

Gradual Arbitrage*

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Abstract

Capital often flows slowly from one market to another in response to buying opportunities. I provide an explanation for this phenomenon by considering arbitrage across two segmented markets when arbitrageurs face illiquidity frictions in the form of price impact costs. I show that illiquidity results in *gradual arbitrage*: mispricings are generally corrected slowly over time rather than instantaneously. The speed of arbitrage is decreasing in price impact costs and increasing in the level of competition among arbitrageurs. This means arbitrage is slower in more illiquid markets for two reasons: First, there is a direct effect, as illiquidity affects the equilibrium trading strategies for a given level of competition among arbitrageurs (*strategic effect*). Second, in equilibrium fewer arbitrageurs stand ready to trade between illiquid markets, further slowing down the speed of arbitrage (*competition effect*). Jointly, these two effects may help explain the observed cross-sectional variation of arbitrage speeds across different asset classes.

Keywords: Illiquidity, Limits to Arbitrage, Strategic Trading, Strategic Arbitrage, Imperfect Competition

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There is now a large body of empirical evidence showing that capital flows only slowly between markets to exploit buying opportunities. As a result, mispricings of financial assets often persist for substantial periods of time, and are arbitrated away only gradually. For example, Mitchell, Pedersen, and Pulvino (2007) document that in the convertible arbitrage and merger arbitrage markets mispricings can persist for months. Gradual price adjustment on a similar time scale have also been documented in the corporate bond markets by Acharya, Schaefer, and Zhang (2008) (in response to the GM/Ford downgrade), and by Newman and Rieron (2003) (in response to supply shocks created by large bond issues). In equity markets, Harris and Gurel (1986), Greenwood (2005), Coval and Stafford (2007) and Andrade, Chang, and Seasholes (2008), among others, document gradual price reversals that can take anywhere from a few days to multiple weeks or even months. These empirical findings are puzzling since classical asset pricing models predict that buying opportunities should be exploited instantly by profit-maximizing arbitrageurs.

What is the reason that so many empirical studies document *gradual* rather than instantaneous arbitrage? And what explains the differences arbitrage speeds across different markets? In this paper, I show that that the observed sluggishness of capital flows and the resulting gradual price adjustments can arise as the equilibrium outcome of a simple model of strategic trading with liquidity frictions, even when the arbitrage opportunity in question is entirely riskless. The intuition is that when markets are illiquid, strategic arbitrageurs, such as hedge funds, will take into account that trading against an arbitrage opportunity too aggressively will reduce profits because they incur losses due to price pressure effects. These price impact costs are incurred both in the markets in which arbitrageurs buy, and—in case they need to free up capital to buy—in the markets in which they sell.

The model proposed in this paper predicts a direct link between market liquidity and the speed of arbitrage: arbitrage is slower in markets that are less liquid. This link is supported by recent empirical evidence by Roll, Schwartz, and Subramahnyam (2007), who show that mispricings in futures markets (the futures-cash basis) persist longer when markets are more

illiquid. The model also predicts that the speed of arbitrage depends crucially on the level of competition among arbitrageurs—the more arbitrageurs are chasing the same opportunity, the more aggressively they trade against it in equilibrium. Finally, the model shows that the level of competition, when endogenized, depends on market liquidity: in more liquid markets, arbitrage is on average more profitable, such that more arbitrageurs enter. This means that, in equilibrium, competition among arbitrageurs is higher in liquid than in illiquid markets.

Through these two channels, illiquidity and the endogenous level of competition among arbitrageurs, the model can help explain the cross-sectional variation of arbitrage speed across different types of markets and asset classes. Consider, for example, quantitative hedge funds that are mainly active in liquid equity markets. The model predicts that mispricings in those markets should be corrected swiftly, since trading costs are low and because funds compete fiercely to exploit opportunities once they open up. In contrast, consider markets for less liquid securities, such as mortgages or infrequently traded bonds. In those markets, arbitrage will be more gradual, both because of the direct effect of higher trading costs on arbitrageurs' trading strategies, and because in equilibrium there is less competition for arbitrage opportunities.

In the model, which is set in continuous time, two assets with identical cash flows are traded in two segmented markets. In each market, there are local investors who can trade only within their own market. In addition, there are arbitrageurs who can trade dynamically between the two markets and thus have the ability to exploit price differentials to make riskless arbitrage profits. When the prices of the two assets diverge due to a demand shock in one of the local markets, arbitrageurs exploit the price differential by selling in one market and buying in the other, taking into account that their trades affect market prices. Illiquidity is captured by the assumption that prices change for two reasons when arbitrageurs trade between the two markets. First, the price of the two assets is affected by the arbitrageurs' aggregate position in each market: When an asset is underpriced and arbitrageurs start buying it, the increase in the arbitrageurs' positions increases the price of that asset. This

is the standard effect that arises in the presence of a downward-sloping residual demand curve for a risky asset. Second, prices also react temporarily to short-term trading pressure. This means, for example, that when arbitrageurs quickly offload a position in one market to capitalize on an opportunity in another market, they temporarily depress the price of the asset they sell more than would be implied by the long-run demand curve. Price pressure of this type is a widely observed empirical phenomenon and is often a first-order concern when hedge funds devise and implement trading strategies.

When trading between the two markets, the arbitrageurs thus face a tradeoff. On the one hand, losses from temporary price pressure give arbitrageurs an incentive to spread their trades and exploit arbitrage opportunities gradually over time. On the other hand, the presence of competing arbitrageurs, who are exploiting the same arbitrage opportunity, gives arbitrageurs an incentive to speed up their trades in order to trade before their competitors can reduce the price differential between the two markets. Solving a continuous-time trading game, I show that in equilibrium mispricings are generally corrected slowly over time, rather than instantly. The model shows that, in equilibrium, the speed of arbitrage is decreasing in illiquidity (the temporary price impact costs that arbitrageurs incur), and increasing in the level of competition among arbitrageurs. Only in the competitive limit are mispricings corrected instantly, as the classical frictionless model would predict.

But what level of competition should one expect in a market with particular liquidity characteristics? To answer this question, I endogenize the number of arbitrageurs that are active between the two markets. When arbitrageurs need to incur upfront costs to stand ready to trade between the two markets—arbitrageurs may need to learn about the assets, hire the required personnel, or set up the necessary trading operations—, the equilibrium number of arbitrageurs is increasing in the probability and the variance of the local demand shocks, but decreasing in illiquidity. This dependence of the level of competition on market liquidity is important, since it implies that in illiquid markets arbitrage is slower for *two* reasons. First, there is a direct effect, since illiquidity affects the equilibrium trading strategies for

a given level of competition (*strategic effect*): holding the number of arbitrageurs constant, an increase in illiquidity implies that each arbitrageur will trade less aggressively against the arbitrage opportunity. Second, a rise in illiquidity reduces the number of arbitrageurs that enter ex ante and stand ready to trade between the two markets when prices diverge. This reduces the intensity of competition between the arbitrageurs, further slowing down the equilibrium speed of arbitrage trading (*competition effect*). It is the combination of these two effects that determines the equilibrium speed of arbitrage. This means that differences in liquidity and the level of competition among arbitrageurs may jointly help explain the cross-sectional variation of arbitrage speeds across markets of different liquidity characteristics.

The paper contributes to the literature on the limits to arbitrage. Most existing papers in this literature focus on mechanisms that force traders to unwind their arbitrage trades at times when this is unprofitable. For example, in DeLong, Shleifer, Summers, and Waldmann (1990), the time at which arbitrageurs unwind their trades is imposed exogenously. This results in noise trader risk, since random supply shocks may increase the mispricing just at the time arbitrageurs need to close out their positions. In Shleifer and Vishny (1997), forced unwinding occurs as an outcome of performance-based arbitrage—interim losses may lead investors to withdraw funds from arbitrageurs in a setup of ‘delegated arbitrage’. In Gromb and Vayanos (2002), Liu and Longstaff (2004), and Kondor (2008), forced unwinding arises through explicit collateral constraints on arbitrageurs.¹ In contrast to these papers, I abstract away from noise trader risk, delegated arbitrage frictions and explicit financial constraints, and focus explicitly on the impact of illiquidity and arbitrageur competition on the speed of arbitrage, which none of the above papers focus on. Moreover, the above papers generally do not analyze the effect of competition on the dynamics of arbitrage (in fact, all of the above papers are set in a perfectly competitive framework). Kondor (2008) informally discusses the effect of competition on arbitrage, yet his proxy for competition—the

¹Other papers in the literature on the limits to arbitrage include Abreu and Brunnermeier (2002), who focus on synchronization risk, and Xiong (2001) and Kyle and Xiong (2001), where arbitrage is limited by wealth effects that arise when arbitrageurs lose money. Stein (2009) proposes a model in which arbitrage is limited because arbitrageurs are uncertain about how many other arbitrageurs have entered the same trade.

aggregate amount of capital arbitrageurs have at their disposal—is not entirely satisfactory. After all, the capital may be deployed by one monopolistic arbitrageur or a large number of competitive arbitrageurs (which is the assumption in Kondor’s paper). The Cournot nature of this model, on the other hand, allows me to explicitly analyze the effect of competition on arbitrage. Moreover, the model allows me to endogenously determine the level of competition among arbitrageurs, as a function of the underlying model parameters. In that regard, the paper that is closest to this paper is Zigrand (2004) who presents a general equilibrium model with an endogenous number of arbitrageurs. In contrast to this paper, however, his model is static and does not focus on illiquidity frictions or the speed of arbitrage.

More broadly, the paper relates to the literature on asset pricing in the presence of illiquidity frictions, which is surveyed in Amihud, Mendelson, and Pedersen (2006). Moreover, in setting up a model of arbitrage in the presence of illiquidity frictions I use tools from the recent literature on strategic trading in illiquid markets. In particular, the arbitrage model I propose is a two-asset extension of the continuous-time trading game developed in Carlin, Lobo, and Viswanathan (2007). Their paper analyzes liquidity provision among strategic traders in a repeated game setting, but does not address how illiquidity affects price dynamics in financial markets through its impact on equilibrium arbitrage strategies.

The remainder of the paper is organized as follows. Section 1 outlines the model setup. Section 2 solves for the equilibrium behavior of the strategic arbitrageurs and the resulting price dynamics and profits. Section 3 endogenizes the number of arbitrageurs that are active between the two markets in free-entry equilibrium. Section 4 concludes. All proofs are contained in the appendix.

1 Model Setup

I consider a stylized setting with two segmented markets. In each of these markets, a risky asset is traded. These two risky assets are labeled $j \in \{1, 2\}$. Time runs continuously and is

indexed by $t \in [0, T]$. At time T , each of the risky assets pays a random final dividend D_j .² The dividends paid at T are identical and have a random distribution across a common mean of D .³ No dividends are paid before maturity. Both risky assets are available in aggregate supply S . A riskless bond, whose net return I normalize to zero, is also traded.

There are two types of traders in the model, local investors and strategic arbitrageurs. Each market is populated by a mass-one continuum of local buy-and-hold investors with a downward-sloping demand curve for the local asset. These local investors can only trade the risky asset in their home market, but not the risky asset traded in the other market. Local investors may be thought of as individual investors or other buy-and-hold investors, such as pension funds. What is critical is that they lack the required expertise or infrastructure to trade across the two markets. In addition to the local investors, there are n risk-neutral arbitrageurs who can buy and sell in both markets to exploit temporary mispricings. The arbitrageurs can be interpreted as hedge funds or other professional traders with the expertise and infrastructure necessary to trade across segmented markets. I initially treat the number of arbitrageurs n as given, but will endogenize it in section 3.

Pricing function. The equilibrium price of the risky asset in each market is determined by the demand of the local investors, the demand of the arbitrageurs, and—to capture illiquidity—short-term selling or buying pressure (which I outline in more detail below). The local investors are price takers, and their aggregate demand for their home asset j is at each point in time given by

$$Z_j(t) = \frac{1}{\gamma}[D - P_j(t)]. \quad (1)$$

Local investors thus demand the risky asset when the expected final dividend D is higher than

²I use subscripts $j = 1, 2$ to denote assets. Later on when I introduce arbitrageurs, I will use superscripts $i = 1, \dots, n$ to denote arbitrageurs.

³This means that a long-short position of the two assets is riskless. One could extend the model to allow for risky arbitrage, for example the case in which the two final dividends are identically distributed, but their realizations may differ. This does not change the model as long as the arbitrageurs are risk-neutral, which is assumed below.

the current price of the asset, and they will buy more of it the more the expected dividend exceeds the price. The parameter γ determines the slope of the local investors' demand curve and is assumed to be equal in both markets.

Modeling a downward-sloping demand curve this way is a common assumption in the literature. For example, a similar residual demand curve is used in Brunnermeier and Pedersen (2005) and Xiong (2001). There are a number of ways to justify a downward sloping demand curve of this type. The local investors could be risk averse. In that case, in order for the local investors to hold more of the risky asset, the price of the asset has to drop in order to compensate them for the additional risk they are bearing. For example, if the final dividend is normally distributed and the local buy-and-hold investors have CARA utility, the resulting demand curve would be given by equation (1), with $\frac{1}{\gamma} = \rho\sigma^2$, where ρ is the coefficient of absolute risk aversion and σ^2 the variance of the final dividend. Alternatively, local long-term investors may be subject to institutional constraints that result in a downward-sloping demand curve.

Absent any selling or buying pressure, the long-term part of the price of asset j is given by the market clearing condition $Z_j(t) + X_j(t) = S$, where $X_j(t) = \sum_{i=1}^n X_j^i(t)$ denotes the aggregate position of the arbitrageurs at time t . Solving for the long-term price component yields $\bar{P}_j(t) = u + \gamma X_j(t)$, where I define $u \equiv D - \gamma S$.⁴ In other words, u is the price the assets would trade at if the total supply S was held by local investors.

To capture temporary buying or selling pressure, I assume that in addition to the level of the arbitrageurs' aggregate position, the price of asset j is also affected by the rate of change in the arbitrageurs' aggregate position, $Y_j(t) = \dot{X}_j(t)$. In other words, the faster the arbitrageurs are buying or selling an asset, the more the price moves into that direction. More specifically, I assume that when arbitrageurs buy or sell asset j at an aggregate rate $Y_j(t) = \sum_{i=1}^n Y_j^i(t)$, this temporarily changes the price of asset j by an amount $\lambda_j Y_j(t)$, where the parameter λ_j measures the short-term price impact—or illiquidity—of asset j .

⁴I implicitly assume that arbitrageurs have limited capital, since otherwise they would hold the entire asset supply, and the price of each risky asset would be trivially equal to the expected payoff, i.e. $P_j(t) = D$.

This temporary price pressure effect means that arbitrageurs do not have immediate access to the long-run demand curve, but rather a ‘tilted’ short run version of the long-run demand curve. Note that while I assume a common γ for both assets, the size of the short-term price pressure effect can differ between assets, i.e. $\lambda_1 \neq \lambda_2$, which may, for example, capture institutional differences between the two markets. Theoretically, short-term price pressure term may be a consequence of search frictions—when it takes time to locate trading partners, arbitrageurs do not have access to the full long-term demand curve at each point in time. Alternatively, the short-term price pressure could be caused by a finite limit order book that at each point in time reflects only part of the long-run demand curve. This limit order book replenishes each instant, but never gives the arbitrageurs access to the entire long-run demand curve. Both of these interpretations would imply that if a trader wants to trade a large amount in a short period of time, he has to walk down a short-run demand curve above and beyond what would be implied by the long-run demand curve.

Combining the permanent effect of the arbitrageurs’ aggregate positions and the temporary effects of the aggregate trading rate on prices, the price of asset j can be written as⁵

$$P_j(t) = u + \gamma X_j(t) + \lambda_j Y_j(t), \quad j = 1, 2 \quad (2)$$

where the aggregate position of the arbitrageurs, $X_j(t)$, is given by their initial position plus the cumulative aggregate trades made from time 0 to t :

$$X_j(t) = X_j(0) + \int_0^t Y_j(s) ds. \quad (3)$$

Both temporary and permanent price impact of large trades are empirically well documented. For example, Shleifer (1986), Chan and Lakonishok (1995), and Wurgler and Zhuravskaya (2002) provide evidence for downward sloping demand curves for stocks. Kraus and

⁵A pricing function of the same type is used by Carlin, Lobo, and Viswanathan (2007) to study episodic breakdowns in liquidity provision.

Stoll (1972), Holthausen, Leftwich, and Mayers (1987), Holthausen, Leftwich, and Mayers (1990), and Madhavan and Cheng (1997) document permanent and temporary price effects for block trades on the New York Stock Exchange. This means that the type of illiquidity effects assumed above are economically significant even for relatively liquid securities, such as U.S. equities. There is also ample evidence that financial institutions take these illiquidity frictions into account when trading—they often rely on trading systems that estimate permanent and temporary price impact parameters, and calculate trading schedules based on these estimates. One example is Citigroup’s Best Execution Consulting Services (BECS) software, which is described in Almgren, Thum, Hauptmann, and Li (2005).

Arbitrage opportunity. To analyze dynamic arbitrage between the two markets, I consider a situation in which the prices of the two risky assets diverge due to differences in local demands. I assume that at time $t = 0$ there is an exogenous negative demand shock of size \tilde{Z} in market 1, and that the arbitrageurs initially hold positions of equal size in the two risky assets. As a natural justification for the equal size of the arbitrageurs positions in the two markets, assume that ex-ante the demand shock is equally likely to occur in either of the two markets. When the demand shock occurs, the difference in local demands creates a price wedge between the two assets, which absent any trading by the arbitrageurs would be of size $\gamma\tilde{Z}$ —the size of the demand shock multiplied by the slope of the demand curve of the asset in the affected market. I also assume that there are no additional demand shocks after time 0. I do this in order to abstract away from noise trader risk (see DeLong, Shleifer, Summers, and Waldmann (1990)).

Since the risky assets provide the same payoffs at time T , the arbitrageurs will trade between the two markets to exploit the price difference between the two assets. I focus on dynamic trading strategies that result in riskless arbitrage profits given the initial position of the arbitrageurs. In other words, arbitrageurs sell in one market, buy in the other, and pocket the difference—while keeping their overall exposures constant. This means that arbitrage involves no risk in addition to that already implied by the arbitrageurs’ initial positions.

This could, for example, be the result of risk management constraints that the arbitrageurs need to observe. Alternatively, the arbitrageurs' combined holdings of the two assets might part of an optimally weighted portfolio whose overall weights are fixed, at least in the short run.

I make two further assumptions to simplify the model. First, I assume that arbitrageurs are fully invested (i.e. the sum of their positions in the two markets equals their desired aggregate holdings in the asset) at time 0. This assumption is meant to capture the fact that it is costly for arbitrageurs to keep spare capital on the side in anticipation of buying opportunities. Consequently, at any moment in time, much of their capital is tied up in investments, which means that often they will have to free up capital in one market to buy in another. Second, I assume that the arbitrageurs' joint positions are large enough to eliminate the price gap completely. I make this assumption to make sure that any delay in arbitrage that occurs in this model is not due a fundamental lack of arbitrageurs' capital. Rather, it is caused by the liquidity cost of moving capital from one market to another. As the following section shows, this liquidity cost of moving capital is an important determinant of the arbitrageurs' trading strategies and the speed of arbitrage.

2 Equilibrium Dynamic Arbitrage

In this section I set up the arbitrageurs' maximization problem and calculate the equilibrium trading strategies. Based on the equilibrium strategies I then characterize the evolution of the equilibrium price gap, and calculate the equilibrium profits to arbitrageurs. The number of arbitrageurs that trade between the two markets is taken as given in this section; in section 3 I endogenize the number of arbitrageurs through the introduction of an entry cost that arbitrageurs need to pay to be active in the two markets.

2.1 Trading Strategies and Price Dynamics

Following from the setup described above, arbitrageurs choose a trading strategy to maximize the profit from trading against the price differential between the two assets, while keeping the overall exposure to the two markets constant. This means that, at each point in time, an arbitrageur must sell as many units in one market as he buys in the other. We can thus denote by Y_t^i the rate at which arbitrageur i sells in market 2 (the overpriced market) and buys in market 1 (the underpriced market), i.e. $Y^i(t) = Y_1^i(t) = -Y_2^i(t)$. Furthermore, denote the aggregate trading rate of all arbitrageurs by $Y(t) = \sum_{i=1}^n Y^i(t)$.

Using this notation, trading against the arbitrage opportunity results in a flow profit of $-\Delta P(t)Y^i(t)$ per dt -interval. Integrating this flow profit over the arbitrage horizon (0 to T), the objective function of arbitrageur i can be written as,

$$\max_{Y^i(t) \in \mathcal{Y}} \int_0^T -\Delta P(t)Y^i(t)dt. \quad (4)$$

Furthermore, using the pricing equation (2), the price gap between the two assets can be rewritten as

$$\Delta P(t) = \gamma \tilde{Z} - 2\gamma \int_0^t Y(s)ds - (\lambda_1 + \lambda_2)Y(t). \quad (5)$$

The above setup constitutes a continuous-time trading game. The price differential between the two assets is affected not only by arbitrageur i 's trading rate, but also by the trading of all other arbitrageurs, and since arbitrageurs take this price impact into account when choosing their trading strategies. This means that arbitrageurs interact strategically through their influence on prices. I derive an equilibrium of this game, in which the arbitrageurs choose trading strategies as functions of time.⁶ The solution approach is similar

⁶Time-dependent strategies imply that the equilibrium is ‘open-loop’, or weakly time consistent. There is no closed-form solution for the closed-loop equilibrium, but Carlin, Lobo, and Viswanathan (2007) show numerically that open-loop and closed-loop equilibria are qualitatively very similar in trading games of this type. For more on different equilibrium concepts in differential games see Dockner, Jorgensen, Long, and Sorger (2000) and Basar and Olsder (1995).

to Carlin, Lobo, and Viswanathan (2007), with two notable differences. First, rather than liquidating one asset, in this model arbitrageurs buy and sell at the same time, such that one has to consider the evolution of the prices of both assets. Second, while in Carlin, Lobo, and Viswanathan (2007) the amount sold buy each trader is fixed, in this model the number of units traded on the interval $[0, T]$ is endogenous and has to be determined through an equilibrium condition.

I impose a number of restrictions on the strategy space \mathcal{Y} to make sure that the maximization problem is well defined. First, I restrict my attention to continuous strategies. Second, for a strategy to be admissible the arbitrageurs' expected profit has to be integrable, i.e. $\int_0^T -\Delta P(t)Y^i(t)dt < \infty$. This is guaranteed, for example, when $\{Y^i(t)\}$ lies in \mathcal{L}^2 (i.e. $\int_0^\infty [Y^i(t)]^2 dt < \infty$). An equilibrium of the dynamic trading game is then defined as follows.

Definition 1 *An equilibrium in time-dependent strategies is given by a set of admissible trading strategies $\{Y^i(t)\}$ for traders $i = 1 \dots n$, that maximize trading profits (4), taking the strategies of all other arbitrageurs, $\{Y^{-i}(t)\}$, as given.*

The equilibrium trading strategies can be calculated using Hamiltonian methods. The details of the calculations are in the appendix.

Proposition 1 *Equilibrium trading schedules. The equilibrium trading strategy of arbitrageur i , when n strategic arbitrageurs trade against the arbitrage opportunity, is to buy the underpriced and sell the overpriced asset at rate $Y^i(t)$, where*

$$Y^i(t) = ae^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t}. \quad (6)$$

The equilibrium value of the constant a is given by

$$a = \frac{n-1}{n+1} \frac{\gamma}{\lambda_1 + \lambda_2} \frac{\tilde{Z}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}. \quad (7)$$

Proof. See appendix. ■

Equation (6) shows that in equilibrium arbitrageurs trade at an exponentially decreasing rate. The initial rate of trading is given by a ; after that the rate of trading decays exponentially at rate $\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1+\lambda_2}$. The rate of decay is determined by the slope of the long run demand curve for the risky asset (γ), the amount of short-run price pressure in the two assets (λ_1 and λ_2), and the number of arbitrageurs active between the two markets.

The equilibrium trading schedules determine equilibrium prices and the evolution of the price differential between the two assets. This is outlined in Proposition 2.

Proposition 2 *Equilibrium price gap.* *The equilibrium price gap, when n arbitrageurs trade against the arbitrage opportunity, is given by*

$$\Delta P(t) = \gamma \tilde{Z} - n \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1+\lambda_2} t}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1+\lambda_2} T}} \gamma \tilde{Z} - n \frac{n-1}{n+1} \frac{e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1+\lambda_2} t}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1+\lambda_2} T}} \gamma \tilde{Z}. \quad (8)$$

Proof. See appendix. ■

Proposition 1 and 2 show how the equilibrium trading strategies and the speed of arbitrage depend crucially on illiquidity and the competition among arbitrageurs. First, note that the number of arbitrageurs determines to what extent racing to the market occurs: While a monopolist arbitrageur sells and buys at a constant rate in order to minimize losses due to illiquidity (shown formally in an example below), competing arbitrageurs race to the market. Competition among arbitrageurs results in more front-loaded arbitrage activity, i.e. trading is heavy in early periods and dies out more rapidly. Thus, the more arbitrageurs are active in the market, the earlier arbitrage activity takes place and the faster the speed of arbitrage.

Short-term price pressure and the slope of the long-run demand curve of the risky assets affect equilibrium trading rates through the ratio $\frac{2\gamma}{\lambda_1+\lambda_2}$. The higher this ratio, the earlier trading will take place. The intuition is as follows. On the one hand, the downward-sloping demand curve γ gives arbitrageurs an incentive to trade against the opportunity before their competitors. The larger γ , the stronger the effect of competition, leading to racing to the market between competing arbitrageurs. On the other hand, temporary price pressure λ_j

gives arbitrageurs an incentive to spread trades over time in order to reduce the costs of short-term price impact. The equilibrium strategy balances these two effects.

The equilibrium price gap at time t consists of three parts, as demonstrated by equation (8). The first part is the price gap that would result in absence of any arbitrage activity. In that case, the price gap would remain constant at the level of the initial shock, until the payment of the final dividend. The second part is the reduction of the price gap due to permanent price changes that result from changes in the positions that arbitrageurs hold in the two markets. As arbitrageurs reduce their position in the overpriced asset and increase their position in the underpriced asset, this permanently reduce the price gap. The third part is the temporary reduction in the price gap due to short-term price impact. This term reflects the fact that in addition to the permanent price changes induced by the change in arbitrageurs' positions, trading by arbitrageurs puts temporary price pressure on the two assets. In particular, the selling of the overpriced asset temporarily depresses its price, while the underpriced asset's price temporarily increases due to buying pressure. This narrows the price gap further, but only at that particular instant.

The above results show that the number of arbitrageurs is an important factor in the determination of the equilibrium trading strategies. This, of course, means that the intensity of competition among arbitrageurs has important implications for the dynamics of the equilibrium price gap and thus determines the *speed of arbitrage*. Through its effect in trading strategies, competition leads to a larger initial decrease in the price gap and leads to a reduction in the price gap between the two assets for any time $t \in [0, T]$. In the limiting case of competitive arbitrage, the price gap is eliminated instantaneously at $t = 0$. Competition among arbitrageurs thus speeds up the arbitrage process. This effect of competition on the equilibrium trading strategies and the equilibrium price gap is illustrated in Figure 1. One thing to note in the right panel of Figure 1 is that the price gap is not always eliminated completely by time T . To see why this is the case, consider a monopolist. At period T , the monopolist weighs the benefit of trading one more unit across the two markets against its

cost. The benefit is that on this marginal unit he will earn the price differential between the two assets. The cost is that the additional price impact cost created by this marginal unit reduces the profit for all other units traded at the same time. This means that the monopolist will not eliminate the price gap entirely at T . As the number of arbitrageurs increases, each arbitrageur trades fewer units per period of time, which reduces concern for inframarginal units at time T .

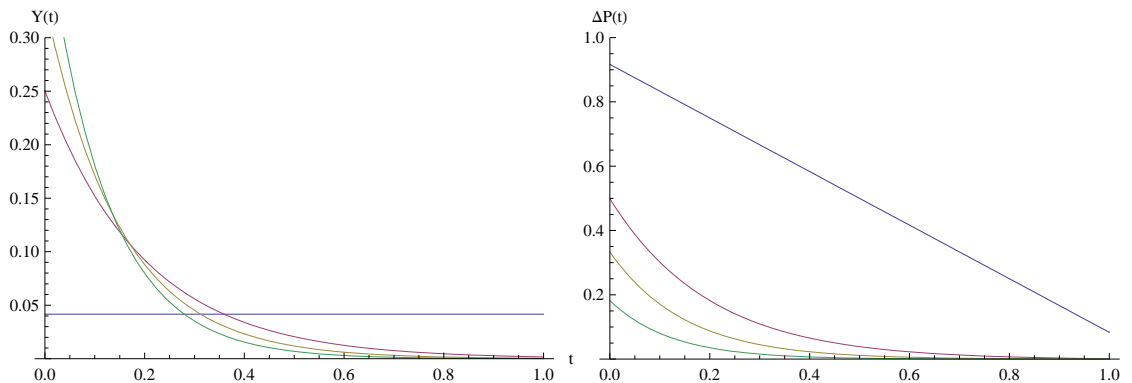


Figure 1: **Equilibrium aggregate trading rates (left) and price gap (right)**. Plotted for $n = 1, 3, 5, 10$. When only one arbitrageur is present, the trading rate is constant and the price gap reduces linearly. As the number of arbitrageurs increases, trading takes place earlier and the price gap decreases exponentially. Competition among arbitrageurs increases the speed of arbitrage. The parameters for this example are $\tilde{Z} = 0.1$, $T = 1$, $\gamma = 10$, $\lambda_1 = \lambda_2 = 1$.

2.2 Two Examples: Monopolistic and Competitive Arbitrage

As an illustration of the equilibrium trading rates and the resulting price gap, it is instructive to consider the limiting cases of monopolistic and competitive arbitrage.

Monopolistic Arbitrage. In this example I present the equilibrium trading strategy and the resulting price gap in the presence of a monopolistic arbitrageur. Corollary 1 shows that a monopolistic arbitrageur will exploit the arbitrage opportunity by shifting assets from market 2 into market 1 at a constant rate in order to minimize his illiquidity costs. This implies that the price gap decreases linearly over time.

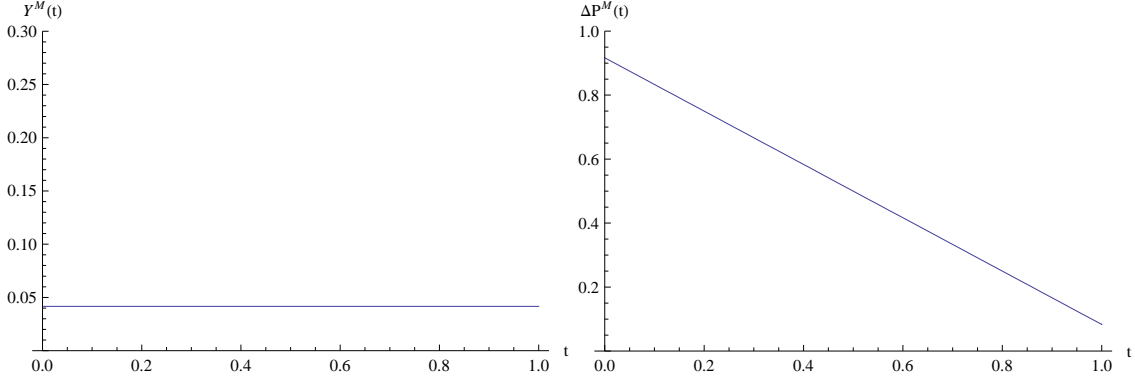


Figure 2: **Monopolistic arbitrage.** Equilibrium trading rate Y_t (left) and equilibrium price differential ΔP_t (right) under monopolistic arbitrage ($n = 1$). The parameters for this example are $\tilde{Z} = 0.1$, $T = 1$, $\gamma = 10$, $\lambda_1 = \lambda_2 = 1$.

Corollary 1 *A monopolistic arbitrageur ($n = 1$) will trade against the arbitrage opportunity with a constant trading intensity*

$$Y^M(t) = \frac{1}{2} \frac{\gamma \tilde{Z}}{\lambda_1 + \lambda_2 + \gamma T}. \quad (9)$$

The equilibrium price gap in the presence of a monopolistic arbitrageur is given by

$$\Delta P^M(t) = \gamma \tilde{Z} - \gamma \frac{t}{(\lambda_1 + \lambda_2 + \gamma T)} \gamma \tilde{Z} - (\lambda_1 + \lambda_2) \frac{1}{2} \frac{1}{(\lambda_1 + \lambda_2 + \gamma T)} \gamma \tilde{Z}. \quad (10)$$

Proof. See appendix. ■

Figure 2 illustrates the equilibrium trading strategy and the equilibrium price gap in the presence of a monopolistic arbitrageur. The monopolistic arbitrageur can exploit the arbitrage slowly over time at a constant trading rate, since he does not face competition from other arbitrageurs who could trade before him and drive down the price differential. This means that the monopolist can minimize the costs from short-term price pressure effects, and that, as a result, the mispricing is only corrected slowly over time.

Competitive Arbitrage. In this example I present the equilibrium aggregate trading strategy and the resulting price gap in the presence of competitive arbitrageurs. Corollary

2 shows that competitive arbitrageurs will race to the market and shift assets from market 2 into market 1 at an exponentially decaying rate. Importantly, perfect competition among arbitrageurs leads to an immediate elimination of the price gap. This is intuitive—each competitive arbitrageur takes the price as given when choosing his trading strategy. This means that if the price gap was downward sloping at any point in time, this would give competitive arbitrageurs an incentive to trade earlier. The only equilibrium is the one in which the price gap is eliminated instantly.

Corollary 2 *In the competitive case ($n = \infty$) arbitrageurs trade against the arbitrage opportunity at an aggregate trading rate of*

$$Y^C(t) = \frac{1}{\lambda_1 + \lambda_2} e^{-\frac{2\gamma}{\lambda_1 + \lambda_2} t} \gamma \tilde{Z}. \quad (11)$$

The price gap is eliminated instantly at $t = 0$, i.e.

$$\Delta P^C(t) = \gamma \tilde{Z} - [1 - e^{-\frac{2\gamma}{\lambda_1 + \lambda_2} t}] \gamma \tilde{Z} - e^{-\frac{2\gamma}{\lambda_1 + \lambda_2} t} \gamma \tilde{Z} = 0. \quad (12)$$

Proof. See appendix. ■

Figure 3 illustrates the equilibrium trading strategies and the equilibrium price gap in the competitive case.

2.3 Equilibrium Trading Profits

I now use the equilibrium trading strategies to determine the profits realized by arbitrageurs as a function of the initial shock, the illiquidity parameters and the level of competition among arbitrageurs. This is summarized in the following proposition.

Proposition 3 *Equilibrium trading profits. Equilibrium aggregate trading profits are*

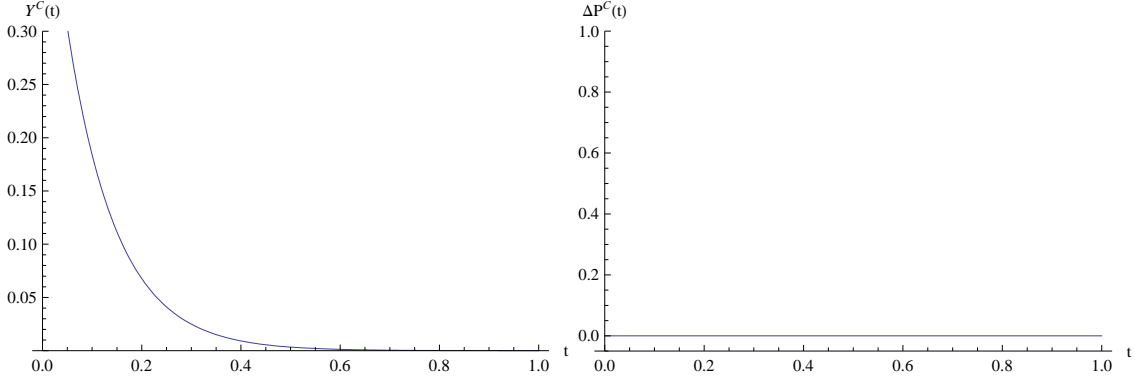


Figure 3: **Competitive arbitrage.** Equilibrium aggregate trading rate Y_t (left) and equilibrium price differential ΔP_t (right) under competitive arbitrage ($n = \infty$). The parameters for this example are $\tilde{Z} = 0.1$, $T = 1$, $\gamma = 10$, $\lambda_1 = \lambda_2 = 1$.

given by

$$\Pi = \frac{n}{n+1} \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \frac{\gamma \tilde{Z}^2}{2}, \quad (13)$$

and equilibrium trading profits for an individual arbitrageur are given by

$$\Pi^i = \frac{1}{n+1} \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \frac{\gamma \tilde{Z}^2}{2}. \quad (14)$$

Proof. See appendix. ■

Equations (13) and (14) can then be used to determine the impact of competition and illiquidity on arbitrage profits.

Corollary 3 *Equilibrium profits are decreasing in the number of arbitrageurs n , both in aggregate and for the individual arbitrageurs:*

$$\frac{\partial \Pi}{\partial n} < 0 \quad \frac{\partial \Pi^i}{\partial n} < 0. \quad (15)$$

Equilibrium profits are decreasing in short term price pressure, λ_j , both in aggregate and for the individual arbitrageurs:

$$\frac{\partial \Pi}{\partial \lambda_j} < 0 \quad \frac{\partial \Pi^i}{\partial \lambda_j} < 0. \quad (16)$$

Proof. See appendix. ■

The reason why aggregate profits are decreasing in the number of arbitrageurs is the change in equilibrium strategies that results from increased competition. Racing to the market by competing arbitrageurs leads to a deadweight loss in terms of illiquidity costs, and thus reduces aggregate profits. This is shown in figure 4, which plots the negative relationship between aggregate and individual profits and the number of arbitrageurs, n . Note that equilibrium trading profits are decreasing in the number of arbitrageurs both in aggregate and for the individual arbitrageur. In other words, competition not only divides up aggregate profits among a larger number of arbitrageurs, it also reduces the ‘size of the pie’: aggregate profits are lower, since arbitrageurs impose a negative externality on each other by racing to the market and raising equilibrium illiquidity costs. In the competitive limit, all individual arbitrageurs (and thus also arbitrageurs as a whole) make zero profits.⁷

3 Entry

The results above show that the level of competition among arbitrageurs is an important determinant of the speed of arbitrage, since competitive pressure leads arbitrageurs to exploit arbitrage opportunities more aggressively. However, up to now the number of arbitrageurs n was exogenously specified. In this section, I endogenize the number of arbitrageurs that connect the two markets and show how the equilibrium level of competition depends on the illiquidity characteristics of the two markets. This allows me to draw conclusions about the level of competition that one may expect among arbitrageurs as a function of asset and market characteristics. In particular, I will show that endogenizing the level of competition

⁷For brevity, I omit the comparative statics with respect to γ in the main text, since they are not the main focus here. They are contained in the appendix.

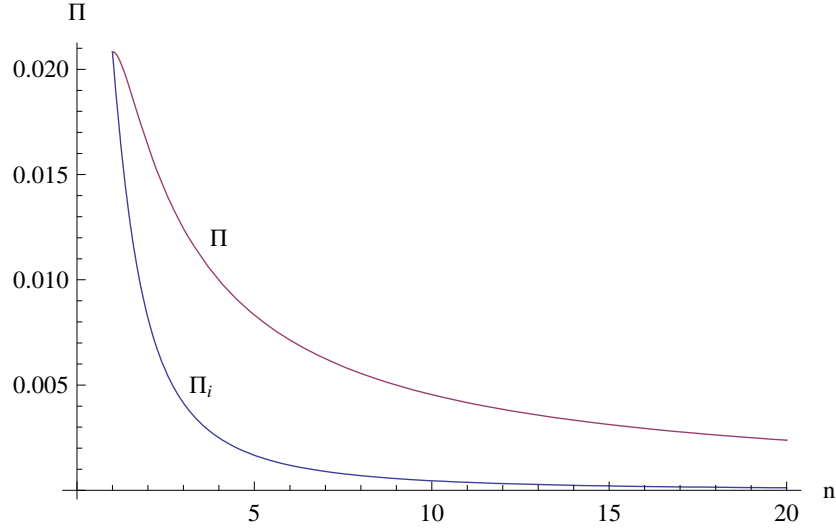


Figure 4: **Equilibrium Profits.** Equilibrium aggregate and individual profits as a function of the number of arbitrageurs, n . The parameters for this example are $\tilde{Z} = 0.1$, $T = 1$, $\gamma = 10$, $\lambda_1 = \lambda_2 = 1$.

reinforces the effects of illiquidity on the speed of arbitrage.

I assume that in order to be active in both markets, arbitrageurs have to pay an ex-ante fixed cost of size F . This fixed cost may, for example, be interpreted as the cost of learning about the markets, the cost of setting up the infrastructure to trade in those markets, or it may stand for the opportunity cost of committing capital. I normalize the fixed cost F such that it already nets out the buy-and-hold profits from being active in both markets. An arbitrageur will then enter the market if the expected profit from trading against a potential arbitrage opportunity exceeds the entry cost F . In free-entry equilibrium with risk-neutral arbitrageurs, the number of arbitrageurs, n^* , is then uniquely pinned down by a zero expected profit condition. For illustrative purposes, assume that arbitrageurs make their entry decision at time $t = -1$, knowing that with probability p a demand shock will occur at time $t = 0$ in one of the two markets. The demand shock \tilde{Z} is random, with mean zero and variance σ_Z^2 . With probability $1 - p$, no demand shock occurs and arbitrageurs only earn buy-and-hold profits. The free-entry equilibrium that emerges in this case is outlined in Proposition 4.

Proposition 4 *Free-entry Equilibrium.* *Given the ex-ante entry cost F , the equilibrium*

number of arbitrageurs in free-entry equilibrium, n^* , solves

$$p \frac{1}{n^* + 1} \frac{1 - e^{-\frac{n^* - 1}{n^* + 1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{n^* - e^{-\frac{n^* - 1}{n^* + 1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \frac{\gamma \sigma_Z^2}{2} = F, \quad (17)$$

Proof. See appendix. ■

The left-hand side of equation (17) is the expected profit to arbitrageur i from being active in the two markets. In equilibrium n^* will be such that this expected profit equal the entry cost (abstracting away from integer constraints). Proposition 4 thus gives an implicit characterization of n^* , the equilibrium number of arbitrageurs that connect two segmented markets. While generally there is no closed-form expression for n^* , taking the limit $T \rightarrow \infty$, gives a closed-form expression, which is given in the following corollary.

Corollary 4 *In the limiting case $T \rightarrow \infty$, the equilibrium number of arbitrageurs can be computed in closed form and is given by*

$$n_{T=\infty}^* = \sqrt{\frac{1}{4} + p \frac{\gamma \sigma_Z^2}{2F}} - \frac{1}{2}.$$

Proof. See appendix. ■

Naturally, in markets in which local demand shocks are frequent—or have large variance—more arbitrageurs enter, and the level of competition among arbitrageurs is higher. The same is true for markets with low entry costs. Yet Proposition 4 also shows that the endogenous level of competition depends crucially on illiquidity. Since expected profits for arbitrageurs are decreasing in the short-term illiquidity parameters λ_j , the equilibrium number of arbitrageurs n^* is also decreasing in λ , i.e. $\frac{\partial n^*}{\partial \lambda_j} < 0$. Thus, the more illiquid a market, the lower the level of competition among arbitrageurs. This effect of illiquidity on arbitrageur competition, which is present as long as the arbitrage horizon T is finite, is illustrated in Figure 5.

Importantly, the effect of illiquidity on competition among arbitrageurs means that asset illiquidity affects the speed of arbitrage in *two ways*. First, there is the direct effect that was shown in section 2: when asset illiquidity is higher, arbitrageurs will trade less aggressively

against the arbitrage opportunity, holding the level of competition among arbitrageurs fixed. This unambiguously raises the price gap at every point in time $t \in [0, T]$. This effect can be interpreted as a purely *strategic effect* that results from the changes in the trading strategies for a given level of competition. However, in addition to this direct effect, there is an indirect effect working in the same direction—as shown, higher illiquidity reduces the equilibrium number of arbitrageurs, and the resulting decrease in competition among arbitrageurs changes equilibrium strategies. In particular, the reduction in competition means that arbitrageurs race to the market less fiercely, which further increases the price gap at each point in time. This effect can be interpreted as a *competition effect*. In other words, in addition to the direct effect of illiquidity on trading strategies, there is an indirect effect that works through the effect of illiquidity on the level of competition among arbitrageurs. This means that for more illiquid assets, arbitrage is not only slower because the price impact costs of trading against arbitrage opportunities are higher, but also because in more illiquid markets the level of competition among arbitrageurs is lower. To the best of my knowledge, this effect on the equilibrium level of competition has not been studied by other papers on the speed of arbitrage. This is stated formally in Proposition 5.

Proposition 5 *Illiquidity reduces the speed of arbitrage in two ways. An increase in temporary price pressure λ_j reduces the speed of arbitrage for two reasons: through its direct effect on strategies and, indirectly, by reducing the equilibrium number of arbitrageurs. This is reflected in the equilibrium trading rate*

$$\frac{dY_t}{d\lambda_j} = \underbrace{\frac{\partial Y_t}{\partial \lambda_j}}_{\text{Direct Effect}} + \underbrace{\frac{\partial Y_t}{\partial n^*} \frac{\partial n^*}{\partial \lambda_j}}_{\text{Competition Effect}}, \quad (18)$$

and in the time t price gap,

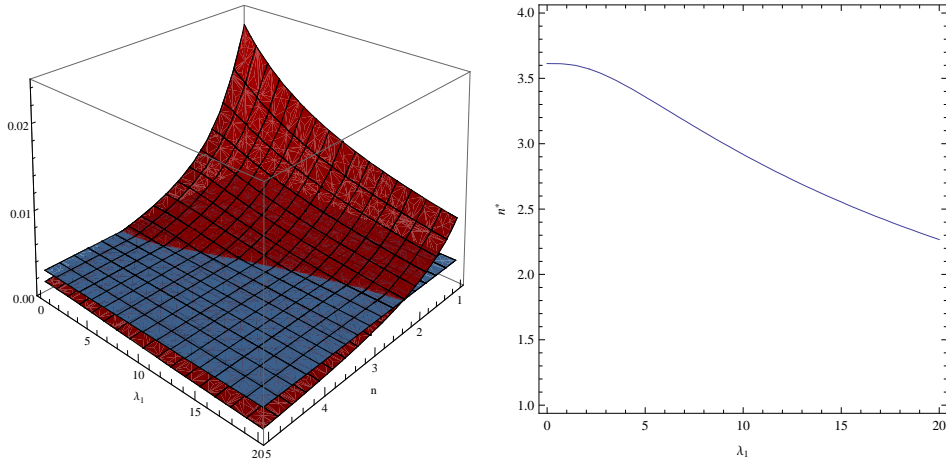


Figure 5: **Endogenous Entry.** The left panel plots expected profits for an individual arbitrageur as a function of the number of arbitrageurs, n , and liquidity of asset 1, λ_1 . The parameters for this example are $p = 0.2$, $\sigma_Z^2 = 0.05$, $T = 1$, $\gamma = 10$, $\lambda_2 = 1$. The intersection with the entry cost plane ($F = 0.003$) maps out the equilibrium number of arbitrageurs present in the market, n^* , abstracting away from integer constraints. This implicit relation is plotted in the right panel.

$$\frac{d\Delta P_t}{d\lambda_j} = \underbrace{\frac{\partial \Delta P_t}{\partial \lambda_j}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \Delta P_t}{\partial n^*} \frac{\partial n^*}{\partial \lambda_j}}_{\text{Competition Effect}} . \quad (19)$$

Proof. See appendix. ■

The strategic effect and the competition effect are illustrated in figure 6. The figure decomposes the difference in equilibrium trading strategies and the equilibrium price gap between a liquid and a relatively less liquid markets into the two effects. The solid line depicts the trading rate (left) and the price gap (right) when arbitrageurs connect two liquid markets ($\lambda_1 = \lambda_2 = 1$). Now consider what happens when we move to arbitrage between more illiquid markets, by increasing λ_1 and λ_2 to 10. The shift from the solid line to the dashed line is the *strategic effect*—for a given level of competition arbitrageurs trade more slowly in more illiquid markets. The mispricing dies out more slowly, except at the very beginning, when the higher value of λ briefly depresses the size of the price gap below that in the liquid market. The shift from the dashed to the dotted line is the *competition effect*. This additional shift

shows that arbitrage slows further when we take into account that level competition among arbitrageurs is lower in more illiquid markets. As evident from the figure, this reduction in competition leads to a further significant reduction in the speed of arbitrage. Taking into account both effects shows that the equilibrium price gap in the illiquid market is larger for any value of $t \in [0, T]$, and that the mispricing is reduced significantly more slowly in the illiquid market relative to the liquid market. As an example, consider quantitative hedge funds that are mainly active in liquid equity markets. In those markets, mispricings should be corrected swiftly, both because trading costs for funds are low and because they compete fiercely to exploit opportunities once they open up. In markets for less liquid securities, such as mortgages or infrequently traded bonds, arbitrage will be more gradual, both because of higher trading costs, and because there is less competition for arbitrage opportunities in those markets.

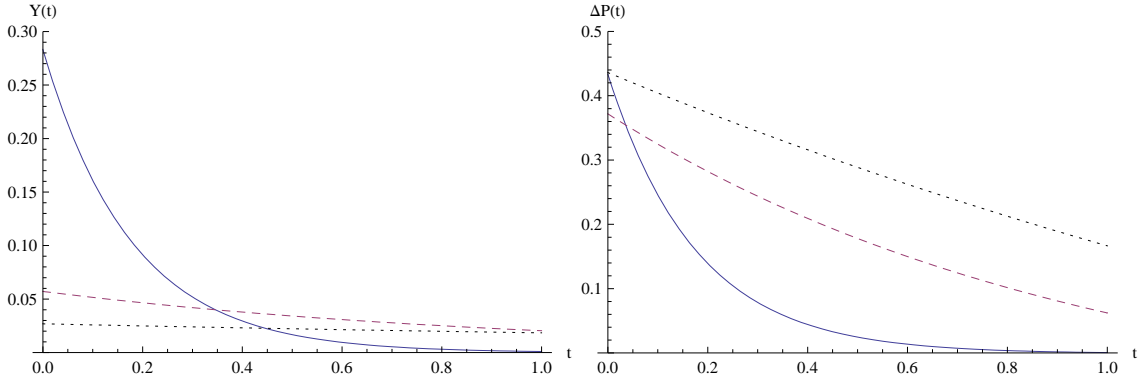


Figure 6: Decomposing the strategic effect and the competition effect. The figure decomposes the difference in the trading rate (left) and price gap (right) in liquid and illiquid markets into the strategic and the competition effect. The solid line depicts arbitrage in a liquid market ($\lambda_1 = \lambda_2 = 1$). Moving to illiquid markets, ($\lambda_1 = \lambda_2 = 10$), arbitrage slows down for a given level of competition. This is the *strategic effect*, illustrated by the shift from the solid line to the dashed line. In addition, the endogenous level of competition in the more illiquid market is lower, further slowing down the speed of arbitrage. This *competition effect* is given by the shift from the dashed line to the dotted line. The parameters in this example are $\gamma = 10, T = 1, \tilde{Z} = 0.1$. To determine the equilibrium number of arbitrageurs it was assumed that $\sigma_{\tilde{Z}}^2 = .05$ and $p = 0.2$. The graph abstracts away from integer constraints.

4 Conclusion

Capital often flows slowly from one market to another in response to buying opportunities. This paper provides a liquidity-based theory for this phenomenon. To exploit buying opportunities in one market, arbitrageurs sell in one market and buy in another. When either or both of these markets are illiquid (in the sense that buying or selling causes short-term price pressure) arbitrageurs exploit arbitrage opportunities gradually over time, rather than instantly. The model shows that the speed of arbitrage depends on the illiquidity of the markets in which the arbitrageurs buy and sell, and on the level of competition among arbitrageurs. In particular, in more illiquid markets, mispricings persist longer for two reasons. First, for a given level of competition an increase in illiquidity results in slower trading against the arbitrage opportunity. Second, in free-entry equilibrium, fewer arbitrageurs are active in illiquid markets, further slowing down the speed of arbitrage.

5 Appendix

Proof of Proposition 1: The derivation of equation (6) is analogous to the proof given in Carlin, Lobo, and Viswanathan (2007). However, there is an additional complication when determining the equilibrium value of the constant a . While in Carlin, Lobo, and Viswanathan (2007) a is determined by an exogenous trading constraint, here a is determined endogenously by profit maximization. More specifically, a is determined by considering the incentive of an individual arbitrageur to unilaterally increase his own a , while all other arbitrageurs keep a fixed.

I first turn to the derivation of equation (6). Arbitrageurs face the following maximization problem:

$$\max_{Y^i(t) \in \mathcal{Y}} - \int_0^T Y^i(t) \Delta P(t) dt$$

subject to

$$\dot{X}_j(t) = \sum_i Y_j^i(t)$$

Denoting by $q(t)$ the costate variable, i.e. the multiplier associated with the evolution of the state variable $X^i(t)$, the Hamiltonian for this problem is given by

$$H = -Y^i(t)[\gamma Z - 2\gamma \sum_{k=1}^n X^k(t) - (\lambda_1 + \lambda_2) \sum_{k=1}^n Y^k(t)] + q(t)Y^i(t),$$

which yields the first-order conditions (after imposing symmetry)

$$\gamma Z - 2n\gamma X(t) - (n+1)(\lambda_1 + \lambda_2)Y(t) + q(t) = 0 \quad (20)$$

$$2\gamma Y(t) - \dot{q}(t) = 0. \quad (21)$$

Taking the time derivative of (20) and substituting in (21) gives

$$2(n-1)\gamma Y(t) + (n+1)(\lambda_1 + \lambda_2)\dot{Y}(t) = 0.$$

Solving this first-order ODE for $Y(t)$ gives

$$Y(t) = ae^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t},$$

which is the expression in equation (6).

We now solve for the endogenous coefficient a . The equilibrium value of a is determined by considering a unilateral deviation in a by one player, keeping all other players' strategies fixed. The equilibrium is given by the value of a for which no player has an incentive to deviate given other players' strategies.

First, use equation (6) to write the profit of arbitrageur i as a function of his own trading constant a^i and that of all other arbitrageurs, a^{-i} . Starting from the expression given in

equation (6), $Y^i(t) = a^i e^{-\frac{n-1}{n+1} \frac{\gamma}{\lambda} t}$, the profit for arbitrageur i can be written as

$$\begin{aligned} \Pi^i &= \int_0^T \left[\gamma \tilde{Z} - 2\gamma \sum_{k=1}^n a^k \frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \left[1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t} \right] - (\lambda_1 + \lambda_2) \sum_{k=1}^n a^k e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t} \right] \\ &\quad \times a^i e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t} dt. \end{aligned}$$

Simplifying this expression yields

$$\begin{aligned} \Pi^i &= \gamma \tilde{Z} a^i \frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \left[1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] - 2\gamma \sum_{k=1}^n a^k a^i \left[\frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \right]^2 \left[1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] \\ &\quad + 2\gamma \sum_{k=1}^n a^k a^i \frac{1}{2} \left[\frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \right]^2 \left[1 - e^{-2\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] \\ &\quad - (\lambda_1 + \lambda_2) a^i \sum_{k=1}^n a^k \frac{1}{2} \frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \left[1 - e^{-2\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right]. \end{aligned}$$

To check the unilateral deviation of trader i , take the derivative of this expression with respect to a^i :

$$\begin{aligned} \frac{\partial \Pi^i}{\partial a^i} &= \gamma \tilde{Z} \frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \left[1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] - 2\gamma \sum_{k=1}^n a^k \left[\frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \right]^2 \left[1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] \\ &\quad + \gamma \sum_{k=1}^n a^k \left[\frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \right]^2 \left[1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] \\ &\quad - (\lambda_1 + \lambda_2) \sum_{k=1}^n a^k \frac{1}{2} \frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \left[1 - e^{-2\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] \\ &\quad - 2\gamma a^i \left[\frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \right]^2 \left[1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] \\ &\quad + \gamma a^i \left[\frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \right]^2 \left[1 - e^{-2\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] \\ &\quad - (\lambda_1 + \lambda_2) a^i \frac{1}{2} \frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \left[1 - e^{-2\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] \end{aligned}$$

Imposing symmetry and equating to zero, this expression simplifies to

$$\begin{aligned}
0 &= \gamma \tilde{Z} \frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \left[1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] - 2\gamma(n+1)a \left[\frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \right]^2 \left[1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] \\
&+ \gamma(n+1)a \left[\frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \right]^2 \left[1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right] \\
&- (\lambda_1 + \lambda_2)(n+1)a \frac{1}{2} \frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \left[1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right]
\end{aligned}$$

After some algebra, solving this expression for a yields

$$a = \frac{n-1}{n+1} \frac{1}{\lambda_1 + \lambda_2} \frac{1}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \gamma \tilde{Z},$$

which is the expression given in (7).

Proof of Proposition 2: The equilibrium price gap can be calculated by substituting (6) and (7) into the expression for the price gap. This yields

$$\begin{aligned}
\Delta P(t) &= \gamma \tilde{Z} - 2\gamma n \int_0^t Y(t) dt - (\lambda_1 + \lambda_2) n Y(t) \\
&= \gamma \tilde{Z} - 2\gamma n \int_0^t \frac{n-1}{n+1} \frac{1}{\lambda_1 + \lambda_2} \frac{1}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \gamma \tilde{Z} e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t} dt \\
&\quad - (\lambda_1 + \lambda_2) n \frac{n-1}{n+1} \frac{1}{\lambda_1 + \lambda_2} \frac{1}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \gamma \tilde{Z} e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t} \\
&= \gamma \tilde{Z} - 2\gamma n \frac{n-1}{n+1} \frac{1}{\lambda_1 + \lambda_2} \frac{1}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \frac{n+1}{n-1} \frac{\lambda_1 + \lambda_2}{2\gamma} \left[1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t} \right] \gamma \tilde{Z} \\
&\quad - (\lambda_1 + \lambda_2) n \frac{n-1}{n+1} \frac{1}{\lambda_1 + \lambda_2} \frac{1}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t} \gamma \tilde{Z} \\
&= \gamma \tilde{Z} - n \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \gamma \tilde{Z} - n \frac{n-1}{n+1} \frac{e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \gamma \tilde{Z},
\end{aligned}$$

which is the expression given in the text. Written like this, the effects of permanent and temporary price affects are separated into individual terms. The expression can be simplified

further to yield

$$\begin{aligned}\Delta P(t) &= \gamma \tilde{Z} \left[1 - n \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} - n \frac{n-1}{n+1} \frac{e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \right] \\ &= \gamma \tilde{Z} \left[1 - \frac{n - \frac{2n}{n+1} e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \right].\end{aligned}$$

Proof of Corollary 1: The expressions are obtained by taking the appropriate limits.

To obtain the trading rate in the monopolist case, note that

$$\begin{aligned}\lim_{n \rightarrow 1} Y^i(t) &= \lim_{n \rightarrow 1} a e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t} = \lim_{n \rightarrow 1} \frac{n-1}{n+1} \frac{1}{\lambda_1 + \lambda_2} \frac{e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \gamma \tilde{Z} \\ &= \frac{1}{2} \frac{\gamma \tilde{Z}}{\lambda_1 + \lambda_2} \lim_{n \rightarrow 1} \frac{n-1}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \\ &= \frac{1}{2} \frac{\gamma \tilde{Z}}{\lambda_1 + \lambda_2} \lim_{n \rightarrow 1} \frac{1}{1 + \frac{2}{(n+1)^2} \frac{2\gamma}{\lambda_1 + \lambda_2} T e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \\ &= \frac{1}{2} \frac{\gamma \tilde{Z}}{\lambda_1 + \lambda_2 + \gamma T}.\end{aligned}$$

The second line follows from the first using Slutsky's Theorem. The third line follows from the second using L'Hôpital's rule. This shows that the rate of trading of a monopolist is constant. Plugging this into the expression for the price gap (8) yields equation (10).

Proof of Corollary 2: To obtain the aggregate trading rate in the competitive case, take the limit as $n \rightarrow \infty$:

$$\begin{aligned}\lim_{n \rightarrow \infty} n Y^i(t) &= \lim_{n \rightarrow \infty} n a e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t} = \lim_{n \rightarrow \infty} n \frac{n-1}{n+1} \frac{1}{\lambda_1 + \lambda_2} \frac{1}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \gamma \tilde{Z} e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t} \\ &= \frac{\gamma \tilde{Z}}{\lambda_1 + \lambda_2} e^{-\frac{2\gamma}{\lambda_1 + \lambda_2} t} \lim_{n \rightarrow \infty} \frac{n}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \\ &= \frac{1}{\lambda_1 + \lambda_2} e^{-\frac{2\gamma}{\lambda_1 + \lambda_2} t} \gamma \tilde{Z}.\end{aligned}$$

Plugging this expression into the expression for the price gap (8) yields equation (12).

Proof of Proposition 3: The expression for the equilibrium trading profits is obtained by inserting (6) and (2) into (4), and simplifying the resulting expression:

$$\begin{aligned}\Pi^i &= \int_0^T \Delta P(t) Y^i(t) dt \\ &= \int_0^T \left[\gamma \tilde{Z} - n \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \gamma \tilde{Z} - n \frac{n-1}{n+1} \frac{e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \gamma \tilde{Z} \right] \\ &\quad \times \frac{n-1}{n+1} \frac{1}{\lambda_1 + \lambda_2} \frac{e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} t}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \gamma \tilde{Z} dt\end{aligned}$$

Simplifying this expression yields

$$\Pi^i = \frac{1}{n+1} \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \frac{\gamma}{2} \tilde{Z}^2.$$

To obtain the aggregate trading profits, multiply by n .

Proof of Corollary 3: First, turn to the comparative statics with respect to the number of arbitrageurs.

We can rewrite the aggregate profit expression (13) as

$$\Pi = \frac{1}{n+1} \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{1 - \frac{1}{n} e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \frac{\gamma}{2} \tilde{Z}^2.$$

$\frac{1}{n+1}$ is decreasing in n . This means that, for the entire expression to be decreasing in n , it is sufficient to show that $\frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{1 - \frac{1}{n} e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}$ is decreasing in n . To show this, start in the monopolistic case: In the limit $n \rightarrow 1$, this expression is equal to one. Now consider increasing n . The expression in the denominator, $1 - \frac{1}{n} e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}$, grows faster than the expression in the numerator, $1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}$. This means that $\frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{1 - \frac{1}{n} e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}$ must be decreasing in n , which completes the proof. Given that aggregate profits are decreasing in competition it follows immediately that individual profits are also decreasing in competition.

Both aggregate and individual profits are decreasing in asset illiquidity λ_j and increasing in the slope of the long-run demand curve, γ . For individual profits:

$$\begin{aligned}\frac{\partial \Pi^i}{\partial \lambda_j} &= -\frac{(n-1)^2}{(n+1)^2} \frac{1}{(\lambda_1 + \lambda_2)^2} \frac{e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} T}{\left[n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T} \right]^2} \gamma^2 \tilde{Z}^2 < 0 \\ \frac{\partial \Pi^i}{\partial \gamma} &= \frac{1}{n+1} \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \frac{\partial}{\partial \gamma} \left[\frac{\gamma}{2} \tilde{Z}^2 \right] + \frac{\partial}{\partial \gamma} \left[\frac{1}{n+1} \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \right] \frac{\gamma}{2} \tilde{Z}^2 > 0\end{aligned}$$

The first expression shows that profits are decreasing in n , both in aggregate and for each individual arbitrageur. Profits are also decreasing in λ_i , i.e. larger price pressure effects make arbitrage less profitable. This is shown in the second equation. The third equation shows that profits are increasing in γ . The expression is positive since both parts of the sum are nonnegative, since $\frac{\partial}{\partial \gamma} \left[\frac{1}{n+1} \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \right] > 0$. The fact that profits are increasing in γ is sensitive to the assumption that the initial shock \tilde{Z} is multiplied by γ , the slope of the long-run demand curve. If the initial mispricing does not depend on γ the expression is ambiguous. A sufficient condition for an increase γ to reduce profits in that case is $\frac{(n+1)^2}{(n-1)^2} < \frac{\gamma}{\lambda} T$.

Proof of Proposition 4: Viewed from the time of the entry decision, a demand shock occurs with probability p . When it occurs, the demand shock has a variance of σ_Z^2 . Under these assumptions, the expected profits from entering, net of buy and hold gains in the two markets, are given by

$$\begin{aligned}E[\Pi^i] &= pE \left[\frac{1}{n+1} \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \frac{\gamma}{2} \tilde{Z}^2 \right] \\ &= p \frac{1}{n+1} \frac{1 - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}}{n - e^{-\frac{n-1}{n+1} \frac{2\gamma}{\lambda_1 + \lambda_2} T}} \frac{\gamma \sigma_Z^2}{2},\end{aligned}$$

where the second line uses the fact that $E[\tilde{Z}^2] = \sigma_Z^2$. Equating this to the fixed cost F determines the number of arbitrageurs that enter in free-entry equilibrium.

Proof of Corollary 4: Taking the limit as $T \rightarrow \infty$ in equation (17) yields

$$\frac{1}{n+1} \frac{1}{n} \frac{\gamma}{2} \sigma_Z^2 = F.$$

This is a quadratic equation with roots $-\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{2\gamma\sigma_Z^2}{F}}$. Since by definition $n \geq 0$, we can ignore the negative root and get

$$n = \sqrt{\frac{1}{4} + \frac{\gamma\sigma_Z^2}{2F}} - \frac{1}{2}.$$

Proof of Proposition 5: The expression follows from taking the derivative of expressions (6) and (8), taking into account that in equilibrium $n = n^*$, and that n^* depends on λ_j through the entry condition (17).

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