is sufficiently high  $c \geq \sigma(r - p)$ , then the supplier's incentive to invest is eliminated,  $\underline{K} = 0$ .

### 3. Optimal Price-Only Relational Contract

Now suppose that the firms produce and sell a succession of distinct products, repeating the game

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theorem to this setting with private information is to recognize that the expected profit after contracting on per unit price  $w$  is  $(r - w)E \min(K, \xi)$  for the buyer and  $(w - p)E \min(K, \xi)$  for the supplier. The private information is embedded in the constant multiplier  $E \min(K, \xi)$  and does not affect the subgame perfect equilibrium.

described in §2 in periods  $t = 1, 2, \dots$ . Because the product produced in each period is distinct, the supplier must make a new capacity "investment" in each period. (This does not necessarily mean that the supplier builds a new production facility every period; instead, the capacity could be thought of as reserved for the buyer's specific product.)

The firms may have alternatives to working with one another. For example, at the beginning of each period, a supplier may instead contract to supply a different product to a different buyer. We denote the supplier's outside option single-period expected profit  $\underline{\Pi}_S$ . Let  $\underline{\Pi}_B$  denote the analogous quantity for the buyer. So that the outside options are viable, we assume that they generate at least the noncooperative equilibrium expected profit derived in §2:  $\underline{\Pi}_B \geq \underline{\Pi}_B$  and  $\underline{\Pi}_S \geq \underline{\Pi}_S$ . The boundary case  $\underline{\Pi}_B = \underline{\Pi}_B$  and  $\underline{\Pi}_S = \underline{\Pi}_S$  captures the situation in which the firms do *not* have viable outside options. To reflect that the potential value that can be created is greater inside rather than outside the supplier-buyer relationship, assume that the sum of the outside option profits is strictly less than the profit of the two-firm integrated system. Although for simplicity we suppose that the supplier is willing to participate if the relational contract gives him at least his per period outside option profit, all our results extend when the supplier insists on receiving a portion of the gain from cooperation (see §6).

With repeated interaction, the firms can adopt an informal agreement about the terms of trade that will give the supplier a greater return on capacity investment than the noncooperative outcome of §2. Under a  $P$  relational contract, the buyer promises to pay the supplier  $w$  for each unit she buys. However, the buyer does not commit to purchase a specific quantity; rather, she is free to order the quantity that maximizes her profit. In maximizing expected profit, one can restrict attention to  $P$  relational contracts in which  $w \in [p, r]$ , so that the buyer will procure the efficient quantity  $\min(K, \xi)$ . (If  $w > r$ , the buyer would order zero, regardless of realized demand. If  $w < p$ , the supplier would produce zero regardless of the buyer's order.)

In each period in which either firm refuses to transact, the firms pursue their outside options, which results in expected profit of  $\underline{\Pi}_B$  for the buyer and  $\underline{\Pi}_S$  for the supplier in that period. In each period

in which both firms transact, the sequence of events is exactly as specified in §2, except the last step is replaced by the following:

3'. If both firms adhere to the proposed terms, they codify price  $w$  per unit in a court-enforceable contract. The buyer orders  $\xi$  and the supplier produces and delivers quantity  $\min(K, \xi)$ . Otherwise, if one or both firms do not adhere, noncooperative bargaining occurs, as in §2: The supplier produces and delivers  $\min(K, \xi)$ , and the buyer pays  $\sigma r + (1 - \sigma)p$  per unit.

The firms are infinitely lived and risk neutral. At the end of each period their game terminates with probability  $\psi$ , and the common discount factor is  $\delta'$ . The termination probability may be a measure of the stability of the firms, economic conditions, or the riskiness of the product market. The discount factor reflects the firms' cost of capital and the length of time between successive products. Short (long) development and production lead times and frequent (infrequent) product introductions will tend to be associated with high (low) discount factors. The effective discount factor is  $\delta = (1 - \psi)\delta'$ .

We assume that both firms employ trigger strategies, as is standard in the economics literature on relational contracts (Baker et al. 2001, 2002; Levin 2003). A *trigger strategy* is to transact and to adhere to the promised payment  $w$  in every period until one firm refuses to do so, and then to refuse transact and refuse to adhere in all subsequent periods. A  $P$  relational contract specifying per unit price  $w$  is *self-enforcing* if the trigger strategies constitute a perfect public equilibrium (PPE). As defined in Fudenberg et al. (1994), a PPE is a profile of public strategies that, for each period  $t$  and public history at the beginning of period  $t$ , constitute a Nash equilibrium from that time onward. If the firms coordinate on a self-enforcing  $P$  relational contract, neither party will subsequently wish to deviate from it unilaterally.

We say that a *relational contract of a particular form is optimal* if it is self-enforcing and no other self-enforcing relational contract of that form generates strictly greater expected profit for the buyer. An *optimal  $P$  relational contract* is characterized by

$$\max_{w \in [p, r]} \pi(w) \quad (2)$$

subject to

$$K^P(w) \equiv \Phi^{-1}([1 - c/(w - p)]^+) \quad (3)$$

$$(w - p)E \min(K^P(w), \xi) - cK^P(w) \geq \underline{\Pi}_S \quad (4)$$

$$(w - p)E \min(K^P(w), \xi) + \delta(1 - \delta)^{-1} \cdot \{(w - p)E \min(K^P(w), \xi) - cK^P(w)\} \\ \geq \sigma(r - p)E \min(K^P(w), \xi) + \delta(1 - \delta)^{-1} \underline{\Pi}_S \quad (5)$$

$$(r - w) \min(K^P(w), \xi) + \delta(1 - \delta)^{-1} \pi(w) \\ \geq (1 - \sigma)(r - p) \min(K^P(w), \xi) + \delta(1 - \delta)^{-1} \underline{\Pi}_B \\ \text{for } \xi \in [0, m], \quad (6)$$

where

$$\pi(w) \equiv (r - w)E \min(K^P(w), \xi)$$

denotes the buyer's expected profit per period when the per unit price is  $w$ . Constraint (3) has the supplier choosing his capacity optimally. To understand this, note that the supplier's expected profit when he builds  $K$  units of capacity is

$$(w - p)E \min(K, \xi) - cK;$$

the supplier faces a newsvendor problem, and the optimal capacity is  $K^P(w)$ ; " $P$ " is mnemonic for price only. Constraint (4) ensures that the supplier's expected single-period profit under the relational contract is greater than his outside option profit. The supplier could refuse to sign a contract employing the proposed unit price, in which case the firms would trade at the noncooperative price  $\sigma r + (1 - \sigma)p$  in that period. Because the firms follow trigger strategies, refusal to adhere to the proposed price would result in expected profit of  $\underline{\Pi}_S$  for the supplier in all subsequent periods. Constraint (5) ensures that the supplier's expected profit when he adheres exceeds his profit when he reneges. After observing the demand, but without knowing the supplier's capacity choice, the buyer could refuse to sign a contract employing the proposed unit price. Constraint (6) ensures that the buyer's expected profit when she adheres exceeds her profit when she reneges.

Our assumption that the firms follow trigger strategies is without loss of generality in maximizing expected profit, because the strongest equilibrium punishment for renegeing on payment terms is refusal

to transact in all subsequent periods. If the firms anticipated that cooperation would resume after a finite number of periods of punishment, the effect on the contract design problem would be to increase the right-hand side of constraints (5) and (6) and thus decrease the optimal objective value.

However, in the event that one firm did violate the relational contract and both firms subsequently refused to transact, the firms could increase their future profits by renegotiating their relational contract and resuming cooperation. This raises a question for experimental and empirical research: Is cooperation restored after a firm breaks a promise? If so, how quickly? In a laboratory experiment using a repeated trust game, Schweitzer et al. (2005) observed that when a subject is deceived by its partner (the partner promises to make a payment in return for cooperative action and breaks that promise), in subsequent periods the subject tends to distrust his partner and to behave noncooperatively. Even when the deceived subject receives a promise, an apology, and a series of cooperative actions from his partner, noncooperative behavior persists.

In a similar vein, Fehr et al. (1997) and references therein provide extensive experimental evidence that people are frequently willing to forgo large amounts of money to punish unfair behavior. According to Helper (1991), U.S. auto manufacturers that violated relational contracts for capacity investment and “cut the legs out from more than one supplier” in the 1970s subsequently had great difficulty building trust and collaborative relationships with suppliers. This experimental and empirical evidence that breaking a promise causes significant and enduring harm provides support for our model formulation with trigger strategies. However, a more complex behavioral model may produce richer managerial insights. Atkins et al. (2006) propose an alternative to trigger strategies, in which the duration of punishment is proportional to the magnitude of deviation from the agreement. An introduction to the economics literature on renegotiation in repeated games is in Abreu and Pearce (1991).

We now turn to characterizing an optimal  $P$  relational contract more precisely. We can without loss of generality restrict attention to  $P$  relational contracts in

which the proposed price exceeds the noncooperative price:

$$w \geq \sigma r + (1 - \sigma)p. \quad (7)$$

If (7) were violated, the supplier would build less than the noncooperative capacity  $\underline{K}$ , which would push the total system expected profit below its level without cooperation  $\underline{\Pi}_B + \underline{\Pi}_S$ .

The supplier’s participation constraint (4) simplifies to a constraint that the promised price be sufficiently large:

$$w \geq \underline{w},$$

where  $\underline{w}$  is the unique solution to

$$(w - p)E \min(K^P(w), \xi) - cK^P(w) = \underline{\Pi}_S,$$

when  $\underline{\Pi}_S > \underline{\Pi}_S$ , and  $\underline{w} = \sigma r + (1 - \sigma)p$  otherwise. Note  $\underline{w} \geq \sigma r + (1 - \sigma)p$ , where the inequality is strict if and only if  $\underline{\Pi}_S > \underline{\Pi}_S$ . Because the promised price is greater than the noncooperative outcome price, the supplier’s adherence constraint (5) is satisfied. The final constraint that remains is the buyer’s adherence constraint (6), which can be rewritten as

$$\begin{aligned} & \delta(1 - \delta)^{-1}[\pi(w) - \underline{\Pi}_B] \\ & \geq [w - \sigma r - (1 - \sigma)p] \min(K^P(w), \xi) \quad \text{for } \xi \in [0, m]. \end{aligned} \quad (8)$$

The left-hand side of (8) is the present value of the gain from the buyer’s ongoing cooperation, and the right-hand side is the buyer’s one-period gain from renegeing. The buyer’s renegeing temptation is most acute when demand is large; the buyer’s maximal one-period gain from renegeing is

$$[w - \sigma r - (1 - \sigma)p]K^P(w). \quad (9)$$

Thus, the  $P$  relational contract design problem can be rewritten as

$$\max_{w \in [\underline{w}, r]} \pi(w) \quad (10)$$

$$\text{subject to } A(w) \geq 0, \quad (11)$$

where

$$\begin{aligned} A(w) \equiv & \delta(1 - \delta)^{-1}[\pi(w) - \underline{\Pi}_B] \\ & - [w - \sigma r - (1 - \sigma)p]K^P(w). \end{aligned}$$

When the wholesale price is contractible, the smallest optimal price for the buyer is

$$w_c^P = \min \left\{ w: w \in \arg \max_{w \in [\underline{w}, r]} \pi(w) \right\};$$

“c” is mnemonic for contractible. Note that if the buyer’s expected profit when the wholesale price is contractible is less than her outside option

$$\pi(w_c^P) \leq \underline{\Pi}_B, \quad (12)$$

then  $P$  relational contracts are ineffective: the buyer cannot increase her profit by simply promising to pay a specified price per unit.

Proposition 1 characterizes an optimal  $P$  relational contract. Define

$$\underline{\delta}^P \equiv \min \{ \delta: \exists w \text{ such that } w > \max(\sigma r + (1-\sigma)p, c+p), \\ w \geq \underline{w} \text{ and } A(w) \geq 0 \}$$

$$\bar{\delta}^P \equiv \min \{ \delta: A(w_c^P) \geq 0 \}.$$

If (12) does not hold, then  $0 < \underline{\delta}^P \leq \bar{\delta}^P < 1$ . The last part of the proposition considers a mild technical restriction on the demand distribution

$$\lim_{x \rightarrow 0} x^2 / \phi(\Phi^{-1}(1-x)) = 0, \quad (13)$$

which is satisfied by commonly used demand distributions (e.g., normal, lognormal, Weibull, gamma, beta, exponential, uniform, triangular, Pareto with finite mean). However, it may be violated by distributions with heavy tails (e.g., Pareto with infinite mean). Equation (13) holds if  $\xi$  has an increasing generalized failure rate, infinite support, a finite second moment, and if  $\lim_{x \rightarrow 0} x\Phi(1-x)$  exists (see Lariviere 2006, Theorem 4).

**PROPOSITION 1.** *If  $\underline{\Pi}_B + \underline{\Pi}_S > \underline{\Pi}_B + \underline{\Pi}_S$  and either (12) holds or  $\delta < \underline{\delta}^P$ , then no self-enforcing  $P$  relational contract exists. Otherwise, an optimal  $P$  relational contract has optimal unit price  $w^P$ , which is increasing in  $\delta$  and satisfies*

$$w^P \begin{cases} = \sigma r + (1-\sigma)p & \text{if } \delta < \underline{\delta}^P \text{ or (12) holds;} \\ & \text{otherwise,} \\ \in (\sigma r + (1-\sigma)p, w_c^P) & \text{if } \delta \in [\underline{\delta}^P, \bar{\delta}^P) \\ = w_c^P & \text{if } \delta \geq \bar{\delta}^P. \end{cases}$$

If (13) holds, then there exists  $\underline{c} > 0$  such that if  $c \leq \underline{c}$ , then (12) holds.

When the capacity cost is small ( $c \leq \underline{c}$ ), the supplier’s noncooperative capacity  $\underline{K}$  is sufficiently large that commitment to pay any price higher than the noncooperative price will reduce the buyer’s profit. Consequently, in this parameter region,  $P$  relational contracts are ineffective. For the remainder of this section, we will focus on the case  $c \in (\underline{c}, r)$ .

Then,  $P$  relational contracts are effective only when the discount factor  $\delta$  is sufficiently high. If  $\delta$  is small, then the buyer’s ongoing expected discounted profit from cooperation is small, so the promise of a high price is not credible, and consequently, the buyer must promise a low price. (Although both firms would prefer a higher price, the resulting reneging temptation for the buyer is too high for such a promise to be credible.) As the discount factor increases, promises of larger wholesale prices become credible and are optimal:  $w^P$  is increasing in  $\delta$ . Consequently, under the optimal  $P$  relational contract, the supplier’s capacity  $K^P(w^P)$  and the buyer and supplier’s expected profits per period, which we denote  $\Pi_B^P$  and  $\Pi_S^P$ , also increase with  $\delta$ . Thus, buyers in industries marked by long development times between successive product generations, high capital costs, and/or low continuation probabilities (resulting from, for instance, uncertain technologies or a turbulent economic environment) should promise relatively low wholesale prices. If any of these factors should change favorably (e.g., the development time shrinks), the buyer should respond by promising a higher wholesale price. This will result in a higher capacity investment by the supplier and greater profits for both firms. Nonetheless, the buyer should always promise a wholesale price that is lower than the price she would offer if the price were contractible:  $w^P \leq w_c^P$ . Further, because  $w^P < r$ , the supplier strictly underinvests in capacity, and total system expected profit is bounded away from the first best even as  $\delta \rightarrow 1$ .

As one would expect, as the supplier’s outside-option profit  $\underline{\Pi}_S$  increases, the promised unit price increases. In contrast, as the buyer’s outside-option profit  $\underline{\Pi}_B$  increases, the promised price *decreases*. Perhaps surprisingly, both of these changes in bargaining positions reduce the buyer’s profit under the relational contract  $\Pi_B^P$ . Having a stronger outside option constrains the buyer’s credible promises and reduces

her profit. When  $\underline{\Pi}_S$  and/or  $\underline{\Pi}_B$  are sufficiently large,  $P$  relational contracts are not effective.

#### 4. Optimal Price-and-Quantity Relational Contract

When demand in a period is low, the supplier suffers under an informal agreement that only specifies a unit price, because he is unable to recoup the costs of his capacity investment. A buyer's informal commitment to purchase a specified quantity protects the supplier from this fate. Under a  $PQ$  relational contract, the buyer promises to purchase  $Q$  units and to pay the supplier  $wQ$ . After capacity has been built but prior to production, the firms contractually specify the terms of trade: If both firms adhere to the promised terms, they codify those terms in a formal contract. The supplier is only able to adhere to the terms if he has sufficient capacity  $K \geq Q$ . If the supplier has built more than  $Q$  units of capacity and demand exceeds  $Q$ , then the firms will split the additional gains from trade according to the noncooperative solution (i.e., the buyer would acquire the additional units at per unit price  $\sigma r + (1 - \sigma)p$ ). Thus, a supplier that builds enough capacity to meet the buyer's promised order will build capacity  $\max(\underline{K}, Q)$ .

An optimal  $PQ$  relational contract is characterized by

$$\max_{w, Q} E[r \min(Q, \xi) - wQ + (1 - \sigma)(r - p) \cdot [\min(\underline{K}, \xi) - Q]^+] \quad (14)$$

subject to

$$(w - p)Q + \sigma(r - p)E[[\min(\underline{K}, \xi) - Q]^+] - c \max(\underline{K}, Q) \geq \underline{\Pi}_S \quad (15)$$

$$(1 - \delta)^{-1} \{ (w - p)Q + \sigma(r - p) \cdot E[[\min(\underline{K}, \xi) - Q]^+] - c \max(\underline{K}, Q) \} \geq \sigma(r - p)E \min(K, \xi) - cK + \delta(1 - \delta)^{-1} \underline{\Pi}_S \quad \text{for } K \in [0, Q] \quad (16)$$

$$(w - p)Q + \delta(1 - \delta)^{-1} \{ (w - p)Q + \sigma(r - p) \cdot E[[\min(\underline{K}, \xi) - Q]^+] - c \max(\underline{K}, Q) \} \geq \sigma(r - p)E \min(Q, \xi) + \delta(1 - \delta)^{-1} \underline{\Pi}_S \quad (17)$$

$$r \min(Q, \xi) - wQ + \delta(1 - \delta)^{-1} \cdot \{ rE \min(Q, \xi) - wQ + (1 - \sigma)(r - p)E[[\min(\underline{K}, \xi) - Q]^+] \} \geq (1 - \sigma)(r - p) \min(Q, \xi) + \delta(1 - \delta)^{-1} \underline{\Pi}_B \quad \text{for } \xi \in [0, m]. \quad (18)$$

Constraint (15) ensures that the supplier's expected single-period profit under the relational contract is greater than his outside option profit. Constraint (16) ensures that the supplier builds enough capacity to meet the buyer's promised purchase quantity. If the supplier failed to do so, the firms would in that period split the gains from trade according to the noncooperative outcome price  $\sigma r + (1 - \sigma)p$ . Because the firms follow trigger strategies, failure to build adequate capacity results in expected profit of  $\underline{\Pi}_S$  for the supplier in all subsequent periods, which explains the right-hand side of (16).

The supplier could renege by refusing to sign a contract to supply  $Q$  units in return for payment  $wQ$ . If the supplier reneges, the firms split the gains from trade according to the noncooperative solution in that period, and the supplier receives expected profit of  $\underline{\Pi}_S$  in all subsequent periods. Constraint (17) ensures that the supplier's expected profit when he adheres exceeds his profit when he reneges. After observing demand, the buyer could renege by refusing to sign a contract to buy  $Q$  units for the per unit price of  $w$  and insisting instead on purchasing at the noncooperative outcome price. Constraint (18) ensures that the buyer's expected profit when she adheres exceeds her profit when she reneges. The buyer's one-period gain from renegeing when she observes demand  $\xi$  is

$$wQ - [\sigma r + (1 - \sigma)p] \min(Q, \xi).$$

The buyer's renegeing temptation is most acute when demand is small (the buyer's maximal one-period gain from renegeing is  $wQ$ ); this is the reverse of the case under the  $P$  relational contract.

It is instructive to compare the buyer's maximal renegeing temptation under a  $PQ$  and  $P$  relational contract. Consider an informal agreement of each type where the capacity investment and unit price are the same. Because the  $PQ$  relational contract involves an additional promise (quantity in addition to price), the buyer's maximal renegeing temptation is higher under the  $PQ$  relational contract.

By following through on the commitment to purchase  $Q$  units, the buyer is able to verify that the supplier has  $Q$  units of capacity. However, this monitoring is costly to the total supply chain in that it requires the supplier to produce  $Q$  even when end demand  $\xi < Q$ . If the production cost is very high, such monitoring will be very costly, rendering  $PQ$  relational contracts ineffective.

We focus on the case where the production cost is sufficiently small that  $PQ$  relational contracts are of potential value. To make this notion precise, it is helpful to specify the maximum expected profit that a buyer could receive under a  $PQ$  relational contract. If the price and quantity were contractible, the contract design problem would be (14)–(15): The constraints ensuring that the firms adhere, (16)–(18), would be dropped. A solution has purchase commitment  $Q_c = \Phi^{-1}([1 - (c+p)/r]^+)$  and per unit payment satisfying  $w_c^{PQ} = c + p + \underline{\Pi}_S/Q_c$ ; “ $c$ ” is mnemonic for contractible. Assume that the maximum profit that the buyer could receive under a  $PQ$  relational contract is larger than her outside option:

$$r \min E(Q_c, \xi) - (c+p)Q_c - \underline{\Pi}_S > \underline{\Pi}_B; \quad (19)$$

this will hold when  $p$ ,  $\underline{\Pi}_B$ , and  $\underline{\Pi}_S$  are sufficiently small.

Any solution to problem (14)–(18) with  $Q < \underline{K}$  can be replicated with  $Q = \underline{K}$  and an appropriately chosen  $w$ ; so without loss of generality we can restrict attention to  $Q \geq \underline{K}$ . Then (14)–(18) simplifies to

$$\max_{w, Q} E[r \min(Q, \xi) - wQ] \quad (20)$$

$$\text{subject to } (w - c - p)Q \geq \underline{\Pi}_S \quad (21)$$

$$\delta(1 - \delta)^{-1}[(w - c - p)Q - \underline{\Pi}_S] \geq \sigma(r - p)E \min(Q, \xi) - (w - p)Q \quad (22)$$

$$\delta(1 - \delta)^{-1}[rE \min(Q, \xi) - wQ - \underline{\Pi}_B] \geq wQ. \quad (23)$$

Because the relational contract must leave the supplier better off than he was under the noncooperative outcome, the promised price is sufficiently high that the supplier will never find it attractive to renege (the right-hand side of (22) is negative). In other words, the supplier’s participation constraint (21) implies the supplier’s adherence constraint (22).

To characterize the optimal  $PQ$  relational contract, we first characterize the optimal promised unit price for any given quantity, and then turn to the optimal promised quantity. If it is optimal to promise a strictly positive quantity, then because as  $w$  decreases, the objective function increases, constraint (21) tightens, and constraint (23) loosens, an optimal  $PQ$  relational contract  $(w^{PQ}, Q^{PQ})$  satisfies

$$w^{PQ} = c + p + \underline{\Pi}_S/Q^{PQ}. \quad (24)$$

Thus, under an optimal  $PQ$  relational contract, the supplier’s expected per period profit is  $\underline{\Pi}_S$ . In identifying a  $PQ$  relational contract that is self-enforcing, the only constraint of (21)–(23) that remains is the buyer’s adherence constraint (23), which simplifies to  $\mathcal{A}(Q) \geq 0$ , where

$$\mathcal{A}(Q) = \delta(1 - \delta)^{-1} \{ rE \min(Q, \xi) - (c+p)Q - \underline{\Pi}_B - \underline{\Pi}_S \} - (c+p)Q - \underline{\Pi}_S.$$

Therefore, the  $PQ$  relational contract design problem can be written as

$$\max_Q \{ rE \min(Q, \xi) - (c+p)Q - \underline{\Pi}_S \} \quad (25)$$

$$\text{subject to } \mathcal{A}(Q) \geq 0. \quad (26)$$

Let

$$\underline{\delta}^{PQ} \equiv \min \{ \delta : \exists Q \text{ such that } rE \min(Q, \xi) - (c+p)Q - \underline{\Pi}_B - \underline{\Pi}_S > 0 \text{ and } \mathcal{A}(Q) \geq 0 \}$$

$$\bar{\delta}^{PQ} \equiv \min \{ \delta : \mathcal{A}(Q_c) \geq 0 \}$$

$$Q(\delta) \equiv \max \{ Q : \mathcal{A}(Q) \geq 0 \}.$$

Note that  $0 < \underline{\delta}^{PQ} < \bar{\delta}^{PQ} < 1$ . Recall that  $\underline{K} > 0$  if and only if  $c < \sigma(r - p)$ .

**PROPOSITION 2.** *If  $\underline{\Pi}_B + \underline{\Pi}_S > \underline{\Pi}_B + \underline{\Pi}_S$  and/or  $c < \sigma(r - p)$  and if  $\delta < \underline{\delta}^{PQ}$ , then no self-enforcing  $PQ$  relational contract exists. Otherwise, an optimal  $PQ$  relational contract  $(w^{PQ}, Q^{PQ})$  has*

$$(w^{PQ}, Q^{PQ}) = \begin{cases} (0, 0) & \text{if } \delta < \underline{\delta}^{PQ} \\ (c + p + \underline{\Pi}_S/Q(\delta), Q(\delta)) & \text{if } \delta \in [\underline{\delta}^{PQ}, \bar{\delta}^{PQ}) \\ (c + p + \underline{\Pi}_S/Q_c, Q_c) & \text{if } \delta \geq \bar{\delta}^{PQ}. \end{cases}$$

If the discount factor is small, then the buyer cannot increase her profit by promising to purchase a specified quantity. However, for larger discount factors, the buyer benefits by promising to purchase a positive quantity. Because  $Q(\delta)$  is increasing in  $\delta$ , both the optimal quantity commitment  $Q^{PQ}$  and the promised payment  $w^{PQ}$  are increasing in  $\delta$ . Thus, under the optimal  $PQ$  relational contract, as the discount factor increases, the buyer promises to buy a larger quantity and pay a larger price. When  $\delta$  is sufficient large and the production cost is zero,  $p=0$ , the promised quantity commitment coincides with the first best capacity investment  $Q^{PQ} = \bar{K}$  and the first best is achieved. In contrast, when  $p > 0$ ,  $Q^{PQ} < \bar{K}$  and the first best is not achieved.

Recall that under the optimal  $P$  relational contract, the promised per unit price  $w^P$  is increasing in  $\delta$ . In contrast, the buyer's promised price per unit in the optimal  $PQ$  relational contract,  $w^{PQ}$ , is decreasing in  $\delta$ . Nonetheless, the contracts are similar in that as the discount factor increases, under both contracts, the supplier's capacity and the buyer's expected per period profit increase.

The next section compares the performance of the simple  $P$  and  $PQ$  relational contracts with one another and with the optimal relational contract derived in Taylor and Plambeck (2003).

### 5. Performance Evaluation of Simple Relational Contracts

When price and quantity are noncontractible, the buyer can increase her profit by making promises about how she will purchase. An important decision facing any buyer is what *kind* of promise to make to the supplier.

Theorem 1 provides clear conditions under which the buyer should make only a unit-price promise and conditions under which the buyer should make a quantity promise in addition to a unit-price promise. We extend our definition of  $\Pi_B^P$  so that it denotes the buyer's expected profit per period under the optimal  $P$  relational contract, if such a contract exists, and denotes the buyer's outside-option profit, otherwise. Let  $\Pi^{PQ}$  denote the analogous quantity under the optimal  $PQ$  relational contract.

**Figure 1** Buyer's Preferred Relational Contract when Production Cost Is Small

Capacity cost $c$	High	Price only	Price and quantity
	Low	Price and quantity	Price and quantity
		Low	High
		Discount factor $\delta$	

**THEOREM 1.** (a) If  $c \leq \underline{c}$ , then

$$\Pi_B^P \leq \Pi_B^{PQ},$$

where the inequality is strict if and only if  $\delta \geq \underline{\delta}^{PQ}$ .

(b) If  $p$  is sufficiently small, then there exists  $\bar{\delta} \in (0, 1)$  such that if  $\delta > \bar{\delta}$ , then

$$\Pi_B^P < \Pi_B^{PQ}.$$

(c) There exists  $\underline{\delta} \in (0, 1)$  such that if  $\delta < \underline{\delta}$ , then

$$\Pi_B^P \geq \Pi_B^{PQ},$$

where the inequality is strict if  $\underline{\Pi}_B = \underline{\Pi}_B^P$ ,  $\underline{\Pi}_S = \underline{\Pi}_S^P$ ,  $c > \sigma(r-p)$  and  $\delta \in (\underline{\delta}^P, \underline{\delta})$ .

If the capacity cost is low, then the buyer should make a unit-price and quantity promise, that is, use a  $PQ$  relational contract (part (a)). Similarly, if the discount factor is high and the unit production cost is low, then the buyer should use a  $PQ$  relational contract (part (b)). In contrast, if the capacity cost is high and the discount factor is low, then the buyer should *not* promise to purchase a specified quantity and instead only commit to paying a specified price per unit, that is, use a  $P$  relational contract (part (c)).<sup>3</sup> Similarly, if the production cost is sufficiently high that (19) is violated, then the buyer should use a  $P$  relational contract. Figure 1 depicts the buyer's weakly preferred relational contract when the production cost is small.

To see the intuition, first consider the case where the capacity cost is low. As discussed above, when the capacity cost is low, the  $P$  relational contract fails to

<sup>3</sup> Although this result is shown only for the case where each firm's outside-option profit is equal to its noncooperative outcome profit, the result is robust to this assumption, provided that the deviations from the noncooperative profits are not too large.

increase the buyer's profit because paying a price that is higher than the noncooperative price reduces the buyer's profit. In contrast, a low capacity cost does not hamper the effectiveness of a  $PQ$  relational contract in the same way. By properly tying the promised payment to a specified quantity, the buyer increases the supplier's capacity investment and the buyer's expected profit.

If the discount factor is high and the production cost is low, then the buyer should use a  $PQ$  relational contract. Observe that as  $\delta \rightarrow 1$ , the  $PQ$  contract design problem essentially reduces to the contract design problem where the price and quantity are contractible (20)–(21). To see the intuition for why, in this context, low production costs favor  $PQ$  relational contracts, consider the special case where the production cost is zero. If the price and quantity are contractible, the buyer can, by formally committing to purchase  $\bar{K}$  units for  $c\bar{K} + \underline{\Pi}_S$ , capture the profit of the integrated system, less the supplier's outside option profit,  $\underline{\Pi}_S$ . In contrast, if the buyer formally commits only to a unit price, the buyer's profit is strictly smaller (Cachon and Lariviere 2001). Thus, when  $\delta$  is large, the buyer is better off promising to purchase a specified quantity.

The intuition continues to apply when the cost of production is positive but small. In this case, although the  $PQ$  relational contract requires wasteful production at times, this cost is relatively small. In contrast, if the production cost is sufficiently high that (19) is violated, then the costs of monitoring the supplier's capacity are too high for a  $PQ$  relational contract to be effective; consequently, the buyer should instead use a  $P$  relational contract.

If the cost of capacity is high and the discount factor is low, then the buyer should use a  $P$  relational contract. When the capacity cost is high, a  $P$  relational contract is effective for the buyer because it is optimal for the buyer to promise to pay a unit price that is higher than the noncooperative price. Although  $PQ$  relational contracts are effective for the buyer when the discount factor is high, they are substantially less effective when the discount factor is low. By committing the buyer to purchase a specified quantity, a  $PQ$  contract entails an additional promise that is not in the  $P$  contract. Because a buyer promises more under

a  $PQ$  contract, she has more to gain by renegeing. Consequently, a  $P$  relational contract, with its lower renegeing temptation, is superior when the discount factor is low.

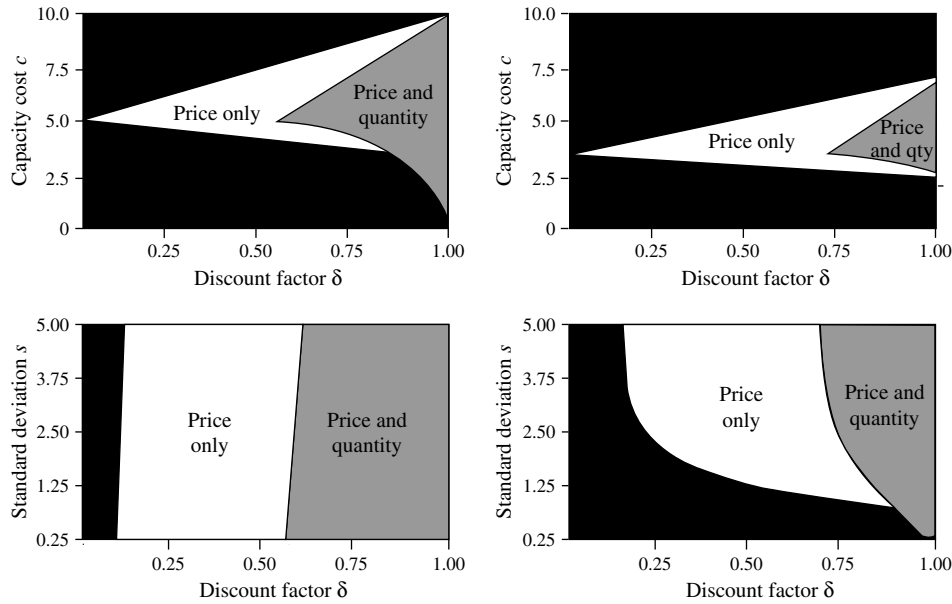
The insights from Theorem 1 are best illustrated and extended by a numerical example. Figure 2 compares the buyer's profit under the optimal  $P$  and  $PQ$  relational contracts. The panels depict the regions in which the buyer strictly prefers either the  $P$  or  $PQ$  relational contract. In the black region, the buyer cannot increase her profit from the noncooperative outcome through either relational contract. In all panels parameters are:  $\underline{\Pi}_B = \underline{\Pi}_B$ ,  $\underline{\Pi}_S = \underline{\Pi}_S$ ,  $r = 10$ ,  $\sigma = 0.5$ , and  $\xi$  is a Normal random variable with mean 5 and standard deviation  $s$ , truncated such that its probability mass is distributed over  $\xi \geq 0$ . The top panels of Figure 2 show the buyer's preferred relational contract as a function of the capacity cost and discount factor; in both panels  $s = 3$ ; the production cost is  $p = 0$  in the left panel and  $p = 3$  in the right panel.

Consistent with Propositions 1 and 2, for any fixed capacity cost, the relational contracts fail to improve over the noncooperative outcome if the discount factor is sufficiently small. This is the case either when capacity is expensive, in which case it is difficult to provide credible incentives for the supplier to invest in capacity, or when capacity is cheap, in which case the gain from cooperation is small.

Consistent with Theorem 1, the top left panel illustrates that if the capacity cost is low or the discount factor is high, then the buyer (weakly) prefers the  $PQ$  relational contract; otherwise, the buyer (weakly) prefers the  $P$  relational contract. Comparing the top right panel with the top left panel shows that when the production cost increases,  $P$  and  $PQ$  relational contracts are effective for a smaller range of parameters, and  $PQ$  relational contracts become comparatively less attractive.

The bottom panels in Figure 2 show the buyer's preferred relational contract as a function of the standard deviation of demand  $s$  and the discount factor  $\delta$ ; in both panels  $p = 0$ ; the capacity cost is  $c = 5.5$  in the left panel and  $c = 4.5$  in the right panel. Again, consistent with Theorem 1, the panels illustrate that if the discount factor is high, the buyer prefers the  $PQ$  relational contract.

Figure 2 Buyer's Preferred Relational Contract



Notes. The regions marked “price only” and “price and quantity” indicate that the buyer’s profit under the optimal relational contract of the specified form strictly exceeds the profit under the optimal relational contract of the other form. In the black region the buyer cannot increase her profit from the noncooperative outcome through either relational contract.

The impact of demand uncertainty on the relative attractiveness of the  $P$  and  $PQ$  relational contracts depends crucially on the firms’ profits outside the relationship  $(\underline{\Pi}_B, \underline{\Pi}_S)$ , and hence on the cost of capacity. When the capacity cost is high ( $c > \sigma(r - p)$ ), the firms’ profits outside of the relationship are zero,  $\underline{\Pi}_B = \underline{\Pi}_S = 0$ , and are unaffected by the standard deviation of demand. Therefore, in the high-capacity-cost case (bottom left panel), the impact of demand uncertainty is relatively easy to understand. At moderate discount factors, increasing the standard deviation makes the  $P$  relational contract more attractive than the  $PQ$  relational contract. As demand uncertainty increases, the buyer’s expected discounted profit under a relational contract decreases, which limits how much the buyer can credibly promise. Thus, it becomes attractive to drop the additional purchase-quantity promise and instead only promise a unit price.

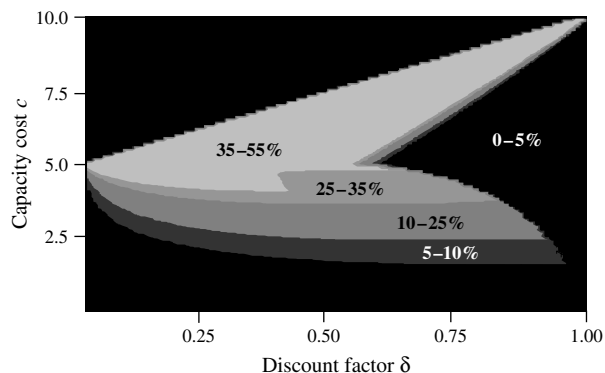
This result may be reversed in the low-capacity-cost case ( $c < \sigma(r - p)$ ), as shown in the bottom right panel. Here, at moderate discount factors, increasing the standard deviation makes the  $PQ$  relational contract more attractive than the  $P$  relational contract. The key to the reversal in results is that when the capacity cost is low, the firms’ profits outside of the

relationship  $(\underline{\Pi}_B, \underline{\Pi}_S)$  are positive and are decreasing in the demand uncertainty. As demand uncertainty increases, it becomes less attractive for the firms to renege, because they will do poorly outside of the cooperative relationship. Thus, making the additional purchase-quantity promise is credible and attractive.

Having compared the optimal  $P$  and  $PQ$  relational contracts in terms of performance, we briefly comment on how they compare in terms of unit prices and capacities. It is intuitive that the optimal unit price would be higher in the  $P$  relational contract  $w^P \geq w^{PQ}$ , because the supplier needs to be compensated for bearing the cost of the imbalance between capacity and demand. It is straightforward to establish this analytically for the case when the supplier lacks a viable outside option  $\underline{\Pi}_S = \underline{\Pi}_S$ , and in an extensive numerical study with  $\underline{\Pi}_S > \underline{\Pi}_S$  we never observed a counterexample. In contrast, the optimal capacity may be higher under the  $P$  or  $PQ$  relational contract (sufficient conditions for each can be established as a corollary to Theorem 1).

Because, as demonstrated in Taylor and Plambeck (2003), the optimal relational contract may be rather complex, we have focused on simple relational contracts that are easy to describe and implement. This

**Figure 3** Loss in Profit from Simple Relational Contracts



*Notes.* The regions indicate the buyer's relative loss in profit from using the best simple ( $P$  or  $PQ$ ) relational contract rather than the optimal relational contract:  $[\Pi^* - \max(\Pi_B^P, \Pi_B^{PQ})] / \Pi^*$ , where  $\Pi^*$  denotes the buyer's expected profit per period under the optimal relational contract.

simplicity comes at a cost, and Figure 3 depicts the size of this loss. The panel indicates the buyer's relative loss in profit from using the best simple ( $P$  or  $PQ$ ) relational contract rather than the optimal relational contract. Parameters are the same as Figure 2's top left panel. In the black region, which constitutes more than half of the parameter region, the loss in profit is less than 5%, and in nearly half of this region, the loss is zero. When the discount factor is high, the  $PQ$  relational contract achieves the first best (or nearly does so), so there is little or no benefit to using a more complex relational contract. When the discount factor is low, no relational contract is effective. However, in the intermediate region, where the discount factor and capacity cost are moderate, the buyer can substantially benefit by employing a more complex relational contract.

Under a  $PQ$  relational contract, the buyer threatens to cease cooperation if the supplier fails to build sufficient capacity to fill the promised order. Purchasing the full promised order is the simplest way to ensure that the supplier has built a target level of capacity. Taylor and Plambeck (2003) show that the optimal relational contract also includes a threat to cease cooperation if the supplier fails to build a target capacity level. However, the optimal scheme to monitor the supplier's capacity may be much more complex, requiring that the buyer order the full target capacity for certain realizations of demand, only the demand for other realizations, and more than the demand but less than the target capacity for other realizations.

Because of this complexity, Taylor and Plambeck (2003) also propose a relational contract that involves a simpler capacity monitoring scheme: The buyer orders the minimum of demand and the target capacity level and ceases cooperation if the supplier fails to fill the order. Taylor and Plambeck (2003) show that this intermediate-complexity relational contract performs very well over a wide range of parameters; for the parameters in Figure 3 the deviation from optimal profit is less than 1%. Thus, buyers that find that simple  $P$  or  $PQ$  relational contracts perform poorly in their environment might consider this relational contract of intermediate complexity. Nonetheless, buyers should be cautious when the demand distribution is bimodal; then the intermediate-complexity relational contract can perform very poorly (achieving zero profit when the optimal relational contract achieves the first best and allocates all profit to the buyer). Furthermore, the intermediate-complexity relational contract and the optimal relational contract in Taylor and Plambeck (2003) require a noncontingent transfer payment at the beginning of each period, before the supplier initiates his capacity investment; managers might be hesitant to make such a payment.

## 6. Extensions

The first purpose of this section is to show that our main result, Theorem 1, extends when several key assumptions are relaxed. The second purpose is to show how  $P$  and  $PQ$  relational contracts should be adapted when the retail price and production cost are stochastic. Proofs of the assertions in this section are in Taylor and Plambeck (2007).

### 6.1. Stochastic Retail Price and Production Cost

Our focus is on products that are ill defined at the time at which the supplier initiates his capacity investment (innovative products), so that the supplier must initiate capacity investment without the benefit of a court-enforceable contract in hand. Because of the evolving product definition, it is natural that the retail price and production cost would be uncertain at this point. Accordingly, suppose that the production cost and retail price ( $p, r$ ) are random variables with joint distribution  $F(p, r)$  and support  $\Omega$ , where all  $(p, r) \in \Omega$  satisfy  $0 \leq p \leq r$ , and that  $(p, r)$  and  $\xi$  are independent. Let  $\bar{r} = \max\{r: (p, r) \in \Omega\}$  and  $\bar{p} = \max\{p: (p, r) \in \Omega\}$ .

$\Omega$ ). The distribution of  $F(p, r)$  is common knowledge, and both firms observe the realization of  $(p, r)$  in Step 2 of the sequence of events (see §2), prior to signing a court-enforceable contract.

It is natural to extend the  $P$  and  $PQ$  relational contracts so that the unit price  $w$  depends on the realization of  $(p, r)$ . After observing the realization of  $(p, r)$ , each firm decides whether to adhere to the relational contract. The formulations for the optimal  $P$  and  $PQ$  relational contracts extend, although the buyer and supplier's adherence constraints must hold for all  $(p, r) \in \Omega$ .

How should the  $P$  relational contract be adapted to reflect the uncertainty in the production cost and retail price? The main insight is that the buyer should promise to pay the supplier a constant premium  $\Delta \geq 0$  over the noncooperative price:

$$w(p, r; \Delta) \equiv \min(\sigma r + (1 - \sigma)p + \Delta, r) \quad \text{for } (p, r) \in \Omega. \quad (27)$$

To see the intuition, as in the base case, the key constraint in the optimal contract design problem is the buyer's adherence constraint (6). The supplier's capacity investment  $K^P$  and the buyer's expected profit under a self-enforcing  $P$  relational contract depend on the promised price function only through its expected value  $Ew(p, r)$ . Analogous to (9), the buyer's maximal gain from renegeing is

$$[w(p, r) - \sigma r - (1 - \sigma)p]K^P(Ew(p, r)), \quad (28)$$

where the definition of  $K^P$  is modified so  $Ep$  replaces  $p$ . In choosing  $w(p, r)$  to satisfy any desired  $Ew(p, r)$ , the contract designer seeks to minimize the maximal renegeing temptation (i.e., the maximal deviation from the noncooperative price) subject to the restriction that  $w(p, r) \leq r$ . This is achieved by the additive adjustment in (27). Consider a change in the joint distribution of  $(r, p)$  that shifts probability mass to the event  $\sigma r + (1 - \sigma)p + \Delta > r$  while maintaining constant  $Ep$  and  $Er$ . This change will reduce  $E\min(\sigma r + (1 - \sigma)p + \Delta, r)$ . Thus, high variance and low covariance in  $(r, p)$  tend to reduce the expected price per unit that the buyer can credibly promise, which reduces the supplier's capacity investment and the buyer's profit under the optimal  $P$  contract.

Proposition 1s characterizes an optimal  $P$  relational contract. In the proposition, the discount factor thresholds  $\underline{\delta}^P$  and  $\bar{\delta}^P$  are defined by natural extension of the definitions in §3,  $\pi(w)$  denotes the buyer's expected profit when the expected price is  $w$ , and  $\Delta_c$  is defined analogously to  $w_c^P$  as the smallest optimal price premium for the buyer when the wholesale price is contractible:

$$\Delta_c = \min \left\{ \Delta : \Delta \in \arg \max_{\Delta \geq 0} \pi(Ew(p, r; \Delta)) \right\}, \quad (29)$$

where  $\underline{\Delta}$  is the unique solution to

$$[Ew(p, r; \Delta) - Ep]E\min(\theta K^P(Ew(p, r; \Delta)), \xi) - cK^P(Ew(p, r; \Delta)) = \underline{\Pi}_S \quad (30)$$

when  $\underline{\Pi}_S > 0$ , and  $\underline{\Delta} = 0$  otherwise.

**PROPOSITION 1s.** *Suppose the production cost and retail price are stochastic. If  $\underline{\Pi}_B + \underline{\Pi}_S > \underline{\Pi}_B + \underline{\Pi}_S$  and either  $\pi(Ew(p, r; \Delta_c)) \leq \underline{\Pi}_B$  or  $\delta < \underline{\delta}^P$ , then no self-enforcing  $P$  relational contract exists. Otherwise, an optimal  $P$  relational contract has*

$$w^P(p, r) = \min(\sigma r + (1 - \sigma)p + \Delta^P, r) \quad \text{for } (p, r) \in \Omega,$$

where  $\Delta^P$  is increasing in  $\delta$  and given by the following: If  $\pi(Ew(p, r; \Delta_c)) \leq \underline{\Pi}_B$  or  $\delta < \underline{\delta}^P$ , then  $\Delta^P = 0$ ; otherwise, if  $\delta \geq \bar{\delta}^P$ , then  $\Delta^P = \Delta_c$ ; if  $\delta \in [\underline{\delta}^P, \bar{\delta}^P)$ , then  $\Delta^P \in (0, \Delta_c)$ . If (13) holds, then there exists  $\underline{c} > 0$  such that if  $c \leq \underline{c}$ , then  $\pi(Ew(p, r; \Delta_c)) \leq \underline{\Pi}_B$  holds.

Under a  $PQ$  relational contract, the buyer promises to purchase  $Q$  units and to pay the supplier  $w(p, r)Q$  when the realized production cost and retail price are  $(p, r)$ . This relational contract is simple in that the buyer promises to buy a deterministic quantity  $Q$ . But stochasticity in the production cost and retail price leads, for some parameters, to complexity in the optimal unit price as a function of the realized production cost and retail price  $w(p, r)$ . Proposition 2s characterizes an optimal  $PQ$  relational contract. In the proposition, the discount factor thresholds  $\underline{\delta}^{PQ}$  and  $\bar{\delta}^{PQ}$  and the quantities  $Q(\delta)$  and  $Q_c$  are defined by natural extension of the definitions in §3. Let

$$\tilde{w}(p, r; Q) = p + \sigma(r - p)E\min(Q, \xi)/Q,$$

and let  $\Gamma$  denote the unique solution to

$$\begin{aligned} & \text{Emin}\left(\tilde{w}(p, r; Q^{PQ}) + \Gamma, \max_{(p, r) \in \Omega} \tilde{w}(p, r; Q^{PQ})\right) \\ & = c + Ep + \underline{\Pi}_S / Q^{PQ}; \end{aligned}$$

note  $\Gamma > 0$ .

**PROPOSITION 2s.** *Suppose the production cost and retail price are stochastic. If  $\underline{\Pi}_B + \underline{\Pi}_S > \underline{\Pi}_B + \underline{\Pi}_S$  and/or  $c < \sigma(r - p)$ , and if  $\delta < \underline{\delta}^{PQ}$ , then no self-enforcing PQ relational contract exists. Otherwise, an optimal PQ relational contract  $(w^{PQ}(p, r), Q^{PQ})$  has*

$$Q^{PQ} = \begin{cases} 0 & \text{if } \delta < \underline{\delta}^{PQ} \\ Q(\delta) & \text{if } \delta \in [\underline{\delta}^{PQ}, \bar{\delta}^{PQ}) \\ Q_c & \text{if } \delta \geq \bar{\delta}^{PQ}. \end{cases}$$

If  $\delta < \underline{\delta}^{PQ}$ , then  $w^{PQ}(p, r) = 0$  for  $(p, r) \in \Omega$ ; otherwise for  $(p, r) \in \Omega$ ,

$$\begin{aligned} & w^{PQ}(p, r) \\ & = \begin{cases} c + Ep + \underline{\Pi}_S / Q^{PQ} & \text{if } \max_{(p, r) \in \Omega} \tilde{w}(p, r; Q^{PQ}) \leq c + Ep + \underline{\Pi}_S / Q^{PQ} \\ \min\left(\tilde{w}(p, r; Q^{PQ}) + \Gamma, \max_{(p, r) \in \Omega} \tilde{w}(p, r; Q^{PQ})\right) & \\ \text{otherwise.} & \end{cases} \quad (31) \end{aligned}$$

In the optimal PQ relational contract price in (31), the expected price satisfies

$$Ew^{PQ}(p, r) = c + Ep + \underline{\Pi}_S / Q^{PQ},$$

so that just as in the case with deterministic production cost and retail price, the supplier's profit is his outside option profit  $\underline{\Pi}_S$ . The optimal PQ relational contract price in (31) is particularly simple in that it does not depend on the realized production cost or retail price when

$$\max_{(p, r) \in \Omega} \tilde{w}(p, r; Q^{PQ}) \leq c + Ep + \underline{\Pi}_S / Q^{PQ}. \quad (32)$$

Then the optimal PQ relational contract only specifies a single price and quantity. Constraint (32) holds when there is limited dispersion in the distribution of the production cost and retail price (as reflected by the maximum production cost  $\bar{p}$  not being too much

larger than  $Ep$  and  $\bar{r}$  not being too much larger than  $Er$ ). When there is substantial dispersion in the distribution of the production cost and retail price, the inequality in (32) is reversed; this implies that the price depends on the realized production cost and retail price and that cooperation is more difficult to sustain (the buyer's maximal renegeing temptation for any fixed  $Q$  is larger).

Our main result, Theorem 1, which compares the relative performance of the  $P$  and  $PQ$  relational contracts, extends. Part (a) extends without modification, part (b) extends when  $p$  is replaced by  $Ep$ , and part (c) extends when  $c > \sigma(r - p)$  is replaced by

$$c > \sigma \bar{r} + (1 - \sigma) \bar{p} - Ep \quad (33)$$

$$E[r - p](c + Ep) / (c - \sigma E[r - p]) > \max_{(r, p) \in \Omega} \{r - p\}. \quad (34)$$

Inequality (33) requires that, as before, the capacity cost be sufficiently high. Inequality (34) requires that maximal unit-contribution  $\max_{(r, p) \in \Omega} \{r - p\}$  not be too large relative to the expected unit-contribution  $E[r - p]$ ; the inequality holds, for example, when the retail price is a fixed markup over the production cost.

## 6.2. Stochastic Yield on Capacity

In addition to uncertainty in the production cost and retail price, there may be uncertainty in the effectiveness of capacity investment. Although the supplier controls the dollar amount he invests in capacity, the number of units that the supplier can produce will depend on the final product definition and the effectiveness of the particular production technologies in which he invested. Both of these are uncertain at the time when the capacity investment is initiated. Accordingly, suppose the effective capacity is a stochastic, multiplicative function of the capacity investment: When the capacity investment is  $K$ , the realized capacity is  $\theta K$ , where the yield  $\theta$  is a continuous random variable with density  $g(\theta)$  and support  $[\theta, \bar{\theta}]$ . Further,  $\theta$ ,  $\xi$ , and  $(r, p)$  are independent; the assumption that  $\theta$  and  $p$  are independent may be reasonable, for example, when the production cost is primarily due to raw materials the price of which is uncertain. Although the distribution of  $\theta$  is common knowledge, only the supplier observes the realization of  $\theta$ ; this reflects the notion that only the

supplier directly observes how his particular technological investments translate into capacity for the final product. The supplier privately observes the realized capacity  $\theta K$  in Step 2 of the sequence of events, prior to signing a court-enforceable contract and hence before he decides whether to adhere to the promised terms.

As before, under a  $P$  relational contract the buyer promises to pay  $w(p, r)$  for each unit she purchases when the realized production cost and retail price are  $(p, r)$ . Proposition 1s, which characterizes the optimal  $P$  relational contract, extends when restriction (13) is generalized to account for stochastic yield.

We generalize the  $PQ$  relational contract to account for stochastic yield as follows: The buyer promises to pay  $w(p, r)Q$  per unit if the supplier delivers  $Q$  units. If the buyer wants to buy more than  $Q$  units, she pays the noncooperative price  $\sigma r + (1 - \sigma)p$  for each additional unit. If the supplier is unable to deliver  $Q$  units, the buyer orders whatever quantity she pleases and pays the noncooperative price per unit. When the yield is deterministic, the optimal contract of this form reduces to the optimal relational contract in Proposition 1s. When the yield is stochastic, the optimal  $PQ$  relational contract is more complex. Nonetheless, it is possible to compare the relative performance of the optimal  $P$  and  $PQ$  relational contracts.

Theorem 1 extends in its entirety when the retail price is deterministic, the production cost is zero, and the yield distribution satisfies the following:  $g(\theta)$  is differentiable and not decreasing too rapidly, that is,  $g'(\theta) \geq -2g(\theta)/\theta$ ,  $g(\bar{\theta})$  is bounded away from zero, and  $\theta g(\theta) = 0$ . Part (a) continues to hold when the retail price and production cost are stochastic and the restrictions on the yield distribution are eliminated. Part (b) continues to hold when the retail price is stochastic. Part (c) continues to hold when the production cost is positive and the restrictions on the yield distribution are eliminated.

Before proceeding, we comment on which assumptions are essential to our analysis. The assumption that both firms observe the retail price and production cost is needed for analytic tractability; without this assumption, the outcome of noncooperative bargaining would not necessarily be the Nash bargaining solution. In particular, if the supplier had private

information about  $p$  or the buyer had private information about  $r$ , then the theorem on page 106 of Rubinstein (1982) would no longer apply. Although we allow for dependence between the uncertain production cost  $p$  and the retail price  $r$ , we have assumed that the random variables  $(p, r)$ ,  $\xi$ , and  $\theta$  are independent. Without this assumption, the newsvendor structure, which is essential for analytic tractability, is lost.

### 6.3. Support of Demand Distribution

We have assumed that the support of the demand distribution is  $[l, m]$  where  $l = 0$ . The analysis and results for the optimal  $P$  and  $PQ$  relational contracts extend to the case where there is a guaranteed minimum level of demand,  $l > 0$ , and the production cost and retail price are uncertain. The main insight is that a guaranteed minimum level of demand makes quantity promises relatively more attractive than price-only promises. This occurs because a guaranteed minimum level of demand weakens the buyer's maximal renegeing temptation under a  $PQ$  relational contract (the maximal renegeing temptation for a buyer facing production cost and retail price  $(p, r)$  is  $w(p, r)(Q - l)$  instead of  $w(p, r)Q$ ), so quantity promises are easier to sustain.

In contrast, the analogous temptation under a  $P$  relational contract is unaffected by a guaranteed minimum demand level. When this level is small, the insights regarding the buyer's preference for a  $PQ$  or  $P$  relational contract carry over from the  $l = 0$  case. When  $l \gg 0$  and  $Ep$  is small, it may be that the buyer always prefers to make a quantity promise.

### 6.4. Private Capacity Cost Information

Our main result, Theorem 1, extends to the case where the supplier has private information about his cost of capacity. In particular, suppose that the per unit cost of capacity is a discrete random variable:  $c = c_i$  with probability  $\lambda_i > 0$ , where  $c_1 < c_2 < \dots < c_N$ . Prior to Step 1 in §2, the supplier observes his cost of capacity, but the buyer only knows the distribution of the capacity cost. When there is asymmetric information about the capacity cost, the firms can make the informal terms contingent on the supplier's reported capacity cost. Under  $P$  relational contracts, there is no gain in doing so, because the supplier would simply report the cost corresponding to the highest expected promised

price. With  $PQ$  relational contracts, the buyer can and should reward a supplier that reports a low capacity cost with a larger quantity commitment, albeit at lower unit price. Specifically, the buyer should offer a menu of price-quantity promises  $\{(w_i, Q_i)\}_{i=1, \dots, N}$ , where the supplier that reports cost  $\tilde{t}$  obtains terms  $(w_{\tilde{t}}, Q_{\tilde{t}})$ , and  $Q_i > Q_j$  implies that  $w_i < w_j$  for  $\tilde{t} \neq j$ . Theorem 1 holds with  $c_N$  replacing  $c$  in part (a) and  $c_1$  replacing  $c$  in part (c).

### 6.5. Supplier Reservation Profit

It is natural that the supplier might insist on capturing a portion of the gain from cooperation. All our results, including those in the extensions above, extend when the supplier is able to demand a reservation expected profit per period under the relational contract of  $\bar{\Pi}_S \geq \underline{\Pi}_S$ ; that is, the supplier demands that he capture at least  $\bar{\Pi}_S - \underline{\Pi}_S$  of the gain from cooperation. In the base case setting considered prior to this section, the only change is that  $\bar{\Pi}_S$  replaces  $\underline{\Pi}_S$  in Proposition 1 and 2 and Theorem 1, and in the definitions of  $\underline{w}$ ,  $A(Q)$ , and  $\delta^{PQ}$ .

$\bar{\Pi}_S$  reflects the supplier's bargaining strength prior to the capacity investment, and  $\sigma$  reflects the supplier's bargaining strength after the capacity investment; our formulation is flexible enough to capture the reality that the bargaining situation may look quite different at these two points in time. One might expect that increasing the supplier's bargaining strength would increase the promised unit price and the supplier's expected profit and decrease the buyer's expected profit under the optimal relational contracts. This is true for the precapacity bargaining strength  $\bar{\Pi}_S$  and both types of relational contracts, as well as for the postcapacity bargaining strength  $\sigma$  and the  $PQ$  relational contract.

However, contrast emerges regarding the impact of  $\sigma$  under the  $P$  relational contract. There, as the supplier's postcapacity bargaining strength  $\sigma$  increases, the promised unit price, buyer profit, and supplier profit may decrease. This occurs when the buyer lacks a viable outside option  $\underline{\Pi}_B = \underline{\Pi}_B$ , so that increasing the supplier's bargaining strength  $\sigma$  increases the buyer's expected profit outside the cooperative relationship (by alleviating the hold-up problem), which, in turn, makes promises of a high unit price (which would benefit both firms) more difficult to sustain.

## 7. Discussion

This paper provides guidance to buyers as to how they should make promises to purchase. Simply promising to pay a per unit price is effective only when the capacity cost is sufficiently high. As the discount factor increases, it is optimal for the buyer to increase its promised price, but the buyer should never promise to pay more than she would if the unit price were contractible. If the buyer's outside alternative to working with her partner supplier improves, she should promise a smaller unit price so as to ensure that her promised price remains credible. In contrast, promising to buy a specified quantity can be effective even when the capacity cost is low. As the discount factor increases, the buyer should promise a lower per unit price and a larger purchase quantity, such that the total payment is larger.

It is useful to compare these results for  $P$  and  $PQ$  promises with what is known about court-enforceable  $P$  and  $PQ$  contracts. The comparison is the most crisp when the production cost is small. If it is possible to sign a court-enforceable contract before the supplier initiates his capacity investment, the buyer is strictly better off signing a  $PQ$  contract rather than a  $P$  contract, because the  $PQ$  contract provides an additional instrument in specifying the contract terms (quantity in addition to price). The preference for  $PQ$  over  $P$  carries over when the buyer must rely on making promises rather than signing contracts prior to the supplier's capacity investment, provided that the discount factor is high. Strikingly, the result is reversed when the discount factor is small (and the capacity cost is sufficiently high that  $P$  promises are effective).

The basic idea is that a quantity commitment is an additional promise, and hence presents a larger temptation for the buyer to renege on the promised terms. The buyer's  $PQ$  promise will only be viewed as credible if the quantity is relatively small, and thus can only induce a small capacity investment by the supplier. In contrast, a properly chosen  $P$  promise provides credible incentives for the supplier to build a larger capacity, with the end result of strictly larger expected buyer profit.

Admittedly, even with the extensions of §6, our model of capacity investment is a simple one: In each period in which the firms commit to working with one another, as the product development effort

evolves, first the supplier's capacity cost is revealed and then later, as the product nears production, the production cost and retail price are revealed. This approach allows us to exploit the structurally appealing aspects of the newsvendor model, a model that is the basis for much of the supply chain contracting literature. However, in reality, at any time, the effective capacity may depend on a complex history of activities and investments by both the buyer and the supplier, and the quality of output may be stochastic and influenced by the efforts of both the buyer and the supplier. For example, Dyer and Chu (2000) document how automakers help their suppliers improve quality and productivity and reduce cost and inventory; these investments play a vital role in building trust and cooperation over time. The optimal relational contract for a general dynamic system, in which a buyer and supplier each make progressive investments over time, is characterized in Plambeck and Taylor (2007).

### Acknowledgments

The authors thank Editor Gerard Cachon, the associate editor, referees, Bob Gibbons, Tunay Tunca, Stefanos Zenios, as well as seminar participants at Columbia University, Cornell University, Dartmouth College, Duke University, Massachusetts Institute of Technology, Purdue University, Stanford University, University of British Columbia, University of North Carolina at Chapel Hill, University of Pennsylvania, University of Southern California, University of Texas at Austin, University of Texas at Dallas, and Washington University for helpful comments. The authors thank Tom Issaevitch for assistance with the numerical analysis. This research was supported by National Science Foundation Grant 0239840. An earlier version of this paper was titled "Simple Relational Contracts for Capacity Investment: Price-Only vs. Quantity Commitment."

### References

- Abreu, D., D. G. Pearce. 1991. A perspective on renegotiation in repeated games. R. Selten, ed. *Game Equilibrium Models II: Methods, Moral, and Markets*. Springer Verlag, Berlin, 44–55.
- Atkins, D., H. Krishnan, X. Zhao. 2006. Renegotiation-proof relational contracts in supply chains. Working paper, Sauder School of Business, University of British Columbia, Vancouver, British Columbia.
- Baker, G., R. Gibbons, K. J. Murphy. 2001. Bringing the market inside the firm. *Amer. Econom. Rev.* **91**(2) 212–218.
- Baker, G., R. Gibbons, K. J. Murphy. 2002. Relational contracts and the theory of the firm. *Quart. J. Econom.* **117**(1) 39–83.
- Cachon, G. P. 2003. Supply chain coordination with contracts. S. Graves, T. De Kok, eds. *The Handbook of Operations Research and Management Science: Supply Chain Management*. North Holland, Amsterdam, 229–339.
- Cachon, G. P., M. A. Lariviere. 2001. Contracting to assure supply: How to share demand forecasts in a supply chain. *Management Sci.* **47**(5) 629–646.
- Cachon, G. P., M. A. Lariviere. 2005. Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Sci.* **51**(1) 30–44.
- Chen, F. 2003. Information sharing and supply chain coordination. S. Graves, T. De Kok, eds. *The Handbook of Operations Research and Management Science: Supply Chain Management*. North Holland, Amsterdam, 341–421.
- Cohen, M. A., T. H. Ho, J. Z. Ren, C. Terwiesch. 2003. Measuring imputed costs in the semiconductor equipment supply chain. *Management Sci.* **49**(12) 1653–1670.
- Corbett, C. J. 2001. Stochastic inventory systems in a supply chain with asymmetric information: Cycle stocks, safety stocks, and consignment stock. *Oper. Res.* **49**(4) 487–500.
- Debo, L., J. Sun. 2004. Repeatedly selling to the newsvendor in fluctuating markets: The impact of the discount factor on supply chain. Working paper, Carnegie Mellon University, Pittsburgh, PA.
- Dyer, J. H., W. Chu. 2000. The determinants of trust in supplier-automaker relationships in the U.S., Japan and Korea. *J. Internat. Bus. Stud.* **31**(2) 259–285.
- Fehr, E., S. Gächter, G. Kirchsteiger. 1997. Reciprocity as a contract enforcement device: Experimental evidence. *Econometrica* **65**(4) 833–860.
- Fudenberg, D., D. Levine, E. Maskin. 1994. The folk theorem with imperfect public information. *Econometrica* **62** 997–1039.
- Grossman, S. J., O. D. Hart. 1986. The costs and benefits of ownership: A theory of vertical and lateral integration. *J. Political Econom.* **94**(4) 691–719.
- Ha, A. 2001. Supplier-buyer contracting: Asymmetric cost information and cutoff level policy for buyer participation. *Naval Res. Logist.* **48**(1) 41–64.
- Helper, S. 1991. The automobile industry. *Bus. History Rev.* **65**(4) 781–824.
- Johnson, B. 2003. Quantifying and managing supply risk and flexibility at Agilent Technologies. Teaching case, Stanford Graduate School of Business, Stanford, CA.
- Lariviere, M. A. 2006. A note on probability distributions with increasing generalized failure rates. *Oper. Res.* **54**(3) 602–604.
- Lariviere, M. A., E. Porteus. 2001. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing Service Oper. Management* **3**(4) 293–305.
- Levin, J. 2003. Relational incentive contracts. *Amer. Econom. Rev.* **93**(3) 835–847.
- Nash, J. 1950. The bargaining problem. *Econometrica* **28**(2) 155–162.
- Özer, Ö., W. Wei. 2006. Strategic commitment for optimal capacity decision under asymmetric forecast information. *Management Sci.* **52**(8) 1238–1257.
- Pasternack, B. A. 1985. Optimal pricing and return policies for perishable commodities. *Marketing Sci.* **4**(2) 166–176.
- Plambeck, E. P., T. A. Taylor. 2006. Partnership in a dynamic production system with unobservable actions and noncontractible output. *Management Sci.* **52**(10) 1509–1527.

- Ren, J. Z., M. A. Cohen, T. H. Ho, C. Terwiesch. 2005. Sharing forecast information in a long-term supply chain relationship. Working paper, University of Pennsylvania, Philadelphia, PA.
- Rubinstein, A. 1982. Perfect equilibrium in a bargaining model. *Econometrica* 50(2) 97–108.
- Sako, M. 1992. *Prices, Quality, and Trust*. Cambridge University Press, Cambridge, England.
- Schweitzer, M. E., J. C. Hershey, E. T. Bradlow. 2005. Promises and lies: Restoring violated trust. *Organ. Behavior Human Decision Processes*. Forthcoming.
- Taylor, T. A. 2002. Coordination under channel rebates with sales effort effects. *Management Sci.* 48(8) 992–1007.
- Taylor, T. A., E. L. Plambeck. 2003. Supply chain relationships and contracts: The impact of repeated interaction on capacity investment and procurement. Working paper, Columbia University, New York.
- Taylor, T. A., E. L. Plambeck. 2007. Technical appendix to “Simple Relational Contracts to Motivate Capacity Investment: Price-Only vs. Price and Quantity.” <http://www.informs.org/Pubs/Supplements/MSOM/1526-5498-2007-09-01-0094-ecompanion.pdf>.
- Tomlin, B. 2003. Capacity investments in supply chains, sharing-the-gain rather than sharing-the-pain. *Manufacturing Service Oper. Management* 5(4) 317–333.
- Tsay, A. A. 1999. The quantity flexibility contract and supplier-customer incentives. *Management Sci.* 45(10) 1339–1358.
- Tunca, T., S. A. Zenios. 2006. Supply auctions and relational contracts for procurement. *Manufacturing Service Oper. Management* 8(1) 43–67.
- Van Mieghem, J. A. 1999. Coordinating investment, production, and subcontracting. *Management Sci.* 45(7) 954–971.