Accrual Accounting in Performance Measurement and the Separation of Ownership and Control

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Abstract. The value implications of managerial decisions depend on uncertain future events and decisions by the manager's successors, often far beyond the manager's own tenure. In this context, performance pay based on accrual accounting can incentivize efficient decisions even when managers' actions are not fully observable. The optimal compensation scheme is not equivalent to selling the firm to the manager, implying that stock-based compensation fails to attain incentive alignment and providing a rationale for the separation of ownership and management commonly seen in practice. Performance pay may in fact show weak or even inverse correlation with stock price and cash flow. Even in enterprises managed by their owners, the prospect of implementing accounting-based incentive compensation and separating ownership from management in the future can induce efficient decision incentives. The change in management must, however, coincide with the sale of the business to a new owner under an accounting-based earn-out agreement.

Keywords: performance measurement; accruals; goal congruence

I. INTRODUCTION

The primary function of accrual-basis accounting lies in measuring the results of an entity's activities over successive, discrete time periods. This performance measurement over finite time intervals is simultaneously necessary and problematic. The necessity arises because most investors, managers and other stakeholders have limited horizons and seek to settle their relationship with the business, whether by selling ownership interests or by receiving compensation for job performance, before the end of the entity's lifespan. At the same time, the value implications of stakeholders' present activities usually depend on uncertain future events and actions, and the activities themselves may be affected by how performance is measured in the first place. Whether and how one can quantify these value implications in a timely yet accurate manner is therefore all but obvious.

This paper considers the finite-period performance measurement problem with finite-horizon actors in a classic stewardship context, where a business owner delegates management duties and seeks to incentivize a better informed manager to commit effort and maximize value creation. Three broad lessons about accounting and organizational design emerge. Firstly, efficient managerial incentive compensation requires accrual accounting, which offers the degrees of freedom needed to design a revelation mechanism with respect to managers' hidden actions and information. To this end, it suffices if accruals are calculated as a function only of the firm's realized cash flows. No assumption is needed that accounting produces information ex nihilo. Secondly, and by contrast, stock-based compensation, or any other form of incentive pay that awards ownership interests to the manager, creates incentive misalignment. Optimal incentive pay can in fact show weak or even negative correlation with stock price. Selling the firm to the agent, normally considered the canonical remedy to intra-firm principal-agent conflicts, would thus fail to resolve the incentive problem, even in the absence of capital constraints and risk aversion. Incentive-compatible performance measurement therefore requires the separation of ownership from management, as is often seen in practice. Thirdly, even in owner-managed enterprises, the mere prospect of instituting separated ownership and management in the future can suffice to induce efficient investment decisions at present. However, the transition to a separated organizational structure must coincide with the sale of the business to a new owner under an accounting-based earn-out agreement, a feature often seen in acquisitions of small private firms by equity investors.

To understand the rationale behind these results, it is useful to view the problem through the paradigm of goal congruence. Goal congruence refers to the alignment of the manager's incentives with the owner's objectives by means of a performance metric that, by design, the manager can only maximize by taking decisions that the owner desires. Existing solution methods in this arena have been derived in settings where the value contributed by an individual manager's actions is well-defined and separable from that of other agents and events, and where agency conflicts between owner and manager are to some degree separable from the goal congruence problem (e.g., Reichelstein 1997; Dutta and Reichelstein 2005a; Baldenius, Dutta and Reichelstein 2007). In the agency model presented here, the joint effects of hidden actions and information, actors with limited horizons, and non-separability of investments create conditions under which these existing solutions are inoperable.

To illustrate the problem, consider a manager who observes decision-relevant information about business conditions and, in response, makes investments that produce cash revenues. These investments consist of both cash expenditures and personal effort. The firm's owner observes the cash inflows and outflows but neither the underlying business conditions nor the manager's effort. Moreover, the manager's term of employment is shorter than the remaining lifespan of the firm, so that the owner must evaluate and reward the manager's performance before all long-term effects of the manager's actions have materialized in verifiable cash flows.[1] Finally, the firm's investment problem is not time-separable, in the sense that the future revenues generated by resources spent today are also a function of past and future investments. Every unit of revenue generated in any given period is therefore the joint product of all past investments working in concert. Performance measurement in this context cannot rely on conventional criteria of optimality, such as the implementation of investments with positive net present value, because the allocation of revenues to individual prior investment decisions is not unique.[2] Even under generic concavity assumptions that imply a unique first-best investing strategy, there exists a multiplicity of ways to credit outcomes to the manager in charge today, as the period-to-period allocation of the surplus value created by that optimal strategy is still, in principle, arbitrary.[3]

[1] That management tenure and horizon have substantive effects on firm performance has been documented empirically numerous times, e.g., by Dechow and Sloan (1991), Cheng (2004) and Ali and Zhang (2015).

[2] See, for example, Barker and Penman (2020) for a discussion of this problem.

[3] By contrast, additively separable investments make incentive-compatibility attainable by deterministic depreciation schedules that match investment costs to revenues in specific periods. For example, Reichelstein (1997) and Dutta

Designing an effective performance metric is feasible despite these limitations because incentivizing managers to make value-maximizing decisions does not require measuring the profitability of their investments. Rather, one can limit the task to inducing sequentially optimal decisions under an incentive-compatibility constraint. To this end, it suffices to localize the problem in each period by treating all past costs as sunk and assuming all subsequent decisions to be exogenously optimized by future incentive contracts. Properly calibrated for each of the investment opportunity sets the manager could face, performance accounting effectively holds the manager responsible for the full incremental value created under the manager's reign, notwithstanding that this value increment partly depends on past and future decisions outside the manager's control. The manager then implements the optimal investing strategy as an equilibrium outcome even though the performance metric does not identify the manager's personal contribution.

At the mechanical level, performance accounting must be a function of available verifiable information, here including the firm's cash expenditures and revenues. Simple cash-basis accounting would, however, set improper incentives, since the manager's investment decisions contribute to revenues beyond the measurement period. Effective performance measurement therefore requires an accrual, or non-cash, component, which, given the verifiability requirement, must itself be a function of the cash flows. This accrual component infers from cash revenues and expenditures the manager's effort and the underlying business conditions and adjusts the performance metric for the value implications of these unobservable factors. Incentive-compatibility obtains if, in equilibrium, the manager maximizes measured performance by generating the cash flow outcomes by which the performance metric infers these unobservable factors correctly.

The magnitude of the accrual component depends on how much of the variation in investment opportunities is explained by cash flows expected after the measurement period. Larger accruals arise when investments have value implications that extend significantly into the future. But even when optimally calibrated with respect to future value implications, the sensitivity of measured performance to the manager's investment decisions does not match the corresponding sensitivity of firm value. In other words, offering to pay compensation equal to the expected incremental value impact of the manager's actions leads to inefficiency. The reason lies in the manager's informational advantage over the owner, who can conjecture what the manager knows

and Reichelstein (2005a) obtain goal congruence by measuring performance by residual income and depreciating investment expenditures over time by the so-called relative benefit rule.

and does based only on observed cash flow outcomes. Incentive-compatibility means rendering it unattractive for the manager to take advantage of this information asymmetry and requires that, around the optimal decision for every possible investment opportunity, a change in business conditions, which the manager observes but cannot influence, has the same marginal effect on measured performance as a change in investment, which the manager controls. Calibrating the pay-for-performance sensitivity with respect to this indifference condition, rather than to value creation, neutralizes managers' temptations to manipulate the performance metric by arbitraging between information and effort. Notably, this construction admits the counterintuitive scenario that an improvement in investment opportunities leads to a decrease in the firm's market value, as the need for incentive-compatibility can force managerial compensation to increase by more than the value of the investment made. This inverse pay-performance relation can obtain even in situations when investment requires only minimal effort exertion on the manager's part.

The non-equivalence of value creation and incentive pay implies that an optimal compensation scheme is not the same as having sold the firm to the manager. Incentive-compatible performance pay therefore requires an organizational design with decision-makers (managers) and residual claimants (owners) as separate economic agents.[4] If, conversely, both ownership and management responsibility were vested in the same individual (an owner-manager), who anticipated selling the firm in the future, this owner-manager would face an incentive to arbitrage between private information and investment decisions at the expense of the uninformed future buyer of the firm, just as a hired manager faces an incentive to arbitrage at the expense of the owner, except that, for an owner-manager, there is no scope for incentive pay to remedy the problem. Divorcing ownership from management affords the additional degrees of freedom needed to devise a compensation scheme that leaves decision incentives uncontaminated by ownership interests. Owners can then hire successive generations of managers by a sequence of incentive-compatible contracts that continually resolve the information asymmetry problem, while ownership shares can be traded freely and efficiently among existing and prospective investors. Without assuming risk aversion or limited individual wealth, one can thus rationalize the separation of ownership

[4] The logic bears some similarity to Holmström's (1982) observation that providing efficient incentives in teams of agents is incompatible with a balanced budget constraint, a problem that can be resolved by introducing a principal who absorbs any budget surplus or shortfall that may arise when agents' incentive pay is calibrated to the optimum. The separation of ownership and management also has strategic implications for competitive equilibria in oligopoly markets (e.g., Fershtman and Judd 1987).

and control typically seen in large, long-lived enterprises.[5] A direct corollary is that stock-based compensation, often thought of as a means to align owners' and managers' interests, is detrimental because it amounts to a remerging of ownership and control and thus brings back the incentive problem that the separation of duties was intended to avoid in the first place.[6]

The separation-of-duties argument nevertheless does not imply that owner-managed enterprises necessarily invest inefficiently, and the reason is directly connected to the possibility of instituting a separated organizational design in the future. Merely hiring an external manager is, however, not an option because an incumbent owner-manager's informational advantage would hamper the negotiation of an optimally efficient compensation contract with the prospective hire. Instead, investment efficiency survives if the owner-managed enterprise is acquired under an earn-out agreement, i.e., for a purchase price that depends on post-acquisition performance of the firm, and if the new owner immediately delegates management duties to a third-party manager and implements incentive-compatible performance accounting. The reason that this triangle deal between seller, buyer and external manager attains full efficiency is that the hired manager is free of the corrupting incentives induced by ownership rights and, given proper incentive pay, invests efficiently during the earn-out period, thus maximizing both firm value for the buyer and the purchase price for the seller. At the same time, all transfer payments contingent on the seller's reported private information can be shifted into the compensation contract between buyer and manager, leaving the seller with no incentive to misrepresent the firm's condition at the acquisition date. The prospect of this acquisition deal in turn is an incentive for the seller to invest efficiently even while the firm is still owner-managed.

By definition of the research problem, the model speaks directly only to the use of accounting in managerial performance evaluation and makes no immediate statement about how its results should interact with general-purpose financial reporting rules. At the same time, independence between accounting for managerial performance and financial reporting is impossible because performance measurement, by determining compensation payments, creates a cash flow effect that financial reporting must account for. For example, it is possible, but not necessary,

[5] Alternative explanations, by contrast, tend to view the agency problem arising from separation as a necessary evil, justified by, for example, improved risk-sharing among a large number of owners or the efficient use of decision-relevant knowledge diffused among the agents in the organization (Fama and Jensen 1983).
[6] Reasons for stock-based compensation may certainly exist outside the limited set of phenomena included in the model presented here. The point is that a simple incentive alignment argument is not sufficient.

that, in the absence of agency problems and incentive pay, better investment opportunities reduce current cash flow, i.e., expenditures may initially outweigh the revenues. With agency problems and incentive pay in play, the net cash flow, including managerial compensation, *always* decreases with better investment opportunities, even if the investment begins to generate high revenues immediately. The performance accounting model may therefore also be of some potential use in framing research problems in financial accounting.

This paper has a close connection to prior work on accrual-based performance measurement. Reichelstein (1997, 2000), Dutta and Reichelstein (2005a), Mohnen and Bareket (2007), and Nezlobin, Reichelstein and Wang (2015) demonstrate that revenue and cost allocation rules, when properly designed in a residual income framework, attain goal-congruence between principal and agent in various transaction settings. Pfeiffer and Schneider (2007) and Johnson, Pfeiffer and Schneider (2013, 2017) show the robustness of these results to the introduction of adverse selection in an intra-firm capital budgeting problem. On the other hand, when the decision problem has option-like features, goal-congruent solutions may not exist (Baldenius, Nezlobin and Vaysman 2016), except in special cases (Livdan and Nezlobin 2017). Dutta and Reichelstein (2002), Dutta and Zhang (2002), Baldenius, Dutta and Reichelstein (2007), and Dutta and Fan (2009) likewise integrate accrual accounting for performance measurement with an explicit agency conflict by interacting the goal congruence objective with a moral hazard problem. These papers consider either single or time-separable investments, which entail deterministic matching of investment costs to future revenues as a solution to the incentive problem. Other models confine attention to linear contracts and accounting variables in reduced form but include leading indicator variables (Dikolli 2001; Smith 2002; Dutta and Reichelstein 2003) or stock price (Dutta and Reichelstein 2005b) in the compensation scheme, or allow for correlation in performance measurement error across time periods (Christensen, Feltham and Sabac 2005) or differences in managers' skills (Dutta 2008).

II. MODEL SETUP

Consider a discrete-time model of a firm that invests resources to generate revenues. Investments take the form of both verifiable cash expenditures and unobservable effort. Expenditures include purchases of, for example, capital assets, inventories and supplies. Effort should be thought of as work performed by employees in creative or managerial positions with complex job descriptions and substantial variety and discretion in their tasks, so that their impact on the firm's revenue cannot be observed directly.[7] The model considers the interaction of two parties: the firm's manager, who determines expenditures and commits effort, and the firm's owner, who hires and compensates the manager and receives the net profits of the business.

Investments are not independent, in the sense that the marginal contribution to future revenue by an investment made today depends on the firm's history of prior investments. Moreover, revenues are not deterministic but depend jointly on investment and on exogenous, random events beyond the manager's control. In particular, the revenue cash flow m in each period $t = 1,2, \ldots$ is a function of a deterministic investment variable, x_t, and the firm's stochastic business environment, θ_t. After its realization at the end of the period, revenue becomes observable and verifiable. The function $m: (x_t, \theta_t) \rightarrow \mathbb{R}^+$ is concave, bounded and continuously differentiable in x_t, with $m_\theta(0, \theta_t) = 0$ for all θ_t.[8]

The environmental state variable $\theta_t \in \mathbb{R}$ summarizes all events outside the firm's control and follows a Markov process with commonly known distributional properties. The state variable can only be discovered by running the operations of the firm. In particular, the firm's manager observes θ_t prior to determining the investment in period t, whereas the owner (or any other outsider) does not learn θ_t in any period, both present and future.[9] One should think of this setting as a simplified representation of the multitude of information that only someone immersed in the day-to-day management is privy to, including, for example, intra-firm personnel politics, the financial situation of customers and suppliers, direct feedback from employees and external business contacts via personal interaction, etc. The only way for the owner to observe θ_t directly is therefore to assume the manager's position and to run the firm personally.

The investment variable x_t is a function of the manager's current investment decision and the firm's history of past investments. In particular, the evolution of x_t follows the process

$$x_t = x_{t-1} + k_t + a_t \tag{1}$$

[7] Examples include basic research and strategic decision-making. Some external vendors' services may also fall into this category, as indicated by the use of performance-based supply contracts in some industries.
[8] Except when indicating time periods, subscripts denote partial derivatives throughout the text.
[9] From the manager's perspective, the current-period cash revenue m therefore becomes deterministic once θ_t has been observed, but, without loss of insight, one can extend the analysis to a scenario in which cash revenue is additionally subject to a random shock that even the manager cannot foresee. Neither is it essential that the manager observes θ_t perfectly. One could alternatively provide the manager with an information signal that is imperfectly correlated with θ_t and restate the analysis as an optimization over expected outcomes conditional on the signal. The critical model feature is the information asymmetry between manager and owner.

where x_{t-1} is the investment value at the end of the prior period, and k_t and a_t denote, respectively, the resource quantity and manager's effort expended in period t. The transition model in (1) implies that investment in period t affects not only contemporaneous revenue but also the marginal return to investment in future periods, and thus future investment decisions.[10] Of the two components of x_t, the resource $k_t \in \mathbb{R}^+$ is verifiable and its cost is paid out of the firm's (and thus the owner's) assets, whereas the effort $a_t \in \mathbb{R}^+$ is incurred privately by the manager and is unobservable to the owner. Let $c: k_t \to \mathbb{R}^+$ denote the monetary cost of k_t, and let $q: a_t \to \mathbb{R}^+$ denote the monetary equivalent of the manager's effort cost. The cost functions c and q are convex, with $c(0) = q(0) = c'(0) = q'(0) = 0$. Both current and future revenues have increasing differences in investment and the environment, i.e.,

$$m_{x\theta}(x_t, \theta_t) > 0$$

and

$$\frac{\partial}{\partial \theta_t} E(m_x(x_{t+1}, \theta_{t+1})|\theta_t) \geq 0$$

for any x_t and θ_t.[11] More favorable conditions thus increase the first-best amount of investment both in the current and, in expectation, in future periods.[12]

The owner hires the manager for a finite number of periods and offers some monetary compensation, which may be contingent on any verifiable information available, including all past and present cash revenues and expenditures. The life of the firm is assumed to be infinite, whereas all actors in the model have a finite horizon and therefore look to liquidate their assets, including any ownership share in the firm, at some finite, future point in time. In particular, the manager retires at the end of the employment contract period and must be rewarded at that time.

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[10] The linear form of (1) simplifies the analysis but is not a necessary condition for the following results. Deriving clean analytical statements from a more general transition function $\{k_i, a_i\}_{i=0}^{t} \to x_t$ would require additional regularity conditions without adding relevant insight.

[11] All expectations of the form $E(m(x_{t+i}, \theta_{t+i})|\theta_t)$, with $i = 1,2, \ldots$, are taken over future states θ_{t+i}, conditional on the current state.

[12] The direction of the interaction between investment and environment in their impact on present and future revenue is not critical as long as monotonicity is maintained. It is, however, important that $m_{x\theta} \neq 0$ in the current period. The reason is that an efficient and incentive-compatible employment contract must amount to a mechanism that reveals θ_t, the manager's private information, as an equilibrium outcome. This inference must come from variations in current cash revenue because m does not become verifiable until realized, and so only the effects of θ_t on present, but not on future, revenues can be used to evaluate the manager's performance. Requiring $m_{x\theta} \neq 0$ in the current period is not a strong assumption, however, as a scenario in which current environmental conditions affect the future but are entirely orthogonal to present outcomes would be implausible.

Compensation must therefore be paid before all future effects of the manager's investment decisions have materialized in observable revenue.[13] The firm's total cash outflow thus consists of investment expenditure and managerial compensation. The firm pays out the net cash flow (revenue minus payments) to the owner at the end of each period. If payments exceed revenues, the owner makes a corresponding contribution of additional capital. Both owner and manager are risk-neutral, and the manager's reservation wage is normalized to zero.[14] As a companion to the main analysis, a parametric illustration of the model can be found in Appendix B.

III. ACCOUNTING AND CONTRACT DESIGN

Efficiency in Owner-Managed Firms

Given risk-neutrality of the actors and the absence of other exogenous impediments, it stands to reason that potential agency problems can be forestalled if the owner either sells the firm to the manager or, equivalently, personally takes control of managing the business. As a benchmark case, consider therefore a hypothetical scenario in which the firm is run by an owner-manager who never retires. No incentive problem arises in this setting because the owner-manager receives all revenues while internalizing all investment costs, ad infinitum. Intrinsic firm value, hereafter denoted by v, therefore equals the first-best optimum

$$v(x_{t-1},\theta_t) = \max_{\{k_i,a_i\}_{i=t}^{\infty}} \sum_{i=t}^{\infty} \gamma^{i-t} E(m(x_i,\theta_i) - c(k_i) - q(a_i)|\theta_t) \tag{2}$$

at the beginning of period t, where $\gamma < 1$ is the firm's discount factor.

The object of interest is the optimal investing policy, consisting of the expenditure and effort levels k^* and a^* that, as functions of x_{t-1} and θ_t, solve (2) in each period t. It is well known that, given the properties of the revenue and cost functions, the value function v is the unique solution to the Bellman equation

$$v(x_{t-1},\theta_t) = \max_{k_t,a_t}\{m(x_t,\theta_t) - c(k_t) - q(a_t) + \gamma E(v(x_t,\theta_{t+1})|\theta_t)\} \tag{3}$$

Further, given the concavity of the firm's net cash flow, v is concave in x_t, and the corresponding optimal investment choices $k^*(x_{t-1},\theta_t)$ and $a^*(x_{t-1},\theta_t)$ are unique in any period t (Lucas

[13] Letting compensation depend on outcomes in some finite number of periods post retirement would not change any insights from the analysis, as long as the manager's investment decisions have effects beyond the point at which compensation is terminally settled.

[14] The zero reservation wage is without loss of generality. If the wage were positive, the owner would incur a cost to hire the manager but would in turn be free to pursue profitable outside employment.

and Stokey 1989).[15] For reference throughout the following discussion, the key properties of the solution to (3) are summarized in Lemma 1 below. All proofs can be found in Appendix A.

Lemma 1. *There exists a unique first-best investment plan* $\{x_{t-1}, \theta_t\} \to \{k^*, a^*\}$ *for all* t. *The associated value function* $v(x_{t-1}, \theta_t)$ *is concave in* x_{t-1}. *Both* k^* *and* a^* *are monotonically increasing in* θ_t *and decreasing in* x_{t-1} *at all* (x_{t-1}, θ_t).

Lemma 1 implies that the infinite-lived owner-manager's expenditure k^* and effort a^* are characterized by the unique solutions to the (necessary and sufficient) first-order conditions

$$m_x(x_t, \theta_t) + \gamma E(v_x(x_t, \theta_{t+1})|\theta_t) - c'(k_t) = 0 \tag{4}$$

and

$$m_x(x_t, \theta_t) + \gamma E(v_x(x_t, \theta_{t+1})|\theta_t) - q'(a_t) = 0 \tag{5}$$

in any period t. That k^* and a^* are increasing as a function of θ_t now follows from the concavity of v and from

$$k_\theta^* = \frac{\big(m_{x\theta} + \gamma E_\theta(v_x)\big)q''}{c''q'' - \big(m_{xx} + \gamma E(v_{xx})\big)(c'' + q'')} \tag{6}$$

and

$$a_\theta^* = \frac{\big(m_{x\theta} + \gamma E_\theta(v_x)\big)c''}{c''q'' - \big(m_{xx} + \gamma E(v_{xx})\big)(c'' + q'')} \tag{7}$$

where

$$E_\theta(v_x(x_t, \theta_{t+1})|\theta_t) \equiv \frac{\partial}{\partial \theta_t} E(v_x(x_t, \theta_{t+1})|\theta_t)$$

and v, m, c and q are all evaluated at $k^*(x_{t-1}, \theta_t)$ and $a^*(x_{t-1}, \theta_t)$.[16] The response functions (6) and (7) will be of relevance for the remainder of the discussion.

Consider now the more realistic scenario of an owner-manager looking to sell the firm to a new owner-manager at some future date $T < \infty$, say, when reaching retirement age. The buyer

[15] Several generalizations of this model are possible, albeit not necessary, for the results to come. In particular, the cost functions $c(\cdot)$ and $q(\cdot)$ could each depend on $\{x_{t-1}, k_t, a_t, \theta_t\}$. Further, the problem need not be stationary, i.e., the revenue and cost functions could also change with t, and revenues and costs could contain period-specific error.

[16] Where used in the text hereafter, the shorthand notation $m \equiv m(x^*(x_{t-1}, \theta_t), \theta_t)$, $v \equiv v(x^*(x_{t-1}, \theta_t), \theta_{t+1})$, $c \equiv c\big(k^*(x_{t-1}, \theta_t)\big)$, etc. always denotes functions evaluated at first-best values.

can observe the firm's history of revenues and expenditures up to $T-1$ and the initial state x_0 but knows neither the incumbent owner-manager's effort a_t nor the state θ_t at any $t = 1, \ldots, T-1$. In a competitive market, the buyer then offers a price equal to a conjectured firm value

$$\hat{v}(\{k_i, m(x_i, \theta_i)\}_{i=1}^{T-1}) \equiv E(v(x_{T-1}, \theta_T) \big| \{k_i, m(x_i, \theta_i)\}_{i=1}^{T-1})$$

at the beginning of period T. The owner-manager receives all of the firm's net cash flows up to the selling date and therefore, in any period $t < T$, chooses an investment plan that solves

$$\max_{\{k_i, a_i\}_{i=t}^{T-1}} \left\{ \sum_{i=t}^{T-1} \gamma^{i-t} E(m(x_i, \theta_i) - c(k_i) - q(a_i) | \theta_t) + \gamma^{T-t} E(\hat{v}(\{k_i, m(x_i, \theta_i)\}_{i=1}^{T-1}) \big| \theta_t) \right\}$$

A necessary equilibrium condition is that the buyer's conjecture $\hat{v}$ correctly anticipates the solution to the above problem, i.e., the owner-manager's investing strategy must be optimal given the buyer's belief $\hat{v}$, and $\hat{v}$ must equal the true firm value v at the selling date T.

Consider whether first-best investment $\{k^*, a^*\}$ by the owner in all periods $t < T$, and a corresponding inference $\hat{v} = v$ by the buyer, could possibly meet this equilibrium condition. In particular, suppose that the buyer believed the expenditure k_t observed in period t to be first-best and hence inferred first-best effort $a^*(k_t, x_{t-1})$ and the associated environment $\theta^*(k_t, x_{t-1})$.[17] (We will generously assume here that the buyer knows the beginning investment balance x_{t-1}. If not, the following argument would hold a fortiori.) The incumbent owner-manager would rationally anticipate these beliefs and consider whether the buyer's belief could be taken advantage of by investing $k_t \neq k^*(x_{t-1}, \theta_t)$. Now, the buyer can tell whether the contemporaneously realized cash revenue m is consistent with the conjectured first-best investment, since m is increasing in all of k_t, a_t and θ_t, and both a^* and θ^* are, by Lemma 1, increasing in k_t. Deviations from first-best investment by the owner-manager can therefore only escape immediate detection by the buyer if it produces an observable revenue cash flow m that mimics the first-best value that the buyer expects on the basis of k_t.[18] The owner-manager therefore needs to choose expenditure and effort such that, given a true state θ_t,

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[17] That x^* and θ^* are unique follows from (6) and (7).

[18] Generating non-first-best-consistent revenue is an out-of-equilibrium strategy for the owner-manager as long as

$$m(x_t, \theta_t) \neq m\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big)$$

induces a sufficiently unfavorable inference by the buyer, e.g.,

$$\hat{v}(k_t, m(x_t, \theta_t), x_{t-1}) \leq \min_{\theta_t}\{v(x_{t-1}, \theta_t)\}$$

in which case the non-first-best strategy would be dominated for all types θ_t by the hypothesized first-best equilibrium actions $k_t = k^*(x_{t-1}, \theta_t)$ and $a_t = a^*(x_{t-1}, \theta_t)$. In any event, however, generating non-first-best-consistent

$$m(x_t, \theta_t) = m\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big) \tag{8}$$

There are two potential ways to exploit (8). Either the owner-manager could overspend and set $k_t > k^*(x_{t-1}, \theta_t)$, while at the same time increasing the effort a_t such that (8) holds, thus making the buyer believe that the state is $\theta^*(k_t, x_{t-1}) > \theta_t$. As further elaboration in the upcoming analysis will confirm, such overinvestment benefits the owner-manager if

$$E(v_x)\frac{m_\theta}{m_x} < E_\theta(v) \Leftrightarrow E(v_x)m_\theta < E_\theta(v)m_x \tag{9}$$

at $x^*(x_{t-1}, \theta_t)$, because then mimicking the effect of a higher θ_t on observable revenue, m_θ, makes a naïve buyer infer a larger increment in future value, $E_\theta(v)$, than the actual increase, $E(v_x)$, per unit of overinvestment required to create the mimicking revenue increase, m_x. In other words, the increase in the buyer's conjectured firm value $\hat{v}$ is large enough to justify the extra investment effort the owner-manager has to expend. If the inequality in (9) holds in reverse, the manager finds generating inflated revenue to increase the buyer's perception of θ_t too costly. Instead, underspending, i.e., $k_t < k^*(x_{t-1}, \theta_t)$, and reducing effort, again such that (8) holds, now becomes attractive. The buyer would infer a lower state $\theta^*(k_t, x_{t-1}) < \theta_t$ but would underestimate the drop in firm value because the real underinvestment effect, $E(v_x)$, is now larger than the effect $E_\theta(v)$ imagined by the buyer, after scaling by the marginal revenue ratio m_θ/m_x at which the owner-manager must invest to manipulate the buyer's beliefs about θ_t. The buyer would thus again end up overpaying for the firm. As a result, the saving in the owner-manager's effort cost outweighs the reduction in the purchase price $\hat{v}$. Now, a rational buyer would of course anticipate the owner-manager's decisions, and hence the hypothesized first-best equilibrium breaks down, except in the knife-edge case when (9) holds as an equality everywhere. Proposition 1 below formalizes this result. A parametric illustration can be found in Appendix B.

Proposition 1. *If $E(v_x)m_\theta \neq E_\theta(v)m_x$ at any $x^*(x_{t-1}, \theta_t)$, then an owner-manager expecting to sell the firm to another owner-manager in period $T > t$ does not make first-best investments.*

---

revenue is always associated with non-first-best investment, which supports the conclusion ultimately reached in Proposition 1 below.

The import of Proposition 1 is that credible commitment to investing optimally is impossible in an owner-managed economy without full observability of investments and economic conditions.[19] Any attempt to solve the problem will need to circumvent two obstacles, namely, that, firstly, the owner-manager bears an investment cost whose benefits will, in part, only be realized after the end of the owner-manager's tenure at the firm, and that, secondly, the transaction between the owner-manager and the buyer of the firm occurs *after* the owner-manager has received private information about the firm's economic condition.[20] Making the transaction price $\hat{v}$ contingent on expenditures and revenues in periods after the sale would not eliminate these issues, as the buyer would then face the same incentives after the sale as the seller did before the sale. (The discussion will return to the role of contingent consideration in the last section.) One can thus conclude that concentrating ownership and control (management responsibility) in the hands of one party creates informational friction and thus a loss in firm value.[21] If a first-best solution exists, it must therefore involve the separation of ownership from management at some point, as summarized in the corollary below.

Corollary 1. *If $E_\theta(v)m_x \neq E(v_x)m_\theta$ at any $x^*(x_{t-1}, \theta_t)$ and if firm ownership is to be transferred at some future date $T > t$, then separation of ownership and control is a necessary condition for implementing first-best investment decisions.*

Separation of Ownership and Management

It remains to be established that there indeed exists an organizational design with separation of duties that attains better, possibly first-best, outcomes than an economy of owner-managed firms. Any such solution has to take the form of a contractual arrangement by which the owner promises a payment to a manager who makes the investment decisions on the owner's behalf. The

[19] A detrimental incentive effect from the limited observability of managerial decisions, combined with a limited horizon problem, also arises in signal jamming and real effects models (see, for example, Stein 1989, and Kanodia 2007). The real effects and signal jamming models do not, however, derive accounting-based incentive mechanisms to solve the problem. Their assumptions also differ in that, in their typical forms, these models do not include private managerial effort and do not involve a decomposition of net cash flow into revenues and expenditures.

[20] These conditions also apply if the owner-manager wanted to retire from managing duties and hire an external manager while retaining ownership. The owner's private knowledge of θ_{t-1} would prevent efficient negotiation of the manager's compensation contract. The discussion will return to this point at the end of the next section.

[21] In practice, this problem seems particularly acute in fields such as private equity, venture capital and hedge fund activism, where short-term ownership interests interfere with operational decision-making. Credit to Evgeny Petrov for these examples.

rules determining the payment amount must be agreed upon *before* the manager obtains private information and must make the manager internalize the owner's objective function, such that maximizing compensation becomes equivalent to maximizing value creation. Specifically, suppose that the manager is hired to run the firm for one period, hereafter referred to as period t.[22] After observing θ_t, the manager determines the expenditure k_t and the effort a_t and, after the realization of the revenue m at the end of the period, receives some compensation p from the owner, which may possibly be contingent on m and k_t. The owner, removed from the daily operations, observes neither the manager's effort nor θ_t. The terms of the manager's compensation contract are visible to any potential future buyer of the firm.

The objective is to design an incentive scheme that implements first-best investment as an equilibrium outcome. Paying a fixed salary would not accomplish this because the manager bears the effort cost q and would therefore set the effort level a_t to the minimum value, regardless of θ_t. Stock-based compensation would likewise fail because the firm's stock price reflects investors' conjectured firm value $\hat{v}$, which recreates the incentive problem underlying Proposition 1 since the manager has now also become an owner.[23] A third simple alternative is paying the manager based on the firm's cash flow, i.e.,

$$p\big(k_t, m(x_t, \theta_t)\big) = m(x_t, \theta_t) - c(k_t) + \bar{q}$$

where $\bar{q}$ is a fixed salary component, set such that the manager's expected payoff, net of effort cost, equals the reservation wage of zero. The cash flow-based scheme would likewise fail to induce first-best investment, in view of the manager's first-order optimality conditions

$$m_x(x_t, \theta_t) - c'(k_t) = 0$$

and

$$m_x(x_t, \theta_t) - q'(a_t) = 0$$

which, compared to (4) and (5), are missing the future value elements. The manager would hence be incentivized to ignore the long-term effects of investment.[24]

[22] No substantive differences arise in a contract spanning several periods, as long as the payment terms are agreed upon ex ante.

[23] Based on similar logic, Dutta and Zhang (2002) demonstrate that the use of mark-to-market accounting in performance measurement fails to achieve incentive alignment. Stock price may, however, be a useful ingredient in a compensation scheme if part of the information it contains is both incentive-relevant and orthogonal to all other contractible variables in the model (Dutta and Reichelstein 2005b).

[24] This problem would persist even if the manager were hired for multiple periods, as long as the manager's tenure is shorter than the life of the firm.

Consider instead paying the manager according to a performance metric p that is calculated using accrual accounting, i.e., based on cash flows that have been adjusted to take the long-term effects of investment into account. Cash revenues and expenditures are verifiable, and hence this accrual-basis performance pay can be any mapping $p: \{m(x_i, \theta_i), c(k_i)\}_{i=1}^{t} \to \mathbb{R}$ or, equivalently, $p: \{m(x_i, \theta_i), k_i\}_{i=1}^{t} \to \mathbb{R}$, since the cost function c is invertible. For an owner expecting to sell the firm at some future date T, the modified optimization problem at the beginning of period $t < T$, before hiring the manager, is thus

$$
\begin{aligned}
&E(v(x_{t-1}, \theta_t)|\theta_{t-1}) \\
&\quad = \max_{p(\cdot)} \left\{ \sum_{i=t}^{T-1} \gamma^{i-t} E\left(m(x_i, \theta_i) - c(k_i) - p\left(\left\{ m(x_j, \theta_j), k_j \right\}_{j=1}^{i} \right) \middle| \theta_{t-1} \right) \right. \\
&\quad \left. + \gamma^{T-t} E(\hat{v}(\{k_i, m(x_i, \theta_i)\}_{i=1}^{T-1}) | \theta_{t-1}) \right\}
\end{aligned}
$$

subject to the manager's incentive-compatibility constraint

$$\{k_i, a_i\} \in \arg\max_{\{k_i, a_i\}} \left\{ p\left(\left\{ m(x_j, \theta_j), k_j \right\}_{j=1}^{i} \right) - q(a_i) \right\}$$

at each θ_i in each period i, the manager's participation constraint

$$E\left(p\left(\left\{ m(x_j, \theta_j), k_j \right\}_{j=1}^{i} \right) - q(a_i) \middle| \theta_{i-1} \right) = 0$$

as of the beginning of each period i, and the future buyer's rationality constraint

$$\hat{v}(\{k_i, m(x_i, \theta_i)\}_{i=1}^{T-1}) = E(v(x_{T-1}, \theta_T)|\theta_{T-1})$$

at *any* possible selling date T.

In critical contrast to the owner-managed firm in Proposition 1, there is no longer any information asymmetry between the owner and the future buyer, since all cash flows and all managers' contracts are observable to the buyer and since the owner has no private information about managers' effort or about θ_i. The purchase price constraint $\hat{v} = E(v)$ is therefore satisfied at any T as long as owner and buyer interpret the observable variables identically. In particular, rationality demands such an identical interpretation if, in equilibrium, the mapping from the manager's effort and the environmental state θ_i to the firm's realized cash flows is one-to-one. We will posit this equilibrium property for now and verify it later.

Now consider the manager's incentive-compatibility condition. If accrual accounting is to avoid the managerial myopia induced by the cash-basis scheme and is to consider all future implications of the manager's actions, it must, for a manager working in period t, include the unobservable future value component $E(v(x_t, \theta_{t+1})|\theta_t)$. However, future value is a function of the investment decisions that the performance measurement scheme itself has induced in the first place. Incentive-compatibility means that the manager maximizes the performance metric p by generating the cash flows based on which p infers $E(v(x_t, \theta_{t+1})|\theta_t)$ correctly. The contractual arrangements between successive generations of owners and managers must sustain a sequentially rational perfect Bayesian equilibrium across periods, such that, in any period t, the owner's and the new manager's interpretation of the cash flows produced by the manager who ran the firm in period $t-1$ leads to correct inferences about the investment state x_{t-1} and the environment θ_{t-1}, based on which the new manager's incentive compensation in period t is determined. In analogy to (3), the owner's optimization problem then reduces to the Bellman equation

$$\begin{aligned} &E(v(x_{t-1}, \theta_t)|\theta_{t-1}) \\ &= \max_{p(\cdot)} E(m(x_t, \theta_t) - c(k_t) - p(k_t, m(x_t, \theta_t), x_{t-1}) + \gamma E(v(x_t, \theta_{t+1})|\theta_t)|\theta_{t-1}) \end{aligned} \tag{10}$$

subject to the incentive-compatibility constraint

$$\{k_t, a_t\} \in \arg \max_{\{k_t, a_t\}} \{p(k_t, m(x_t, \theta_t), x_{t-1}, \theta_{t-1}) - q(a_t)\}$$

for each θ_t, and the participation constraint

$$E(p(k_t, m(x_t, \theta_t), x_{t-1}) - q(a_t)|\theta_{t-1}) = 0$$

as of the beginning of period t.

If first-best investment were the unique solution to (10), then the owner, the manager to be hired in period $t+1$, and any potential buyer of the firm would, in view of Lemma 1, all infer x_t and θ_t correctly from the expenditure k_t and the revenue m in period t and in turn construct an analogous optimal contract in $t+1$. Since t is arbitrary, the feasibility of incentive-compatible, first-best optimal compensation contracts in all periods would then follow by induction, as long as some initial state $\{x_0, \theta_0\}$ (say, at the firm's founding date) is commonly known.[25]

To test the viability of this conjecture, suppose, tentatively, that an incentive-compatible compensation scheme was successfully implemented in period $t-1$ and that the owner and the

[25] Revealing the initial state publicly poses a non-trivial problem when an existing owner-managed firm attempts to implement separation of ownership and management. The discussion will examine this problem in the next section.

new manager to be hired in period t have correct beliefs about x_{t-1} and θ_{t-1}. (Recall that they know the terms of the predecessor manager's compensation arrangement.) Now posit that incentive-compatibility can be achieved in period t if the manager is paid

$$\begin{aligned} &p(k_t, m(x_t,\theta_t), x_{t-1}, \theta_{t-1}) \\ &\quad = r(k_t, x_{t-1})m(x_t,\theta_t) - h(k_t, x_{t-1}) - z(k_t, m(x_t,\theta_t), x_{t-1}) \\ &\quad + \bar{q}(x_{t-1}, \theta_{t-1}) \end{aligned} \tag{11}$$

where r, h and z are continuously differentiable functions, yet to be specified, and $\bar{q}$ is a fixed salary component, chosen so that expected compensation, as of the beginning of period t and conditional on x_{t-1} and θ_{t-1}, equals the manager's reservation wage of zero.[26] The z-function will serve as a disciplinary device and will be designed to have a local minimum of $z = 0$ on the level set of expenditure-revenue pairs that are consistent with the observable cash flow outcomes intended by the owner (and, conversely, to impose a suitably high penalty $z > 0$ on all other outcomes, which the manager will therefore not implement in equilibrium).[27] Obviously, (11) would reduce to the cash-basis accounting scheme if $r(k_t, x_{t-1}) = 1$ and $h(k_t, x_{t-1}) = c(k_t)$ everywhere, and hence the accrual component of (11) is, up to a constant, equal to

$$(r(k_t, x_{t-1}) - 1)m(x_t,\theta_t) - h(k_t, x_{t-1}) + c(k_t)$$

First-best investment clearly would solve (10) if, for all θ_t, the performance metric in (11) were to have its unique maximum at the same solution

$$\{k^*(x_{t-1},\theta_t), a^*(x_{t-1},\theta_t)\} = \arg\max_{k_t,a_t}\{m(x_t,\theta_t) - c(k_t) - q(a_t) + \gamma E(v(x_t,\theta_{t+1})|\theta_t)\}$$

as the unconstrained problem in (2). To verify the feasibility of this result, let $z = z_k = z_m = 0$ if and only if (8) holds. The set of compensation-maximizing investment choices then includes all k_t and a_t that solve the first-order necessary conditions

$$\begin{aligned} &p_m(k_t, m(x_t,\theta_t), x_{t-1}, \theta_{t-1})m_x(x_t,\theta_t) - q'(a_t) \\ &\quad = r(k_t, x_{t-1})m_x(x_t,\theta_t) - q'(a_t) = 0 \end{aligned} \tag{12}$$

and

[26] Note that x_{t-1} and θ_{t-1} have been realized by the time the compensation contract is agreed, so that $\bar{q}(x_{t-1},\theta_{t-1})$ is constant with respect to decisions and outcomes in period t.

[27] If the firm's observable cash flows were realized with error, the local minimum condition would instead apply to the expected value of z. Note that the penalty mechanism alone will not be sufficient to force first-best efficiency since, in view of Proposition 1, different combinations of effort choices a_t and environments θ_t can generate the same observable cash flow outcomes.

$$
\begin{aligned}
&p_k(k_t, m(x_t,\theta_t), x_{t-1}, \theta_{t-1}) + p_m(k_t, m(x_t,\theta_t), x_{t-1}, \theta_{t-1}) m_x(x_t,\theta_t) \\
&\quad = r_k(k_t, x_{t-1}) m(x_t,\theta_t) + r(k_t, x_{t-1}) m_x(x_t,\theta_t) - h'(k_t, x_{t-1}) = 0
\end{aligned}
\tag{13}
$$

For each θ_t, the solution set to (12) and (13) needs to include the unique first-best solution to (4) and (5). Now recall that, by Lemma 1 and in view of (6) and (7), the state transition function can be inverted to obtain, for any given expenditure k_t, the unique first-best effort $a^*(k_t, x_{t-1})$ and the associated state $\theta^*(k_t, x_{t-1})$. Evaluating (12) at these values implies that

$$
r(k_t; x_{t-1}) = \frac{q'\big(a^*(k_t, x_{t-1})\big)}{m_x\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big)}
\tag{14}
$$

where

$$
x^*(k_t, x_{t-1}) = x_{t-1} + k_t + a^*(k_t, x_{t-1})
$$

and, similarly, evaluating (13) at these values yields

$$
\begin{aligned}
&h(k_t, x_{t-1}) \\
&\quad = \int \Big(r(k_t, x_{t-1}) m_x\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big) \\
&\quad + r_k(k_t, x_{t-1}) m\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big) \Big)\, dk_t \\
&\quad = c(k_t) + \int r_k(k_t, x_{t-1}) m\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big)\, dk_t
\end{aligned}
\tag{15}
$$

where the second equality follows from (4) and (5). The fixed wage component is therefore

$$
\begin{aligned}
&\bar{q}(x_{t-1}, \theta_{t-1}) \\
&\quad = E\big(q\big(a^*(x_{t-1}, \theta_t)\big) \big| \theta_{t-1}\big) \\
&\quad - E(r(k^*(x_{t-1}, \theta_t), x_{t-1}) m(x^*(x_{t-1}, \theta_t), \theta_t) - h(k^*(x_{t-1}, \theta_t), x_{t-1}) | \theta_{t-1})
\end{aligned}
\tag{16}
$$

It remains to be determined whether first-best investment is indeed the only solution to (12) and (13). To narrow the manager's scope for suboptimal behavior, investments that violate (8), i.e., that yield revenue-expenditure pairs inconsistent with *any* possible first-best outcome, can be rendered unattractive through the penalty function z. For arbitrary revenue and cost functions, a necessary and sufficient condition to this end is that

$$
\begin{aligned}
&z(k_t, m(x_t, \theta_t), x_{t-1}) \\
&\quad \geq \Big(1 - \mathbf{I}_{m^*(k_t)}\big(m(x_t, \theta_t)\big)\Big) \Big(r(k_t, x_{t-1}) m(x_t, \theta_t) - h(k_t, x_{t-1}) - \underline{p}(x_{t-1}) \Big)
\end{aligned}
\tag{17}
$$

where

$$
\underline{p}(x_{t-1}) = \inf\{ r(k^*(\theta_t), x_{t-1}) m(x^*(\theta_t), \theta_t) - h(k^*(\theta_t), x_{t-1}) \}
$$

is the lowest possible (variable) compensation the manager could obtain when investing first-best optimally, and where the indicator function $\mathbf{I}_{m^*(k_t)}$ equals unity when the realized revenue m equals its expected first-best value $m\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big)$ for the observed expenditure k_t, and zero otherwise.[28] In particular, if (17) holds with equality, managerial pay reduces to

$$\begin{aligned} & p(k_t, m(x_t, \theta_t), x_{t-1}, \theta_{t-1}) \\ & \quad = \mathbf{I}_{m^*(k_t)}\big(m(x_t, \theta_t)\big)\big(r(k_t, x_{t-1})m(x_t, \theta_t) - h(k_t, x_{t-1})\big) \\ & \quad + \Big(1 - \mathbf{I}_{m^*(k_t)}\big(m(x_t, \theta_t)\big)\Big)\underline{p}(x_{t-1}) + \bar{q}(x_{t-1}, \theta_{t-1}) \end{aligned}$$

Proposition 2 below establishes that, among the remaining, first-best-mimicking actions the manager is left to choose from, the actual first-best investment is the unique choice to maximize p, and that (11), with its components defined in (14) through (17), indeed constitutes the only incentive-compatible accounting method with this property. A simple parametric illustration of the general result can be found in Appendix B.

Proposition 2. *A first-best managerial compensation scheme is incentive-compatible if and only if performance is measured by (11), with r, h, $\bar{q}$ and z defined by (14) through (17).*

The design plan underlying Proposition 2 rests on two self-fulfilling premises. First, the compensation contract localizes the problem by declaring all past and future managers' actions exogenously fixed at their corresponding first-best values. This assumption effectively holds the current manager responsible for the full agency cost of the firm's value optimization problem. Secondly, any observable outcome during the incumbent manager's term in office is treated as the product of first-best optimal decision-making on the part of that manager, which, by Lemma 1, implies a one-to-one inference mapping from outcomes to (otherwise unobservable) effort. Now, in any given environment θ_t, the manager faces the same choice set as the owner-manager in Proposition 1: there exists a continuum of expenditure and effort choices that, by equation (8), generate revenue figures apparently consistent with first-best investment but are, except for one, inefficient. The compensation formula p in (11) aligns incentives in that, for each θ_t, the actual first-best investment from this continuum is indeed the only investment choice that maximizes p.

[28] On a technical note, the indicator function in (17) is not differentiable in the classical sense but that one can use the Dirac delta function as its derivative, in which case z has all required properties.

To extend this local incentive-compatibility to past and future periods and thus validate the initial localization restriction, it now suffices to apply the compensation scheme across all periods, i.e., the manager's predecessors and successors indeed act as assumed if they are likewise compensated according to (11). Observable cash flow outcomes then reveal x_t and θ_t and permit sequentially optimal contracting as the owner hires successive managers.

The incentive pay in (11) is designed to cure the inefficiency inherent in owner-managed enterprises, as laid out in Proposition 1, but the conjecture-and-verify approach leading to the solution in Proposition 2 might not fully convey the intuition how the underlying accounting leads to the separation of ownership and control as a first-best efficient organizational design. The formal logical connection between the problem and its solution becomes more apparent if one compares value creation and managerial pay in equilibrium. Consider the marginal net benefit accruing to the manager when the environment improves. Incentive-compatibility of the compensation scheme implies optimal investment, i.e., $k_t = k^*(x_{t-1}, \theta_t)$ and $a_t = a^*(x_{t-1}, \theta_t)$, at all θ_t. An increase in θ_t therefore changes the manager's payoff, net of incremental effort cost, by

$$\begin{aligned} \frac{d}{d\theta_t}\Big(p\big(k_t, m(x_t, \theta_t)\big) - q(a_t)\Big) &= \frac{\partial p\big(k_t, m(x_t, \theta_t)\big)}{\partial \theta_t} = r(k_t) m_\theta(x_t, \theta_t) \\ &= m_\theta(x_t, \theta_t) + \gamma E(v_x(x_t, \theta_{t+1})|\theta_t) \frac{m_\theta(x_t, \theta_t)}{m_x(x_t, \theta_t)} \end{aligned} \tag{18}$$

where the first equality follows from the envelope theorem and the final equality obtains because

$$r(k_t) = 1 + \frac{\gamma E(v_x(x_t, \theta_{t+1})|\theta_t)}{m_x(x_t, \theta_t)} \tag{19}$$

after substitution of the first-order optimality condition in (5) into (14).[29] (For better readability, the notation indicating dependence on x_{t-1} is omitted from hereon, as x_{t-1} is known in equilibrium and constant with respect to the decisions and outcomes in t.) Now compare (18) to the total marginal surplus value accruing to owner and manager combined. Managerial pay drops out of the calculation as a transfer payment, leaving net value creation of

$$\begin{aligned} \frac{d}{d\theta_t}\Big(v(x_{t-1}, \theta_t) + p\big(k_t, m(x_t, \theta_t)\big) - q(a_t)\Big) &= \frac{\partial v(x_{t-1}, \theta_t)}{\partial \theta_t} \\ &= m_\theta(x_t, \theta_t) + \gamma E_\theta(v(x_t, \theta_{t+1})|\theta_t) \end{aligned} \tag{20}$$

[29] Changes in θ_t are the proper unit of analysis because, given the incentive-compatibility of p, all variation in managerial decisions and related outcomes arises from θ_t in equilibrium. Examining the level, rather than changes, of p would not be insightful since the level is determined by the manager's reservation wage, which is exogenous.

If the incentive contract had amounted to the canonical remedy of selling the firm to the agent, (18) and (20) would coincide. Instead, one can readily see that equivalence obtains if and only if

$$E(v_x)m_\theta = E_\theta(v)m_x \tag{21}$$

which is familiar from Proposition 1 as the knife-edge case in which even an owner-managed firm attains first-best efficiency.

Since (21) is necessary for incentive-compatibility but investment opportunities in reality do not come with that property except by fortuitous coincidence, the incentive scheme in Proposition 2 effectively turns the manager into the owner of a virtual enterprise in which (21) holds artificially by dint of purpose-designed accrual accounting. Incentive-compatibility now obtains when the value-maximizing investment decisions in this virtual enterprise are calibrated to match those of the real firm. The matching of decision incentives, however, leaves no degree of freedom to match payoff *levels*, so that real and virtual firms produce different amounts of surplus value, as indicated by the difference

$$\begin{aligned}\frac{dv(x_{t-1},\theta_t)}{d\theta_t} &= \frac{\partial v(x_{t-1},\theta_t)}{\partial\theta_t} - \frac{\partial p\big(k_t, m(x_t,\theta_t)\big)}{\partial\theta_t} \\ &= \gamma E_\theta(v(x_t,\theta_{t+1})|\theta_t) - \gamma E(v_x(x_t,\theta_{t+1})|\theta_t)\frac{m_\theta(x_t,\theta_t)}{m_x(x_t,\theta_t)}\end{aligned} \tag{22}$$

between (18) and (20). Incentive-compatibility obtains if and only if the excess or shortfall in (22) accrues to a party that is separate and distinct from the decision-maker, necessitating the presence of the owner as a residual claimant.[30]

To see how accrual-based performance accounting controls the incentives for over- and underinvestment discussed earlier in the context of Proposition 1, recall that an incentive to overinvest obtains when (21) is violated in the direction of

$$E(v_x)\frac{m_\theta}{m_x} < E_\theta(v)$$

as shown in (9). If managerial compensation were naively based on imputed incremental future firm value, overinvestment would earn the manager extra pay at a rate of $E_\theta(v)$ while actually only creating value at a rate of $E(v_x)$, scaled by the ratio m_θ/m_x at which the manager must in-

[30] One can also analogize the employment contract as renting, rather than selling, the firm to the manager, under an incentive-compatible profit-sharing scheme that leaves residual claims to the owner.

vest in order to mimic the effect of a unit-size increase in θ_t on observable revenue. The overinvestment scenario thus applies to business environments in which, in relative terms, the investment decisions under the manager's control impact revenue more strongly in the current period while exogenous factors impact revenue more strongly in future periods. By paying the manager only according to the investment effect and not for the imputed environmental factor, incentive-compatible performance measurement undercompensates the manager relative to incremental firm value and thereby removes the incentive to overinvest. This is why the performance measure in (11) depends on $E(v_x)$ but not on $E_\theta(v)$. The excess value accrues to the owner, whose return in (22) therefore increases in the degree to which $E_\theta(v)$ exceeds $E(v_x) \cdot m_\theta/m_x$, resulting in a dampened rise in the manager's welfare per unit of stock price appreciation.

In the converse case, $E(v_x) \cdot m_\theta/m_x > E_\theta(v)$, the future value implications of θ_t are lower, so that the incentive problem is reversed. The manager now would, if paid according to imputed incremental firm value, invest below the optimum, saving effort cost while incurring only a moderate reduction in pay because the naïve performance measurement would underestimate the decrement in firm value. Using $E(v_x)$ instead of $E_\theta(v)$ in measuring performance again renders this deviation unattractive. Performance accounting in this case accentuates the equilibrium sensitivity of the manager's welfare to incremental firm value in order to deter shirking and under-expenditure, forcing the manager to internalize a larger loss in future value than is actually induced by a decrease in θ_t. Notably, the manager is now also over-rewarded on the upside, relative to the amount of actual value created, leaving a negative residual value increment in (22) to be absorbed by the owner.[31] Firms with this latent underinvestment problem operate in environments in which the investment decisions under their managers' control affect revenue beyond the current period more strongly than exogenous factors do. Rewarding managers for good outcomes even to the point where the marginal reward exceeds the managers' marginal productivity is efficient in this case. Negative correlation between managerial pay and stock price is therefore not necessarily a sign of organizational capture by management or other forms of inefficiency.[32]

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[31] Recall, however, that this does not imply a negative return ex ante, as the owner extracts the expected surplus value via a suitably low fixed compensation component.

[32] In terms of the standard covariance formula, with expectations taken over θ_t, one has

$$E\Big(\big(v - E(v)\big)\big(p - E(p)\big)\Big) = E\Big(v\big(p - E(p)\big)\Big) < 0$$

when (22) is negative, since v is non-negative and decreasing in θ_t, p is increasing in θ_t, and $E\big(p - E(p)\big) = 0$.

Now, the sensitivity of the manager's net welfare in (18) is not amenable to empirical testing because the effort cost q is unobservable, but the conclusion extends readily to just the (observable) compensation payment p, sans the effort cost, since

$$\frac{dp}{d\theta_t} = rm_\theta + q'a_\theta^* = m_\theta + m_x a_\theta^* + \gamma E(v_x)\left(a_\theta^* + \frac{m_\theta}{m_x}\right) \tag{23}$$

is also independent of $E_\theta(v)$.

As a corollary to its relation with stock price, managerial pay appears similarly 'misaligned' in relation to the firm's contemporaneous cash flow, net of compensation. As a benchmark, suppose that effort were contractible, in which case the owner would pay the manager for the actual effort cost q incurred. Net cash flow then responds to an increase in θ_t by

$$\begin{aligned}\frac{d}{d\theta_t}(m - c - q) &= m_\theta + m_x(k_\theta^* + a_\theta^*) - c'k_\theta^* - q'a_\theta^* \\ &= m_\theta - \gamma E(v_x)(k_\theta^* + a_\theta^*)\end{aligned} \tag{24}$$

in view of (4) and (5). Absent agency problems, an improvement in environmental conditions, with the attendant increase in compensated effort, can thus lead to either an increase or a decrease in cash flow, depending on whether the incremental current-period revenue outweighs the additional expenditures made in response to the improved investment opportunities. Consider now the effect of incentive pay. Replacing q by p in (24) yields

$$\frac{d}{d\theta_t}(m - c - p) = -\gamma E(v_x)\left(k_\theta^* + a_\theta^* + \frac{m_\theta}{m_x}\right) \tag{25}$$

which is negative for all θ_t because the need to solve the limited-horizon agency problem requires paying out the revenue increment m_θ in (24) to the manager as incentive compensation, turning the possibility of a negative correlation between net cash flow and compensation into a certainty.[33] A unit increase in the firm's cash flow before compensation produces more than a unit increase in managerial pay. In contrast to its relation with stock price, managerial compensation thus *always* correlates negatively with the firm's net cash flow, not only when $E_\theta(v)/m_\theta < E(v_x)/m_x$.[34] The preceding observations are summarized below.

[33] The difference between (24) and (25) of course equals the manager's incremental net reward in (18).
[34] This observation comes with a critical caveat. Negative correlation obtains on within-period variation in θ_t, i.e., conditional on the expected values of compensation and cash flow. Expected cash flow levels and managerial pay are, however, not independent across time periods because investment and environment in the current period affect expected investment, environment and revenue in future periods. Before pooling data across multiple periods for empirical testing, it is therefore essential to subtract the period-specific expected values.

Corollary 2. *Managerial incentive pay correlates*

- *positively (negatively) with stock price if the payoff from investment is realized with more (less) delay than the payoff from environmental factors; and*
- *negatively with the firm's net cash flow after compensation.*

The accounting for managerial performance solves both a moral hazard problem and an adverse selection problem simultaneously, as the manager must not only be induced to commit unobservable effort in general but also to set the effort level in first-best response to a specific environmental state that only the manager can observe. Accordingly, compensation is part indemnity for effort and part incentive to reveal private information, as illustrated in (23): at any increment of θ_t, the manager receives a reimbursement of $q' a_\theta^*$ for the additional effort cost, and a bonus of rm_θ for staying on the first-best investment path. Substituting (14) into (23) to obtain

$$\frac{dp}{d\theta_t} = rm_\theta + q' a_\theta^* = q' \left(\frac{m_\theta}{m_x} + a_\theta^* \right)$$

shows that the information-related component rm_θ is itself partially dependent on the optimal effort choice but also increases in m_θ / m_x, the revenue contribution of θ_t relative to x_t. Hence, the larger the effect of changes in exogenous environmental factors, θ_t, to the firm's success, the less incentive pay is explained by the manager's own actions. In firms operating in volatile environments, much of the variation in managerial compensation may thus arise solely because outsiders, including investors, cannot distinguish managers' accomplishments from the effects of exogenous factors. High amounts of incentive pay observed in practice therefore need not necessarily be a reward for hard work but may rather constitute an ex-post rent extraction that investors must tolerate to make their managers react optimally to changing business environments.[35]

A further implication of (23) is that, as a function of θ_t, managerial pay has a higher variance than the underlying cost of effort q. To see why, observe that the difference in the variances of p and q, as functions of θ_t, is

$$E\big(p - E(p)\big)^2 - E\big(q - E(q)\big)^2 = E\big((p + q)(p - q)\big) > 0$$

[35] But recall that owners recapture this ex-post rent ex ante by a fixed adjustment to compensation. By contrast, agents earn strictly positive rents if they observe their private information before signing the contract (see, for example, Dutta and Reichelstein 2002).

where the final inequality follows from

$$\frac{dp}{d\theta_t} = rm_\theta + q'a_\theta^* > q'a_\theta^* = \frac{dq}{d\theta_t} > 0$$

and the initial equality follows from $E(p) = E(q)$. Recall further that investment consists of expenditures, at a cash cost of c, and managerial effort, at a cost of q. Since effort is unobservable, the firm's actual cash outlay for effort is the compensation payment p. Now observe that, in view of the necessary conditions (4) and (5), the marginal expenditure and effort costs, c' and q', always coincide at first-best investment, and hence, if c and q showed similar convexity for the average firms, the variances of c and q across θ_t in equilibrium would be about the same. That p has a higher variance than q would then imply that payments for observable investments, c, have a lower variance than payments to reimburse unobservable effort, p. In practice, one might therefore expect a firm relying predominantly on non-verifiable investment in human capital to have a higher variance in its cash outlays than a firm investing primarily in physical assets.

An implicit but critical condition underlying all observations made thus far is that the manager does not anticipate renewing the employment contract at the end of its term. The contract terms offered by the owner to the manager's successor in period $t+1$ are an equilibrium outcome based on inferences about the end-of-period investment variable x_t and the environmental state variable θ_t, neither of which is directly observable. If the new contract were to be offered to the incumbent manager and the latter had rational expectations during the original contract period, the anticipated renewal terms would factor into the manager's original investment decision. In all but the most fortuitous of circumstances, a distortion of incentives away from the originally intended, first-best equilibrium behavior then becomes unavoidable, as the next result shows.

Proposition 3. *Rehiring the manager after the initial term of employment makes first-best investment unattainable as an incentive-compatible outcome if*

$$\frac{\partial \bar{q}_{t+1}}{\partial \theta_t} \neq -E(m_x(x_{t+1}, \theta_{t+1}) + E(v_x(x_{t+1}, \theta_{t+2})|\theta_{t+1})|x_t, \theta_t)\frac{m_\theta(x_t, \theta_t)}{m_x(x_t, \theta_t)}$$

for any θ_t.

A manager expecting to receive a renewal contract in period $t+1$ faces two considerations in the investment decision in period t that would not arise in the absence of renewal. First, over-or underinvesting in period t would change the marginal return to future investment under the anticipated renewal contract in period $t+1$ and mislead the owner about the investment state x_t. This effect is the right-hand side of the inequality in Proposition 3. Second, over- or underinvestment in period t would alter the owner's beliefs about the state of the environment to values above or below the actual θ_t and thus make the owner increase or decrease the fixed pay component $\bar{q}$ of the offered renewal contract. This effect is the left-hand side of the inequality. Only if these two anticipation effects fortuitously neutralize each other exactly in all states can contamination of the manager's decision incentives in period t be avoided, but such neutralization requires an exceedingly improbable knife-edge scenario. As in Proposition 1, this knife-edge condition requires that marginal returns to investment and to environmental conditions are exactly equal, except that now the equivalence must straddle neighboring time periods.

One might intuit that designing a modified version of the original performance metric p could remedy the problem. This endeavor fails because the manager can accept or decline the renewal contract discretionarily at the beginning of the second period, and hence any modified performance metric used in the first period would have to be incentive-compatible under both renewal and non-renewal. Since declining an offered renewal contract is equivalent to the baseline one-period contract, the only possible solution is therefore the original performance metric p from Proposition 2, which, by Proposition 3, does not work except by fortuitous accident. In contrast to the baseline scenario, the renewal problem can therefore not be solved via strategically designed incentive pay. Generally, corruption of decision incentives can thus only be avoided if a new manager is hired at the end of the incumbent manager's employment contract.[36] Note, however, that the owner could still hire a manager for multiple successive periods, provided that the contract terms are fixed at the outset rather than renegotiated periodically, consistent with the common practice of multi-year employment contracts in higher-level management.

[36] Dutta and Reichelstein (2003) similarly conclude that a series of employment contracts only yields optimal investment if the manager is replaced in each new contract. While their model setup differs substantially from what is presented here, the central cause is analogous: after signing the original contract, the agent takes actions unobservable to the principal, and the resulting ex-post information asymmetry leaves incentive-compatibility intact only if the agent cannot exploit this informational advantage in a renewed interaction with the principal. Renewal contracts can also suffer efficiency losses from ratchet effects arising when a principal learns performance-relevant information about an agent over time (see, for example, Indjejikian and Nanda 1999).

Proposition 3 features an informed manager and an uninformed owner, but the result also applies, symmetrically, if the roles are reversed. In particular, consider an incumbent owner-manager looking to delegate management duties to a third party. The informational disadvantage of the new hire is the same as that of an owner renewing an incumbent manager's contract, and the same inefficiency problem arises. Hence, although separation of ownership and management, with incentive pay according to Proposition 2, is efficient, an existing owner-managed firm cannot attain this efficiency by simply hiring an external manager. This transition problem is formally summarized in the following corollary and explored in the next section.

Corollary 3. *An owner-manager expecting to hire a third-party manager to run the firm in period $t+1$ does not make first-best investments in period t if*

$$\frac{\partial \bar{q}_{t+1}}{\partial \theta_t} \neq -E(m_x(x_{t+1},\theta_{t+1}) + E(v_x(x_{t+1},\theta_{t+2})|\theta_{t+1})|x_t,\theta_t)\frac{m_\theta(x_t,\theta_t)}{m_x(x_t,\theta_t)}$$

for any θ_t.

Ownership Transfers with Contingent Consideration

The discussion began by observing in Proposition 1 that an owner-manager faces incentives to invest inefficiently when anticipating to sell the firm to another owner-manager in the future. Separation of ownership from management can, by Proposition 2, remedy the underlying information asymmetry problem and, one might expect, should therefore be the norm in practice. Yet, owner-managed enterprises in reality account for the vast majority of business entities. Even corporate-style organizations with separated ownership and management tend to have started out as owner-managed businesses, raising the question how they accomplished the transition despite the impossibility result in Corollary 3. These seeming inconsistencies between theory and practice could be reconciled if one were able to solve the transition problem by an alternative route: if an owner-manager sells the firm not to another owner-manager but to a buyer who then delegates decision-making to a third-party manager, does there exist an equilibrium in which the anticipation of such a deal incentivizes first-best investment prior to the sale? A solution of this form would redeem the owner-managed firm as a first-best efficient organizational design and link this efficiency causally to the prospect of instituting separation of ownership and management at the time when the firm is sold to a new owner.

To this end, consider an owner-manager looking to sell the firm at the end of period $t-1$. The seller reports the firm's investment state x_{t-1} and its environment θ_{t-1} to a prospective buyer as $\hat{x}_{t-1}$ and $\hat{\theta}_{t-1}$, respectively. (Recall that x_{t-1} and θ_{t-1} are sufficient to determine firm value but are unobservable to outsiders.) Suppose that the buyer commits to hiring a manager, independent of both buyer and seller, to run the firm in period t, with an incentive compensation contract as given in Proposition 2, based on the reported $\hat{x}_{t-1}$ and $\hat{\theta}_{t-1}$. Suppose further that seller and buyer agree on a purchase price in the form of contingent consideration, equal to

$$\begin{aligned}&\hat{v}(k_t, m(x_t, \theta_t), \hat{x}_{t-1})\\ &\quad = m(x_t, \theta_t) + \gamma E\big(v\big(x^*(k_t, \hat{x}_{t-1}), \theta_{t+1}\big|\theta^*(k_t, \hat{x}_{t-1})\big) - c(k_t)\\ &\quad - q\big(a^*(k_t, \hat{x}_{t-1})\big) - \beta z(k_t, m(x_t, \theta_t), \hat{x}_{t-1})\end{aligned} \tag{26}$$

and to be paid by the buyer after revenue and expenditure in period t have been realized. The penalty function z in (26) is the same as in the manager's compensation in Proposition 2, scaled by some factor $\beta > 0$, and thus reduces the purchase price whenever

$$m(x_t, \theta_t) \neq m\big(x^*(k_t, \hat{x}_{t-1}), \theta^*(k_t, \hat{x}_{t-1})\big)$$

i.e., when realized revenue is inconsistent with the expected first-best value given the observed expenditure k_t and the $\hat{x}_{t-1}$ reported by the seller. Proposition 4 below shows that this contract design solves the transition problem.

Proposition 4. *An owner-manager makes first-best investments and can sell the firm at first-best value at the end of any period* $t-1$ *if*

- *the selling price consists of contingent consideration in period* t*, equal to (26); and*
- *the buyer hires a third-party manager in period* t *whose compensation is given by (11), based on* $\hat{x}_{t-1}$ *and* $\hat{\theta}_{t-1}$ *reported by the owner-manager at the time of sale.*

The critical innovation in this buy-and-delegate transition scheme, relative to the impossibility result in Proposition 1, clearly lies in the introduction of the third-party manager. To see why hiring an external manager is a necessary condition for efficiency, consider the contrary case of making the purchase price $\hat{v}$ conditional on post-sale outcomes in period t while letting the buyer manage the firm. The buyer would then solve

$$\max_{\{k_i,a_i\}_{i=t}^{T}} \left\{ -\hat{v}(k_t, m(x_t,\theta_t), \{k_i, m(x_i,\theta_i)\}_{i=1}^{t-1}) + \sum_{i=t}^{T-1} \gamma^{i-t} E(m(x_i,\theta_i) - c(k_i) - q(a_i)|\theta_t) \right.$$
$$\left. + \gamma^{T-t} E(\hat{v}(\{k_i, m(x_i,\theta_i)\}_{i=t}^{T-1}) \middle| \theta_t) \right\}$$

where T is the date when the buyer in turn expects to sell the firm. If the solution were first-best, it would have to be the same regardless of the selling price $\hat{v}$ because first-best investment is independent of whether the firm has been sold. Hence, $\hat{v}$ must have a stationary point at first-best investment. Now, θ_t and the buyer's effort a_t are unobservable to anyone but the buyer, who therefore faces the same incentive to deviate as the seller did before period t. Incentive-compatibility therefore requires either a constant $\hat{v}$ or, in view of Proposition 2, that $\hat{v}_k = p_k$ and $\hat{v}_m = p_m$ everywhere. In either case, exact ex-post matching of firm value and selling price, i.e., achieving $v = \hat{v}$ at all θ_t, is impossible.[37] Selling price and actual value can only be equated in expectation, which requires $\hat{v}$ to contain a constant term such that $E(\hat{v}) = E(v)$ as of the selling date $t-1$. This constant term must necessarily depend on the seller's private information about θ_{t-1} at time $t-1$, creating again the very incentive problem underlying Proposition 1. Conversely, if the seller, rather than the buyer, were to act as manager in period t and had been given the optimal compensation p, the incentive problem from Proposition 3 would arise: expecting to be hired, the seller would deviate from first-best investment in periods $t-1$ and earlier in order to manipulate the terms of the anticipated compensation contract. Hence, giving management responsibility to either buyer or seller fails to implement first-best investment in equilibrium, even with a variable purchase price.

The impasse arises because a contract between only buyer and seller forces the payments between the two parties to depend on the seller's private information θ_{t-1}. Given the zero-sum nature of the transaction, any loss to the buyer, when the seller reports $\hat{\theta}_{t-1} \neq \theta_{t-1}$, must necessarily be the seller's gain. Hiring a third-party manager breaks this budget constraint. Now the seller's reported $\hat{\theta}_{t-1}$ affects only the manager's compensation, which is paid by the buyer, but not the purchase price. Misreporting θ_{t-1} would now merely induce a wealth transfer between buyer and manager without changing the seller's own payoff. The seller is paid the actual ex-post firm value v, as inferred in (26) from realized revenue and expenditure outcomes in period t. The

[37] The only exception is again the knife-edge edge case where $E_\theta(v)m_x = E(v_x)m_\theta$ at all first-best outcomes.

reason that the valuation in (26) is correct in equilibrium and unaffected by the seller's reported $\hat{\theta}_{t-1}$ is that the manager's decision incentives are independent of $\hat{\theta}_{t-1}$, as $\hat{\theta}_{t-1}$ affects only the fixed component $\bar{q}$ of managerial pay, but not its variable parts. (Recall that $\bar{q}(x_{t-1}, \theta_{t-1})$ is the only part of p in (11) that depends on θ_{t-1}.) Incentive-compatibility is therefore unaffected by $\hat{\theta}_{t-1}$ and the manager invests optimally as long as the seller reports x_{t-1} correctly. Making misreporting of x_{t-1} unattractive is the role of the penalty term βz in (26). The manager's incentive pay in (11), including the penalty z, is calibrated toward the first-best outcomes expected under $\hat{x}_{t-1} = x_{t-1}$. If the seller misreported $\hat{x}_{t-1} \neq x_{t-1}$, generating these expected first-best outcomes would become inefficiently costly for the manager, who, for any continuously differentiable z, would now prefer to incur some amount of penalty and deviate from the first-best outcomes imputed under $\hat{x}_{t-1}$. The observed inconsistency between expected and realized revenue and expenditure outcomes would in turn trigger the penalty βz in the seller's payoff. For sufficiently large β, the seller will then not find it profitable to misreport x_{t-1} in the first place. The third-party manager thus serves two roles, first, as an arbiter to determine first-best investment, with decision incentives undistorted by ownership interests, and, second, as a third-party recipient (or 'budget breaker') whose payoff absorbs all variation in $\hat{\theta}_{t-1}$ that would otherwise accrue to the seller, thus neutralizing the seller's incentive to exploit the private knowledge of θ_{t-1}. The latter point also shows why this mechanism requires an ownership transfer. Without the buyer in play, the seller would have to pay the manager's compensation personally and would therefore be incentivized to manipulate $\hat{\theta}_{t-1}$ at the manager's expense, as shown in Corollary 3.

The purchase contract in Proposition 4 rationalizes the common practice of including contingent consideration (or earn-out clauses) in acquisition deals. Consistent with the model, earn-out agreements in practice tend to rely on detailed, accounting-based measures of post-deal performance and occur most frequently when the acquired entity is small, privately held and owner-managed, conditions under which the information asymmetry between buyer and seller is high. At the same time, earn-out agreements in practice frequently let the selling entrepreneur stay on as a manager, an arrangement that, in view of Proposition 3, leads to inefficiency. Hiring a seller should therefore only occur in the presence of significant overriding concerns outside the model, such as specific human talent or knowledge that a third-party manager could not provide.

That owner-managed firms have incentives to invest first-best optimally after all, given well-designed ownership transfer contracts, is not a contradiction to the need for separation of ownership and control observed earlier. Rather, the prospect of creating a separated organizational structure at the next ownership transfer is precisely what sustains the incentive for first-best behavior in owner-managed firms in the first place. Proposition 4 thus predicts an asymmetry in the organizational transformations accompanying business takeovers. Owner-managed entities should, absent overriding concerns, become run by hired managers after an acquisition. By contrast, separated organizations, when acquired, may either remain separated or be converted into owner-managed entities by the acquirer, in either case without loss of efficiency.

IV. CONCLUSION

Managers not only have privileged access to information about their companies and take decisions that investors can only imperfectly observe, but they also tend to receive their compensation well before investors get to see the long-term outcomes of their managers' job performance. Incentive pay based on properly designed performance accounting can address this horizon mismatch problem. To achieve alignment with investors' goal of maximizing value creation, performance accounting must be constructed to accrue the putative future value impact of the manager's current actions in a manner that neutralizes the manager's incentives to manipulate the measurement process through an inefficient use of hidden actions and information. The results of this design exercise have been laid out in this paper in a simple dynamic model and are noteworthy in several respects. First, optimally calibrated incentive pay is not equivalent to selling the firm to the manager, providing a rationale for the separation of ownership and management often seen in practice and questioning the efficacy of stock-based compensation. Optimal managerial compensation can, in fact, correlate negatively with stock price. Second, managerial pay is in part compensation for effort and in part inducement to respond optimally to private information. The more important the information aspect, the less of the variation in pay is explained by managers' own effort, but such rewarding of exogenous factors does not signify an inefficient compensation scheme. Finally, the mere prospect of separating ownership from management and putting performance accounting in place in the future can induce optimal decision incentives even in businesses presently managed by their owners. The organizational transition must, however, coincide with the sale of the firm to a new owner under an accounting-based earn-out agreement.

None of these results assume that accounting produces otherwise unavailable information by fiat. Rather, performance accounting is merely a transformation of realized cash flows, but strategically designed based on an understanding of the underlying incentive problem.

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APPENDIX A – PROOFS

Proof of Lemma 1. The uniqueness of the investment policy function $(x_{t-1}, \theta_t) \rightarrow (k^*, a^*)$ and the concavity of the value function v follow from Stokey and Lucas (1989), chapter 9, theorems 9.6 and 9.8, and chapter 12.6, lemma 12.14. To establish that k^* and a^* are increasing in θ_t, differentiate the first-order conditions (4) and (5) totally. By the implicit function theorem, the response functions k_θ^* and a_θ^* are then characterized by

$$\big(m_{xx}(x_t,\theta_t) + \gamma E(v_{xx}(x_t,\theta_{t+1})|\theta_t)\big)\big(k_\theta^*(x_{t-1},\theta_t) + a_\theta^*(x_{t-1},\theta_t)\big) + m_{x\theta}(x_t,\theta_t)$$
$$+ \gamma E_\theta(v_x(x_t,\theta_{t+1})|\theta_t) - c''(k_t)k_\theta^*(x_{t-1},\theta_t) = 0$$

and

$$\big(m_{xx}(x_t,\theta_t) + \gamma E(v_{xx}(x_t,\theta_{t+1})|\theta_t)\big)\big(k_\theta^*(x_{t-1},\theta_t) + a_\theta^*(x_{t-1},\theta_t)\big) + m_{x\theta}(x_t,\theta_t)$$
$$+ \gamma E_\theta(v_x(x_t,\theta_{t+1})|\theta_t) - q''(a_t)a_\theta^*(x_{t-1},\theta_t) = 0$$

where

$$E_\theta(v_x(x_t,\theta_{t+1})|\theta_t) \equiv \frac{\partial}{\partial\theta_t}E(v_x(x_t,\theta_{t+1})|\theta_t)$$

and v, m, c and q are all evaluated at the first-best solution values $k^*(x_{t-1},\theta_t)$ and $a^*(x_{t-1},\theta_t)$. Rearranging yields (6) and (7), which are positive in view of the concavity of v. An analogous argument shows that k^* and a^* are decreasing in x_{t-1}.

Proof of Proposition 1. Suppose that the owner-manager has made first-best investment choices through period $t-1$ and that a potential outside buyer has correctly inferred x_{t-1} from the history of expenditures and revenues. Under a first-best investment policy, any observed pair of expenditure k_t and revenue m in period t then must be consistent in the sense that

$$m(x_t,\theta_t) = m\big(x^*(k_t,x_{t-1}),\theta^*(k_t,x_{t-1})\big)$$

A deviation from first-best expenditure then appears consistent with first-best outcomes if the owner-manager adjusts effort by some amount a_k such that, for the actual θ_t observed,

$$\begin{aligned} &m_x(x_t,\theta_t)\big(1 + a_k(x_{t-1},\theta_t)\big) \\ &\quad = m_x\big(x^*(k_t,x_{t-1}),\theta^*(k_t,x_{t-1})\big)\big(1 + a_k^*(x_{t-1},\theta_t)\big) \\ &\quad + m_\theta\big(x^*(k_t,x_{t-1}),\theta^*(k_t,x_{t-1})\big)\theta_k^*(k_t,x_{t-1}) \end{aligned}$$

where a_k^* and θ_k^* are implied by (6) and (7). Then

$$\begin{aligned} & m_x\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big)\big(a_k(x_{t-1}, \theta_t) - a_k^*(x_{t-1}, \theta_t)\big) \\ & \quad = m_\theta\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big)\theta_k^*(k_t, x_{t-1}) \end{aligned} \tag{A1}$$

for any possible deviation from the path of first-best outcomes $x = x^*$. Anticipating to sell the firm in period $t + 1$ to a buyer who naively believes that first-best investment has occurred, the owner would set the investment expenditure to solve the first-order condition

$$m_x(1 + a_k) + E(v_x)(1 + a_k^*) + E_\theta(v)\theta_k^* = c' + q' a_k \tag{A2}$$

where a_k is characterized by (A1). For the solution to (A2) to coincide with the first-best investment policy, the owner-manager's choice of k_t and a_t must simultaneously also solve (4) and (5), which, after substitution into (A2), yields

$$E(v_x)(a_k^* - a_k) + E_\theta(v)\theta_k^* = 0 \Leftrightarrow E_\theta(v)m_x = E(v_x)m_\theta \tag{A3}$$

where the equivalence relation follows from (A1). Since the first-best continuation value v anticipates first-best investment in all future periods, this necessary condition must hold for all possible θ_t in all periods t in order for first-best investment to be incentive-compatible. To see that incentive-compatibility cannot be restored by conditioning the selling price on outcomes in periods after t, consider a selling price $\hat{v}$ contingent on outcomes in period $t + 1$, the first period managed by the new owner, who now faces the optimization problem

$$\begin{aligned} \max_{\{k_i, a_i\}_{i=t+1}^{T-1}} & \Bigg\{ -\hat{v}(k_{t+1}, m(x_{t+1}, \theta_{t+1}), \{k_i, m(x_i, \theta_i)\}_{i=1}^{t}) \\ & + \sum_{i=t+1}^{T-1} \gamma^{i-t-1} E(m(x_i, \theta_i) - c(k_i) - q(a_i) | \theta_t) + \gamma^{T-t-1} E\big(\hat{v}(\{k_i, m(x_i, \theta_i)\}_{i=1}^{T-1}) \big| \theta_{t+1}\big) \Bigg\} \end{aligned}$$

Since first-best investment is independent of whether the firm has been sold, the selling price $\hat{v}$ must have a stationary point at $x^*(\theta_{t+1})$ and offer no incentive to deviate, which requires

$$\hat{v}_x(a_k^* - a_k) + \hat{v}_\theta \theta_k^* = 0$$

at all θ_{t+1}. Then $\hat{v} \neq v$ unless (A3) holds, while at the same time $\hat{v}$ is now fixed up to a constant. Setting $\hat{v}$ equal to firm value at time t in expectation can then only be achieved by fixing this constant at time t, which reintroduces the seller's original incentive problem. The same argument applies when conditioning on any other periods $t + i$, for $i > 1$.

Proof of Proposition 2. After substitution of (14) and (15) into the first-order conditions (12) and (13), the compensation-maximizing choices of effort and expenditure must solve

$$r(k_t, x_{t-1})m_x(x_t, \theta_t) - q'(a_t) = \frac{q'\big(a^*(k_t, x_{t-1})\big)m_x(x_t, \theta_t)}{m_x\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big)} - q'(a_t) = 0 \quad \text{(A4)}$$

and

$$\begin{aligned} &r(k_t, x_{t-1})\left(m_x(x_t, \theta_t) - m_x\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big)\right) \\ &\quad + r_k(k_t, x_{t-1})\left(m(x_t, \theta_t) - m\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big)\right) \\ &\quad - z_k(k_t, m(x_t, \theta_t), x_{t-1}) - z_m(k_t, m(x_t, \theta_t), x_{t-1})m_k(x_t, \theta_t) = 0 \end{aligned} \quad \text{(A5)}$$

By design of the penalty function z, producing revenue

$$m(x_t, \theta_t) \neq m\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big)$$

fails to maximize compensation, and so the manager creates expenditure and revenue consistent with first-best outcomes. In view of $z_k = z_m = 0$ at first-best outcomes, (A5) then reduces to

$$r(k_t, x_{t-1})\left(m_x(x_t, \theta_t) - m_x\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big)\right) = 0$$

to which the only solution is investing $x = x^*$ because m is increasing and concave in x and because $m_{x\theta} > 0$. The second-order condition

$$\begin{aligned} &-r(k_t, x_{t-1})\left(m_{x\theta}\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big) \cdot \theta_k^*(k_t, x_{t-1}) + m_{xx}\big(x^*(k_t, x_{t-1}), \theta^*(k_t, x_{t-1})\big)\right. \\ &\quad \left. \cdot \big(a_k^*(k_t, x_{t-1}) - a_k(k_t, x_{t-1})\big)\right) < 0 \end{aligned}$$

confirms that $x = x^*$ indeed attains a maximum. Given $x = x^*$, the only solution to (A4) is the first-best effort choice a^*, which implies that the expenditure k must also be first-best. To establish that any incentive-compatible performance metric must be equivalent to (11), observe that first-best effort and expenditure are unique for each θ_t, and thus designing p to solve (12) and (13) is both necessary and sufficient.

Proof of Proposition 3. The manager can discretionarily accept or decline any offered renewal contract. The performance metric in period t must therefore be incentive-compatible regardless of whether renewal occurs. By Proposition 2, p is the only incentive-compatible performance metric for a one-period contract. A first-best optimal contract therefore exists if and only if p is incentive-compatible when the manager anticipates renewal. After forming beliefs about x_t and

θ_t, the owner offers a renewal contract in period $t+1$ that yields a net reward equal to the manager's reservation wage, i.e.,

$$E\big(p\big(k_{t+1}, m(x_{t+1}, \theta_{t+1})\big) - q(a_{t+1})\big|\hat{x}_t, \hat{\theta}_t\big)$$
$$= E\big(r(k_{t+1})m(x_{t+1}, \theta_{t+1}) - h(k_{t+1}) + \bar{q}_{t+1} - q(a_{t+1})\big|\hat{x}_t, \hat{\theta}_t\big) = 0$$

where $\hat{x}_t = \hat{x}\big(k_t, m(x_t, \theta_t)\big)$ and $\hat{\theta}_t = \hat{\theta}\big(k_t, m(x_t, \theta_t)\big)$ are the owner's inferences about x_t and θ_t, and $\bar{q}_{t+1}$ is the fixed wage component in period $t+1$. If both the original and the renewal contract were incentive-compatible, an observed increase in k_t in period t would induce changes in contract terms in period $t+1$ that leave expected compensation unaltered, i.e.,

$$\begin{aligned}
&\frac{d}{dk_t} E\big(p\big(k_{t+1}, m(x_{t+1}, \theta_{t+1})\big) - q(a_{t+1})\big|\hat{x}_t, \hat{\theta}_t\big) \\
&= E\big(r_k(k_{t+1})m(x_{t+1}, \theta_{t+1}) + r(k_{t+1})m_x(x_{t+1}, \theta_{t+1}) - h_k(k_{t+1})\big|\hat{x}_t, \hat{\theta}_t\big)\frac{dk_{t+1}}{dk_t} \\
&+ E\big(r(k_{t+1})m_x(x_{t+1}, \theta_{t+1}) - q'(a_{t+1})\big|\hat{x}_t, \hat{\theta}_t\big)\frac{da_{t+1}}{dk_t} \\
&+ \frac{\partial}{\partial \hat{x}_t} E\big(p\big(k_{t+1}, m(x_{t+1}, \theta_{t+1})\big) - q(a_{t+1})\big|\hat{x}_t, \hat{\theta}_t\big)\frac{d\hat{x}_t}{dk_t} \\
&+ \frac{\partial}{\partial \hat{\theta}_t} E\big(p\big(k_{t+1}, m(x_{t+1}, \theta_{t+1})\big) - q(a_{t+1})\big|\hat{x}_t, \hat{\theta}_t\big)\frac{d\hat{\theta}_t}{dk_t} = 0
\end{aligned} \tag{A6}$$

for all k_t. The first two terms of the right-hand side in (A6) equal zero by construction of p, in view of (12) and (13). The final two terms would each equal zero if the manager followed the first-best investment plan. Suppose now that the owner offered compensation in period t based on p in (11), believing the contract to be incentive-compatible. Then over- or underinvestment by the manager, relative to first-best, would make inferred effort differ from actual effort and change the inference $\hat{\theta}_t$ although the true state remains unaltered, so that (A6) reduces to

$$\begin{aligned}
&\frac{d}{dk_t} E\big(p\big(k_{t+1}, m(x_{t+1}, \theta_{t+1})\big) - q(a_{t+1})\big|x_t, \theta_t\big) \\
&= E(r(k_{t+1})m_x(x_{t+1}, \theta_{t+1})|x_t, \theta_t)\left(\frac{\partial a_t^*}{\partial k_t} - \frac{\partial a_t}{\partial k_t}\right) - \frac{\partial \bar{q}}{\partial \hat{\theta}_t}\frac{d\hat{\theta}_t}{dk_t} \\
&= -\left(E(r(k_{t+1})m_x(x_{t+1}, \theta_{t+1})|x_t, \theta_t)\frac{m_\theta(x_t, \theta_t)}{m_x(x_t, \theta_t)} + \frac{\partial \bar{q}_{t+1}}{\partial \theta_t}\right)\frac{d\hat{\theta}_t}{dk_t}
\end{aligned} \tag{A7}$$

where $\partial a_t^*/\partial k_t$ and $\partial a_t/\partial k_t$ are characterized by (A1), since any deviation from k_t^* must be accompanied by a concurrent change in effort that maintains outcomes appearing consistent with first-best investment decisions. The claimed result now obtains after replacing r in (A7) by (19). Given rational expectations by the owner in inferring x_t and θ_t, $\hat{x}_t = x_t^*$ and $\hat{\theta}_t = \theta_t^*$ in all states can only hold if (A7) is zero for all θ_t.

Proof of Proposition 4. Consider first the hired manager's decision incentives after the realization of θ_t, given compensation of p based on some reported investment state $\hat{x}_{t-1}$ and actual investment state x_{t-1}. Let

$$m^* \equiv m\big(x^*(k_t, \hat{x}_{t-1}), \theta^*(k_t, \hat{x}_{t-1})\big)$$

denote first-best revenue given the observed k_t and the reported $\hat{x}_{t-1}$, and let $m \equiv m(x_t, \theta_t)$ denote actual revenue. The manager chooses expenditure such that

$$p_k = r m_x + r_k m - z_k - h_k = 0 \Leftrightarrow r(m_x - m_x^*) + r_k(m - m^*) - z_k = 0$$

By construction of the penalty function z, the solution $m = m^*$ and $m_x = m_x^*$ is unique if $\hat{x}_{t-1} = x_{t-1}$. Since $m_{x\theta} > 0$ and $m_{xx} < 0$, overreporting x_{t-1} requires $\theta_t < \theta^*$ in order for $m_x = m_x^*$ to hold, but then $m < m^*$. The converse applies to underreporting x_{t-1}. Hence, first-best investment cannot meet the necessary condition for maximizing the manager's compensation if $\hat{x}_{t-1} \neq x_{t-1}$. For any continuously differentiable penalty z sufficiently large at $m \neq m^*$ such that there exists no profitable deviation given correctly reported x_{t-1}, managers will therefore invest the first-best amount if $\hat{x}_{t-1} = x_{t-1}$ but will find it optimal to deviate from first-best and incur a non-zero penalty if $\hat{x}_{t-1} \neq x_{t-1}$. Then if the purchase price penalty factor β is set sufficiently high, the seller can only maximize the purchase price by reporting x_{t-1} correctly. The reported $\hat{\theta}_{t-1}$ only affects the fixed component $\bar{q}$ of managerial pay, in view of (11), but not the purchase price $\hat{v}$, and hence the seller has no incentive to misreport θ_{t-1}. Then

$$E\big(q\big(a^*(k_t, \hat{x}_{t-1})\big)\big|\hat{x}_{t-1} = x_{t-1}; \theta_{t-1}\big) = E\big(p(k_t, m(x_t, \theta_t), \hat{x}_{t-1})\big|\hat{x}_{t-1} = x_{t-1}; \hat{\theta}_{t-1} = \theta_{t-1}\big)$$

and hence the manager is willing to accept the employment contract, which in turn implies

$$E(\hat{v}(k_t, m(x_t, \theta_t), \hat{x}_{t-1})|\hat{x}_{t-1} = x_{t-1}; \theta_{t-1}) = E(v(x_{t-1}, \theta_t)|\theta_{t-1})$$

and hence both seller and buyer are willing to accept the pricing terms. Then since the true firm value is revealed in the process, the seller invests to maximize firm value in all periods prior to t.

APPENDIX B – PARAMETRIC ILLUSTRATION

As a parametric illustration of the model mechanics, consider a two-period version in which the original owner sells the firm at the end of period $t = 1$. The owner exits the game at this point, and therefore the selling price must be settled in $t = 1$ without recourse and cannot depend on outcomes in period $t = 2$. The firm's revenue is

$$m(x, \theta_i) = \theta_i x$$

in period $i = 1,2$. In both periods, the investment variable $x = k + a$ is the sum of expenditure k and effort a made in period $t = 1$.[38] The environmental state $\theta_i \in \Theta \subset \mathbb{R}^+$ has unconditional mean $\mu > 0$ and follows a stochastic process with

$$E(\theta_2|\theta_1) = \beta\mu + (1-\beta)\theta_1$$

where $\beta \in \left(\frac{-\inf\{\Theta\}}{\mu - \inf\{\Theta\}}, 1\right)$. For simplicity, the discount factor is normalized to $\gamma = 1$, and expenditure and effort have quadratic cost functions. First-best firm value at date $t = 1$ is thus

$$v(\theta_1) = \max_{k,a}\big(m(x,\theta_1) + E(m(x,\theta_2)|\theta_1)\big) = \max_{k,a}\big((\beta\mu + (2-\beta)\theta_1)x - k^2 - a^2\big)$$

with an optimal investing policy of

$$k^* = a^* = \frac{\beta\mu + (2-\beta)\theta_1}{2} \tag{B1}$$

and so

$$v(\theta_1) = \frac{(\beta\mu + (2-\beta)\theta_1)^2}{2}$$

Suppose now that a naïve prospective buyer, having observed k, assumes that first-best investment has occurred. The buyer infers effort of $\hat{a} = k$ and, by inverting (B1), an environment of

$$\hat{\theta}_1(k) = \frac{2k - \beta\mu}{2 - \beta}$$

provided that the observed revenue appears consistent with first-best investment, i.e., if

$$m(x,\theta_1) = \hat{\theta}_1\hat{x} = \hat{\theta}_1(k + \hat{a}) = 2\hat{\theta}_1 k = \frac{4k^2 - 2\beta\mu k}{2-\beta} \tag{B2}$$

The naïve buyer would then offer to buy the firm at a price of

$$E\big(m(\hat{x},\theta_2)\big|\hat{\theta}_1\big) = \big(\beta\mu + (1-\beta)\hat{\theta}_1\big)\hat{x} = \big(\beta\mu + (1-\beta)\hat{\theta}_1\big)(k + \hat{a})$$

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[38] Investment in period $t = 2$ is not subject to an incentive problem and is therefore, without loss of insight, normalized to zero to avoid unnecessary extra notation.

An owner-manager running the firm in $t = 1$ would thus receive a total net value of

$$\begin{aligned}\hat{v}(x,\theta_1) &= m(x,\theta_1) + E\big(m(\hat{x},\theta_2)\big|\hat{\theta}_1\big) - k^2 - a^2 \\ &= \theta_1 x + \big(\beta\mu + (1-\beta)\hat{\theta}_1\big)\hat{x} - k^2 - a^2\end{aligned} \tag{B3}$$

subject to making an effort choice such that

$$m(x,\theta_1) = \theta_1(k+a) = \frac{4k^2 - 2\beta\mu k}{2-\beta} \Leftrightarrow a = \frac{4k^2 - 2\beta\mu k}{(2-\beta)\theta_1} - k \tag{B4}$$

in order to produce the first-best-mimicking revenue in (B2) that the buyer expects. Inserting (B4) into (B3) yields

$$\hat{v}(x,\theta_1) = \left(3 - \left(\frac{4k - 2\beta\mu}{(2-\beta)\theta_1} - 1\right)^2\right)k^2$$

and thus the owner-manager's value-maximizing investment is given by the first-order condition

$$\hat{v}_k(x,\theta_1) = 2k\left(3 - \left(\frac{4k - 2\beta\mu}{(2-\beta)\theta_1} - 1\right)^2\right) - \frac{8k^2}{(2-\beta)\theta_1}\left(\frac{4k - 2\beta\mu}{(2-\beta)\theta_1} - 1\right) = 0$$

whose only economically plausible solution is

$$k = \frac{\beta\mu}{2} + \frac{(2-\beta)\theta_1}{16}\left(\sqrt{\left(\frac{2\beta\mu}{(2-\beta)\theta_1} + 1\right)^2 + 24} - \frac{2\beta\mu}{(2-\beta)\theta_1} + 3\right) \tag{B5}$$

One can readily see that (B5) coincides with the first-best solution (B1) if and only if $\beta = 0$, and indeed $\beta = 0$ is the unique solution to the knife-edge condition

$$\frac{E(v_x)}{m_x} = \frac{E_\theta(v)}{m_\theta} \Leftrightarrow \frac{\beta\mu + (1-\beta)\theta_1}{\theta_1} = 1 - \beta \Leftrightarrow \beta = 0$$

in Proposition 1 of the general model, under which an owner-manager invests optimally. Incentives for overinvestment (underinvestment) obtain when $\beta < 0$ ($\beta > 0$). In line with Proposition 1, the hypothesized first-best equilibrium therefore breaks down. Consider instead employing a third-party manager and offering the incentive pay in Proposition 2. The manager then receives

$$p\big(k, m(x,\theta_1)\big) = r(k)m(x,\theta_1) - h(k) - z\big(k, m(x,\theta_1)\big) + \bar{q}$$

where

$$r(k) = 1 + \frac{E(v_x)}{m_x} = \frac{2(2-\beta)k}{2k - \beta\mu}$$

in view of (18), and

$$h(k) = k^2 + \int r_k(k)m\big(x^*(k),\theta^*(k)\big)\,dk = k^2 - \beta\mu(2k + \beta\mu\ln(2k - \beta\mu))$$

in view of (14). The penalty function z can be set to zero in this case because there are no profitable deviations that violate (B2). Straightforward algebra shows that the first-best solution in (B1) is indeed the unique investment choice that maximizes p for the manager. One may also note the following properties.

- The revenue adjustment factor r decreases in the investment expenditure k if $\beta > 0$ and increases if $\beta < 0$, in view of

$$r_k(k) = \frac{-2(2-\beta)\beta\mu}{(2k-\beta\mu)^2} \Rightarrow r_k\big(k \big| k = k^*(\theta_1)\big) = \frac{-2\beta\mu}{(2-\beta)\theta_1^2}$$

  When $\beta > 0$, θ_2 follows a mean-reverting process and hence the marginal impact of investment on revenue is smaller in $t = 2$ than in $t = 1$. The higher θ_1, the larger the magnitude of this difference, so that, to account for the future value impact of investment, current revenue m must be scaled by a smaller factor r as θ_1 (and thus, in equilibrium, k) increases. When $\beta < 0$, the argument applies in reverse.

- The revenue adjustment factor r increases in β if $k < \mu$ (and thus, in equilibrium, if $\theta_1 < \mu$) and decreases if $k > \mu$ (and thus if $\theta_1 > \mu$), in view of

$$r_\beta(k) = \frac{4k(\mu-k)}{(2k-\beta\mu)^2} \Rightarrow r_\beta\big(k \big| k = k^*(\theta_1)\big) = \frac{(\beta\mu + (2-\beta)\theta_1)(\mu-\theta_1)}{(2-\beta)\theta_1^2}$$

  A higher β in $E(\theta_2|\theta_1)$ gives greater weight to the expected value μ and less to θ_1. Hence, if $\theta_1 < \mu$, the weight of the expected revenue in $t = 2$, relative to the revenue in $t = 1$, increases in β, requiring a higher adjustment factor r. (Recall that $E(\theta_1) = E\big(k^*(\theta_1)\big) = \mu$.) Conversely, if $\theta_1 > \mu$, the value contribution of revenue in $t = 1$, relative to revenue in $t = 2$, increases in β, and the adjustment factor r decreases accordingly.

- The revenue adjustment factor r increases in μ if $\beta > 0$ and decreases if $\beta < 0$, in view of

$$r_\mu(k) = \frac{2\beta(2-\beta)k}{(2k-\beta\mu)^2} \Rightarrow r_\mu\big(k \big| k = k^*(\theta_1)\big) = \frac{\beta(\beta\mu + (2-\beta)\theta_1)}{(2-\beta)\theta_1^2}$$

  If the weight β is positive, a higher mean μ makes the expected revenue in $t = 2$ contribute more to firm value, relative to revenue in $t = 1$, necessitating a higher adjustment factor r. The converse applies when the weight β is negative.