

# A Dynamic Model of Consumer Replacement Cycles in the PC Processor Industry

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As high-tech markets mature, replacement purchases inevitably become the dominant proportion of sales. Despite the clear importance of product replacement, little empirical work examines the separate roles of adoption and replacement. A consumer's replacement decision is dynamic and driven by product obsolescence because these markets frequently undergo rapid improvements in quality and falling prices. The goal of this paper is to construct a model of consumer product replacement and to investigate the implications of replacement cycles for firms.

To this end, I develop and estimate a dynamic model of consumer demand that explicitly accounts for the replacement decision when consumers are uncertain about future price and quality. Using a unique data set from the PC processor industry, I show how to combine aggregate data on sales and product ownership to infer replacement behavior. The results reveal substantial variation in replacement behavior over time, and this heterogeneity provides an opportunity for managers to tailor their product introduction and pricing strategies to target the consumers of a particular segment that are most likely to replace in the near future.

*Key words:* durable goods; replacement; structural estimation; dynamic programming; innovation; upgrades; PC processor; CPU; technology products

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## 1. Introduction

As high-tech markets mature, replacement purchases inevitably become the dominant proportion of sales. In 2004, replacement purchases accounted for 55% of digital camera sales, 63% of cell phone sales, and 82% of computer sales.<sup>1</sup> Because these markets frequently undergo rapid improvements in quality and falling prices, a consumer's replacement decision most often stems from product obsolescence as opposed to wear and tear. This decision is dynamic because a consumer who forgoes a purchase today might buy a potentially better product tomorrow for less money. Despite the clear importance of product replacement, little work examines the separate roles of adoption and replacement, especially from an empirical perspective. The goal of this paper is to construct a model of consumer product replacement and to investigate the implications of replacement cycles for firms.

To address these issues, I develop and estimate a structural model of dynamic demand. The model allows for both product adoption and replacement decisions when consumers are uncertain about future price and quality. Each period, consumers in the model choose whether to keep their existing product

(if any) or to replace it with one of the new products available. Thus, the consumer's replacement decision depends on the quality of the product she currently owns. This introduces an additional source of heterogeneity because a consumer's past purchase determines the present value of her outside option.

I apply the model to a unique data set from the PC processor industry. This industry is particularly interesting because two firms, Intel and Advanced Micro Devices (AMD), control roughly 95% of the market, and intense technological innovation and price competition have driven sales. As market penetration has grown from 28% in 1993 to 74% in 2004, the firms have increasingly relied on product replacement purchases (Computer Industry Almanac 2005).

This paper makes both methodological and empirical contributions. First, the paper develops a dynamic structural model of adoption and replacement, whereas previous work typically focused on adoption. Incorporating replacement into a dynamic model of high-tech goods is difficult because in the absence of individual data, the replacement decision is unobserved. To address this problem, I show how to use additional microdata—in the form of aggregate survey data on the installed base of processor ownership over time—to augment the estimation of the model. The estimation results show that the

<sup>1</sup> InfoTrends (2006), IC Insights (2005), and Computer Industry Almanac (2005).

additional information the ownership data provide is necessary for capturing the dynamics of replacement. A model restricted to product adoption results in biased parameter estimates that yield incorrect pricing and other strategic recommendations.

Second, I demonstrate that heterogeneity in product ownership and replacement cycles provides an opportunity for managers to tailor their product introduction and pricing strategies to target the consumers of a particular segment that are most likely to replace in the near future. The model may be used as a broad tool for firms to help guide their overall marketing strategy based on a comparison of the relative profitability of targeting the different segments. For example, if a large segment of more price-sensitive consumers is likely to replace its products in the near future, a firm could release a more value-oriented product to coincide with this event.

To estimate the model, I construct a comprehensive data set of prices, characteristics, sales, and ownership of PC processors manufactured by Intel and AMD from January 1993 to June 2004. The data set includes unit shipments from an industry research firm and proprietary survey data on ownership by processor type from a consumer research company. The survey data are only available at the aggregate level, but they allow me to estimate the distribution of ownership by processor quality in a period. Changes in the ownership distribution from one period to the next and the current period's sales make inferring replacement behavior at the aggregate level possible. In addition, I collect detailed information on the history of processor prices and characteristics from old press releases, news reports, and industry periodicals. Finally, I use a processor speed benchmark to create a single index variable for processor quality. Using these data, I estimate the model using generalized method of moments (GMM) as part of a nested fixed-point algorithm to match a set of simulated moment conditions to their empirical counterparts.

The results reveal the importance of replacement in shaping demand within this technologically dynamic industry. First, estimates from the structural model imply significant variation in the distribution of replacement cycles across consumers both within a period and over time. Replacement cycles lead to the endogenous formation of different distributions of consumer tastes in different time periods. Consumers with higher valuations for a product are more likely to buy early on and leave lower valuation consumers in the market until new features are introduced, which will also draw back repeat consumers. Second, the marginal effect of innovation on the length of the replacement cycle has decreased over time, implying that PC hardware manufacturers may not always be able to rely on quality improvements to generate

replacement sales in the future. Third, a comparison of the benchmark dynamic model to a version with myopic consumers yields substantially different results. The myopic model underestimates the length of consumer replacement cycles by roughly one year, implying that consumers replace their products more frequently than the dynamic model implies. The myopic model also underestimates price elasticities by up to 45%. These findings demonstrate that accounting for the strategic behavior of consumers is particularly important for firms in high-tech durable goods markets.

The extant literature examines models of demand for durable goods in a variety of contexts, but few studies consider the replacement of durable goods, especially using a structural model.<sup>2</sup> In marketing, much of the existing work on product adoption follows in the tradition of the Bass diffusion model (Bass 1969). Some relevant extensions consider the diffusion of successive generations of a technology product (Norton and Bass 1987) and the optimal introduction timing of new product generations (Wilson and Norton 1989). Some incorporate product replacement into forecasting models of durable goods sales (Bayus 1988, Bayus et al. 1989, Steffens 2003).<sup>3</sup> Song (2007) models demand for PC processors using a static pure characteristics model.

This work contributes to the recent stream of research that uses structural models to study strategic consumer demand for high-tech durable goods. Melnikov (2001) constructs a model of dynamic demand for durable goods and applies it to computer printers. Song and Chintagunta (2003) and Carranza (2007) apply similar models to examine the introduction of digital cameras. Erdem et al. (2005) model consumer learning and information search about the choice of a computer platform (e.g., IBM compatible or Mac) using panel data that track the information sources consumers use to help make their decisions. Nair (2007) studies the optimal pricing problem for a monopolist selling video game consoles to forward-looking consumers. A common feature of these papers is that they formulate the consumer's problem as an optimal stopping problem.<sup>4</sup> In this formulation, the consumer decides on the optimal time period to enter

<sup>2</sup> I limit this discussion to infrequently purchased durable goods. The replacement decision for frequently purchased products such as paper towels or cereal is fundamentally different because of the lack of obsolescence. See, for example, Gönül and Srinivasan (1996), Sun et al. (2003), Mehta et al. (2004), and Sun (2005).

<sup>3</sup> Ratchford et al. (2000) provide an excellent review of the literature on diffusion models with replacement and multiple purchases.

<sup>4</sup> Horsky (1990) and Chatterjee and Eliashberg (1990) take a different approach by constructing an aggregate diffusion curve based on the individual-level adoption decisions derived from a consumer's maximization problem.

the market, makes a single purchase from among the available alternatives, and then permanently exits the market. Although suitable for applications with only adoption, extending this approach to include replacement purchases is difficult.

The key difference between these papers and the present work is that I study both the adoption and replacement decisions in a market with technological innovation. The presence of innovation distinguishes this model from Rust (1987), who studies the replacement of a durable good stemming from wear and tear rather than innovation. The characteristics of the single product are fixed, allowing Rust (1987) to formulate the model as a regenerative optimal stopping problem. Prince (2008) creates a model of demand for PCs to quantify the effects of subsidies to first-time buyers, but his model assumes consumers have perfect foresight about future product quality and price. Gowrisankaran and Rysman (2007) allow consumers to replace their products, but they assume that consumer expectations—concerning all future product characteristics and prices—can be reduced into a single variable.

## 2. The Data

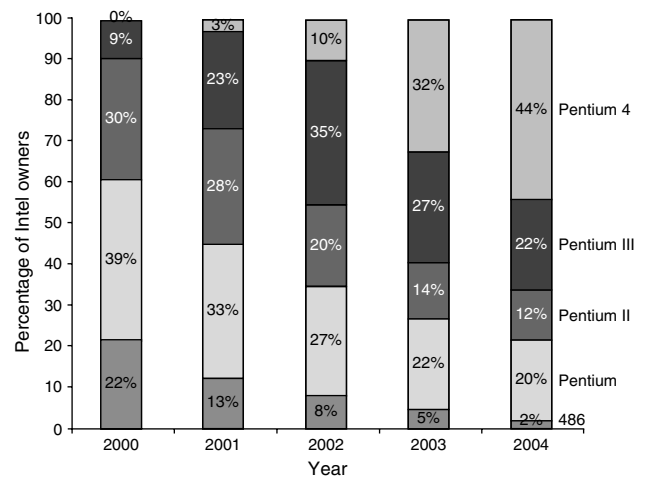
The data set focuses on the desktop PC processor market and consists of PC processor unit shipments, consumer PC ownership, manufacturer prices, and quality measurements, by processor, over the period January 1993 to June 2004.<sup>5</sup>

### 2.1. Unit Sales

Quarterly unit shipment data were obtained from In-Stat/MDR (2003a, b), an industry research firm that specializes in the microprocessor industry. In-Stat/MDR uses a combination of contacts in various distribution channels and a detailed knowledge of manufacturing costs to back out estimates of shipment data by processor type. I obtained data from the Computer Industry Almanac (2005) on the U.S. portion of global chip sales, the portion of sales for replacement machines, and the market size. I use this information to convert the global shipment figures from In-Stat/MDR to U.S. quantities. PC manufacturers have strong incentives to minimize their inventories, so the delay between the shipment of a PC processor and the subsequent purchase of a PC containing that processor by an end user should be negligible. The data presumably include a component of unit sales that go to consumers who purchase multiple computers, but data limitations make it difficult to incorporate multi-PC households into the current model.

<sup>5</sup> Historically, server and mobile processors have occupied a small share of the overall PC market, although laptop sales have significantly increased in the last few years.

**Figure 1** PC Processor Ownership by Intel Architecture from 2000 to 2004



Note. Intel introduced the Pentium 4 processor in April 2001.

### 2.2. Ownership

Aggregate information on consumer PC ownership and penetration rates come from the Homefront study created by Odyssey, a consumer research firm that specializes in technology products. The firm conducts semiannual telephone surveys using a nationally representative sample ranging from 1,500 to 2,500 households. The households do not belong to a prechosen panel, and the firm draws a new sample for each wave of the study. The survey data are available neither at the household level nor in panel form. To increase the accuracy of the relevant sample, approximately 500 additional households that own a PC are oversampled.

The survey asks respondents about the characteristics of their primary or most recently purchased PC. This ensures that the most relevant PC information is collected in the case of a household with multiple PCs. The survey gathers basic information on the PC, such as the CPU manufacturer, architecture, and processor speed. These data allow me to estimate the percentage of consumers in the population who own a CPU from a particular speed range, such as Pentium III processors operating between 500 and 800 MHz.<sup>6</sup>

Figure 1 displays a sample of the ownership data, aggregated to the processor generation level. Figure 1 covers the period from 2000 to 2004 and shows the share of households that own an Intel 486, Pentium, Pentium II, Pentium III, or Pentium 4 processor. The Pentium III was released in late 1999 and the Pentium 4 in late 2001. The other processors were no longer being sold. The graph reveals several interesting points. First, despite the fact that Intel has not

<sup>6</sup> The survey operators are instructed to help the consumers find the relevant information if needed.

sold a 486 processor since 1995, a significant portion of households still owned 486-based PCs in 2000. Second, more households own a PC with a Pentium chip compared to a Pentium II, even though the Pentium II is the more advanced technology and neither chip is available for sale. Third, the rate of decline in the ownership shares for the Pentium and Pentium II leveled off in recent years, suggesting perhaps that the remaining owners are less likely to replace their products in the near future.

### 2.3. Prices

I collected the complete price history for the set of processors from a number of sources. The manufacturer prices are quoted in chip quantities of 1,000 units. These quotes represent the official prices, although it is possible that the actual prices Intel and AMD charge to PC manufacturers are different. Nevertheless, Intel and AMD adjust their official prices frequently (between 5 and 10 times per year), which implies that the posted prices can still serve as adequate indicators. The primary data sources for prices were news websites, In-Stat/MDR, historical Intel and AMD press releases, technology newsgroups, and other sources. Data from these sites were also supplemented with information from historical issues of PC-related magazines and periodicals. Although wholesale prices should serve as a consistent measure of the aggregate retail prices faced by consumers, some noise is likely to exist.

### 2.4. Quality

Processor frequency does not adequately capture the computational power of a CPU because of differences in chip architecture and characteristics.<sup>7</sup> To account for such differences, I use a processor benchmark to generate composite quality ratings for each chip. A benchmark measures a processor's speed based on its actual computational performance on a common set of tasks, facilitating speed comparisons between different processors. The CPU Scorecard (<http://www.cpuscorecard.com>) provides a comprehensive list of benchmarks that adequately covers the sample period. The list of processor speed ratings from the CPU Scorecard does not contain all the processors in the data set: 74 of 217 processors did not have benchmarks (38 from AMD and 36 from Intel). To fill in the missing values, I impute the missing benchmark based on the available ratings.<sup>8</sup>

<sup>7</sup> For example, the AMD Athlon XP 3000+ processor has a frequency of 2.16 GHz, but it performs comparably to an Intel Pentium 4 at 3.0 GHz.

<sup>8</sup> The imputed ratings are treated as known and fixed in the rest of the analysis; I do not account for any errors in measuring performance. The imputed values are likely accurate measures of quality

## 3. The Model

This section presents the model of consumer demand. The first subsection describes the structure of the product market and defines the consumer's dynamic optimization problem. The second subsection describes the particular forms of consumers' expectations over future product qualities and prices. The third subsection derives the aggregate demand functions and provides the laws of motion for the distribution of product ownership.

I make the following assumptions both to keep the model tractable and because of data limitations. First, a consumer owns a single product. This assumption is necessary because the survey data only contain information on the primary PC in each consumer's household. Second, old products are kept or discarded—no secondhand market exists. Some durable goods markets such as automobiles have established used goods markets. Only a fraction of purchases of durable goods with rapid innovation, such as CPUs and consumer electronics, transact in used markets.<sup>9</sup> Third, depreciation or upkeep costs do not exist, implying that products retain their original quality. From a modeling perspective, including product depreciation is trivial, but estimating the rate of depreciation is likely to require additional assumptions. Fourth, in the empirical application, I view processors as summary statistics for the overall quality of a PC. According to a 2004 Forrester Research study, more consumers indicated that processor speed was a significant motivating factor in deciding to buy a new PC than any other characteristic.<sup>10</sup> Last, firms' behavior is exogenous: consumers are small relative to the firms and take product characteristics as given.

Although some of these assumptions may appear strong, I believe they are reasonable given the type of products this paper considers. I discuss potential extensions to the model that relax some of these assumptions in §7.

### 3.1. Basic Setup

Products are represented using a single, composite quality attribute  $q_{jk} \in Q = \{1, 2, \dots, \bar{q}\}$  for product  $j$  of firm  $k$ , with  $p_{jk} \in \mathbb{R}_+$  the associated price. I measure quality on a log scale such that product improvements

because a processor's speed is a deterministic function of its characteristics. Regressing the existing speed ratings against processor frequency and brand dummies produces an  $R^2$  of 97.4%. Adding other processor characteristics, such as bus frequency, cache size, and dummies for the processor architecture, increases the  $R^2$  to 99.8%.

<sup>9</sup> Esteban and Shum (2007) develop a dynamic oligopoly model of the automobile industry with secondhand markets under the assumption that consumers have perfect foresight with respect to future prices and product qualities.

<sup>10</sup> Forrester Research (2004).

are proportional increases in quality.<sup>11</sup> The market contains  $K$  firms and each sells  $J$  products.<sup>12</sup> The vector  $\mathbf{q}_t \in Q^{KJ}$  denotes the set of products available in a period, and the vector  $\mathbf{p}_t \in \mathbb{R}_+^{KJ}$  is the associated set of prices. Product qualities and prices evolve according to exogenous stochastic processes. No restrictions are placed on the product market configuration. Either or both firms may have the highest quality product in a given period, and one firm's frontier product may be of lower quality than the other firm's product. The time subscript will be dropped when possible.

I model the consumer's decision to purchase a new product as a dynamic optimization problem under price and quality uncertainty. A consumer must decide whether to keep her existing product (if any) or to purchase a new product. Consumers who do not own a product face a technology adoption decision: enter the market now or stay out. Consumers who own a product face a replacement decision: purchase a more advanced product today or keep the existing product. The quality of the products available in the market changes over time as new products enter and old products exit.

This object simply tracks the history of ownership shares of over time.

Each consumer owns a product  $\tilde{q}_{kt}$  from firm  $k$  at time  $t$ . This product comes from the set of products offered from time  $\tau = 1, \dots, t$ , defined as

$$\tilde{Q}_t = \left\{ \tilde{q}_k: \tilde{q}_k \in \left\{ \bigcup_{\tau=1}^t \mathbf{q}_\tau \right\} \cup \{0\} \right\},$$

where  $\tilde{q} = 0$  represents a consumer who owns no product.  $\tilde{Q}_t$  represents the set of all products that have been sold (and are currently being sold) up until time  $t$  such that  $\tilde{Q}_t \subseteq \tilde{Q}_{t+1}$ .

Each consumer  $i$  belongs to one of  $I$  unobserved segments. The period utility function for a consumer in segment  $i$  who purchases some product  $q_{jk} \in \mathbf{q}$  is

$$u_{ijk} = \gamma_i q_{jk} - \alpha_i p_{jk} + \xi_{ik} + \varepsilon_{ijk}, \quad (1)$$

where  $\gamma_i$  is consumer  $i$ 's taste for quality,  $\alpha_i$  is the marginal utility of income,  $\xi_{ik}$  is a brand fixed effect, and  $\varepsilon_{ijk}$  represents unobservable factors that influence the consumer's utility. I place no restrictions on which product a consumer can purchase; a consumer with a high-quality product may replace it with a lower-quality product.<sup>13</sup>

<sup>11</sup> Using a log scale makes more sense than a linear scale in the context of the processor industry because consumers value proportional gains in computer power.

<sup>12</sup> The model could be extended to allow each firm to sell a different number of products  $J_k$ . See Appendix A.1 for more details.

<sup>13</sup> Consumers may purchase a product of lower quality than they own because of the unbounded support of the distribution of  $\varepsilon_{ijk}$ . Every period, a small number of consumers receives large enough idiosyncratic shocks to rationalize purchasing a lower-quality product.

A consumer may instead choose to retain her existing product. The period utility for a consumer who owns  $\tilde{q}_k$  and does not make a purchase is

$$u_{i\tilde{q}_k} = \begin{cases} \gamma_i \tilde{q}_k + \xi_{ik} + \varepsilon_{ik} & \text{if } \tilde{q}_k > 0, \\ \varepsilon_{i0} & \text{if } \tilde{q}_k = 0. \end{cases} \quad (2)$$

If the consumer owns a product ( $\tilde{q}_k > 0$ ), she receives the utility associated with it without having to pay any additional cost. I normalize the utility to zero in the case where a consumer does not own a product ( $\tilde{q}_k = 0$ ). This specification demonstrates the fact that a consumer's existing product represents her state-specific outside option. The utility of the outside option is a function of both a consumer's exogenously determined segment  $i$  and the endogenously determined past choice  $\tilde{q}_k$ .

The state space for the dynamic decision problem is  $\{\tilde{q}_k, \mathbf{q}, \mathbf{p}, \varepsilon\}$ , consisting of the product the consumer currently owns, the set of currently available product qualities and their prices, and the vector of unobserved taste shocks. Consumers have rational expectations about the stochastic processes governing the evolution of prices and qualities, which follow a regular Markov transition kernel  $\Pi(\cdot | \cdot)$ . The next subsection provides the specification for  $\Pi$ . The consumer is endowed with an initial product  $\tilde{q}_0 \in \tilde{Q}_0$  at  $t = 0$ . The consumer then chooses a sequence of product purchases that maximize the sum of discounted expected future utility over the infinite horizon:

$$\max_{\{q_t \in \mathbf{q}_t \cup \tilde{q}_{kt}\}_{t=0}^{\infty}} E \left\{ \sum_{t=0}^{\infty} \beta^t [u_i(\tilde{q}_{kt}, \mathbf{q}_t, \mathbf{p}_t, \varepsilon_t) \cdot I_{\{q_t = \tilde{q}_{kt}\}} + u_i(\tilde{q}_{kt}, \mathbf{q}_t, \mathbf{p}_t, \varepsilon_t) \cdot (1 - I_{\{q_t = \tilde{q}_{kt}\}})] \mid \tilde{q}_{kt}, \mathbf{q}_t, \mathbf{p}_t, \varepsilon_t \right\}, \quad (3)$$

where  $E \equiv E_{\mathbf{s}, \varepsilon}$  and the discount factor  $\beta$  is fixed. The recursive form of the consumer's optimization problem is written as

$$\begin{aligned} & V_i(\tilde{q}_k, \mathbf{q}, \mathbf{p}, \varepsilon) \\ &= \max \left\{ u_i(\tilde{q}_k, \mathbf{q}, \mathbf{p}) + \beta E[V_i(\tilde{q}_k, \mathbf{q}', \mathbf{p}', \varepsilon'_{i0}) \mid \mathbf{q}, \mathbf{p}], \right. \\ & \quad \left. \max_{q_{jl} \in \mathbf{q}} \{ u_i(q_{jl}, \mathbf{q}, \mathbf{p}) + \beta E[V_i(q_{jl}, \mathbf{q}', \mathbf{p}', \varepsilon'_{ijl}) \mid \mathbf{q}, \mathbf{p}] \} \right\}, \quad (4) \end{aligned}$$

where the expectation is defined as

$$\begin{aligned} & E[V_i(q, \mathbf{q}', \mathbf{p}', \varepsilon') \mid \mathbf{q}, \mathbf{p}] \\ &= \int_{\mathbf{q}', \mathbf{p}', \varepsilon'} V_i(q, \mathbf{q}', \mathbf{p}', \varepsilon') \Pi(\mathbf{q}', \mathbf{p}' \mid \mathbf{q}, \mathbf{p}) \nu(\varepsilon'). \quad (5) \end{aligned}$$

The structure of Equation (4) shows that a consumer decides whether to keep her existing product or to purchase a new product (the outer maximum); plus, conditional on choosing a new product, she decides

which product to choose (the inner maximum). Equation (5) defines the expected value function obtained after integrating over future product qualities, prices, and the unobservables.

### 3.2. Price and Quality Expectations

I model consumer expectations about future prices and qualities independently such that  $\Pi(\mathbf{q}', \mathbf{p}' | \mathbf{q}, \mathbf{p}) = \Pi_q(\mathbf{q}' | \mathbf{q}) \Pi_p(\mathbf{p}' | \mathbf{p})$ . This assumption can be relaxed to allow for correlations between price and quality changes.<sup>14</sup>

**3.2.1. Price Expectations.** The highly competitive nature of the CPU industry creates a significant amount of interdependence between processor prices. To capture these complex relationships, prices follow a first-order vector autoregressive (VAR) process:

$$\log(\mathbf{p}_t) = A_0 + A_1 \log(\mathbf{p}_{t-1}) + z_t, \quad z_t \sim N(0, \Sigma), \quad (6)$$

where  $A_0$  and  $z_t$  are  $(KJ \times 1)$  vectors, and  $A_1$  and  $\Sigma$  are  $(KJ \times KJ)$  matrices. The cross-terms in each regression equation account for price competition between Intel and AMD, and the off-diagonal elements of  $\Sigma$  capture the covariance between different product prices. Allowing for correlation in the random shocks further captures the comovement of prices of the competing firm. This reduced-form representation of prices is intended to provide a reasonable approximation to consumers' expectations and memories. This VAR could be viewed as a reduced-form representation of the firms' joint policy functions.<sup>15</sup>

**3.2.2. Quality Expectations.** Processors experience frequent incremental improvements and the occasional large innovation. Small quality improvements usually stem from production refinements that result in faster chip frequencies. Large improvements stem from significant changes in the fundamental architecture of a chip. To capture both innovation processes, I model changes in product quality as discrete improvements, or "jumps," on a quality ladder. First, I discretize the continuous measure of quality obtained from the speed benchmark data. Second, I use a modified version of a zero-inflated Poisson (ZIP) process and separately model the probability that a product's quality changes or remains the same. Two benefits result from this approach: the ZIP process helps account for the underdispersion in the number of zeros (periods in which the product quality does not change), and I allow product qualities to change by more than one quality unit.

Let  $\phi(q_{jkt}) = q_{jkt} - q_{jk, t-1}$  be the change in a product's quality from one period to the next, where  $\phi(q_{jkt}) \geq 0$  for all  $j, k, t$ . The probability that a product's quality does not change from one period to the next depends on the current product quality:

$$\Pr(\phi(q_{jkt}) = 0 | q_{jk, t-1}) = \kappa_0 + \kappa_1 q_{jk, t-1} + \varepsilon_{\phi_{jkt}}, \quad (7)$$

where  $\varepsilon_{\phi_{jkt}}$  is normally distributed. The dependence on current product quality is necessary to capture the fact that product life cycles for Intel and AMD have decreased over time.

The probability of a positive quality change is derived from a standard Poisson distribution:

$$\Pr(\phi(q_{jkt}) = z | q_{jk, t-1}) = \frac{1 - \phi_0(q_{jkt})}{1 - e^{-\lambda_{jk}}} \frac{e^{-\lambda_{jk}} \lambda_{jk}^z}{z!} \quad \text{for } z > 0, \quad (8)$$

where  $\phi_0(q_{jkt}) \equiv \Pr(\phi(q_{jkt}) = 0 | q_{jk, t-1})$  and the first fraction is required as a normalization. Let  $I_{\phi(q_{jkt})=0}$  be the indicator function representing no innovation.

Let  $\phi(\mathbf{q}_t) = \{q_{jkt}\}$  for all  $j$  and  $k$ . Assuming the innovation processes are independent across products, the transition kernel for all product qualities is

$$\begin{aligned} \Pi_q(\mathbf{q}_{t+1} | \mathbf{q}_t) &= \Pr(\phi(\mathbf{q}_t) | \mathbf{q}_t) \\ &= \prod_{q_{jkt} \in \mathbf{q}_t} \left[ \phi_0(q_{jk, t+1} | q_{jkt})^{I_{\phi(q_{jkt})=0}} \right. \\ &\quad \left. \cdot \Pr(\phi(q_{jk, t+1}) = q_{jk, t+1} - q_{jkt} | q_{jkt})^{1 - I_{\phi(q_{jkt})=0}} \right]. \quad (9) \end{aligned}$$

### 3.3. Demand

The market size  $M_t$  is observed and evolves deterministically. The solution to the consumer's decision problem for each type  $(i, \tilde{q}_k)$  and the distribution of product ownership over consumer types determine aggregate demand for a product. Consumer demand in this period determines the distribution of product ownership in the next period: what consumers own today and what they buy today determines the distribution of what consumers own tomorrow. For some period  $t$ , let  $\Delta_{ikqt}$  be the fraction of consumers who belong to segment  $i$  and own product  $\tilde{q}_k$ , let  $\Delta_{kqt} = \sum_{i \in I} \Delta_{ikqt}$  be the fraction of all consumers who own product  $\tilde{q}_k$ , and let  $\Delta_{it|kq} = \Delta_{ikqt} / \Delta_{kqt}$  be the fraction of consumers who belong to segment  $i$  conditional on owning product  $\tilde{q}_k$ .

Following Rust (1987), I assume  $\{\varepsilon_{ijk}\}$  are drawn from a multivariate extreme value distribution. This assumption produces the standard multinomial logit formula for product demand from consumers of type  $(i, \tilde{q}_l)$ , for  $l = 1, \dots, K$  who purchase some  $q_{jk} \in \mathbf{q}_t$ :

$$d_{jkt}(\tilde{q}_l, i) = \frac{\exp\{\bar{V}_i(q_{jk}, \tilde{q}_l, \mathbf{q}_t, \mathbf{p}_t)\}}{\sum_{q' \in \mathbf{q}_t \cup \tilde{q}_l} \exp\{\bar{V}_i(q', \tilde{q}_l, \mathbf{q}_t, \mathbf{p}_t)\}}, \quad (10)$$

<sup>14</sup> See the alternative model of expectations in Appendix A.1.

<sup>15</sup> This approach is similar to Adda and Cooper (2000), who use a VAR to model the stochastic evolution of prices, and to Erdem et al. (2003), who model the price process for multiple products as contemporaneous functions of the price of a base product.

where  $\bar{V}_i(q_{jk}, \tilde{q}_l, \mathbf{q}_t, \mathbf{p}_t)$  is the product-specific value function obtained after integrating out the unobserved consumer heterogeneity:

$$\bar{V}_i(q_{jk}, \tilde{q}_l, \mathbf{q}_t, \mathbf{p}_t) = u_i(q_{jk}, \tilde{q}_l, \mathbf{q}_t, \mathbf{p}_t) \quad (11)$$

$$+ \beta \int_{\mathbf{q}_{t+1}, \mathbf{p}_{t+1}} \log \left( \sum_{q' \in \mathbf{q}_{t+1} \cup q_{jk}} \exp \{ \bar{V}_i(q', q_{jk}, \mathbf{q}_{t+1}, \mathbf{p}_{t+1}) \} \right) \cdot \Pi(\mathbf{q}_{t+1}, \mathbf{p}_{t+1} | \mathbf{q}_t, \mathbf{p}_t). \quad (12)$$

Let  $\tilde{d}_{kt}(\tilde{q}_k, i)$  denote the set of consumers of type  $(i, \tilde{q}_k)$  who choose to retain their existing product and not make a purchase.

Integrating over consumer preferences and summing over all other existing products determines demand for a product, which yields

$$x_{jkt} = M_t \sum_{\substack{\tilde{q}_l \in \tilde{Q}_t \\ \tilde{q}_l \neq q_{jk}}} \Delta_{lqt} \sum_{i \in I} d_{jkt}(\tilde{q}_l, i) \Delta_{it|lq}. \quad (13)$$

Note that this quantity represents the total new demand for a product but not the proportion of consumers who will own the product next period. Consumers who already own this product and choose not to purchase anything must be accounted for in the future distribution of product ownership. The number of consumers who do not purchase a new product and retain their existing product is

$$\tilde{x}_t = M_t \sum_{\tilde{q}_k \in \tilde{Q}_t} \Delta_{kqt} \sum_{i \in I} \tilde{d}_{kt}(\tilde{q}_k, i) \Delta_{it|kq}. \quad (14)$$

Market shares for current products are

$$\mu_{jkt} = \frac{x_{jkt}}{\tilde{x}_t + \sum_{q_{j'k'} \in \mathbf{q}_t} x_{j'k't}}. \quad (15)$$

The proportion of consumers who own a product in the following period is the sum of those who purchased the product in the previous period plus those who already owned the product and did not make a new purchase. For all  $q_k \in \mathbf{q}_t \cap \mathbf{q}_{t+1}$ , the share of consumers who own  $q_k$  next period is

$$\Delta_{kq(t+1)} = x_{jkt} / M_t + \Delta_{kqt} \sum_{i \in I} \tilde{d}_{kt}(\tilde{q}_k, i) \Delta_{it|kq}. \quad (16)$$

The law of motion for the marginal distribution over products that are no longer sold such that  $\tilde{q}_k \notin \mathbf{q}_{t+1}$  is

$$\Delta_{kq(t+1)} = \Delta_{kqt} \sum_{i \in I} \tilde{d}_{k,t}(\tilde{q}_k, i) \Delta_{it|kq}. \quad (17)$$

Define the law of motion for the conditional distribution of segment membership over existing products

that are in the product market in the next period as follows. For all  $q_k \in \mathbf{q}_t \cap \mathbf{q}_{t+1}$ , let

$$\Delta_{it|kq} = \frac{\tilde{d}_{kt}(\tilde{q}_k, i) \Delta_{kqt} \Delta_{it|lq} + \Delta_{lqt} \sum_{\tilde{q}_l \in \tilde{Q}_t} d_{jkt}(\tilde{q}_l, i) \Delta_{it|lq}}{\Delta_{lq(t+1)}}. \quad (18)$$

This expression captures consumers of type  $i$  who retained  $q_k$  and those substituting ownership of this product for a more advanced product. One must also update the conditional distribution differently for products not currently being sold. For any  $\tilde{q}_k \notin \mathbf{q}_t$ , the next period conditional distribution is

$$\Delta_{i(t+1)|kq} = \frac{\tilde{d}_{k,t}(\tilde{q}_k, i) \Delta_{it|kq}}{\sum_{i' \in I} \tilde{d}_{k,t}(\tilde{q}_k, i') \Delta_{i't|kq}}. \quad (19)$$

## 4. Estimation

I estimate the model using GMM as part of a nested fixed point. This procedure sets parameters that make the moments of the simulated model as close as possible to their empirical counterparts. This section begins with an explanation of the approach to product aggregation, followed by a discussion of the identification of the structural parameters and then the details of the estimation.

The market of interest is consumers in the United States; thus, the market size is set to the number of households. The time period length is one month. This choice reflects the need to accommodate the frequent introduction of new products and changes in prices. The frequency of the data vary: prices are available continuously, shipments quarterly, and ownership information semiannually. Because the shipment and ownership data are less volatile than prices, I convert all the series to monthly observations for a total of  $T = 138$  time periods. I set prices to the mean price observed in a month, distribute shipments evenly over the quarter, and use cubic splines to interpolate ownership shares.<sup>16</sup>

### 4.1. Creating Composite Products

The model allows each firm to sell multiple products, and the data set contains information on each firm's complete product line. Estimating the model is, however, computationally challenging with the full product lines. Thus, I aggregate the product lines to create composite frontier and nonfrontier processors for each firm. Allowing for two products per firm still permits an analysis of consumer purchasing behavior and product competition with multiproduct firms while reducing the computational burden.

I define composite products in terms of the current period product offerings for each firm. For a given

<sup>16</sup> Tests using other conversion methods produce qualitatively similar results.

period and firm, I divide each firm's product line into groups of processors above and below the median quality product. I form the frontier product by taking the average price and quality of the upper processor group, and assign the nonfrontier product the average price and quality of the lower group.<sup>17</sup> The quality and price of these composite products change as the underlying set of products and their corresponding prices change. These changes occur as new products enter and old products exit and as the prices of existing products change. I calculate composite market shares in a similar fashion.<sup>18</sup>

Figure 2 plots the quality and price of the frontier product over time. The quality plots show that Intel started as the dominant technology provider, but when AMD released its Athlon processor in mid-1999, the two firms entered into close technological competition. A similar story exists in the price graphs. Intel starts as the high-cost and high-quality provider, whereas AMD serves as the lower-cost and lower-quality provider. After mid-1999, the price differences for each firm's products narrows significantly.<sup>19</sup>

#### 4.2. Identification

The ideal data set to study product adoption and replacement is individual-level consumer panel data with information on which product a consumer owns and which product she replaces it with. This panel data would allow the researcher to directly observe the replacement decision. In the context of durable goods markets, such data are difficult to obtain and expensive to collect because of the length of replacement cycles. At the opposite extreme, one could try to estimate a model of replacement using only sales data, but additional assumptions and restrictions on the data-generating process may be required.

This paper takes an intermediate approach by indirectly inferring replacement decisions through a combination of two market-level data sets: (1) quantities sold and (2) the current installed base in each period.<sup>20</sup> This hybrid approach raises two issues. First,

the combination of data sources is not equivalent to individual-level panel data; the data help us uncover the overall replacement rate for each type of owner but not the conditional purchase probabilities. Nevertheless, the data provide information in terms of the patterns of ownership the model must rationalize to estimate the parameters. Second, estimating the model using GMM results in an efficiency loss relative to what one could achieve using maximum likelihood estimation (MLE) with the panel data. I conduct a Monte Carlo simulation study in §4.4 to evaluate the magnitude of this efficiency loss and find that the aggregate model still performs well.

With the discount factor fixed, I identify the dynamic parameters through the combination of demand and ownership data. Variation in price and quality, as shown in Figure 2, identifies the consumer's sensitivity to money and product quality. Replacement behavior is inferred through the relationship between changes in the distribution of ownership and period sales. I include two additional moments to help empirically separate adoption versus replacement behavior: the overall market penetration rate and share of period sales because of replacement. These moments help ensure that the model places the appropriate utility on the outside option of no purchase, conditional on being a nonowner making an adoption decision or an owner making a replacement decision.

Consumer heterogeneity changes endogenously as the distribution of ownership evolves. I identify consumer heterogeneity through the joint variation in the product availability and changes in the installed base. For example, suppose a large improvement occurs in the quality of the frontier product. If the installed base changes more for consumers who own more recent products, this suggests consumers with a higher preference for quality and a lower price sensitivity are purchasing the new product. If not, the data would show a relatively larger portion of consumers who own older products shifting to this new product. Additional variation through intertemporal substitution patterns also helps identify the parameters.

#### 4.3. Implementation

I use the method of moments to estimate the parameters based on the moment equations:

$$E[\mathbf{m}_t(\theta_0)' \mathbf{Z}_t] = 0. \quad (20)$$

The parameter vector  $\theta \in \mathbb{R}^d$  contains both the structural parameters and a set of initial conditions. The vector of moments  $\mathbf{m}_t(\cdot) \in \mathbb{R}^w$ , with  $w \geq d$ , specifies the differences between the observed and simulated quantities. Price endogeneity is often a concern when a product characteristic exists that consumers and firms observe but the econometrician does not.

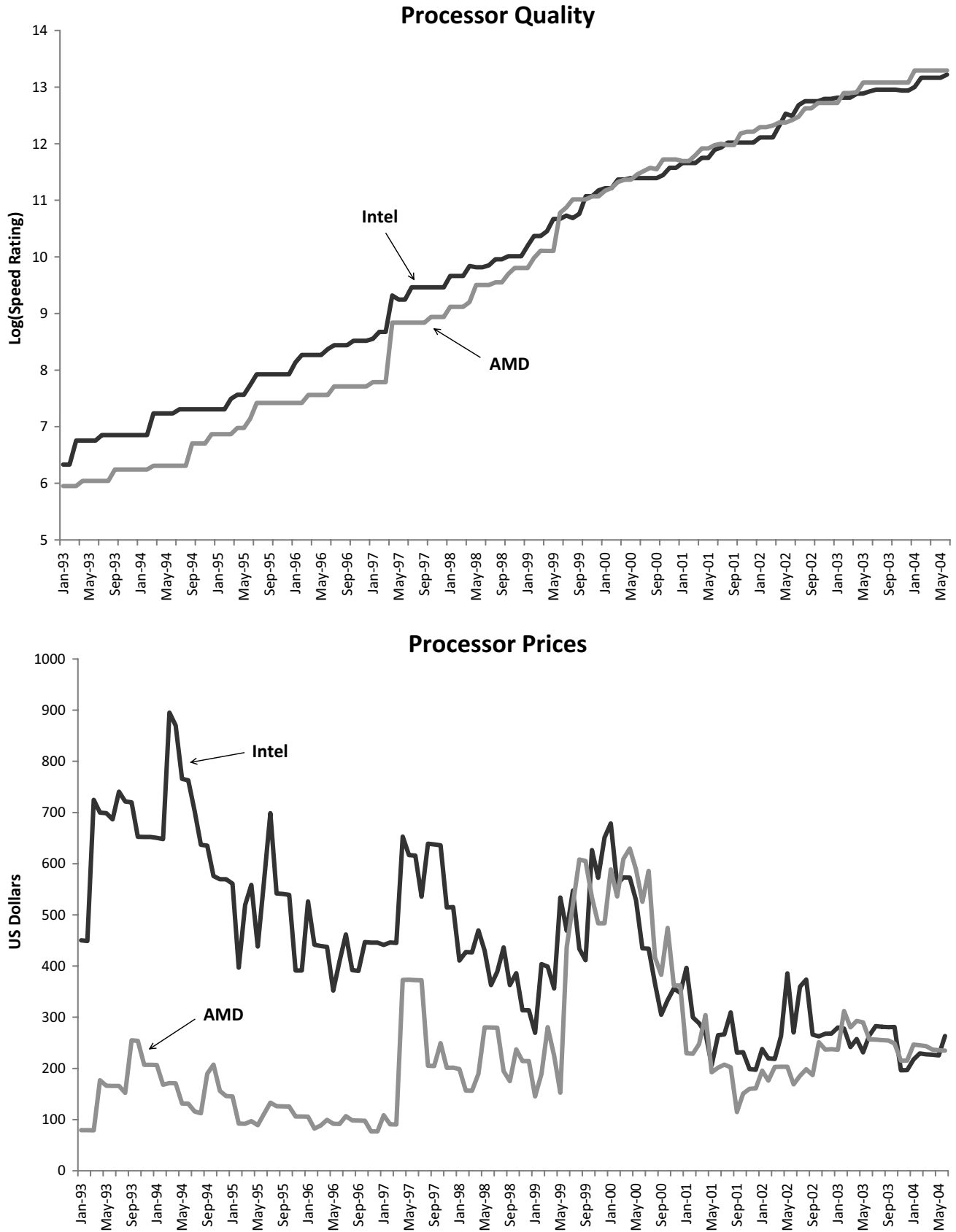
<sup>17</sup> One alternative would be to use the top  $K$  processors in each subgroup, using the average price and quality of these to form the composite products. The model produces qualitatively similar results with  $K = 3$ .

<sup>18</sup> Product line length increased over time as the firms reduced their product life cycles and tried to more finely segment the market. Although I used other aggregation methods that produced qualitatively similar results, the model does not account for this additional aspect of the product market.

<sup>19</sup> A plot of the quality and price of the nonfrontier product can be found in the Technical Appendix, which can be found at <http://mktsci.pubs.informs.org>.

<sup>20</sup> Others have also used additional micromoments to aid in the estimation of dynamic structural models, such as Luan (2005) and Gowrisankaran and Rysman (2007).

Figure 2 Composite Frontier Qualities and Prices for Intel and AMD



Note. Prices are in constant January 2000 dollars.

In contrast, the processor speed benchmark provides an accurate measure of product quality, implying that the effects of any unobservable product characteristics should be minimal. To correct for potential price endogeneity, I use a set of exogenous instruments  $\mathbf{Z}_t \in \mathbb{R}^w$ . As in Berry et al. (1995), I include in  $\mathbf{Z}_t$  functions of observable characteristics and exploit variables that affect the price-cost margin. I include the own firm’s product characteristics, the mean of the competitor’s product characteristics, and the number of products in each firm’s observed product lines. These variables help take into account the position of the product relative to others in the characteristic space, which should affect substitutability and the price-cost margin across products.<sup>21</sup>

Two types of moment conditions exist: those relating to demand and those relating to the distribution of ownership. For the demand shares  $\boldsymbol{\mu}_t(\theta) = \{\mu_{jkt}(\theta)\}$ , I match the within-market shares associated with each composite product and the shares of sales stemming from replacement purchases. For the ownership shares  $\mathbf{G}_t(\theta) = \{G_{i\tilde{q}}(\theta)\}$ , I match the share of owners who own a particular composite processor and the overall market penetration rate. Matching the penetration rate and the share of replacement sales are important because they help identify the value of the no-purchase option.

The moment conditions can be written as

$$\mathbf{m}_t(\theta) = \begin{bmatrix} \boldsymbol{\mu}_t(\theta) - \hat{\boldsymbol{\mu}}_t \\ \mathbf{G}_t(\theta) - \hat{\mathbf{G}}_t \end{bmatrix}, \quad (21)$$

where a generalized method of moments estimator,  $\hat{\theta}$ , minimizes the weighted quadratic form:

$$Q_T(\theta) = \min_{\theta \in \mathbb{R}^d} \frac{1}{2} \left[ \frac{1}{T} \sum_{t=1}^T \mathbf{m}_t(\theta) \right]' \mathbf{Z}_t \Omega \mathbf{Z}_t' \left[ T^{-1} \sum_{t=1}^T \mathbf{m}_t(\theta) \right], \quad (22)$$

where  $\Omega$  is an  $w \times w$  positive semidefinite weighting matrix. Under the assumption that  $\Omega \xrightarrow{p} \Omega_0$ , define the  $w \times d$  matrix  $\mathbf{M}_0 = E[\nabla_{\theta} \mathbf{m}_t(\theta_0)]$ . Let  $\Lambda_0 = E[\mathbf{m}_t(\theta_0) \mathbf{m}_t(\theta_0)']$  and substitute a consistent estimator for  $\Lambda_0^{-1}$  into the weighting matrix. Under the standard assumption that  $\mathbf{m}_t(\theta)$  is independent across  $t$ , we have that

$$\sqrt{T}(\hat{\theta} - \theta_0) \xrightarrow{d} N(0, (\mathbf{M}_0' \Lambda_0^{-1} \mathbf{M}_0)^{-1} / T). \quad (23)$$

I include a set of initial conditions in the parameter vector. These conditions are the unobserved distribution of consumer segments over product ownership at

$t = 0$ . Given these initial conditions, I use the laws of motion for the ownership distribution and the current period market shares defined in Equations (16)–(19) to calculate the next period conditional distribution of ownership. Although the data contain information on the share of consumers who own particular products, I do not observe the share of a particular segment who own a certain product. That is, for each segment  $I$  and each owned product  $\tilde{q} \in Q_0$ , I would need to estimate  $(I - 1)(JK + 1)$  parameters. To reduce the number of parameters, I assume that the initial distribution of consumers across segments is identical for both firms for a given product.<sup>22</sup>

The discount factor is fixed at  $\beta = 0.98$  so time periods correspond to roughly one month.<sup>23</sup> Estimation proceeds as follows. First, I estimate the price and quality processes using maximum likelihood. I treat these estimates as known and substitute them into the consumer’s dynamic optimization problem. Second, a nested fixed-point procedure minimizes a GMM objective function in the outer loop and computes the value function in the inner loop. For a given parameter vector, I solve the consumer’s fixed-point problem for all consumer types  $(i, \tilde{q}_k)$ . Starting at  $t = 1$ , I use Equations (10) and (15) to compute aggregate consumer demand, followed by Equations (16)–(19) to calculate the implied distribution of ownership at  $t = 2$ . I repeat this process until  $t = T$  over the sequence of *observed* states. I form moments based on the simulated and empirical values and minimize the GMM objective function. I use simulation to compute the expectation with respect to the quality process and Rust’s (1997) randomization approach to integrate over the continuous price vector. Details of the computation are in the Appendix B.

#### 4.4. Monte Carlo Study: Comparing Aggregate vs. Individual Data

I conduct a simulation study to assess the ability of the model (a) to recover the unobserved consumer segment structure and (b) to evaluate the efficiency loss from using aggregate data as opposed to a model based on individual panel data. I simulate data at the consumer level and compare the estimates obtained

<sup>21</sup>In unreported results, I included cost shifters as instruments using data from In-Stat/MDR at the processor die level (e.g., McKinley, Prescott), broken down into estimates of the untested die cost, the package cost, and the packing and testing costs. Using these cost-based instruments did not have a significant impact on the results.

<sup>22</sup>There are  $JK$  products that people could own at  $t = 0$  as well as one outside good ( $\tilde{q} = 0$ ). The segment shares must sum to one. I assume that the share of consumers who own the frontier product and nonfrontier product for each firm is the same across segments. I also need to estimate the proportion of each segment who do not own any product,  $\Delta_{0i|\tilde{q}=0}$ . Alternatively, one could parameterize this initial unobserved conditional distribution using some parametric form.

<sup>23</sup>This monthly discount factor is roughly consistent with the weekly value of 0.995 often used in the literature (cf. Erdem and Keane 1996). I evaluated the model using a monthly beta ranging from 0.9 to 0.99, implying a yearly discount factor ranging from 0.28 to 0.87, and found that the results are qualitatively unchanged.

**Table 1** Monte Carlo Simulations

	True values	Aggregate data		Panel data	
		Est.	Std. dev.	Est.	Std. dev.
Quality (Seg 1)	0.500	0.536	0.058	0.511	0.026
Quality (Seg 2)	0.750	0.712	0.073	0.748	0.029
Price (Seg 1)	-2.000	-2.130	0.120	-2.045	0.061
Price (Seg 2)	-1.500	-1.427	0.099	-1.519	0.040
Intel (Seg 1)	2.000	2.114	0.105	2.042	0.035
Intel (Seg 2)	3.000	2.970	0.125	2.986	0.048
AMD (Seg 1)	0.250	0.242	0.031	0.248	0.013
AMD (Seg 2)	0.500	0.561	0.038	0.520	0.018
Initial conditions					
Nonowner	0.900	0.932	0.052	0.906	0.018
Frontier	0.600	0.582	0.038	0.591	0.025
Nonfrontier	0.800	0.809	0.033	0.803	0.017
Mean repl. cycle	3.534	3.611	0.194	3.470	0.082
Std. dev. of repl. cycle	2.380	2.568	0.320	2.292	0.126
Mean upgrade percent	317.834	339.382	26.909	309.472	12.845
Std. dev. of upgrade percent	192.651	201.300	16.123	190.665	7.963

*Notes.* Mean parameter estimates and standard deviations for a set of Monte Carlo simulations evaluating the recoverability of the parameters. The estimates under “Aggregate data” estimate the model as described in the body of the paper using GMM based on matching aggregate moments. The estimates under “Panel data” use individual-level data to estimate the model using MLE.

using aggregate and panel data. As discussed in §4.2, the combination of aggregate data sets is not equivalent to consumer panel data but still allows us to observe overall replacement rates conditional on product ownership. The goal here is to understand how well the aggregate model is able to identify the relevant patterns of replacement behavior compared to a model using individual data.

I conduct the simulation with two consumer segments who differ in their price, quality, and firm fixed-effects coefficients.<sup>24</sup> First, I use the actual price and quality time series from the data. Second, I use the actual initial distribution of product ownership and assume that, conditional on product owned, consumers are evenly distributed across segments. Third, I solve each type’s dynamic programming problem and draw a random set of Type I extreme value error terms,  $\varepsilon_{ijkt}$ , to arrive at a simulated value for  $V_{ijkt}(\tilde{q}_{tk}, \mathbf{p}_t, \mathbf{q}_t, \varepsilon_{ijkt})$ . I use these values to simulate the purchasing behavior of 10,000 consumers. Fourth, based on the optimal choices, I aggregate the outcomes across the consumers to form the market shares and distribution of product ownership in each time period.

I use the simulated data to estimate the aggregate model using GMM and estimate an individual-level model using MLE. For the GMM case, I follow the estimation approach §4.3 details, excluding the

instruments. For the MLE case, I maximize the log-likelihood function based on the individual choice probabilities in Equation (10). Table 1 presents the parameter estimates and replacement statistics. The first column shows the true parameter values for the simulated data. This column also contains some descriptive statistics concerning consumer replacement behavior that are a result of the given parameter values: the mean replacement cycle length (in years) and the mean percent quality upgrade. The other columns report the mean and standard deviation of the estimates over 50 simulations.<sup>25</sup>

The results, reported in Table 1, demonstrate the ability of the model to recover the structure of consumer heterogeneity and replacement behavior from aggregate data. The true parameter values lie within the 95% confidence intervals of the estimates. As expected, the standard deviations of the mean parameter estimates are smaller for the individual data models compared to the aggregate data models. This difference reflects the standard efficiency gains associated with using MLE. The standard deviations using individual data are roughly one-third the magnitude of those using the aggregate data. Imprecise estimates of the initial conditions create an additional source of error, despite the fact that the initial conditions are adequately recovered. The Monte Carlo results also demonstrate the ability of the model to recover the replacement behavior of the consumers. Both the mean replacement cycle length and the mean percentage of quality upgrade fall within the 95% confidence intervals of the estimates, as do the standard deviations of these statistics.

As expected, there is an efficiency loss relative to using MLE, but the difference is not too large. One significant drawback, however, of a model with aggregate data is the difficulty in specifying a rich structure of heterogeneity.<sup>26</sup> Individual-level data could make it easier to specify a simpler consumer model.

## 5. Results

This section presents an evaluation of the model’s fit, discusses the parameter estimates, and provides results from several policy simulations.

<sup>25</sup> Because of the computational burden of estimating the model, I have limited the number of simulated replications to 50.

<sup>26</sup> Bodapati and Gupta (2004) demonstrate the difficulty of estimating latent-class models with aggregate data. One particular concern in their paper is that the presence of measurement error from forming market shares based on store-level data makes separating the preferences of the latent segments difficult, especially when many products have small shares. In contrast, the market share data in this paper contain fewer products, and I assume that In-Stat/MDR estimates the shares to within a reasonable degree of accuracy.

<sup>24</sup> The conclusions of the Monte Carlo study are similar with homogeneous consumers.

### 5.1. Model Fit and Comparison

To evaluate the model, I report estimates from five different specifications in Table 2. The columns in Table 2 correspond to the following: (1) a homogeneous myopic model with  $\beta = 0$ , (2) a homogeneous dynamic model in which consumers only make an adoption purchase and then exit the market, (3) a dynamic model with homogeneous consumers estimated without the additional ownership moments, (4) a dynamic homogeneous model estimated with the ownership moments, and (5) a two-segment dynamic model. Appendix A describes two additional specifications that were excluded here for sake of brevity.

Table 2 reports the parameter estimates, the objective function values and  $J$ -statistics, and the distance metric (DM) statistics. I determine the number of segments by incrementally adding segments until a new segment's preference parameters are not statistically different from those of an existing segment. A three-segment model results in two segments with statistically equivalent parameter estimates and does not significantly improve the objective function value. The

$p$ -values for the  $J$ -statistics show that the data do not reject any of the models.

I use the DM statistic to formally compare the models, where  $DM = 2T[Q_T(\hat{\theta}_{an}) - Q_T(\hat{\theta}_{\theta_n})] \sim \chi_r^2$  is based on differences in the GMM objective function between unconstrained and constrained estimators.<sup>27</sup> The DM statistics and  $p$ -values in Table 2 compare the two-segment dynamic model (alternative) against each of the first four models (nulls). The  $p$ -values show that the first four models are strongly rejected against the two-segment model. Given that the two-segment model is preferred versus models (1)–(4), I use the two-segment dynamic model as the benchmark specification for the rest of the results.

Further examination of the results from Table 2 reveals three additional points. First, the myopic model (first column) performs the worst. This poor performance stems from the fact that a myopic model has difficulty explaining data that an inherently dynamic process generates. The myopic model has a particular problem explaining why many consumers waited so long to make their first purchase.

Second, a dynamic model that restricts consumers to make only one purchase (second column) does not account for heterogeneity in the value of consumers' outside options and yields less sensible parameter estimates. A model with replacement can account for the fact that some consumers who purchase later in time may be replacing an existing product. An adoption-only model would conclude that these consumers placed a high value on not owning anything, leading to smaller product coefficients and a larger relative variance of the idiosyncratic error term. This causes the adoption-only model to underestimate the quality and firm-specific coefficients, because it assigns too much importance to the outside option.<sup>28</sup>

Third, estimating the dynamic homogeneous model without the additional ownership moments generally yields less precise parameter estimates. Understanding the means in which excluding the ownership moments produces this set of parameters with larger standard errors is difficult, but the objective function value shows that the fit is inferior to the model with the ownership moments.

Figure 3 compares the market penetration rate from the data and benchmark model. I calculate the predicted rates at each time period given the

**Table 2** Structural Parameter Estimates

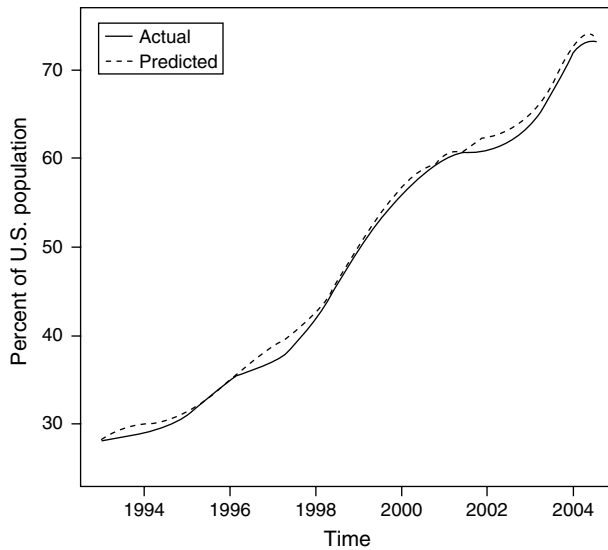
	Myopic	Adoption only	One segment		Two segments	
			w/o moments	One segment	Seg 1	Seg 2
Quality	1.425 (0.365)	0.085 (0.031)	0.448 (0.194)	0.618 (0.243)	0.582 (0.071)	0.679 (0.79)
Price	-0.849 (0.471)	-0.283 (0.185)	-1.512 (0.599)	-1.734 (0.376)	-1.803 (0.423)	-1.620 (0.430)
Intel	3.940 (0.812)	1.103 (0.353)	1.809 (0.661)	2.205 (0.487)	2.084 (0.392)	2.568 (0.386)
AMD	0.671 (0.106)	0.116 (0.026)	0.216 (0.068)	0.292 (0.038)	0.257 (0.050)	0.341 (0.054)
Initial conditions						
Nonowners	—	—	—	—	0.923 (0.125)	0.077
Frontier	—	—	—	—	0.684 (0.096)	0.316
Nonfrontier	—	—	—	—	0.761 (0.131)	0.239
Segment size	—	—	—	—	0.863 (0.062)	0.137
Obj. value	0.172	0.193	0.158	0.121	0.081	
$J$ -statistics	23.736	26.634	21.804	16.698	11.178	
$p$ -value	0.206	0.114	0.294	0.405	0.264	
DM statistics	25.116	30.912	21.252	11.040	—	
$p$ -value	0.000	0.000	0.000	0.026	—	

*Notes.* Parameter estimates for different specifications of the model. The models in the columns correspond to a homogeneous model with  $\beta = 0$ , a dynamic model in which consumers only make one purchase and then exit the market, a dynamic model with homogeneous consumers estimated without the ownership moments, a dynamic homogeneous model estimated with the ownership moments, and a two-segment dynamic model.

<sup>27</sup> Because of Newey and West (1987), the DM statistic is the GMM counterpart to the likelihood ratio test in an MLE setting. The DM statistic requires the use of two-step GMM and, under the null hypothesis, has a limiting  $\chi^2$  distribution with degrees of freedom  $r$  equal to the dimension of the null hypothesis that the restricted coefficients are zero.

<sup>28</sup> These findings are broadly consistent with those in Gowrisankaran and Rysman (2007).

Figure 3 Actual vs. Predicted Market Penetration Rate



observed level. Variation in the rate of product adoption depends on the rate of innovation and changes in prices: a decline in quality-adjusted prices will lead to increased penetration, whereas stagnant prices or minor quality improvements will result in fewer adoptions. The model does a good job fitting the penetration rate, implying that the model is able to recover adequately the value of the outside option over time.

To validate the model, I use data from two surveys conducted by Forrester Research on PC ownership in North America. The first survey (Forrester Research 2000) was conducted in 2000 using a sample of 63,927 responses, and the second survey (Forrester Research 2002) was conducted in 2002 using a sample of 50,517 responses. Each survey asked respondents about the age of their most recent PC purchase. These surveys allow me to compare the age distribution of ownership that the model implies to an external estimate for this distribution based on data that is not used to estimate the model. Figure 4 compares the distributions from the Forrester surveys to the distributions the model implies. The plot shows that the model does a good job of fitting the distribution of ownership in both years. Although there are some differences between the levels the model predicts and those found in the survey, the overall shape and magnitude of each bin is consistent with the data. This suggests that the model provides a reasonable fit to aggregate consumer replacement behavior.<sup>29</sup>

<sup>29</sup> A  $\chi^2$  test rejects the null hypothesis that the simulated and sample proportions are equal because of the large size of the Forrester surveys. However, the plot shows that the model provides a good qualitative fit to the data.

Table 3 Vector Autoregression Process for Prices

	Intel frontier( $t$ )	Intel nonfrontier( $t$ )	AMD frontier( $t$ )	AMD nonfrontier( $t$ )
Intel frontier( $t - 1$ )	0.8151 (0.0752)	0.3249 (0.0689)	-0.1342 (0.0387)	0.0187 (0.0472)
Intel nonfrontier( $t - 1$ )	0.1072 (0.0425)	0.4179 (0.0916)	-0.1397 (0.0302)	-0.1836 (0.1117)
AMD frontier( $t - 1$ )	-0.0150 (0.0316)	-0.0485 (0.0226)	0.8853 (0.0486)	0.0414 (0.0353)
AMD nonfrontier( $t - 1$ )	-0.1016 (0.0654)	-0.0470 (0.0599)	-0.0161 (0.1007)	0.6836 (0.0731)
Constant	1.0984 (0.4065)	1.1952 (0.3724)	0.8620 (0.6258)	1.9313 (0.4544)
$R^2$	0.8367	0.7361	0.7953	0.6359
-LL	61.3607	73.3306	2.2381	46.0780

Notes. All quantities are in logs. Standard errors in parentheses. LL refers to log-likelihood.

Table 4 Product Quality Process Estimates

	Intel frontier	Intel nonfrontier	AMD frontier	AMD nonfrontier
$\kappa_0$	0.4912 (0.0289)	0.4877 (0.0272)	0.4501 (0.0190)	0.4537 (0.0203)
$\kappa_1$	0.0218 (0.0053)	0.0250 (0.0069)	0.0334 (0.0041)	0.0329 (0.0048)
$\lambda$	1.5656 (0.0572)	1.4406 (0.5320)	1.5248 (0.4798)	1.3717 (0.0412)
-LL	15.3830	16.2184	15.5011	16.8874

Notes. All quantities are in logs. Standard errors in parentheses. LL refers to log-likelihood.

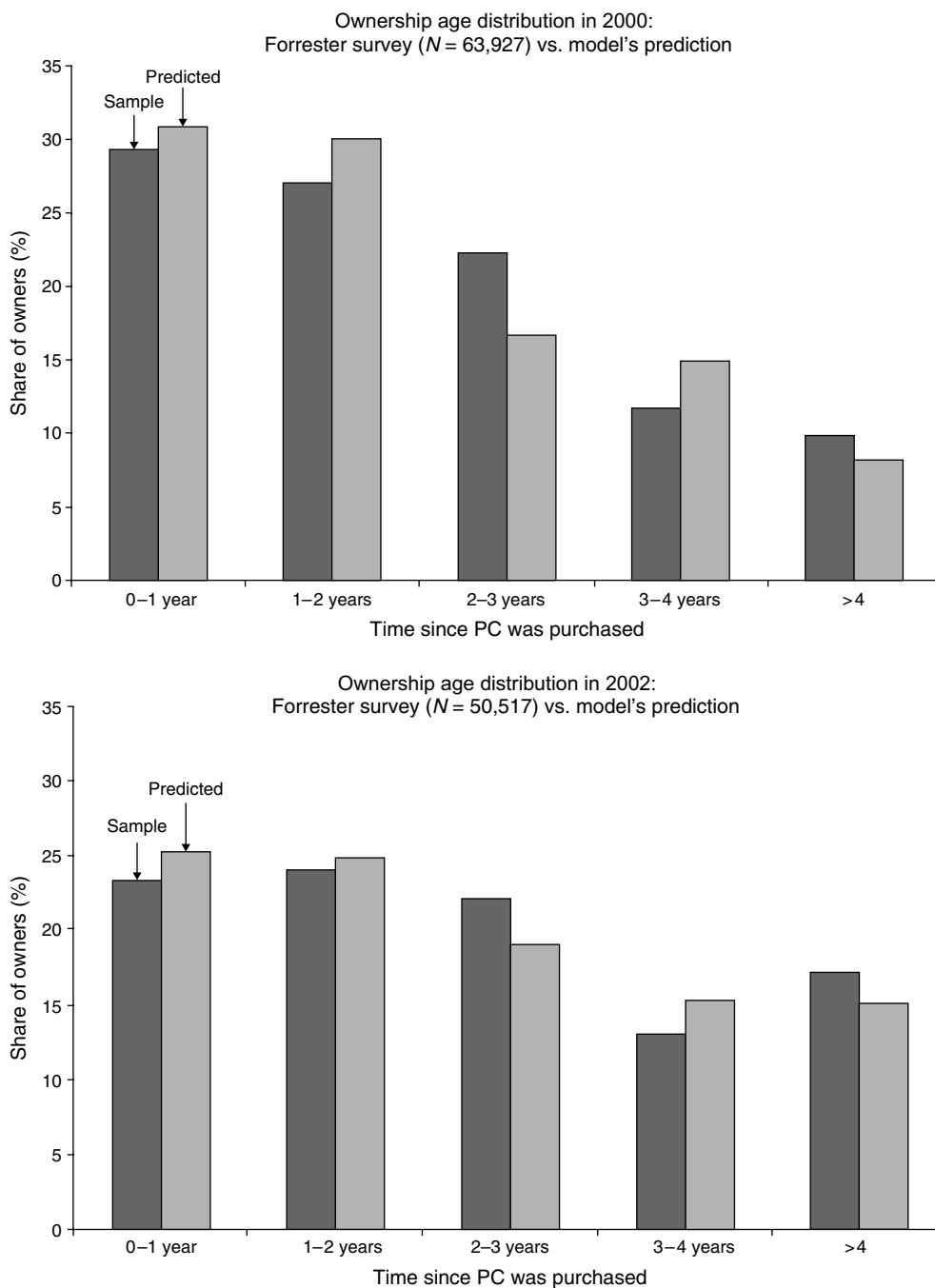
## 5.2. Parameter Estimates

Tables 3 and 4 report the parameter estimates for the price and quality processes. Overall, both fit well. All the own-price coefficients in the price VAR are significant at the 99% level. Several of the cross-price coefficients are also significant, suggesting a competitor's prices have some effect on a firm's price decisions. Price competition appears to be asymmetric: Intel's prices have a larger effect on AMD's prices than the converse. The estimates are robust to higher-order lags and including the set of product qualities as exogenous regressions, which only increases the  $R^2$  by 0.2%. All the roots lie inside the unit circle, indicating that the VAR is stable.<sup>30</sup>

The parameters of the quality process are precisely estimated. The significance of the  $\kappa_1$  parameters indicates some nonstationarity in the probability of a product-changing quality, although the magnitude of this parameter suggests that the effect is small. The

<sup>30</sup> To understand why adding product quality has such a small effect, recall that prices are already in "quality-adjusted" terms. The actual quality of each composite product changes over time, as does its associated price. The fact that quality does not significantly add predictive power to the price VAR might indicate that there is a smooth relationship between composite price and quality.

Figure 4 Comparison of the Ownership Distribution in 2000 and 2002 to Out-of-Sample Survey Data

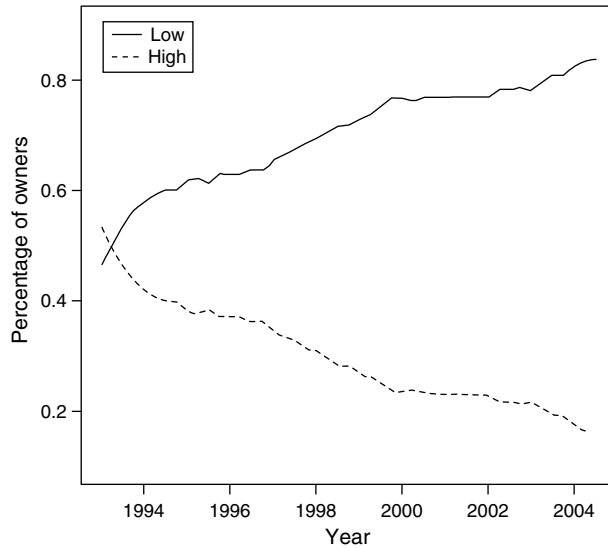


values of  $\kappa_1$  for AMD's products are larger because AMD tended to introduce smaller innovations more frequently than Intel, especially later in the sample period.

The structural parameter estimates in Table 2 are all statistically significant. Comparing the estimates for the benchmark model, segment 1 is more price sensitive and less quality sensitive, and segment 2 is less price sensitive and more quality sensitive. For the remainder of the discussion, I refer to the first segment as the low-valuation segment and the second

segment as the high-valuation segment. I calculate the size of each consumer segment by combining the estimates of the initial conditions with the initial distribution of ownership. The benchmark dynamic model estimates that the low-segment consumers make up approximately 86.3% of the population. The initial conditions imply that at the beginning of 1993, 92.3% of nonowners were low-segment consumers. High-segment consumers, although only 13.7% of the population, owned disproportionately large shares of both frontier and nonfrontier processors.

Figure 5 Distribution of Consumer Segments Among Owners



Recall that the price is only paid once but the fixed effects represent flow utility in a given month. Then, the difference in value associated between two equal quality processors, one from each brand, can be computed as the difference in the discounted stream of utility, given by  $(\xi_{i, \text{Intel}} - \xi_{i, \text{AMD}})/\alpha_i(1 - \beta)$ . The estimates imply that the low and high segments are willing to spend about \$51 and \$69 extra for an Intel processor, respectively. The estimated difference in the fixed effects is required for the model to rationalize the difference in the firms' market shares and prices.

Although consumer segments are static, the mix of segments among owners varies over time. Figure 5 displays the evolution of the conditional distribution of segment membership for consumers who already own a product. The plot reveals a familiar story: high-segment consumers made up the majority of owners early in the market's history and declined as a portion of all owners over time. At the beginning of the sample period, the high-segment consumers represented slightly more than half of all owners, despite the fact that they only represent 13.7% of the population. As the market penetration increased, the share of owners who belonged to the high segment declined.<sup>31</sup>

### 5.3. Market Structure, Innovation, and Product Replacement

This section uses the parameter estimates to analyze consumer behavior in the model. First, I compute price elasticities from a permanent price change and compare the results from the myopic and benchmark dynamic models. Second, I characterize the

<sup>31</sup> As penetration increases, the share of high-segment consumers in the plot will eventually asymptote to the proportion of high-segment consumers in the overall population.

Table 5 Summary of Price Elasticities

	Dynamic model		Myopic model	
	Mean	Std. dev.	Mean	Std. dev.
Intel				
Own-elasticities	-5.70	1.09	-3.68	0.81
Cross-elasticities	2.70	0.41	1.63	0.32
AMD				
Own-elasticities	-5.186	1.32	-3.40	0.95
Cross-elasticities	2.187	0.16	1.59	0.17

Note. Average value of elasticities for a permanent 10% change in prices.

replacement behavior of consumers and conduct policy simulations of replacement behavior under alternative rates of technological innovation. Third, I show how consumer heterogeneity in both preferences and replacement behavior can be useful for altering a manager's strategies.

**5.3.1. Price Elasticities.** I calculate the elasticity estimates using *permanent* changes in the price of a product to capture the long-term effects of the change on the consumer's expectations. I generate elasticities as follows. First, I use the observed quantities to solve the consumer problem and estimate a baseline level of demand. Second, I create the new time series for prices by permanently adjusting the price for one product starting in a period until the end of the sample. The price process is reestimated using the new time series. All other prices remain fixed. Finally, I solve for the optimal consumer behavior given these alternate time series and compare the new demand estimates to the baseline. I repeat this process for each time period. The reported estimates are the average of the elasticities calculated in each period using the observed quantities.<sup>32</sup>

Using a price change of 10%, Table 5 compares summaries of the price elasticities for the dynamic and myopic models. The estimates show that a myopic model underestimates price elasticities by 30% to 40%. Myopic consumers do not consider the future utility associated with owning a product, which leads to downward biased price and quality coefficients, and thus myopic consumers underreact to a permanent price change.

Table 6 decomposes these elasticity values according to each potential consumer choice. The last two columns in Table 6 contain the cross-elasticities for each product with respect to a consumer's no-purchase option. First, the results reveal an asymmetry in the market structure: Intel's products have a larger impact on AMD's products than the converse. One potentially counterintuitive result is that the

<sup>32</sup> Erdem et al. (2003) and Hendel and Nevo (2006) estimate elasticities in a similar fashion.

**Table 6 Average Price Elasticities**

	Intel frontier	Intel non- frontier	AMD frontier	AMD non- frontier	No purchase	
					Owners	Non- owners
Intel frontier	-5.238	3.485	3.206	2.232	2.246	3.025
Intel nonfrontier	2.841	-6.155	2.648	1.793	1.899	2.657
AMD frontier	3.384	2.810	-5.692	1.631	2.813	3.533
AMD nonfrontier	1.690	2.033	1.572	-4.679	1.680	2.104

*Notes.* All estimates are statistically significant; standard errors are not reported. Each entry is the average percent change in demand for the column product given a permanent 10% change in the price of the row product. The last two columns on the right are the change in demand for the outside option (no purchase) given a 10% change in the price of the row product. These elasticities are the average value of the elasticity calculated in each sample period.

more established brand, Intel, has higher own-price elasticities for the nonfrontier products compared to the lesser-known brand, AMD. Second, nonowners are more sensitive than owners because nonowners must have a larger marginal return for product adoption than an owner does for product replacement. The price change has a larger impact on nonowners' probability of purchasing because the value of their outside option is strictly less than the value of not purchasing for owners.

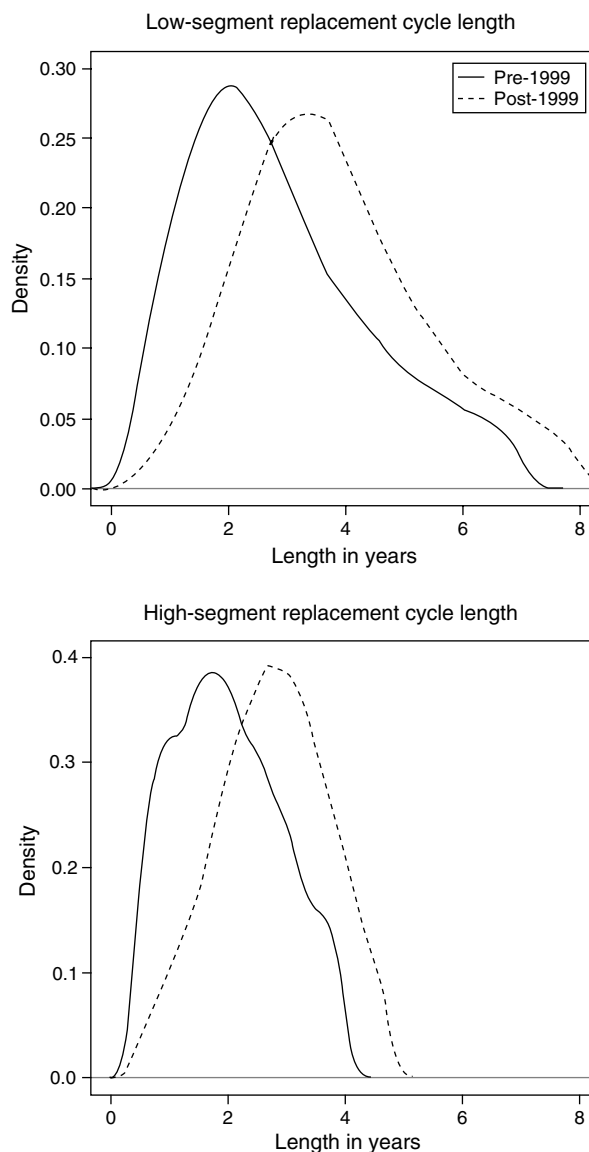
**5.3.2. Product Replacement.** Table 7 reports the average replacement cycle length by consumer segment over the entire sample in the benchmark and myopic models. The benchmark results indicate that consumers replace their existing processors about every 3.3 years on average. This figure is consistent with estimates from the market researcher firm Gartner (Pruitt 2004). As expected, consumers in the high segment have a shorter replacement cycle than those in the low segment. The myopic consumer

**Table 7 Mean Replacement Cycle Length by Period**

	Benchmark model		Myopic model	
	Mean	Std. dev.	Mean	Std. dev.
Full sample				
Low segment	3.451	1.503	2.564	1.315
High segment	2.583	1.029	2.208	0.973
All	3.302	1.427	2.485	1.244
Pre-1999				
Low segment	2.892	1.481	2.066	1.405
High segment	2.033	0.987	1.780	0.934
All	2.710	1.303	1.929	1.348
Post-1999				
Low segment	3.939	1.686	2.994	1.464
High segment	2.836	0.935	2.635	0.925
All	3.651	1.448	2.952	1.457

*Note.* Mean replacement cycle lengths, in years, for the dynamic two-segment model and the myopic model.

**Figure 6 Distribution of CPU Replacement Cycles for Low- and High-Segment Consumers**



model underestimates replacement cycle length, suggesting that consumers replace their products more frequently than the dynamic model implies. Table 7 shows that the average replacement cycle length increased from 2.71 years before 1999 to 3.65 years after 1999.<sup>33</sup>

To demonstrate the degree of variation across consumers, Figure 6 displays the distribution of replacement cycle lengths in the first and second half of the sample for each consumer segment. The mean and variance of each replacement distribution has

<sup>33</sup> I generate these figures by tracking product replacement within a given time period until every consumer who owned a product in the beginning had replaced their product. Thus, a consumer who purchased a product in 1998 and replaced it in 2002 was counted under the pre-1999 period.

**Table 8** Effects of Different Innovation Rates on Replacement Cycle Length

	Benchmark model		+25% Innovation		+50% Innovation	
	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
Low segment	3.342	1.532	3.075	1.551	2.588	1.412
High segment	2.590	1.035	2.203	0.996	1.704	0.964
All	3.239	1.464	2.956	1.474	2.467	1.351

*Note.* Mean replacement cycle lengths, in years, over the entire sample for the benchmark model and counterfactual cases.

increased over time, with both distributions shifting to the right. Less variation exists in the replacement behavior of the high-segment consumers because they represent a smaller part of the consumer population. The distribution for the low-segment consumers is particularly skewed to the right, illustrating the fact that some consumers rarely replace their products.

Given the apparent increase in the length of replacement cycles, I conduct a policy simulation to examine the effects of alternative rates of innovation on consumer upgrades. Empirically, average product quality tends to double about every two years. I alter the parameters of the quality process to increase the rate of innovation by 25% and 50%, which implies that product qualities should double every 1.5 years and 1 year, respectively. All prices are fixed.

Table 8 provides a comparison of the benchmark replacement cycle estimates to those from the policy simulations. A 25% increase in the rate of technological innovation lowers the mean replacement time by 8.7% (or about 3.4 months), and a 50% increase lowers the mean by 23.8% (or about 9.3 months). In both cases, the effect of the faster innovation rates is greater for the high-segment consumers than the low-segment consumers. For the high-segment consumers, the alternative innovation rate reduces the mean replacement cycle length by 14.9% and 34.2%, compared to reductions of 7.4% and 22.0% for the low-segment consumers. These results suggest that technology innovations can have a significant impact on the replacement cycle, especially for consumers who place a premium on processor quality.

**Table 9** Effects of Different Innovation Rates on Replacement

Year	Benchmark	+25% Innovation		+50% Innovation	
	Mean	Mean	% Change	Mean	% Change
1995	2.14	1.89	-11.83	1.32	-38.24
1997	2.47	2.21	-10.77	1.63	-34.23
1999	2.86	2.62	-9.01	1.99	-30.74
2001	3.30	3.05	-7.66	2.38	-27.72
2003	3.68	3.41	-7.43	2.83	-23.00

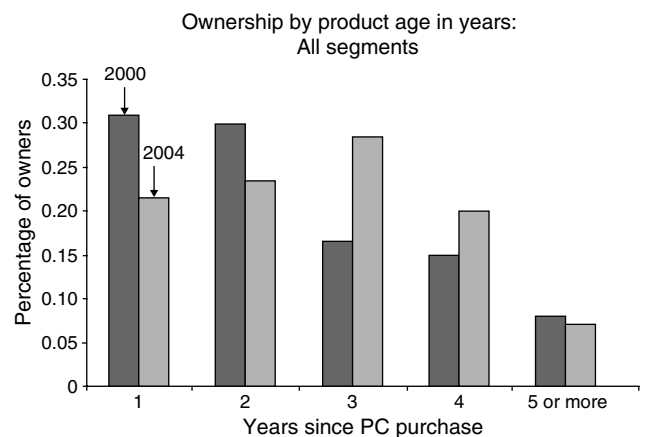
*Note.* Mean replacement cycle lengths, in years, in the benchmark and counterfactual models.

Table 9 examines how the length of the replacement cycle responds over time to changes in the innovation rate. I fix the innovation parameters at their default values until a particular period. After this period, I increase the rate of innovation for all subsequent periods and examine the resulting change in the length of the replacement cycle. One column in Table 9 shows the percent change in replacement cycle length between the benchmark and alternative models. The declining percentages in the column show that the marginal effect of innovation on the length of the replacement cycle decreases over time. This finding implies that as the market matures, the pace of innovation slows and consumers have fewer reasons to upgrade. It also implies that the incentives to replace because of innovation have declined.

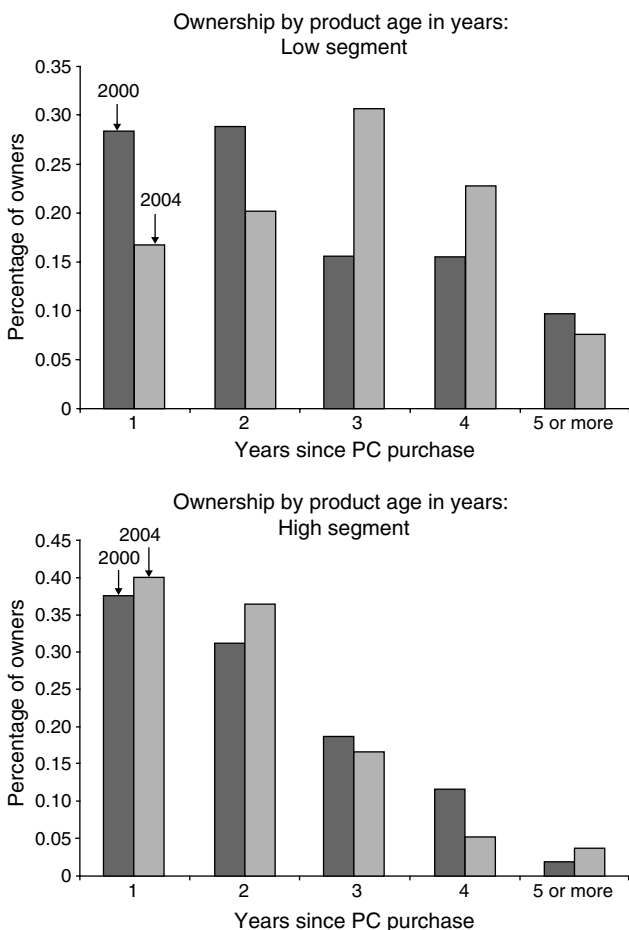
One issue with this policy simulation is that prices and innovation rates are endogenously determined in equilibrium. One might expect the firms to alter their pricing strategies given the new rates of innovation and consumer demand. An equilibrium model of competition such as Goettler and Gordon (2008) could account for the endogenous adjustment of other variables.

**5.3.3. Segmenting Replacement Cycles.** The previous results demonstrate the variation in replacement behavior over time and across consumer segments. This variation arises endogenously as the marginal distribution of ownership across the two segments responds differently to product prices and qualities. A large increase in product quality might trigger many high-valuation consumers to upgrade—despite the price being high—and thus shift the segment-specific distribution of ownership toward newer products.

Figure 7 plots the distribution of ownership by product age implied by the model for the years 2000 and 2004. For example, in 2000, Figure 7 shows that

**Figure 7** The Distribution of Product Ownership Over All Consumers for the Years 2000 and 2004

**Figure 8** The Distribution of Product Ownership by Each Consumer Segment for the Years 2000 and 2004



roughly 30% of owners purchased within the last year, roughly 30% purchased in the last one to two years, roughly 16% purchased in the last two to three years, and so on. The key observation is that the mode of the distribution of ownership shifted. In 2000, a large number of consumers bought in the past two years, but in 2004, more consumers owned a product that was about three years old. Given that the mean replacement cycle length in this period is close to 3.5 years, we would expect a significant number of consumers to replace their products starting in 2005.

Figure 8 decomposes the distribution of ownership into the segments implied by the model. A comparison of the upper and lower graphs demonstrates the asymmetry in each segment's distribution of product ownership and replacement. In 2000, a large number of high-segment consumers (lower graph) had recently replaced their products, but in 2004, a larger number had replaced their products in the past two years. Given the estimates of the high segment's replacement cycle length, the high-segment consumers were unlikely to replace their products in

early 2005 because many had recently purchased new products.

The shape of the ownership distribution for the high-segment consumers should be contrasted with the distribution for the low-segment consumers (upper graph). In 2000, a large portion of low-segment consumers had recently purchased, but in 2004, many of the low-segment consumers had products between three and four years old. This implies that a disproportionately large number of low-segment consumers at the end of 2004 were increasingly likely to replace.

## 6. Managerial Implications

In high-tech markets, firms should consider the impact of their current decisions on the future distribution of product ownership and replacement cycles. A firm can increase sales today but often at the expense of reduced demand tomorrow. To mitigate this aspect of durable goods markets, firms often rely on their ability to continually reduce prices and improve quality. This approach represents an effective price discrimination strategy in the early stages of a market when most consumers are first-time adopters. Past research, however, has ignored the impact of such strategies on future product replacement behavior, which inevitably becomes the primary source of sales as a market matures.

Why do replacement cycles represent a useful dimension for segmenting consumer preferences? Consider the following illustrative example with two consumers. Assume Consumer A's last purchase was in 2005 and Consumer B's last purchase was in 2002. Lacking additional information, we know little about the likelihood that either will upgrade in the near term. Suppose Consumer A previously upgraded in 2002 and Consumer B upgraded in 1996. This reveals information about their respective replacement cycle lengths and their likely segment membership: given their replacement cycles and the timing of their purchases, Consumer A's behavior might be more consistent with an early innovator, whereas Consumer B's purchasing behavior might be more consistent with a late adopter. A model that ignores replacement might classify both consumers into the same segment. A model with replacement allows for the necessary heterogeneity in ownership that helps drive the timing of consumers' replacement decisions and lets the model more effectively separate consumers.

Whereas the model is specified at the aggregate level and thus is unable to target individual consumers, the intuition from the example above still applies to an aggregate setting. The model provides a broad tool that a firm can use to help guide its strategies, depending on the relative size and characteristics of each consumer segment and the potential

profitability of targeting these segments at the market or channel level. The results from §5.3.3 show that a manager can take advantage of variation in consumer replacement cycles across segments to adjust product introduction and pricing strategies. First, the distribution plots in Figure 8 suggest that a more value-oriented product could have been released in 2005 to target the replacement cycle of the low-segment consumers. Second, the menu of prices in the firms' product lines could have been adjusted: nonfrontier product prices could have increased and frontier product prices could have decreased. These actions could increase profit on the nonfrontier set of products while encouraging some low-segment consumers to upgrade to the frontier products.

The second implication relates more specifically to the PC market. Processor life cycles have shrunk over the years as Intel and AMD have strived to bring ever-faster processors to the market. The results from §5.3.2 show that not only has the replacement cycle for PCs increased but that more recent improvements in processor performance have not translated into upgrades as effectively as they once did.<sup>34</sup> This suggests that the processor manufacturers may not be able to rely on incremental quality improvements to spur consumers to replace their products at the same rate in the future as they did in the past. More significant changes in raw processor speed or innovation in other product dimensions may be needed. This does not imply that processor makers should reduce their research and development investments; instead, they should focus their efforts along product dimensions most likely to spur upgrades. Some recent evidence suggests that Intel and AMD have begun to pursue such a strategy, as both firms have shifted away from producing faster processors and toward creating processors with multiple cores and improved energy efficiency. The ability of Intel and AMD to effectively communicate the value of these new product dimensions to consumers—directly through marketing campaigns or indirectly through hardware manufacturers—will be critical for their ultimate success.

## 7. Conclusions, Limitations, and Future Research

Replacement sales inevitably surpass adoption sales as high-tech markets mature. This paper presents a structural model of dynamic demand that explicitly accounts for replacement and uncertainty about future product characteristics. The model helps provide an understanding of the impact of price and quality changes on consumer replacement behavior and provides a richer basis to study consumer

replacement cycles, planned obsolescence, and incentives to innovate in such markets. The results show that accounting for consumer heterogeneity in both preferences and product ownership can impact firms' strategies.

The analysis relies on several critical assumptions. Some assumptions were necessary because of data limitations, and others were done for modeling convenience. Relaxing some of these assumptions might be possible avenues for future research and could enrich our understanding of strategic consumer behavior in high-tech markets.

One set of assumptions simplified the consumer model by focusing attention on the purchase of a unidimensional product. First, the model ignores the role of computer software in a consumer's hardware purchase decision. A model that links a consumer's decision to purchase a computer with his software demand might be able to shed some light on the historical relationship between the hardware and software industries. Second, computers are multi-dimensional products, consisting of a processor, hard drive, random-access memory, multimedia drive, and other features. Modeling the consumer's choice of the entire PC could reveal additional insights for the firm in terms of which product characteristics have a relatively larger influence on a consumer's replacement.<sup>35</sup> Third, the number of households with multiple PCs has increased over the years as has consumer ownership of laptop computers. If suitable data were available, a model could examine the effects of existing PCs on replacement behavior and the potential substitution effects between desktop PCs and laptops.

Another set of assumptions were required because of the aggregate nature of the data. The model assumes that consumers are forward looking with preferences that are fixed. If individual-level data were available, a myopic model of consumers with dynamic preference parameters might yield different results while still being consistent with the nature of the data-generating process. Individual-level data would also allow for a more flexible structure of unobserved consumer heterogeneity, which could help relate demographic variables to replacement patterns.

Last, product prices and qualities are exogenous. The results of the policy simulations could change if the variables were able to adjust endogenously. We need an equilibrium model to fully control for the endogeneity of price and investment decisions. Goettler and Gordon (2009) construct a dynamic oligopoly model with durable goods and endogenous

<sup>34</sup> See Wilcox (2001) and Fisher (2004).

<sup>35</sup> One additional dimension is advertising. Intel provides substantial marketing support dollars to PC manufacturers to help promote the Intel brand, and this "ingredient branding" affects consumer purchase decisions.

innovation, and they apply it to the PC processor market with the goal of assessing the importance of competition in high-tech markets.

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## Appendix A

### A.1. Alternative Model of Consumer Expectations

This section presents an alternative formulation for consumer expectations that relaxes the independence assumption between changes in product qualities and prices. With a slight abuse of notation, let  $\psi_t = [\ln(P_t), \ln(P_{At}), \ln(Q_{It}), \ln(Q_{At})]'$  be the vector of log prices and qualities for each firm’s frontier product. Consumer expectations over frontier product qualities and prices follow a first-order VAR:

$$\psi_t = A_0 + A_1\psi_{t-1} + w_{t-1}, \quad w_{t-1} \sim N(0, \Sigma_w). \quad (A.1)$$

The system above describes the process governing frontier product characteristics. Following Erdem et al. (2003), I model the price and quality process for nonfrontier products as contemporaneous functions of the frontier price and quality. That is, the price and quality of Intel’s nonfrontier processor depend, respectively, on the price and quality of Intel’s frontier processor. Denoting the nonfrontier price and quality using lowercase, the expectations for firm  $k$  are given by

$$\ln(p_{kt}) = a_0 + a_1 \ln(P_{kt-1}) + u_{t-1}, \quad (A.2)$$

$$\ln(q_{kt}) = b_0 + b_1 \ln(Q_{kt-1}) + v_{t-1}, \quad (A.3)$$

where the errors are normally distributed.

Note that this alternative parameterization differs from the benchmark approach in two ways. First, this approach explicitly models the dependency between each firm’s frontier price and quality, allowing for both correlations within a firm and across the firms. The VAR still captures the effects of competition on frontier prices and qualities. Second, the number of products per firm  $J_k$  can be increased without changing the size of the state space and can be made firm specific. I use ordinary least squares to estimate the parameters for each nonfrontier product.

The estimation procedure is the same as in the benchmark model. To facilitate model comparison, I fix the initial conditions—and thus the segment sizes—at the values

estimated in the benchmark. The estimates from this alternative specification can be found in Table 1 of the Technical Appendix, which can be found at <http://mktsci.pubs.informs.org>, under the heading *Model with Alternative Expectations*. The structural parameters exhibit minimal change under the alternative structure of consumer expectations. This finding suggests that both specifications are sufficiently flexible to be able to adequately capture the relevant dynamics of prices and qualities over time. The invariance of the results to the form of consumer expectations may stem from the use of aggregate data; results from a model with consumer-level data might be more sensitive to the way expectations are operationalized.

### A.2. Incorporating Aggregate Preference Shocks

The model assumes that consumers’ preferences for product quality are constant over time. However, the benefits associated with a unit of processor performance are likely to have changed. These changes could stem from discrete events (e.g., the release of new operating systems) or to continuous improvements in available software (e.g., the Internet). In either case, the evolving role of software suggests that over time a consumer may receive more utility from owning a processor (or, equivalently, a PC).

To capture aggregate changes in preferences for using a processor, suppose the quality coefficient  $\gamma$  follows a simple stochastic process:

$$\gamma_t = \gamma_0 + \gamma_{t-1} + u_t, \quad (A.4)$$

given  $\gamma_0 > 0$  and  $u_t > 0$ , where  $u_t$  is distributed i.i.d. log normal with single parameter  $\sigma_\gamma$ . To reduce the number of parameters the quality process introduces, I assume that the mean of the log normal is zero and estimate the variance. We can interpret the shock  $u_t$  as a permanent upward shift in consumers’ marginal willingness to pay for products of higher quality. Because the shock is permanent, we must include  $\gamma$  in the state space for the dynamic programming problem. The revised choice-specific value function for the consumer is

$$\begin{aligned} & \bar{V}_i(\gamma, q_{jk}, \tilde{q}_i, \mathbf{q}, \mathbf{p}) \\ &= u_i(q_{jk}, \gamma, \tilde{q}_i, \mathbf{q}, \mathbf{p}) \\ &+ \beta \int \log \left( \sum_{q' \in q' \cup q_{jk}} \exp\{\bar{V}_i(\gamma + u, q', q_{jk}, \mathbf{q}', \mathbf{p}')\} \right) \\ & \cdot \Pi(\mathbf{q}', \mathbf{p}' | \mathbf{q}, \mathbf{p}) dG(u). \end{aligned} \quad (A.5)$$

I reestimate the model using the above specification assuming two consumer segments, with a separate  $\sigma_{\gamma_i}$  for each segment. To facilitate comparison with the benchmark results, I fix the initial conditions—and thus the segment sizes—for this alternative model at those obtained from the benchmark. The estimates from this alternative specification can be found in Table 1 in the Technical Appendix, which can be found at <http://mktsci.pubs.informs.org>, under the heading *Model with Aggregate Shocks*.

The estimate for preference shock distribution parameter is significant for segment 1 at the 95% level, but the estimate is not significant for segment 2. The relatively low significance levels of these coefficients is not surprising given

the difficulty of estimating dynamic preference parameters in a forward-looking consumer model with aggregate data. Incorporating the aggregate preference shock reduces the quality preference parameter estimates by roughly 10%. The lower objective function values suggest that allowing consumer quality preferences to stochastically adjust over time adds some flexibility to the model, but this flexibility comes at the cost of increased modeling and estimation complexity. The resulting parameter estimates are not statistically different from those obtained with the benchmark two-segment dynamic model.

**Appendix B. Computational Details**

This section discusses the computational details associated with estimating the model. There are three issues to address: (1) integrating over the continuous price vector, (2) integrating over the quality vector, and (3) computing the approximation to the value function.

Computing the fixed point requires integrating over the continuous four-dimensional price vector. I use the randomization technique that Rust (1997) develops to compute an approximate value function.<sup>36</sup> After integrating out the idiosyncratic components of the utility function, I replace the continuous form of the value function with a discretized equivalent. Let  $P \subset \mathbb{R}_+^4$  be the compact space of price vectors, and let  $\{\tilde{p}_1, \dots, \tilde{p}_{N_p}\}$  be a set of  $N_p$  uniform random draws from  $P$ . I replace the original continuous transition function with the discrete probability densities  $\Pi_{N_p}(\tilde{\mathbf{p}}' | \tilde{\mathbf{p}})$  constructed using the normalization:

$$\Pi_{N_p}(\tilde{\mathbf{p}}'_a | \tilde{\mathbf{p}}) = \frac{\Pi(\tilde{\mathbf{p}}'_a | \tilde{\mathbf{p}})}{\sum_{i=1}^{N_p} \Pi(\tilde{\mathbf{p}}'_i | \tilde{\mathbf{p}})}, \tag{B.1}$$

which guarantees that the discretized probabilities are sufficiently smooth and sum to one.

A second issue is that the size of the quality state space is  $\bar{q}^{KJ}$ . The number of states becomes prohibitive for even relatively small values of  $\bar{q}$ . Because the quality state space contains many product market configurations that are not observed in the data, I normalize all the products in a period relative to the most technologically advanced product in a period,  $q_i^* = \max\{\hat{\mathbf{q}}_i\}$ . All other products are represented as discrete differences between the quality index and their actual product qualities. That is, we can represent a state in the quality state space using a combination of  $q_i^*$  and a set of nonnegative integers  $\omega_i \in \mathbb{N}^{KJ}$  such that the current product vector can be rewritten as  $\{q_i^* - \omega_{kit}\}$  for each  $k \in K$  and  $j \in J$ . Note that at least one element of  $\omega_i$  will be zero because this product will correspond to  $q_i^*$ .

To reduce the computational burden, I compute the model on a subset of the state space using a maximum bound of  $\bar{\omega}$ . This makes the size of the effective state space  $\bar{q} \cdot (\bar{\omega})^{KJ}$ , significantly smaller than the original size. In practice,  $\bar{\omega}$  corresponds to a difference in product quality of 200%, which is twice as large as the maximum difference in qualities observed in the data. I set  $\bar{q}$  to be sufficiently high such that solving the model at  $\max_i\{q_i^*\}$  is unaffected by the bound.

<sup>36</sup> Standard Monte Carlo integration techniques are not appropriate because the value function has to be evaluated at arbitrary random points, which may lie off the predefined state-space grid.

I use simulation to compute the integration over quality. First, for each  $j, k$ , and  $t$ , draw a normal random  $\hat{\varepsilon}_{\phi_{jkt}}$  based on the variance of the error estimated in Equation (7). Second, use  $\hat{\varepsilon}_{\phi_{jkt}}$  to compute  $\phi_0(q_{jkt}) \equiv \phi_0(q_i^* - \omega_{kit}) = \Pr(\phi(\omega_{kit}) = 0)$ . Third, draw a set of  $\hat{r}_{kjt} \sim U(0, 1)$  variables and compare them to the probabilities computed in the second step. If  $\hat{r}_{kjt} < \phi_0(q_{jkt})$ , then product  $q_{kjt}$  experienced a quality change in period  $t$ . Fourth, conditional on a change in product quality, draw  $M = 50$  random values  $\hat{\omega}_{kjitm} \sim Pois(\hat{\kappa}_{kj0}, \hat{\kappa}_{kj1}, \hat{\gamma}_{kj})$  from Equation (8) using  $\phi_0(q_{jkt})$  and the parameter estimates in Table 4. This sequence of creating the random quality draws is held fixed throughout the process of value function iteration.

Combining the approximations for price and quality above, denote  $\hat{V}_{i, N_p}(q_{jk}, \tilde{q}_l, q^*, \omega, \mathbf{p})$  as the discretized version of the product-specific value function for  $q_{jk} \in \mathbf{q} \cup \tilde{q}_l$ . This quantity can be computed based on the expectation over the product-specific value function:

$$\begin{aligned} \hat{V}_{i, N_p}(q_{jk}, \tilde{q}_l, q^*, \omega, \mathbf{p}) &= U_i(q_{jk}, \tilde{q}_l, \mathbf{s}) \\ &+ \beta \sum_{m=1}^M \sum_{a=1}^{N_p} \hat{v}_{i, N_p}(q_{jk}, \tilde{q}_l, q_m^*, \hat{\omega}_m, \tilde{\mathbf{p}}'_a) \Pi_{N_p}(\tilde{\mathbf{p}}'_a | \tilde{\mathbf{p}}), \end{aligned} \tag{B.2}$$

where  $q_{jk} = q^* - \hat{\omega}_{jk}$ ,  $q_m^* = q^* + \max_{k,j}(\hat{\omega}_{kjm})$ , and

$$\begin{aligned} \hat{v}_{i, N_p}(q_{jk}, \tilde{q}_l, q_m^*, \hat{\omega}_m, \tilde{\mathbf{p}}'_a) &= \log \left( \sum_{\omega' \in \hat{\omega}_m \cup q_{jk}} \exp\{\bar{V}_i(q', q_{jk}, q_m^*, \hat{\omega}_m, \tilde{\mathbf{p}}'_a)\} \right). \end{aligned} \tag{B.3}$$

The randomized Bellman approach is appealing because it does not require interpolation, and the random grid points are drawn once and then remain fixed at successive iterations.

Finally, I implement the random multigrid algorithm in Rust (1997) to solve the value function. The algorithm consists of a set of outer iterations  $k = 1, 2, \dots$ , where a number  $N^{(k)}$  of uniform random sample points  $\{\tilde{p}_1, \dots, \tilde{p}_{N^{(k)}}\}$  is drawn at each iteration independent of the sample drawn at previous iterations  $k - 1, k - 2, \dots$ . The basic idea is to start with relatively few sample points  $N^{(0)}$  at  $k = 0$  and to successively increase the number of sample points according to the rule  $N^{(k)} = 2^{2k} N^{(0)}$ . Within each outer iteration  $k$ , I take a number  $T(k)$  of successive approximation steps using the random Bellman operator  $\hat{\Gamma}_{N^{(k)}}$ . Let  $\hat{V}^{(k)}$  denote the value function produced after  $T(k)$  steps at outer iteration  $k$ . Using nonparametric regression, I form estimates of the value function at iteration  $k + 1$  based on the final values obtained from iteration  $k$ . This process leads to the recursion  $\hat{V}_i^{(k+1)} = \hat{\Gamma}_{N^{(k)}}(\hat{V}_i^{(k)})$ , where the starting point for the value function at iteration 0 is the maximum of the period utility function. Because  $\hat{\Gamma}_{N_p^{(k)}}$  is self-approximating, evaluating it at arbitrary points in the state space is possible without the need for interpolation. In practice, I set  $N^{(0)} = 100$  and find that the multigrid algorithm converges after three or four iterations.

I use a two-step GMM procedure to produce efficient standard error estimates. I use the nonderivative-based Nelder-Mead algorithm to get within a neighborhood of the optimal parameters and then switch to a quasi-Newton method. I also perform a check of the numerical condition for local

identification. Let  $\hat{m}_i^s(\theta)$  be a subvector of  $m_i(\theta)$  such that  $\dim(\hat{m}_i^s) = \dim(\theta)$ . Then, a local identification condition requires that  $\det(\partial \hat{m}_i^s / \partial \theta) \neq 0$ . Roughly interpreted, if the determinant of the Jacobian is nonzero, then the moments  $m_i$  are informative about the structural parameters  $\theta$ .

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