

## Covenants, Interest Rates, and the Cost of Debt

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**ABSTRACT:** In debt financing, borrowers' and lenders' misaligned preferences over risk-return tradeoffs create an agency cost that is jointly determined by interest rates and covenants. Under basic conditions, the optimal, cost-minimizing combination of the two is unique and, depending on borrower characteristics, can lie anywhere between low interest rates with strict covenants and high interest rates with loose covenants. Contrary to this fact, empirical research in accounting has frequently conflated low interest rates with efficient contracting and a low cost of debt. At the same time, predictable regularities in optimally designed debt contracts exist. The optimized cost of debt is concave and non-monotonic in profitability and leverage. Debt contracts are most efficient when leverage and profitability are inversely aligned, consistent with the empirically observed negative correlation between the two. If the borrower has greater scope than the lender to affect risk and return, low interest rates and restrictive covenants are optimal.

**Keywords:** *debt contracts; covenants; cost of debt; asset substitution*

## 1. INTRODUCTION

The functioning of debt markets is a long-standing research topic of interest to accounting and corporate finance. Of particular interest are the optimal design of debt contracts and the properties and interactions of the contract terms, such as interest rates and covenants. This analysis is non-trivial when, as is usually the case in practice, information is incomplete and the interactions of borrower and lender produce an agency cost that the contract design must address. This paper seeks to point out how empirical studies in accounting have often applied an incomplete interpretation to this agency problem and, by way of a simple analytical model, to provide some results that are relevant to the design and interpretation of past and future empirical work in this area.

Several well-known themes are central to the debt contract design problem. First, it has long been recognized that debt financing incurs a latent inefficiency due to a misalignment of risk preferences between borrowers and lenders, often encountered in the form of asset substitution. In settings with incomplete information, potential consequences of this agency problem include business decisions involving suboptimal risk-return tradeoffs, or the necessity for costly ex-post renegotiation of contract terms in an effort to avert such outcomes. Second, much attention has been devoted to the role of creditor rights, particularly in the form of covenants, in mitigating the preference misalignment problem. As contract clauses agreed upon ex ante, covenants can channel decisions toward the socially optimal outcome through the strategic assignment of contingent control rights to either contracting party. At the same time, stricter covenants transfer wealth from the borrower to the lender and therefore require compensatory adjustments to the stated interest rate. Interest rates, in turn, interact with the preference misalignment problem and thus with the optimal covenant design choice. Contract terms must therefore be optimized jointly with respect to both efficiency and the division of payoffs between borrower and lender.

The solution to this optimization problem has implications relevant for both the design and the interpretation of empirical research findings on debt financing. Optimal covenants and interest rates are uniquely and jointly determined by the debt amount and the characteristics of the borrower, and the optimal covenant restrictiveness and interest rate level increase jointly in the

debt amount.<sup>1</sup> Neither covenant nor interest rate can therefore be interpreted in isolation. Moreover, optimal contract terms can vary across a range from restrictive covenants with low interest rates to loose covenants with high interest rates, depending on the characteristics of the investment to be financed. Observing a particular configuration of contract terms does therefore not permit any immediate inference about the magnitude of the agency cost of the debt. In particular, the stated interest rate does not signal whether debt is more or less ‘expensive’ but merely indicates which balancing of contract terms is most efficient given the economic fundamentals of the borrower. Numerous past studies in accounting research have conflated interest rates with the cost of debt (e.g., Sengupta 1998; Ahmed et al. 2002; Beatty et al. 2002; Francis et al. 2005; Zhang 2008; Beatty et al. 2012; Spiceland et al. 2016; Bonsall and Miller 2017; Sunder et al. 2018; Badertscher et al. 2019).

Notwithstanding this inference problem, the optimal choice of covenants and interest rates and the resulting agency cost of debt exhibit some analytically predictable regularities. In particular, the agency cost of debt in a contract with covenants is a non-monotonic, concave function of both the profitability of the borrower’s investment and the borrower’s financial leverage. Covenant-free models in prior research, by contrast, predict monotonically higher agency costs as leverage increases (Green and Talmor 1986; Décamps and Faure-Grimaud 2002). Moreover, leverage and profitability move the optimal contract terms in opposite directions, suggesting that borrowers optimally align their debt-to-capital ratio inversely to their profitability. Rajan and Zingales (1995) document empirically that leverage and profitability indeed appear to be negatively correlated in practice. Existing theoretical models, on the other hand, have generally proposed a positive relationship between profitability and leverage.<sup>2</sup> Moreover, when borrowers can change the risk-return profile of the investment *after* signing the contract, e.g., by changing some fundamental properties of the project in which to invest, the optimal contract features a lower interest rate and a stricter covenant than would obtain if the choice were contractible, consistent with the high frequency of restrictive covenants and covenant violations seen in practice.

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<sup>1</sup> This point is borne out in the data (Callen, Chen, Dou and Xin 2016) but not always considered in research design. For example, Matvos (2013) estimates a model with debt principal and covenants as choice variables but a fixed maturity value, which forces a suboptimal decrease in the interest rate as the amount of principal increases.

<sup>2</sup> Theoretical predictions about the relationship between profitability and leverage generally derive from investor-manager conflicts and suggest a positive relationship (for example, Ross 1977; Harris and Raviv 1990; Stulz 1990). An exception is Chang (1999), who derives a negative relationship in a model of costly state verification. See also Harris and Raviv (1991) for a review of the earlier literature.

These results derive from only two basic premises. First, borrowers and lenders respond to incentives to maximize their individual payoffs by sacrificing welfare-maximizing outcomes in exchange for outcomes with a higher or lower variance.<sup>3</sup> Second, these incentives affect realized outcomes and are therefore relevant to contract design ex ante. In particular, even if ex-post renegotiation could avert socially inefficient outcomes, an optimal contract must heed the incentive problem whenever renegotiation costs exceed the potential benefits of renegotiation in at least some circumstances. Under these conditions, covenants and interest rates act at cross purposes in their bearing on the agency problem and on the division of payoffs between borrower and lender. A higher interest rate in exchange for a looser covenant gives the lender less control rights but brings the lender's incentives closer to the social optimum, while the borrower receives more control but, simultaneously, an added incentive to diverge from socially optimal decisions. There exists a unique interior point at which these forces are in balance.

There is extensive empirical evidence supporting the posited agency conflict between borrower and lender. As lenders obtain the right to influence the borrower's business decisions in the wake of a covenant violation, borrowers tend to reduce their capital expenditures (Chava and Roberts 2008; Nini et al. 2012), issue less debt (Roberts and Sufi 2009), and have lower leverage and higher CEO turnover (Nini et al. 2012). Begley and Feltham (1999) observe that the existence and restrictiveness of debt covenants are negatively correlated with the degree to which CEOs' compensation structure is aligned with the incentive of shareholders rather than lenders. Beneish and Press (1993) document that debt contract renegotiations following a covenant violation tend to result in higher interest rates, increases in collateral, the prohibition of certain investing or financing transactions, or the divesting of capital assets in order to meet payment obligations.<sup>4</sup> Borrowers therefore often make accounting choices designed to avoid covenant violations (Dichev and Skinner 2002; Beatty and Weber 2003), and lenders charge lower interest rates to firms willing to give up the flexibility to make these accounting choices (Beatty et al. 2002).

The problem of optimizing debt contracts with respect to interest rates and covenant-induced allocations of control rights has been analyzed in a number of prior studies, albeit with critical

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<sup>3</sup> Borrowers, as residual claimants, effectively hold a call option on the income generated by their borrowed capital and therefore benefit from increased volatility of the income at the expense of the lender (Merton 1974). The incentive implication to sacrifice return for risk have long been recognized (see, for example, Jensen and Meckling 1976; Gavish and Kalay 1983; Green and Talmor 1986).

<sup>4</sup> Chen and Wei (1993) observe that covenant waivers are generally only given to firms with low leverage and a low probability of bankruptcy.

differences to the analysis presented here, in both setup and research objectives. Gârleanu and Zwiebel (2009) model a reduced-form incentive problem in which decision preferences, and thus the optimal covenant, do not depend on the debt maturity value. The agency cost of debt financing is independent of the borrower's capital needs and leverage in this case.<sup>5</sup> Sridhar and Magee (1997) permit endogenously chosen covenants and interest rates but conclude that the optimum balance between the two is not generally unique and provide no explicit results about the agency cost.<sup>6</sup> Anderson and Sundaresan (1996) condense interest payment and covenant into one choice variable by equating a transfer of control rights to the lender with the borrower's failure to pay. Gigler et al. (2009) recognize the interaction between maturity values and incentives but focus on the effects of asymmetrically precise accounting information on decision efficiency in a setting with complete contracts that fully resolve the agency problem. Caskey and Hughes (2012) model covenant and maturity value as choice variables in a project selection problem but limit the covenant to one of five pre-specified control allocation schemes.<sup>7</sup> Other models have focused on the efficiency implications of the borrower's incentives only (Gavish and Kalay 1983; Green and Talmor 1986; Leland 1998; Décamps and Faure-Grimaud 2002) or on the properties of accounting information underlying the covenant (Göx and Wagenhofer 2010; Li 2013).

The problem of misaligned risk preferences is specific to the payoff structure of debt. As is common in research on agency conflicts in debt contracts, the model is not intended to explain the optimality of debt relative to other forms of capital but takes the financing choice as given. Prior research has identified a large number of settings in which debt is preferable to other forms of capital, including, among others, the presence of information asymmetry in capital markets, the need for costly state verification, or the existence of private benefits to inside equity owners.<sup>8</sup>

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<sup>5</sup> Gârleanu and Zwiebel study the effect of information asymmetry and conclude that the resulting adverse selection problem generally makes control allocation to the lender optimal.

<sup>6</sup> Sridhar and Magee aim their analysis at the possibility that borrowers may manipulate the information variable on which the covenant-based control allocation operates.

<sup>7</sup> See section 3.3 for a discussion of project selection and connections to the results by Caskey and Hughes.

<sup>8</sup> An initial theory explaining the heterogeneity in debt-to-equity ratios seen in practice has centered on the tradeoffs associated with the prioritization of debt in bankruptcy (Scott 1977), bankruptcy costs (Baxter 1967), and the tax benefits associated with debt due to heterogeneity in the taxation of bond interest, dividends, and corporate profits (Miller 1977; DeAngelo and Masulis 1980). Subsequent attempts to motivate the role of debt financing have focused on frictions between lenders, owners, managers, and other constituents. These frictions include private benefits (Jensen and Meckling 1976; Aghion and Bolton 1992), the potential diversion of funds by equity owners (Hart and Moore 1998), information asymmetry in capital markets (Myers and Majluf 1984; Noe 1988), costly state verification (Townsend 1979; Gale and Hellwig 1985; Harris and Raviv 1990), signaling via capital structure choices (Ross 1977), and debt as a commitment device in a moral hazard problem (Grossman and Hart 1982).

While not modeled explicitly, these settings provide a motivation for debt financing to be a desirable. This paper therefore differs from studies such as Aghion and Bolton (1992) or Leland (1994) in that it solves neither a complete security design problem (which would require endogenously chosen payoff functions) nor a complete capital structure problem (which would require modeling the cost and benefits of alternative forms of financing). Rather, this paper seeks to point out how contract design choices interact with the incentive conflict inherent in debt financing and to describe the properties of the resulting agency cost.

## 2. MODEL SETUP

A firm, hereafter referred to as the borrower, requires an amount of capital  $k \in \mathbb{R}^+$  in order to finance a venture. The investment yields a single cash flow  $c \in \mathbb{R}^+$  upon completion. The cash flow is uncertain at the time of investment but sufficiently high in expectation to yield a positive net present value, i.e.,  $E(c) \geq k$ . For lack of own funds, the borrower must obtain the capital from an outside investor, hereafter referred to as the lender. The financing is initially assumed to consist of debt only. The effects of including equity are examined later in the analysis.<sup>9</sup> The lender's payoff is defined by a maturity value  $m$ , which the borrower must pay upon completion of the project.<sup>10</sup> If  $c < m$ , the lender receives only  $c$ , and thus the borrower's and lender's payoff functions are  $\max(c - m, 0)$  and  $\min(c, m)$ , respectively. Borrower and lender are risk-neutral, the risk-free interest rate is normalized to zero, and capital markets are competitive. Hence, the lender agrees to any contract that yields an expected payment of at least  $k$ .<sup>11</sup>

The debt contract is signed at date  $t = 0$ , at which time the capital is committed to the investment project. The cash flow  $c$  is realized at date  $t = 2$ . At the interim date  $t = 1$ , the state of nature  $\theta \in \Theta$  is realized and, upon realization of  $\theta$ , an action  $a \in A$  must be taken. The action should be thought of as an operational decision made in response to an evolving business envi-

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<sup>9</sup> Explaining the optimality of debt over other forms of financing is not the objective of the model, but one could readily introduce features that make debt superior without altering the substance of the analysis. For example, if, in the spirit of Jensen and Meckling (1976), the entrepreneur can divert cash flows in excess of a certain level  $\bar{c}$  for private benefits and  $\bar{c}$  is just slightly above the maturity value of an optimal debt contract that permits the outside financier to break even, then debt is the only feasible form of outside capital.

<sup>10</sup> The terms interest rate and maturity value are used interchangeably in this text, as the interest rate associated with a given maturity value  $m$  is simply the ratio of  $m - k$  to  $k$  and thus linear in  $m$ .

<sup>11</sup> Giving the lender a required payoff in excess of  $k$  is equivalent to raising the amount of capital the borrower needs. The implications of higher capital amounts are discussed later as part of the analysis.

ronment, such as switching production to a different product, adopting a new technology, or targeting a new market segment (or refraining from any of these changes).<sup>12</sup> Both  $\theta$  and  $a$  affect the distribution of  $c$ . Borrower and lender both observe  $\theta$ , and either party is capable of undertaking the action  $a$ . An extension in Section 3.3 considers a setting in which the borrower has more scope for action-taking than the lender.<sup>13</sup>

As in Leland (1998), the action space  $A$  is assumed to be so large that prescribing actions contractually is infeasible or exceedingly costly (and debt contracts in reality certainly do not make any such attempt). The contract may, however, include a covenant that awards the decision right over the action to either the borrower or the lender, possibly conditioning the control allocation on verifiable information at  $t = 1$ .<sup>14</sup> To model the agency problem as parsimoniously as possible, the analysis will assume that the state variable  $\theta$  is verifiable and that the covenant can therefore be a function of  $\theta$  directly. This simplification should not be mistaken as inconsistent with the assumed non-contractibility of the action  $a$ . In reality,  $\theta$  would very likely be too complex and high-dimensional to be verifiable, and the covenant would instead be written on a lower-dimensional information variable that is imperfectly correlated with  $\theta$ .<sup>15</sup> Introducing such an information variable would not impact any results but would needlessly clutter the notation.<sup>16</sup>

As a formal representation of the covenant, let  $r$  denote the covenant variable based on which the allocation of control is implemented. In practice,  $r$  might correspond to a financial ratio calculated from the borrower's accounting records. Generally,  $r$  can be any arbitrary function of the state variable  $\theta$  and any other contract terms. As control allocation is a binary outcome, it is without loss of generality to restrict the codomain of  $r$  to  $\mathbb{R}$  and to define a covenant violation

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<sup>12</sup> Prior models have often assumed  $A$  to be binary, in which case at least one contracting party prefers the first-best action in any given state. The setup in this model admits a larger state space and thus the possibility that neither party has an incentive to choose the first-best  $a$ .

<sup>13</sup> In reality, managerial decisions are likely made by the borrower, while the lender usually does not take a direct managing role but may force the borrower's actions in a certain direction, for example, by interdicting new investments (Nini et al. 2009). As will become clear during the analysis, the lender prefers actions leading to low-risk outcomes, which is consistent with refraining from new initiatives and thus aligns well with the more passive 'interceptor' role lenders typically take in practice. Sridhar and Magee (1997) and Gârleanu and Zwiebel (2009) model this role more explicitly by permitting the lender to deny the borrower, in certain circumstances, the implementation of a new investment.

<sup>14</sup> A contract requiring the parties to agree on an action at  $t = 1$ , rather than assigning decision rights to one party, would amount to a joint ownership structure rather than debt financing and would require additional assumptions about the bargaining process over the decision. See Aghion and Bolton (1992) for an analysis of joint ownership.

<sup>15</sup> It can also be shown that, with a coarse information variable, allocating decision rights can be more efficient than prescribing actions directly even if prescribing actions is feasible.

<sup>16</sup> See Dou (2019) for an empirical analysis of the precision of financial accounting metrics used in covenants.

(also referred to as technical default), and thus a transfer of decision rights to the lender, as an outcome  $r < 0$ .<sup>17</sup> The borrower retains control for any outcome  $r \geq 0$ .<sup>18</sup> The specification of  $r$  is agreed upon at time  $t = 0$  as part of the contract terms.<sup>19</sup> The debt contract is thus a triple  $\{r, m, k\}$ , where  $k$  is, for now, exogenous, and  $r$  and  $m$  are choice variables.

Under any contract agreed on ex ante at  $t = 0$ , borrower and lender may, upon realization of  $\theta$  at  $t = 1$ , prefer different actions ex post, and either party's preferred action may differ from the first-best action that maximizes the investment's total expected cash flow. (The requisite definitions and assumptions are stated below.) Hence, there may be scope for a mutually beneficial renegotiation of contract terms at  $t = 1$ , which the parties may undertake if they choose to. The renegotiation process creates a deadweight cost, say, because debt contract renegotiations are protracted and time-consuming procedures that occupy key decision-makers and result in delays that impact operations adversely.<sup>20</sup> The cost can depend on  $\theta$  and on the controlling party's preferred action, and it may possibly exceed the benefit of renegotiation in some states.<sup>21</sup> Further structure and details of the renegotiation process are discussed in Section 3.2.<sup>22</sup> The complete timeline of events is summarized in Figure 1.

### [FIGURE 1]

To flesh out the incentive misalignment problem between borrower and lender in tractable form, let  $\Theta$  be a subset of  $\mathbb{R}^n$ , with joint cumulative distribution function  $G(\theta)$ , and let  $A \subset \mathbb{R}$  be sufficiently large such that the preferred actions of borrower and lender lie in the interior of  $A$  for

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<sup>17</sup> Defining  $r \in \{0,1\}$  instead and mapping  $\theta$  either to  $r = 0$  in case of technical default or to  $r = 1$  otherwise leads to a mathematically identical characterization of the covenant but would require additional notation later on.

<sup>18</sup> In the language of Christensen and Nikolaev (2011), a covenant implementing state-contingent control allocation would be called a 'performance covenant.' By contrast, a restriction on financing transactions, such as issuing dividends or new debt, would constitute a 'capital covenant,' which is not the focus of this paper.

<sup>19</sup> In practice, a distinction is made between the tightness of a given covenant (say, the minimum level of a financial ratio the borrower must maintain in order to avoid technical default) and the number of covenants in a contract. For modeling purposes, this distinction is unnecessary because tightness and number of covenants are equivalent in that they both determine the likelihood of violation, which is the only outcome of interest here. A change in the contract to award more decision rights to the lender therefore has a dual interpretation as either a tightening of existing covenants or as the introduction of additional covenants.

<sup>20</sup> Indirect effects can be a further source of renegotiation costs. For example, a major customer of the borrower might become aware of the renegotiation, interpret the event as a sign of financial difficulty, and seek a new supplier that does not have a potential going concern problem.

<sup>21</sup> It is of no particular relevance which party suffers the cost at  $t = 1$ . Given the lender's break-even constraint, renegotiation costs are ultimately borne by the borrower via appropriate adjustments to the contract terms ex ante.

<sup>22</sup> The case of zero renegotiation cost is, of course, trivial as, by the Coase theorem, the initial allocation of decision rights would be irrelevant, save to ensure that the lender receives the minimum required payment in expectation. The omnipresence of covenants in practice, however, makes the zero-cost scenario implausible.

all  $\theta$ . Let  $F(c|a, \theta)$  denote the cumulative distribution function  $c$ , conditional on  $\theta$  and  $a$ . For analytical convenience,  $F$  is assumed to be differentiable in  $a$  and  $\theta$ , and  $F$  and  $G$  have atomless densities. The action  $a$  affects the cash flow distribution in the following manner.

**Assumption 1a.** For any  $\theta$  and  $a$ , there exists some  $z \in \mathbb{R}^+$  such that  $F_a(c|a, \theta) \geq 0$  at all  $c \leq z$  and  $F_a(c|a, \theta) < 0$  at all  $c > z$ .<sup>23</sup>

**Assumption 1b.** For any  $\theta$  and  $a$ ,  $F_{aa}(c|a, \theta) \geq 0$  at all  $c$ .

Part 1a formalizes the risk-return tradeoff underlying the preference misalignment between borrower and lender. The characterization can be read as a single-crossing property: a small increase in  $a$  rotates the cash flow distribution clockwise and thereby shifts probability mass toward extreme outcomes. A higher  $a$  thus corresponds to a decision to pursue a riskier course of action. At the same time, the assumption makes no statement as to whether higher  $a$  increase or decrease the expected cash flow and therefore imposes no directional effect on investment returns. Assumption 1a is thus more general than classic asset substitution models in which higher risk always comes at the expense of expected return (Jensen and Meckling 1976; Green and Talmor 1986).<sup>24</sup> The global concavity property imposed in part 1b is a technical convenience and implies that there exists a unique optimal action choice.

### 3. OPTIMAL CONTRACTS AND THE COST OF DEBT

The source of inefficiency in the contract to be designed is the possibility that, in any given state  $\theta$ , either borrower or lender may prefer an action  $a$  other than the one that maximizes total expected cash flow. This inefficiency, hereafter referred to as the cost of debt, may arise either because the controlling party literally takes an inefficient action or because averting the inefficient action by renegotiation is costly, and the following discussion considers both scenarios. The contract signed at  $t = 0$  anticipates the agency conflict and seeks to minimize the cost of debt, subject to the constraint that the lender's expected payoff equal or exceed the initial loan amount  $k$ .

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<sup>23</sup> Throughout the text, subscripts denote partial derivatives.

<sup>24</sup> The classic asset substitution problem can be thought of as selecting an outcome distribution that is inferior in the sense of second-order stochastic dominance, i.e., that has a higher variance but a lower mean. Assumption 1a preserves the variance aspect of second-order stochastic dominance without imposing a mean-ordering.

It will be convenient to work backward from the decision problem at the interim date  $t = 1$ . The following notation will be used throughout the remainder of the paper. The first-best action that maximizes the total expected cash flow in a given state  $\theta$  is denoted by

$$a^*(\theta) = \arg \max_a E(c|a, \theta) \quad (1)$$

which serves as the benchmark against which the cost of debt is measured. The global concavity property in Assumption 1b implies that  $a^*$  is unique for each  $\theta$ . The borrower, if left to its own devices, has a preferred action

$$a^b(m, \theta) = \arg \max_a E(\max(c - m, 0) | a, \theta) \quad (2)$$

which is likewise unique, in view of Assumption 1b. Similarly, the lender would unilaterally optimize its own payoff by choosing

$$a^l(m, \theta) = \arg \max_a E(\min(c, m) | a, \theta) \quad (3)$$

As a baseline case, consider first a scenario in which renegotiation is either infeasible or too costly in all states  $\theta$ . Then if left in control at date  $t = 1$ , the borrower would take its preferred action  $a = a^b$  and obtain an expected payoff of

$$b(a^b, m, \theta) = E(\max(c - m, 0) | a^b(m, \theta), \theta) \quad (4)$$

while the lender would receive

$$l(a^b, m, \theta) = E(\min(c, m) | a^b(m, \theta), \theta) \quad (5)$$

The cost of debt, i.e., the welfare loss relative to the socially optimal outcome, is the difference between the expected cash flow obtained under the first-best action  $a^*$  and the sum of (4) and (5) and is denoted by

$$q(a^b, m, \theta) = E(c|a^*(\theta), \theta) - E(c|a^b(m, \theta), \theta) \quad (6)$$

Replacing  $b$  in (4), (5) and (6) with  $l$  yields the payoffs and debt cost if the lender is in control.

The covenant variable  $r$  determines whether  $b$ ,  $l$  and  $q$  are realized from action  $a^b$  or from action  $a^l$ . In a slightly abusive but efficient recycling of the notation  $b$  and  $l$ , the parties' payoffs, given a realization of  $r$ , will be denoted by

$$b(r, m, \theta) = b(a^b, m, \theta)\mathbf{I}_{r \geq 0}(r) + b(a^l, m, \theta)(1 - \mathbf{I}_{r \geq 0}(r)) \quad (7)$$

and

$$l(r, m, \theta) = l(a^b, m, \theta)\mathbf{I}_{r \geq 0}(r) + l(a^l, m, \theta)(1 - \mathbf{I}_{r \geq 0}(r)) \quad (8)$$

where the indicator function

$$\mathbf{I}_{r \geq 0}(r) = \begin{cases} 1 & \text{if } r \geq 0 \\ 0 & \text{if } r < 0 \end{cases}$$

designates whether a covenant violation has occurred. Similarly, the debt cost becomes

$$q(r, m, \theta) = q(a^b, m, \theta)\mathbf{I}_{r \geq 0}(r) + q(a^l, m, \theta)(1 - \mathbf{I}_{r \geq 0}(r)) \quad (9)$$

The payoffs  $b$  and  $l$  of the two parties must naturally add up to the total expected cash flow, and so, by construction, payoffs and debt cost reconcile via the ‘balance sheet equation’

$$E(c|a^*(\theta), \theta) - q(r, m, \theta) = b(r, m, \theta) + l(r, m, \theta)$$

with assets, i.e., total investment value net of expected inefficiency, on the left and liabilities  $l$  plus equity  $b$  on the right, or, equivalently,

$$b(r, m, \theta) = E(c|a^*(\theta), \theta) - q(r, m, \theta) - l(r, m, \theta) \quad (10)$$

In view of (10), the optimal contract design can now be cast as a solution to the problem

$$\max_{r, m} B(r, m)$$

over the borrower’s residual claims, subject to the lender’s break-even constraint

$$L(r, m) \geq k$$

where

$$B(r, m) = \int_{\Theta} b(r, m, \theta) dG$$

denotes the equity value and

$$L(r, m) = \int_{\Theta} l(r, m, \theta) dG$$

denotes the debt value, both in expectation as of date  $t = 0$ . Substituting (10) for  $b$  and writing the program in Lagrange form yields

$$\max_{r, m, \lambda} \mathcal{B}(r, m, \lambda, k) = \max_{r, m, \lambda} \int_{\Theta} \mathcal{b}(r, m, \lambda, k, \theta) dG \quad (11)$$

where

$$\mathcal{b}(r, m, \lambda, k, \theta) = E(c|a^*(\theta), \theta) - q(r, m, \theta) + (\lambda - 1)l(r, m, \theta) - \lambda k \quad (12)$$

and  $\lambda$  denotes the Lagrange multiplier. The second-best contract obtained as a solution to (11) will henceforth be referred to as an ‘optimal contract.’ For distinction, the benchmark social optimum obtained under the first-best decision policy  $a^*$  will be referred to as ‘first-best.’

The dependence of the borrower’s and lender’s preferred actions  $a^b$  and  $a^l$  on the maturity value  $m$  suggests that neither action choice should generally coincide with the first-best action  $a^*$ , which is independent of  $m$ . Such preference for a socially suboptimal decision necessarily

implies that the preferred action results in a wealth transfer to the decision-maker at the expense of the counterparty, and that this wealth transfer exceeds the decision-maker's share of the resulting efficiency loss. As higher actions induce higher risk in the cash flow distribution, the following well-known regularities obtain. All proofs can be found in the Appendix.

**Lemma 1.** *In any state  $\theta$  and under any contract  $\{r, m, k\}$ , the borrower's and lender's preferred actions are ordered  $a^l \leq a^* \leq a^b$ .*

**Lemma 2.** *For any  $\theta$ , both the borrower's preferred action  $a^b$  and the lender's preferred action  $a^l$  increase in the maturity value  $m$ .*

Both results are well-known properties of debt financing but deserve explicit discussion here, as they underlie the central results to follow. Lemma 1 states the standard incentive misalignment arising from the parties' asymmetric payoff functions. As the residual claimant, the borrower effectively holds a call option whose value, in analogy to the behavior of option prices, increases in the volatility of the underlying cash flow (Merton 1974). The borrower thus prefers an outcome distribution with inefficiently high volatility, whereas the lender prefers an outcome distribution with inefficiently low volatility. Lemma 2 links these incentives to the maturity value. In the option price analogy, the maturity value  $m$  acts as the strike price of the borrower's 'call option' and, as is well known, the option value benefits more from increased volatility if the strike price is high. A debt contract that charges the borrower a higher rate of interest therefore induces an action  $a^b$  that deviates further from the social optimum  $a^*$ , while the lender's preferred action  $a^l$ , again by complementarity of payoffs, moves closer to  $a^*$ . In a covenant-free loan agreement that leaves all decision rights to the borrower, an increase in the interest rate then indeed corresponds to a higher cost of debt. In the presence of covenants that permit the lender to intervene under certain circumstances, on the other hand, the effect of an interest rate increase on the debt cost is neither mechanical nor obvious.<sup>25</sup> Lemmas 1 and 2 nonetheless imply a useful property of the

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<sup>25</sup> Given Assumption 1a, lenders always impose their preferred action on borrowers after a covenant violation and never waive their rights to do so. However, a simple extension would produce the covenant waivers often seen in practice, without affecting the substance of the results. If the covenant could only be based on a coarse information signal about the actual state  $\theta$ , a given signal realization might occur for  $\theta$  with varying degrees of preference misalignment. Then if the lender, being an outsider to the business, had to incur even a minimal cost to implement its

cost of debt: the higher the maturity value, the more efficient the assignment of control to the lender, formally summarized in the following corollary.

**Corollary 1.** *For any  $\theta$ , the cost of debt  $q(r, m, \theta)$  is supermodular in  $r$  and  $m$ .*

While critical for the remainder of the analysis, Corollary 1 does not yet by itself provide an immediate prescription with respect to optimal contract design because it merely implies that giving all payments and control rights to one party would achieve first-best efficiency, but neither borrower nor lender would be so generous to its counterparty. At the same time, a given division of payoffs between the parties can be achieved either by assigning the lender extensive decision rights, paired with a low value of  $m$ , or by leaving more decisions rights with the borrower but compensating the lender through a higher maturity value. In either case, the change in control rights favors the party whose decision incentives are at the same time exacerbated by the concurrent adjustment to  $m$ . While raising more debt and paying an appropriately higher interest rate would clearly result in increased inefficiency if covenants were not available, with covenants in play, neither the efficiency implications of higher amounts of debt capital nor the optimal balancing of covenant and maturity value for a given level of debt capital are trivial.<sup>26</sup>

To complicate matters, the identity in (10) implies that the borrower benefits not only from reducing the debt payment  $l$  but also from reducing the cost of debt  $q$ , i.e., any improvement in efficiency arising from changes in contract terms accrues to the borrower. It therefore remains to be established whether the borrower might optimally concede higher debt payments to the lender in order to induce more efficient actions, i.e., whether an increase in  $l$  could lead to a decrease in  $q$  of larger magnitude. One might intuit that the answer should be negative because the borrower can extract any potential lender surplus by retaining additional control rights (which has no incentive effect), and indeed this rationale justifies why, as the stated formally in the following lemma, strategically overpaying the lender is never part of an optimal second-best contract.

**Lemma 3.** *The lender's break-even constraint  $L(r, m) \geq k$  binds in any optimal debt contract.*

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preferred action whereas the borrower did not, the lender would strictly prefer to waive its decision rights in states where the covenant is violated but the preference misalignment between the contracting parties is small.

<sup>26</sup> Décamps and Faure-Grimaud (2002) document the former outcome in a setting without covenants, in which the borrower (and only the borrower) can decide when to liquidate the investment funded by the debt capital.

The solution to (11) now follows from Lemmas 1 to 3. To understand the underlying logic, consider first the problem of setting the covenant variable  $r$ . Intuitively, one might expect decision rights to be optimally vested with the party whose action preference is most closely aligned with  $a^*$  in a given state  $\theta$ , but Lemma 3 implies at the same time that the chosen allocation gives the lender no more than its required minimum payoff  $k$ . The second-best optimal covenant thus strikes a compromise between minimizing inefficiency and minimizing expected debt payments, a tension that turns out to be the principal driver of the cost of debt. An optimal covenant variable  $r$  induces technical default, i.e., a covenant violation, in all states  $\theta$  in which lender control is comparatively better with respect to this dual objective. Quantitatively, the difference in the borrower's equity value between ceding and retaining control in a given state  $\theta$  is

$$h(m, \lambda, \theta) = q(a^l, m, \theta) - q(a^b, m, \theta) + (1 - \lambda)(l(a^l, m, \theta) - l(a^b, m, \theta)) \quad (13)$$

The formal derivation of  $h$  can be found in the proof of Proposition 1 in the Appendix. For any  $\theta$  at which  $h$  is negative, the borrower is better off with a covenant that cedes control to the lender at this  $\theta$ , with the reverse conclusion for any  $\theta$  where  $h$  is positive. In economic terms,  $h$  thus represents the borrower's net cost or benefit of technical default. Since technical default has been defined as an outcome  $r < 0$ , an optimally designed covenant variable  $r$  must therefore be such that  $r < 0$  if and only if  $h < 0$ , which yields the following result.

**Proposition 1.** *A debt covenant is optimal if and only if  $r(m, \lambda, \theta) = (jh)(m, \lambda, \theta)$ , where  $j$  is any positive real function.*

Since the covenant must distinguish only between  $r < 0$  and  $r \geq 0$ , the functional form of the optimal covenant variable  $r$  is clearly not unique: unless the current state  $\theta$  is at the boundary where  $r = 0$ , a small increase or decrease in  $r$  does not alter the allocation of decision rights and hence the realizations of  $b$ ,  $l$  and  $q$ . In practice, this means that a small change in a financial ratio underlying a covenant would not alter outcomes unless the current value of the ratio happens to be exactly at the contractually specified threshold that triggers technical default. Any  $r$  that is sign-consistent with  $h$  solves (11), and it is therefore without loss of generality to impose the normalization  $r = h$ , i.e., to set  $j(m, \lambda, \theta) = 1$  at all  $\theta$ .

Consider now the threshold states  $\{\theta: h(m, \lambda, \theta) = 0\}$  that separate the borrower's and lender's spheres of control. These threshold states form the frontier along which control changes when the covenant is marginally tightened or loosened. With an optimal covenant in place, there exists no way to rearrange control rights along this frontier such that the lender's break-even constraint is maintained while the debt cost decreases. A necessary condition is that the efficiency gain or loss from switching control to the lender is the same in all states with  $h = 0$  relative to the amount of additional debt payments conceded by the control transfer. Formally, this necessary condition amounts to requiring a constant ratio

$$\frac{q(a^l, m, \theta) - q(a^b, m, \theta)}{l(a^l, m, \theta) - l(a^b, m, \theta)} = \lambda - 1 \quad (14)$$

in all states with  $h = 0$ , in view of (13). In economic terms,  $\lambda - 1$  can thus be thought of as the borrower's marginal cost of debt: raising an additional unit of debt capital reduces efficiency by  $\lambda - 1$ . The discussion will return to this point later, including the possibility that  $\lambda < 1$ .

### [FIGURE 2]

Figure 2 provides a graphical illustration for a state space  $\Theta \subset \mathbb{R}^2$ . The two surfaces correspond to the inefficiency caused by the borrower,  $q(a^b, m, \theta)$ , and the lender,  $q(a^l, m, \theta)$ , if the respective party were in control. The level set  $\{\theta: h(m, \lambda, \theta) = 0\}$ , indicated by the solid dark line, identifies the boundary between the states in which inefficiency is determined by the borrower's surface and the states in which inefficiency is determined by lender's surface, both shaded in dark gray. If the choice of the boundary were unconstrained, it would optimally follow the intersection of the two surfaces, but the binding of the lender's break-even constraint generally prevents this outcome. The boundary instead follows the unique path along which the distance between the surfaces, scaled by the denominator on the left-hand side of (14), is equal to  $\lambda - 1$ . Figure 2 shows that the highest inefficiency is realized around the boundary, i.e., the agency problem takes its heaviest toll when the borrower has either violated the covenant by a narrow margin or just barely avoided a violation. By contrast, little inefficiency is incurred if the covenant variable is either well above or far below the threshold value  $r = 0$ .

Proposition 1 is only a partial result. The optimal control allocation is uniquely identified by  $h$ , which makes the multiplicity of the covenant variable  $r$  irrelevant, and the discussion will accordingly refer to an optimal covenant as unique if the optimal  $r$ , normalized to  $r = h$ , is unique. However,  $h$  itself depends on  $m$  and  $\lambda$ , both of which are choice variables, and thus Proposition

1 only identifies a conditional optimum. To complete the solution to the contract design problem, it remains to be established whether the borrower could face a choice of several maturity values that deliver equally efficient outcomes. Proposition 2 rules out this possibility.

**Proposition 2.** *The optimal debt contract  $\{r, m, k\}$  is unique.*

The uniqueness result follows from the supermodularity of the agency cost  $q$  in  $r$  and  $m$ : as the maturity value rises, switching control to the borrower becomes increasingly inefficient. To understand the relevance of this property, recall first that, by Lemma 3, the lender's break-even constraint must bind. Secondly, adjusting either the maturity value or the covenant in the lender's favor increases expected debt payments, i.e., the two contract dimensions are substitutes with respect to the lender's payoff.<sup>27</sup> As a heuristic, one can therefore evaluate all solution candidates by raising the maturity value  $m$  in small increments while simultaneously loosening the covenant stringency such that the break-even condition  $L(r, m) = k$  is maintained. (The reader is reminded here that the debt capital  $k$  is still exogenously imposed at this time and therefore not subject to optimization.) By Lemma 2, higher maturity values align the lender's incentives closer to the social optimum while exacerbating the borrower's, which leads to two effects. First, expanding the borrower's sphere of control means that the detrimental impact of higher  $m$  on the borrower's incentives affects a growing set of states while the beneficial effect on the lender's incentives affects a shrinking set of states. Second, raising  $m$  has a progressively unfavorable effect on the marginal agency cost incurred from pushing the control frontier in favor of the borrower. In other words, the marginal agency cost from raising the maturity value increases in covenant slack and, conversely, the marginal agency cost from loosening the covenant increases in the maturity value. This mutual reinforcement means that there exists a unique stationary point along the continuum of break-even contracts.<sup>28</sup>

### [FIGURE 3]

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<sup>27</sup> Handing a given state  $\theta$  over to the lender increases expected debt payments because borrower and lender prefer different actions, and thus  $l(a^l, m, \theta) > l(a^b, m, \theta)$ .

<sup>28</sup> In a geometric analogy, one might think of two squares whose respective areas represent the inefficiency caused by the borrower and the lender. The base of each square reflects the incentive effect of the maturity value and the height reflects the extent of the respective party's sphere of control. The break-even constraint requires that one square expands along both dimensions when the maturity value is increased, while the other square shrinks. The unique solution to minimizing the total area is to make both squares of equal size.

Figure 3 provides some visual intuition. For illustrative purposes, the state space is condensed to a one-dimensional  $\theta$ , where higher  $\theta$  correspond to states in which the lender's preferred action is closer to the first-best action. The optimal covenant thus takes the form of a threshold value below which the borrower is in technical default. The range of possible threshold values is shown, in reverse order, on the x-axis, and maturity values are plotted on the y-axis. For each possible combination of threshold and maturity value, the level of the surface reflects the magnitude of the associated agency cost. The set of break-even contracts follows the solid line crossing the surface. Contracts below the line fail to provide the lender with sufficient expected payments to recover the loan amount, while contracts beyond the line would overpay the lender, which is not in the borrower's interest. Hence, the southwest and northeast corners, which assign full control and claims to all cash flows to one party and are therefore most efficient, are infeasible. The set of feasible (break-even) contracts, by contrast, runs in northwest-to-southeast direction because maintaining break-even means that more cash flow claims to one party must be compensated by additional control rights to the other. The northwest and southeast corners make the least efficient contracts as they assign full decision rights to one and full payoff claims to the other party and thus create the worst possible incentive situations. Contracts between these extremes achieve a better balancing, and hence the optimal contract finds the interior point on the break-even curve that attains the lowest cost value. Consistent with the preceding discussion, the break-even curve is quasiconvex and hence its lowest point is unique.

Proposition 2 is economically relevant in several respects. First, the result clarifies the interdependence of covenants and interest rates with respect to contract efficiency. A *ceteris paribus* optimization over one dimension alone cannot deliver useful insights about optimal contract design, and hence an assessment of the value implications of including covenants in a debt contract (e.g., Matvos 2013) would likely yield better results if the maturity value were not imposed exogenously. At the same time, the unique optimum suggests predictability. In practice, one should expect borrowers with the same economic characteristics to have the same (or very similar) debt contract terms. What economic characteristics make a debt contract with a stricter covenant and a lower interest rate preferable to a contract with the reverse features has yet to be discussed, likewise the question whether the optimized debt cost is predictably higher or lower in given cir-

cumstances. In any event, however, Proposition 2 indicates that the common practice of interpreting interest rates or loan spreads as the cost of debt financing (e.g., Francis et al. 2005; Zhang 2008) is a conflation of two clearly distinct concepts.<sup>29</sup>

A few comments regarding the generality of the uniqueness result are in order. The model considers only one of the possible frictions that may arise in debt financing. Additional complications, such as information asymmetry (see, e.g., Gârleanu and Zwiebel 2009), would likely change the optimal contract terms. It is, however, not obvious why introducing an additional friction should predictably lead to a multiplicity of optimal contracts. Under a more technical aspect, the clean derivation of the uniqueness result rests on Lemma 2: a higher maturity value leads to a monotonic shift in the borrower's and lender's action preferences. The risk-ordering in Assumption 1a and the global concavity in Assumption 1b are sufficient but by no means necessary conditions for this outcome. As long as Lemma 2 holds, uniqueness extends to settings in which the set of available actions does not obey these assumptions globally.

### *3.1. Leverage and Profitability*

The debt principal  $k$  has thus far been treated as exogenous. Consider now the role of capital structure: how would the cost of debt respond if the borrower had some equity capital available and could finance the investment with lower leverage? To facilitate the discussion, it will be useful to operationalize the notion of a 'stricter' covenant by the following definition.

**Definition 1.** *A covenant based on  $r$  is said to be stricter than a covenant based on  $\hat{r}$  if, for any given maturity value  $m$ ,  $L(r, m) > L(\hat{r}, m)$ .*

A stricter covenant means an increase in expected debt payments as a result of a control reallocation, with the maturity value held fixed. Given the preference misalignment, this increase in debt payments implies that the reallocation favors the lender. The real-world analogue to Defini-

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<sup>29</sup> Beatty et al. (2002) view the cost of debt as consisting of contracting (agency) costs and interest costs. Proposition 2 shows that such a separation is not possible because interest rates interact with the agency problem and therefore do not constitute a separable component of the cost of debt.

tion 1 can be either a tightening of technical default rules, say, an increase in the level of net income below which the borrower is considered in violation of the covenant, or an increase in the number of covenants.<sup>30</sup> Further, let

$$k = k^d + k^e$$

denote the new capital structure, where  $k$  is now the total amount, financed by external debt in the amount of  $k^d$  and by the borrower's own equity capital in the amount of  $k^e$ .

An increase in the debt component  $k^d$  has two distinct interpretations. If the borrower decides to finance the investment by less equity and replace the shortfall by debt, the increase in  $k^d$  means an increase in the borrower's financial leverage. If, on the other hand, the investment cost has increased and the borrower decides to finance the increase proportionally by debt and equity, leverage remains the same while, *ceteris paribus*, the net present value of the investment decreases.<sup>31</sup> Hence, higher debt can be interpreted as an increase in leverage, a decrease in profitability, or any combination of these. Regardless of interpretation, however, the implications with respect to contract design and efficiency, shown in Proposition 3 below, are identical.

**Proposition 3.** *An increase in the amount of debt capital  $k^d$  implies an optimal debt contract with a higher maturity value and a stricter covenant.*

To see why loosening the covenant (lowering the interest rate) and overcompensating by a large increase the interest rate (severe tightening of the covenant) is never an optimal response to an increase in  $k^d$ , recall that raising the maturity value brings the lender's preferred action closer to the first-best action in every state  $\theta$ , while the opposite is true for the borrower. Accordingly, the marginal agency cost from expanding the lender's sphere of control decreases. Compensating the lender for providing additional capital to the borrower is therefore optimally accomplished by adjusting both maturity value and covenant in the lender's favor simultaneously.<sup>32</sup> Proposition 3

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<sup>30</sup> A stronger notion of covenant restrictiveness is  $\{\theta: r(\theta) < 0\} \supset \{\theta: \hat{r}(\theta) < 0\}$ , i.e., the lender gains control over some additional  $\theta$  without losing control rights elsewhere, which implies the criterion  $L(r, m) > L(\hat{r}, m)$  in Definition 1, whereas the converse is not true. Meeting this stricter definition would, for most results, require additional regularity conditions without adding insight.

<sup>31</sup> Modeling a decrease in profitability instead by holding  $k^d$  constant and reducing the cash flow proportionally in all states would be equivalent.

<sup>32</sup> The increase in maturity value also implies an increase in the interest rate if  $m$  is convex in  $k^d$ , or, equivalently, if

$$\frac{dm}{dk^d} > \frac{m}{k^d}$$

thus suggests that empirical estimation of the role of covenants in permitting firms to raise debt capital (e.g., Matvos 2013) should be carried out without constraining the maturity value.

Proposition 3 provides a rationale for some empirical findings on the interaction of debt contract terms with borrower characteristics. There is evidence that highly levered firms use restrictive covenants more frequently (Nikolaev 2010), violate their covenants more frequently (Beneish and Press 1993), pay higher interest rates (Zhang 2008), and use more conservative accounting practices, which tend to accelerate covenant violations (Ahmed et al. 2002; Zhang 2008; Nikolaev 2010). More profitable firms obtain lower interest rate spreads (Zhang 2008) and violate their covenants less frequently (Beneish and Press 1993). Empirical findings by Bharat et al. (2008) show that borrowers with poor accounting quality face both higher interest rates and more restrictions in other contract terms.

It remains to be established how the contract term adjustments in Proposition 3 impact the agency cost of the debt. Given that leverage is a choice variable, this question connects to the larger problem of the optimal capital structure, i.e., whether there exists a unique optimal  $k^d$  for a given investment opportunity. The limited setting studied here cannot provide a general answer because the agency problem under consideration is debt-specific while inefficiencies associated with equity are not modeled.<sup>33</sup> Without such opportunity cost, the trivial optimum would, of course, be full equity financing for *any* investment opportunity. However, examining the agency cost implications of changing  $k^d$  can nonetheless deliver some useful insights into the capital structure problem because the cost of debt constitutes one of its inputs and therefore cannot be

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The inequality holds for all  $m$  and  $k^d$  under first-best decision-making because then

$$\frac{dm}{dk^d} = \frac{1}{p} > \frac{m}{k^d}$$

in view of the break condition

$$L(r, m) = pm + (1 - p)E(c|r, m, c < m) = k^d$$

where

$$p = \Pr(c > m|r, m)$$

If action preferences are considered but control allocation is held fixed as  $m$  increases, the inequality holds a fortiori because the incentive effect of a higher  $m$  reduces  $L$ , by the envelope theorem, and therefore requires a greater increase in  $m$ . The gradual transfer of decision rights to the lender at higher  $m$  mitigates the incentive effect, and establishing global convexity analytically would require additional regularity conditions, but the preceding arguments, along with the observation that  $m = 0$  when  $k^d = 0$  and  $m = \infty$  when  $k^d = E(c)$ , suggest that convexity holds.

<sup>33</sup> Equity is effectively a sunk cost here and does not change incentives or give rise to additional constraints in the contract design problem. While not the object of interest here, agency conflicts associated with equity financing do, however, exist. For example, insiders to the firm may divert resources for private consumption at the expense of outside equity owners, and debt financing is one possible means to address the problem (Jensen and Meckling 1976).

ignored even in a more comprehensive model.<sup>34</sup> Recall the claim made earlier that the borrower's marginal cost of debt is  $\lambda - 1$ . To understand why, observe that total enterprise value at time  $t = 0$  is the sum of the equity value  $\mathcal{B}(r, m, \lambda, k^d)$  and the debt value  $L(r, m) = k^d$  and thus changes in  $k^d$  by

$$\frac{d\mathcal{B}(r, m, \lambda, k)}{dk^d} + 1 = -\lambda + 1$$

in view of the envelope theorem. Décamps and Faure-Grimaud (2002) observe that, in a world without covenants, higher leverage exacerbates the agency problem, so that the cost of debt increases monotonically, which would mean that  $\lambda > 1$  for all  $k^d$ . Proposition 4 below shows that introducing covenants makes the agency cost non-monotonic, i.e., there exist values of  $k^d$  at which  $\lambda < 1$  and thus the cost of debt can be locally decreasing in leverage.

**Proposition 4.** *The cost of debt is concave and non-monotonic in the amount of debt capital  $k^d$ .*

Figure 3 helps to develop a visual understanding of the result. As noted before, the optimal contract is the unique minimum on the solid line of break-even contracts traversing the contract space of possible combinations of covenants and maturity values. The path of the break-even line is determined by the debt level  $k^d$ . When the debt level rises, the break-even isoquant shifts upward toward the northeast corner of the graph as the lender needs a combination of stricter covenants and higher maturity values to obtain the break-even payoff. The location of the optimal contract therefore travels along a curve from the southwest to the northeast corner as leverage increases. The endpoints of this curve correspond to the first-best efficient extreme cases in which one party is vested with full control and claims to all cash flows. The agency cost therefore rises as  $k^d$  increases from low to intermediate values but declines again from there. The high agency cost around intermediate debt levels can thus be thought of as a kind of costly compromise over opposing decision preferences between borrower and lender. The difference to the monotonicity result by Décamps and Faure-Grimaud (2002) thus arises because, as leverage increases, the presence of covenants permits a concurrent transfer of decision rights to the lender

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<sup>34</sup> A simple cost-benefit tradeoff with an optimal interior leverage ratio arises, for example, if one considers the tax shield advantage of debt to offset against the agency cost (Mello and Parsons 1992).

that curbs the impact of the borrower's increasingly detrimental incentives. Proposition 4 therefore implies that it is neither true that borrowers with more covenants must have higher agency costs, as conjectured by Beatty et al. (2008), nor that higher leverage monotonically intensifies the agency conflict, as assumed by Ahmed et al. (2002).

As observed earlier, leverage and profitability have symmetric effects on the optimal contract terms. Increasing both simultaneously therefore moves the agency cost in opposite directions along the concave cost curve. Then if the marginal cost of equity financing were unrelated to the level of debt, the optimal choice of leverage would be inversely aligned with the firm's profitability: highly profitable firms would finance themselves with low levels of debt while firms with low profit margins would be highly levered. A robust analytical result to this end, and certainly any quantitative statements as to how much agency cost the borrower optimally tolerates, would require a more elaborate model, but the logic for an inverse alignment does appear to be borne out in the empirical findings by Rajan and Zingales (1995), who document that leverage and profitability in a large worldwide sample of firms are negatively correlated. At the same time, even if harmonizing leverage with investment returns were the principal recipe for determining capital structure, the non-monotonicity of the agency cost implies that a given cost level can generally obtain for two different choices of  $k^d$ . Hence, even with the cost of alternative forms of capital ignored, a simple model of debt cost suggests that the optimal leverage is not generally unique. Proposition 4 thus contributes another possible reason why pinning down an answer to the capital structure problem has proven difficult.

### *3.2. Renegotiation*

The results up to this point have been obtained under the assumption that no renegotiation occurs at the interim date  $t = 1$ . In practice, however, debt contracts are frequently renegotiated, and whether the cost of debt behaves differently in renegotiable contracts is therefore an economically relevant question.<sup>35</sup> To explore the problem, borrower and lender will hereafter be given the opportunity to renegotiate the contract at date  $t = 1$ , prior to taking action. The objective is to show that, under plausible circumstances, the insights obtained so far continue to apply.

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<sup>35</sup> Chava and Roberts (2008) document that about 25% to 37% of borrowers violate their covenants at some point during the loan period, which in most cases leads to a renegotiation of the contract. Nikolaev (2015) finds that borrowers in private debt markets, on average, face a 37% probability of renegotiating at least one of their contracts in any given year.

There are several reasons to expect the results from the renegotiation-free setting to hold relevance even when renegotiation is possible. The pervasiveness of covenants in practice suggests that there is a need for strategic allocation of control rights, which, in turn, implies that renegotiation is unlikely to be costless, as an ex-ante allocation of control rights would, in line with the Coase theorem, be irrelevant in the absence of renegotiation costs. A strictly positive cost of debt and a covenant-based allocation of decision rights thus continue to be critical elements of the model. Importantly, the cost of debt is still connected to the borrower's and lender's preference misalignment problem because there would be no need for costly renegotiation if incentives were aligned. The underlying agency problem therefore remains relevant. Finally, although common, renegotiation does not occur all of the time, suggesting that the cost of renegotiating can outweigh the potential efficiency gains.

Formally, the model setup and the sequence of events are the same as before, except for the following modification. After the realization of  $\theta$  at time  $t = 1$  but prior to taking action, the controlling side may, in exchange for an adjustment of payment terms, offer its counterparty to implement an action other than the inefficient one it would otherwise take. The controlling party is assumed to have full bargaining power and can therefore make a take-it-or-leave-it offer that extracts the entire welfare gain.<sup>36</sup> Without loss of generality, the payment term adjustment takes the form of an increase or decrease in the maturity value from  $m$  to a new value  $\hat{m}$ .<sup>37</sup> The renegotiation incurs a deadweight cost, which reduces the welfare gain and may vary across states.<sup>38</sup> In practice, this cost may arise from the time spent on the process and from the need to monitor whether the agreed-upon action is indeed implemented.<sup>39</sup> To avoid unnecessary complications, the renegotiation cost is assumed to be the same regardless of the action the parties agree on, so that renegotiation always leads to implementation of the first-best action  $a^*$ .

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<sup>36</sup> Alternative assumptions about bargaining power would not affect any conclusions qualitatively.

<sup>37</sup> Since the action to be taken has been agreed upon at this time, the maturity value has no incentive effect anymore and does therefore not affect the bargaining surplus. One can also readily verify that, since  $\hat{m} \in (0, \infty)$ ,  $\hat{m}$  can implement any possible division of the final cash flow between the two parties.

<sup>38</sup> Letting the cost reduce the welfare gain effectively imposes the renegotiation cost on the controlling party. Alternative assumptions can be made, but the initial contract terms would then simply be adjusted such that the lender still breaks even in expectation.

<sup>39</sup> Costly monitoring of whether the agreed-upon action is implemented can also serve as a motivation for the assumption that the full action space  $A$  is not contractible: if verifying an action is costly and  $A$  is large, the cost of anticipating all possible actions in a contract at time  $t = 0$  would be prohibitive.

The deadweight cost must be connected to the latent inefficiency  $q$  from the renegotiation-free setting because borrower and lender would only undertake renegotiation in states where the maximum possible welfare gain from eliminating  $q$  covers or exceeds the associated cost. When  $q$  is so small that the cost of renegotiation would exceed the potential welfare gain, the parties would rationally forgo renegotiation and incur the inefficiency  $q$  instead. Hence, the agency cost  $p$  in a setting with renegotiation must be bounded by  $q$ , or

$$p \leq q(a^i, m, \theta) = E(c|a^*(\theta), \theta) - E(c|a^i(m, \theta), \theta) \quad (15)$$

where  $i = b, l$ . A cost of  $p < q$  means that renegotiation has occurred and a cost of  $p = q$  means that renegotiation would have been too costly and therefore has been forgone. Since the right-hand side of (15) depends on which party has control rights during the bargaining process, one can also readily see that control allocation continues to impact efficiency. To emphasize this dependence on the underlying agency problem, the cost of debt will be denoted  $p(q)$ .

After a covenant violation, the lender offers the borrower to undertake the first-best action  $a^*$  and extracts the resulting surplus by demanding a higher maturity value  $\hat{m}$  such that

$$l(a^*, \hat{m}, \theta) = l(a^l, m, \theta) + q(a^l, m, \theta) \quad (16)$$

Since  $l$  is monotonic in the maturity value, there exists a unique  $\hat{m}$  that solves (16). At the same time, the lender bears the associated renegotiation cost  $p(q)$ . Conversely, the borrower has the bargaining power in the absence of a covenant violation and therefore offers to implement  $a^*$  in exchange for a lower maturity  $\hat{m}$  such that

$$l(a^*, \hat{m}, \theta) = l(a^b, m, \theta)$$

and thereby captures all surplus, net of the renegotiation cost. The lender's expected payoff in state  $\theta$  in a setting with a renegotiable debt contract is therefore

$$\begin{aligned} l(r, m, \theta) &= l(a^*, \hat{m}, \theta) \mathbf{I}_{r \geq 0}(r) \\ &\quad + \left( l(a^*, \hat{m}, \theta) - p(q(a^l, m, \theta)) \right) (1 - \mathbf{I}_{r \geq 0}(r)) \\ &= l(a^b, m, \theta) \mathbf{I}_{r \geq 0}(r) \\ &\quad + \left( l(a^l, m, \theta) + q(a^l, m, \theta) - p(q(a^l, m, \theta)) \right) (1 - \mathbf{I}_{r \geq 0}(r)) \end{aligned} \quad (17)$$

In situations where renegotiation is optimally forgone, i.e.,  $p(q) = q$ , (17) reduces to the original expression in (8). The optimal renegotiable debt contract now solves the program

$$\max_{r,m,\lambda} \mathcal{B}(r, m, \lambda, k) = \max_{r,M,\lambda} \int_{\Theta} \mathcal{b}(r, m, \lambda, k, \theta) dG \quad (18)$$

where

$$\mathcal{b}(r, m, \lambda, k, \theta) = E(c|a^*(\theta), \theta) - p(q(r, m, \theta)) + (\lambda - 1)l(r, m, \theta) - \lambda k \quad (19)$$

and

$$p(q(r, m, \theta)) = p(q(a^b, m, \theta))\mathbf{I}_{r \geq 0}(r) + p(q(a^l, m, \theta))(1 - \mathbf{I}_{r \geq 0}(r)) \quad (20)$$

The solution to (18) obviously depends on the properties of  $p$ . In particular, inspection of (19) shows that the agency cost retains the supermodularity property in Corollary 1 if  $p'(q) > 0$ , i.e., if the agency cost  $p$  is comonotonic with the latent inefficiency  $q$ .<sup>40</sup> As before, raising the maturity value makes control allocation to the borrower less efficient, so that the derivations underlying Propositions 1 through 4 remain the same. Proposition 5 gives a formal summary.

**Proposition 5.** *For any renegotiation cost  $p(q)$  with  $p'(q) > 0$ , the optimal debt contract is unique. An increase in debt capital implies a higher optimal maturity value and a stricter optimal covenant. The cost of debt is concave and non-monotonic in the amount of debt capital.*

Proposition 5 relies on the comonotonicity between the latent inefficiency  $q$  and the realized cost  $p$ , but the assumption is plausible for several reasons. First, the relationship automatically obtains on any subset of  $\Theta$  where  $p(q) = q$ , i.e., where renegotiation is not worthwhile. Moreover, the boundary condition in (15) means that higher  $q$  extend the range of possible  $p$  upward, i.e., high values of  $p$  are only realized if  $q$  is also at least as high. The real-world analogue of a high  $q$  is a situation in which the controlling party has preferences that differ substantially from the first-best course of action. Given the strong incentive to deviate from the social optimum, it is reasonable to think that deliberations over contract modifications then take longer and monitoring whether the new agreed-upon action is implemented is costlier than in situations in which the controlling party's preferred action is already close to first-best.

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<sup>40</sup> In practice, renegotiation typically occurs when a covenant has been violated, i.e., in states where the lender has control. While the model does not provide a direct motivation why renegotiation in borrower-controlled states should be less common, lender-control-only renegotiation, however justified, could readily be accommodated by setting the agency cost in borrower-controlled states to  $p = q$  everywhere, which would change (20) to

$$p(q(r, m, \theta)) = q(a^b, m, \theta)\mathbf{I}_{r \geq 0}(r) + p(q(a^l, m, \theta))(1 - \mathbf{I}_{r \geq 0}(r))$$

The supermodularity property, and hence the conclusions in Proposition 5, then continue to hold.

To obtain a better understanding how renegotiation affects contract design and the cost of debt, it is instructive to consider the limit case when the renegotiation cost approaches a constant value  $\bar{p}$  for all  $\theta$ , which means that  $p'(q) \rightarrow 0$  at all  $p(q) < q$ , and, in the limit,

$$p(q(r, m, \theta)) = \min(q(r, m, \theta), \bar{p})$$

Graphically, the area around the intersection of the two surfaces depicting the agency cost in Figure 2 now approaches a plateau corresponding to the subset  $\{\theta: p(q(r, m, \theta)) = \bar{p}\}$ , whose size depends on the magnitude of  $\bar{p}$ .<sup>41</sup> Consider the borrower's net cost or benefit of transferring control to the lender in a given state  $\theta$ . When  $p(q) = q$ , one obtains the same value  $h$  as in (13) in the renegotiation-free case. When  $p(q) = \bar{p}$ ,  $h$  becomes

$$h(m, \lambda, \theta) = (1 - \lambda) \left( l(a^l, m, \theta) + q(a^l, m, \theta) - \bar{p} - l(a^b, m, \theta) \right) \quad (21)$$

As before, the first-order condition for an optimal allocation of control rights is  $h = 0$ , and the set of states where  $h = 0$  identifies the frontier that separates the two parties' spheres of control. If *any* point of the frontier lies on the plateau,  $h = 0$  can only hold if  $\lambda = 1$  because then the second parenthetical term on the right-hand side of (21) is always positive. In other words, the borrower's marginal cost of debt,  $\lambda - 1$ , becomes zero. The outcome is intuitive: the borrower could raise additional debt capital in exchange for additional control rights to the lender without having to change the interest rate and thus impacting the agency cost, as long as there is scope for such control concessions within  $\{\theta: p(q(r, m, \theta)) = \bar{p}\}$ . Hence, borrowers facing only small variation in renegotiation costs have a range of leverage ratios over which the cost of debt is nearly constant, which further speaks to the difficulty of pinpointing the optimal capital structure.

### 3.3. Project Selection

Borrower and lender have so far been modeled as identical in their ability to influence the distribution of the cash flow. In reality, the borrower likely has greater scope than the lender to make operational decisions that change the nature of the financed investment, as such decisions require knowledge of technology, information about market prices and opportunities, and interpersonal relationships with employees, customers and suppliers, to which an outside lender, such as a

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<sup>41</sup> When the renegotiation cost is constant, the allocation of control rights on the plateau set  $\{\theta: p(q(r, m, \theta)) = \bar{p}\}$  has no effect on the agency cost and can therefore be arbitrary as long as the lender's break-even constraint is maintained. The optimal covenant would no longer be unique in this case, but uniqueness holds as long as  $p'(q) > 0$ , regardless how small the magnitude of  $p'$ , and hence the optimal covenant has a unique limit as  $p'(q) \rightarrow 0$ .

commercial bank, may not have ready access. Moreover, these decisions may be too numerous and subtle to specify contractually *ex ante*. Suppose therefore that, in addition to the action  $a$  that either borrower or lender can undertake, the borrower has the exclusive opportunity to take an additional action that the lender cannot prevent or influence.<sup>42</sup>

The timeline of events remains the same as before, except for the following insertion. After the contract is signed but before the state  $\theta$  is realized, the borrower takes an action  $s \in S$  that affects the distribution of the investment's terminal cash flow.<sup>43</sup> The action is irreversible, non-contractible and can only be taken by the borrower. To maintain consistency with prior research on this type of problem (Caskey and Hughes 2012), the additional action will be referred to as a project selection, i.e., the borrower has some latitude in how to invest the debt capital. The realization of  $\theta$ , the covenant-based allocation of control rights, and the subsequent action  $a$  by the controlling party occur as in the original setting. The cumulative distribution function of the cash flow, conditional on  $a$ ,  $s$ , and  $\theta$ , will be denoted by  $F(c|a, s, \theta)$ .

As a baseline, consider a setting in which the first-best project, hereafter denoted by  $\underline{s}$ , can be prescribed contractually. Solving for the optimal contract is then identical to solving the original optimization problem in (11), and the properties of this contract are consistent with all results obtained thus far. Now consider how contract terms and the cost of debt are impacted if prescribing  $\underline{s}$  were impossible and the borrower could instead choose a project  $s \neq \underline{s}$  after the contract has been signed. To make the comparison as transparent as possible, the project choice set is simplified to  $S = \{\underline{s}, \bar{s}\}$ , where  $\bar{s}$  is an inferior project in the sense that  $E(c|\underline{s}) > E(c|\bar{s})$ .<sup>44</sup>

The project selection problem is obviously of no relevance if the borrower has no incentive to choose  $\bar{s}$ . Some formal structure is therefore needed to identify circumstances when  $\bar{s}$  is inferior while the borrower nonetheless has an incentive to choose it. Principally, the incentives that determine the borrower's preferred project selection, hereafter denoted by  $s^b$ , are the same as those underlying the borrower's preferred action  $a^b$ : the borrower might sacrifice welfare in exchange for a higher cash flow variance and thereby benefit at the expense of the lender. The economically interesting scenario therefore features project choices that can be ordered by the risk in

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<sup>42</sup> The action space  $A$  considered so far should thus be thought of as limited to those decisions that even an outsider could make, such as approving a new product development or shutting down an operating division.

<sup>43</sup> Whether  $s$  occurs before or after the realization of  $\theta$  is not of critical importance for the following results.

<sup>44</sup> Modeling  $s$  as continuous would lead to equivalent conclusions, but the discrete case makes a more insightful comparison to the baseline model.

the associated cash flow in a manner similar to the action space  $A$ . Unlike the action  $a$ , however, the risk ordering associated with project selection matters in two distinct respects. First, project selection has a direct effect on the cash flow distribution by changing the parameter  $s$  in  $F(c|a, s, \theta)$ . Secondly, and in contrast to  $a$ ,  $s$  can have an indirect effect on the cash flow distribution because the sequential occurrence of  $s$  and  $a$  means that project selection may change the marginal effect of the subsequent action  $a$  and thus alter the borrower's and lender's preferred actions  $a^b$  and  $a^l$ , respectively. The following assumption addresses both aspects.

**Assumption 2a.** For any  $\underline{s} < \bar{s}$ , there exists some  $z \in \mathbb{R}^+$  such that, given any  $a$  and  $\theta$ ,  $F(c|a, \bar{s}, \theta) \geq F(c|a, \underline{s}, \theta)$  at all  $c \leq z$  and  $F(c|a, \bar{s}, \theta) < F(c|a, \underline{s}, \theta)$  at all  $c > z$ .

**Assumption 2b (neutral).** For any  $\theta$ ,  $a$  and  $\underline{s} < \bar{s}$ ,  $F_a(c|a, \underline{s}, \theta) = F_a(c|a, \bar{s}, \theta)$  at all  $c$ .

Part 2a mirrors the single-crossing property imposed on  $a$  in Assumption 1: higher values of  $s$  rotate the cumulative cash flow distribution function clockwise and thereby increase its variance. Higher  $s$  might, for example, mean selling at higher margins to a customer group with lower credit quality or replacing an existing product with stable sales by an innovative one for which demand is highly uncertain. Part 2b addresses the question how a change in  $s$  affects the marginal impact of the action  $a$ . In principle, selecting an investment with higher risk can either dampen or reinforce the effect of the subsequent action  $a$ , and one can argue that either scenario is plausible. A dampening (or substitutive) effect could be justified if the project selection already exhausts much of the scope for risk-taking, so that, the riskier the project choice, the lesser the incremental risk created by later actions. Alternatively, a reinforcing (or complementary) effect could obtain if choosing to invest in a more volatile product market increases the scope for later actions that involve risk-taking.<sup>45</sup> Part 2b chooses the neutral middle ground by assuming that the marginal effect of  $a$  is the same regardless whether the project choice is  $\underline{s}$  or  $\bar{s}$ , so that

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<sup>45</sup> For example, suppose a consulting firm must decide first whether to focus on financial consulting or on strategy consulting (the project selection), and then whether to pursue small clients or large clients (the action choice). For argument's sake, let strategy consulting and catering to small clients constitute the respective high-risk options. A dampening (substitutive) effect means that choosing small clients over large clients creates only a minor increase in the variance of the firm's profit if the firm is in strategy consulting but a major increase if the firm is in financial consulting. A reinforcing (complementary) effect would describe the reverse relationship.

project choice does not affect the borrower's and lender's preferred actions. The implications of switching the assumption in either direction are discussed at the end of this section.

The borrower chooses  $s$  after the contract has already been signed and can therefore act opportunistically. In particular, the borrower opts for the optimal project  $\underline{s}$  if

$$B(\underline{s}, r, m) \geq B(\bar{s}, r, m) \quad (22)$$

where

$$B(s, r, m) = \int_{\Theta} \left( b(a^b, s, m, \theta) \mathbf{I}_{r \geq 0}(r) + b(a^l, s, m, \theta) (1 - \mathbf{I}_{r \geq 0}(r)) \right) dG$$

and

$$b(a, s, m, \theta) = E(\max(c - m, 0) | a, s, \theta) = E(c | a, s, \theta) - m + \int_0^m F(c | a, s, \theta) dc$$

are the equity values, as in the original model, except that the cash flow distribution  $F(c | a, \theta)$  is replaced by  $F(c | a, s, \theta)$ .<sup>46</sup> The contract design problem is therefore

$$\max_{r, m} B(s^b, r, m) \quad (23)$$

subject to the lender's break-even constraint and to the incentive compatibility (project selection) constraint (22). There are three possible solution scenarios. In the first case, the incentive constraint does not bind, the borrower chooses  $s^b = \underline{s}$ , and the solution contract is the same as in the baseline model. Project selection does not limit contract design in this case. The second possibility is that the incentive constraint binds, in which case  $s^b = \underline{s}$  still obtains but covenant and maturity value are set in a compromise between ensuring optimal project selection and minimizing the original agency cost arising from misaligned preferences over  $a$ . As a third outcome, the parties may find that incentivizing the borrower to choose  $\underline{s}$  is either costlier than accepting  $s^b = \bar{s}$  or outright infeasible, and therefore optimize the contract conditional on  $\bar{s}$ .

The second scenario, in which (22) binds, holds the key to understanding when and how project selection constrains debt contract design. The object of interest is the set of contracts

$$\{r, m, k\}: B(\underline{s}, r, m) = B(\bar{s}, r, m)\}$$

that demarcates the boundary between the set of contracts under which  $s^b = \underline{s}$  and the set of contracts under which  $s^b = \bar{s}$ . Under any contract in this boundary set, the borrower is indifferent between  $\underline{s}$  and  $\bar{s}$ , and so (22) holds with equality. Now, changing the maturity value  $m$  has the

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<sup>46</sup> Without loss of generality, the borrower is assumed to resolve indifference between  $\underline{s}$  and  $\bar{s}$  in favor of  $\underline{s}$ .

same impact on the borrower's preferred project as it does on the borrower's preferred action, with the same rationale: the borrower's payoff behaves like a call option with a strike price of  $m$ , and the higher the strike price, the more the option value benefits from increased cash flow volatility. The result is a direct analogue to Lemma 2.

**Lemma 4.** *There exists a unique maturity value  $\bar{m}$  such that the borrower prefers the first-best action  $\underline{s}$  if and only if  $m \leq \bar{m}$ .*

Given Lemma 4, one can readily tell which of the three solution scenario sketched out above obtains. In particular, if the optimal contract in the baseline scenario has a low maturity value of  $m \leq \bar{m}$ , implementing this contract does not distort the borrower's project selection preference away from the optimal project  $\underline{s}$ . Hence, the solution to (23) is the same as the solution to the baseline problem in (11). An optimal baseline contract with a high maturity value  $m > \bar{m}$ , on the other hand, cannot be replicated when project selection is not contractible because the borrower would choose  $\bar{s}$  instead of  $\underline{s}$ . In this case, either the parties accept that the borrower implements the inefficient project  $\bar{s}$  and optimize accordingly, or they agree on a contract that incentivizes the borrower to choose  $\underline{s}$  and accept that this contract has a suboptimally low maturity value  $m = \bar{m}$ .<sup>47</sup> The following proposition gives a formal summary.

**Proposition 6.** *When the project selection constraint binds, the optimal debt contract has a lower maturity value, a stricter covenant and a higher cost of debt than it would if project selection were contractible.*

Figure 4 provides a graphical illustration. For expositional simplicity, the state space is reduced to one dimension, so that the covenant takes the form of a threshold value below which the borrower is deemed to be in technical default. The graph shows the mapping from maturity values to the corresponding, conditionally optimal covenant thresholds when the chosen investment is  $\underline{s}$ .

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<sup>47</sup> A maturity value  $m < \bar{m}$  is never optimal in this case. By Proposition 2, the optimal baseline contract is unique as the contract design problem in (11) has a unique interior maximum. Hence, the cost of debt increases monotonically in the distance between  $m$  and the unique optimum, so that setting  $m = \bar{m}$  is more efficient than any  $m < \bar{m}$ . If  $m = \bar{m}$  is infeasible because the lender would not receive the required break-even payoff even if given full control rights, a maturity value  $m > \bar{m}$  is chosen by necessity and the parties must optimize over the inferior project  $\bar{s}$ .

The vertical line at  $\bar{m}$  splits this curve into the solid segment of incentive-compatible contracts with low maturity values, and the dashed segment of contracts that cannot be implemented because the borrower would choose  $\bar{s}$  instead of  $\underline{s}$  at these high maturity values.

As noted earlier, a lower maturity value, and thus a lower interest rate, is not equivalent to a lower cost of debt. Proposition 6 not only reinforces this observation but describes a situation in which, to the contrary, a lower interest rate is in fact indicative of a higher financing cost. The need to maintain incentive compatibility by lowering the maturity value, and thus the interest rate, below the optimal baseline level is what makes a debt contract with discretionary project selection more costly than the benchmark setting with contractible project selection.

By Proposition 3, borrowers with high financial leverage or low profitability set higher optimal maturity values in the baseline model. It is these types of borrowers that suffer increases in their cost of debt when project selection is added to the contracting problem since the efficient high maturity values are not incentive-compatible. One might therefore expect firms to finance themselves with high debt-to-equity ratios or to seek debt financing for barely profitable ventures only when the use of the debt capital can be specified contractually in some detail or when the borrower's scope for taking on high risks *ex post* is limited.

In the baseline model, the incentives and actions of borrower and lender are symmetric: the borrower has worse incentives when highly levered, whereas the lender has worse incentives at low leverage; the borrower has worse incentives when profitability is low, whereas the lender has worse incentive when profitability is high; etc. Hence, the baseline model provides no immediate reason to predict debt covenants, in general, to favor one party over the other. The project selection problem breaks the symmetry. Stricter covenants than in the baseline model arise from the need to put greater emphasis on aligning the borrower's incentives with the social optimum. The direction of the asymmetry is consistent with the conclusion by Gârleanu and Zwiebel (2009), who also predict covenants to favor the lender, albeit for a different reason. Rather than giving borrowers greater scope for action-taking *ex post*, Gârleanu and Zwiebel endow borrowers with superior information *ex ante*, and the optimal contract to address the resulting adverse selection problem includes, on balance, a control allocation favoring the uninformed lender.

The conclusions in Proposition 6 are consistent with the results from the project selection model by Caskey and Hughes (2012), who also conclude that control allocation should favor the

lender. For Caskey and Hughes, however, control allocation favoring the lender is optimal because the lender can then abandon an inefficiently risky project, which creates a disincentive for the borrower to choose the inefficient, high-risk project in the first place. In Proposition 6, on the other hand, the borrower's project selection incentives, in view of Lemma 4, are determined by the maturity value, not by the allocation of control rights. Although strict covenants are part and parcel of optimal contract design, the expansion of the lender's sphere of control only serves to compensate the lender for agreeing to a reduced maturity value and makes no direct contribution to aligning the borrower's incentives more closely with the social optimum.

There are two reasons for this difference in rationale. First, Caskey and Hughes model the direct effect of project selection, as reflected in Assumption 2a, in a reduced form in which the borrower's project selection incentives do not interact with the maturity value of the debt. Hence, the rationale underlying Lemma 4 is not applicable to their setting. The second reason lies in the indirect effect that project selection has by changing the marginal impact of the subsequent action  $a$  and thus the parties' action preferences, as specified in Assumption 2b. As noted earlier, the indirect effect can either reinforce or dampen the effect of  $a$ , and thus either drive the borrower's and the lender's preferred actions away from the social optimum or align them more closely. In Proposition 6, the indirect effect is neutral, i.e., the contracting parties' action preferences are unaffected by project selection. Caskey and Hughes, on the other hand, assume that the high-risk project only creates inefficiency if the subsequent action is also risky, which corresponds to the following variant of Assumption 2b.<sup>48</sup>

**Assumption 2b (complementary).** For any  $\underline{s} < \bar{s}$ ,  $F_a(c|a, \underline{s}, \theta) \leq F_a(c|a, \bar{s}, \theta)$  at  $c \leq z$  and  $F_a(c|a, \underline{s}, \theta) \geq F_a(c|a, \bar{s}, \theta)$  at  $c > z$ , where  $z$  is the same as in Assumption 2a.

The complementarity version of Assumption 2b implies that choosing the riskier project  $\bar{s}$  increases the incremental risk created by higher actions  $a$ . Graphically, a higher action  $a$  rotates the cumulative distribution function of the cash flow more strongly when  $s = \bar{s}$  than it does

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<sup>48</sup> In particular, Caskey and Hughes model a binary action with project continuation as the high-risk option and project abandonment as the low-risk option. Choosing the inefficient high-risk project only benefits the borrower if the subsequent action is to continue the project.

when  $s = \underline{s}$ . One might intuit that the borrower would then prefer a higher action under  $\bar{s}$  than under  $\underline{s}$ , and indeed, given an action  $a^b$  optimized for  $\underline{s}$ ,

$$\begin{aligned} & b_a(a^b(\underline{s}), \bar{s}, m, \theta) - b_a(a^b(\underline{s}), \underline{s}, m, \theta) \\ &= \int_m^\infty \left( F_a(c|a^b(\underline{s}), \underline{s}, \theta) - F_a(c|a^b(\underline{s}), \bar{s}, \theta) \right) dc \geq 0 \end{aligned} \quad (24)$$

i.e., the borrower would benefit from increasing  $a$  further given  $\bar{s}$ , and so  $a^b(\bar{s}) > a^b(\underline{s})$ . The lender's preferences naturally go in the opposite direction, as is readily verified by observing that

$$\begin{aligned} & l_a(a^l(\underline{s}), \bar{s}, m, \theta) - l_a(a^l(\underline{s}), \underline{s}, m, \theta) \\ &= \int_0^m \left( F_a(c|a^l(\underline{s}), \underline{s}, \theta) - F_a(c|a^l(\underline{s}), \bar{s}, \theta) \right) dc \leq 0 \end{aligned} \quad (25)$$

and thus  $a^l(\bar{s}) \leq a^l(\underline{s})$ . Tightening the covenant to give more control to the lender now makes project  $\bar{s}$  less attractive to the borrower, as the benefit of switching from  $\underline{s}$  to  $\bar{s}$  decreases by

$$\begin{aligned} & b(a^b(\underline{s}), \underline{s}, m, \theta) - b(a^b(\bar{s}), \bar{s}, m, \theta) - b(a^l(\underline{s}), \underline{s}, m, \theta) + b(a^l(\bar{s}), \bar{s}, m, \theta) \\ &= b(a^b(\underline{s}), \bar{s}, m, \theta) - b(a^b(\bar{s}), \bar{s}, m, \theta) + b(a^l(\bar{s}), \bar{s}, m, \theta) - b(a^l(\underline{s}), \bar{s}, m, \theta) \\ &+ \int_{a^l(\underline{s})}^{a^b(\underline{s})} \int_m^\infty \left( F_a(c|a, \bar{s}, \theta) - F_a(c|a, \underline{s}, \theta) \right) dc da \leq 0 \end{aligned}$$

A stricter covenant thus discourages the borrower from choosing  $\bar{s}$  because  $\bar{s}$  induces the lender to take an action even lower than under  $\underline{s}$ , which is detrimental to the borrower. Under the complementarity variant of Assumption 2b, tightening the covenant thus not only compensates the lender for the reduced maturity value but, consistent with the observations by Caskey and Hughes, also reinforces the borrower's incentive to select the socially optimal project.<sup>49</sup>

The counterpart to this scenario is a substitutive relationship between  $s$  and  $a$ , modeled by reversing the inequalities from the complementary scenario. One can see immediately that the substitutive variant reverses the signs of (24) and (25), so that restricting the covenant now encourages the borrower to choose  $\bar{s}$ . Intuitively, the substitution version means that, by choosing  $\bar{s}$ , the borrower can maneuver the business into a position where the subsequent action  $a$  has a lesser impact on business risk than under  $\underline{s}$ . Given  $\bar{s}$ , the lender is therefore willing to undertake a higher action  $a$ , while the borrower finds raising  $a$  less beneficial. Incentivizing the borrower

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<sup>49</sup> Proving formally that the cutoff maturity value  $\bar{m}$  is still unique requires additional regularity conditions because increasing  $m$  now changes  $a^b$  and  $a^l$  differentially between  $\underline{s}$  and  $\bar{s}$ , but the qualitative conclusions from Proposition 6 remain the same.

to select  $\underline{g}$  becomes more difficult as a result, as the incentive effect of lowering the maturity value is counteracted by the concurrent tightening of the covenant.

#### 4. CONCLUSION

Debt is costly because it distorts borrowers' and lenders' preferences over risk-return tradeoffs away from the social optimum. The model presented in this paper is intended to shed some light on the effects of this misalignment problem on debt contract design, particularly on the interaction between interest rates and covenants and their bearing on the agency cost of the debt. A central theme is that interest rates and covenants determine the cost of debt jointly and should not be viewed as contract terms to be optimized independently. Given a set of borrower characteristics, there exists a unique optimal debt contract somewhere on the spectrum between contracts with tight covenants and low interest rates and contracts with loose covenants and high interest rates. The most efficient, lowest-cost contract is generally not the one with the lowest interest rate, and hence interest rates or spreads should not be equated with the cost of debt.

Several results are worth highlighting. The cost of debt is concave and non-monotonic in the borrower's profitability and financial leverage. With covenants included in debt contracts, it is therefore not generally the case that the agency problem between borrower and lender is exacerbated by higher leverage, even though more highly levered or less profitable borrowers face higher interest rates and tighter covenants. Leverage and profitability shift the cost of debt in opposite directions along the cost curve, so that debt contracts are most efficient when leverage and profitability are inversely aligned. Empirical research indeed suggests that leverage and profitability show negative correlation in practice. These conclusions apply in settings with and without renegotiation, provided that the renegotiation cost is comonotonic with the latent inefficiency of the underlying agency problem. Adding a project selection problem, and thereby giving the borrower greater scope than the lender to affect risk and return, tends to reduce the optimal interest rate and tighten the optimal covenant while increasing the borrower's cost of debt. Asymmetry between borrower and lender in their scope and ability to affect risk and return may therefore explain in part why debt covenants in practice are often set tightly and violated frequently.

## APPENDIX

**Proof of Lemma 1.** The lender's preferred action  $a^l$  solves the first-order condition

$$-\int_0^m F_a(c|a^l, \theta) dc = 0 \quad (\text{A1})$$

and is unique in view of  $F_{aa} > 0$  by Assumption 1b. The first-best action  $a^*$  solves

$$-\int_0^\infty F_a(c|a^*, \theta) dc = 0$$

and so

$$-\int_0^m F_a(c|a^*, \theta) dc \leq 0$$

since  $F_a(c|a, \theta) \geq 0$  at all  $c \leq z$  and the converse at all  $c > z$  for some  $z > 0$  by Assumption 1a. Then, again in view of  $F_{aa} > 0$ , (A1) can only hold if  $a^l \leq a^*$ . A symmetric argument applies to the borrower's preferred action  $a^b$ .

**Proof of Lemma 2.** The adjustment to the lender's preferred action  $a^l$  in response to an increase in the maturity value  $m$  is implicitly defined as the solution to

$$-F_a(m|a^l, \theta) - \int_0^m F_{aa}(c|a^l, \theta) \frac{\partial a^l}{\partial m} dc = 0$$

Assumption 1a implies that the first-order condition in (A1) can only hold if  $F_a(m|a^l, \theta) < 0$ . Then, in view of  $F_{aa} > 0$  by Assumption 1b, it must be that the adjustment to the lender's preferred action is  $\frac{\partial a^l}{\partial m} > 0$ . A symmetric argument applies to the borrower's preferred action  $a^b$ .

**Proof of Lemma 3.** In any state  $\theta$ , the borrower's payoff is higher in the absence of a covenant violation than in case of technical default, i.e.,

$$b(a^b, m, \theta) > b(a^l, m, \theta)$$

because, by construction,

$$a^b = \arg \max_a b(a, m, \theta)$$

The converse relationship applies to the lender's payoff  $l(a, m, \theta)$ . Then for any contract  $\{r, m, k\}$  such that  $L(r, m) > k$ , there either exists a covenant variable  $\hat{r}$  such that  $\{\theta: r(m, \lambda, \theta) < 0\} \supset \{\theta: \hat{r}(m, \lambda, \theta) < 0\}$  and  $L(\hat{r}, m) = k$ , in which case  $\mathcal{B}(\hat{r}, m, \lambda, k) > \mathcal{B}(r, m, \lambda, k)$  and so the contract  $\{r, m, k\}$  cannot have been optimal, or a covenant variable  $\hat{r}$

such that  $\{\theta: \hat{r}(m, \lambda, \theta) < 0\} = \emptyset$  and a maturity value  $\hat{m} < m$  such that  $L(\hat{r}, \hat{m}) = k$ , in which case  $B(\hat{r}, \hat{m}, \lambda, k) > B(r, m, \lambda, k)$  again holds. The lower maturity value  $\hat{m}$  increases in the borrower's payoff at all  $\theta$  because the lender never obtains control and, by the envelope theorem,

$$\frac{db(a^b, m, \theta)}{dm} = b_m(a^b, M, \theta) > 0$$

in borrower-controlled states.

**Proof of Proposition 1.** For any optimal covenant variable  $r$ , the first variation

$$\left. \frac{d}{d\varepsilon} \int_{\Theta} \mathcal{L}(r + \varepsilon z(\theta), m, \lambda, k, \theta) dG \right|_{\varepsilon=0} = \int_{\Theta} z(\theta) \mathcal{L}_r(r, m, \lambda, k, \theta) dG \quad (\text{A2})$$

must be zero for any continuous test function  $z(\cdot)$ . By the fundamental lemma of calculus of variations, (A2) is zero if and only if the Euler-Lagrange equation

$$\mathcal{L}_r(r, m, \lambda, k, \theta) = h(m, \lambda, \theta) \delta(r) = 0 \quad (\text{A3})$$

holds for all  $\theta$ , where

$$h(m, \lambda, \theta) = q(a^l, m, \theta) - q(a^b, m, \theta) + (1 - \lambda) \left( l(a^l, m, \theta) - l(a^b, m, \theta) \right)$$

and  $\delta(\cdot)$  is the Dirac delta distribution. The delta distribution is zero at any  $r \neq 0$  but assigns point mass when  $r = 0$ , and so (A3) holds if and only if  $h = 0$  whenever  $r = 0$ . Then  $r(m, \lambda, \theta) = (jh)(m, \lambda, \theta)$ , where  $j(m, \lambda, \theta) > 0$  everywhere, is sufficient to set (A2) to zero. It remains to be shown that this solution maximizes  $B$ . Indeed,  $h > 0$  implies that the equity value is higher if  $\theta \in \{\theta: r(m, \lambda, \theta) \geq 0\}$ , while  $h < 0$  implies the opposite, which corresponds to the control allocation that  $r(m, \lambda, \theta) = (jh)(m, \lambda, \theta)$  implements. In order to establish necessity, it suffices to note that an alternative covenant variable  $\hat{r}$  cannot be written in the form  $r(m, \lambda, \theta) = (jh)(m, \lambda, \theta)$  only if  $\text{sgn}(\hat{r}) \neq \text{sgn}(r)$  for at least some  $\theta$ , but then the control allocation under  $\hat{r}$  is not optimal for any such  $\theta$ , and hence  $\hat{r}$  cannot be optimal.

**Proof of Proposition 2.** As noted in the text, an optimal covenant will be considered unique if the normalized form  $r(m, \lambda, \theta) = h(m, \lambda, \theta)$  is unique. A sufficient condition for the contract to achieve a local maximum is

$$\int_{\Theta} z^2(\theta) \cdot \det(\mathbf{W}) dG > 0 \quad (\text{A4})$$

where

$$\mathbf{W} = \begin{bmatrix} 0 & L_m & l_r \\ L_m & \mathcal{B}_{mm} & \mathcal{B}_{rm} \\ l_r & \mathcal{B}_{rm} & \mathcal{B}_{rr} \end{bmatrix} \quad (\text{A5})$$

is the bordered Hessian matrix at a given  $\theta$  and  $z(\theta)$  is an arbitrary, continuous test function.

Then (A4) holds if and only if

$$\det(\mathbf{W}) = -L_m^2 \mathcal{B}_{rr} + 2l_r L_m \mathcal{B}_{rm} - l_r^2 \mathcal{B}_{mm} \geq 0$$

for all  $\theta$ , with strict inequality for at least some  $\theta$ . Since  $b(a^b, m, \theta) > b(a^l, m, \theta)$ , the change in the lender's payoff

$$l_r(r, m, \theta) = \left( l(a^b, m, \theta) - l(a^l, m, \theta) \right) \delta(r)$$

induced by an increase in  $r$  must be negative for all  $\theta$  at which  $h(m, \lambda, \theta) = 0$  and zero elsewhere. Conversely, the change in the lender's payoff

$$L_m(r, m) = \int_{\Theta} l_m(r, m, \theta) dG$$

from raising  $m$  must be positive by the following argument. Let  $m(k)$  denote the optimal maturity value for a given level of  $k$ . For  $k = 0$ ,  $\{\theta: r(m, \lambda, \theta) < 0\} = \emptyset$  and  $m(0) = 0$ , and so

$$L_m(r, m)|_{m=0} = \int_{\Theta} l_m(a^l, m, \theta) dG > 0$$

by the envelope theorem. Increase  $m$  and  $k$  and adjust the control allocation  $\{\theta: r(m, \lambda, \theta) < 0\}$  to its unique conditional optimum for each  $(m, k)$ , and consider any path on which  $m = m(k)$  everywhere. If  $L_m(r, m) < 0$  for any  $m_1 > 0$  on this path, continuity implies that  $L_m(r, m) = 0$  for some  $m_0 < m_1$  because  $L_m(r, m)|_{m=0} > 0$ . Then  $\mathcal{B}_m(r, m, \lambda, k) = 0$  at  $m_0$  if and only if  $Q_m(r, m) = 0$ , which implies  $B_m(r, m) = 0$  because  $B_m(r, m) + L_m(r, m) = -Q_m(r, m)$  in view of (10). Hence,  $m_0$  must maximize both  $B$  and  $L$ , which implies that both would decrease if  $m$  were increased beyond  $m_0$ . Then  $m_1$  cannot meet the first-order necessary condition for an optimal contract, and so  $L_m(r, m) > 0$  at any possible  $m(k)$ . Next, the term  $B_{mm}(r, m, \lambda, k)$  must be negative. Otherwise, the borrower could raise  $m$  and obtain a higher equity value without violating the lender's break-even constraint because  $L_m(r, m) > 0$ , but the break-even constraint must bind by Lemma 3. Hence, the related allocation of decision rights could not be optimal. Next, the term

$$\mathcal{B}_{rr}(r, m, \lambda, k, \theta) = h(m, \lambda, \theta) \frac{d\delta(r)}{dr} = h(m, \lambda, \theta) \delta'(h(m, \lambda, \theta)) = -\delta(h(m, \lambda, \theta))$$

is non-negative when evaluated at the optimum  $r(m, \lambda, \theta) = h(m, \lambda, \theta)$ .<sup>50</sup> Finally, the cross-partial derivative with respect to  $r$  and  $m$  is

$$\mathcal{B}_{rm}(r, m, \lambda, k, \theta) = h_m(m, \lambda, \theta)\delta(r)$$

whose sign at any critical  $\theta$  where  $r = 0$  equals the sign of

$$h_m(m, \lambda, \theta) = q_m(a^l, m, \theta) - q_m(a^b, m, \theta) + (1 - \lambda)(l_m(a^l, m, \theta) - l_m(a^b, m, \theta))$$

By Lemma 2, increasing  $m$  reduces inefficiency in lender-controlled states and increases inefficiency in borrower-controlled states. Hence, the difference between the two leading terms on the right-hand side is negative and so  $\mathcal{B}_{rm}(r, m, \lambda, k, \theta) < 0$  for  $r = 0$  and  $\lambda = 1$ . To rule out a reversal of the inequality for  $\lambda \neq 1$ , consider moving from frontier states with  $h = 0$  to neighboring states where  $h$  is the same as would be obtained at frontier states if  $m$  were marginally increased, i.e.,

$$\nabla \mathcal{B}_r(r, m, \lambda, k, \theta) \cdot \iota = \mathcal{B}_{rm}(r, m, \lambda, k, \theta)$$

where  $\iota$  is a vector in  $\Theta$ . Then if  $\mathcal{B}_{rm} > 0$  anywhere, raising  $m$  is equivalent to moving to a  $\theta$  in the borrower's sphere of control. At the same time, a move in the direction of the marginal inefficiency of control transfer to the borrower would mean moving to a  $\theta$  in the lender's control sphere. Hence, if  $\mathcal{B}_{rm} > 0$  for some  $\theta$  at the frontier, there would exist states over which borrower and lender could swap control rights such that break-even is maintained while inefficiency decreases, and so the control allocation cannot have been optimal to begin with. In combination, the preceding observations imply that  $\det(\mathbf{W})$  must be non-negative for all  $\theta$  and strictly positive at  $\theta$  where  $h = 0$ . Hence, any contract that meets the first-order necessary conditions attains a maximum. Multiple maxima require the existence of saddle points or minima, and so the maximum must be unique.

**Proof of Proposition 3.** Given the uniqueness of the optimal contract, the implicit function theorem implies that the optimal adjustment to  $\mathcal{B}$  in response to an increase in  $k$  is characterized by

$$\frac{d\mathcal{B}_r}{dk} = \mathcal{B}_{rr}r_k + \mathcal{B}_{rm}m_k + l_r\lambda_k = 0 \tag{A6}$$

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<sup>50</sup> Equivalently, one could define  $\frac{dh(m, \lambda, \theta)}{dr} = \nabla h(m, \lambda, \theta) \cdot \iota$  as the directional derivative of  $h$ , where  $\nabla$  is the gradient with respect to  $\theta$ , and  $\iota$  is any vector in  $\Theta$  that corresponds to a unit increase in  $r$ . Graphically,  $\iota$  thus points from default to no-default states.

for all  $\theta$  where  $h = 0$ , where  $\ell_k$ ,  $m_k$  and  $\lambda_k$  denote the contract term adjustments. The signs of all remaining partial derivatives follow from Proposition 2. The contract adjustments must further solve

$$\frac{dB_m}{dk} = B_{mm}m_k + L_m\lambda_k + \int_{\Theta} \ell_{rm}r_k dG = 0 \quad (\text{A7})$$

and

$$\frac{dL}{dk} = L_m m_k + \int_{\Theta} l_r r_k dG = 1 \quad (\text{A8})$$

A stricter covenant obtains if the integral term in (A8) is positive. If  $m_k, \lambda_k > 0$ , (A6) implies that  $r_k < 0$  at all  $\theta$  where  $h = 0$ , and thus claimed result obtains immediately. If  $m_k, \lambda_k < 0$ , (A6) implies that  $r_k > 0$ , which is infeasible because then (A8) cannot hold. For the remaining cases, solve (A6) for  $r_k$  and substitute into (A7) and (A8) to obtain

$$B_{mm}m_k + L_m\lambda_k - \int_{\Theta} \frac{\ell_{rm}^2}{\ell_{rr}} m_k dG - \int_{\Theta} \frac{\ell_{rm}l_r}{\ell_{rr}} \lambda_k dG = 0 \quad (\text{A9})$$

and

$$L_m m_k - \int_{\Theta} \frac{\ell_{rm}l_r}{\ell_{rr}} m_k dG - \int_{\Theta} \frac{l_r^2}{\ell_{rr}} \lambda_k dG = 1 \quad (\text{A10})$$

If  $m_k < 0$  and  $\lambda_k > 0$ , the integral terms in (A9) must net to a negative value, which implies that the integral terms in (A10) must also net to a negative value since, for the inner product of any two positive vectors  $x$  and  $y$ ,  $\langle x, x \rangle > \langle x, y \rangle \Leftrightarrow \langle x, y \rangle > \langle y, y \rangle$ , but then (A10) cannot hold. Finally, if  $m_k > 0$  and  $\lambda_k < 0$ , the integral terms in (A10) must net to a positive value, and so the claimed result holds.

**Proof of Proposition 4.** Since  $\frac{dB}{dk} = -\lambda$  by the envelope theorem, the cost of debt is concave in  $k$  if  $\lambda_k < 0$ . Given the uniqueness of the optimal contract terms, the contract term adjustments in response to an increase in  $k$  are optimal if and only if

$$\int_{\Theta} \mathbf{W} \cdot \begin{bmatrix} \lambda_k \\ m_k \\ r_k \end{bmatrix} dH = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (\text{A11})$$

where  $\mathbf{W}$  is as defined in Proposition 2. In each state  $\theta$  where  $h = 0$ , the integrand of (A11) is

$$\mathbf{W} \cdot \begin{bmatrix} \lambda_k \\ m_k \\ r_k \end{bmatrix} = \begin{bmatrix} 1 + \varepsilon_1 \\ \varepsilon_2 \\ 0 \end{bmatrix} \quad (\text{A12})$$

for some  $\varepsilon_i$ , where  $E_\theta(\varepsilon_i) = 0$  for  $i = 1, 2$ . The third element on the right-hand side of (A12) is always zero because the first-order condition  $\mathcal{L}_r = 0$  must hold for every  $\theta$ . Solving (A12) yields

$$\det(\mathbf{W}) \lambda_k = (\mathcal{B}_{mm} \mathcal{L}_{rr} - \mathcal{L}_{rm}^2)(1 + \varepsilon_1) + (l_r \mathcal{L}_{rm} - L_m \mathcal{L}_{rr}) \varepsilon_2$$

and

$$\det(\mathbf{W}) m_k = (l_r \mathcal{L}_{rm} - L_m \mathcal{L}_{rr})(1 + \varepsilon_1) - l_r^2 \varepsilon_2$$

for all  $\theta$  where  $h = 0$ , where all terms on the right-hand sides are as given in the proof of Proposition 2. Since  $E_\theta(\varepsilon_2) = 0$ , continuity implies that  $\varepsilon_2 = 0$  for some  $\theta$ . Then if  $m_k > 0$ , it must be that  $\varepsilon_1 > -1$  at this  $\theta$ , in which case  $\lambda_k$  is negative if

$$\mathcal{B}_{mm} \mathcal{L}_{rr} - \mathcal{L}_{rm}^2 < 0 \quad (\text{A13})$$

It remains to be shown that both  $m_k > 0$  and (A13) hold for all  $k$ . First,  $m = 0$  when  $k = 0$  but  $m > 0$  when  $k > 0$ , and so by continuity,  $m_k < 0$  can only hold if  $m_k = 0$  for some  $k$ . The multiplier of  $\varepsilon_1$  in  $m_k$  is always nonzero, and so  $m_k = 0$  can only hold if  $\varepsilon_1 = -1$  when  $\varepsilon_2 = 0$ , which in turn implies  $\lambda_k = 0$ . Then (A7) can only hold if  $r_k = 0$  for all  $\theta$  where  $h = 0$ , but then all contract terms remain unchanged while  $k$  has increased, in which case the break-even constraint is violated. Hence,  $m_k \leq 0$  cannot hold for any level of  $k$ . To establish (A13), one can note that its left-hand side is equal to the determinant of the Hessian matrix in the unconstrained optimization program

$$\max_{r,m} (-q(r, m, \theta) + (\lambda - 1)l(r, m, \theta)) \quad (\text{A14})$$

over  $m$  and  $r$ , which differs from program (8) in that  $\lambda$  is fixed,  $\Theta$  is reduced to a single state and  $l$  is unconstrained. The unique maximum of (A14) is always a corner solution with either  $m = 0$  and  $r > 0$  when  $\lambda < 1$ , or  $m = \infty$  and  $r < 0$  when  $\lambda \geq 1$  because i)  $q = 0$  in both cases, ii)  $l$  attains its minimum or maximum when all payoffs and control rights are allocated to one party only, and iii) the interior stationary point given by  $m$  and  $r$  must be unique by Proposition 2, and so local interior maxima cannot exist. Further,  $\mathcal{B}_{mm} < 0$  and  $\mathcal{L}_{rr} < 0$  imply that  $m$  and  $r$  cannot yield a minimum either, and therefore (A14) must be at a saddle point, which implies that (A13) holds. The non-monotonicity claim follows from the observation that the agency cost is zero in the extreme cases  $k = 0$  and  $k = E(c)$ .

**Proof of Proposition 5.** The proofs of Propositions 2 through 4 can be replicated with renegotiation by replacing  $q$  with  $p$ . Since  $l_r < 0$  and  $l_m > 0$  continue to hold and since  $p'(q) > 0$ , the directionality of all results remains unchanged.

**Proof of Lemma 4.** Equality in (22) is equivalent to

$$\int_{\Theta} \int_m^{\infty} \left( F(c|a^i, \bar{s}, \theta) - F(c|a^i, \underline{s}, \theta) \right) dc dG = 0 \quad (\text{A15})$$

for  $i = b, l$ , where the actions  $a^i$  are the same under both  $\underline{s}$  and  $\bar{s}$  in view of Assumption 2b. In order for (A15) to hold, the crossing point of the cash flow distribution functions in the integrand must lie above  $m$ . Then the left-hand side change in  $m$  by

$$F(m|a^i, \underline{s}, \theta) - F(m|a^i, \bar{s}, \theta) < 0$$

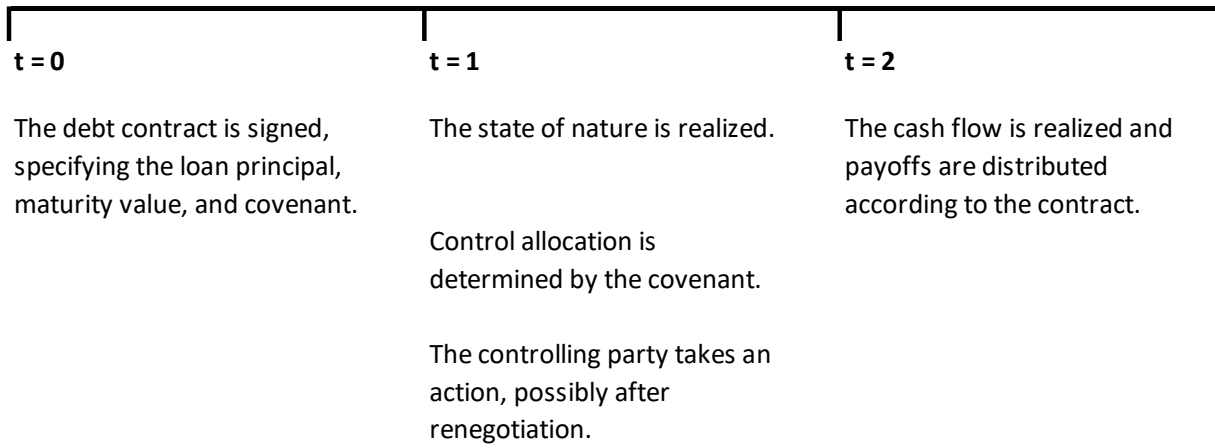
and thus there exists a unique maturity value  $\bar{m}$  such that, ceteris paribus, the borrower prefers  $\underline{s}$  for any  $m \leq \bar{m}$  and  $\bar{s}$  for any  $m > \bar{m}$ . The existence of  $\bar{m}$  follows from the observation that the left-hand side of (15) is negative at  $m = 0$  and positive at sufficiently large  $m$  in view of Assumption 2a.

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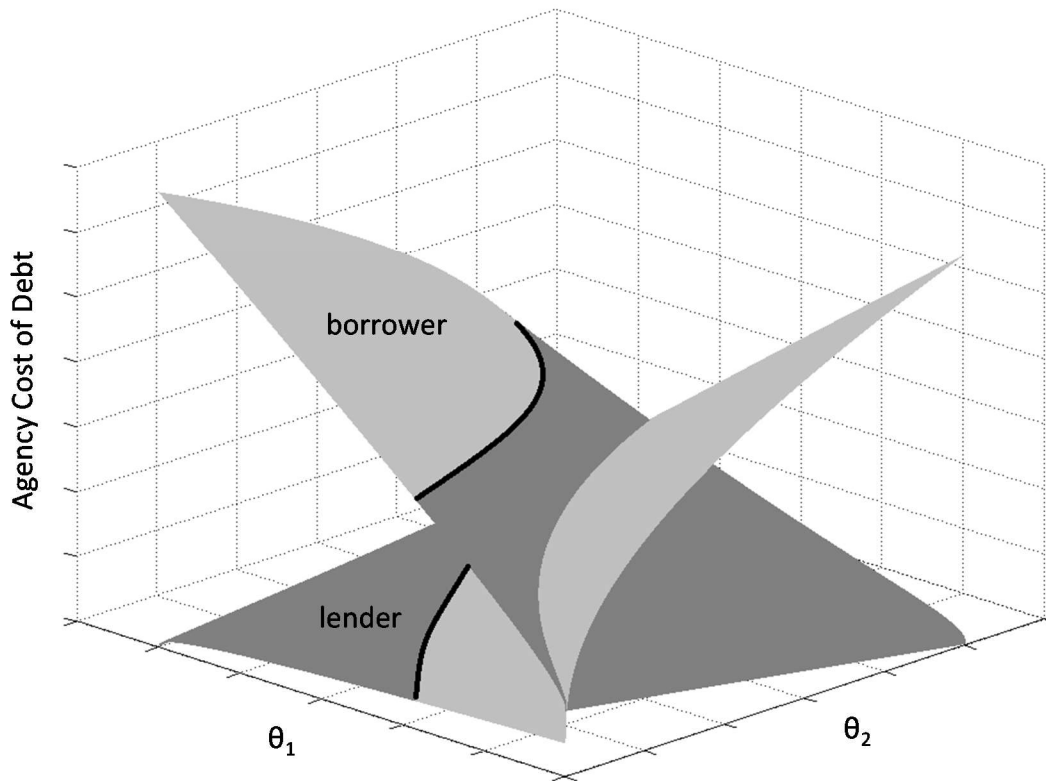
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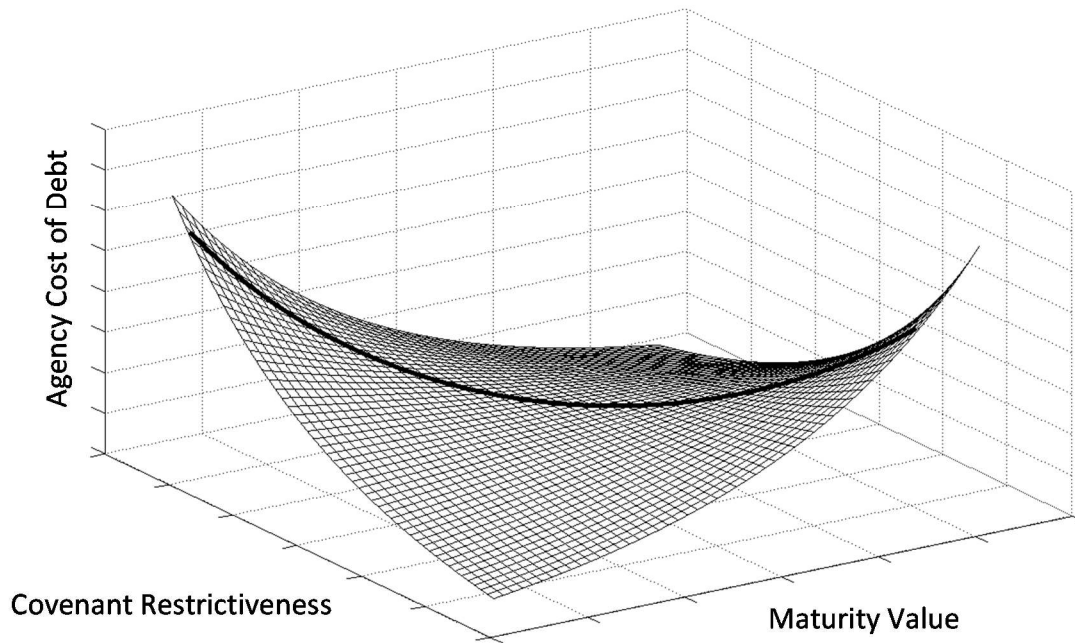
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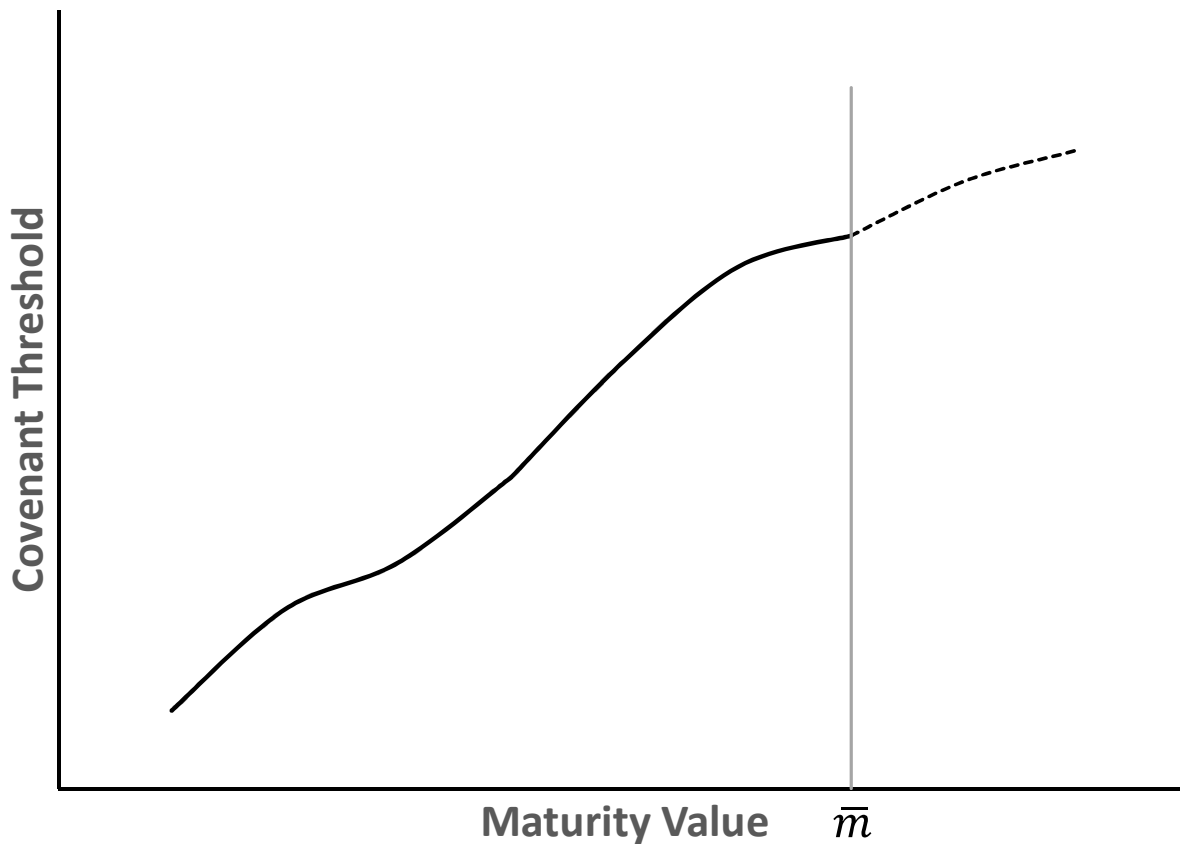
**Figure 1.** Timeline of events.



**Figure 2.** Agency cost as a function of control allocation in a two-dimensional state space  $\Theta \subset \mathbb{R}^2$ . The two surfaces show the potential agency cost incurred if control is allocated to the borrower and the lender, respectively. The solid black line is the set of frontier states  $\{\theta: h(m, \lambda, \theta) = 0\}$  that separates the contracting parties' spheres of control. The dark gray part of each party's surface shows the agency cost in states when that party has control. The inefficiency in the light gray part of each surface is not incurred because the other side has control in these states.



**Figure 3.** Agency cost as a function of maturity value and covenant restrictiveness. Higher restrictiveness values correspond to tighter covenants and thus a larger sphere of control for the lender. The solid line is the set of break-even contracts for a given loan amount. The unique optimal contract is the lowest point on this line.



**Figure 4.** The graph shows the set of break-even contracts for the socially optimal project in a project selection problem. The state space is  $\Theta \subset \mathbb{R}$  and the covenant takes the form of a threshold value, where higher covenant thresholds correspond to looser covenants and thus a larger sphere of control for the borrower. Contracts on the solid segment, with maturity values below  $\bar{m}$ , are incentive-compatible as the borrower prefers the socially optimal project; contracts on the dashed segment are infeasible as the borrower prefers to implement the inferior project.