Technical Note—Impact of Inventory on Quota-Bonus Contracts with Rent Sharing

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We study the impact of limited inventory on optimal sales-force compensation contracts. We adopt a principal-agent framework, characterized by limited liability and rent sharing with the agent. A commonly invoked assumption in the inventory management literature is that the demand distribution satisfies the increasing failure rate (IFR) property. Under this assumption, however, past research has established that a quota-bonus contract—a widely adopted sales-force compensation contract in the practice—cannot sustain in equilibrium. We show that because of demand censoring in the presence of limited inventory (i.e., demand realizations higher than the inventory level are unobservable), a quota-bonus contract is the optimal equilibrium contract, and it exists, even for a demand distribution with the IFR property. Since most well-known distributions satisfy the IFR property, and inventory constraints are operative in many real-world situations, our results significantly extend the scope of the optimality of quota-bonus contracts and underscore the importance of considering the inventory aspect while making sales-force compensation decisions.

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1. Introduction

Firms frequently use quota-bonus contracts to compensate their sales forces (Joseph and Kalwani 1998, Steenburgh 2008, Misra and Nair 2011, Chung et al. 2014). Several theoretical studies explain the prevalence of quota-bonus contracts (Park 1995, Kim 1997, Oyer 2000). Specifically, Oyer (2000) derives the well-known result that when the salesperson has limited liability and there is rent sharing (i.e., the participation constraint does not bind), the optimal compensation plan, if it exists, will be a quota-bonus contract. Oyer considers two types of demand distributions: those with increasing-to-decreasing hazard rates, and those with continuously increasing hazard rates (also known as IFR distributions, i.e., increasing failure rate distributions). For the first type of distributions, Oyer shows that an equilibrium contract exists with an interior quota. For the second type of distributions, an equilibrium quota-bonus contract does not exist, even though the optimality result holds. This is because the firm has the incentive to increase the quota until it reaches the upper bound of the support of the demand distribution, at which point the quota is not achievable by the agent.

The framework in Oyer (2000) assumes (as does almost all of classical sales-force compensation literature) that the firm satisfies all of the demand induced by the salesperson's effort.

However, in many real-world situations, this assumption does not hold and the firm cannot sell more units than the inventory it stocks. For instance, consider a firm selling office products, such as stationery and furniture. The firm typically purchases a particular quantity of these products from an overseas supplier with a lead time of several weeks or months, and stocks them in a local warehouse. The field sales agents employed by the firm generate demand for these products, but short-run sales are limited by the inventory stocked. Furthermore, if realized demand is greater than inventory, it is often not possible to keep track of demand that was realized but was not fulfilled, or that could have been realized but was not because, instead of backordering, customers chose to not order the product or to postpone their purchases. This leads to the phenomenon of demand censoring (Besbes and Muharremoglu 2013; Dai and Jerath 2013, 2015), that is, the firm cannot observe the actual demand level in excess of the inventory.[1]

In this paper, we incorporate the inventory component into the framework of Oyer (2000). Noting that the inventory management literature often invokes the assumption that the demand distribution satisfies the IFR property (Cheng and Sethi 1999, Gavirneni et al. 1999, Lariviere and Porteus 2001), we focus on IFR demand distributions. In modeling limited inventory, we explicitly address demand censoring

and show that it can significantly affect the optimal strategy for sales-force compensation.

We find that, in the presence of limited inventory, a quota-bonus contract is the optimal equilibrium contract, and it exists even for demand distributions that have the IFR property (unlike in Oyer 2000). In the optimal contract, the firm chooses the quota at which the salesperson receives the bonus to be equal to the inventory level. This is an equilibrium contract because the salesperson's probability of achieving the quota is exactly the firm's stock-out probability, which is strictly positive. This result is a significant one, both theoretically and practically, because of the following reasons.

First, most well-known distributions have the IFR property, so our results extend the optimality and existence of quota-bonus contracts to a large family of demand distributions. These include the uniform, normal, truncated-normal, logistic, log-normal, exponential, Laplace, and Weibull distributions, to name several popular ones, and all translations and convolutions of these distributions. Previously, Oyer (2000) had shown that an equilibrium quota-bonus contract exists only for distributions with increasing-then-decreasing hazard rates, a property not satisfied by most common distributions. Second, inventory constraints and demand censoring are realistic in many situations and so this is a natural and useful setting to consider; Chen (2000), Plambeck and Zenios (2003), Chen (2005), Chu and Lai (2013), and Dai and Jerath (2013, 2015), for example, jointly study inventory and sales-force compensation issues. Third, the contract form we obtain is a remarkably simple one—the salesperson will be paid the bonus if he clears the inventory stocked, otherwise he will be paid only the fixed wage—and it reflects the structure of contracts sometimes used in the industry (Dai and Jerath 2013).

The rest of this note is organized as follows. In the next section, we describe the model. In §3, we present the optimal incentive contract for demand distributions that satisfy the IFR property in the presence of limited inventory. In §4, we conclude.

2. The Model

Following the modeling framework in Oyer (2000), we assume that there is a single risk-neutral firm that employs a risk-neutral salesperson to enhance demand. When the salesperson exerts an effort level e, it leads to demand given by

$$\xi = h(e) + \eta, \tag{1}$$

where $h'(e) > 0$, and the random noise $\eta \geq 0$. The salesperson's disutility from exerting effort e is given by $v(e)$, which is convex increasing in e, i.e., $v'(e) \geq 0$ and $v''(e) \geq 0$. The demand ξ has a density of $g(\xi \mid e)$ and a cumulative distribution function of $G(\xi \mid e)$, where g is differentiable in both ξ and e, and the upper bound of the demand distribution is ∞. The density function $g(\xi \mid e)$ is unimodal and

$G_e(\xi \mid e) < 0$ for any allowed demand realization ξ, i.e., the demand is stochastically increasing in effort. Therefore, the hazard rate (also known as the failure rate) of the demand distribution, given by $g(\xi \mid e)/[1 - G(\xi \mid e)]$, will be either monotonically increasing or increasing-to-decreasing; our focus is on the former case. We assume that the salesperson has limited liability (Park 1995, Kim 1997, Oyer 2000, Dai and Jerath 2013) and, without loss of generality, normalize the limited liability to zero. Furthermore, we assume that the participation constraint is not binding (i.e., there is rent sharing with the agent), say due to the salesperson's job-specific skills.

Extending the framework in Oyer (2000), we assume that the quantity sold is limited by the inventory level that the firm stocks. Specifically, we denote the firm's inventory level by I, where I lies within the support of the demand distribution, and denote the sales by y; then, $y = \min\{I, \xi\}$. When the realized demand exceeds the inventory level, the firm cannot observe the exact realization of demand, and thus has to provide the salesperson with the same compensation for any realization of demand equal to or above the inventory level.

We assume that the firm incurs a unit marginal cost of c_o to manufacture/procure the product, and collects a revenue of p per unit of sales.

We denote by $s(y)$ the compensation given for observed sales of y, and assume that $s(y)$ is nondecreasing in y. The firm's problem can be written as

$$\max_{s(\cdot),\, e,\, I} \quad \left\{ E[p \cdot y - s(y)] - c_o \cdot I \right\},$$

subject to: $y = \min\{\xi, I\}, \tag{2}$

$$e \in \arg\max E[s(y)] - v(e), \tag{3}$$

$$s(y) \geq 0. \tag{4}$$

In the formulation above, (2) is due to the firm's inventory constraint, (3) is due to the salesperson's incentive compatibility constraint, and (4) is due to the salesperson's limited liability constraint.

3. Optimal Contracts for IFR Distributions

3.1. Optimal Contract without Inventory Considerations

We first restate a result from Oyer (2000), which shows that (without inventory considerations) a quota-bonus contract cannot sustain in equilibrium when the demand distribution satisfies the IFR property.

PROPOSITION 1 (OYER 2000). *When the hazard rate of the demand distribution, $g(\xi \mid e)/[1 - G(\xi \mid e)]$, is monotonically increasing in ξ, there is no equilibrium quota-bonus contract at which the participation constraint does not bind.*

The insight driving the above result is that, under an IFR distribution, the firm has the incentive to set the sales quota

as high as possible, and the equilibrium quota-bonus contract, if it exists, would have a quota equal to the upper bound of the support of the demand distribution. This, however, would lead to zero probability of payment (and zero expected payment) for the salesperson.

3.2. Optimal Contract with Exogenous Inventory Level

We now consider the case where the firm chooses the optimal compensation plan for the salesperson when the inventory level, I, is exogenous. The following proposition states that a quota-bonus contract is optimal, and it exists, and that the optimal sales quota is equal to the inventory level.

PROPOSITION 2. *When the hazard rate of the demand distribution, $g(\xi \mid e)/[1 - G(\xi \mid e)]$, is monotonically increasing in ξ, the optimal compensation scheme, if it exists, is to pay a discrete bonus if sales equal the inventory level (i.e., realized demand equals or exceeds the inventory level). Formally, there is a bonus, b, and a quota, χ, equal to the inventory level, I, such that*

$$s(y) = \begin{cases} 0 & \text{for } y < \chi = I, \\ b & \text{otherwise.} \end{cases}$$

PROOF. The firm's problem can be rewritten as

$$\max_{s(\cdot), e} \left\{ p \cdot \left[\int_0^I \xi \cdot g(\xi \mid e)\, d\xi + \int_I^\infty I \cdot g(\xi \mid e)\, d\xi \right] \right.$$
$$\left. - \left[\int_0^I s(\xi) g(\xi \mid e)\, d\xi + \int_I^\infty s(I) g(\xi \mid e)\, d\xi \right] - c_o \cdot I \right\} \quad (5)$$

subject to

$$\int_0^I s(\xi) g_e(\xi \mid e)\, d\xi + \int_I^\infty s(I) g_e(\xi \mid e)\, d\xi - v'(e) = 0, \quad (6)$$

$$s(\xi) \geqslant 0 \quad \text{for all } \xi \geqslant 0, \quad \text{and} \quad (7)$$

$$s(\xi_1) = s(\xi_2) \quad \text{for any } \xi_1, \xi_2 \geqslant I. \quad (7)$$

Note that (6) is obtained from (3) by applying the first-order condition. (We assume, following Oyer 2000, that the first-order approach is valid.) Because $s(0)$ is absent in (6), a higher $s(0)$ does not motivate a higher effort level from the salesperson. Therefore, it is always optimal for the firm to set $s(0) = 0$. The firm's problem of determining its incentive scheme $s(\cdot)$ is thus equivalent to the choice of the incremental payment at different demand levels, defined by

$$r(\xi) = s(\xi) - \sup[s(\xi') \mid \xi' < \xi].$$

The constraint (7) implies that $r(\xi) = 0$, for all $\xi > I$. Thus, the firm only needs to determine $r(\xi)$ for $0 \leqslant \xi \leqslant I$. The Lagrangian for (5)–(7) is

$$L(r(\xi), e)$$
$$= p \cdot \left[\int_0^I \xi \cdot g(\xi \mid e)\, d\xi + \int_I^\infty I \cdot g(\xi \mid e)\, d\xi \right]$$

$$- \left\{ \int_0^I \left[\int_0^\xi r(x)\, dx \right] g(\xi \mid e)\, d\xi \right.$$
$$\left. + \int_I^\infty \left[\int_0^I r(x)\, dx \right] g(\xi \mid e)\, d\xi \right\}$$
$$- c_o \cdot I + \lambda \cdot \left\{ \int_0^I \left[\int_0^\xi r(x)\, dx \right] g_e(\xi \mid e)\, d\xi \right.$$
$$\left. + \int_I^\infty \left[\int_0^I r(x)\, dx \right] g_e(\xi \mid e)\, d\xi - v'(e) \right\}$$
$$+ \int_0^I \mu(\xi) r(\xi), \quad (8)$$

where λ and μ terms are the Lagrange multipliers for the incentive compatibility and limited liability constraints, respectively. Integrating by parts, we obtain the following two equations:

$$\int_0^I \left[\int_0^\xi r(x)\, dx \right] g(\xi \mid e)\, d\xi + \int_I^\infty \left[\int_0^I r(x)\, dx \right] g(\xi \mid e)\, d\xi$$
$$= \int_0^I [1 - G(\xi \mid e)] r(\xi)\, d\xi, \quad \text{and} \quad (9)$$

$$\int_0^I \left[\int_0^\xi r(x)\, dx \right] g_e(\xi \mid e)\, d\xi + \int_I^\infty \left[\int_0^I r(x)\, dx \right] g_e(\xi \mid e)\, d\xi$$
$$= -\int_0^I G_e(\xi \mid e) r(\xi)\, d\xi. \quad (10)$$

After substituting (9) and (10) into (8), and applying point-wise optimization of $r(\xi)$ for $0 \leqslant \xi \leqslant I$, we obtain the following equation:

$$1 = \lambda \left[\frac{-G_e(\xi \mid e)}{1 - G(\xi \mid e)} \right] + \mu(\xi) \left[\frac{1}{1 - G(\xi \mid e)} \right] \quad (11)$$

for $0 \leqslant \xi \leqslant I$. The Kuhn-Tucker condition for $\mu(\xi)$ gives $\mu(\xi) r(\xi) = 0$ for $0 \leqslant \xi \leqslant I$. Therefore, there is a positive incremental payment at any ξ only if $\mu(\xi) = 0$. This means that $r(\xi)$ is positive only at the point where $-G_e(\xi \mid e)/[1 - G(\xi \mid e)]$ is maximized.

By (1), we have $-G_e(\xi \mid e) = h'(e) g(\xi \mid e)$. So χ should maximize $g(\xi \mid e)/[1 - G(\xi \mid e)]$. But since the demand distribution is IFR, $g(\xi \mid e)/[1 - G(\xi \mid e)]$ is monotonically increasing in ξ. Therefore, it must be the case that the firm pays the salesperson a positive salary if and only if the realized demand is greater than or equal to the inventory level I, i.e., the realized sales are equal to the inventory level I. Q.E.D.

In contrast to Proposition 1, and because of the assumption of limited inventory availability, Proposition 2 states that a quota-bonus contract emerges as the optimal compensation contract for an IFR demand distribution. The reason is that even though the firm has the incentive to increase the quota as much as possible, the inventory level serves as a natural level to set the quota at, i.e., there is no reason to increase the quota beyond this point. Therefore, in the optimal contract, the firm uses the stocked inventory level as the sales quota,

and the salesperson is paid a bonus when the inventory is cleared. The salesperson has the incentive to accept the contract in equilibrium because, although the sales quota is equal to the highest observable demand level, it is still below the upper limit of the support of the demand distribution. Therefore, the salesperson's probability of achieving the quota is equal to the probability of stock out, which is strictly positive. We have the following corollary, which follows from Proposition 2.

COROLLARY 1. *Under the optimal quota-bonus contract with a bonus of b^* and a quota of I^*, the salesperson's expected compensation is $[1 - G(I^* \mid e)] \cdot b^*$, which is always strictly positive.*

3.3. Optimal Contract with Endogenous Inventory Level

We now consider the case where the firm jointly determines the compensation plan and the inventory level—this is essentially a sales-force compensation problem in a newsvendor-like model (Porteus 2002). We can decompose the firm's problem into a two-stage problem, with the inventory level decided in the first stage and the compensation contract decided in the second stage. Clearly, in the second stage, given the inventory level, the optimal contract is the same quota-bonus contract as specified in Proposition 2. We obtain the following interesting result for the inventory decision.

PROPOSITION 3. *The firm chooses the optimal inventory level such that the stock level (given by $G(I^* \mid e^*)$) is $(p - c_o)/p$, which is equal to the stock level in the first-best scenario (given by $G(I^{FB} \mid e^{FB})$).*

PROOF. Under the first-best scenario, the firm's problem is

$$\max_{e, I} \left\{ p \cdot \left[\int_0^I \xi \cdot g(\xi \mid e) \, d\xi + \int_I^\infty I \cdot g(\xi \mid e) \, d\xi \right] - v(e) - c_o \cdot I \right\},$$

which, by the first-order condition with respect to I, gives $G(I^{FB} \mid e^{FB}) = (p - c_o)/p$. With inventory, we have from the first-order condition of (8) with respect to I that $p[1 - G(I \mid e)] - c_o - r(I) \cdot [1 - G(I \mid e) + \lambda G_e(I \mid e)] = 0$, which gives $p[1 - G(I \mid e)] = c_o$ because

$$1 - G(I \mid e) + \lambda G_e(I \mid e)$$
$$= [1 - G(I \mid e)] \cdot \left\{ 1 - \lambda \left[\frac{-G_e(I \mid e)}{1 - G(I \mid e)} \right] \right\}$$
$$= [1 - G(I \mid e)] \cdot \mu(I) \left[\frac{1}{1 - G(I \mid e)} \right] = 0,$$

where the last two equations are obtained from (11). Therefore, the firm chooses $G(I \mid e) = (p - c_o)/p = G(I^{FB} \mid e^{FB})$. Q.E.D.

Interestingly, Proposition 3 shows that although the first-best outcome cannot be attained because the participation constraint is nonbinding, the firm, as in the first-best scenario, will set the stocking level according to the "critical fractile" solution (Porteus 2002). The proposition also implies that the probability that the salesperson meets the quota is c_o/p because, under the optimal quota-bonus contract the stock-out probability, given by $1 - G(I \mid e) = c_0/p$, is equal to the salesperson's probability of meeting the quota. Finally, this proposition shows that for every induced effort level, there is an optimal inventory level that uniquely corresponds to it (and serves as the sales quota in the optimal contract).

4. Conclusions

We consider a sales compensation scenario in which sales are limited by inventory and demand realizations above inventory are censored, i.e., they cannot be observed. We follow the framework of Oyer (2000), characterized by limited liability and a nonbinding participation constraint. We show that, if the demand distribution satisfies the IFR property, a simple quota-bonus contract in which the salesperson is paid a bonus if he clears all the inventory stocked and otherwise is paid the minimum wage, is optimal and it exists. This is in contrast to the result in Oyer (2000) that (without inventory considerations) an equilibrium optimal contract does not exist for a demand distribution with the IFR property.

Endnote

1. We note that in some situations it may be possible to track unmet demand, especially if customers can place backorders (though some noise would almost always be introduced).

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