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Optimal Cross Holding with Externalities and Strategic Interactions*

I. Introduction

Cross holding, defined as one firm having an equity position (long or short) in another firm, arises in various industries.¹ Northwest Airlines, for example, acquired a 14% equity position in Continental Airlines on January 27, 1998, with a payment of \$500 million in cash and stock.² Microsoft announced on August 6, 1997, the purchase of \$150 million of nonvoting preferred stock of Apple Computer, which Microsoft later converted into common equity. In another instance, Microsoft held a 10% position in RealNetworks in July 1997, which it sold off due to product incompatibility.³ In the telecommunications industry, Telefonica de Espana holds a 3.5% stake in Portugal Telecom,

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1. Throughout the paper, when we refer to a firm holding an equity position, this equity position is assumed to be in the other firm affected by the externality.

2. Northwest Airlines also relinquished the voting right associated with its equity position.

3. See *Wall Street Journal*, November 19, 1998. Microsoft also purchased a 5% stake in VDO Net Corp. (see *Wall Street Journal*, August 19, 1997).

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We analyze a two period setting where firms first choose equity positions in each other and second engage in operating activities that cause externalities. Firms facing positive externalities optimally choose long equity positions to increase their profits. Firms facing negative externalities encounter a prisoners' dilemma, where each firm optimally chooses short positions in the first period, committing to a more aggressive operating stance that results in lower profits. In contrast to the prior literature, regulation restricting cross holdings reduces consumer surplus and economic welfare when the number of firms is fixed. However, such regulation can increase entry, improving net welfare.

which in turn, holds the option on a 5% stake in a unit of Telefonica de Espana.⁴ This paper analyzes the role of cross holding among publicly traded firms that generate externalities.

We initially examine cross holding in nonvoting stock, assuming that firms maximize equity value. We proceed to explore how voting and control rights affect optimal cross-holding decisions. We analyze a setting where two firms first choose their level of cross holding, then make an operational decision that generates an externality upon the other firm. This externality can be pecuniary or nonpecuniary, with positive or negative effect. Consider the case of positive externalities: we find that long (short) positions induce value-maximizing firms to increase (decrease) the operational decision causing the positive externality. With a short position, the firm can make additional profits by decreasing the value of the corresponding firm. This occurs if the firm decreases its operational decision that has a positive externality. In the case of negative externalities, the opposite results obtain.

The optimal cross holding depends on the effect of the externality on the marginal profitability of each firm. Assuming that a positive externality increases the marginal profitability associated with operational decisions, we find that firms which trigger positive externalities optimally hold long positions. Thus, cross holding among firms internalizes the externality and allows firms to increase their profits. When operational decisions trigger negative externalities that also decrease marginal profitability, a Prisoner's Dilemma type problem arises: firms choose short positions to commit to a more aggressive operational decision, which in turn decreases profits relative to the absence of cross holdings.

Such externalities arise when firms produce related goods or services in an imperfectly competitive market. As an illustration, consider a sequential game in which two managers first choose cross holdings and subsequently compete in a Cournot duopoly. If the products are complements, they generate positive externalities; if the products are substitutes, they generate negative externalities. We find that, when products are complements and cross holding is allowed, the product market equilibrium exhibits higher quantity, lower price, higher profits, and higher consumer surplus than in the absence of cross holdings. Production increases because cross holdings cause firms to internalize the positive externality resulting from an increase in output on the competitor's profitability. When products are substitutes, the product market equilibrium exhibits higher quantity, lower price, lower profits, and higher consumer surplus than in the absence of cross holding. Moreover, the interaction between cross holding and product markets increases economic welfare.⁵

4. See Nairn (1998) and *Wall Street Journal*, March 3, 1998.

5. Ignoring the effect on entry deterrence, our results should alleviate concerns raised by the European Union Commission regarding cross holdings in the airlines industry and concerns raised by the Department of Justice regarding Microsoft's investment strategy, provided Microsoft's products complement Apple's.

We demonstrate that optimal cross holding, when the firms generate positive externalities, entails long equity positions, a finding consistent with the prevalence of cross holdings among airlines. The airlines' products are often complements,⁶ and although the industry competes on price, the necessity of a considerable capital investment (in aircraft) suggests that the Cournot outcome is appropriate (see Kreps and Scheinkman 1983). This implies that the product market decisions are complements and thus, if one airline increases price, this produces a positive externality on other airlines.

To investigate the robustness of these results we consider the scenario where one firm can incur a fixed cost to enter the monopoly market of another firm. Our finding that prohibiting cross holding decreases the likelihood of entry, firm profits, and consumer surplus when products are complements, renders the previous results robust in the case of complements. Moreover, our finding that prohibiting cross holding encourages entry when the products are substitutes suggests that the effect on consumer surplus of prohibiting cross holdings depends on the level of entry costs.

A substantial literature analyzes cross holdings and imperfect competition when products are substitutes. Reynolds and Snapp (1986) and Farrell and Shapiro (1990) compare the product market equilibrium in the absence of cross holdings to the simultaneous deviation by both firms to positive cross holding. Their findings suggest that, when firms take long equity positions with substitute products, the resulting product market equilibrium is characterized by less quantity, higher prices and profits for both firms and lower consumer surplus than when there is no cross holding. We show, however, that this is not a Nash equilibrium in cross holding choice. In particular, a firm can increase its profits by unilaterally deviating from positive cross holding to no cross holding.

Farrell and Shapiro (1990) explore when a firm might rationally increase its initial exogenous equity position in a rival. Although operating profits decline for any firm that increases its equity position, the firm makes a profit on the initial cross holding in the rival if the rival firm's profitability increases. Thus, a firm increases its equity position if the gain on the initial position in the rival's stock outweighs its own lost operating profits. Flath (1991) demonstrates that, if two firms start without cross holding, neither firm will initiate long equity positions in the setting considered by Farrell and Shapiro. The conditions under which a firm increases its cross holding (in Farrell and Shapiro) are thus driven entirely by the assumed initial cross holding position. Flath (1991) also characterizes optimal cross holding under Bertrand competition.

Hansen and Lott (1995) demonstrate that cross holding can affect the entry decision. They analyze equity positions taken prior to entry and

6. For example, Northwest and Continental overlap on only eight routes worldwide.

demonstrate that a firm considering entry can profit from trading in an incumbent firm's stock because the prospective entrant has private information about its own entry decision. They assume that, if the entrant does enter the product market, the stock price of the incumbent falls and otherwise it rises. The potential entrant earns profits by taking a short position in the stock prior to entering the product market and otherwise taking a long position. Thus, their rationale for trading in equity is to profit from private information about the incumbent's future stock price. In contrast, our model presumes that equity transactions occur in an efficient, full-information, securities market such that neither profit nor loss is realized from equity trading. The motive to cross hold in our model is based exclusively on how cross holdings affect the firms' subsequent decisions.

Hansen and Lott (1996) investigate the proper objective of a firm when operational decisions generate externalities and demonstrate that maximizing firm value need not be in the best interest of shareholders. Instead, fully diversified shareholders prefer joint value maximization. However, if some shareholders are not diversified, this leads to a conflict among the shareholders as to the preferred operating activities. Nonetheless, if managers are compensated with stock or stock options, their incentives lead them to maximize equity value over shareholder wealth. We show that cross holding leads the firm to internalize these externalities and can allow value maximization to become more aligned with the intent of diversified shareholders.

We add to the existing literature in several ways. First, we determine the optimal cross-holding positions endogenously. In our model, the rationale for cross holding is its strategic effect on the operational decisions. We show that, for imperfect product market competition, long positions arise when products are complements and short positions when products are substitutes. Second, we demonstrate that these cross-holding positions increase consumer surplus in both cases. Third, we show conditions under which our results on cross holding apply to any externality created between firms, not just imperfect product markets. The key factor regarding the optimal cross holding is the effect of the externality on the marginal profits of the other firm.

In Section II, we introduce the general model and characterize the incentives for cross holding given no initial equity position in a rival. Section III presents closed-form solutions for firm profitability, consumer surplus, and economic welfare when the demand curve is linear. In Section IV, we endogenize the market structure by considering the effect of restrictions on cross holding on the likelihood of entry. Section V discusses the impact of control rights and shareholder value maximization (as opposed to equity value maximization) on our results. Our findings are summarized in Section VI. All proofs are in the appendix.

II. The Model

Two all-equity firms, denoted by i and k , each is assumed to make operational decisions that impose externalities. Prior to these operational decisions the firms simultaneously choose equity positions (long or short). At each stage, firm managers make decisions that maximize the equity value of their firms.⁷ We assume that firms have no debt and ignore any moral hazard problem that might arise from a shareholder-manager conflict. Maximizing equity value is not necessarily commensurate with maximizing shareholder value in this situation. This is because the other firm may be a shareholder. Due to the externality, maximizing the other firm's value (i.e., that particular shareholder's value) may mean selecting an action that does not maximize the firm's own equity value. In addition, maximizing equity value need not coincide with maximizing shareholder value when shareholders are diversified between the two companies, see Hansen and Lott (1996). We assume that firm managers ignore these concerns and maximize equity value.⁸ In Section V we analyze the effects of maximizing shareholder value and the conflicts that arise. We assume that firm managers observe the cross holding prior to the second stage decision and we solve for the subgame perfect equilibrium. The model has no underlying uncertainty. Firm profits are based solely on the decision of each firm.

A. Operating Profit and Assumptions

In the first stage, each firm chooses an equity position. Let α^i (α^k) denote the equity position of firm i (k). Initially, all assumptions are symmetric and we perform analyses only with respect to firm i . The equity positions are publicly disclosed and the firms' shares trade in a competitive, full-information capital market. We assume that investors ignore discounting. These assumptions ensure that the terminal value of the equity positions equals the initial acquisition price of these equity positions during the first stage; that is, neither profit nor loss is associated with transactions in equity. Abstracting from the control rights of long equity positions, our initial analyses apply to silent interests (see Section V for issues relating to control rights). These assumptions allow us to accentuate the strategic effect of equity positions on operational decisions; that is, our model examines equity positions exclusively for their effect on operational decisions that induce externalities. In the second stage, the firms simultaneously make operational decisions, represented by x^i and x^k , that induce externalities. At the end of the second stage, profits are realized.

7. Although the model considers only two firms, our results generalize quite naturally to industries with many firms, provided that each has some degree of market power.

8. In addition to the commonly stated goal of equity value maximization, we believe this is an important case, given the large amount of incentive compensation managers receive, which is normally tied to stock price.

Operating profits, $\pi^i(x^i, x^k)$ are assumed to be jointly determined by x^i and x^k .

Each firm's strategy comprises a first stage equity position and a second stage operational decision that depends on both firms' equity positions chosen during the previous stage. We consider the subgame perfect equilibrium to ensure that both firms' decisions regarding first stage equity position correctly anticipate the rival's simultaneous choice of equity position and the induced effect on the second stage operational decisions. Equilibrium strategies are determined using backward induction; thus, we first establish the equilibrium operational decisions for any possible level of cross holding, then solve for equilibrium cross holding.

B. Operational Decision Equilibrium

The second stage operational decisions are chosen simultaneously, taking the equity positions of both firms, α^i and α^k , as given. Each firm makes a decision that maximizes its firm value. The total value of firm i is based on the sum of its operating profits and the α^i -fraction of the operating profits of firm k . Firm i makes its operational decision to maximize

$$V^i = \pi^i + \alpha^i \pi^k, \quad (1)$$

where V^i represents the total value of firm i .⁹ The first-order condition that must be satisfied for firm i is

$$V_i^i = \pi_i^i + \alpha^i \pi_i^k = \frac{\partial \pi^i(x^i, x^k)}{\partial x^i} + \alpha^i \frac{\partial \pi^k(x^i, x^k)}{\partial x^i} = 0, \quad (2)$$

where the subscripts i represent derivatives with respect to x^i . The second-order condition that must be satisfied to ensure an interior maximum is $V_{ii}^i < 0$. The following assumption suffices to ensure the existence and stability of the Nash equilibrium:¹⁰

$$V_{kk}^k V_{ii}^i - V_{ik}^i V_{ki}^k > 0, \quad (3)$$

where the subscript ik represents the derivatives with respect to both x^i and x^k .

9. Although the sum of firm values exceeds the sum of the industry's operating profits, this objective function is appropriate under cross holdings, as noted by Breshnahan and Salop (1986), Reynolds and Snapp (1986), Farrell and Shapiro (1990), and Flath (1991), among others.

10. This is a common assumption for these models (Brander and Lewis 1986). Intuitively, this assumes that firm value is affected more strongly by firm-initiated than by competition-induced changes in quantity.

THEOREM 1. If the firms generate positive externalities, that is, $\partial \pi^k / \partial x^i > 0$, then an increase in α^i leads to an increase in x^i . If the firms generate negative externalities, that is, if $\partial \pi^k / \partial x^i < 0$, then an increase in α^i leads to a decrease in x^i .

THEOREM 2. If the operational decisions of a firm without cross holding generate positive externalities, an increase in α^i causes an increase in x^k if and only if $[\pi_{ki}^i + \alpha^i \pi_{ki}^k] > 0$; if the operational decisions generate negative externalities, an increase in α^i causes an increase in x^k if and only if $[\pi_{ki}^i + \alpha^i \pi_{ki}^k] < 0$.

Theorem 1 states the effect on equilibrium operational decisions of a change in the equity position of one firm. The impact on the operational decisions of a change in cross holding depends on the type of externality generated. When increasing its cross holding, α^i , firm i increases its emphasis on the profits of firm k when making operational decisions. If the firm generates positive externalities, firm i wants to increase its operational decision, x^i , which has a positive effect on the profits of firm k . Firm k then increases or decreases its operational decision, depending on the sign of $[\pi_{ki}^i + \alpha^i \pi_{ki}^k]$. If $\alpha^i > 0$, then $[\pi_{ki}^i + \alpha^i \pi_{ki}^k]$ is positive and an increase in α^i leads to an increase in x^k .

If the operational decisions cause negative externalities, an increase in α^i leads firm i to decrease x^i , which has a positive effect on the profits of firm k . Again, the reaction of firm k to an increase in α^i depends on the sign of $[\pi_{ki}^i + \alpha^i \pi_{ki}^k]$. If $[\pi_{ki}^i + \alpha^i \pi_{ki}^k]$ is negative, an increase in α^i leads firm k to increase x^k . Note that this is true when α^i is close to zero.¹¹

C. Equity Market Equilibrium

Prior to the operational decisions, each firm establishes an equity position. The equity positions are chosen simultaneously. These positions are announced and procured in a competitive capital market. Each firm's choice of equity position correctly anticipates its rival's choice and reflects full knowledge of how the chosen equity positions affect each firm's operational decision. At this stage, total firm value is the sum of the firm's operating profit and the net profit from acquiring the fraction, α , of the other firm's equity. Put formally,

$$V^i = \pi^i + (\alpha^i \pi^k - \text{cost of firm } i\text{'s equity position in firm } k). \quad (4)$$

Recall that, there being no uncertainty and no discounting and investors having rational expectations, the cost of acquiring the equity position equals the second-stage payoff from the equity position.¹² Thus, in the first stage, the firm chooses its equity position to maximize operating profits:

$$V^i = \pi^i(x^i, x^k). \quad (5)$$

11. These conditions hold globally in our special case in Section III.

12. See Grossman and Hart (1980).

The first-order condition for firm i is

$$\frac{\partial V^i}{\partial \alpha^i} = \frac{\partial \pi^i}{\partial x^i} \frac{\partial x^i}{\partial \alpha^i} + \frac{\partial \pi^i}{\partial x^k} \frac{\partial x^k}{\partial \alpha^i} = 0. \quad (6)$$

In general, we cannot determine the extent of cross holding (long or short) from this first-order condition. We can show, however, that when firms' operational decisions generate externalities (positive or negative), the absence of cross holding on the part of either firm is not an equilibrium.

THEOREM 3. Given no cross holding, each firm manager has an incentive to take a long (short) position if the operational decisions generate a positive (negative) externality and the externality also increases (decreases) the marginal profitability of the operational decision, that is, for positive externalities $\partial^2 \pi^i(x^i, x^k) / \partial x^i \partial x^k > 0$ and for negative externalities $\partial^2 \pi^i(x^i, x^k) / \partial x^i \partial x^k < 0$.

Since previous explanations rely on information-based trading (Diamond and Verrecchia 1987; Committee on Government Operations 1991; Hansen and Lott 1995), this suggests a new rationale for short selling.

III. Example of Externality: Product Market Competition

In this section, we derive the closed-form solution to the model assuming the externality is created because firms compete in an imperfect product market with constant marginal cost, C , and the following inverse demand function for firm i :

$$p^i = A - q^i - Bq^k,$$

where p^i is the unit price for firm i , q^i is the quantity produced by firm i , and A and B are constants. The constant A captures the general level of market demand; B characterizes the relation between the firms' products. We assume that a change in its own output has a greater effect on a firm's price than a change in the competing firm's output; that is, $|B| < 1$.

A. Product Market Equilibrium

From the first-order condition, equation (2), we derive the following reaction function for firm i :

$$q^i = \frac{(A - C)}{2} - \frac{(1 + \alpha^i)Bq^k}{2}.$$

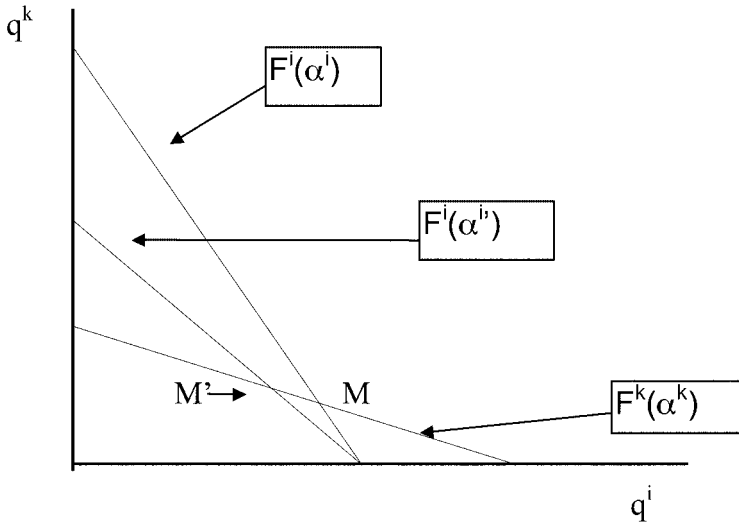


FIG. 1.—The output market reaction functions for each firm when the products are substitutes, $B > 0$. $F^k(\alpha^k)$ is the reaction of firm k , for an arbitrary amount of cross holding, α^k . Likewise, $F^i(\alpha^i)$ is the reaction of firm i , given cross holding of α^i . This graph shows how the reaction of firm i changes when it increases its cross holding from α^i to $\alpha^{i'}$. When firm i increases its cross holding, it increases the weight it puts on its competitor's profits when choosing its own quantity. This causes it to compete less aggressively and moves the output equilibrium from M to M' , which results in lower output from firm i and higher output from firm k .

Solving the two corresponding reaction functions simultaneously yields the equilibrium quantity for any previously chosen equity positions:

$$q^i = \frac{(A - C)[2 - (1 + \alpha^i)B]}{[4 - (1 + \alpha^i)(1 + \alpha^k)B^2]} \tag{7}$$

The second order condition for an optimum holds.

COROLLARY 1. If the products are substitutes, an increase in α^i causes a decrease in q^i and an increase in q^k . If the products are complements, an increase in α^i causes increases in q^i and q^k .

Corollary 1 illustrates Theorems 1 and 2. When products are substitutes ($B > 0$), an increase in the equity position of firm i induces firm i to decrease quantity, which has a positive effect on the profits of firm k (see figure 1). Likewise, when products are complements ($B < 0$), an increase in the equity position of firm i induces firm i to increase quantity, which also has a positive effect on the profits of firm k (see figure 2). In both cases, added profitability induces firm k to increase its quantity.

B. Equity Market Equilibrium

In the first stage, each firm manager chooses the equity position that maximizes operating profits, anticipating correctly that, in the second

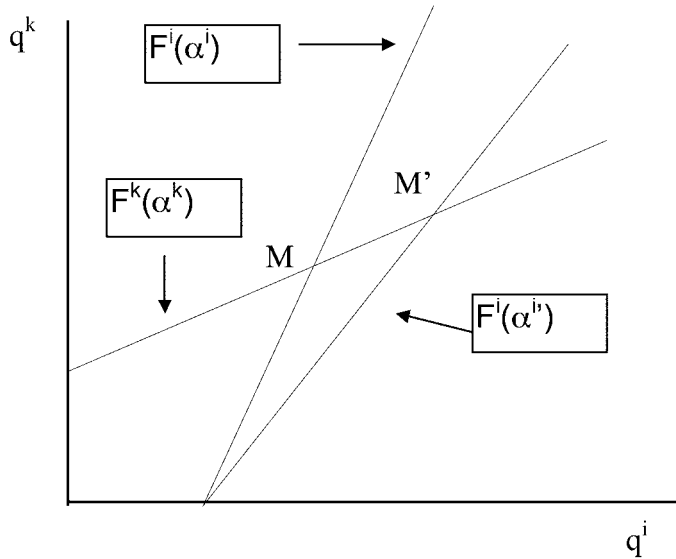


FIG. 2.—The output market reaction functions for each firm when the products are complements, $B < 0$. This graph shows how the reaction of firm i , $F^i(\alpha^i)$, changes when it increases its cross holding from α^i to $\alpha^{i'}$. When firm i increases its cross holding, it increases the weight it puts on the competitor's profits when choosing its own quantity. When the firms' products are complements, this causes the firm to increase its production, which has a positive effect on the rival firm's profits. The equilibrium moves from M to M' , which results in higher output from both firms.

stage, the quantities each firm chooses will satisfy equation (7). Substituting equation (7) and the corresponding quantity for firm k into equation (5), taking the first-order condition, and solving yields the following reaction function for equity position:

$$\alpha^i = \frac{(1 + \alpha^k)(2 - B)B}{(-4 + 2B + B^2) + \alpha^k(2B + B^2)}. \quad (8)$$

Solving the corresponding reaction functions simultaneously yields the equilibrium equity position for each firm.¹³

THEOREM 4. The equilibrium amount of cross holding is $\alpha^i = -B/(2 + B)$ and each firm produces $q^i = (A - C)[2 + B]/4(1 + B)$. Further, $dq^i/dB < 0$, $dp^i/dB < 0$, and $d\pi^i/dB < 0$.

Recalling that $|B| < 1$, it follows immediately from Theorem 4 that the equilibrium level of cross holding consists of a long equity position if the products are complements ($B < 0$) and a short position if the products are

13. This results in a second-order polynomial with two roots, the second, $\alpha^i = (2/B) - 1$, being a minimum.

substitutes ($B > 0$). This result is in contrast to Flath (1991), who precludes short selling and therefore finds, under Cournot competition, no strategic role for cross holding when products are substitutes. Note that the quantity increases and prices and profits decrease as the degree of substitution increases.

We next examine the welfare implications associated with equilibrium cross holding in the setting of imperfect product markets.

THEOREM 5. Firm profits are lower when cross holding is allowed if and only if the products are substitutes. Further, consumer surplus and economic welfare are higher when cross holding is allowed than when it is prohibited.

This theorem indicates that, in our setting, regulatory restrictions on cross holding always decrease consumer surplus and economic welfare. This result is in stark contrast to the common view that limiting cross holding increases consumer surplus. The difference arises because past literature focused on out-of-equilibrium changes in exogenous cross holdings. Farrell and Shapiro (1990) identified conditions for exogenous cross holdings to increase consumer surplus. In contrast, we focus on cross holdings that arise in equilibrium when equity positions are strategic variables chosen optimally by firm managers.

IV. Cross Holding and Entry

In this section, we consider an alternative scenario, where the strategic effect of cross holding affects entry. Hansen and Lott (1995) also examine how cross holding affects entry. They assume that, if there is no entry, the incumbent earns monopoly profits. If there is entry, they assume the incumbent earns lower profits. This incumbent profit level, however, is assumed to change only based on entry or no entry and is unaffected by the extent of cross holding. In contrast, we explicitly model the strategic role of cross holding and how that affects entry. In our scenario, the level of cross holding directly affects the actions taken in the product market, which in turn determine the profits of each firm.

Let firm i be the market incumbent and firm k have the ability to enter the market by incurring a fixed cost, E . For simplicity, we ignore the possibility of other entrants. This augments the previous model by allowing firm k to decide whether to enter. If firm k enters the two stages proceed as in Section III; otherwise, firm k earns zero profits and firm i monopolizes the product market.

Note first that, in the absence of entry, cross holding does not affect firm profits. Second, consider the entry decision when cross holding is prohibited. Firm k , upon entry, receives total profits of $\{(A - C)^2/[2 + B]^2 - E\}$. Entry thus occurs whenever $E < (A - C)^2/[2 + B]^2$. Third, consider the entry decision when subsequent equilibrium cross holding is fully anticipated by the entrant. The equity market and product market

equilibrium that results if firm k enters is characterized in Theorem 4. Firm k , upon entry, receives total profits of $\{(A - C)^2[4 - B^2]/16(1 + B) - E\}$. Consequently, firm k will enter the market if $E < (A - C)^2[4 - B^2]/16(1 + B)$. This leads to the following theorem.

THEOREM 6. Prohibiting cross holdings deters entry when the products are complements and promotes entry when products are substitutes.

The intuition of Theorem 6 is straightforward. Given entry, allowing cross holding increases firm profits if the products are complements. Entry is thus more likely when cross holding is allowed than when cross holding is prohibited. Further, since cross holding increases both the probability of entry and consumer surplus conditional on entry, it follows that, if the products are complements, consumer surplus and economic welfare are higher when cross holding is allowed than when cross holding is prohibited. In contrast, if the products are substitutes, each firm earns lower product market profits when cross holding is allowed making entry less likely than when cross holding is prohibited. Consumer surplus, however, is higher conditional on entry when cross holding is allowed. The overall effect of allowing cross holding on consumer surplus is thus ambiguous. Consider the case of perfect substitutes. For sufficiently small entry costs, $E < (A - C)^2[4 + B^2]/16(1 + B)$, entry occurs regardless of whether cross holding is allowed, consequently cross holding increases consumer surplus and economic welfare. In contrast, allowing cross holding deters entry for intermediate levels of entry costs, $(A - C)^2[4 + B^2]/16(1 + B) < E < (A - C)^2/(2 + B)^2$, thereby decreasing consumer surplus and economic welfare. Hence, we demonstrated that the rationale for prohibiting cross holding should incorporate the effect of cross holding on entry in addition to the effect of cross holding on equilibrium product market decisions.

V. Cross Holding, Control Rights, and Shareholder Value vs. Firm Value

Thus far, we assumed that managers strive to maximize equity value and not necessarily shareholder value. Equity value and shareholder value coincide when no shareholders have any ownership in the rival firm.¹⁴ Of course, as soon as long cross holding positions exist, this is not the case. In addition, if individual shareholders are diversified and hold direct claims on both firms, they may prefer a strategy of joint profit maximization. In this case, the shareholders benefit if one firm makes decisions that decrease its equity value if the result is a larger gain in the rival's equity value. In either case, we must make assumptions about the equity positions of the shareholders and the mechanism through which conflicts between groups of shareholders are resolved to fully investigate how control rights influence optimal decisions and firm value.

14. It is also necessary that the equity holders not be consumers of the firms' products.

A. Fully Diversified Individual Shareholders

When fully diversified across the two related companies, shareholders prefer joint profit maximization regardless of the level of cross holding between the firms. Under this assumption the shareholders desire strategies from each firm that satisfies the condition

$$\frac{\partial \pi^i(x^i, x^k)}{\partial x^i} + \frac{\partial \pi^k(x^i, x^k)}{\partial x^i} = 0. \tag{9}$$

In this case, the firms choose their operational decisions to satisfy equation (9) and maximize joint profits regardless of any cross holding positions. Thus, each firm internalizes the externality (positive or negative) due to its shareholders' interests in joint profit maximization. Cross holding lends no additional strategic effect.

B. Nondiversified Individual Shareholders

We next consider the case where individual shareholders are not diversified and assume that all individual shareholders hold stock in either firm i or firm k but not both. If the firms have no cross holdings, then all shareholders are individual investors who have no stake in the rival firm. Thus, maximizing equity value (as in Section II) is the same as maximizing shareholder value. The preceding argument also applies when the firms hold short positions, since again all shareholders are individual investors. If, however, both firms hold long positions, then there are two sets of shareholders with divergent preferences regarding the strategic decisions. The individual investors still want firm i to maximize its equity value, as previously, while the rival, firm k , which is also a shareholder in firm i , wants firm i to choose the operational decision that maximizes the equity value of firm k .¹⁵

Assuming that firms take long equity positions α^i and α^k , the individual shareholder still want the firm to maximize equity value. For firm i , the shareholders want the strategic decision (x^i) to be chosen to maximize equity value as outlined in equation (2), Section II.B:

$$V_i^i = \pi_i^i + \alpha^i \pi_i^k = \frac{\partial \pi^i(x^i, x^k)}{\partial x^i} + \alpha^i \frac{\partial \pi^k(x^i, x^k)}{\partial x^i} = 0.$$

Firm k is also a shareholder in firm i and wants firm i to make a strategic decision, x^i , that maximizes the equity value of firm k :

$$V^k = \pi^k + \alpha^k \pi^i, \tag{10}$$

$$V_i^k = \pi_i^k + \alpha^k \pi_i^i = \frac{\partial \pi^k(x^i, x^k)}{\partial x^i} + \alpha^k \frac{\partial \pi^i(x^i, x^k)}{\partial x^i} = 0. \tag{11}$$

15. For example, firm i is now a shareholder in firm k . Firm i wants firm k to pick an operational decision, x^k , that maximizes firm i 's value, not firm k 's value.

Suppose x^{i*} satisfies equation (2) and x^{i**} satisfies equation (11). In general, when there are externalities, x^{i*} and x^{i**} need not coincide. A formal voting model should now be applied to determine the optimal strategic decisions. While we do not develop a formal model in this paper, reasonable voting models would generate some strategic decision between these two desired choices.¹⁶

Consider the effect on optimal cross holding. First, note that this situation occurs only when the optimal cross holding position involve long positions. Recall that long positions arise optimally only when the strategic decisions produce positive externalities. This increases the benefit from cross holding over the previous result. The firm's decision, characterized by equation (2), arises if the cross-holding firm has no direct influence on the other firm's decision. This, in itself, is beneficial and results in positive cross holding. In this setting, the firm exerts influence over the other firm's decision and can move it from (2) toward (11), which increases its propensity for cross holding. Therefore, we would expect larger cross holding in this situation.

Under these conditions, the direction (long or short) of the optimal cross holding remains the same. When short positions are optimal (that is, when the operational decisions induce negative externalities), there is no change in cross holding. However, when long positions are optimal (when the operational decisions induce positive externalities), we expect to see larger cross-holding positions.

This shows that, if firms have nondiversified shareholders, the propensity for firms to cross hold is similar to the original model. Only in the case where the shareholders are diversified and management chooses to maximize shareholder value instead of firm value does cross holding become irrelevant in this model. However, as long as management has incentives to maximize firm value or some shareholders are not fully diversified then cross holding is relevant.¹⁷

VI. Conclusion

This paper analyzes the incentives and consequences of cross holding. The common argument that allowing cross holding reduces consumer surplus is based on models that examine exogenous cross holding with substitute products. We demonstrate that, when cross holding is a strategic decision by firms, allowing equilibrium cross holding enhances consumer surplus.

We add to the literature in other ways. First, we analyze cross holding for companies that produce both complementary and substitute products.

16. For example, the support maximization approach by Peltzman (1976).

17. If management's performance reviews are based on firm value or if management compensation includes stock or stock options (all of which is common), we would expect management decision to be based on maximizing firm value.

Second, we analyze both short and long equity positions. Third, we fully endogenize the cross-holding choice and analyze equilibrium cross holding conditional on the type of competition. This analysis reveals a new role for short selling, namely, to commit a firm to an aggressive product market stance. Fourth, we analyze how equilibrium cross holding affects consumer surplus and economic welfare. We find that, when firms engage in equilibrium cross holding, economic welfare improves relative to the benchmark of no cross holdings, although firm profits decrease when the products are substitutes.

Finally, we investigate the effect of cross holding on entry. We demonstrate that our results are robust to endogenizing the industry structure when products are complements. When products are substitutes, however, cross holding decreases entry, giving rise to an ambiguous effect on consumer surplus. These results yield new insights into whether cross holding can be regulated for the benefit of consumers.

Appendix

Proof of Theorems 1 and 2

Totally differentiating equation (2) and the corresponding first-order condition for firm k with respect to x^i , x^k , and α^i gives the following equations:

$$\begin{aligned} V_{ii}^i dx^i + V_{ik}^i dx^k + V_{i\alpha(i)}^i d\alpha^i &= 0, \\ V_{ki}^k dx^i + V_{kk}^k dx^k + V_{k\alpha(i)}^k d\alpha^i &= 0. \end{aligned}$$

Since $V_{k\alpha(i)}^k = 0$, the second equation simplifies to

$$V_{ki}^k dx^i + V_{kk}^k dx^k = 0.$$

Solving these equations simultaneously yields

$$dx^i/d\alpha^i = -V_{kk}^k V_{i\alpha}^i/H, \tag{A.1}$$

$$dx^k/d\alpha^i = V_{ki}^i V_{i\alpha}^i/H, \tag{A.2}$$

where $H = V_{kk}^k V_{ii}^i - V_{ik}^i V_{ki}^k$. Note that $H > 0$ by assumption and $V_{kk}^k < 0$ by the second-order condition.

Let us examine the situation when there are positive externalities. First, we determine the sign of (A.1). Since $H > 0$ by assumption and $V_{kk}^k < 0$ by the second-order condition, the sign of (A.1) is the same as the sign of $V_{i\alpha}^i$. However, $V_{i\alpha}^i = \pi_i^k > 0$ by assumption of positive externalities. Thus, $(A.1) = dx^i/d\alpha^i > 0$. Second, we determine the sign of (A.2). Again, $H > 0$ by assumption and we now know that $V_{i\alpha}^i > 0$, therefore, the sign of (A.2) is determined by the sign of V_{ki}^i :

$$V_{ki}^i = \pi_{ki}^i + \alpha^i \pi_{ki}^k. \tag{A.3}$$

Thus, if $\pi_{ki}^i + \alpha^i \pi_{ki}^k > 0$, then $dx^k/d\alpha^i > 0$; and if $\pi_{ki}^i + \alpha^i \pi_{ki}^k < 0$, then $dx^k/d\alpha^i < 0$.

We next consider the situation when the operational decisions induce negative externalities. First, we determine the sign of (A.1). As before, $H > 0$ and $V_{kk}^k < 0$, but $V_{i\alpha}^i = \pi_i^k$ becomes negative. The sign of (A.1) therefore is negative. Second, we consider the sign of (A.2). We know that $H > 0$ and $V_{i\alpha}^i < 0$, therefore, the sign of (A.2) is opposite that of $V_{ki}^i = \pi_{ki}^i + \alpha^i \pi_{ki}^k < 0$. Therefore, if $V_{ki}^i = \pi_{ki}^i + \alpha^i \pi_{ki}^k < 0$, (A.2) is greater than zero; and if $V_{ki}^i = \pi_{ki}^i + \alpha^i \pi_{ki}^k > 0$, (A.2) is less than zero.

Proof of Theorem 3

Here, we examine the situation where a firm has zero cross holding (i.e., $\alpha^i = 0$). If the operational decisions induce positive externalities, we know from Theorem 1 that $dx^k/d\alpha^i > 0$ and by assumption that $\pi_k^i > 0$. We also know from equation (2) that $\pi_i^i(x^i, x^k) = 0$. Thus, when $\alpha^i = 0$, equation (6) reduces to

$$dV^i/d\alpha^i = (d\pi^i/dx^k)(dx^k/d\alpha^i) > 0,$$

meaning that the manager can increase firm value by increasing cross holding from zero (i.e., by taking a long position the other firm).

When the operational decisions induce negative externalities and $\alpha^i = 0$, we know from Theorem 1 that $\partial x^k/\partial \alpha^i > 0$ and by assumption that $\pi_k^i < 0$. We also know from equation (2) that $\pi_i^i(x^i, x^k) = 0$. Thus, when $\alpha^i = 0$, equation (6) reduces to

$$dV^i/d\alpha^i = (d\pi^i/dx^k)(dx^k/d\alpha^i) < 0,$$

meaning that the firm can increase shareholder value by decreasing its cross holding from zero (i.e., taking a short position in the other firm).

Proof of Theorem 5

First, we solve the model for the case where cross holding is prohibited, that is, $\alpha^i = \alpha^k = 0$, indexing the corresponding equilibrium values by superscript 0. Substituting into equation (3) yields the quantity $q^{i0} = [(A - C)/(2 + B)]$ and price $p^{i0} = A - (1 + B)(\frac{A-C}{2+B})$.

Second, we prove that prices are always lower or, equivalently, quantities are always higher when cross holding is allowed:

$$\begin{aligned}
 A - (1 + B)q^i &= p^i < p^{i0} = A - (1 + B)q^{i0} \\
 \text{if and only if } \frac{(A - C)[2 + B]}{4(1 + B)} &= q^i > q^{i0} = \left(\frac{A - C}{2 + B}\right) \\
 \text{if and only if } [2 + B]^2 &> 4(1 + B) \\
 \text{if and only if } B^2 &> 0,
 \end{aligned}$$

which is true.

Third, we identify sufficient and necessary conditions for firm profits to be lower when short selling is allowed than when it is prohibited.

$$\frac{(A - C)^2(4 - B^2)}{16(1 + B)} = \pi^i < \pi^{i0} = \frac{(A - C)^2}{(2 + B)^2}$$

if and only if $(4 - B^2)(4 + 4B + B^2) < 16(1 + B)$

if and only if $(16 + 16B + 4B^2) - (4B^2 + 4B^3 + B^4) < 16 + 16B$

if and only if $-(4B^3 + B^4) < 0$

if and only if $B > 0$.

Finally, defining economic welfare as

$$W^i = S^i + \pi^i = \frac{1}{2}(A - p^i)(q^i - 0) + (p^i - C)q^i,$$

where S^i is the consumer surplus, we demonstrate that economic welfare is always lower when firms are precluded from cross holding, that is, when $W^i > W^{i0}$:

$$\begin{aligned} \frac{1}{2}(1 + B)(q^i)^2 + (p^i - C)q^i &= W^i > W^{i0} \\ &= \frac{1}{2}(1 + B)(q^{i0})^2 + (p^{i0} - C)q^{i0}. \end{aligned}$$

By substitution, we find that

$$\begin{aligned} \frac{1}{2}(1 + B) \left\{ \frac{(A - C)[2 + B]}{4(1 + B)} \right\}^2 + \frac{(A - C)^2[4 - B^2]}{16(1 + B)} \\ > \frac{1}{2}(1 + B) \left(\frac{A - C}{2 + B} \right)^2 + \left(\frac{A - C}{2 + B} \right)^2. \end{aligned}$$

This reduces to

$$(2 + B)^4 + 2[4 - B^2](2 + B)^2 > 16(1 + B)^2 + 32(1 + B)$$

and

$$48 + 64B + 24B^2 - B^4 > 48 + 64B + 16B^2$$

and

$$8B^2 > B^4.$$

Proof of Theorem 6

The entrant’s profit after entry is higher when cross holding is allowed than when it is prohibited, if and only if

$$\frac{(A - C)^2(4 - B^2)}{16(1 + B)} > \frac{(A - C)^2}{[2 + B]^2},$$

or, recalling that $|B| < 1$,

$$(4 - B^2)[2 + B]^2 > 16(1 + B).$$

This reduces to $4B^3 + B^4 < 0$, or $B < 0$. Likewise, the probability of entry is lower when cross holding is allowed than when it is prohibited when the products are substitutes ($B > 0$).

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