Leverage Dynamics under Costly Equity Issuance[*]

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June 29, 2021

Abstract

We propose a parsimonious model of leverage and investment dynamics featuring a jump-diffusion cash-flow process, retained earnings, short-term debt, and external equity. Crucially equity issuance is costly. We show that firms' efforts to avoid incurring equity issuance costs generate empirically plausible target leverage and nonlinear leverage dynamics. Paradoxically, it is the fixed cost of equity issuance that causes the firm to keep leverage low, in contrast to the predictions of Modigliani-Miller and Leland tradeoff and Myers' pecking-order theories. The marginal source of external financing on an on-going basis is debt, but when leverage gets too high, the firm optimally deleverages by issuing equity at a cost. When leverage is low, it tends to revert to the target, but when leverage is high, the firm is caught in a debt death spiral. When the firm is at its target leverage, profits are paid out, but losses cause leverage to drift up. When leverage is high and the firm is hit by a large jump loss, it defaults.

Keywords: tradeoff theory, pecking order, financial slack, costly equity issuance, costly default, risk seeking, q theory of investment

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1 Introduction

In his AFA presidential address DeMarzo (2019) states that "Capital structure is not static, but rather evolves over time as an aggregation of sequential decisions." In this paper we take a similar dynamic perspective to capital structure choices and propose a tractable dynamic model that accounts for CFO's top four considerations in determining their leverage choices according to Graham and Harvey's (2001) survey: 1.) financial flexibility, 2.) credit rating, 3.) earnings and cash flow volatility, and 4.) insufficient internal funds. The key driving force in our model is external equity financing costs.

To demonstrate how costly external equity financing fundamentally alters empirical predictions, we proceed in two steps: first developing a tradeoff model with costless equity financing and then incorporating the cost of equity issuance.

As in standard tradeoff models, debt has a funding advantage over equity (DeMarzo, 2019)[1] but may cause financial distress as in Modigliani and Miller (1961) and Leland (1994).[2] Importantly, our costless-equity-issuance model differs from Leland (1994) in two key aspects.

First, debt is short term as in Abel (2018) and many portfolio choice and asset pricing models following Merton (1971) and Black and Scholes (1973). As emphasized by Abel (2018), since short-term debt can be adjusted continuously, it makes salient the recurrent nature of the debt financing decision, in contrast to the once-and-for-all debt financing decision in Leland (1994) and occasional debt issuance decisions in Goldstein, Ju, and Leland (2001), where debt issuance is costly. Our risky short-term debt corresponds to the one-period debt in discrete-time models (e.g., Hennessy and Whited, 2007).

Second, we model the firm's earnings before interest and taxes (EBIT) process by using a jump-diffusion process.[3] Downward EBIT jump shocks can cause default and thus induce positive credit spreads in equilibrium *ex ante* even when debt is short term. Our EBIT process is non-stationary and nests the geometric Brownian motion (GBM) process widely used in the contingent-claim literature (e.g., Fischer, Heinkel, and Zechner, 1989; Leland, 1994; and Goldstein, Ju, and Leland, 2001) as a special case. Additionally, jumps generate

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[1]We follow his work to allow for two sources for debt funding advantages: cheaper cost of capital (when shareholders are more impatient than creditors) and tax benefit of debt.

[2]As in the literature, we assume that shareholders are protected by limited liability and can declare default at any time. The absolute priority rule (APR) applies when the firm defaults. Creditors liquidate the firm and collect the liquidation recovery value, which is assumed to be a fraction of unlevered firm value as in Leland (1994). Creditors ex ante price the firm's default risk.

[3]Malenko and Tsoy (2021) use the same EBIT process with downward jumps to study optimal time-consistent debt policies.

negatively skewed and fat tailed EBIT growth as observed in the data and allow us to better calibrate our model to data.

We show that in our tradeoff model with no equity issuance costs, the firm optimally keeps its leverage at a constant target at all time until it chooses to default. This target leverage strategy is implemented by continuously issuing equity when making losses and paying out dividends when making profits, as in standard contingent-claim capital structure models. However, these continuous and active equity (issuance and payout) adjustments are empirically counter-factual. Firms rarely issue equity and when they do, they issue lumpy amounts (Donaldson, 1961; Shyam-Sunder and Myers, 1999).

In order to generate empirically plausible joint dynamics of leverage, equity issuance, and payout, we deviate from Leland (1994) and the contingent-claim capital structure literature by incorporating costly equity issuance into our costless-equity-issuance tradeoff model introduced above. Our model with costly external equity financing generates the following results and predictions on optimal capital structure and leverage dynamics.

First, our model generates a dynamic pecking-order prediction in that the firm prefers using internally generated cash flows (retained earnings) first, then external debt, and finally equity as the last resort (Myers, 1984, and Myers and Majluf, 1984). When issuing equity, the firm significantly deleverages its balance sheet and afterwards reverts to using retained earnings and debt financing. This process continues until a jump shock arrives that causes a sufficiently large EBIT loss that the firm chooses to default. The firm's default decision depends on both its pre-jump-arrival leverage and the realized EBIT loss.

Second, we show that depending on the level of its debt-EBIT ratio,, the firm is in one of four mutually exclusive regions separated by three endogenous thresholds: 1.) the *payout region*, where the firm borrows to pay dividends so as to reach its target leverage; 2.) the *debt financing region*, where the firm exclusively relies on retained earnings and debt financing to manage its leverage dynamics; 3.) the *equity issuance region*, where the firm issues equity to deleverage; and 4.) the *default region*, where it is optimal for shareholders to default.

Third, we show that the higher the equity issuance cost, the lower is target leverage, which seems paradoxical. Higher equity issuance costs, far from encouraging the firm to rely more on cheaper debt financing, make the firm prudent with its debt policy. This is because taking on too much debt increases the likelihood that the firm, after incurring persistent losses, is forced to issue costly external equity to deleverage. Moreover, increasing equity issuance costs also lowers the firm's debt capacity which in turn reduces its financial

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flexibility. To minimize the likelihood of getting itself into this "debt death spiral," the firm rationally chooses a prudent level of target leverage to preserve its financial flexibility. This is why equity issuance costs are key to explaining the relatively low observed corporate leverage. Our dynamic model challenges the conventional reasoning that the firm should rely more on the cheaper source of external financing.

Fourth, we show that target leverage coincides with the firm's payout boundary. At its inception, the firm maximizes its value by raising just enough debt and distributing the proceeds to shareholders so that the firm's (endogenous) marginal value of debt equals the marginal cost of raising debt. Therefore, the optimality condition for target leverage is the same as the one for the optimal payout.

However, the firm does not stay at its target leverage at all time. After setting its leverage at the target level, a negative EBIT (diffusion or jump) shock decreases the firm's enterprise value and mechanically increases leverage.[4] The only way for the firm to bring its leverage back to its target level is to issue equity. However doing so in response to a small EBIT loss is suboptimal because equity issuance is costly. The standard option value of waiting reasoning implies that inaction (for equity issuance and payouts) is optimal for a range of values of the debt-EBIT ratio (our state variable), provided that leverage is not too high.

Fifth, our model generates persistent, highly nonlinear leverage dynamics and provides an explanation for the leverage-profitability puzzle.[5] In the debt-financing region where the firm's leverage is not too high, the firm behaves as a credit-card revolver by passively reacting to EBIT shocks: leverage ramps up following a negative EBIT shock and decreases following a positive shock.[6] When the firm's EBIT is hit by a sufficiently large downward jump shock,

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[4]If the firm receives a positive EBIT shock at its target, it pays out just enough dividends so that its leverage stays at its target.

[5]See Titman and Wessels (1988), Myers (1993), and Rajan and Zingales (1995) among others, on the leverage-profitability puzzle. Fama and French (2002), Leary and Roberts (2005), and Lemmon, Roberts, and Zender (2007) show that leverage is persistent, and that firms tend to increase leverage if it is below its target leverage, and decrease leverage when it is high.

[6]Denis and McKeon (2012) find that the evolution of a firms leverage ratio depends primarily on whether or not it produces a financial surplus and firms tend to cover their deficits predominantly with more debt even though leverage ratios for these firms are already well above their target levels. DeAngelo, Gonçalves, and Stulz (2018) show that firms pay down their debt when they receive a positive earnings shock and increase their debt when they have no choice to do otherwise. Their main conclusion is that "Debt repayment typically plays the main direct role in deleveraging." Our results are also consistent with the findings of Korteweg, Schwert, and Strebulaev (2019), who show that firms tend to cover operating losses by drawing down a line of credit, giving rise to similar leverage dynamics as in our model. Over a longer time horizon our leverage dynamics are also consistent with the findings of DeAngelo and Roll (2015), who emphasize that leverage is far from time invariant.

passively rolling over debt is no longer optimal. Instead, the firm could find itself in the equity-financing region and tap costly external equity to reduce its leverage.[7] As a result, the firm's financial situation is much improved but old shareholders are significantly diluted.[8]

Sixth, we show that the firm can be either endogenously risk averse or risk seeking depending on the level of its debt-EBIT ratio. When leverage is low or moderate, leverage drifts towards the target level and firm value is concave in the debt-EBIT ratio, as the firm is averse to costly external equity financing– this is a standard result for firms facing costly external financing. In contrast, when leverage is sufficiently high, the firm becomes a risk seeker as in Jensen and Meckling (1976) and firm value is convex in the debt-EBIT ratio, which is caused by the firm's incentive to economize on the fixed cost of issuing equity. Rather than reverting to the target, leverage diverges into a debt death spiral. This convex firm value region has received relatively less attention in the dynamic corporate finance literature.[9] In sum, even though the firm spends very little time in the equity-issuance region, the option of issuing costly equity makes leverage dynamics and marginal value of debt in the debt-financing region highly nonlinear and non-monotonic.

Seventh, recapitalization target, to which the firm returns after issuing equity, is the same regardless of the pre-equity-issuance level of debt, because the marginal cost of equity issuance is constant. Moreover, the firm's recapitalization target is higher than its target leverage. This is because external equity is the marginal source of financing for the recapitalization decision but debt is the marginal source of financing for the firm's target leverage choice. Since external equity is more costly than debt, the two optimality conditions imply that the firm's recapitalization target must be higher than its target leverage.

We extend our analysis by endogenizing the EBIT process with a capital accumulation and adjustment cost technology from the neoclassical q theory of investment (e.g., Hayashi, 1982, and Abel and Eberly, 1994) to study the joint investment and leverage dynamics. This generalized q model delivers highly nonlinear and non-monotonic predictions about corporate investment, marginal q, and average q, challenging the neoclassical q theory.

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[7]This result is consistent with findings of Fama and French (2005): Firms tap equity markets before exhausting its financing capacity.

[8]DeAngelo, DeAngelo, and Stulz (2010) find that when the firm issues equity in order to delever, existing shares are highly diluted.

[9]Hugonnier, Morellec, and Malamud (2015) show that firm value can be concave or convex depending on the level of cash holdings in a model with lumpy investment and capital supply uncertainty. Della Seta, Morellec, and Zucchi (2020) develop a model showing that short-term debt and rollover losses can cause risk-taking when firms are close to financial distress.

When leverage is low or moderate, the firm is endogenously risk averse and investment decreases with leverage. This is due to debt overhang (Myers, 1977) caused by the firm's liquidity management considerations. But when leverage is high firm value is convex in leverage and investment increases with leverage. This risk seeking result (Jensen and Meckling, 1976) is due to the firm's incentive to economize on the fixed equity issuance cost. Additionally, in contrast to the predictions of the neoclassical q theory under perfect capital markets, investment and marginal q move in exactly the opposite direction with leverage in both concave and convex firm value regions, because of fixed equity issuance costs.[10] This is because investment is determined by the ratio between marginal q and the marginal cost of debt financing, both of which move in the same direction with leverage, and moreover, the marginal cost of debt is more sensitive to leverage than marginal q.

Our calibrated model (using COMPUSTAT data) suggests that EBIT growth is subject to both substantial downward jump and diffusion risks. A jump is expected to arrive once every ten months and each jump on average causes an expected 13% reduction in EBIT. And the diffusion volatility of EBIT growth is about 40% per annum. The fixed equity issuance costs are economically significant and are necessary to explain infrequent equity issuances (which is about once every fourteen years for COMPUSTAT firms.) With the calibrated parameter values, our model predicts an average market leverage of 24%, within the range of estimates reported in Strebulaev and Whited (2011), a standard deviation of 12%, and only 1% of firms with market leverage higher than 63%.

We also find that when equity issuance is too costly, so that it does not ever issue any equity, then the firm chooses a leverage target very close to zero, suggesting an explanation for the zero-leverage puzzle (Strebulaev and Yang, 2013). Even a small fixed equity issuance cost generates a wide equity-inaction region and significantly lowers target leverage and average leverage. Furthermore, target leverage is highly sensitive to cash flow risk, in particular jump risks. However, we find moderate effects of taxes on target leverage and essentially no effects of changes in liquidation recovery value on target leverage. These results are consistent with the survey findings of Graham and Harvey (2001).

Related literature. Our emphasis on costly equity financing is a key departure from the contingent-claim capital structure theory literature following Fischer, Heinkel, and Zech-

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[10]Bolton, Chen, and Wang (2011) derive a similar result in the region where the firm uses line of credit.

ner (1989) and Leland (1994, 1998).[11] Goldstein, Ju, and Leland (2001) generalize Leland (1994) to allow for dynamic recapitalization with costly debt issuance. Strebulaev (2007) emphasizes the role of liquidity ratios on dynamic capital structure decisions. In effect, this literature typically parameterizes term debt with a geometric amortization schedule (including perpetual risky debt in Leland (1994) as a special case) and assumes that this term debt is costly to adjust but equity is costless to issue. Therefore, the marginal source of financing in these models on a on-going basis is by assumption equity and the firm retains no earnings, continuously issues equity and makes payouts to shareholders, which are counterfactual.

Abel (2018) develops a dynamic tradeoff model with short-term debt and a stationary EBIT process that is constant for a random duration of time and then changes to a new value (drawn from a given time-invariant distribution) at a stochastic moment governed by a Poisson process. As in Leland (1994), The firm can issue equity at no cost and hence there is no need for it to retain earnings. The costless-equity-issuance version of our model is closely related to Abel (2018) as debt is short-term and default is triggered by sufficiently large downward EBIT jumps in both models. A key difference is that the EBIT process is non-stationary in our costless-equity-issuance model but stationary in Abel (2018). Also our main model focuses on how costly external equity determines leverage dynamics.

Our work is also related to the debt ratcheting literature. DeMarzo (2019) and DeMarzo and He (2020) analyze equilibrium leverage dynamics in Leland-style trade-off models in which the firm can continuously adjust leverage but cannot commit to a policy ex ante.[12] They show that firms never choose to actively reduce leverage. Moreover, the firm's lack of commitment and incentives to dilute existing debt dissipate all value gains from trading in equilibrium (the intuition is similar to the one for Coase conjecture in the context of durable-goods monopolists.) Unlike these models which feature non-exclusive debt, our model features short-term debt and hence there are no debt dilution incentives. Also, debt issuance in their model is smooth but is stochastic in our model.[13]

Cooley and Quadrini (2001), Gomes (2001), Hennessy and Whited (2005, 2007), Gamba and Triantis (2008), and DeAngelo, DeAngelo, and Whited (2011) develop discrete-time dynamic capital structure models with investment. An important methodological difference of our analysis is the continuous-time formulation of the firm's problem. Our continuous-time

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[11]Kane, Marcus, and McDonald (1984) is an early important contribution.

[12]DeMarzo, He, and Tourre (2021) analyze the debt ratcheting problem in a sovereign debt context.

[13]Note that shrinking the debt maturity in their models to zero yields a different type of short-term debt from the one in our model.

formulation allows for a sharper characterization of the underlying economic tradeoffs and of the firm's highly nonlinear, non-monotonic, state-contingent, path-dependent leverage, payout, equity issuance, and corporate investment policies in the four mutually exclusive interior regions including various endogenous boundaries.[14]

Additionally, our model features constant returns to scale and capital adjustment costs, and therefore predicts that firm size (e.g., capital stock) grows exponentially with a stochastic, endogenous drift. Exponential growth together with endogenous default (firm exit) allows our model to generate empirically observed a fat-tailed distribution (power law) for firm size.[15] which is challenging for decreasing-returns-to-scale-based investment models to generate. Our model is complementary to these discrete-time models.

Strebulaev and Whited (2011) survey the two literatures "Dynamic Contingent Claims Models" and "Discrete-time Investment Models" on dynamic corporate policies. We generate empirically plausible predictions and new insights about leverage and investment dynamics in a tractable model by integrating key building blocks and insights from these two literatures.

Our paper is also related to the continuous-time corporate liquidity and risk management literature, e.g., Decamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011), Hugonnier, Malamud and Morellec (2015), and Abel and Panageas (2020). These papers focus on cash and corporate liquidity management but not leverage dynamics.[16]

More broadly, our model is related to the dynamic contracting and optimal dynamic security design literature, e.g., DeMarzo and Sannikov (2006), Biais, Mariotti, Plantin, and Rochet (2007), DeMarzo and Fishman (2007), Biais, Mariotti, Rochet, and Villeneuve (2010), and Malenko (2019).[17] The optimal contracts in these models can be implemented via a combination of debt, (inside and outside) equity, and corporate liquidity.

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[14]Moreno-Bromberg and Rochet (2018) provide a textbook treatment of continuous-time finance, banking, and insurance. In their survey, Brunnermeier and Sannikov (2016) discuss the advantages of continuous-time modeling in dynamic contracting and macro-finance contexts.

[15]Luttmer (2007) develops a tractable model of balanced growth consistent with observed size distribution of firms. Gabaix (2009) and Luttmer (2010) survey the literature on power law and firm dynamics.

[16]When they include debt, these models assume an exogenous debt capacity with risk-free debt. There is no notion of target leverage nor of nonlinear/nonmonotonic leverage dynamics, e.g., Bolton, Chen, and Wang (2011). In our model, both debt capacity and credit spreads are endogenously determined, and the firm has a target leverage.

[17]Biais, Mariotti, and Rochet (2013) and Sannikov (2013) provide recent surveys on this subject.

2 Model

We begin by introducing the process for the firm's earnings before interest and taxes (EBIT). We then describe the firm's financing choices and its dynamic financial policy problem. We complete the model description by formulating the firm's recursive optimization problem.

Earnings process. The firm's EBIT Y_t evolves according to the following geometric jump-diffusion process:

$$\frac{dY_t}{Y_{t-}} = \mu dt + \sigma d\mathcal{B}_t - (1-Z)d\mathcal{J}_t, \quad Y_0 > 0, \tag{1}$$

where μ is the drift parameter, σ is the diffusion-volatility parameter, $\mathcal{B}$ is a standard Brownian motion process, and $\mathcal{J}$ is a pure jump process with a constant arrival rate λ. We denote the sequence of independent jump arrival moments by $\{T^{\mathcal{J}}\}$. This process (1) is widely used in the macro-finance rare-disasters literature to model aggregate consumption or GDP.[18]

This jump-diffusion process generalizes the geometric Brownian motion process commonly used in the contingent-claim capital structure literature, e.g., Fischer, Heinkel, and Zechner (1989), Leland (1994), and Goldstein, Ju, and Leland (2001).[19] Since the diffusion shock $\mathcal{B}$ is continuous, if a jump does not occur at date t ($d\mathcal{J}_t = 0$), we have $Y_t = Y_{t-}$ (where $Y_{t-} \equiv \lim_{s\uparrow t} Y_s$ denotes the left limit of the firm's earnings).

If a jump arrives at date t ($d\mathcal{J}_t = 1$), EBIT changes from Y_{t-} to $Y_t = ZY_{t-}$, where $Z \in [0,1]$ is a random variable with a well-behaved cumulative distribution function $F(\cdot)$.[20] Since the expected percentage EBIT loss upon a jump arrival is given by $(1 - \mathbb{E}(Z))$, the expected EBIT growth rate, g, is given by:

$$g = \mu - \lambda(1 - \mathbb{E}(Z)). \tag{2}$$

Equity and debt investors. We assume that capital markets are perfectly competitive and that all investors (debt and equityholders) are risk neutral.[21] However, the firm faces

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[18]This framework has proved useful in modeling various asset-pricing and macroeconomic phenomena. Examples include Rietz (1988), Barro (2006), Barro and Jin (2011), Bhamra and Strebulaev (2011), Gabaix (2012), Gourio (2012), Wachter (2013), and Rebelo, Wang, and Yang (2021), among others.

[19]Malenko and Tsoy (2021) use the same EBIT process with downward jumps to study optimal time-consistent debt policies.

[20]Since the focus of our model is on how jumps can generate default risk we only consider downward jumps for simplicity. We could also allow for upward earnings jumps. We leave this extension out for brevity.

[21]We can generalize our model by allowing investors to be risk averse and well diversified. For example, we can account for investors' risk aversion by using the stochastic discount factor (SDF) to price the firm's free cash flows. For brevity, we leave this extension out.

external financing costs. As in DeMarzo and Sannikov (2006), Biais, Mariotti, Plantin, and Rochet (2007), DeMarzo and Fishman (2007a, 2007b), DeMarzo (2019), and others, we assume that the firm's shareholders are weakly impatient relative to debt investors. Shareholders discount future payouts at the rate of γ, larger than the risk-free rate r: $\gamma \geq r > 0$. The wedge $\gamma - r$ describes in a simple way the idea that debt is a cheaper source of financing than equity and generates a meaningful payout policy.[22] Before specifying the firm's financing choices and financial distress costs, we introduce the first-best benchmark with no financial distress.

First-best benchmark. As being (weakly) more patient than equity investors, debtholders value the firm the most. Let $\Pi(Y)$ denote the present value of expected EBITs under first best:

$$\Pi(Y) = \pi Y, \tag{3}$$

where π is the EBIT multiple:

$$\pi = \frac{1}{r-g}. \tag{4}$$

Equations (3)-(4) map to a standard Gordon growth valuation model, where g is the expected growth rate of EBIT given in (2). To ensure that $\Pi(Y)$ is finite, we require $r > g$.

Financing choices. The firm's financing choices involve both internal and external sources of funds. Internal funds come from accumulated retained earnings. External funds are short-term debt, which we assume to be costless for simplicity,[23] and costly equity. Let X_t denote the firm's debt position at time t. We assume that debt is short-term, as in most discrete-time dynamic corporate finance models (e.g. Hennessy and Whited, 2007), asset pricing, and macro finance.[24]

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[22]This impatience assumption could be preference based or could arise indirectly because shareholders have other attractive investment opportunities. This relative impatience makes shareholders prefer early payouts, *ceteris paribus*.

[23]While in practice firms incur transaction costs in issuing debt or securing a line of credit, these costs are small relative to the costs firms incur when they issue equity. What is important for our analysis is that equity issuance costs are higher than debt issuance costs, not whether debt issuance is costly or not. This is why we set debt issuance costs to zero for simplicity.

[24]Black and Scholes (1973) and Merton (1973) use time-varying, short-term, risk-free debt positions in a replicating portfolio to value a call option on a stock. In Merton (1971) the investor optimally holds a levered market portfolio position financed with short-term, risk-free debt (when her risk aversion is sufficiently low or Sharpe ratio is sufficiently high). Lucas (1978) and Breeden (1979) assume that debt is short term in their equilibrium asset pricing models. Kiyotaki and More (1997) and Brunnermeier and Sannikov (2014) also use short-term debt.

Short-term creditors may be exposed to default risk because EBIT is subject to downward jump shocks which are not hedgeable. Thus, following a sufficiently bad loss the firm's shareholders may default on their debt obligations. Of course, creditors in equilibrium rationally price this default option held by shareholders.

Let $T^{\mathcal{D}}$ denote the firm's optimal default timing and $\mathbf{1}_t^{\mathcal{D}}$ be an indicator function that takes the value of one if and only if the firm defaults at date t and the value zero otherwise ($\mathbf{1}_t^{\mathcal{D}} = 1$ if and only if $t = T^{\mathcal{D}}$). We assume for simplicity that following a default the firm is liquidated and that the absolute priority rule (APR) holds in bankruptcy whereby creditors are repaid before shareholders. Let $L_{T^{\mathcal{D}}}$ denote the firm's liquidation value at the moment of default $T^{\mathcal{D}}$. Because the APR applies in bankruptcy and liquidation is inefficient, equityholders only default if it is no longer in their interest to keep the firm going; that is, when the firm's equity value is equal to zero. Creditors then receive the entire liquidation proceeds at $t = T^{\mathcal{D}}$, and the loss given default for creditors is $(X_{t-} - L_t)$, the difference between promised repayment X_{t-} and liquidation value L_t.

We model the deadweight losses caused by bankruptcy as in Leland (1994) by specifying that the liquidation recovery value at $T^{\mathcal{D}}$ is equal to a fraction $\alpha \in (0,1)$ of $\Pi(Y)$, the firm's enterprise value under the first-best given in (3): That is, $L_{T^{\mathcal{D}}}$ is given by:

$$L_{T^{\mathcal{D}}} = \alpha \Pi(Y_{T^{\mathcal{D}}}). \tag{5}$$

We may write equivalently that the firm's liquidation recovery value $L_{T^{\mathcal{D}}}$ as

$$L_{T^{\mathcal{D}}} = \ell Y_{T^{\mathcal{D}}}, \tag{6}$$

where

$$\ell = \alpha \pi. \tag{7}$$

As long as α is not too high, corporate bankruptcy causes deadweight losses and we have a meaningful dynamic tradeoff theory with benefits and costs of debt.

Debt pricing: credit spreads. Let C_{t-} denote the contractually agreed interest payments for the short-term debt issued at date $t-$. We write C_{t-} as

$$C_{t-} = (r + \eta_{t-})X_{t-}, \tag{8}$$

where r is the risk-free rate and η_{t-} is a time-varying equilibrium credit spread. To compensate debtholders for the credit losses they may bear if the firm defaults at t, the equilibrium

credit spread η_{t-} must satisfy the following zero-profit condition for creditors:

$$X_{t-}(1 + rdt) = (X_{t-} + C_{t-}dt)\left[1 - \lambda\mathbb{E}_{t-}\left(\mathbf{1}_t^{\mathcal{D}}\right)dt\right] + \mathbb{E}_{t-}\left(L_t\,\mathbf{1}_t^{\mathcal{D}}\right)\lambda dt\,. \tag{9}$$

The first term on the right side of (9) is equal to the product of the total contractual repayment to creditors, $(X_{t-} + C_{t-}dt)$, and the probability $\left[1 - \lambda\mathbb{E}_{t-}\left(\mathbf{1}_t^{\mathcal{D}}\right)dt\right]$ at time $t-$ that the firm won't default at time t. The second term gives the creditors' expected payoff if the firm defaults. In sum, the equilibrium condition (9) states that creditors' expected rate of return (including both default and no-default scenarios) is equal to the risk-free rate r, which is captured by the left side of (9).

Corporate Taxes. We next introduce the standard tax benefit of debt: Interest payments are tax deductible at the corporate level. The detailed interest tax exemption rules can be quite complicated.[25] To simplify the tax schedule, we only incorporate corporate taxes and leave out personal debt and equity income taxation out of the model.[26] We also ignore tax loss carryforward.

We specify the firm's tax payment (Θ_t) as a function of its interest obligations (C_t) and its EBIT (Y_t): $\Theta_t = \Theta(C_t, Y_t)$ that satisfies the following homogeneity property:

$$\Theta(C_t, Y_t) = \theta(c_t)Y_t\,, \tag{10}$$

where $c_t = C_t/Y_t$ and $\theta(c)$ is decreasing in c.

Next, we introduce an important special case of (10), which we use in our quantitative analysis in Section 5. The firm pays income taxes at a constant rate $(\tau > 0)$, when malking a profit $(Y_t > C_t)$, and pays no taxes when i incurring a loss $(Y_t \leq C_t)$. This specification captures asymmetric tax treatments for profits and losses in a simple way. The scaled tax payment is:

$$\theta(c) = \tau(1 - c)\mathbf{1}_{c<1}\,, \tag{11}$$

where $\tau > 0$ is the constant profit tax rate and $\mathbf{1}_{c<1} = 1$ if and only if $c < 1$ and zero otherwise.

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[25]Hennessy and Whited (2005) model taxes with a richer, more realistic schedule.

[26]In their influential survey, Graham and Harvey (2001) find very little evidence that firms directly consider personal taxes.

Costly external equity issuance. Firms rarely issue external equity, and when they do, they incur significant costs, due to a combination of asymmetric information, managerial incentive issues, and underwriting costs. For convenience we capture adverse selection costs, underwriting fees, and other floatation costs with a simple reduced form function. A large empirical literature has sought to quantify these costs, including the costs arising from the negative stock price reaction in response to the announcement of a new equity issue.[27] Building on the findings of this literature, we assume that the firm incurs both fixed and proportional costs when it issues equity. Fixed costs are what generates the observed infrequent and lumpy equity issuance.

Let N_t denote the firm's (undiscounted) cumulative net amount of external equity financing up to time t, and H_t denote the corresponding (undiscounted) cumulative costs of external equity financing up to time t. To preserve the model's homogeneity we further assume that the fixed cost is proportional to the firm's earnings Y_t, so that $h_0 Y_t$ denotes the total fixed equity-issuance cost. This form of fixed equity-issuance costs ensures that the firm never grows out of the fixed cost, preserving the first-order consideration of equity issuance costs . The firm also incurs variable equity-issuance costs: For each dollar the firm raises it incurs a marginal cost of h_1. Therefore, when raising net proceeds M_t, the firm incurs a total equity issuance cost of $h_0 Y_t + h_1 M_t$. Hennessy and Whited (2007) allow for fixed, proportional, and convex costs and find that the fixed and proportional costs are the more important ones.

Equity payouts. In addition to managing debt and equity issuance, the firm also decides if and when to make distributions to its shareholders. There is an economically meaningful payout policy in our model given that shareholders are *impatient* compared with creditors ($\gamma \geq r$) and that debt financing has tax advantages. We denote by U_t the firm's cumulative (nondecreasing) payout to equityholders up to time t. Equityholders are protected by limited liability at all time, so that payouts to them cannot be negative at any time t. This means that dU_t, the incremental payout over time interval dt, has to be non-negative. We show that the optimal target leverage, a key concept in capital structure theory, is closely tied to the firm's payout to shareholders.

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[27] Altinkilic and Hansen (2000) estimate underwrite fee schedules. Asquith and Mullins (1986) measure the indirect costs of external equity using seasoned equity offerings announcements. Hennessy and Whited (2007) use simulated method of moments to infer the magnitude of financing costs and their estimates support the view that external equity issuances involve large indirect costs and firms are sensitive to these costs.

Retained earnings and debt dynamics. Since it is costly to issue equity the firm has an incentive to maintain a liquidity buffer. This way it avoids having to return to equity markets too often. When the firm is solvent (when $t < T^{\mathcal{D}}$), its debt level X_t evolves according to the following law of motion:

$$dX_t = (C_{t-} + \Theta_{t-} - Y_{t-})\, dt - dN_t + dU_t - \mathbf{1}_t^{\mathcal{D}}\,(X_{t-} - L_t)\, d\mathcal{J}_t. \tag{12}$$

The first term on the right side of (12) describes how debt evolves absent equity issuance, payout, and default: if the firm's interest and tax payments $(C_{t-} + \Theta_{t-})$ exceed its contemporaneous EBIT Y_{t-}, the firm's debt balance X_t increases by the net amount: $C_{t-} + \Theta_{t-} - Y_{t-}$. This term captures the firm's behavior under normal circumstances. It pays down debt if making profits and builds up debt otherwise, behaving as a credit card revolver. Importantly, the interest rate at which the firm revolves its credit is endogenously determined.

The second term dN_t describes how the firm may use equity issuance to recapitalize its balance sheet: the firm's net debt decreases one to one in response to its net equity issuance dN_t. The third term dU_t shows how distributions to shareholders increase its debt. Firms typically respond to profits and losses by adjusting their debt level, and only occasionally use payouts $(dU_t > 0)$ or equity issuance $(dN_t > 0)$ given that raising equity is costly.[28] Moreover, it is never optimal to be simultaneously active on the equity issuance and payout margins since equity financing is costly. Payout (when $dU_t > 0$) and equity issuance (when $dN_t > 0$) are two very different margins.

The last term in (12) captures the consequence of bankruptcy. When a sufficiently large jump loss arrives at t (i.e. $t = T^{\mathcal{D}}$ and $d\mathcal{J}_t = 1$) it causes the firm to default (i.e. $\mathbf{1}_t^{\mathcal{D}} = 1$), so that its debt level decreases from its pre-default value X_{t-} to its liquidation value L_t given in (3). Combining all four margins, earnings retention, equity issuance, equity payouts, and default, we obtain a complete description of the firm's debt dynamics. In contrast, without equity issuance costs, debt is not a state variable but rather a control variable as we show in Section 3.

Optimality. The firm chooses: i) incremental payouts to shareholders $(dU_t \geq 0)$; ii) net external equity issuance $dN_t \geq 0$; and, iii) the default timing $T^{\mathcal{D}}$ to maximize shareholder

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[28]In contrast, in Leland (1994) and later models without equity issuance costs, the firm continuously engages in equity issuance and payouts. Only the free cash flow $(dU_t - dN_t)$ is economically meaningful in these models. Absent default, $dU_t - dN_t = (Y_{t-} - C_{t-})\, dt$ and the gross flows dU_t and dN_t are indeterminate.

value:

$$\mathbb{E}_t \left[\int_t^{T^{\mathcal{D}}} e^{-\gamma(s-t)} \left(dU_s - dN_s - dH_s \right) \right], \tag{13}$$

subject to debt dynamics given in (12), the equilibrium credit risk pricing equation (9), and a transversality condition. Note that since equity issuance is costly ($dH_t > 0$ whenever $dN_t > 0$) we need to subtract the equity issuance cost dH_t in (13). The optimal earnings retention are implied by debt dynamics given in (12). Since APR applies in bankruptcy, shareholders receive nothing upon default, which explains why there is no payoff to shareholders from time $T^{\mathcal{D}}$ onward.

Value function and homogeneity property. Both EBIT Y_t and debt X_t are state variables for the optimization problem, so that the firm's equity value P_t is given by the function $P(Y_t, X_t)$ that solves the problem defined in (13). The firm's enterprise value V_t, given by the sum of its equity and debt values, is also a function of EBIT Y_t and debt X_t:

$$V_t = V(Y_t, X_t) = P(Y_t, X_t) + X_t. \tag{14}$$

We exploit the homogeneity property of our model to reduce the two-dimensional optimization problem in Y_t and X_t to a one-dimensional problem where the debt-EBIT ratio

$$x_t \equiv X_t / Y_t \tag{15}$$

is the state variable for leverage dynamics. Accordingly, let p_t denote the firm's equity value-EBIT ratio ($p_t = P_t / Y_t$) and v_t denote firm value-EBIT ratio ($v_t = V_t / Y_t = p_t + x_t$). Also let ML_t denote the firm's *market leverage*, the ratio of debt value X_t to enterprise value V_t:[29]

$$ML_t \equiv \frac{X_t}{V_t} = \frac{x_t}{v_t}. \tag{16}$$

A common liquidity metric used by practitioners to measure a firm's ability to service its debt is the *interest coverage ratio*, which we denote by ICR_t:

$$ICR_t = \frac{Y_t}{C_t}. \tag{17}$$

The ICR_t measures the firm's ability to service its debt interest payments out of current EBIT. We can also evaluate the firm's ability to meet the sum of interest and tax payments with its contemporaneous EBIT with the following *liquidity coverage ratio*:

$$LCR_t = \frac{Y_t}{C_t + \Theta_t}. \tag{18}$$

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[29]This ratio is known as the loan-to-value (LTV) ratio in real estate finance.

When $LCR_t > 1$, the firm's debt balance X_t automatically decreases.[30]

3 Tradeoff Theory With Costless Equity Issuance

Before analyzing the firm's dynamic financing problem when equity issuance is costly, we present the solution of the special case where equity issuance is costless. A static version of this tradeoff model is widely taught in MBA classrooms. This costless equity issuance benchmark sets the stage to understand how costly external equity financing fundamentally affects both the firm's optimal target leverage and its leverage dynamics.

3.1 Solution

We show that the optimal financing policy is as follows. The firm sets its debt-EBIT ratio at a constant target level x^*, as equity issuance is costless and the firm's EBIT growth rate is $i.i.d.$ It makes dividend payments to shareholders whenever it books a profit and issues costless equity to repay its outstanding debt whenever it makes a loss, provided that the loss is not too high. The firm defaults if a jump arrives that causes a large enough percentage loss $(1 - Z)$. We characterize this solution via backward induction in three steps.

Step 1: Equity holders' default decision when debt is due. Since default is costly and equity/debt issuance is costless, shareholders will only use default as a last resort.[31] Given the debt-EBIT ratio x_{t-} at time $t-$, suppose that a jump arrives at t causing the firm's EBIT to drop from Y_{t-} to $Y_{t-}Z$, where $Z \in [0, 1]$ is a random draw from the distribution function $F(Z)$. The post-jump debt-EBIT ratio then mechanically increases to $x_t^{\mathcal{J}} = x_{t-}/Z$. Let $\overline{x}$ denote the post-jump debt-EBIT ratio x at which the firm is indifferent between defaulting or not following a jump arrival. As the firm's EBIT growth is $i.i.d.$, this default threshold $\overline{x}$ is invariant over time. Let $\overline{Z}(x_{t-})$ denote the corresponding default threshold

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[30]Strebulaev (2007) defines financial distress as situations where the firm's ICR_t falls below one and requires the firm to take corrective actions (e.g., inefficient asset sales) to alleviate debt burden under financial distress. We share his view that liquidity measures such as ICR and LCR are critically important for capital structure decisions. However, in Strebulaev (2007) the firm continuously issues equity or makes dividend payments to shareholders and there is no earnings retention. Leverage dynamics in his model are fundamentally different from ours in which earnings retention, debt rollover, equity issuance, and payout jointly determine leverage dynamics.

[31]It is optimal for equity holders to repay the existing debt when debt matures absent a jump arrival. This is because diffusion shocks are locally continuous and it is more efficient to issue equity and/or roll over debt in response to diffusion shocks.

for the EBIT recovery fraction upon a jump arrival. Rewriting $x_t^{\mathcal{J}} = x_{t-}/Z$ with $x_t^{\mathcal{J}} = \overline{x}$ and $Z = \overline{Z}(x_{t-})$, we obtain

$$\overline{Z}_t = \overline{Z}(x_{t-}) = x_{t-}/\overline{x}\,, \tag{19}$$

which characterizes the default threshold.

Since it is costless to adjust both debt and equity at any time before default, the firm's total value $v(x) = p(x) + x$ must be constant for all values of $x \leq \overline{x}$. This is because the firm can always adjust x to x^*, the level that maximizes $v(x)$, which we formalize in Step 3. Moreover, since equity value is zero at default $(p(\overline{x}) = 0)$ and $v(x)$ is constant for all $x \leq \overline{x}$, we have

$$v(x_{t-}) = \overline{x}\,, \quad \text{for} \ \ x_{t-} \leq \overline{x}\,. \tag{20}$$

Step 2: Equilibrium credit spread and the firm's enterprise value. Given the default threshold $\overline{Z}_t = \overline{Z}(x_{t-})$, we have the following expression for the expected liquidation value upon default:

$$\mathbb{E}_{t-}\left(L_t\,\mathbf{1}_t^{\mathcal{D}}\right) = \mathbb{E}_{t-}\left(\ell Y_t\,\mathbf{1}_{Z<\overline{Z}_t}\right) = \ell Y_{t-}\,\mathbb{E}_{t-}(Z\,\mathbf{1}_{Z<\overline{Z}_t}) = \ell Y_{t-}\int_0^{\overline{Z}_t} Z dF(Z)\,. \tag{21}$$

Combining (21) with the zero-profit condition (9) for debtholders, we obtain the following pricing equation for the credit spread $\eta_{t-} = \eta(x_{t-})$:

$$\eta(x_{t-}) = \lambda\left[F(\overline{Z}(x_{t-})) - \left(\frac{\ell}{x_{t-}}\right)\int_0^{\overline{Z}(x_{t-})} Z dF(Z)\right]\,. \tag{22}$$

This equation ties the equilibrium credit spread to the firm's default strategy and the debt-EBIT ratio x_{t-}. The credit spread is lower than the probability of default $\lambda F(\overline{Z}(x_{t-}))$ as creditors' recovery in default is positive, $\ell > 0$.[32]

The homogeneity property together with costless equity issuance imply that the optimal leverage policy is time invariant. A solvent firm that sets the debt level to x at all time then has an enterprise value of $v(x) = p(x) + x$ given by:

$$v(x) = \frac{1}{\gamma - g}\left[1 + (\gamma - r)x - \theta(c(x)) - \lambda(v(x) - \ell)\left(\int_0^{\overline{Z}(x)} Z dF(Z)\right)\right]\,, \tag{23}$$

where the expected EBIT growth rate g is given in (2), the liquidation recovery per unit of EBIT ℓ is given in (7), and the default threshold $\overline{Z}(x)$ is given in (19).

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[32]For the special case where creditors recover nothing upon default, so that $\ell = 0$, the equilibrium spread η is simply given by $\lambda F(\overline{Z}(x_{t-}))$.

Step 3: Optimal leverage choice. Equityholders choose x to maximize the firm's enterprise value by solving the following problem:

$$\max_x \; v(x) = p(x) + x \,. \tag{24}$$

Note that this optimization problem takes into account equityholders' default option after debt is issued and the ex ante equilibrium credit spread. Let x^* denote the optimal level of x, the maximand for the optimization problem defined in (24).

3.2 Adjusted Present Value (APV)

When equity issuance costs are set to zero we can also derive a simple APV formula (Myers, 1974), which decomposes the costs and benefits of debt in an intuitive way. Let $pb(x)$ denote the (scaled) present value of debt financing:

$$pb(x_t) = \frac{1}{Y_t}\mathbb{E}_t\left[\int_t^{T^{\mathcal{D}}} e^{-r(s-t)}\big[\underbrace{(\tau - \theta(c(x_s)))\,Y_s}_{\text{tax shields}} + \underbrace{(\gamma - r)X_s}_{\text{cheaper debt}}\big]ds\right]. \tag{25}$$

Similarly, let $pc(x)$ denote the (scaled) present value of financial distress costs. Upon default, the firm's creditors receive a liquidating payoff $L_{T^{\mathcal{D}}} = \ell Y_{T^{\mathcal{D}}}$ given in (6) but the firm permanently loses its unlevered continuation value $V(Y_{T^{\mathcal{D}}},0)$. Therefore, the realized cost of financial distress at $T^{\mathcal{D}}$ is the difference between $V(Y_{T^{\mathcal{D}}},0)$ and $L_{T^{\mathcal{D}}}$, and the PV of distress cost is then given by:

$$pc(x_t) = \frac{1}{Y_t}\mathbb{E}_t\left[e^{-r(T^{\mathcal{D}}-t)}\left(V(Y_{T^{\mathcal{D}}},0) - \ell Y_{T^{\mathcal{D}}}\right)\right]. \tag{26}$$

To gain further insight into the APV formula, it is helpful to introduce the truncated EBIT process, $\widehat{Y}_t$ such that for $t < T^{\mathcal{D}}$, $\widehat{Y}_t = Y_t$, but for $t \geq T^{\mathcal{D}}$, $\widehat{Y}_t = 0$. This process allows us to focus on the EBIT growth for a solvent firm. Let $\widehat{g}_{t-} = \widehat{g}(x_{t-})$ denote the expected EBIT growth rate for this truncated EBIT $\widehat{Y}_t$ process:

$$\widehat{g}(x_{t-}) = \frac{1}{dt}\mathbb{E}_{t-}\left(\frac{d\widehat{Y}_t}{\widehat{Y}_{t-}}\right) = \mu - \lambda\left(1 - \int_{\overline{Z}(x_{t-})}^1 Z dF(Z)\right). \tag{27}$$

Note that $\widehat{g}_{t-} = \widehat{g}(x_{t-})$ is lower than the EBIT growth rate g given in (2) for an unlevered firm. This is because $\widehat{Y}_t = 0$ when a jump with $Z < \overline{Z}(x_{t-})$ arrives and triggers the levered firm to default. In contrast, the firm if unlevered would continue to operate regardless of the size of Z.

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Solving (25), we obtain the value of debt financing:

$$pb(x) = \frac{(\tau - \theta(c(x))) + (\gamma - r)x}{\gamma - \widehat{g}(x)}, \tag{28}$$

The numerator on the right side of (28) captures the two flow benefits of debt financing: 1.) $(\gamma - r)x$ due to shareholders' impatience and 2.) the tax shield, which is $(\tau - \theta(c(x)))$ per period. When the firm makes a profit ($Y > C$ and equivalently $c(x) < 1$), the tax shield is $\tau - \tau(1 - c(x)) = \tau c(x)$, where $c(x)$ is the scaled interest payment. This is the standard tax deduction of interest payments. When the firm incurs a loss, the firm pays no taxes ($\theta(c(x)) = 0$) and hence the tax shield is maximized at τ per unit of EBIT. The denominator uses the shareholders' discount rate γ and the growth rate for the truncated EBIT process, as debt financing benefits stop when the firm defaults.

Using (19) and (20) and solving (26), we obtain the present value of financial distress costs:

$$pc(x) = \frac{\lambda(v(0) - \ell) \int_0^{x/v(x)} Z dF(Z)}{\gamma - \widehat{g}(x)}, \tag{29}$$

where $\widehat{g}(x)$ is given in (27). The numerator on the right side of (29) is the flow loss of firm value upon default. Note that the firm only defaults and hence incurs losses when $Z < \overline{Z}(x) = x/v(x)$, which is the upper limit of the integral. The denominator for $pc(x)$ is the same as that for $pb(x)$.

Rewriting (23), we obtain the following intuitive APV formula for firm value:

$$v(x) = v(0) + pb(x) - pc(x), \tag{30}$$

where the first term in (30) is the unlevered value of the firm

$$v(0) = \frac{1 - \tau}{\gamma - g}, \tag{31}$$

the second term $pb(x)$ is given by (28), and the third term $pc(x)$ is given by (29). Note that the APV formula (30) is an implicit function of $v(x)$, which takes into account the endogenous default threshold $\overline{Z}(x) = x/v(x)$, the equilibrium interest payment $c(x)$, and the tax payment $\theta(c(x))$.

3.3 Summary

In sum, the APV formula (30) states that the firm's value $v(x)$ is given by the sum of the unlevered enterprise value (the first term) and the PV of debt financing benefits (the second

term) minus the PV of financial distress costs. (the third term). The optimal debt-EBIT ratio x^* solves the problem defined in (24), which is to choose x to maximize the implicit equation $v(x)$ given in (30) , (31), (28), and (29).

Optimal market leverage is $ML^* = x^*/v(x^*)$, where $v(x^*)$ is the firm value under optimal leverage. The firm only defaults when a jump arrives and the realized recovery fraction Z is lower than $\overline{Z}(x^*) = x^*/v(x^*) = x^*/\overline{x}$. The ex post optimal default threshold $\overline{x}$ is equal to $v(x^*)$, which follows from 1.) firm value is constant for all values of x (as shareholders can always costlessly adjust leverage and therefore are indifferent between any two levels of x in this interval); 2.) by continuity, $v(x^*) = v(\overline{x}) = \overline{x} + p(\overline{x})$; and 3.) equity is worthless upon default ($p(\overline{x}) = 0$).

Finally, with no equity issuance costs, diffusion volatility σ has no effect on leverage. This prediction is inconsistent with the survey finding that CFOs consider earnings and cash flow volatility to be a major factor influencing debt policies (Graham and Harvey, 2001).

Before analyzing the impact of costly equity issuance on financing policies, we highlight a few key differences between our costless equity issuance model and the standard contingent-claim capital structure model of Leland (1994) and Goldstein, Ju, and Leland (2001).

First, debt is short term in our model but a perpetuity in Leland (1994) and term debt in Leland and Toft (1996). Second, the default mechanisms are different: in our costless-equity-issuance model the firm can adjust both its debt and equity continuously, so only a sufficiently large EBIT (jump) loss causes the firm to default, whereas in Leland (1994) the cost of adjusting debt after issuance at time 0 is essentially infinite and hence diffusion shocks can trigger default. Finally, the equilibrium credit spread in our model is always positive, as jumps are unpredictable but is zero just prior to default in Leland (1994) and other contingent-claim models with term debt and diffusion shocks.

4   Leverage Dynamics, Equity Issuances, and Payouts

We now show how incorporating equity issuance costs fundamentally alters leverage dynamics. In contrast to the case when equity issuance is costless, for which x is a *control variable*, when equity issuance is costly x becomes a *state variable*.

At each moment t, the firm is in one of four mutually exclusive regions, separated by three endogenously determined threshold values: a.) the payout boundary $\underline{x}$; b.) the equity-issuance boundary $\widehat{x}$; and c.) the default boundary $\overline{x}$. Naturally, we have $\underline{x} < \widehat{x} \leq \overline{x}$.
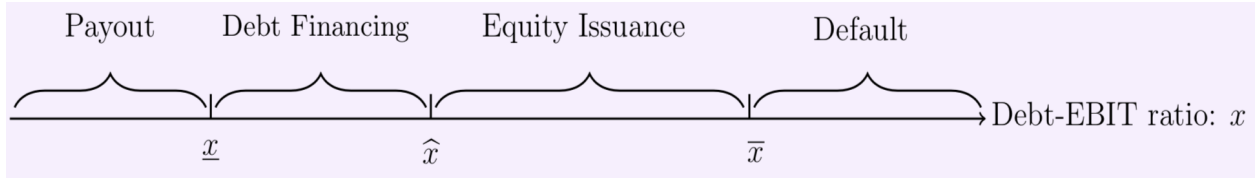
Figure 1: This figure demonstrates the four mutually exclusive regions for debt-EBIT ratio x. For low leverage ($x \leq \underline{x}$), the firm makes a one-time lumpy dividend payout ($\underline{x} - x$). For $\underline{x} \leq x \leq \widehat{x}$, the firm behaves as a creditor card revolver. For $\widehat{x} \leq x \leq \overline{x}$, the firm issues external equity to bring down x to an endogenous target level $\widetilde{x} \in (\underline{x}, \widehat{x})$ in the debt-financing region. Finally, for $x > \overline{x}$, the firm defaults. The three thresholds satisfying $\underline{x} < \widehat{x} \leq \overline{x}$ are endogenously determined. The payout boundary $\underline{x}$ is the firm's optimal target debt-EBIT ratio. The recapitalization target $\widetilde{x}$ in the debt financing region is where the firm's x_t starts after any equity issuance regardless of its pre-issuance level of $x_{t-} \in (\widehat{x}, \overline{x})$.

The four regions are: 1.) the *payout region*: $x < \underline{x}$, where the firm makes a one-time lumpy dividend payout ($\underline{x} - x$); 2.) the *debt financing region*: $\underline{x} \leq x < \widehat{x}$, where the firm exclusively relies on earnings and debt to manage its leverage dynamics; 3.) the *equity issuance region*: $\widehat{x} \leq x \leq \overline{x}$, where the firm issues equity to reduce its debt-EBIT ratio x to an endogenously determined recapitalization target, which we denote by $\widetilde{x}$; and 4.) the *default region*: $x > \overline{x}$, where it is optimal for shareholders to default. Figure 1 illustrates these four regions.

4.1   Payout Region

When x is below the endogenous payout boundary $\underline{x}$, the firm makes a lump-sum payment $(\underline{x} - x)Y$ to shareholders, so that

$$p(x) = p(\underline{x}) + \underline{x} - x, \quad \text{for} \quad x < \underline{x}. \tag{32}$$

Since (32) holds for x close to $\underline{x}$, we obtain the following smooth-pasting condition for $\underline{x}$:

$$p'(\underline{x}) = -1, \tag{33}$$

by taking the limit $x \to \underline{x}$. At $\underline{x}$, the firm is indifferent between reducing debt by one dollar and distributing this dollar to shareholders, so that the marginal benefit of paying out dividends equals one: $-p'(\underline{x}) = 1$. Since the payout boundary $\underline{x}$ is an optimal choice, we also have the following super-contact condition (see, e.g., Dumas (1991)):

$$p''(\underline{x}) = 0. \tag{34}$$

As we show below, at the payout boundary $\underline{x}$, the firm is at its optimal target leverage (ML): $\underline{x}/v(\underline{x})$, where firm value $v(x)$ is the highest. The intuition for this result is as follows. When choosing the optimal target leverage at its inception, the firm uses debt as its marginal source of financing as external equity is more costly than debt. Moreover, since the debt issuance cost is zero, at the optimal target leverage, the firm must be borrowing to the level of $\underline{x}$ that finances dividend distribution to shareholders. While the firm would optimally stay at $\underline{x}$, keeping x permanently at $\underline{x}$ is infeasible and suboptimal, as EBIT shocks are unhedgeable and equity issuance is costly.

4.2 Debt Financing Region

We first introduce the leverage dynamics in this region and then solve the equity value $p(x)$.

Leverage dynamics. When $x > \underline{x}$ it is suboptimal to make payouts to shareholders. Doing so increases the firm's debt burden but the marginal cost of debt exceeds one in this region. Therefore, when $\underline{x} \leq x \leq \widehat{x}$ (where $\widehat{x}$ is the endogenous equity-issuance boundary), the firm behaves like a *credit card revolver*: It simply lets x evolve passively in response to profits and losses (after interest and tax payments) and issues no equity, pays no dividends, and does not default. If it books a profit it reduces its debt balance, and if it incurs a loss it increases its leverage.

Using Ito's Lemma, we obtain the following dynamics for the debt-EBIT ratio $x_t = X_t/Y_t$:

$$dx_t = \mu_x(x_{t-})\,dt - \sigma x_{t-}d\mathcal{B}_t + \left(x_t^{\mathcal{J}} - x_{t-}\right)\,d\mathcal{J}_t\,, \tag{35}$$

where $\mu_x(x_{t-})$, the first term in the law of motion (35), is the drift of x absent jumps:

$$\mu_x(x_{t-}) = [c(x_{t-}) + \theta(c(x_{t-})) - 1] - \left(\mu - \sigma^2\right)x_{t-} \tag{36}$$

and $x_t^{\mathcal{J}}$ is the post-jump debt-EBIT ratio given by

$$x_t^{\mathcal{J}} = \mathbf{1}_t^{\mathcal{P}}\ell + \left(1 - \mathbf{1}_t^{\mathcal{P}}\right)\frac{x_{t-}}{Z}\,. \tag{37}$$

Equation (36) shows that the higher the EBIT growth parameter μ the lower is $\mu_x(x_{t-})$. Also note that the higher the EBIT growth diffusion volatility σ the higher the function $\mu_x(x_{t-})$ as x is convex in Y (by Jensen's inequality). The second term in (35), $-\sigma x_{t-}$, captures the effect of EBIT diffusion shocks on x_{t-}. A positive shock increases the firm's value

and lowers x_t explaining the negative sign, consistent with the observed negative leverage-profitability relation.

The last term in (35) describes the effect of a jump shock on x. Upon arrival of a jump loss at t, the debt-EBIT ratio changes from x_{t-} to the post-jump level of $x_t^{\mathcal{J}}$. Equation (37) describes the two possible scenarios for $x_t^{\mathcal{J}}$: 1.) if the firm does not default ($\mathbf{1}_t^{\mathcal{D}} = 0$), the debt-EBIT ratio mechanically and passively increases from x_{t-} to $x_{t-}^{\mathcal{J}} = x_{t-}/Z$ and market leverage accordingly increases from $ML_{t-} = x_{t-}/v(x_{t-})$ to $ML_t = x_t/v(x_t) = x_{t-}/(Zv(x_{t-}/Z)) > ML_{t-}$; 2.) if the jump triggers a default ($\mathbf{1}_t^{\mathcal{D}} = 1$), then creditors receive the entire proceeds from liquidating the firm and $x_t^{\mathcal{J}} = \ell$.

In sum, the firm often lets its leverage drift in response to realized EBIT shocks. Leverage increases following a negative EBIT shock and decreases following a positive shock, consistent with the empirical literature on the negative relation between leverage and profitability in the cross-section.[33] Next, we characterize the firm's equity value in this region.

Equity value $p(x)$. The scaled equity value, $p(x)$, satisfies the following ODE:

$$(\gamma - \mu)\, p(x) = (c(x) + \theta(c(x)) - 1 - \mu x)\, p'(x) + \frac{\sigma^2 x^2}{2} p''(x) + \lambda \left[\int_{\overline{Z}(x)}^1 Z p(x/Z) dF(Z) - p(x) \right] \tag{38}$$

for $x \in (\underline{x}, \widehat{x})$. Note that $p(x)$ is a decreasing function of x. Less obviously, firm value $v(x) = p(x) + x$ also decreases with x.

The first and second term on the right side of (38) capture the drift and volatility effects of x on $p(x)$, respectively. The last term in (38) captures the effect of jump shocks on equity value. A jump causes its debt-EBIT ratio to increase from x_{t-} to $x_t^{\mathcal{J}} = x_{t-}/Z$ and the equity value to decrease from $p(x_{t-})$ to $p(x_t^{\mathcal{J}}) = p(x_{t-}/Z)$. Since jumps link $p(x)$ to $p(x/Z)$ for $Z \geq \overline{Z}(x)$ we need to solve $p(\cdot)$ globally. The solution method for our jump-diffusion model is different from pure diffusion models, which only require local information around x.

There are three scenarios upon a jump arrival. First, if $Z < \overline{Z}(x_{t-})$, where $\overline{Z}(x_{t-}) = x_{t-}/\overline{x}$ is given in (19)), the post-jump debt is so high ($x_t^{\mathcal{J}} = x_{t-}/Z > x_{t-}/\overline{Z}(x_{t-}) = \overline{x}$) that

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[33]This is consistent with the evidence on the profitability puzzle in Titman and Wessels (1988), Myers (1993), Rajan and Zingales (1995), and Welch (2004). Danis, Rettl, and Whited (2014) find a positive relation in the cross-section between leverage and profitability for firms that make significant payouts to shareholders. Their finding is also consistent with our predictions, since in our model firms with $x_t < \underline{x}$ increase their borrowing (thereby increasing their leverage) to pay out the difference $(\underline{x} - x_t)Y_t$ to shareholders.

the firm defaults, resulting in $p(x_t^{\mathcal{J}}) = 0$. Second, if the jump loss is small or moderate, the firm rolls over its debt as a credit-card revolver. Third, if the jump loss is somewhat large the firm issues costly equity to repay its debt and bring its leverage down to a moderate level, which we refer to as the recapitalization target. We next characterize the firm's equity issuance/recapitalization decisions.

4.3   Equity Issuance Region

The equity issuance option, although costly, is valuable as it allows the firm to avoid an even worse outcome: a costly default. We show that there is an equity-issuance region characterized by $\widehat{x} \leq x \leq \overline{x}$. Let $m_t = M_t/Y_t$ denote the scaled net proceeds from an equity issue and let $\widetilde{x}_t$ denote the "recapitalization target debt-EBIT ratio" after the equity issue:[34]

$$\widetilde{x}_t = x_t - m_t. \tag{39}$$

As the firm's equity value must be continuous before and after issuance, the following value-matching condition holds:

$$p(x) = p(x - m) - (h_0 + m + h_1 m). \tag{40}$$

That is, $p(x_t)$, is equal to the equity value after issuance, $p(\widetilde{x}_t)$, after paying for the sum of net equity issuance m_t and the issuance costs $(h_0 + h_1 m_t)$. In addition, conditional on paying the fixed equity issuance cost, the optimal net equity issuance amount, m, satisfies the following FOC:

$$-p'(\widetilde{x}) = -p'(x - m) = (1 + h_1). \tag{41}$$

The optimal amount m is such that the marginal cost of debt at the post-issuance debt-EBIT ratio $\widetilde{x}$, given by $-p'(\widetilde{x}) = -p'(x - m)$, is equal to the marginal cost of equity issuance, $(1 + h_1)$.

Inverting (41), we find that the "recapitalization target" debt-EBIT ratio $\widetilde{x}$ is constant, independent of the firm's pre-issuance level of x. Note that as long as the marginal equity issuance cost is strictly positive $(h_1 > 0)$, the "recapitalization target" $\widetilde{x}$ is larger than the optimal target debt-EBIT ratio, $\underline{x}$, which is also the optimal payout boundary, as $-p'(\underline{x}) = 1$.

The intuition is as follows. Debt is the marginal source of financing when the firm determines its optimal target leverage but external equity is the marginal source of financing

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[34]Here, we focus on the case where the cost of a seasoned equity offering (h_0 and h_1) are not too high, which can be made precise by comparing value functions (with and without using equity issuance options.)

when the firm deleverages its balance sheet to reach the optimal recapitalization target $\widetilde{x}$. As the marginal cost of debt issuance is lower than the marginal cost of equity issuance, we have $\underline{x} < \widetilde{x}$.

The firm enters into the equity-issuance region only from the debt-issuance region. When it does so, it immediately issues equity to reduce its debt-EBIT ratio to the recapitalization target level of $\widetilde{x}$ in the debt financing region. Even though the firm spends almost no time in the equity-issuance region, the option of issuing costly equity to deleverage in the future has profound implications for the firm's leverage dynamics, as we have highlighted above.

Next we determine the firm's optimal equity issuance boundary $\widehat{x}$. Equations (40) and (41) together imply that $p(x)$ is linear in x in the equity-issuance region:

$$p(x) = p(\widehat{x}) - (1 + h_1)(x - \widehat{x}), \quad \widehat{x} \leq x < \overline{x}. \tag{42}$$

The intuition is as follows. The firm always returns to the recapitalization target $\widetilde{x}$ from any level of x in this region. Also, the marginal equity issuance cost (h_1) is constant.[35] Since $p(x)$ is continuous and differentiable at its endogenous equity-issuance boundary $\widehat{x}$, the following value-matching and smoothing-pasting conditions must hold at $\widehat{x}$:

$$p(\widehat{x}+) = p(\widehat{x}-) \quad \text{and} \quad p'(\widehat{x}+) = p'(\widehat{x}-), \tag{43}$$

where $\widehat{x}+$ and $\widehat{x}-$ denote the right and left limits of $\widehat{x}$. The equity-issuance FOC (41) and the smoothness of $p(x)$ imply that at $\widehat{x}$:

$$p'(\widehat{x}) = -(1 + h_1). \tag{44}$$

In effect, since equity issuance is costly the firm has a "pecking order" in its preferred financing options, first using its internally generated cash flows to service its debt, increasing its debt level if necessary to cover its funding gap (as long as its debt-EBIT ratio $x < \widehat{x}$), and issuing equity only when its debt-EBIT ratio $x \geq \widehat{x}$ to delever.

4.4 Default Region

The firm only uses default as a last resort. When its debt is so large that it exceeds its endogenous default boundary $\overline{x}$ it defaults and shareholders are wiped out. Because of

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[35]To be precise, for any $x \in [\widehat{x}, \overline{x})$, we have

$$p(x) = p(\widetilde{x}) - h_0 - h_1(x - \widetilde{x}).$$

Using the preceding equation, we obtain $p(x) - p(\widehat{x}) = -(1 + h_1)(x - \widehat{x})$ given in (42).

limited liability, shareholder value is then equal to zero:

$$p(x) = 0\,, \text{ when } x \geq \overline{x}\,, \tag{45}$$

and creditors get the firm's liquidation value $\ell Y_{T^\mathcal{D}}$.

Substituting $p(\overline{x}) = 0$ into the linear equity valuation equation (42), we obtain the following relation between the default boundary $\overline{x}$ and the equity-issuance boundary $\widehat{x}$:

$$\overline{x} - \widehat{x} = \frac{p(\widehat{x})}{1 + h_1}\,. \tag{46}$$

In words, the length of the support of the equity-issuance region, $\overline{x} - \widehat{x}$, is equal to the equity value $p(\widehat{x})$ divided by the marginal value of equity $(1 + h_1)$. This follows from the combination of the linearity of $p(x)$ in the equity-issuance region and $p(\overline{x}) = 0$.

4.5   Summary

Our solution features four mutually exclusive regions: payout, debt financing, equity financing, and default regions with three endogenously determined thresholds: a.) the payout boundary $\underline{x}$; b.) the equity-issuance boundary $\widehat{x}$; and c.) the default boundary $\overline{x}$. The firm's equity value $p(x)$ satisfies (32) in the payout region where $x < \underline{x}$, the ODE (38) in the debt-financing region where $\underline{x} < x < \widehat{x}$, (42) in the equity-issuance region where $\widehat{x} < x < \overline{x}$, and $p(x) = 0$ in the default region where $x > \overline{x}$, subject to the boundary conditions given by (33)-(34) for the payout threshold $\underline{x}$ (also the target debt-EBIT ratio), (43)-(44) for the equity-issuance threshold $\widehat{x}$, and (46) for the default threshold $\overline{x}$. Finally, the target recapitalization debt-EBIT ratio $\widetilde{x}$ solves (41).

5   Quantitative Analysis

In this section, we explore the quantitative properties of our model.

5.1   Parameter Choices: Baseline Case

We set the annual risk-free rate r to 6% as in Leland (1994) and the subsequent contingent-claim capital structure research. We set shareholders' discount rate to the risk-free rate: $\gamma = r = 6\%$ in our baseline case which focuses on the tax advantage of debt. We set the corporate tax rate τ at 21%, the current federal rate in the US. The firm pays taxes only when making profits. We leave aside personal taxation and other considerations for simplicity.

As in this literature, we assume that the cumulative distribution function $F(Z)$ for the EBIT recovery fraction Z is governed by a power law:[36]

$$F(Z) = Z^{\beta}, \quad 0 \leq Z \leq 1. \tag{47}$$

The lower is the exponent β the more fat-tailed is the distribution of $(1 - Z)$.

We calibrate the diffusion volatility parameter σ, and the two parameters for the jumps, β and λ, by matching the model-implied variance, skewness, and excess kurtosis of the annual EBIT logarithmic growth rate to 0.223, -0.252, and 0.326, respectively for the COMPUS-TAT data.[37] We obtain an annual diffusion volatility (σ) of 40.6%, a jump arrival rate λ of 1.25 per annum, and $\beta = 6.57$, which implies that each jump arrival causes an expected (percentage) EBIT loss of $(1 - \mathbb{E}(Z)) = 1/(\beta + 1) = 13.2\%$. That is, the firm's EBIT on average jumps downward about once every 9.6 months ($\lambda = 1.25$ per annum) and the expected annual percentage reduction in EBIT caused by jumps equals $\lambda(1 - \mathbb{E}(Z)) = 1.25 \times 13.2\% = 16.5\%$. These numbers indicate that the EBIT process is subject to both significant jump and diffusion risks.

We set the expected EBIT growth rate g to 2% per annum as in Huang and Huang (2012). This choice also yields a sensible firm value-to-EBIT multiple for an all-equity financed firm: $\pi = 1/(r - g) = 25$, as $r - g = 4\%$. Using the formula for g given in (2), we obtain $\mu = g + \lambda\mathbb{E}(1 - Z) = 2\% + 16.5\% = 18.5\%$.

For the equity issuance costs, we set $h_1 = 0.06$, which is in line with empirical estimates (see Hennessy and Whited, 2007; Eckbo, Masulis and Norli, 2007) and within the range of values used in the literature.

As in Huang and Huang (2012) and others, we target the recovery value for creditors when the firm defaults to 51% of the debt face value. The equity issuance probability is 7.3% per annum (based on the same COMPUSTAT sample that we use to calibrate σ, β, and λ). Calibrating to these two targets, we obtain an estimate for the fixed equity issuance cost parameter of $h_0 = 0.5$ and for the liquidation recovery parameter of $\ell = 5.6$. Creditors

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[36]See Gabaix (2009) for a survey on power laws in economics.

[37]The dataset is Compustat North American Fundamentals for publicly traded firms in North America, which contains balance sheets reported annually by companies between 1970 and 2020. We drop firms with missing total assets, negative sales, negative cash and short-term investments, negative capital expenditures, negative debt in current liabilities, negative depreciation and amortization, negative R&D expenses, missing SIC, zero common shares outstanding, or negative common equity. We focus on firms incorporated in the United States. Financial firms (SIC code 6000-6999) and utility firms (SIC codes 4900-4999) are excluded from the sample. The final sample comprises 91,657 firm-year observations. We calculate the logarithmic EBIT growth rates and winsorize at 5% on both sides.

Table 1: PARAMETER VALUES

This table summarizes the parameter values for our baseline analysis. Whenever applicable, parameter values are annualized.

Parameters	Symbol	Value
risk-free rate	r	6%
shareholders' discount rate	γ	6%
corporate tax rate	τ	21%
diffusion volatility	σ	40.6%
jump arrival rate	λ	1.25
jump recovery parameter	β	6.57
drift parameter	μ	18.5%
liquidation recovery scaled by EBIT	ℓ	5.6
equity issue fixed cost	h_0	0.5
equity issue proportional cost	h_1	0.06

recover about 5.6 times the firm's EBIT upon liquidation. Table 1 reports the parameter values used for our baseline solution.

We begin our quantitative analysis with the case where equity issuance is costless.

5.2 Costless Equity Issuance

Figure 2 plots the solution when equity issuance is costless. In Panel A, we determine the optimal debt-EBIT ratio x^* by plotting the (scaled) firm value as a function of the debt-EBIT ratio x. Panel A is our dynamic version of the standard tradeoff theory where optimal leverage is determined as a choice variable. As one would expect, $v(x)$ (the solid blue line) first increases and then decreases with x. In Panel B, we plot the implied market leverage ML as a function of x (the solid blue line).

The optimal debt-EBIT ratio is $x^* = 9.05$ where firm value is maximized with a value of $v(x^*) = 22.22$. The corresponding target market leverage is $ML^* = x^*/v(x^*) = 9.05/22.22 = 41\%$. The red dots in the two panels depict the optimal solutions. The dashed black line in Panel A depicts the constant firm value-EBIT ratio $v(x) = 25$ under the first-best (with no taxes and no financial distress).

If the firm is all equity financed ($x = 0$), its enterprise value is $v(0) = 19.75$, which is 11% lower than $v(x^*) = 22.22$ under optimal leverage. Targeting a market leverage ML
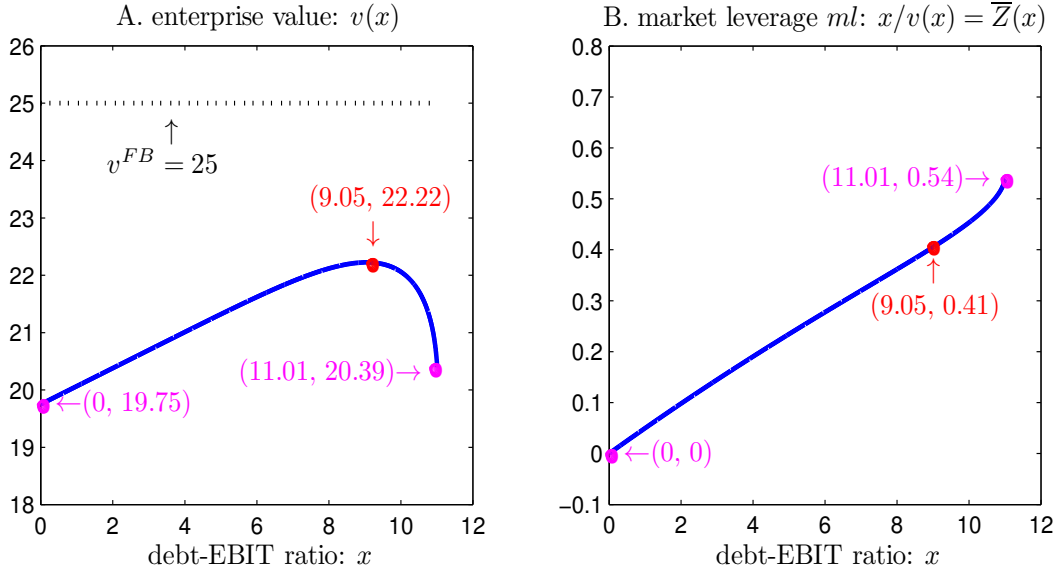
Figure 2: **Classical capital structure tradeoff theory: No equity issuance costs**
$(h_0 = h_1 = 0)$. Panel A plots the scaled firm value $v(x)$ as a function of the debt-EBIT ratio
x and Panel B plots market leverage $ML = x/v(x)$ as a function of x. The optimal debt-
EBIT ratio is $x^* = 9.05$ which implies an optimal target market leverage of $ML^* = 41\%$.
The scaled enterprise value under the first-best v^{FB} is 25. All parameter values other than
$h_0 = h_1 = 0$ are given in Table 1.

higher than 41% is suboptimal as the marginal cost of financial distress then exceeds the
marginal benefit of debt financing. Note that the maximally attainable market leverage is
only 54%, which is supported by the corresponding maximally attainable debt-EBIT ratio
of $x = 11.01$. This is due to the significant downward jump risk. Note also that volatility
has no impact on market leverage when equity issuance is costless.

We can calculate the PV of debt financing, $pb(x)$, evaluated at the optimal x^*:

$$pb(x^*) = \frac{\tau c(x^*)}{\gamma - \widehat{g}(x^*)} = \frac{21\% \times 0.568}{6\% - 1.88\%} = 2.89 \tag{48}$$

and the PV of distress costs, $pc(x)$, evaluated at x^*:

$$pc(x^*) = \frac{\lambda(v(0) - \ell) \int_0^{\overline{Z}(x^*)} Z dF(Z)}{\gamma - \widehat{g}(x^*)} = \frac{1.25 \times (19.75 - 5.6) \int_0^{0.41} Z dF(Z)}{6\% - 1.88\%} = 0.42 \tag{49}$$

using $\gamma = r = 6\%$, $v(0) = 19.75$, $\ell = 5.6$, $x^* = 9.05$, $c(x^*) = 0.568$, $\overline{Z}(x^*) = 0.41$, and
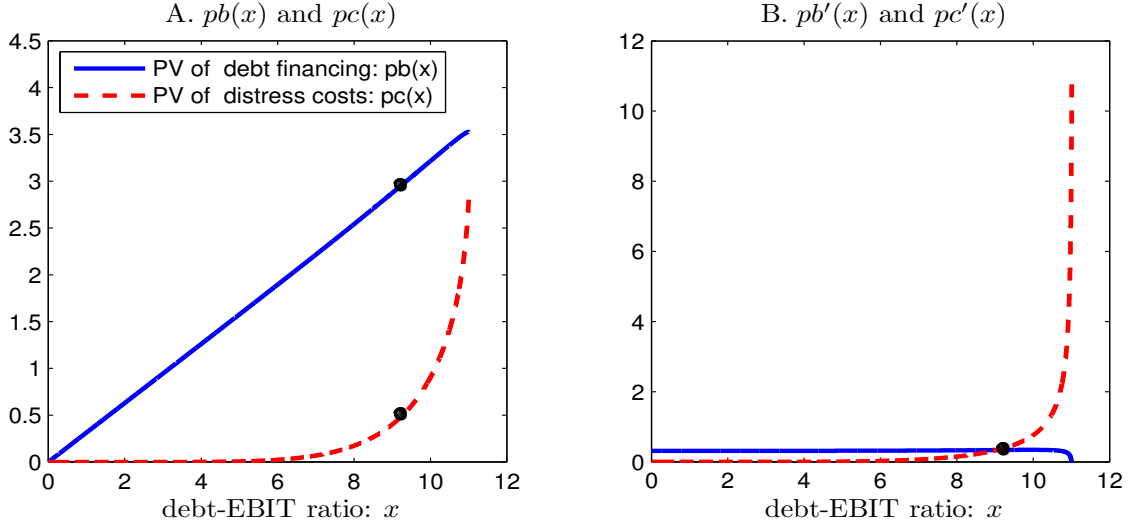$\widehat{g}(x^*) = 1.88\%$. We plot $pb(x)$ and $pc(x)$ in Panel A of Figure 3, and $pb'(x)$ and $pc'(x)$ in

Figure 3: **PV of distress costs versus PV of debt financing: No equity issuance costs** $\left(h_0 = h_1 = 0\right)$. Panel A plots $pb(x)$ and $pc(x)$. Panel B plots the marginal value of debt financing pb'^*) and the marginal cost of distress costs $pc(x) =$. At the optimal debt-EBIT ratio $x^* = 9.05$, where the optimal target market leverage is $ML^* = 41\%$, we have $pb(x^*) = 2.89$, $pc(x^*) = 0.42$, and $pb'(x^*) = pc'(x^*) = 0.338$ (see the black dots in the two panels.)

Panel B. As can be seen, financial distress costs rise sharply when x is set beyond $x^* = 9.05$, while the marginal benefits of debt $pb'(x)$ remains flat.

We next show how the introduction of equity issuance costs gives predicted leverage levels and dynamics that are in line with Graham and Harvey (2001) survey and evidence.

5.3 Costly Equity Issuance

Figure 4 plots the firm's equity value $p(x)$, enterprise value $v(x)$, net marginal cost of debt $-v'(x)$, and marginal value of earnings V_Y. Together the four panels provide a rich characterization of the firm's optimal financing choices, leverage dynamics, and firm value.

Equity value $p(x)$ and enterprise value $v(x)$. In Panels A and B, we plot the firm's equity value $p(x)$ and enterprise value $v(x)$, respectively. Note that $v(x)$ is maximized at the target debt-to-EBIT ratio $\underline{x} = 2.32$ as we discussed earlier and decreases with x for $x \geq \underline{x}$. If at inception x is below $\underline{x} = 2.32$, the firm optimally issues just enough debt to bring x to the target debt-EBIT ratio $\underline{x} = 2.32$. The corresponding target market leverage
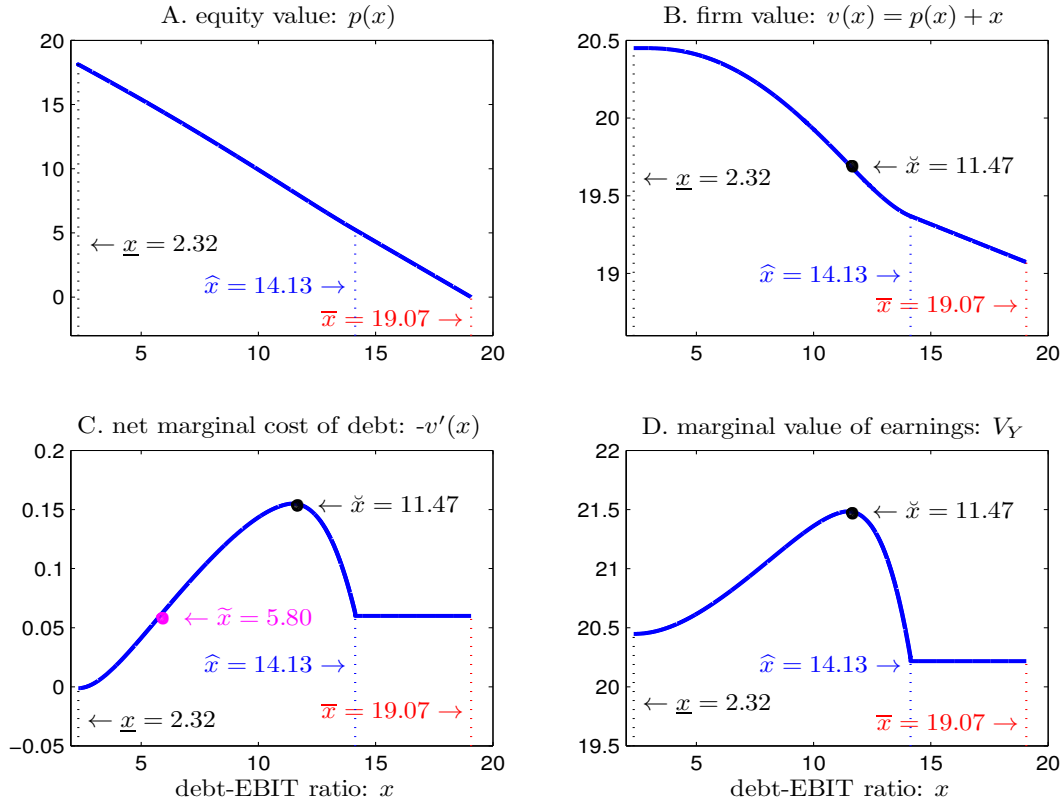
Figure 4: **Equity value, $p(x)$, firm value, $v(x) = p(x) + x$, net marginal cost of debt, $-v'(x)$, and marginal value of EBIT, $V_Y = v(x) - xv'(x)$.** The endogenous target debt-EBIT ratio, also the payout boundary, is $\underline{x} = 2.32$ where market leverage is $ML(\underline{x}) = \underline{x}/v(\underline{x}) = 11.3\%$. The endogenous (upper) default boundary is $\overline{x} = 19.07$, where $p(\overline{x}) = 0$. The equity-issuance boundary is $\widehat{x} = 14.13$, where $p(\widehat{x}) = 5.24$ and market leverage is $ML(\widehat{x}) = \widehat{x}/v(\widehat{x}) = 73.0\%$. The firm's post-equity-issuance target level of x is $\widetilde{x} = 5.80$, with an implied market leverage of $ML(\widetilde{x}) = \widetilde{x}/v(\widetilde{x}) = 29\%$. And the inflection point is $\breve{x} = 11.47$ and market leverage is $ML(\breve{x}) = \breve{x}/v(\breve{x}) = 58.2\%$. All parameter values are given in Table 1.

is $ML(\underline{x}) = \underline{x}/v(\underline{x}) = 11.3\%$, which is significantly lower than the target market leverage $ML^* = 41\%$ in the costless equity issuance case. Once the firm is off the ground running, it makes dividend payments to shareholders, bringing x back to $\underline{x} = 2.32$, whenever its cumulative profits cause its x_t to fall below $\underline{x} = 2.32$ otherwise.

In the event that it accumulates losses over time such that x_t exceeds $\underline{x} = 2.32$, it is optimal to let x_t drift passively and stochastically in response to realized EBIT shocks, rather than constantly issue costly equity to keep x at $\underline{x} = 2.32$. This passive leverage management

policy is optimal for a wide range of debt-EBIT ratios: $x \in (\underline{x}, \widehat{x}) = (2.32, 14.13)$.

Even though its enterprise value $(v(x))$ is maximized at the target leverage $ML(\underline{x}) = 11.3\%$, the firm barely spends any time at $\underline{x} = 2.32$. This is because keeping its debt-to-EBIT ratio at $\underline{x}$ all the time is too expensive. A fundamental observation here is that paradoxically low target leverage is not caused by costly debt, but rather by costly equity.

When the debt-to-EBIT ratio x exceeds the equity issuance boundary $\widehat{x}$ but stays below the default boundary $\overline{x}$ (when $x_t \in (\widehat{x}, \overline{x}) = (14.13, 19.07)$), market leverage is very high: $ML(x) \in (ML(\widehat{x}), ML(\overline{x})) = (73.0\%, 100\%)$.[38] In this equity-issuance region, it is optimal to recapitalize by issuing costly external equity and bring the debt-to-EBIT ratio to the recapitalization target $\widetilde{x} = 5.80$ (the pink solid dot), and a post-recapitalization market leverage of $ML(\widetilde{x}) = \widetilde{x}/v(\widetilde{x}) = 29\%$, regardless of the value of the pre-equity-issuance level of x. Note the substantial deleveraging from a market leverage higher than 73%. Note also that although the aim of the recapitalization is to reduce leverage, it is suboptimal to bring market leverage down all the way to the target leverage $ML(\underline{x})$, which is only 11.3%.

To better understand the wedge between the recapitalization target $ML(\widetilde{x})$ and target market leverage $ML(\underline{x})$, it is helpful to introduce the (net) marginal cost of debt financing for the firm:

$$-\frac{\partial V(Y, X)}{\partial X} = -v'(x) \geq 0. \tag{50}$$

Note the minus sign in the definition.

When choosing its recapitalization target $\widetilde{x}$, the firm optimally equates the firm's marginal cost of debt, $-v'(x)$, to the marginal equity issuance cost h_1. In contrast, when choosing its target debt-EBIT ratio $\underline{x}$ it equates $-v'(x)$ to zero (as the marginal debt issuance cost is zero). Because external equity is more costly than debt at the margin, $\widetilde{x}$ is larger than the target level $\underline{x}$. Next, we analyze the curvature of $v(x)$.

Concavity and convexity of $v(x)$. An important result that emerges from our model, depicted in Figure 4 Panel B, is that depending on the level of x firm value can be either concave or convex in x. We also plot $-v'(x)$ in Panel C. Note that while always positive, $-v'(x)$ is not monotonic: it is first increasing in x, reaching a maximum value at $\breve{x} = 11.47$, and then decreasing in x. That is, $v(x)$ is concave in x for $x \in (\underline{x}, \breve{x}) = (2.32, 11.47)$ and convex for $x \in (\breve{x}, \widehat{x}) = (11.47, 19.07)$. And $\breve{x}$ is the inflection point where $v''(x) = 0$ and

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[38]This follows from $ML(\widehat{x}) = \widehat{x}/v(\widehat{x}) = 73.0\%$ and $p(\overline{x}) = 0$.

the corresponding market leverage is $ML(\check{x}) = \check{x}/v(\check{x}) = 11.47/19.71 = 58.2\%$. When $x \in (2.32, 11.47)$, market leverage (ML) is in the range of $(11.3\%, 58.2\%)$ and the firm is averse to raising costly external equity. As a result, the firm behaves prudently (in an endogenously risk averse manner, with $v''(x) < 0$.)

In contrast, when $x \in (11.47, 19.07)$, market leverage lies in the range of $(58.2\%, 73.0\%)$ and the firm is close to the point when it may have to issue costly external equity. In this region, increasing earnings volatility creates value for the firm. As in Jensen and Meckling (1976), a risk-seeking motive emerges when leverage is sufficiently high. This gambling for resurrection is caused by the fixed cost of issuing equity. This convex region of $v(x)$ has received less attention in dynamic models of financially constrained firms, which typically feature a concave firm value function caused by costly external financing (see e.g., Bolton, Chen, and Wang, 2011). Our solution reveals that the endogenous risk aversion force dominates when a firm's leverage is low, but the risk seeking incentive dominates when the firm's leverage is sufficiently high. Hugonnier, Morellec, and Malamud (2015) show that firm value can be concave or convex depending on the level of the firm's cash holdings in a model with stochastic capital supply and lumpy investment.

Panel C also confirms two key results discussed earlier: 1) In the equity issuance region where $x_t \in (\widehat{x}, \overline{x}) = (14.13, 19.07)$, the (net) marginal cost of debt always equals the marginal cost of issuing equity: $-v'(x) = h_1 = 0.06$; 2.) firm value $v(x)$ is maximized at the payout boundary $\underline{x}$, as $v'(x) = 0$ only when $x = \underline{x}$.

Marginal value of EBIT $V_Y(Y, X)$. In Panel D of Figure 4, we plot the marginal value of EBIT:

$$\frac{\partial V(Y, X)}{\partial Y} = v(x) - v'(x)x. \tag{51}$$

Note that the marginal value of EBIT is always greater than the average value of EBIT: $V_Y \geq v(x)$, as $V_Y - v(x) = -xv'(x)$ and $v'(x) < 0$. This is because increasing Y lowers the probability of an equity issue or default, both of which are costly. Reducing distress likelihood (and distress costs) creates value for shareholders. The solid curve for $V_Y(Y, X)$ in Panel D is always above the solid curve for $v(x)$ in Panel B. That is, the marginal value of EBIT equals the average value of EBIT, $V_Y = v(\underline{x})$, only at the payout boundary, where the firm is not financially constrained, $v'(\underline{x}) = 0$. Note finally that both the net marginal cost of debt $(-v'(x))$ and marginal value of EBIT $(V_Y(Y, X))$ peak at the inflection point: $\widetilde{x} = 11.47$, where $v''(\widetilde{x}) = 0$.
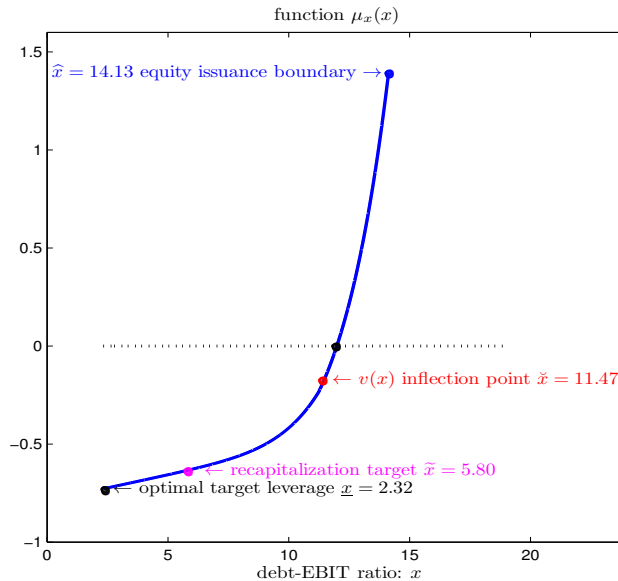
Figure 5: This figures shows that debt-EBIT ratio drifts towards its target $\underline{x}$, i.e., $\mu_x(x) < 0$ when $x \leq 11.99$. In contrast, when $x > 11.99$, leverage is in an (upward) death spiral, as $\mu_x(x) > 0$. Downward jumps further speed up leverage increase. All parameter values are given in Table 1.

Reverting to the target and diverging to the debt death-spiral. In Figure 5 we plot the function $\mu_x(x)$ given in (36) over the debt financing region $x \in (\underline{x}, \widehat{x}) = (2.32, 14.13)$. When $x \leq 11.99$ (i.e., when market leverage $ML \leq 61.1\%$), the debt-EBIT ratio and market leverage decrease in expectation: $\mu_x(x) < 0$ even absent jumps. The lower is the debt-to-EBIT ratio x, the faster this ratio drifts down towards the target $\underline{x} = 2.32$ where $v(x)$ is maximized and the market leverage is only 11.3%. That is, firms with low leverage are able to decrease their leverage faster as $\partial \mu_x(x) / \partial x > 0$. It is easier for a firm with low leverage to service its debt, keep its leverage low, and pay dividends to shareholders. Therefore, as long as x is not too high, leverage dynamics exhibit reversion to the target $ML(\underline{x})$.

In contrast, when $x > 11.99$, the firm's debt-to-EBIT ratio x is expected to drift up: $\mu_x(x) > 0$ even absent jumps. This is because fixed equity issuance costs significantly discourages the firm from issuing equity. Note that the point at which the drift of $\mu_x(x)$ turns positive ($x = 11.99$) is close to the $v(x)$ inflection point: $\breve{x} = 11.47$. Aversion to issuing costly equity together with the effect of downward EBIT jumps on leverage drives the firm into a debt death-spiral in expectation. In this region leverage dynamics exhibit diversion from the target $ML(\underline{x})$. As a large debt servicing burden makes the firm more indebted,

leaving little room to keep its debt under control, the odds to raise costly external equity or default thus rise significantly as x increases.

6 Simulation

To gain further insight into the leverage dynamics resulting from costly equity issuance we illustrate the joint dynamics of equity issuance, debt financing, payouts, and default along a simulated path. This simulation compares the financial behavior of the firm under respectively positive and zero equity issuance cost.
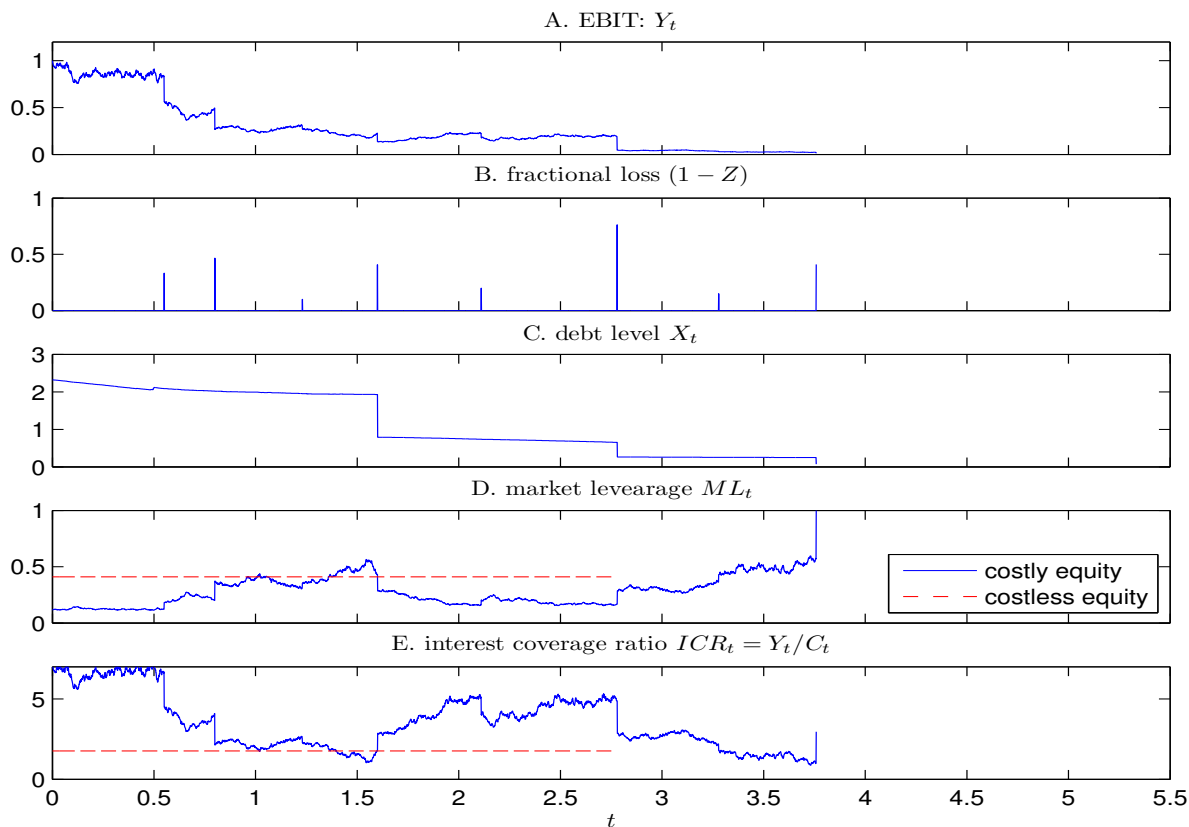

Figure 6: **Simulation.** Panel A displays a sample path of EBIT Y_t starting with $Y_0 = 1$. Panel B plots eight realized EBIT (percentage) losses for the path in Panel A. Panel C plots the firm's debt (X_t) dynamics. Panels D and E plot the implied market leverage (ML) and interest coverage ratio (ICR), respectively. The red dashed lines in Panels D and E correspond to the costless equity issuance case where $ML = 41\%$ and $ICR = 1.76$ for all t before default. The blue solid lines in Panels D and E are for the costly equity issuance case.

Panel A of Figure 6 displays the sample path of EBIT Y_t starting with $Y_0 = 1$. Panel B

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singles out the eight jumps (EBIT losses $(1 - Z_t)$) of the Y_t sample path in Panel A. These jumps occur at times $t = 0.55, 0.8, 1.23, 1.6, 2.11, 2.78, 3.28$, and 3.76. The corresponding percentage losses are $(1 - Z) = 0.33, 0.47, 0.1, 0.41, 0.2, 0.76, 0.15$, and 0.41. We include eight jumps of varying sizes to illustrate the richness of leverage dynamics. The last jump at $t = 3.76$ causes the firm to default.

Dynamic evolution of debt X_t. The blue solid line in Panel C plots the evolution of debt X_t in response to realized EBIT jump-diffusion shocks (when equity issuance costs are strictly positive). The firm begins by setting its initial debt-to-EBIT ratio to the optimal target level $x_0 = \underline{x} = 2.32$. Since $Y_0 = 1$, this means that the firm borrows $X_0 = \underline{x}Y_0 = 2.32$ at $t = 0$ and makes a one-time dividend payment to shareholders.

Note that debt X_t drops discontinuously three times. These events represent two recapitalizations (via equity issuance) and a default. The first discrete debt adjustment occurs at $t = 1.6$ when the EBIT drops by 41% (the fourth EBIT jump). The pre-jump debt-EBIT ratio is at 8.40, (corresponding to a 42% ML); the jump drop in EBIT then causes the debt-EBIT ratio to increase to 14.18, which exceeds the equity-issuance threshold of $\hat{x} = 14.13$. As a result, the firm responds by recapitalizing its balance sheet and raising a net amount $M_{1.6} = 1.14$, bringing down its debt from 1.93 to $X_{1.6} = 0.79$. The gross amount of equity raised is 1.28, which includes issuance costs of $h_0 Y_{1.6} + h_1 M_{1.6} = 0.14$, causing an *equity dilution* of 65% for the existing shareholders. That is, the new shareholders own 65% of the firm, leaving old shareholders with 35% of the firm after this equity issue.[39] Finally, the third discrete debt adjustment occurs at $t = 3.76$ when the EBIT drops by $(1 - Z) = 41\%$. Given that the pre-jump market leverage is 57.6%, this EBIT drop brings the debt-EBIT ratio to 19.28, exceeding the default threshold of $\bar{x} = 19.07$, so that the firm declares default and equity holders are completely wiped out.

For the other five EBIT jumps, X_t is continuous and smooth as the firm optimally rolls over its debt like a credit card revolver by passively following $dX_t = (C_{t-} + \Theta_{t-} - Y_{t-})dt$. Note that even when it is hit by a large negative earnings jump the firm can still roll over its debt (provided of course that the pre-jump leverage is not too high). For example, even when its EBIT drops by 47% at $t = 0.8$, the firm optimally rolls over its debt since its

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[39]The second discontinuous debt reduction occurs at time $t = 2.78$, when EBIT decreases by 76% (the sixth EBIT jump). Following the recapitalization X_t decreases from 0.65 to 0.26 and again shareholders are significantly diluted.

pre-jump market leverage is relatively low (at $ML_{0.8-} = 0.20$).[40]

In sum, debt X_t adjusts discretely only when the firm is unable to roll over its debt like a credit card revolver. In this case, the firm must issue external equity to deleverage. If equity issuance is not an option, the firm must default. In all other circumstances, its debt dynamics are continuous and smooth.

Next we analyze the evolution of market leverage (ML) and the interest coverage ratio (ICR). First, we summarize the results when equity issuance costs zero.

Costless equity issuance. The red dashed lines in Panels D and E show that both market leverage and interest-coverage ratio are constant over time with $ML_t = x^*/v(x^*) = 41\%$ and $ICR_t = 1/c^* = 1.76$ until it defaults.[41] The firm is able to keep its ML at 41% (and equivalently ICR at 1.76 and LCR at 1.52) at all time by issuing just enough equity to pull its leverage back to 41% (when making losses) and distributing just enough dividends to bring its leverage up to 41% (when making profits). The firm defaults at $t = 2.78$ because it is the first moment when the realized drop in EBIT (which is 78%) exceeds the default threshold $(1 - \overline{Z}^*) = 59\%$. The ICR may seem a bit low, but that is precisely because equity issuance is costless. We show next that when external equity is costly the firm aims for a much higher ICR.

Dynamics of market leverage ML_t and interest-coverage ratio ICR_t. Panels D and E plot respectively the ML and ICR dynamics under costly equity issuance (the blue solid lines) providing a sharp contrast against the ML and ICR dynamics when equity issuance is costless (flat red dashed lines.) The target market leverage at $t = 0$ is only 11.3%, which is only about a quarter of the target market leverage (41%) when equity issuance is costless. Similarly, the corresponding ICR at $t = 0$ is 7.18, which is 4 times the ICR (1.76) when equity issuance is costless.

At $t = 0.55$, the 33% EBIT drop causes market leverage to leap from the pre-jump level of 0.12 to the post-jump level of 0.18. The firm keeps rolling over its debt, but the permanent negative earnings shock results in a large drop in market value.[42]

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[40]Recall that the first equity issuance occurs at $t = 1.6$ following a smaller percentage EBIT loss (41%). This is because the pre-jump market leverage then is much higher (at 0.42).

[41]The *liquidity coverage ratio* until the firm defaults is also constant: $LCR = Y/(C + \Theta) = 1.52$, which also exceeds one.

[42]The jump at $t = 0.8$ causes a $(1 - Z) = 47\%$ drop of EBIT, and market leverage increases mechanically from the pre-jump level of 0.20 to the post-jump level of 0.37. The same passive debt rollover occurs at

In contrast, the jump at $t = 1.6$, which causes a 41% decrease in EBIT, pushes the firm to respond with a recapitalization to bring market leverage down to $\widetilde{x}/v(\widetilde{x}) = 29\%$. Absent the equity issue, the firm's debt-to-EBIT ratio would have risen to 14.3 and market leverage would have been 73%, which is suboptimal for the firm.[43] Even after actively deleveraging, the post-SEO recapitalization target ($ML = 29\%$) is still much higher than the optimal target leverage of 11%. This is because external equity, the marginal source of financing for recapitalization, is more costly than debt, the marginal source of financing that pins down the optimal target leverage. Finally, this post-SEO recapitalization target ($ML = 29\%$) is still significantly lower than the target leverage ($ML = 41\%$) predicted by the classical tradeoff theory with no equity issuance costs.

In sum, a firm facing external equity issuance costs lets its market leverage bounce around in response to EBIT shocks, passively rolling over its debt most of the time, and only resorts to equity issuance when its leverage spirals out of control, i.e., $ML \geq ML(\widehat{x}) = 73\%$. Beyond that point the firm actively deleverages to bring its ML to 29%.

Next we compute the long-run distribution of x and ML using 100,000 simulated paths.[44]

Distribution of debt-to-EBIT ratios and market leverage. Table 2 reports the mean, the standard deviation, and the quantile distribution of x and $ML(x)$. With our baseline parameter values, average market leverage is 24%, which is within the range of $20\% - 25\%$ for COMPUSTAT firms reported in Strebulaev and Whited (2011), and the median leverage is 20%, with a standard deviation of 12%. The vast majority of firms has low market leverage. For example, only 5% of firms have market leverage larger than 49%, and only 1% of firms have market leverage larger than 63%.

In sum, our calculations further illustrate how incorporating external equity issuance costs into a classical dynamic tradeoff theory dramatically changes leverage dynamics, makes the firm much more conservative with its debt policy, and yields more empirically plausible predictions about market leverage and ICR.

$t = 1.23$, $t = 2.11$, and $t = 3.28$.

[43]Similarly, the jump at $t = 2.78$ causes a 76% decrease of EBIT, which prompts the firm to issue costly external equity to bring down market leverage again to $\widetilde{x}/v(\widetilde{x}) = 0.29$.

[44]Each path starts with the target leverage $ML(\underline{x}) = 11\%$ and ends when either the firm defaults or it reaches 100 years.

Table 2: STATIONARY DISTRIBUTIONS OF x AND MARKET LEVERAGE ML. ALL PARAMETER VALUES ARE GIVEN IN TABLE 1.

	Mean	Std. dev.	1%	5%	25%	50%	75%	95%	99%
x	4.78	2.40	2.34	2.41	2.88	3.99	5.99	9.87	12.31
ML	0.24	0.12	0.11	0.12	0.14	0.20	0.29	0.49	0.63

Table 3: COMPARATIVE STATIC ANALYSIS: THE DISTRIBUTIONS OF MARKET LEVERAGE ML. Parameter values for the baseline case are given in Table 1. For all other rows, we vary one parameter (shown in the first column) from the baseline case.

	Mean	Std. dev.	1%	5%	25%	50%	75%	95%	99%
baseline	0.24	0.12	0.11	0.12	0.14	0.20	0.29	0.49	0.63
$h_0 = 0.001$	0.32	0.11	0.17	0.18	0.22	0.30	0.41	0.51	0.55
$h_0 = 2$	0.16	0.11	0.08	0.08	0.09	0.12	0.19	0.41	0.62
no issuance option	0.04	0.02	0.03	0.03	0.03	0.04	0.04	0.07	0.13
$h_1 = 0$	0.27	0.11	0.16	0.16	0.18	0.23	0.32	0.52	0.64
$h_1 = 0.2$	0.18	0.12	0.08	0.08	0.10	0.13	0.23	0.44	0.60

7 Comparative Statics

To better understand the quantitative effects of our model with costly external equity financing, we conduct comparative static exercises for debt-EBIT ratio x and market leverage ML with respect to key parameters: the equity issuance costs (h_0 and h_1), tax rate τ, liquidation recovery ℓ, and EBIT parameters (σ, λ, β, and μ). We report the mean and quantile distribution for market leverage ML in Table 3. The row labeled "baseline" reproduces the results for our baseline case. Table 1 reports the baseline parameter values.

7.1 Equity Issuance Costs

How sensitive is the firm's financing policy to changes in equity issuance costs? When we reduce the fixed equity issuance cost parameter from $h_0 = 0.5$ to essentially zero, $h_0 = 0.001$ (500 times smaller), the firm is still very prudent with its leverage policy. For example, Table 3 reports that the median ML increases from 20% (for our baseline) to only 30%, and at

the 95% quantile market leverage barely increases from 49% (for our baseline) to 51%. That is, the conservative leverage prediction of our model depend mostly on the existence of the fixed equity issuance cost, not on the size of the fixed cost. A small fixed cost goes a long way in generating a wide equity-inaction region.

Going in the opposite direction, increasing the fixed equity issuance cost by four times from $h_0 = 0.5$ to $h_0 = 2$, however, lowers market leverage significantly: the median of ML decreases from 20% to 12%. When the firm has no option to issue equity (e.g., when equity issuance costs are high, $h_0 \geq 12$) it becomes quite risk averse and keeps its market leverage essentially at zero (the average market leverage is only 4% and just 1% of firms have market leverage larger than 13%). This result highlights how increasing equity issuance costs has a first-order effect on leverage. The marginal equity issuance cost h_1 overall has a moderate effect on the distribution of debt-EBIT ratio x and ML.

In sum, we find that 1.) fixed equity issue costs, even when they are small, have a significant effect on leverage distribution and dynamics; 2.) even if it is costly, the option of equity issuance can be highly valuable, allowing the firm to survive against large downward earnings shocks; and 3.) without an option to issue equity, firms barely take on any debt.

7.2   Tax Benefits and Financial Distress

How much does the firm's debt vary with the corporate tax rate? A little but not nearly as much as predicted by standard capital structure models, e.g., Leland (1994). Table 4 reports that increasing the tax rate τ from 21% to 30% only raises the average market leverage from 24% to 27% and increases the median market leverage from 20% to 23%. The quantile distribution shifts up slightly. Decreasing the tax rate τ from 21% to 10% causes the average ML to decrease from 24% to 18%.

The default recovery value parameter ℓ has an even smaller and negligible effect than taxes on leverage. Decreasing ℓ from 5.6 to 2 or increasing it to 8 barely has any effect on the average ML and its distribution as we see from Table 4.

Our results on the effects of the corporate tax rate shed a new light on Miller (1977)'s famous "rabbit" versus "horse" critique. Unlike in standard tradeoff models (without equity issuance costs), which predict higher leverage than is empirically plausible, our model with costly external equity generates empirically plausible low market leverage for a wide range of parameter values.

Table 4: Comparative static analysis: the distributions of market leverage ML. Parameter values for the baseline case are given in Table 1. For all other rows, we vary one parameter (shown in the first column) from the baseline case.

	Mean	Std. dev.	1%	5%	25%	50%	75%	95%	99%
baseline	0.24	0.12	0.11	0.12	0.14	0.20	0.29	0.49	0.63
$\tau = 10\%$	0.18	0.11	0.08	0.09	0.10	0.13	0.22	0.42	0.57
$\tau = 30\%$	0.27	0.13	0.14	0.14	0.17	0.23	0.33	0.53	0.66
$\ell = 2$	0.23	0.12	0.11	0.12	0.14	0.19	0.29	0.48	0.61
$\ell = 8$	0.24	0.13	0.12	0.12	0.14	0.20	0.30	0.51	0.65

Table 5: Comparative static analysis: the distributions of market leverage ML. Parameter values for the baseline case are given in Table 1. For all other rows, we vary one parameter (shown in the first column) from the baseline case.

	Mean	Std. dev.	1%	5%	25%	50%	75%	95%	99%
baseline	0.24	0.12	0.11	0.12	0.14	0.20	0.29	0.49	0.63
$\beta = 19$	0.37	0.16	0.19	0.20	0.24	0.33	0.46	0.69	0.83
$\lambda = 0.5$	0.29	0.14	0.14	0.15	0.18	0.25	0.36	0.58	0.72
$\mu = 0.21$	0.24	0.11	0.10	0.10	0.14	0.21	0.30	0.46	0.55
$\sigma = 10\%$	0.31	0.09	0.23	0.23	0.24	0.29	0.36	0.50	0.59
$\sigma = 60\%$	0.20	0.14	0.07	0.07	0.09	0.15	0.26	0.49	0.65

7.3 EBIT Process Parameters

Jump arrival rate λ and power law exponent β. The effects of changes in the jump arrival rate λ and the power law exponent for the tail distribution β on the level and distribution of ML are significant. Table 5 shows that a reduction in the fractional loss $\mathbb{E}(1 - Z) = 1/(\beta + 1)$ conditional on a jump from 13.2% to 5% substantially increases average ML from 24% to 37%, and at the 95th percentile ML increases significantly from 49% to 69%. A reduction of the arrival frequency from $\lambda = 1.25$ (once every 9.6 months) to $\lambda = 0.5$ (once every two years) increases the average ML from 24% to 29%.

Diffusion volatility σ. The effects of changes in diffusion volatility are also visible. A decrease in the volatility parameter σ from 40.6% (for our baseline) to 10% causes the average ML to increase from 24% to 31%. A reduction in earnings volatility reduces risk for shareholders, as they are endogenously risk averse when equity issuance is costly. Our comparative statics results with respect to σ are in sharp contrast with those for the costless equity issuance case, where diffusion volatility has no effect at all on the firm's market leverage.

Drift parameter μ. In contrast, the effect of changes in μ on leverage is limited. An increase in μ from 18.5% (in our baseline) to 21%, which implies that the expected EBIT growth rate g increases from 2% (in our baseline) to 4.5% per annum, has a small effect on average ML and the distribution of ML, as can be seen in Table 5.

In sum, our comparative static results imply that financial flexibility and internal funds (both of which critically depend on equity issuance costs) and cash flow risk (captured by jumps and diffusion risks) are more important drivers than taxes for corporate debt policies, consistent with the survey findings in Graham and Harvey (2001).

8 q-Theory of Investment and Leverage Dynamics

What are the implications of costly external equity on leverage and investment dynamics? To answer this question, we generalize our model of Section 2 by endogenizing the EBIT process via a capital accumulation process and capital adjustment costs following Hayashi (1982), Abel (1983, 2016), Abel and Eberly (1994) and other q models of investment.[45]

8.1 Capital Accumulation, Production, and Endogenous EBIT

Let K and I denote the level of the capital stock and gross investment, respectively. At each time t, the firm uses its capital stock K_t to produce cash flows proportional to its contemporaneous K_t at the rate of AK_t per unit of time, where A is a constant that measures the firm's productivity.[46] The price of capital is normalized to one, so that the firm's unlevered

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[45]The q−theory of investment is natural for our model as it has the homogeneity property and implies the debt-capital ratio is the natural state variable. Our theory of dynamic corporate finance and investment is a complement to Hennessy and Whited (2005, 2007) and other discrete-time dynamics models of investment, which assume decreasing returns to scale.

[46]We can interpret this linear production function as one that features a constant-return-to-scale production function also involving other factors of production. For instance, suppose the firm has a Cobb-Douglas

free cash flow net of investment is given by:

$$Y_t = AK_t - I_t. \tag{52}$$

We assume that the capital stock evolves according to the following process:[47]

$$dK_t = \Psi(I_{t-}, K_{t-})dt + \sigma_K K_{t-}d\mathcal{B}_t^K - (1-Z)K_{t-}d\mathcal{J}_t^K. \tag{53}$$

There are three terms contributing to the change in capital stock dK_t. The first term in (53), $\Psi(I_{t-}, K_{t-})$, corresponds to the rate of capital accumulation over time interval dt in the absence of diffusion shocks and jumps. As in the q-theory of investment (Lucas and Prescott, 1971, Hayashi, 1982, Abel and Eberly, 1994, and Jermann 1998), we assume that the firm incurs capital-adjustment costs and that the capital stock depreciates over time. The function $\Psi(I_{t-}, K_{t-})$ captures both the costs of installing new capital and capital stock depreciation. Let $\Psi(I, K)$ be homogeneous of degree one in I and K, so that

$$\Psi(I, K) = \psi(i) \cdot K, \tag{54}$$

where $i = I/K$ denotes the investment-capital ratio, and $\psi'(i) > 0$ ($\psi(i)$ is concave and continuously differentiable).[48]

The second term in (53) describes the Brownian shock, where σ_K is the diffusion-volatility parameter and $\mathcal{B}^K$ is a standard Brownian motion. These continuous shocks can be thought of as stochastic capital depreciation shocks.

The third term in (53) describes discrete downward jumps in the capital stock, where $\mathcal{J}^K$ is a jump process with a constant arrival rate $\lambda_K > 0$. Let $T_K^{\mathcal{J}}$ denote the jump arrival time. If a jump does not occur at time t, so that $d\mathcal{J}_t^K = 0$, we have $K_t = K_{t-}$, where $K_{t-} \equiv \lim_{s\uparrow t} K_s$ is the left limit of K_t. If a jump occurs at time t, so that $d\mathcal{J}_t = 1$, the capital stock drops from K_{t-} to $K_t = ZK_{t-}$. As before we assume that Z is distributed

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production function with capital and labor as inputs. Let w denote the constant wage rate for a unit of labor and $\widehat{A}$ denote the firm's productivity with this Cobb-Douglas production function. The firm then has the following embedded static tradeoff within its dynamic optimization problem: $\max_{N_t} \widehat{A}K_t^\nu N_t^{1-\nu} - wN_t$, which yields optimal labor demand: $N_t^* = \left(\frac{(1-\nu)\widehat{A}}{w}\right)^{1/\nu} K_t$, proportional to the capital stock K_t. Using this expression for N_t^*, we then obtain the revenue net of labor cost AK_t, where $A = \frac{\nu}{1-\nu}\left((1-\nu)\widehat{A}\right)^{1/\nu} w^{\frac{\nu-1}{\nu}}$.

[47]Pindyck and Wang (2013) use this process in a general-equilibrium setting to quantify the economic cost of catastrophes. Brunnermeier and Sannikov (2014) and Barnett, Brock, and Hansen (2019) use the same capital accumulation process given in (53) with no jumps.

[48]Also see Boldrin, Christiano and Fisher (2001), Jermann (1998), and Brunnermeier and Sannikov (2014) among others for this widely-used specification.

according to a well-behaved cumulative distribution function $F(Z)$. We let financial distress and equity issuance costs be functions of K. The firm's liquidation value at the moment of default $T_K^{\mathcal{D}}$, is now given by

$$L_{T_K^{\mathcal{D}}} = \ell_K K_{T_K^{\mathcal{D}}}, \tag{55}$$

where, ℓ_K is the market recovery value per unit of capital. Default generates deadweight losses if ℓ_K is sufficiently low as in our EBIT-based model.

To preserve the homogeneity property of our model, as in Bolton, Chen, and Wang (2011), we also assume that external equity financing costs are proportional to the capital stock K_t, so that $h_0^K K_t$ denotes the fixed equity-issuance cost and $h_1^K M_t$ refers to the proportional equity-issuance cost, where $h_0^K \geq 0$ and $h_1^K \geq 0$, and M_t is the net amount raised via external equity issuance.

8.2 Solution

As K_t replaces Y_t as the state variable measuring firm size, we use v_t to denote the firm's total value scaled by capital (Tobin's average q) and i_t to denote the investment-capital ratio:

$$v_t = V_t/K_t = V(K_t, X_t)/K_t \quad \text{and} \quad i_t = I_t/K_t.$$

Let $p_t = P_t/K_t = P(K_t, X_t)/K_t$. Note that the marginal firm value of capital is equal to the marginal equity value of capital, which we refer to as the *marginal q* and denote by q_m:

$$q_m = \frac{\partial V(K, X)}{\partial K} = v(x) - xv'(x) = p(x) - xp'(x). \tag{56}$$

8.2.1 Costless Equity Issuance

The firm's average q at the optimal target leverage x^*, $v(x^*)$, satisfies the same Gordon-growth formula given in (23)-(24). Compared with our EBIT-based model of Section 2, we have an additional control variable, investment, which satisfies the standard FOC for investment in q models:

$$\frac{1}{\psi'(i^*)} = v(x^*) = x^*. \tag{57}$$

Equation (57) equates its marginal cost of investing, $1/\psi'(i^*)$, with $v(x^*)$, its marginal q. At the optimal target leverage, we have $q_m(x^*) = v(x^*)$. This follows from $p'(x^*) = 1$ (or equivalently $v'(x^*) = 0$) under costless equity issuance. It also explains why the firm's marginal q is equal to its average q, as in Hayashi (1982).

8.2.2 Costly External Equity

When equity issuance is costly, the solution again features four mutually exclusive regions: payout, debt financing, equity financing, and default region as in our EBIT-based model.

In the debt financing region, the scaled equity value, $p(x)$, satisfies the following ODE:

$$
\gamma p(x) = \max_i \quad -\left[A - i - c(x) - \theta(c(x))\right] p'(x) + \psi(i)(p(x) - xp'(x)) + \frac{\sigma_K^2 x^2}{2} p''(x)
$$
$$
+\lambda_K \left[\int_{\overline{Z}(x)}^1 Zp(x/Z)dF(Z) - p(x)\right] , \tag{58}
$$

where $c(x) = C(K, X)/K$ is the scaled debt coupon payment. We note the following three key differences between the ODE (58) for $p(x)$ in our q model and the ODE for $p(x)$ given in (38) in our EBIT-based model in Section 2.

First, as x is debt X scaled by endogenous K rather than by exogenous EBIT Y, the scaled free cash flow to the entire firm is $A - i(x)$ in our q model as opposed to unity (by definition) in our EBIT-based model.

Second, investment depends on not only marginal q but also marginal cost of debt, as debt is the marginal source of financing for investment. The following FOC holds:

$$
\frac{1}{\psi'(i(x))} = \frac{q_m(x)}{-p'(x)} = \frac{p(x) - xp'(x)}{-p'(x)} \tag{59}
$$

in our q model with costly external equity. Importantly, because $\psi(\cdot)$ is concave ($\psi''(\cdot) < 0$), the optimal i increases with $q_m(x)/-p'(x)$, the ratio between the marginal q in the numerator of (59) and the marginal cost of debt, $-p'(x) > 0$ in the denominator.[49]

Third, we replace the exogenous drift parameter, μ, in our EBIT-based model with the endogenous function $\psi(i(x))$ in the pricing equation (58) for $p(x)$. While behaving as a credit card revolver, the firm also actively adjusts its investment (and asset sale, if $i < 0$) to manage its leverage dynamics.

Finally, for brevity, we omit value functions in the other three regions and equity issuance conditions together with various boundary conditions, as they are essentially the same as in our EBIT-based model (after adjusting the definition for x.)

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[49]The marginal q and $-p'(x)$ are correlated in our model. Also, as we show, the marginal cost of debt financing is greater than one, $-p'(x) \geq 1$.

8.3 Quantitative Analysis

For our quantitative analysis, we focus on the shareholder impatience channel, with $\gamma > r$. For brevity, we leave out any tax considerations.

Table 6: PARAMETER VALUES FOR THE q MODEL

This table summarizes the parameter values for our q model of investment and leverage dynamics in Section 8. Whenever applicable, parameter values are annualized.

Parameters	Symbol	Value
shareholders' discount rate	γ	7%
risk-free rate	r	6%
diffusion volatility	σ_K	40.6%
jump arrival rate	λ_K	1.25
jump recovery parameter	β_K	6.57
capital productivity	A	28%
liquidation recovery scaled by capital	ℓ_K	0.33
adjustment cost parameter	ξ	2
equity issue fixed cost	h_0^K	0.01
equity issue proportional cost	h_1^K	0.06

We use the following functional form for the investment-efficiency function $\psi(\cdot)$:

$$\psi(i) = i - \frac{\xi}{2}i^2, \tag{60}$$

with $\xi > 0$. As in our EBIT-based model, we assume that the cumulative distribution function for the recovery fraction of the capital stock, Z, also satisfies $F(Z) = Z^{\beta_K}$, where $Z \in [0,1]$ and $\beta_K > 0$ is the parameter. To ease comparison with our EBIT-based model specification, we use the same parameter values for the risk-free rate, the diffusion and jump parameters (β_K, σ_K, and λ_K) and the marginal cost of equity issuance h_1^K as in our EBIT-based model: $r = 6\%$, $\lambda_K = 1.25$, $\sigma_K = 40.6\%$, $\beta_K = 6.57$, and $h_1^K = 0.06$.

As for the other five parameters, we set $h_0^K = 1\%$, $\xi = 2$, $A = 28\%$, $\gamma = 7\%$, and $\ell_K = .33$ so as to target: i) an equity issuance frequency of 7% per annum; ii) an average investment-capital ratio i at 10%; iii) a mean average q of 1.25; iv) average market leverage ML at 27%; and, v) a debt recovery upon default of 51% of the debt face value.[50] These

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[50]While we pin down the five parameters with five empirical targets, consistent with our intuition, we

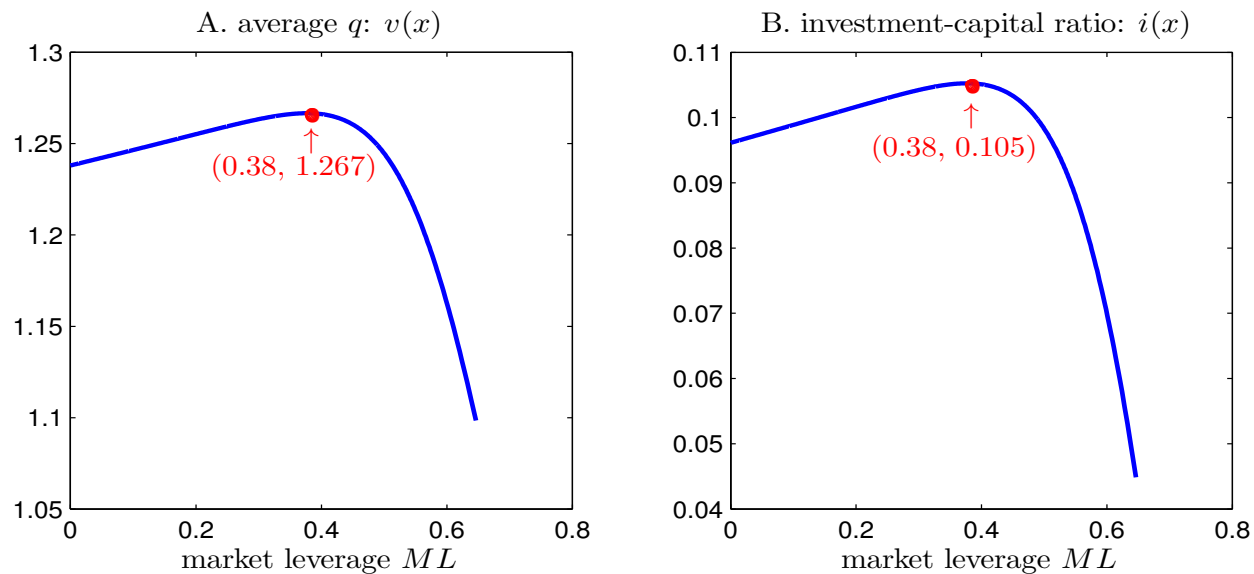parameter values are broadly in line with the values used in dynamic corporate finance and q theory literatures.

Figure 7: **Optimal capital structure, investment and Tobin's average q: Classic tradeoff theory with costless equity issuance:** $h_0^K = h_1^K = 0$. Panels A and B plot the firm's average q, $v(x)$, and investment i with respect to market leverage $ML = x/v(x)$. The firm optimally keeps its market leverage at the constant value-maximizing level: $ML^* = 38\%$ at all time until it defaults when a jump arrives and causes its EIBT to decrease by more than $1 - Z^* = 62\%$. The optimal investment-capital ratio is $i^* = 10.5\%$ per annum and the corresponding average q is $q^* = 1.267$. All parameter values are given in Table 6.

8.3.1 Classical Tradeoff Theory with Costless Equity Issuance.

In Panels A and B of Figure 7, we plot the firm's average q and investment i for the admissible range of market leverage ML, respectively. The optimal market leverage is $ML^* = 38\%$ (corresponding to a debt-capital ratio of $x^* = 0.48$), the optimal investment-capital ratio is $i^* = 10.5\%$ per annum, and the Tobin's average q is $q^* = 1.267$ at all time, until the firm defaults (it does so when a jump arrives that causes its capital to decrease by more than $1 - Z^* = 62\%$). As in our EBIT-based model, the firm constantly issues equity and pay out

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find that the fixed equity issuance cost h_0^K is most sensitive to the equity issuance frequency, the recovery parameter ℓ_K is most sensitive to the creditors' liquidation recovery value, the productivity A and adjustment cost parameter ξ respond mostly to the average q and investment targets, and finally, the cost of capital wedge $\gamma - r$ is most sensitive to the average market leverage target.

Table 7: STATIONARY DISTRIBUTIONS OF x AND MARKET LEVERAGE ML. ALL PARAMETER VALUES ARE GIVEN IN TABLE 6.

	Mean	Std. dev.	1%	5%	25%	50%	75%	95%	99%
x	0.33	0.12	0.23	0.23	0.25	0.28	0.38	0.59	0.72
ML	0.27	0.09	0.18	0.19	0.20	0.23	0.30	0.47	0.59

to shareholders so that the firm's market leverage stays at the target $ML^* = 38\%$ at all time before it defaults, which is clearly counter-factual.

8.3.2 Costly-equity-issuance Case

Table 7 reports the predicted distributions for the debt-capital ratio x and market leverage ML. The main results for the distribution are similar to our EBIT-based model. Firms manage leverage prudently. Average market leverage is 27%, the median market leverage is 23%, and only 1% firms have leverage higher than 59%.

In Figure 8, we plot our q model solution when equity issuance is costly. As in our EBIT-based model, there are four mutually exclusive regions divided by three endogenous thresholds: 1.) the payout threshold, $\underline{x} = 0.23$, which corresponds to the optimal target leverage of $ML(\underline{x}) = 18.4\%$; 2.) the equity-issuance threshold $\widehat{x} = 0.84$, which corresponds to market leverage of $ML(\widehat{x}) = 68.6\%$; and 3.) the default threshold $\overline{x} = 1.20$. When market leverage is in the range of $(ML(\widehat{x}), ML(\overline{x})) = (68.6\%, 100\%)$, the firm issues equity to bring its market leverage down to $ML(\widetilde{x}) = 41.3\%$ in the debt-financing region, where $v(x)$ is equal to the marginal cost of equity financing h_1, i.e., $-v'(\widetilde{x}) = h_1 = 0.06$.

Average q versus marginal q. Panel A plots the average q, $v(x) = p(x) + x$, which is nonlinear and decreases with x. The gap between the constant first-best $v^{FB} = 1.336$ and $v(x)$ reflects the significant costs of equity issuance and financial distress.

Panels B shows that the marginal q, $q_m(x)$, is non-monotonic. It increases in the concave $v(x)$ region $x \in (\underline{x}, \breve{x}) = (0.23, 0.71)$, reaches the maximal value of 1.307 at the $v(x)$ inflection point $\breve{x} = 0.71$, and decrease in the convex region $x \in (\breve{x}, \widehat{x}) = (0.71, 0.84)$.

Note that marginal q is always larger than the average q in the debt region $x > 0$:
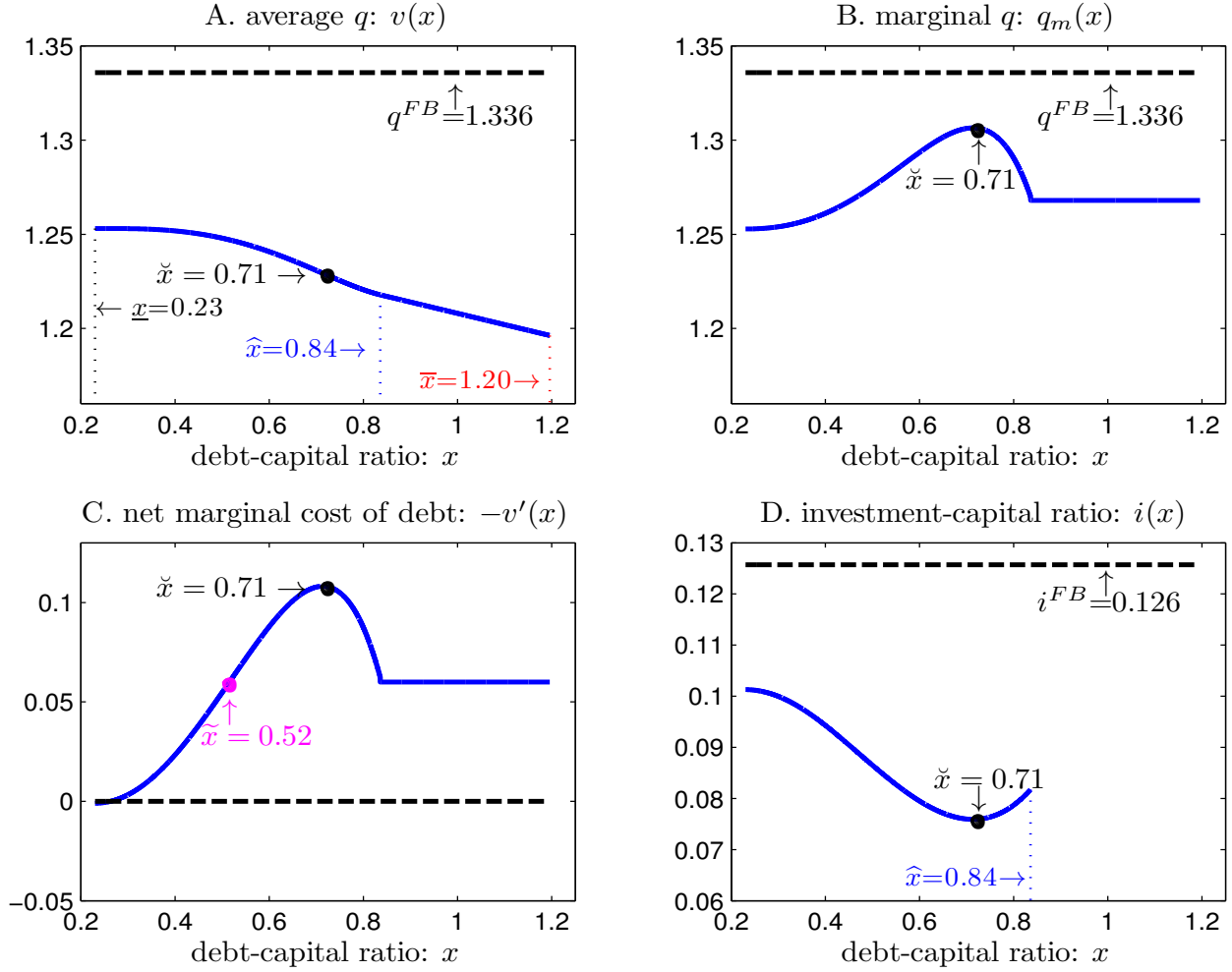
$$q_m(x) - v(x) = -xv'(x) \geq 0. \tag{61}$$

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Figure 8: **Tobin's average** q $(v(x))$, **marginal** q, $(q_m(x))$, **(net) marginal cost of debt** $(-v'(x))$, **and investment-capital ratio** $(i(x))$. The first-best investment-capital ratio is $i^{FB} = 0.126$ and the first-best average q is $q^{FB} = 1.336$. The endogenous (lower) payout boundary is $\underline{x} = 0.23$ and the endogenous (upper) default boundary is $\overline{x} = 1.20$. The equity issuance boundary is $\widehat{x} = 0.84$ and the recapitalization target is $\widetilde{x} = 0.52$. Finally, the $v(x)$ inflection point is $\breve{x} = 0.71$, below which average q, $v(x)$, is concave and above which $v(x)$ is convex. All parameter values are given in Table 6.

This follows from $-v'(x) \geq 0$ and $x > 0$, as a unit of capital makes the firm less levered and hence less financially constrained (decreasing x) making the marginal value of capital (marginal q) to be larger than the average value of capital (average q). That is, the solid blue line in Panel B is above the solid blue line in Panel A for all levels of x. Marginal q and average q coincide only at the payout boundary where $x = \underline{x}$: $q_m(\underline{x}) = v(\underline{x}) = 1.253$.

Endogenous risk aversion versus risk seeking and marginal cost of debt, $-v'(x)$.
Panel C shows that the marginal cost of debt, $-v'(x)$, increases in the region where $x \in (\underline{x}, \breve{x}) = (0.23, 0.71)$, reaches the maximal value of 0.108 at the $v(x)$ inflection point $\breve{x} = 0.71$, and decreases in the region where $x \in (\breve{x}, \widehat{x}) = (0.71, 0.84)$. Note that $-v'(x)$ has the same monotonicity property with respect to x as the marginal q. This is because $q'_m(x) = -xv''(x)$ and the firm is in debt $(x > 0)$. That is, the firm is endogenously risk averse (and $v''(x) < 0$) where $x \in (\underline{x}, \breve{x}) = (0.23, 0.71)$ but endogenously risk seeking where $x \in (\breve{x}, \widehat{x}) = (0.71, 0.84)$.

Corporate investment $i(x)$. Panel D shows that $i(x)$ is at its highest level when the firm is financially unconstrained at its payout boundary: $\underline{i} = i(\underline{x}) = 0.105$, decreases with x in the concave $v(x)$ region $x \in (\underline{x}, \breve{x}) = (0.23, 0.71)$, reaches the minimal value of 0.076 at the $v(x)$ inflection point $(\breve{x} = 0.71)$, and then increases in the convex $v(x)$ region $\in (\breve{x}, \widehat{x}) = (0.71, 0.84)$. Note that there is no investment policy $i(x)$ in the equity-issuance region $x > \widehat{x} = 0.84$ as the firm immediately leaves this region by issuing equity to bring its debt-EBIT ratio x to the recapitalization target $\widetilde{x} = 0.52$.

Note that investment and marginal q move in exactly the opposite direction with respect to x in both concave and convex $v(x)$ regions.[51] This relation is the opposite of what neoclassical q-theory of investment (e.g., Hayashi, 1982) predicts: investment increases with marginal q, which is only valid when the Modigliani-Miller financing irrelevance conditions hold (and capital adjustment cost is convex.) The intuition for this result is as follows.

First, marginal q not only depends on the firm's investment opportunity but also the firm's balance sheet (e.g., leverage) in a world when the firm is financially constrained (e..g, due to costly external financing.) Second, investment for a financially constrained firm is determined by the ratio between marginal q and the marginal cost of debt financing $(-v'(x) > 0)$, as shown in (59). Differentiating (59) with respect to x, and using the

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[51]In the credit line section, Bolton, Chen, and Wang (2011) derive similar results about investment and marginal q. Key features of our model that differ from that paper include 1.) debt capacity and credit risk pricing are endogenous in our model; 2.) shocks are permanent rather than transitory (i.i.d.)), and 3.) the convex $v(x)$ region where the firm is a risk seeker.

concavity of $\psi(\,\cdot\,)$, we obtain:[52]

$$i'(x) = -\frac{(\psi'(i(x)))^2}{\psi''(i(x))} \frac{\partial}{\partial x} \left[\frac{q_m(x)}{-p'(x)} \right] = \underbrace{-\frac{(\psi'(i(x)))^2}{\psi''(i(x))} \frac{p(x)}{(p'(x))^2}}_{\text{positive as } \psi(\,\cdot\,) \text{ is concave}} p''(x) \,. \tag{62}$$

Therefore, $i(x)$ decreases with x in the concave $v(x)$ region where the firm is endogenously risk averse $p''(x) = v''(x) < 0$ and $i(x)$ increases with x in the convex $v(x)$ region where the firm is a risk seeker $p''(x) = v''(x) > 0$.

Finally, in the region where $x \in (\widehat{x}, \overline{x}) = (0.84, 1.20)$, the firm immediately issues costly external equity to bring its debt-capital ratio back to $\widetilde{x} = 0.52$. This equity issuance region is the flat region in Panels B and C. The firm only enters into this region via a jump but immediately issues equity to exit from the region barely spending any time there.

Summary. In sum, investment and leverage dynamics exhibit both *debt overhang* (Myers, 1977) and *risk shifting* (Jensen and Meckling, 1976) aspects, however, not for the usual reasons. Investment policy serves two roles, value creation (via productive capital accumulation) and prudent leverage management (via underinvestment) to avoid a costly equity issue or default.

Debt overhang is driven by liquidity management considerations. When leverage is low or moderate ($\underline{x} \le x \le \breve{x}$), the firm's endogenous marginal cost of debt financing increases with x, as there is then a greater chance that the firm will be pushed into a costly recapitalization or default. As a result the higher the firm's debt-capital ratio x, the more the firm underinvests: $i'(x) < 0$.

Beyond the inflection point of $v(x)$, i.e., $x \ge \breve{x}$, risk-shifting incentives kick in, but not because losses are borne disproportionately by debt holders (debt is short term and always fairly priced). Risk shifting is caused by the fixed costs of equity issuance that the firm is likely to pay in the foreseeable future. Anticipating that the firm's leverage will be much lower after recapitalization and hence capital stock will be much more valuable (less debt overhang), the firm's incentive to mitigate underinvestment when $x \ge \breve{x}$ becomes stronger. Since the firm is prepared to issue equity should its leverage exceed $ML(\widehat{x}) = 68.6\%$, it is thus optimal for the firm to reduce its underinvestment and build a larger capital stock in

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[52]In the concave $v(x)$ region, marginal q increases with x at a slower rate than the marginal cost of financing; investment thus decreases with marginal q. In the convex $v(x)$ region, marginal q decreases with x at a slower rate than the marginal cost of financing, so that investment $i(x)$ increases with marginal q.

the convex $v(x)$ region where $ML \in (ML(\breve{x}), ML(\widehat{x})) = (58.1\%, 68.6\%)$ to prepare for the future after it deleverages its balance sheet. Again, our analysis conveys the significance of nonlinearity and non-monotonicity for both investment and leverage dynamics caused by costly external equity issuance.

Finally, we note that all other key predictions of our EBIT-based model remain valid in our q-theory formulation.

9   Conclusion

We develop a dynamic capital structure model for a financially constrained firm that faces costly external equity. Although parsimonious, our model yields rich and empirically valid predictions that contribute to our understanding of the key role played by equity issuance costs in determining corporate leverage dynamics, earnings retention, payouts, equity issuance, and investment policies observed in practice. Our model can also explain why leverage dynamics is persistent, tends to revert to the target level, when it is relatively low, and diverges into a debt death-spiral, when it is sufficiently high.

Paradoxically, the explanation for the observed low corporate leverage is not that debt is costly, but that equity is costly. One would think that when equity is costly firms would want to rely more on debt. But that is a static intuition. From a dynamic perspective firms seek to avoid costly equity issuance in the future and therefore maintain financial slack today by keeping leverage low. When equity issuance is so costly that it is not a viable option for the firm, we find that the firm barely takes on any debt.

There has been a long-running and still unresolved empirical debate between the tradeoff and pecking-order theories of capital structure since Myer's 1984 AFA presidential address.[53] In effect, our model combines insights from both theories by adding equity issuance costs to a simple dynamic tradeoff theory model. Our model predictions, however, differ from both static theories in very important ways, such as the highly nonlinear, non-monotonic, path-dependent leverage dynamics, endogenously risk averse and risk seeking behaviors, and the occasional recapitalization through equity issuance to bring down leverage.

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[53]See Fama and French (2002) and Frank and Goyal (2008) for example.

Appendix (to be Added)

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