



## Communication in Settings with No Transfers

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# Communication in settings with no transfers

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and

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*Consider a setting commonly found in intrafirm, regulatory, and political relationships wherein an uninformed decision maker, attempting to elicit information from an informed party affected by his decision, is unable to use transfers. This article examines whether both parties will agree on the introduction of communication-based organizational structures; results in prior research suggest they will. In contrast we demonstrate that when there is enough disagreement between the two parties, introducing communication benefits the decision maker but leads to a loss for the informed party. This result holds both when the decision maker can commit to a decision rule (as in Holmström (1977)) and when he cannot (as in Crawford and Sobel (1982)). We also show that in the commitment case, with enough disagreement, the optimal communication-based decision rule is discontinuous. Our results suggest a possible rationale for observed legal and regulatory limitations on the information decision makers can elicit from other parties, as well as for attempts by various parties to avoid preplay communication.*

## 1. Introduction

■ A central feature of economic and political relationships is that information, potentially useful for decision making, is dispersed among various parties having conflicts of interest over the ultimate decision. An important objective in the design of institutions is to facilitate the communication of private information to decision makers. There are two major potential roles for the communication of private information. The first, a planning role, is to improve an uninformed party's decision choices through elicitation of relevant private information. The second, a control role, particularly applicable in agency relationships, is to influence agents' unobservable action choices.

Research on the roles communication plays in various relationships has focused on settings differing along two prominent dimensions: whether transfers between the uninformed party and the informed party are feasible and whether the uninformed party can credibly

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commit to his decision rule. The main research issues in settings involving both transfers and commitment have been to identify conditions under which incorporating the communication of an agent's private information into contracts results in a strict improvement for a principal and to characterize the nature of an optimal communication-based contract.<sup>1</sup>

This study focuses on communication in settings with no transfers both when commitment to a decision rule is possible (as in Holmström (1977) and (1984)) and when this commitment is not possible (as in Crawford and Sobel (1982)).<sup>2</sup> Our objective is to question the Pareto optimality of introducing communication into these settings. Further, we are interested in characterizing the nature of communication in no-transfer settings as commitment ability varies.

Settings with *no transfers* characterize salient aspects of some intrafirm interactions (as in Holmström (1977), and Kanodia and Gigler (1988)), certain regulatory relationships (such as the ones between the Securities and Exchange Commission and the Financial Accounting Standards Board studied in Melumad and Shibano (1991) and between the Federal Reserve and investors studied in Stein (1989)), and various political relationships (for example, the interactions between the President and the Congress analyzed in Matthews (1989) and Ferejohn and Shipan (1990), between a congressional committee and the Congress modelled by Gilligan and Krehbiel (1987), and between legislators as studied in Austen-Smith (1988)). The ability to commit in these relationships varies depending on legal environment (e.g., feasible contracts and allowable penalties), information structure (e.g., observability and verifiability of decisions), duration of the relationship, reputational forces, and other factors. Thus, an analysis of the polar cases of commitment and no-commitment could provide useful benchmarks for studying these and other relationships.

In no-transfer settings, an uninformed decision maker, often called the receiver, tries to utilize information available only to an informed agent, often referred to as the sender. Since the sender is assumed not to make an action choice, communication serves only a planning role. Intuitively, communication may be valuable to the receiver because the sender may be interested in conveying part of her private information to the receiver in an attempt to affect the latter's decision.

A result common to earlier work analyzing the commitment and no-commitment settings is that the receiver as well as the sender are *ex ante* better off with communication (see Holmström (1977) and Crawford and Sobel (1982)). As Crawford and Sobel (hereafter, CS) accordingly argue, if we believe there is a "tendency for efficient outcomes to prevail in economic situations" and if an equilibrium is selected *ex ante*, then "a strong case can be made for the equilibrium with the finest partition."<sup>3</sup> This suggests that members of an organization will *ex ante* coordinate on the installation of organizational structures involving communication rather than structures involving no communication.

In this article, we show that the Pareto efficiency of adopting communication-based arrangements is not robust to certain extensions of the above analyses. We illustrate our point by studying a special case of the Holmström and CS models. In our commitment model we allow the decision rule to be *discontinuous*; this is in contrast to Holmström, who restricts his analysis to continuous decisions.<sup>4</sup> In our no-commitment model, we consider a more general notion of preference divergence between the receiver and the sender than

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<sup>1</sup> For work addressing these issues see, for example, Baiman and Evans (1983), Baron and Besanko (1987), Caillaud, Guesnerie and Rey (1989), Christensen (1981), Demougin (1989), Dye (1983), Laffont and Tirole (1986), McAfee and McMillan (1987), Melumad and Reichelstein (1989), Penno (1984), and Picard (1987).

<sup>2</sup> A related literature is the one on "cheap talk" (see, e.g., Farrell (1990), Farrell and Gibbons (1989), and Stein (1989)).

<sup>3</sup> Crawford and Sobel (1982).

<sup>4</sup> Holmström fully recognizes that this restriction might entail a loss of performance; however, given the generality of his setting, the restriction is necessary for mathematical tractability.

the one adopted by CS. These two unrelated extensions lead to an identical result: communication of the sender's private information may make the sender *ex ante* worse off than without communication.<sup>5</sup> In other words, there will not necessarily be *ex ante* unanimity regarding the installation of communication-based organizational structures.

More specifically, in the commitment model we find, surprisingly, that when the players' sensitivities to the private information are highly disparate, the optimal communication-based decision rule is a one-jump-discontinuity decision rule. This implies that it is optimal for the receiver to utilize the information communicated very coarsely; the decision rule distinguishes between only two subsets of all possible sender environments. In this case, the sender is often *ex ante* worse off with communication compared with no communication. On the other hand, when the players' sensitivities are similar, the optimal decision rule is continuous and the sender is shown never to be worse off.

In the no-commitment setting, communication is valuable only for a subset of cases for which it is valuable in the commitment setting; the lack of commitment leads to a strict loss to the receiver. Here we get a welfare result stronger than the one in the commitment setting. If the players' sensitivities to the private information are highly disparate, the sender is always *ex ante* worse off in an equilibrium with communication than in an equilibrium with no communication. We examine the viability of the no-communication equilibrium by applying Farrell's (1990) neologism-proof refinement.

Our results provide a possible explanation for observed attempts by parties to avoid communication with others. For instance, some firms try to commit credibly not to respond to financial analysts' speculations about their private information; these firms try to establish a reputation for "not communicating" by formally announcing a long-term nondisclosure policy.<sup>6</sup> This phenomenon is also consistent with political parties or Supreme Court nominees that decline to formulate a clear statement of position on a particular issue (i.e., a choice not to sort out its "type"), in order to avoid alienating voters on either end of the political spectrum. An example of such an issue in the United States is abortion, and in Israel, readiness to compromise on the Palestinian issue.

Furthermore, our article suggests a rationale for legal limitations imposed by legislators and regulators on the information that receivers can require senders to disclose. For instance, employers are generally not allowed to ask job applicants certain questions about physical characteristics, age, race, sexual preferences, religion, and health (though in some situations such questions are legitimate). A second example arises in the context of insurance and immigration, where we observe prohibitions against questions on sexual preference or the results of an AIDS test. A possible interpretation is that regulators, concerned with the welfare of "senders" who are made worse off with communication, might prohibit some forms of disclosure desired by "receivers."

The article is organized as follows. In Section 2 we present the commitment model, characterize incentive-compatible decision rules, and identify cases in which the optimal decision rule is continuous/discontinuous. We then analyze the optimal decision rule in each case, provide an alternative delegation implication of our results, and study the welfare implications of introducing communication. In Section 3, we turn to the no-commitment case. We characterize equilibria of the game, examine the welfare implications of alternative equilibria differing in the fineness of their communication, and apply Farrell's equilibrium refinement. We conclude the article by discussing various extensions of our analysis.

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<sup>5</sup> The extensions we consider make the Holmström and CS models more comparable. The preference divergence we consider for our no-commitment model is allowed in Holmström's commitment setting. The discontinuous decision rules we allow in our commitment model are an integral part of the equilibria of the CS no-commitment model.

<sup>6</sup> For a recent discussion of this phenomenon, see "How Much Should Companies Tell?" *New York Times*, July 17, 1990.

## 2. The commitment model

■ There are two players, a receiver and a sender. The sender has private information about her environment,  $t \in T \subseteq R^1$ . The receiver makes a decision,  $x \in X \subseteq R^1$ . The preferences of the receiver and sender are defined over the decision and the environment.

We consider a game in which the receiver first commits to a decision rule  $x(t)$ . The sender, privately informed about her environment, then submits a report regarding the environment to the receiver, who then adopts the decision prescribed by the decision rule  $x(t)$ . In the analysis below we adopt the following terminology. When a decision rule distinguishes between environments (i.e., there exists some  $t_1$  and  $t_2$ ,  $t_1 \neq t_2$ , for which  $x(t_1) \neq x(t_2)$ ), we say the decision rule is *communication-dependent*; otherwise, it is *communication independent*. Communication is said to be *valuable* if there does not exist any communication-independent decision rule that attains the same performance for the receiver.

We focus our attention on settings in which there are potential gains to communication between a privately informed sender and a receiver who has ultimate power over decision making. To capture this partial conflict of interests we assume utility functions that are nonmonotone (in the decision).<sup>7</sup>

For simplicity, the prior probability distribution over environments is uniform over  $T \equiv [0, 1]$ , and the receiver's and sender's preferences are represented, respectively, by the (nonmonotone) quadratic utility functions

$$U^r(x, t) = -(x - k - at)^2$$

and

$$U^s(x, t) = -(x - t)^2,$$

where  $k$  and  $a$  are scalars.<sup>8</sup> We further assume that, for any  $t$ , there exists an interior decision  $x^* \in X$  that maximizes each player's utility, i.e.,  $\left. \frac{\partial U^i(x, t)}{\partial x} \right|_{x=x^*} = 0$ ,  $i = r, s$ . The first-best decisions for the receiver and sender, then, are

$$x^r(t) = k + at$$

and

$$x^s(t) = t.$$

The parameter  $k$  measures the players' preference divergence that is unconditional on the environment, while the  $a$ -1 measures the conditional preference divergence. Together,  $k$  and  $a$  characterize the conflict of interest between the players. We call the parameter  $a$  the receiver's *sensitivity* to the environment and normalize the sender's sensitivity to one.<sup>9</sup> We say that the receiver is more (less) sensitive to the sender's environment than the sender if  $a > 1$  (if  $a < 1$ ), and that players' preferences have similar (disparate) sensitivities if  $a \in (0, 2]$  (if  $a > 2$ ). Furthermore, we say there is a *preference reversal*

<sup>7</sup> Note that in a setting with no transfers, when either the receiver's or the sender's utility is monotone, communication cannot be valuable. If the receiver's utility were monotone, his optimal decision would be independent of the sender's environment. If the sender's utility were monotone, for all environments, she would respond to a communication-dependent decision rule with the same report.

<sup>8</sup> These utilities are single-peaked (i.e., there is a unique maximizing decision for each environment) and have the single-crossing property (i.e.,  $U_{12}^s > 0$  and  $U_{12}^r > (<) 0$  for  $a > (<) 0$ ) where subscripts denote partial derivatives. Note that a single-peaked utility function can be thought of as a one-dimensional representation of two-dimensional preferences monotone in both variables when they are jointly constrained. Most of our analysis can be extended to a more general version of single-peaked preferences with the single-crossing property.

<sup>9</sup> Alternatively, we could normalize the receiver's sensitivity and allow the sender's sensitivity to vary. Our results are qualitatively unaffected if we adopt this alternative formulation. We prefer our formulation because it simplifies both exposition and calculation.

if the receiver’s first-best decision is higher than the sender’s first-best decision over some environments and lower over the others. Formally, preference reversal occurs when  $k \in (0, 1 - a)$  for  $a < 1$  or when  $k \in (1 - a, 0)$  for  $a > 1$ . Note that if there is a preference reversal, there is an environment in which the players’ first-best decision coincides.

We do not impose an individual rationality constraint; this enables us to focus on the maximum potential value of communication.<sup>10</sup> The interpretation of not having this constraint is that the sender cannot make her utility independent of the receiver’s decision (i.e., the sender has no “quit” option).<sup>11</sup>

Invoking the revelation principle we can, with no loss of generality (performance-wise), represent the optimization problem as

$$\max_{x(t)} - \int_0^1 (x(t) - k - at)^2 dt \tag{1}$$

$$\text{subject to } -(x(t) - t)^2 \geq -(x(\hat{t}) - t)^2, \forall t, \hat{t}. \tag{2}$$

The optimization is carried out over the space of squared-integrable functions with discontinuities of the first kind only.<sup>12</sup>

In the following proposition we show that an incentive-compatible (IC) decision rule has a very simple structure. We define  $x^-(\tau) \equiv \lim_{t \rightarrow \tau^-} (x(t))$  and  $x^+(\tau) \equiv \lim_{t \rightarrow \tau^+} (x(t))$ .

*Proposition 1.* An incentive-compatible decision rule  $x(t)$  must satisfy the following: (i)  $x(t)$  is weakly increasing. (ii) If  $x(t)$  is strictly increasing and continuous on an open interval  $(t_1, t_2)$ , then  $x(t) = x^s(t)$  on  $(t_1, t_2)$ . (iii) If  $x(t)$  is discontinuous at  $\tau$ , the discontinuity must be a jump discontinuity that satisfies

- (a)  $U^s(x^-(\tau), \tau) = U^s(x^+(\tau), \tau)$ ,
- (b)  $x(t) = x^-(\tau), \quad \forall t \in [x^-(\tau), \tau)$   
 $x(t) = x^+(\tau), \quad \forall t \in (\tau, x^+(\tau)]$ ,
- (c)  $x(\tau) \in \{x^-(\tau), x^+(\tau)\}$ .

*Proof.* See the Appendix.<sup>13</sup>

The results of Proposition 1 can be graphically illustrated by a decision rule  $x(t)$  mapping environments to decisions. Figure 1 depicts a general IC decision rule.

Note that the diagonal represents the sender’s first-best decision rule. An IC decision rule is weakly increasing and consists of (1) segments where the decision is independent of the sender’s report (i.e., “flat” segments) and (2) segments where the decision prescribed by the IC decision rule is equal to the senders’ first-best decision. Proposition 1 further implies that an IC decision rule can involve discontinuities only if they are symmetric around the sender’s first-best decision line. Part (iiib) of Proposition 1 says that, both to the left and to the right of any discontinuity point, there must be a flat segment. Note that an IC decision rule may start or end with a flat segment.

<sup>10</sup> Clearly, when an optimal unconstrained communication-dependent decision rule is infeasible under the individual rationality constraint, the value of communication is, in general, reduced. The effect of the individual rationality constraint on our analysis is discussed in the concluding section.

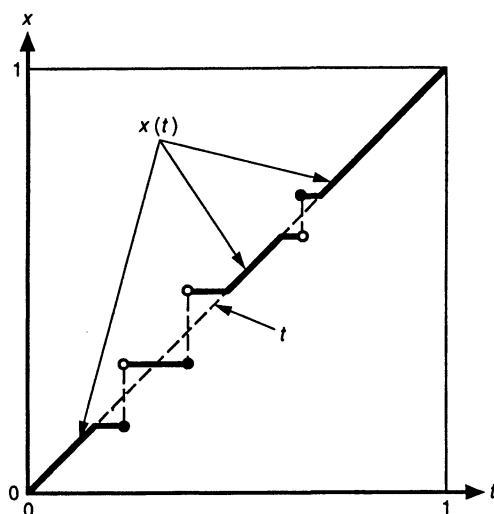
<sup>11</sup> This formulation is descriptive of certain institutional relationships. In a regulatory setting, for example, a regulatee cannot avoid being affected by a regulator’s decision.

<sup>12</sup> Existence of a solution is easy to establish given Proposition 1 below, which narrows the set of admissible functions and enables us to identify the optimal solution.

<sup>13</sup> The proposition extends to general distributions and general single-peaked, single-crossing utility functions. The Appendix provides the general proof.

FIGURE 1

THE GENERAL IC DECISION RULE



We now study conditions necessary for communication to be valuable and characterize the optimal communication-dependent decision rule. We first identify circumstances under which the optimal IC decision rule must be continuous. Proposition 1 is instrumental in establishing the following result.

*Proposition 2.* (i) For  $a < 2$ , any optimal IC decision rule is everywhere continuous. (ii) For  $a = 2$ , any optimal IC decision rule is equivalent (for the receiver) to an everywhere continuous IC decision rule.

*Proof.* See the Appendix.

Before turning to communication-based decision rules, note that the optimal communication-independent decision rule is  $x = k + \frac{a}{2}$ , i.e., the optimal decision corresponds to the average environment.

We start by observing that when the receiver's and sender's sensitivities to the environment are negatively related, communication can never be valuable.

*Observation 1.* When the players' sensitivities are negatively related (i.e.,  $a \leq 0$ ), the optimal IC decision rule is communication-independent.

*Proof.* See the Appendix.

Given this observation, our analysis of settings with continuous decision rules can focus on  $a \in (0, 2]$ . Propositions 1 and 2 imply that for  $a \in (0, 2]$ , the optimal communication-dependent IC decision rule is the solution to the following program:

$$\max_{t_1, t_2} - \int_0^{t_1} (t_1 - k - at)^2 dt - \int_{t_1}^{t_2} (t - k - at)^2 dt - \int_{t_2}^1 (t_2 - k - at)^2 dt \quad (3)$$

subject to

$$(i) \quad t_2 \geq t_1$$

$$(ii) \quad t_1, t_2 \in [0, 1].$$

If the first constraint is not met, the decision rule violates the monotonicity requirement in Proposition 1 (i) for incentive compatibility. The second constraint reflects an admissibility requirement that  $t_1$  and  $t_2$  belong to the set of environments  $T$ .<sup>14</sup>

Based on the simple optimization problem in (3), we can establish the following result regarding value of communication.

*Proposition 3.* When the players' preferences have similar sensitivities (i.e.,  $a \in (0, 2]$ ), communication is valuable if and only if  $k \in \left(-\frac{a}{2}, 1 - \frac{a}{2}\right)$ . When communication is valuable, the optimal IC decision rule takes the following form:

$$x(t) = \begin{cases} t_1 & \text{for } t \in [0, t_1) \\ t & \text{for } t \in [t_1, t_2] \\ t_2 & \text{for } t \in (t_2, 1], \end{cases}$$

where  $t_1 = \max\left\{0, \frac{2k}{2-a}\right\}$  and  $t_2 = \min\left\{\frac{2k+a}{2-a}, 1\right\}$ .

*Proof.* See the Appendix.

The requirement  $k \in \left(-\frac{a}{2}, 1 - \frac{a}{2}\right)$  of Proposition 3 implies that some environment  $t \in (0, 1)$  exists in which the sender's first-best decision coincides with the receiver's optimal no-communication decision. Note that although preference reversal implies this requirement, the reverse does not hold.

The implication of Proposition 3 is that all environments corresponding to the lower and upper regions (possibly degenerate) are pooled together in the sense that the optimal response in these regions is independent of the environment. By contrast, in the intermediate region there is perfect separation in that the decision rule prescribes a different decision for each environment in that region.

Our framework has an alternative delegation interpretation similar to that found in Holmström (1977). In our setting, the performance of communication-based centralized decision making is equivalent to the performance induced by some decentralized mechanism whereby the receiver delegates to the sender the choice of a decision from a limited subset of decisions.<sup>15</sup> Proposition 3 implies that the optimal delegated decision subset, when players' sensitivities are similar, is a connected subset, i.e., the interval

$$\left[ \max\left\{0, \frac{2k}{2-a}\right\}, \min\left\{\frac{2k+a}{2-a}, 1\right\} \right].$$

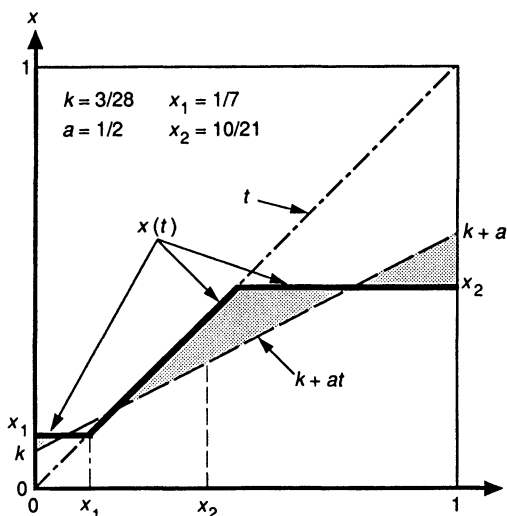
In contrast, when sensitivities are disparate, delegation of a connected subset will be shown to be suboptimal.

Figures 2 and 3 below illustrate the optimal decision rule for two distinct cases with similar sensitivities. While Figure 2 seems to suggest that for sender types whose first-best decision is close to that of the receiver, the optimal decision rule fully distinguishes among different environments, Figure 3 indicates this need not be the case. In particular, the optimal decision rule in Figure 3 pools types whose ideal decision is close to that of the receiver and fully distinguishes among those whose ideal decision is farther from that of the receiver.

<sup>14</sup> If constraint (ii) is not met,  $x(t)$  cannot be incentive compatible (see Proposition 1).

<sup>15</sup> Holmström (1984) presents his results in terms of delegation but points out that, in his setting, "delegation of authority to an agent" is equivalent to "asking the agent for information and promising to act on the information in a particular way" (i.e., communication). Green and Stokey (1981) formalize Holmström's argument. We note, however, that in an agency model involving hidden information, hidden action, and a publicly observed decision, communication generally dominates delegation; see Melumad and Reichelstein (1987).

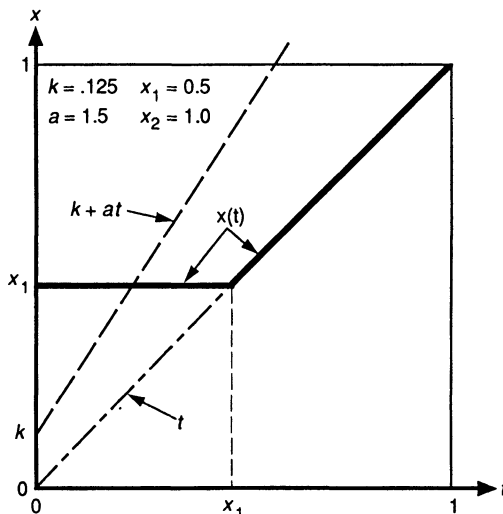
FIGURE 2  
 COMMITMENT WITH SIMILAR SENSITIVITIES: EXAMPLE 1



The difference between the two examples is that while in Figure 2 there is a reversal of the players' preferences (at the environment  $\frac{k}{1-a}$ ), in Figure 3 the receiver prefers a higher decision for all environments. In Figure 2, when the players' preferences are relatively similar (i.e., for  $t \in [x_1, x_2]$ ), the optimal decision rule prescribes the sender's first-best decision. When the preferences are sufficiently divergent (i.e., for  $t < x_1$  or  $t > x_2$ ), the optimal decision is independent of the environment over the low or high regions. In Figure 3, however, when the players' preferences are relatively similar (i.e., for  $t < x_1$ ), the optimal decision rule is independent of the environment over that region. When the preferences are sufficiently divergent (i.e., for  $t > x_1$ ), the optimal decision is the sender's first best.

Communication was demonstrated to be valuable for cases in which the sensitivities

FIGURE 3  
 COMMITMENT WITH SIMILAR SENSITIVITIES: EXAMPLE 2



of the players to the environment were similar. The following proposition demonstrates that similarity of sensitivities is not a necessary condition. As long as the players' preferences exhibit the reversal property, communication is valuable. The nature of the optimal IC decision rule, however, differs greatly from the one optimal for the case of similar sensitivities.

*Proposition 4.* When players' preferences have disparate sensitivities (i.e.,  $a > 2$ ), communication is valuable if and only if there is a preference reversal, i.e.,  $k \in (1 - a, 0)$ . The optimal IC decision rule is a one-jump discontinuity decision rule of the form

$$x(t) = \begin{cases} x_1 & \text{for } t \in [0, \tau] \\ x_2 & \text{for } t \in (\tau, 1], \end{cases} \tag{4}$$

where

$$x_1 = (a - 2)\tau^2 + 2(k + 1)\tau - k - \frac{a}{2}, \quad x_2 = 2\tau - x_1 \tag{4a}$$

and  $\tau$  is determined by the cubic equation

$$2(a - 2)^2\tau^3 + 3(a - 2)(1 + 2k)\tau^2 + (4k^2 + 8k - 2ak + 2a - a^2)\tau - k(a + 2k) = 0. \tag{4b}$$

*Proof.* See the Appendix.

Proposition 4 implies that confining attention to continuous decision rules generally results in a loss of performance.<sup>16</sup> Furthermore, when the receiver's and the sender's sensitivities to the environment are disparate, the optimal IC decision rule only distinguishes between two subsets of all possible environments. The relative size of each environment subset depends on the specific parameters.

It is interesting to note how the optimal mechanism changes as we increase the sensitivity parameter  $a$ . As is clear from Proposition 3, for a given  $k$ , as  $a$  increases from 0 to 1, the interval for which we have full separation increases. For all  $a \in [1, 2]$ , the optimal mechanism always involves full separation, i.e.,  $x(t) = t$ , for all  $t$ . Beyond  $a = 2$ , the optimal mechanism changes drastically to the discontinuous decision rule described in Proposition 4.

Interpreting Proposition 4 in a delegation framework, we see that for disparate sensitivities the performance of an IC direct revelation mechanism cannot be replicated by delegating to the sender the choice of a decision from a connected subset. The performance-equivalent decentralized mechanism is the delegation of choice out of a binary subset  $\{x_1, 2\tau - x_1\}$ .

The solution for the case of disparate sensitivities cannot be expressed in closed form. Below we demonstrate the implications of Proposition 4 via two numerical examples based on the expressions in (4a) and (4b). In the first example,  $a = 3$  and  $k = -1.0$ ; the solution is  $\tau = 0.5$ ,  $x_1 = -0.25$ , and  $x_2 = 1.25$  (see Figure 4). In the second example,  $a = 3$  and  $k = -0.35$ ; the solution (illustrated in Figure 5) is  $\tau = 0.289$ ,  $x_1 = -0.69$ , and  $x_2 = 1.277$ .

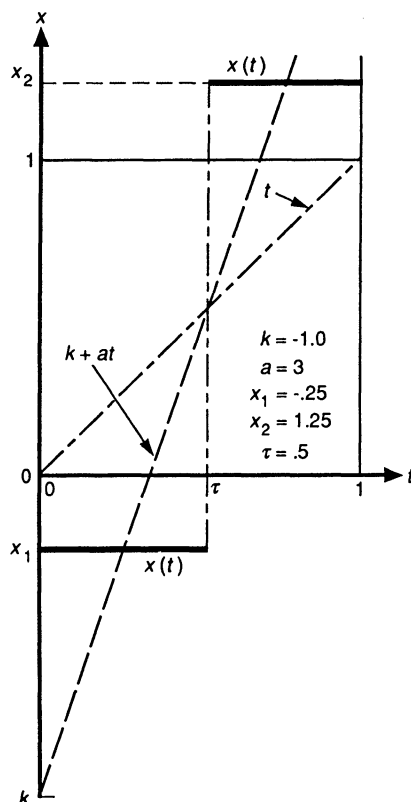
The first example is a symmetric case.<sup>17</sup> Here, the optimal decision rule partitions the environments into two equal regions. The decision rule prescribes the receiver's average first-best decision over each region. On the other hand, when the parameters are not symmetric, the trade-off inherent in the incentive-compatibility requirement results in an optimal decision rule that does not prescribe the receiver's average first-best decision. This is high-

<sup>16</sup> In the context of social choice rule implementation, Reichelstein (1984) shows via an example that smoothness requirements on allowable mechanisms restrict the class of implementable choice rules.

<sup>17</sup> By the symmetric case, we mean that the sender's and the receiver's preferences coincide at the middle environment  $t = .5$ . The corresponding parameters, referred to as symmetric parameters, are  $(k, a)$  such that  $k = \frac{(1 - a)}{2}$ .

FIGURE 4

COMMITMENT WITH DISPARATE SENSITIVITIES: EXAMPLE 1



lighted in the second example, where the receiver forgoes potential gains from a higher  $x_1$  along the lower region in order to choose a decision  $x_2$  better tailored to his preferences in the upper region. In fact, the decision  $x_1$  over the lower region is uniformly lower than any of the receiver's first-best decisions along that region.

We now turn to identifying the welfare implications of introducing communication. When the receiver's and the sender's sensitivities to the environment are similar, introducing communication is a Pareto improvement. When the receiver's and the sender's sensitivities are disparate, installing communication may make the sender strictly worse off.

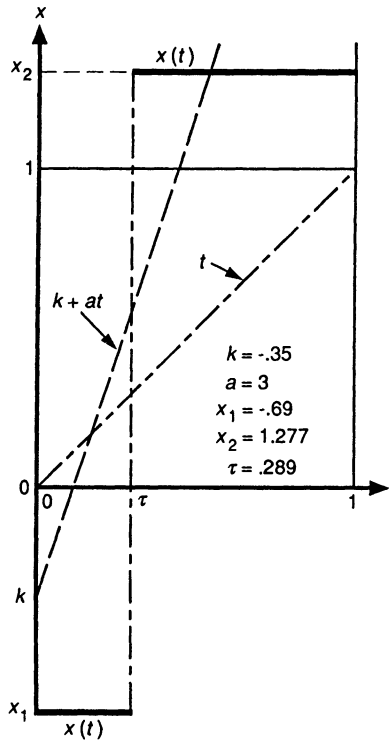
*Proposition 5.*

- (i) When the players' preferences have similar sensitivities and communication is valuable, both the sender and receiver are strictly better off with the optimal communication-dependent decision rule than with the optimal no-communication decision rule.
- (ii) When the players' preferences have disparate sensitivities and communication is valuable, the receiver is always strictly better off with communication but the sender is worse off for a generic class of parameters.

*Proof.* See the Appendix.

In the proof of Proposition 5(ii) we show that the sender is always strictly worse off in a symmetric case as well as in a nontrivial neighborhood of the symmetric parameters. The region over which the sender is worse off is typically a significant subset of the parameters for which communication is valuable. To illustrate this we consider the case of  $a = 3$ . The

FIGURE 5  
 COMMITMENT WITH DISPARATE SENSITIVITIES: EXAMPLE 2



range of  $k$  values for which communication is valuable is  $(-2, 0)$ ; for  $k$  in  $(-1.65, -.35)$ , the sender is worse off.

Proposition 5(ii) implies that communication enables the receiver to increase his welfare, while the sender loses. Interestingly, for commitment settings allowing transfers and involving hidden action, Melumad and Reichelstein (1989) demonstrate a related result. In that study, a privately informed agent is required to submit a report regarding his information. The principal has no interest in the agent's information per se; the sole purpose of communication is to improve control over the agent's action. Melumad and Reichelstein (Proposition 4) show that in some cases communication allows the principal to reduce the agent's informational rent while leaving the latter's action unchanged. Thus, as in our setting, communication enables the principal to gain while the agent loses relative to when no communication is allowed.

### 3. The no-commitment model

■ In the absence of commitment ability on the part of the receiver, an insight emerges similar to that found in a full-commitment setting: the sender is often worse off with the introduction of communication. Apart from the assumption of no-commitment, the setting in this section parallels that of the commitment model. Our analysis of the no-transfer and no-commitment setting borrows heavily from Crawford and Sobel (1982) (CS). The only substantive extension is that we consider a generalized notion of preference divergence. Specifically, we allow the preferences of the sender and the receiver to exhibit the preference reversal property and conditional preference divergence.<sup>18</sup>

<sup>18</sup> For  $a = 1$  and  $k < 0$ , our model has the same structure as the CS example. If we set  $a = 1, t = m, k = -b$  and  $x = y - b$ , then our  $U^R(x, t) \equiv -(x - k - at)^2 = -(y - m)^2$ , which is the receiver's utility in CS. Similarly, our  $U^S(x, t) \equiv -(x - t)^2 = -(y - (m + b))^2$ , which is the sender's utility in CS.

In the no-commitment setting, unlike the commitment setting in the previous section, the receiver cannot credibly promise to limit his use of the sender's information transmission. The sender, privately informed about her environment  $t \in T$ , chooses a reporting rule  $\hat{i}(t)$  regarding the environment, and the receiver chooses a decision rule  $x(\hat{i})$ . A Bayesian-Nash equilibrium consists of a reporting rule  $\{\hat{i}(t), t \in T\}$  for the sender and a decision rule  $x(\hat{i})$  for the receiver such that<sup>19</sup>

$$\text{For every environment } t, \hat{i}(t) = \underset{i \in \mathcal{T}}{\operatorname{argmax}} - (x(\hat{i}) - t)^2, \quad (5)$$

where the sets of feasible reports are the Borel sets  $\mathcal{T}$  on  $T \equiv [0, 1]$ , and

$$\text{For every report } \hat{i} \in \mathcal{T}, x(\hat{i}) = \underset{\tilde{x} \in X}{\operatorname{argmax}} E[-(\tilde{x} - k - at)^2 | \hat{i}(t)]. \quad (6)$$

Condition (5) says that for a given environment, the sender chooses a report which is optimal given the receiver's decision rule. Condition (6) says that given the sender's reporting rule, the receiver updates his priors on the sender's environment and chooses the decision that maximizes his expected utility. The Bayesian-Nash equilibrium concept guarantees that the receiver's conditional probabilistic beliefs based on each report are self-confirming.

We define a partition equilibrium of size  $N$  ( $N$  finite) to be the partition  $[t_0, t_1, \dots, t_N]$  induced on the interval  $[0, 1]$  where  $0 = t_0 < t_1 < \dots < t_N = 1$  and where each sender with an environment in  $[t_i, t_{i+1}]$  uniformly randomizes her report over that interval. This reporting strategy is equivalent to another reporting strategy whereby the sender reports the pool to which her type belongs and the receiver's beliefs are uniform over the types in each pool.<sup>20</sup> Thus, for a partition equilibrium of size  $N$ , we can confine attention to  $N$  distinct reports. We denote the class of reports that indicate membership in  $[t_i, t_{i+1}]$  by  $\hat{i}_{i+1}$  and the set of reports by  $\hat{T}^N \equiv \{\hat{i}_1, \dots, \hat{i}_N = 1\}$ . We say that "a communication-dependent equilibrium exists" if there exists a partition equilibrium with two or more reports. If a communication-dependent equilibrium yields utility for the receiver strictly higher than the communication-independent equilibrium, communication is said to be *valuable*. We also maintain the terminology introduced in the previous section regarding sensitivity and preference reversal.

We focus on equilibria with a finite number of reports for mathematical tractability.<sup>21</sup> Our first step is to show that for any equilibrium involving finitely many reports, we can confine our attention to a partition equilibrium.

*Observation 2.* Any communication-dependent equilibrium involving a finite number  $N$  of different decisions is essentially equivalent to a partition equilibrium of size  $N$ .

*Proof.* See the Appendix.

The following lemma characterizes the partition equilibria.<sup>22</sup>

<sup>19</sup> Without loss of generality, we restrict attention to pure strategy equilibria for the receiver. The receiver never finds it worthwhile to randomize since  $U_{i1}^r < 0$ . Furthermore, there can exist no neighborhood in which the decision rule discriminates among all environments because it would then violate either incentive compatibility or sequential rationality. Therefore, given  $U_{i1}^s < 0$  and  $U_{i2}^s > 0$ , only a set of measure zero of senders would be indifferent between (at most) two decisions.

<sup>20</sup> For a formal statement of this equivalence argument see CS.

<sup>21</sup> Lemma 1 of CS, which guarantees finiteness of the number of distinct equilibrium reports, does not apply in our analysis. In fact, we can show for the symmetric case there exists an equilibrium with infinitely many reports. We note however that any equilibrium with an infinite number of reports must involve pooling of types.

<sup>22</sup> The methodology of the proof is similar to that of Theorem 1 in CS. Their equilibrium characterization, however, does not directly apply to our equilibria because we allow players' preference reversal, while they do not.

*Lemma 1.* If a partition equilibrium of size  $N$  exists, it involves the partition  $[0 = t_0, t_1, \dots, t_i, \dots, t_N = 1]$ , where

(i) For  $a \in (0, 1) \cup (1, 2) \cup (2, \infty)$ :

$$t_i = \frac{k}{1-a} B^i + \frac{1 - \frac{k}{1-a} B^N}{C^N} C^i,$$

where  $B^i = \frac{b_2 b_1^i - b_1^i + b_2^i - b_1 b_2^i + b_1 - b_2}{b_1 - b_2}$ ,  $C^i = \frac{b_1^i - b_2^i}{b_1 - b_2}$ ,  $i = 0, 1, \dots, N$ ,

$$\text{and } b_1, b_2 = \frac{2-a}{a} \pm \frac{2\sqrt{1-a}}{a} \text{ (} b_1 \text{ and } b_2 \text{ may be complex numbers);}$$

(ii) For  $a = 1$  (CS example):

$$t_i = \frac{i}{N} - 2ki(i - N), \quad \text{where } i = 0, 1, \dots, N;$$

(iii) For  $a = 2$ :<sup>23</sup>

$$[t_0 = 0, t_1 = 1/2, t_2 = 1].$$

*Proof.* See the Appendix.

Before analyzing communication-dependent equilibria, we note that a communication-independent equilibrium always exists and the optimal communication-independent decision rule is the same as in the commitment model (i.e.,  $x = k + \frac{a}{2}$ ).

We are now in a position to study conditions necessary for communication to be valuable. In Proposition 6 we focus on two-report equilibria, as they provide the simplest means of illustrating the existence of communication-dependent equilibria. The welfare results of Proposition 7, on the other hand, are established for general partition equilibria of size  $N$ .

*Proposition 6.* (i) When the players' preferences have similar sensitivities, there exists a communication-dependent equilibrium of size 2 if and only if  $k \in \left(-\frac{a}{4}, 1 - \frac{3a}{4}\right)$ . (ii)

When the players' preferences have disparate sensitivities, there exists a communication-dependent equilibrium of size 2 if and only if  $k \in \left(1 - \frac{3a}{4}, -\frac{a}{4}\right)$ . The equilibrium partition

in both cases is  $\left[0 = t_0, t_1 = \frac{a + 4k}{4 - 2a}, t_2 = 1\right]$ . Communication in these equilibria is valuable.

(iii) When the players' sensitivities are negatively related, the only equilibrium is communication independent, i.e.,  $\max N = 1$ .

*Proof.* See the Appendix.

Note that the absence of commitment ability reduces the range of parameters for which communication is valuable. For  $a \in (0, 2)$ , for example, the range changes from  $k \in \left(-\frac{a}{2}, 1 - \frac{a}{2}\right)$  (see Proposition 3) to  $k \in \left(-\frac{a}{4}, 1 - \frac{3a}{4}\right)$ . Furthermore, it can be

<sup>23</sup> This is a nongeneric equilibrium that exists only for  $k = -1/2$  and involves only two reports.

shown that when communication is valuable, its value is strictly lower than in the commitment case. The requirement  $k \in \left(1 - \frac{3a}{4}, -\frac{a}{4}\right)$  for the case of disparate sensitivities implies preference reversal, yet the latter property is not sufficient for valuable communication.

Figures 6 and 7 below illustrate Proposition 6 for a case with similar sensitivities ( $a = 1/2$  and  $k = 3/28$ ) and a case with disparate sensitivities ( $a = 3$  and  $k = -7/8$ ).

In the case described in Figure 6, within each pool, the sender, depending on her specific environment, may prefer a decision that is higher or lower than the equilibrium decision corresponding to the pool. In the case described in Figure 7, in contrast, every sender in the upper (lower) pool prefers a lower (higher) decision than the corresponding equilibrium decision. In fact, we can show that in this example the sender is *ex ante* worse off with communication. This observation is generalized in Proposition 7.

We examine the welfare implications of communication-dependent equilibria for the general  $N$ -report case. This welfare comparison is strikingly similar to that of the commitment case (Proposition 5).

*Proposition 7.* (i) When the players' preferences have similar sensitivities and a communication-dependent equilibrium (of any size) exists, both the sender and receiver are strictly better off in the communication-dependent equilibrium than in the communication-independent equilibrium. (ii) When the players' preferences have disparate sensitivities and a communication-dependent equilibrium (of any size) exists, the receiver is strictly better off and the sender is strictly worse off in the communication-dependent equilibrium than in the communication-independent equilibrium.

*Proof.* See the Appendix.

Part (ii) of Proposition 7 implies that communication enables the receiver to increase his welfare at the expense of the sender. Thus, when a sender has an (*ex ante*) option of not participating in the receiver's decision-making process, she may exercise that option.

FIGURE 6  
NO COMMITMENT WITH SIMILAR SENSITIVITIES

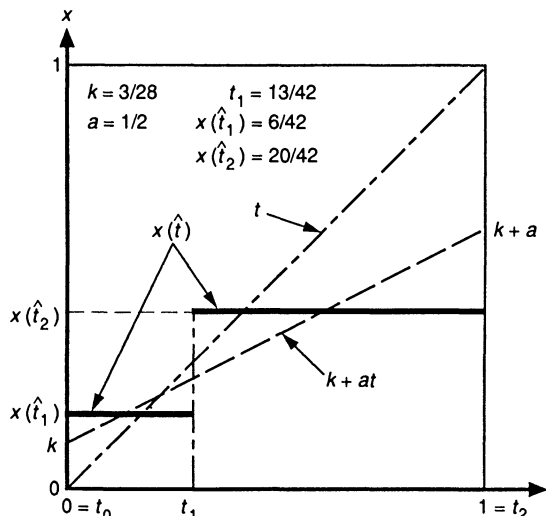
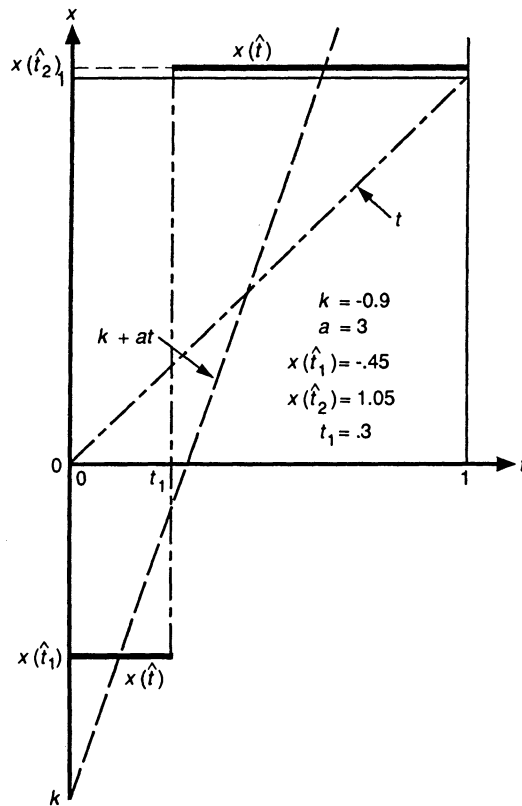


FIGURE 7  
NO COMMITMENT WITH DISPARATE SENSITIVITIES



This result seems at odds with the CS suggestion that *ex ante* the players are likely to coordinate on the equilibrium with the finest partition.<sup>24</sup>

Given the multiplicity of equilibria in the no-commitment model, a natural question is “Could we eliminate some equilibria using an equilibrium refinement?” When both parties are better off in a communication-based equilibrium, one might conjecture that the parties will attempt to coordinate their equilibrium choices for the purpose of ending up in a Pareto-dominant equilibrium. While Pareto dominance is a desired property for a predicted equilibrium, it is not an acceptable equilibrium selection criterion. We utilize Farrell’s (1990) neologism-proof refinement of cheap-talk equilibria.<sup>25</sup> Loosely speaking, an equilibrium is neologism-proof if there exists no report that is sent by precisely those types expected by the receiver, when the receiver (1) believes the report, (2) responds optimally to the report, and (3) continues to respond to reports from other types with the equilibrium decisions.<sup>26</sup> Such a report is referred to as a *credible neologism*. An interesting result emerges regarding the viability of the pooling equilibrium.

<sup>24</sup> Pitchik and Schotter (1987) point out in a bimatrix game that the equilibrium probability of “honesty” might decrease as players become “more similar.” They then argue that their observation, made in a significantly different setting using entirely different notions of informativeness and similarity of preferences, seems at odds with the CS result.

<sup>25</sup> An alternative approach is presented in Rabin (1990).

<sup>26</sup> See Farrell (1990) for the formal definition of neologism-proofness.

*Proposition 8.* Consider the class of parameters in Proposition 6. The communication-independent equilibrium is neologism-proof if and only if the sender is *ex ante* worse off with communication, (i.e.,  $a > 2$ ).

*Proof.* See the Appendix.

This result shows a close parallel between the *ex ante* welfare characteristics identified in Proposition 7 and the neologism-proof property. When sensitivities are disparate, the sender is *ex ante* better off in a pooling equilibrium and this equilibrium is neologism-proof. On the other hand, when the sender *ex ante* prefers a communication equilibrium, the pooling equilibrium is not neologism-proof. This seems to indicate the viability of the pooling equilibrium in the disparate sensitivity case.<sup>27</sup> Note that while the *ex ante* desirability is an “average” property, neologism-proofness is a property pertaining to subsets of sender types.

#### 4. Discussion and extensions

■ We have shown that the Pareto efficiency of communication-based mechanisms depends to a large extent on the relative sensitivities of the players’ preferences to their environment. When the receiver’s and sender’s sensitivities to variations in the environment are highly disparate, the sender might be worse off in a communication-based relationship relative to no communication. This result is robust to the polar cases of full-commitment ability and no-commitment ability for the receiver. The value of communication, however, is shown to be dependent on the receiver’s ability to commit. In the absence of commitment, communication is valuable in fewer instances, and, when valuable, communication yields a lower expected utility to the receiver relative to the commitment case.

In different organizational arrangements the ability to commit might be limited for various reasons, such as legal and political restrictions, observability constraints, and communication costs. Our analysis provides a benchmark for evaluating the relative efficiency of observed institutional arrangements where commitment ability seems to be limited and contingent transfers are not a prominent feature.

An example of such an application is Melumad and Shibano (1991), where we study the informational rationale of the *observed* veto-based delegation arrangement between the Securities and Exchange Commission (SEC) and the Financial Accounting Standards Board (FASB). We model a veto-based mechanism as one in which the SEC delegates the policy choice to the FASB and either (1) accepts the FASB policy choice or (2) vetoes the FASB policy choice, choosing instead a sequentially rational policy choice from a prespecified reversion set.<sup>28</sup> We find conditions under which veto mechanisms, utilizing up to two optimally chosen reversions, implement the performance of any full-commitment mechanism. This efficiency property of veto-based mechanisms suggests that, in certain instances, veto-based mechanisms are optimal.

Another extension of our analysis is considering the effect of imposing an individual rationality (IR) constraint in the commitment model. When the IR constraint is *ex ante* (i.e., there is a minimum requirement on the sender’s expected utility) and the unconstrained communication-independent decision rule is feasible, the value of communication is unaffected when sensitivities are similar, and it is lower than in the unconstrained case when sensitivities are disparate. When the IR constraint is *ex post* (i.e., there is a minimum requirement on the sender’s utility for every possible environment) and the unconstrained

<sup>27</sup> An open question beyond the scope of this article is whether there is a neologism-proof communication equilibrium in the similar or disparate sensitivity cases. What we can show is that the two-report communication equilibrium is not neologism-proof in either case.

<sup>28</sup> Another example of such a relationship is the one between congressional committees and the congressional “floor” under “closed-rule” voting.

independent decision rule is feasible, the value of communication is in general lower than in the unconstrained case. In both the *ex ante* and *ex post* settings, when the unconstrained communication-independent decision rule is not feasible in the constrained problem, we can show that the value of communication in the constrained problem might be higher than in the unconstrained one.<sup>29</sup>

Other interesting extensions involve allowing both parties to have decision-making power and/or private information (as in Kanodia and Gigler (1988), Newman and Sansing (1991), and Seidmann (1990)), and allowing multiple senders and receivers. We have examined the case in which both the receiver and sender have private information at the beginning of the game and the sender's utility is independent of the receiver's private information; we found that the results of our article remain essentially intact. The extent to which the other extensions affect our results is yet to be explored.

**Appendix**

■ The proofs of Propositions 1–8, Observations 1 and 2, and Lemma 1 follow.

*Proof of Proposition 1.* We prove our result for general twice-differentiable von Neumann-Morgenstern utility functions  $U^i(x, t)$ ,  $i = r, s$ , with the single-peaked property, i.e.,  $U^i_1(x, t) = 0$ , for some  $x$  for each  $t$ , and  $U^i_{11} < 0$ , as well as the single-crossing property, i.e.,  $U^i_{12} > 0$ .

Let the first-best decision for the receiver and sender be

$$x^r(t) \equiv \operatorname{argmax}_x U^r(x, t) \quad \text{and} \quad x^s(t) \equiv \operatorname{argmax}_x U^s(x, t).$$

We adopt the following notation:  $x_t = x(t)$ ,  $x^i_t = x^s(t)$ ,  $x^-(\tau) = \lim_{t \rightarrow \tau^-}(x(t))$ ,  $x^+(\tau) = \lim_{t \rightarrow \tau^+}(x(t))$ . We first establish the following Lemmas to be used below.

*Lemma A1.*  $x^s(t)$  is strictly increasing.

*Proof.* Since  $U^i_{11} < 0$  and  $U^i_{12} > 0$ , it is clear the inverse  $x^s(t) \equiv (U^s_1)^{-1}(0|t)$  exists and is strictly increasing in  $t$ . *Q.E.D.*

*Lemma A2.* If  $x^s(t) > x_1 > x_2$  or  $x^s(t) < x_1 < x_2$  then  $U^s(x_1, t) > U^s(x_2, t)$ .

*Proof.* Immediate from  $U^s_1(x^s(t), t) = 0$  and  $U^s_{11} < 0$ ,  $\forall t$ . *Q.E.D.*

*Part (i).* Assume  $t_2 > t_1$  and  $x_1 > x_2$ . By Lemma A1, the following cases exhaust all possibilities:

Case 1:  $x^s_1 \geq x_1 > x_2$ . By Lemma A1,  $x^s_2 > x_1 > x_2$ . Therefore, by Lemma A2,  $x_2$  is not incentive compatible.

Case 2:  $x_1 > x_2 \geq x^s_1$  (for any  $x^s_2$ ). By Lemma A2,  $x_1$  is not incentive compatible.

Case 3a:  $x^s_2 \geq x_1 > x^s_1 > x_2$ . By Lemma A2,  $x_2$  is not incentive compatible.

Case 3b:  $x_1 > x^s_2 > x^s_1 > x_2$ . If  $U(x_2, t_1) > U(x_1, t_1)$ , then  $x(\cdot)$  is not IC at  $t_1$ . Suppose then that  $U(x_1, t_1) \geq U(x_2, t_1)$ . Then we want to show that  $U(x_1, t_2) > U(x_2, t_2)$ , which violates IC. Define  $x^c(x|t)$  such that  $U(x^c(x|t), t) = U(x, t)$ , where  $x^c(x|t) = x$  if and only if  $x = x^s(t)$ . Such an  $x^c(x|t)$  always exists by the strict concavity of  $U(\cdot, t)$ . Then if  $U(x_1, t_1) > U(x_2, t_1)$  and  $x_2 < x^s_1 < x_1$ , then  $x_2 \leq x^c(x_1|t_1) < x^s_1$ . The single-crossing property implies  $U_1(x, t_2) > U_1(x, t_1)$ , for all  $x$ , thus,

$$\int_{x^c(x_1|t_1)}^{x_1} U_1(x, t_2) dx > \int_{x^c(x_1|t_1)}^{x_1} U_1(x, t_1) dx, \quad \text{or,}$$

$$U(x_1, t_2) - U(x^c(x_1|t_1), t_2) > U(x_1, t_1) - U(x^c(x_1|t_1), t_1).$$

By definition of  $x^c$ , LHS =  $U(x^c(x_1|t_2), t_2) - U(x^c(x_1|t_1), t_2) > 0 =$  RHS. Combined with Case 3(b), this implies  $x_2 \leq x^c(x_1|t_1) < x^c(x_1|t_2) < x^s_2$ ; thus  $t_2$  will prefer  $x_1$  to  $x_2$ , i.e.,  $x_2$  is not IC.

*Part (ii).* Suppose not. If  $x(t) < x^s(t)$  for some  $t \in (t_1, t_2)$ , then since  $x(t)$  is continuous and strictly increasing in  $t$ , there exists an  $\epsilon > 0$  such that  $x(t) < x(t + \epsilon) < x^s(t)$ . Therefore, by Lemma A2,  $U^s(x(t + \epsilon), t) > U^s(x(t), t)$ , so  $x(\cdot)$  is not incentive compatible at  $t$ . A similar argument holds for  $x(t) > x^s(t)$ .

*Part (iii).* This is directly a requirement of incentive compatibility.

*Part (iiib).* Note that the inverse  $(x^s)^{-1}(\cdot)$  of  $x^s(t)$  is well defined. Consider  $t \in [(x^s)^{-1}(x^-(\tau)), \tau)$ . By construction,  $x^-(\tau) < x^s(t)$ . By Proposition 1(i),  $x(t) \leq x^-(\tau)$ . Suppose that  $x(t) < x^-(\tau)$ . By Lemma A2,

<sup>29</sup> Note that in this case, while a constrained communication-dependent solution always exists, a constrained communication-independent solution may not.

$U^s(x^-(\tau), t) > U^s(x(t), t)$ , that is,  $x(\cdot)$  is not IC at  $t$ . Thus,  $x(t) = x^-(\tau), \forall t \in [(x^s)^{-1}(x^-(\tau)), \tau)$ . Similarly,  $x(t) = x^+(\tau), \forall t \in (\tau, (x^s)^{-1}(x^+(\tau))]$ .

*Part (iiic).* Since  $U^s(x^-(\tau), \tau) = U^s(x^+(\tau), \tau), x^-(\tau) < x^s(\tau) < x^+(\tau)$ . Suppose  $x(\tau) < x^-(\tau)$ . Then, by Lemma A2,  $x(\cdot)$  is not IC at  $\tau$ . Similarly, if  $x(\tau) > x^+(\tau)$ , then Lemma A2 implies  $x(\cdot)$  is not IC at  $\tau$ . Suppose  $x(\tau) \in (x^-(\tau), x^s(\tau))$ . Then, for every (but not only)

$$t \in [(x^s)^{-1}(x(\tau)), \tau), U^s(x(\tau), t) > U^s(x^-(\tau), t) \geq U^s(x(t), t).$$

Thus,  $x(\cdot)$  is not IC at  $\tau$ . Similarly, if  $x(\tau) \in (x^s(\tau), x^+(\tau))$ ,  $x(\cdot)$  is not IC at  $\tau$ . *Q.E.D.*

*Proof of Proposition 2.* Suppose to the contrary that the optimal IC decision rule  $x(t)$  is discontinuous at  $\tau$ . Then by Proposition 1, there exists a  $\underline{t}$  such that

$$x(t) = \begin{cases} \underline{t}, & \forall t \in [\underline{t}, \tau) \\ 2\tau - \underline{t}, & \forall t \in (\tau, 2\tau - \underline{t}]. \end{cases}$$

We confine our analysis to the interval  $[\underline{t}, \bar{t}]$ , where  $\bar{t} = 2\tau - \underline{t}$ .

*Part (i).* The IC property in our setting is a pointwise property (see Proposition 1). Therefore, replacing any segment of an IC decision rule by some other IC segment does not affect the IC property of the decision rule. We now show that by increasing  $\underline{t}$  to some  $t_1 \leq \tau$  while maintaining incentive compatibility, we increase the receiver's expected utility in contradiction to the assumed optimality of  $x(t)$ . The above variation creates two new subsegments  $[\underline{t}, t_1)$  and  $(2\tau - t_1, 2\tau - \underline{t}]$ ,  $t_1 \in [\underline{t}, \tau]$  in which  $x(t) = t$ . The receiver's expected utility is, then,

$$-\int_{\underline{t}}^{t_1} (t - k - at)^2 dt - \int_{t_1}^{\tau} (t_1 - k - at)^2 dt - \int_{\tau}^{2\tau - t_1} (2\tau - t_1 - k - at)^2 dt - \int_{2\tau - t_1}^{2\tau - \underline{t}} (t - k - at)^2 dt.$$

The first derivative with respect to  $t_1$  is  $2(\tau - t_1)^2(2 - a)$ . Note that for any  $a < 2$  and  $t_1 \neq \tau$ , the derivative is positive. Thus, given that  $t_1 \in [\underline{t}, \tau]$ , the optimal  $t_1 = \tau$ ; that is, the optimal IC decision rule is continuous. This implies everywhere continuity of the optimal IC decision rule, because arbitrarily adding discontinuities, other than those ruled out above, will destroy the IC property, as shown in Proposition 1.

*Part (ii).* In the case of  $a = 2$ , it is readily verified that

$$-\int_{\underline{t}}^{\tau} (t - k - 2t)^2 dt - \int_{\tau}^{2\tau - \underline{t}} (t - k - 2t)^2 dt + \int_{\underline{t}}^{2\tau - \underline{t}} (t - k - 2t)^2 dt = 0.$$

This means that the receiver is indifferent between having the above-mentioned discontinuity and the sender's first-best decision. *Q.E.D.*

*Proof of Observation 1.* Assume to the contrary that the optimal IC decision  $x(t)$  is communication-dependent. From Proposition 2, we know  $x(t)$  must be continuous. Thus, by Proposition 1(ii), there is a region  $\bar{T}$  in which  $x(t) = t$ . Let  $t^m \in \underset{t \in \bar{T}}{\operatorname{argmax}} -(x(t) - k - at)^2$ . Consider an alternative decision rule  $x^m(t) \equiv x(t^m), \forall t$ . We can

readily verify that this decision rule is IC and yields higher expected utility for the receiver because it is uniformly closer to the receiver's first-best decision rule  $x^*(t)$ . This contradicts the optimality of  $x(t)$ . *Q.E.D.*

*Proof of Proposition 3.*

*"If" part. Step 1.* For the above parameters,  $k + \frac{a}{2} \in (0, 1)$ . The value of communication is evident as the first derivative of expression (3) with respect to  $t_1$ , i.e.,  $-t_1((2 - a)t_1 - 2k)$  (first derivative with respect to  $t_2$ , i.e.,  $-(1 - t_2)((2 - a)t_2 - 2k - a)$ ), evaluated at the optimal no-communication decision, i.e.,  $t_1 = t_2 = k + \frac{a}{2}$ , is negative (positive). Thus some communication-dependent decision rule strictly outperforms the communication-independent decision rule.

*Step 2.* We now identify the optimal levels  $t_1$  and  $t_2$ . We first solve the unconstrained optimization problem and then verify that the solution meets constraints (i) and (ii) for the specified region  $k \in \left(-\frac{a}{2}, 1 - \frac{a}{2}\right)$ . The first- and second-order conditions with respect to  $t_1$ , i.e.,  $t_1((2 - a)t_1 - 2k) = 0$  and  $-2((2 - a)t_1 - k) < 0$  respectively, imply that

$$t_1 = \begin{cases} 0 & \text{if } k < 0 \\ \frac{2k}{2 - a} & \text{if } k \geq 0. \end{cases}$$

Thus  $t_1 = \max \left\{ 0, \frac{2k}{2 - a} \right\}$ .

The first- and second-order conditions with respect to  $t_2$ , i.e.,  $-(1 - t_2)((2 - a)t_2 - 2k - a) = 0$  and  $2((2 - a)t_2 - k - 1) < 0$  respectively, imply that

$$t_2 = \begin{cases} 1 & \text{if } k > 1 - a \\ \frac{2k + a}{2 - a} & \text{if } k \leq 1 - a. \end{cases}$$

Thus  $t_2 = \min \left\{ \frac{2k + a}{2 - a}, 1 \right\}$ .

Admissibility is obvious for  $t_1 = 0$  and  $t_2 = 1$ . The solution  $t_1 = \frac{2k}{2 - a}$  is admissible for  $k < 1 - \frac{a}{2}$  and the solution  $t_2 = \frac{2k + a}{2 - a}$  is admissible for  $k > -\frac{a}{2}$ .

Let  $(t_1, t_2)$  be a solution pair. Incentive compatibility is trivially met for the solution pairs  $(0, 1)$  and  $\left(\frac{2k}{2 - a}, \frac{2k + a}{2 - a}\right)$ . The solution pairs  $\left(0, \frac{2k + a}{2 - a}\right)$  and  $\left(\frac{2k}{2 - a}, 1\right)$  are incentive compatible for  $k > -\frac{a}{2}$  and for  $k < 1 - \frac{a}{2}$ , respectively.

“Only if” part. For  $k \geq 1 - \frac{a}{2}$  and  $a \leq 2$  (with at least one strict inequality), the first derivative of expression (3) with respect to  $t_1$ ,  $-t_1((2 - a)t_1 - 2k)$ , is positive for all  $t_1 \in (0, 1]$ . (Note that by the second-order condition,  $t_1 = 0$  is a local minimum.) Incentive compatibility requires, however, for any given  $t_2$ ,  $t_1 \leq t_2$ . Therefore,  $t_1 = t_2$  is optimal, i.e., communication is not valuable.

For  $k = 1 - \frac{a}{2}$  and  $a = 2$ , the above derivative is zero for all  $t_1 \in (0, 1]$ , thus the value of the objective function is independent of  $t_1$ . In particular,  $t_1 = t_2$  is optimal, therefore there is no value to communication.

For  $k \leq -\frac{a}{2}$  and  $a \leq 2$  (with at least one strict inequality), the first derivative of expression (3) with respect to  $t_2$ ,  $-(1 - t_2)((2 - a)t_2 - 2k - a)$ , is negative for all  $t_2 \in [0, 1)$ . (Note that  $t_2 = 1$  is a local minimum.) Incentive compatibility requires, however, for any given  $t_1$ ,  $t_2 \geq t_1$ . Therefore,  $t_2 = t_1$  is optimal, i.e., communication is not valuable. For  $k = -\frac{a}{2}$  and  $a = 2$ , the first and second derivatives are zero for all  $t_2 \in [0, 1)$ , and we can verify that the value of the objective function is independent of  $t_2$ . Again, there is no value to communication. *Q.E.D.*

*Proof of Proposition 4.*

“If” part. *Step 1.* We first show that the optimal IC decision rule cannot have any subinterval  $(\underline{t}, \bar{t})$  such that  $x(t) = t$  on that interval. Suppose it does. Fix an arbitrary  $\tau \in (\underline{t}, \bar{t})$  and choose  $t_1$  such that  $\underline{t} \leq t_1 < \tau < 2\tau - t_1 \leq \bar{t}$ . Consider the following discontinuous decision rule:

$$\hat{x}(t) = \begin{cases} x(t) & \text{for } t \in [0, \underline{t}) \cup (\bar{t}, 1] \\ t & \text{for } t \in [\underline{t}, t_1) \cup (2\tau - t_1, \bar{t}] \\ t_1 & \text{for } t \in [t_1, \tau) \\ 2\tau - t_1 & \text{for } t \in [\tau, 2\tau - t_1]. \end{cases}$$

This decision rule is IC according to Proposition 1.

As in the proof of Proposition 2, the receiver’s expected utility over  $[\underline{t}, \bar{t}]$  is

$$-\int_{\underline{t}}^{t_1} (t - k - at)^2 dt - \int_{t_1}^{\tau} (t_1 - k - at)^2 dt - \int_{\tau}^{2\tau - t_1} (2\tau - t_1 - k - at)^2 dt - \int_{2\tau - t_1}^{\bar{t}} (t - k - at)^2 dt.$$

The first derivative with respect to  $t_1$  is  $2(\tau - t_1)^2(2 - a)$ . Note that for any  $a > 2$  and  $t_1 \neq \tau$ , this derivative is negative. Thus, the decision rule  $\hat{x}(t)$ , discontinuous at  $\tau$ , dominates the continuous decision rule  $x(t)$ . This contradicts the assumed optimality of  $x(t)$ . An optimal IC decision rule therefore cannot have any subinterval  $(\underline{t}, \bar{t})$  such that  $x(t) = t$  on that interval.

*Step 2.* Proposition 1 combined with step 1 implies that the optimal  $x(t)$  is an increasing  $n$ -jump function,  $n \geq 0$ . We now argue that  $n > 0$ ; specifically, we show the optimal communication-independent decision rule  $x(t) = k + \frac{a}{2}$  is dominated by some (nonoptimal!) discontinuous decision rule. There are two cases to consider.

Case (i).  $k \in \left(\frac{1-a}{2}, 0\right)$ . Here consider the following:

$$\hat{x}(t) = \begin{cases} k + \frac{a}{2} & \text{for } t \in \left[0, \frac{k}{1-a}\right) \\ k \frac{1+a}{1-a} - \frac{a}{2} & \text{for } t \in \left[\frac{k}{1-a}, 1\right]. \end{cases}$$

Note this decision rule is IC and, for  $k \in \left(\frac{1-a}{2}, 0\right)$ , involves an interior discontinuity.

The receiver's gain from  $\hat{x}(t)$  relative to  $x(t)$  (for the assumed parameters) is

$$\int_{k/(1-a)}^1 \left\{ -\left(k \frac{1+a}{1-a} - \frac{a}{2} - k - at\right)^2 + \left(k + \frac{a}{2} - k - at\right)^2 \right\} dt = a^2 \left(\frac{2k}{1-a} - 1\right) \left(1 - \frac{k}{1-a}\right)^2 > 0.$$

Case (ii).  $k \in \left(1-a, \frac{1-a}{2}\right)$ . Here consider the following:

$$\hat{x}(t) = \begin{cases} k \frac{1+a}{1-a} - \frac{a}{2} & \text{for } t \in \left[0, \frac{k}{1-a}\right) \\ k + \frac{a}{2} & \text{for } t \in \left[\frac{k}{1-a}, 1\right]. \end{cases}$$

Note this decision rule is IC and, for  $k \in \left(1-a, \frac{1-a}{2}\right)$ , involves an interior discontinuity.

The receiver's gain from  $\hat{x}(t)$  relative to  $x(t)$  (for the assumed parameters) is

$$\int_0^{k/(1-a)} \left\{ -\left(k \frac{1+a}{1-a} - \frac{a}{2} - k - at\right)^2 + \left(k + \frac{a}{2} - k - at\right)^2 \right\} dt = a^2 \left(1 - \frac{2k}{1-a}\right) \left(\frac{k}{1-a}\right)^2 > 0.$$

*Step 3.* From the above two steps, we know the optimal IC decision rule is discontinuous. We now show it has only a single-jump discontinuity. Suppose to the contrary that the optimal  $x(t)$  has  $n$  jumps,  $n \geq 2$ . We show it is strictly dominated by some  $n - 1$ -jump IC decision rule  $\hat{x}(t)$ .

Consider three adjacent steps,  $x_1$ ,  $x_2$ , and  $x_3$ , where  $x_1 < x_2 < x_3$ , of the assumed optimal  $x(t)$ . For  $t \in [x_1, x_3]$ ,

$$x(t) = \begin{cases} x_1 & \text{for } t \in \left[x_1, \frac{x_1+x_2}{2}\right) \\ x_2 & \text{for } t \in \left[\frac{x_1+x_2}{2}, \frac{x_2+x_3}{2}\right) \\ x_3 & \text{for } t \in \left[\frac{x_2+x_3}{2}, x_3\right]. \end{cases}$$

Consider now

$$\hat{x}(t) = \begin{cases} x_1 & \text{for } t \in \left[x_1, \frac{x_1+x_3}{2}\right) \\ x_3 & \text{for } t \in \left[\frac{x_1+x_3}{2}, x_3\right]. \\ x(t) & \text{otherwise.} \end{cases}$$

We show that the above  $n - 1$ -jump decision rule  $\hat{x}(t)$  dominates  $x(t)$ . Evaluating the difference in receiver's expected utility from using  $\hat{x}(t)$  instead of  $x(t)$  yields

$$\int_{x_1}^{(x_1+x_3)/2} -(x_1 - k - at)^2 dt + \int_{(x_1+x_3)/2}^{x_3} -(x_3 - k - at)^2 dt - \int_{x_1}^{(x_1+x_2)/2} -(x_1 - k - at)^2 dt - \int_{(x_1+x_2)/2}^{(x_2+x_3)/2} -(x_2 - k - at)^2 dt - \int_{(x_2+x_3)/2}^{x_3} -(x_3 - k - at)^2 dt = (x_3 - x_2)(x_2 - x_1)(x_3 - x_1) \left(\frac{a-2}{4}\right) > 0.$$

The same argument shows that any  $n - j$ -jump decision rule dominates an  $n - j - 1$ -jump decision rule, for all

$j = 1, \dots, n - 3$ . Recall from step 2 that a one-jump decision rule dominates a zero-jump decision rule. Thus, a one-jump decision rule is optimal.

*Step 4.* Recall that Proposition 1 requires, for a lower constant level  $x_1$  and a jump point  $\tau$ , the upper constant level to be  $2\tau - x_1$ . Therefore, steps 1–3 imply the receiver chooses  $x_1$  and  $\tau$  to maximize

$$\int_0^\tau -(x_1 - k - at)^2 dt + \int_\tau^1 -(2\tau - x_1 - k - at)^2 dt.$$

The optimality conditions for  $x_1$  and  $\tau$  yield expressions (4a) and (4b).

“Only if” part

Assume  $k \notin (1 - a, 0)$ . If communication is valuable, then steps 1, 3, and 4 above hold, and the difference in the receiver’s utility between the optimal communication-dependent decision rule and the optimal communication-independent decision rule must be positive, i.e.,

$$\int_0^\tau \left\{ -(x_1 - k - at)^2 + \left( a \left( \frac{1}{2} - t \right) \right)^2 \right\} dt + \int_\tau^1 \left\{ -(2\tau - x_1 - k - at)^2 + \left( a \left( \frac{1}{2} - t \right) \right)^2 \right\} dt > 0.$$

Let  $G(t) \equiv -(a - 2)(1 + 2k)t^2 + (-4k^2 - 8k + 2ak - 2a + a^2)t + 3k(a + 2k)$ . It can be shown (via tedious calculations) that conditions (4a) and (4b) imply that the above requirement holds if and only if  $G(\tau) < 0$ . We distinguish between two cases. For  $k > 0$ ,  $G(0) > 0$ ,  $G(1) > 0$ , and, for all  $t \in (0, 1)$ ,  $G''(\cdot) < 0$ . Therefore,  $G(t) > 0$  for all  $t \in (0, 1)$  and in particular  $G(\tau) > 0$ , thus communication is not valuable. For  $k < 1 - a$ ,  $G(1) > 0$ ,  $G'(1) < 0$ , and, for all  $t \in (0, 1)$ ,  $G''(\cdot) > 0$ , so  $G(\cdot)$  achieves its minimum on  $(0, 1)$  at  $t = 1$ . Thus  $G(t) > 0$  for all  $t \in (0, 1)$  and in particular  $G(\tau) > 0$ , thus communication is not valuable. *Q.E.D.*

*Proof of Proposition 5. Part (i).* In step 1 of the proof of Proposition 3, we establish that the receiver is strictly better off with communication. The sender’s incremental expected utility with communication relative to no communication,

$$-\int_0^{t_1} (t_1 - t)^2 dt - \int_{t_1}^{t_2} (t - t_1)^2 dt - \int_{t_2}^1 (t_2 - t)^2 dt - \int_0^1 \left( k + \frac{a}{2} - t \right)^2 dt,$$

is positive because Proposition 3 implies that  $k + \frac{a}{2} \in (t_1, t_2)$ .

*Part (ii).* The reduced form of equation (4b) is

$$y^3 + py + q = 0, \quad \text{where} \tag{A1}$$

$$y = \tau + \frac{(1 + 2k)}{2(a - 2)} \tag{A2}$$

$$p = \frac{4k^2 + 8k - 2ak + 2a - a^2}{2(a - 2)^2} - \frac{3(1 + 2k)^2}{4(a - 2)^2} \tag{A3}$$

$$q = \frac{(1 + 2k)^3}{4(a - 2)^3} - \frac{(1 + 2k)(4k^2 + 8k - 2ak + 2a - a^2)}{4(a - 2)^3} - \frac{k(a + 2k)}{2(a - 2)^2}. \tag{A4}$$

The solutions to (A1), as long as  $p < 0$  and  $D = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2 \leq 0$ , are

$$y_1 = -2R \cos\left(\frac{\phi}{3}\right), \quad y_2 = -2R \cos\left(\frac{\phi}{3} + \frac{2\pi}{3}\right), \quad \text{and} \quad y_3 = -2R \cos\left(\frac{\phi}{3} + \frac{4\pi}{3}\right), \tag{A5}$$

where  $R = \text{sgn}(q) \sqrt{\frac{|p|}{3}}$  and  $\cos(\phi) = \frac{q}{2R^3}$ .

We first consider the class of symmetric cases (i.e.,  $k = \frac{1 - a}{2}$ ). Let  $k^s(a) \equiv \frac{1 - a}{2}$ .

*Lemma A3.* In the symmetric case,  $q = 0$ . Therefore, the solutions are

$$y_1 = -\sqrt{-p}, \quad y_2 = \sqrt{-p}, \quad y_3 = 0.$$

*Proof.* Substituting  $k = \frac{1 - a}{2}$  into (A4) yields  $q = 0$  and results in the above solutions to (A1). The lemma and (A2) imply that for the symmetric case,  $\tau_i = y_i + \frac{1}{2}$ ,  $i = 1, 2, 3$ . We now argue that the only admissible solution is  $\tau_3 = \frac{1}{2}$ . In the symmetric case, (A3) takes the following form:

$$p = \frac{a^2 - 5a + 5}{2(a - 2)^2} - \frac{3}{4}. \tag{A6}$$

The value of  $p$  in (A6), for  $a > 2$ , is monotone increasing and  $\lim_{a \rightarrow \infty} p(a) = -1/4$ . We further observe that in the symmetric case  $D < 0$ , since  $q = 0$  and  $p < 0$ . Therefore, the solutions in (A5) hold. *Q.E.D.*

By Lemma A3,  $\tau_1 < 0$  and  $\tau_2 > 1$ , and thus the only admissible solution under communication is  $\tau_3 = 1/2$  (and  $x_1 = \frac{1}{2} - \frac{a}{4}$ ). From Observation 1, the optimal solution under no-communication is  $x = 1/2$ .

Let the expected utilities of the players in the communication and no-communication cases be denoted as follows:

$$EU_{com}^s(a, k, \tau) = \int_0^\tau -(x_1 - t)^2 dt + \int_\tau^1 -(2\tau - x_1 - t)^2 dt.$$

$$EU_{com}^r(a, k, \tau) = \int_0^\tau -(x_1 - k - at)^2 dt + \int_\tau^1 -(2\tau - x_1 - k - at)^2 dt.$$

$$EU_{nc}^s(a, k) = \int_0^1 -\left(k + \frac{a}{2} - t\right)^2 dt.$$

$$EU_{nc}^r(a, k) = \int_0^1 -a^2\left(\frac{1}{2} - t\right)^2 dt.$$

We first note:

*Claim.* In the symmetric case with  $a > 2$ , while the receiver is better off with communication, the sender is worse off.

*Proof.* Evaluating the receiver's and sender's expected utilities at  $\tau_3 = 1/2$  yields

$$EU_{com}^r\left(a, k^s(a), \frac{1}{2}\right) = -\frac{a^2}{96} > EU_{nc}^r(a, k^s(a)) = -\frac{a^2}{12}$$

and

$$EU_{com}^s\left(a, k^s(a), \frac{1}{2}\right) = -\frac{a^2}{16} + \frac{a}{8} - \frac{1}{12} < EU_{nc}^s(a, k^s(a)) = -\frac{1}{12}. \quad \text{Q.E.D.}$$

Given the continuity of  $p, q$ , and  $D$  in  $a$  and  $k$ , there exists a neighborhood  $\eta_1$  of  $(a, k^s(a))$  such that for all  $(a', k') \in \eta_1$ ,  $p < 0$  and  $D < 0$ . Therefore, the solutions in (A5) hold in  $\eta_1$ . Depending on slope  $a$ , the value of  $q$  could be positive or negative for a given  $(a', k') \in \eta_1$ .

*Case 1.* If  $q$  is weakly positive, then these solutions are continuous in  $(a, k)$  and in particular are continuous at  $(a, k^s(a))$ . Thus, there exist neighborhoods  $\eta_2$  and  $\eta_3$  of  $(a, k^s(a))$  such that  $y_1 < 0$  for all  $(a', k') \in \eta_2$  and  $y_2 > 1$  for all  $(a', k') \in \eta_3$ . Consequently, in  $\eta^* = \cup_{i=1}^3 \eta_i$ , the only admissible  $\tau$  is  $\tau_3$ .

*Case 2.* If  $q$  is negative, then  $\lim_{(a', k') \rightarrow (a, k^s(a))} y_1 = y_2((a', k') = (a, k^s(a))) = \sqrt{-p}$  and

$$\lim_{(a', k') \rightarrow (a, k^s(a))} y_2 = y_1((a', k') = (a, k^s(a))) = -\sqrt{-p}.$$

Thus, there exist neighborhoods  $\eta_5$  and  $\eta_6$  of  $(a, k^s(a))$  such that  $y_2 < 0$  for all  $(a', k') \in \eta_5$  and  $y_1 > 1$  for all  $(a', k') \in \eta_6$ . Consequently, in  $\eta^* = \cup_{i=1,5,6} \eta_i$ , the only admissible  $\tau$  is  $\tau_3$ .

The above analysis implies that there exists a neighborhood  $\eta^*$  of  $(a, k^s(a))$  in which the optimal  $\tau = \tau_3$  which is continuous in  $a$  and  $k$ . Thus,  $EU_{com}^r(a, k, \tau_3)$  and  $EU_{com}^s(a, k, \tau_3)$  are continuous in  $a$  and  $k$  for all  $(a', k') \in \eta^*$ . In light of the above Claim, it follows that for a generic subset of parameters for which communication is valuable, the sender is worse off with communication. *Q.E.D.*

*Proof of Observation 2.* Let  $X^r$  be the set of decisions induced in a communication-dependent equilibrium. Let  $x_1, x_2 \in X^r$ ,  $x_1 < x_2$ , and  $x^{-1}(x) \equiv \{t \in T | x \in \text{argmax}_{x' \in X^r} U^s(x', t)\}$ . Each type sends a report resulting in her preferred

decision  $x \in X^r$ . Note that, due to  $U_{i1}^1 < 0$  and  $U_{i2}^2 > 0$ , for any two given decisions, there is only one type who might be indifferent between them. Suppose  $t_1 < t_2 < t_3$  and  $t_1, t_3 \in x^{-1}(x_1)$ ,  $t_2 \in x^{-1}(x_2)$ ,  $t_2 \notin x^{-1}(x_1)$ , and without loss of generality  $t_1 \notin x^{-1}(x_2)$ . So  $t_2$  prefers  $x_2$  to  $x_1$ . By  $U_{i1}^1 < 0$ , there must exist a  $x_0$  between  $x_1$  and  $x_2$  such that  $U_{i1}^1(x_0, t_2) > 0$ . Since  $U_{i2}^2 > 0$ ,  $U_{i1}^1(x_0, t_3) > U_{i1}^1(x_0, t_2) > 0$ . But since  $U_{i1}^1 < 0$ , this implies that  $t_3$  prefers  $x_2$  to  $x_1$ , and thus sends a report resulting in  $x_2$ , in contradiction to the assumption. *Q.E.D.*

*Proof of Lemma 1.* For partition  $[0 = t_0, t_1, \dots, t = 1]$  to be an equilibrium, we need (1) the receiver's decision  $x(\cdot)$  to be a best response to the report  $[t_i, t_{i+1}]$ ,  $\forall i$ , and (2) the sender's report  $\hat{t}_{i+1} \equiv [t_i, t_{i+1}]$  to be a best response to  $x(\cdot)$ ,  $\forall i$ .

If the sender's report is  $\hat{t}_{i+1}$ , the receiver's best response is  $x(\hat{t}_{i+1}) = k + \frac{a}{2}(t_i + t_{i+1})$ . If the receiver decision rule is  $x(\hat{t}_{i+1}) = k + \frac{a}{2}(t_i + t_{i+1})$ , condition (5) requires that the sender  $t_i$  be indifferent between sending report  $\hat{t}_i$  and  $\hat{t}_{i+1}$ . The indifference condition for sender  $t_i$  is

$$-\left(k + \frac{a}{2}(t_{i-1} + t_i) - t_i\right)^2 = -\left(k + \frac{a}{2}(t_i + t_{i+1}) - t_i\right)^2, \quad \text{or} \quad (A7)$$

$$\frac{a}{2}t_{i+1} + (a - 2)t_i + \frac{a}{2}t_{i-1} = -2k. \quad (A7a)$$

Part (i). For  $a \in (0, 1) \cup (1, 2) \cup (2, \infty)$ , the indifference condition for sender  $t_i$  in (A7a) becomes

$$t_{i+1} + \frac{2a - 4}{a}t_i + t_{i-1} = -\frac{4k}{a}. \quad (A8)$$

This is a second-order linear difference equation with constant coefficients and constant term. Let  $b_1, b_2 = \frac{2 - a}{a} \pm \frac{2\sqrt{1 - a}}{a}$ , where  $b_1$  and  $b_2$  may be complex. The general solution to (A8) is  $t_{ic} = A_1b_1i + A_2b_2i$ . A particular solution to (A8) is  $t_{ip} = d$ , where  $d$  is a constant. Substituting into (A8), we solve for  $d$  as follows:  $d - d[2a - 4]/a + d = -4k/a$  implies  $d = \frac{k}{(1 - a)} \equiv A_3$ . So the solution to (A8) is

$$t_i = t_{ic} + t_{ip} = A_1b_1i + A_2b_2i + A_3.$$

Solve for  $A_1, A_2$ , and  $A_3$  using the boundary condition  $t_0 = 0 = A_1 + A_2 + A_3$  and  $t_1 = A_1b_1 + A_2b_2 + A_3$ . Tedious calculation yields

$$t_i = \frac{k}{1 - a}B^i + t_1C^i.$$

For an equilibrium of size  $N$ ,  $t_N = 1 = \frac{k}{1 - a}B^N + t_1C^N$ , so  $t_1 = \frac{1 - \frac{k}{1 - a}B^N}{C^N}$ .

Therefore,

$$t_i = \frac{k}{1 - a}B^i + \frac{1 - \frac{k}{1 - a}B^N}{C^N}C^i$$

$$B^i = \frac{b_2b_1^i - b_1^i + b_2^i - b_1b_2^i + b_1 - b_2}{b_1 - b_2}$$

$$C^i = \frac{b_1^i - b_2^i}{b_1 - b_2}. \quad Q.E.D.$$

Part (ii). See Crawford and Sobel's (1982) quadratic case on pages 1441-1442.

Part (iii). If  $a = 2$ , then (A7) becomes  $t_{i+1} + t_{i-1} = -2k$ . Thus, for equilibrium of size  $N$ , when  $i = 1$ , then  $t_2 = -2k$ , and when  $i = N - 1$ , then  $1 + t_{N-1} = -2k$ . So  $t_2 = 1 + t_{N-1}$ , or  $t_2 = 1$  and  $t_{N-2} = 0 \equiv t_0$ . The largest  $N$  for which this holds is  $N = 2$ , and it must be that  $k = -1/2$ . Q.E.D.

Proof of Proposition 6. Part (i).  $a \in (0, 2)$ . Assume that the sender adopts the specified partition. Then the receiver's best response is  $x(\hat{t}_1) = k + a \frac{a + 4k}{8 - 4a}$  and  $x(\hat{t}_2) = k + a \frac{4 - a + 4k}{8 - 4a}$ .

Given the receiver's best response, we need only check that the sender of environment  $t_1 = \frac{a + 4k}{4 - 2a}$  is indifferent between sending report  $\hat{t}_1$  and  $\hat{t}_2$ , i.e.,  $\left(k + a \frac{a + 4k}{8 - 4a} - \frac{a + 4k}{4 - 2a}\right)^2 = \left(k + a \frac{4 - a + 4k}{8 - 4a} - \frac{a + 4k}{4 - 2a}\right)^2$ , which is easily verified. Therefore, the partition forms an equilibrium.

The partition involves communication when  $0 < t_1 < 1$ , or  $k \in \left(-\frac{a}{4}, 1 - \frac{3a}{4}\right)$ . Value for communication is established because the receiver's expected utility in the equilibrium of size  $N = 2$  is higher than in the communication-independent equilibrium, as shown by the following positive difference:

$$\int_0^{t_1} -\left(k + a \frac{t_1}{2} - k - at\right)^2 dt + \int_{t_1}^1 -\left(k + a \frac{1+t_1}{2} - k - at\right)^2 dt - \int_0^1 -\left(k + \frac{a}{2} - k - at\right)^2 dt$$

$$= t_1(1 - t_1)\left(\frac{a}{2}\right)^2 > 0, \quad \forall a > 0. \quad (A9)$$

Part (ii).  $a > 2$ . The proof for Case 1 goes through intact, except the partition involves communication when  $0 < t_1 < 1$ , or  $k \in \left(1 - \frac{3a}{4}, -\frac{a}{4}\right)$ .

Part (iii). Assume to the contrary that  $a \leq 0$  and that there exists an equilibrium of size  $N > 1$  with partition  $[0 = t_0, t_1, \dots, t_N = 1]$ .

By condition (6), if the sender's report is  $[t_i, t_{i+1}]$ , the receiver's expected utility is  $\int_{t_i}^{t_{i+1}} - (x(\hat{t}_{i+1}) - k - at)^2 dt$ . Thus his best response is  $x(\hat{t}_{i+1}) = k + \frac{a}{2}(t_i + t_{i+1})$ . From (5), the dominance condition for sender  $t_i + \epsilon, \epsilon > 0$  is

$$-\left(k + \frac{a}{2}(t_{i-1} + t_i) - (t_i + \epsilon)\right)^2 \leq -\left(k + \frac{a}{2}(t_i + t_{i+1}) - (t_i + \epsilon)\right)^2, \quad \text{or}$$

$$\left(\frac{a}{2}t_{i+1} + (a-2)t_i + \frac{a}{2}t_{i-1} + 2k - 2\epsilon\right)\left(\frac{a}{2}(t_{i-1} - t_{i+1})\right) \geq 0.$$

Substituting in (A7a) yields  $-2\epsilon\left(\frac{a}{2}(t_{i-1} - t_{i+1})\right) \geq 0$ , which is satisfied only for  $a \geq 0$ , which is a contradiction. *Q.E.D.*

*Proof of Proposition 7.*

Part (i). The difference between the sender's expected utility in a communication-dependent equilibrium of size  $N$  and in the communication-independent equilibrium is

$$\sum_{i=1}^N \int_{t_{i-1}}^{t_i} -\left(k + \frac{a}{2}(t_{i-1} + t_i) - t\right)^2 dt - \int_0^1 -\left(k + \frac{a}{2} - t\right)^2 dt$$

$$= \frac{a}{2} \sum_{i=1}^N (1 - t_{i-1} - t_i) \left(2k + \frac{a}{2}(1 + t_{i-1} + t_i) - t_{i-1} - t_i\right) (t_i - t_{i-1}) = -\frac{a}{2} \left(\frac{a}{2} - 1\right) \sum_{i=1}^N t_{i-1}t_i(t_i - t_{i-1}). \quad (A10)$$

It is clear that  $\forall a \in (0, 2), \forall N$ , expression (A10) is positive, thus the sender is better off in any communication-dependent equilibrium.

Similarly, the difference between the receiver's expected utility in a communication-dependent equilibrium of size  $N$  and in the communication-independent equilibrium is

$$\sum_{i=1}^N \int_{t_{i-1}}^{t_i} -\left(\frac{a}{2}(t_{i-1} + t_i) - at\right)^2 dt - \int_0^1 -\left(\frac{a}{2} - at\right)^2 dt = \frac{a^2}{4} \sum_{i=1}^N (1 - t_{i-1} - t_i)(1 - t_{i-1} - t_i)(t_i - t_{i-1})$$

$$= \frac{a^2}{4} \sum_{i=1}^N t_{i-1}t_i(t_i - t_{i-1}). \quad (A11)$$

Again,  $\forall a \in (0, 2), \forall N$ , (A11) is positive, thus the receiver is better off in any communication-dependent equilibrium.

Part (ii). This follows because, for all  $a > 2$ , (A10) is negative and (A11) is positive. *Q.E.D.*

*Proof of Proposition 8.* Observe that, by the monotonicity of the sender's first-best decision line, if  $t$  (weakly) prefers a decision  $y$  to  $k + \frac{a}{2}$  where  $y > (<) k + \frac{a}{2}$ , then so will all  $t' \in (t, 1] (t' \in [0, t))$ .

"Only if" part. We show that, for  $a < 2$ , there is a credible neologism in the communication-independent equilibrium.

Consider the neologism  $T = (t^*, 1]$ , where  $t^* > k + \frac{a}{2}$ . By the above observation,  $T$  is credible if the sender of type  $t^*$  is indifferent between using  $T$  and using the communication-independent equilibrium report.  $T$  then would induce the receiver to choose the decision  $y^* = k + a\left(\frac{t^* + 1}{2}\right)$ ; recall also that the communication-independent equilibrium decision is  $y = k + \frac{a}{2}$ . The sender's indifference condition is  $k + a\left(\frac{t^* + 1}{2}\right) - t^* = t^* - \left(k + \frac{a}{2}\right)$  or

$t^* = \frac{2(2k+a)}{4-a}$ . For the neologism to be feasible, we should have  $k + \frac{a}{2} < t^* < 1$ . The first inequality is trivially met, and the second amounts to  $k < 1 - \frac{3a}{4}$ , which is met by condition (i) of Proposition 6. Therefore, for  $a < 2$ , the communication-independent equilibrium is not neologism-proof.

"If" part. We show that, for  $a > 2$ , there is no credible neologism in the communication-independent equilibrium. Assume to the contrary that there exists a credible neologism. As argued above, if a credible neologism involves a type  $t > (<) k + \frac{a}{2}$ , then it must involve all types larger (smaller) than  $t$ . It is also necessary that there exists a sender  $t^*$ , where  $t^* < t$  ( $t^* > t$ ), who is indifferent between using the neologism  $T = (t^*, 1]$  ( $T = [0, t^*)$ ) and using the communication-independent equilibrium report. As above, this indifference condition implies  $t^* = \frac{2(2k+a)}{4-a} \left( t^* = \frac{4k-a}{3} \right)$ . For the neologism to be feasible, a necessary condition is  $t^* < 1$  ( $t^* > 0$ ). This amounts to  $k < 1 - \frac{3a}{4} \left( k > -\frac{a}{4} \right)$ , which is in contradiction with required conditions of Proposition 6(ii).

Therefore, for  $a > 2$ , the communication-independent equilibrium is neologism-proof. *Q.E.D.*

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