# Strategic Communication: Prices versus Quantities*


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**Abstract**

We examine how cheap talk communication between managers within the same firm depends on the type of decisions that the firm makes. A firm consists of a headquarters and two operating divisions. Headquarters is unbiased but does not know the demand conditions in the divisions' markets. Each division manager knows the demand conditions in his market but is also biased towards his division. The division managers communicate with headquarters which then sets either the prices or quantities for each division. The quality of communication depends on whether headquarters sets prices or quantities. This is the case even though, once communication has taken place, expected profits are the same whether headquarters sets prices or quantities.



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# 1 Introduction

"If HP knew what HP knows, we would be three times as profitable."[1] This observation by Lew Platt, the former CEO of Hewlett-Packard, suggests that even in the age of emails and video conferencing – and even in a high-tech company such as Hewlett-Packard – there are still significant barriers that limit the flow of information between managers. And it suggests that these barriers have a significant effect on the efficiency with which firms operate. What are these barriers and to what extent do they limit the performances of firms? One answer to these questions lies in the old saying that "knowledge is power." If managers are biased, they have an incentive to distort the information they share to influence decision making in their favor. And if they distort the information they share, and their counterparts understand that they are doing so, valuable information is lost (Crawford and Sobel 1982). The firms' managers and their biased incentives are then the barriers that limit the flow of information within firms. The managers' incentives to distort information depend on various features of the environment that the managers and their firms operate in. They depend, for instance, on the distribution of decision rights across managers (Alonso et al. 2008 and Rantakari 2008) and on the degree of product-market competition (Alonso et al. 2009). Managers' incentives to distort information also depend on the type of decisions that firms make. This link between the type of decisions that firms make and managers' incentives to distort information is the focus of this paper. Our central result shows that the flow of information between managers, and thus the performance of their firm, depends crucially on whether the firm sets prices or quantities. This is true even when the firm's performance does not depend on whether it sets prices or quantities once communication has taken place.

Our model is a simplified and reduced-form version of the model in Alonso et al. (2009). A firm consists of a headquarters and two operating divisions. Each operating division manufactures and sells a good in its own market. An increase in production by one division increases the total and marginal costs of the other, for instance, because it uses up scarce resources. Headquarters cares about overall firm profits but does not observe the demand conditions. Each division manager, in contrast, observes the demand conditions in his market but is also biased towards his division. After the division managers observe the demand conditions in their markets they communicate with headquarters. Headquarters then makes the decisions that maximize expected profits. The decisions that headquarters makes are either the prices that the divisions charge or the quantities that they produce. Note that since headquarters does not

[1] "Getting Tacit Knowledge to Work," *Financial Times*, March 28, 2004.

commit to decision or transfer rules, communication takes the form of cheap talk.

In our setting, the firm would be indifferent between setting prices or quantities if headquarters could perfectly observe the demand conditions in both markets.[2] Differences in expected profits are therefore only due to differences in the quality of communication. The quality of communication depends on the extent to which division managers have an incentive to misrepresent their information to influence headquarters' decision making in their favor. To understand what determines the quality of communication, we therefore need to understand division managers' incentives to misrepresent their information. To do so, note first that the division managers' incentives to misrepresent their information do not depend on there being two interacting and strategic managers. Indeed, the incentives of any one division manager to misrepresent his information would be the same if headquarters could perfectly observe the demand shock in the other market. What is crucial, however, is that by misrepresenting his information, each division manager is trying to influence two decisions, headquarters' decision in his own market and its decision in the other market.

To see this, suppose first that headquarters' sets quantities and consider the incentives of Manager 1, the manager in charge of Division 1, to misrepresent his information. If Manager 1 truthfully reported the demand conditions in his market, then, from his biased perspective, headquarters would produce too little in Market 1 and too much in Market 2. To induce headquarters to produce more in Market 1 and less in Market 2, Manager 1 would like headquarters to believe that demand conditions in his market are stronger than they actually are. Manager 1 therefore always has an incentive to overstate the demand conditions in his market.

Suppose next that headquarters sets prices and consider again the incentives of Manager 1 to misrepresent the demand conditions in his market. If Manager 1 truthfully reported his information, then, from his biased perspective, headquarters would set prices that are too high in Market 1 and too low in Market 2. Holding constant the price in Market 2, Manager 1 would like headquarters to believe that demand conditions in his market are weaker than they actually are. Headquarters would then set a lower price in Market 1. Holding constant the price in Market 1, however, Manager 1 would like headquarters to believe that

[2] There are a number of papers that examine a firm's choice between setting prices and quantities in the absence of informational problems. Meyer and Klemperer (1986) and Reisinger and Ressner (2009), for instance, identify factors which favor price- or quantity setting if a firm needs to adapt to demand shocks. In the absence of uncertainty, Singh and Vives (1984) show that there is a strategic advantage for oligopolistic firms of committing to fix a quantity rather than a price. To focus attention on the role of information, we deliberately abstract from any factors that favor price- or quantity setting in the absence of informational problems.

demand conditions in his market are stronger than they actually are. Headquarters would then expect Division 2's marginal costs to be high and thus set a higher price in Market 2. Manager 1 therefore faces countervailing incentives: on the one hand, he wants to understate demand to reduce the price in his own market but, on the other, he wants to overstate demand to increase the price in the other market. Because of these countervailing incentives, Manager 1 wants to understate demand if conditions are below average and overstate demand if conditions are above average. If conditions are exactly average, however, Manager 1 has no incentive to misrepresent demand.

These differences in the division managers' incentives to misrepresent their information translate into differences in how the division managers communicate and how much information headquarters receives in equilibrium. The crucial difference is that the division managers always want to misrepresent their information when headquarters sets quantities but not if it sets prices. As a result, headquarters receives more information if it sets prices rather than quantities. And because expected profits depend on the quality of communication, headquarters also expects to realize higher profits if it sets prices rather than quantities.

## 2 The Model

A firm consists of a headquarters and two operating divisions. Divisions 1 and 2 produce $q_1$ and $q_2$ units of their respective goods. Demand for the good produced by Division $j = 1, 2$ is characterized by the inverse demand function $p_j = \alpha_j - bq_j$, where $\alpha_j > 0$ and $b > 0$.

The production costs of Division $j = 1, 2$ are given by $c_j = cq_j + gq_1q_2$, where $c \geq 0$ and $g = b/2$.[3] An increase in production by Division 1 therefore increases the total and marginal costs of Division 2 and vice versa. This may be the case, for instance, because both divisions use a common input with a price that is increasing in the firm's total demand for the input.

Each division is run by a single manager. The utility function of Manager 1 − the manager in charge of Division 1 − is given by $U_1 = \lambda\pi_1 + (1 - \lambda)\,\pi_2$, where $\pi_1$ and $\pi_2$ are the profits of Divisions 1 and 2 and $\lambda \in (1/2, 1]$ is a parameter that captures the extent to which Manager 1 is biased towards his own division. Similarly, the utility function of Manager 2 is given by $U_2 = \lambda\pi_2 + (1 - \lambda)\,\pi_1$. We follow the modelling approach in Alonso et al. (2008, 2009) and Rantakari

[3]Our main result also holds for other values of $g$. Setting $g = b/2$ facilitates the characterization of the communication equilibrium under price setting since it ensures that the point of congruence is at $\alpha_j = \mu$ for $j = 1, 2$.

(2008) and assume that the own-division bias is exogenously given. The utility function of the manager in charge of headquarters is given by $U_{HQ} = \pi_1 + \pi_2$.

The firm must either decide on the prices or the quantities of the two divisions. We assume that the goods that the divisions produce, and thus the prices and quantities, cannot be contracted on ex ante or ex post. We focus on the case in which headquarters has the right to make all the decisions.

The demand conditions in Market $j = 1, 2$ are summarized by the intercept of the inverse demand function $\alpha_j$. It is common knowledge that $\alpha_1$ and $\alpha_2$ are independently drawn from a uniform distribution with support $[\mu - s,\ \mu + s]$, where $\mu > c$ and $s \leq (\mu - c)/3$.[4] Manager 1 learns the realization of $\alpha_1$ and Manager 2 learns the realization of $\alpha_2$. After the division managers learn the demand conditions in their respective markets, they simultaneously send a message to headquarters. Headquarters then updates its beliefs about the demand conditions and sets the prices or quantities that maximize the firm's expected profits. Since headquarters does not commit to a decision or transfer rule, communication takes the form of cheap talk.

The timing is as follows. First, each division manager learns the demand condition in his market. Second, the division managers simultaneously send a message to headquarters. Third, headquarters sets prices or quantities. Finally, payoffs are realized and the game ends.

# 3 Decision Making

We start by examining the decisions that headquarters makes for any given communication rule.

## 3.1 Quantities

Suppose headquarters receives messages $m_1$ and $m_2$ from the division managers. It then sets the quantities that maximize the firm's expected profits conditional on $m_1$ and $m_2$. The first order conditions for this problem are given by

$$q_1 = \frac{1}{2b}\left(\mathrm{E}\left[\alpha_1 | \, m_1\right] - c\right) - \frac{1}{2} q_2 \quad \text{and} \quad q_2 = \frac{1}{2b}\left(\mathrm{E}\left[\alpha_2 | \, m_2\right] - c\right) - \frac{1}{2} q_1. \tag{1}$$

Solving the first order conditions (1) we find that headquarters' quantity rules are

$$q_j = \frac{2}{3b}\left(\mathrm{E}\left[\alpha_j | \, m_j\right] - c\right) - \frac{1}{3b}\left(\mathrm{E}\left[\alpha_i | \, m_i\right] - c\right) \quad \text{for } i, j = 1, 2 \text{ and } i \neq j. \tag{2}$$

[4] The second inequality ensures that the firm's production levels are always positive.

Given these quantity rules, the firm's expected profits are given by

$$\mathrm{E}\left[\pi_1+\pi_2\right]=\frac{(\mu-c)^2}{3b}+2\frac{\sigma^2-V}{3b}, \tag{3}$$

where $\sigma^2=s^2/3$ is the variance of the demand conditions in each market and $V=\mathrm{E}\left[\left(\alpha_j-\mathrm{E}\left[\alpha_j\left|m_j\right.\right]\right)^2\right]$ for $j=1,2$ is the residual variance that measures the quality of communication. If $V=0$, the quality of communication is perfect and if $V=\sigma^2$, no information is communicated. Below we derive the equilibrium value of the residual variance and show that it depends on whether headquarters sets prices or quantities.

### 3.2 Prices

Suppose next that headquarters sets prices rather than quantities. After receiving messages $m_1$ and $m_2$ headquarters then sets the prices that maximize the firm's expected profits conditional on $m_1$ and $m_2$. The first order conditions for this problem are given by

$$p_j=\frac{1}{2}\left(\mathrm{E}\left[\alpha_j\right|m_j\right]+c\right)+\frac{1}{2}\left(\mathrm{E}\left[\alpha_i\right|m_i\right]-p_i\right)\quad \text{for } i,j=1,2 \text{ and } i\neq j. \tag{4}$$

Solving the first order conditions we find that the prices that headquarters' charges are given by

$$p_1=p_2=\frac{1}{3}\left(\mathrm{E}\left[\alpha_1\right|m_1\right]+\mathrm{E}\left[\alpha_2\right|m_2\right]+c\right). \tag{5}$$

Given these pricing rules, the firm's expected profits are again given by (3). For any given quality of communication expected profits therefore do not depend on whether the firm sets prices or quantities. This is the case because, for given prices, profits are linear in each division's quantity and, for given quantities, they are linear in each division's price. It can also be shown that expected profits are the same conditional on any posterior belief that headquarters might have about the demand conditions. After it has received the division managers' message, headquarters is therefore indifferent between setting prices and quantities.

## 4 Incentives to Misrepresent Information

To understand communication between the division managers and headquarters, we first need to understand the division managers' incentives to misrepresent their information.

## 4.1 Quantities

Suppose that Manager 1 can credibly misrepresent the demand conditions in his market. In other words, suppose that he can simply choose the posterior belief $z_1 \equiv \mathrm{E}[\alpha_1 | m_1]$ that headquarters has about the market conditions in Market 1. Manager 1 would then choose the posterior belief $z_1^*$ that solves

$$\max_{z_1} \mathrm{E}\left[\lambda \pi_1 + (1-\lambda)\pi_2 | \alpha_1\right], \tag{6}$$

subject to the quantities being set according to (2). In equilibrium it has to be the case that $\mathrm{E}_{m_2}[\mathrm{E}[\alpha_2 | m_2]] = \mathrm{E}[\alpha_2] = \mu$. Assuming that this relationship holds, the posterior $z_1^*$ that solves the maximization problem satisfies $z_1^* - \alpha_1 = b_Q$, where

$$b_Q \equiv \frac{2\lambda - 1}{4\lambda}(\mu - c) > 0 \tag{7}$$

is the "communication bias" under quantity setting. Manager 1 therefore always wants to overstate the demand conditions in his market to induce headquarters to increase production by his division and reduce production by the other division. Note that the amount by which he wants to overstate demand does not depend on the true demand conditions. We will see below that because of this feature the communication equilibria under quantity setting are analogous to those of the constant bias example in Crawford and Sobel (1982).

## 4.2 Prices

Suppose next that headquarters sets prices and assume again that Manager 1 can choose headquarters' posterior belief $z_1 \equiv \mathrm{E}[\alpha_1 | m_1]$ about the demand conditions in Market 1. Manager 1 would then choose the posterior belief $z_1^*$ that solves (6) subject to prices being set according to the pricing rules (5). Assuming again that $\mathrm{E}_{m_2}[\mathrm{E}[\alpha_2 | m_2]] = \mu$, the posterior belief $z_1^*$ that Manager 1 would choose then satisfies $z_1^* - \alpha_1 = b_P(\alpha_1 - \mu)$, where

$$b_P \equiv \frac{1}{2}(2\lambda - 1). \tag{8}$$

is the communication bias under price setting. If $\alpha_1 < \mu$, Manager 1 has an incentive to understate the demand conditions to lower $p_1$. And if $\alpha_1 > \mu$, he has an incentive to overstate the demand conditions to increase $p_2$. If $\alpha_1 = \mu$, however, Manager 1 has no incentive to misrepresent the demand conditions in his market.[5] In contrast to the quantity setting case, therefore, the extent to

[5]Note that that this is the case even though for any $\alpha_1$, including $\alpha_1 = \mu$, headquarters makes different decisions than Manager 1 would like it to make.

which Manager 1 wants to misrepresent his information depends on the true demand conditions.

## 5 Strategic Communication

We can now describe the Perfect Bayesian Equilibria of the game. To do so, we denote by $\mu_j(m_j \mid \alpha_j)$ the communication rule of Manager $j = 1, 2$ and we denote by $g_j(\alpha_j \mid m_j)$ headquarters' belief functions that state the probability of $\alpha_j$ given $m_j$. Headquarters' decision rules are given by (2) if it sets quantities and they are given by (5) if it sets prices.

All communication equilibria are partition equilibria in which the state space is divided into intervals and the division managers only reveal which interval the demand conditions belong to. Under quantity setting we denote by $a_j^N = (a_{j,0},\ a_{j,1}, ...,\ a_{j,N})$ the partitioning of the state space $[\mu - s,\ \mu + s]$ of Division $j = 1, 2$ into $N$ intervals, where $a_{j,0} = \mu - s$ and $a_{j,N} = \mu + s$. The next proposition describes the equilibria of the communication game when headquarters' sets quantities.

PROPOSITION 1. *Suppose headquarters sets quantities. Then, there exists a positive integer $N(b_Q)$ such that for every integer $1 \leq N_j \leq N(b_Q)$, $j = 1, 2$, there exists at least one equilibrium $(\mu_1(\cdot), \mu_2(\cdot)\ , q_1(\cdot), q_2(\cdot), g_1(\cdot), g_2(\cdot))$, where*

*i.* $\mu_j(m_j \mid \alpha_j)$ *is uniform, supported on* $[a_{j,i-1}, a_{j,i}]$ *if* $\alpha_j \in (a_{j,i-1}, a_{j,i})$,

*ii.* $g_j(\theta_j \mid m_j)$ *is uniform supported on* $[a_{j,i-1}, a_{j,i}]$ *if* $m_j \in (a_{j,i-1}, a_{j,i})$,

*iii.* $a_{j,i+1} - a_{j,i} = a_{j,i} - a_{j,i-1} + 4b_P$ *for* $i = 1, ...,\ N_j - 1$*, and*

*iv.* $q_1(\cdot)$ *and* $q_2(\cdot)$ *are given by (2).*

*Moreover, all other equilibria have relationships between $\alpha_1$ and $\alpha_2$ and headquarters' choices of $q_1$ and $q_2$ that are the same as those in this class for some value of $N_1$ and $N_2$; they are therefore economically equivalent.*

As anticipated above, therefore, the communication equilibria in the case of quantity setting are analogous to those of the constant bias example in Crawford and Sobel (1982). Essentially, the state space is partitioned into a finite number of intervals which grow in size as the demand conditions improve. The amount by which they grow is given by $4b_Q$, where $b_Q$ is the communication bias under quantity setting and is given by (7). Intuitively, because the division managers have an incentive to overstate demand, less information is communicated when the division managers report strong demand than when they communicate weak demand.

We focus on the most informative equilibrium in which the number of partitions is maximized. The residual variance for this equilibrium is well known from Crawford and Sobel (1982) and is reproduced in the following proposition.

PROPOSITION 2. *Suppose headquarters sets quantities. In the most informative equilibrium the residual variance is given by* $V_Q = \sigma^2/N(b_Q)^2 + b^2\left(N(b_Q)^2 - 1\right)/3$, *where* $N(b_Q)$ *is the largest integer that does not exceed* $\left(1+\sqrt{1+4s/b_Q}\right)/2$.

Under price setting all communication equilibria are still partition equilibria but they take a somewhat different form than under quantity setting. To describe the equilibria, we denote by $a_j^{2N} \equiv (a_{j,-N}, ..., a_{j,-1}, a_{j,0}, a_{j,1}, ..., a_{j,N})$ and $a_j^{2N-1} \equiv (a_{j,-N}, ..., a_{j,-1}, a_{j,1}, ..., a_{j,N})$ the partitioning of the state space $[\mu - s, \mu + s]$ of Manager $j = 1, 2$ into $2N$ and $2N-1$ intervals respectively, where $a_{j,-N} = \mu - s$, $a_{j,0} = \mu$ and $a_{j,N} = \mu + s$. Thus, $a_j^{2N}$ corresponds to finite interval equilibria with an even number of intervals and $a_j^{2N-1}$ corresponds to those with an odd number of intervals. As will be shown in the next proposition, the intervals are symmetrically distributed around $\mu$, that is, $a_{j,i} - \mu = \mu - a_{j,-i}$ for all $i \in \{1, ..., N\}$. The following proposition characterizes the finite communication equilibria.

PROPOSITION 3. *Suppose headquarters sets prices. Then for every positive integer* $N_j$, $j = 1, 2$, *there exists at least one equilibrium* $(\mu_1(\cdot), \mu_2(\cdot), p_1(\cdot), p_2(\cdot), g_1(\cdot), g_2(\cdot))$, *where*

*i.* $\mu_j(m_j \mid \alpha_j)$ *is uniform, supported on* $[a_{j,i-1}, a_{j,i}]$ *if* $\alpha_j \in (a_{j,i-1}, a_{j,i})$,

*ii.* $g_j(\alpha_j \mid m_j)$ *is uniform supported on* $[a_{j,i-1}, a_{j,i}]$ *if* $m_j \in (a_{j,i-1}, a_{j,i})$,

*iii.* $a_{j,i+1} - a_{j,i} = a_{j,i} - a_{j,i-1} + 4b_P\left(a_{j,i} - \mu\right)$ *for* $i = 1, ..., N_j - 1$,

$a_{j,-(i+1)} - a_{j,-i} = a_{j,-i} - a_{j,-(i-1)} + 4b_P\left(\mu - a_{j,-i}\right)$ *for* $i = 1, ..., N_j - 1$, *and*

*iv.* $p_1(\cdot)$ *and* $p_2(\cdot)$ *are given by (5).*

*Moreover, all other finite equilibria have relationships between* $\alpha_1$ *and* $\alpha_2$ *and the managers' choices of* $p_1$ *and* $p_2$ *that are the same as those in this class for some value of* $N_1$ *and* $N_2$*; they are therefore economically equivalent.*

The communication equilibria are similar to those examined in Alonso et al. (2008). Essentially, the state space is partitioned into intervals which are small when the demand conditions are close to the average $\mu$ and grow as demand conditions become more extreme, that is, as $|\alpha_j - \mu|$ increases. The amount by which they grow is proportional to $4b_P$ , where $b_P$ is the communication bias under price setting (8). Intuitively, a lot of information is communicated when demand conditions are close to average because the division managers' incentives are then closely aligned with headquarters. As demand conditions become more extreme, however, less information is communicated because division managers' incentives to over- or understate demand become stronger.

We again focus on the most informative equilibrium in which the number of partitions is maximized. In contrast to the quantity setting case, under price setting the number of partitions can go to infinity. In this case, there is an accumulation point at $\alpha_j = \mu$ in the neighborhood of which partitions are infinitesimally small. Alonso et al. (2008) characterize these communication equilibria and determine the residual variance of the most informative equilibrium. The next proposition applies their results to our setting.

PROPOSITION 4. *Suppose headquarters sets prices. In the most informative equilibrium in which $N_j \to \infty$, for $j = 1, 2$, the residual variance is given by $V_P = [(2\lambda - 1) / (8\lambda + 2)] \sigma^2$.*

We can now compare the quality of communication under quantity and price setting.

PROPOSITION 5. *In the permissible parameter range, that is, when $\mu > c$ and $s \leq (\mu - c)/3$, the quality of communication is better under price setting than under quantity setting, that is, $V_P \leq V_Q$.*

We saw above that the difference in expected profits only depends on the quality of communication. This proposition therefore implies that expected profits are larger under price than under quantity setting.

# 6 Conclusions

In this paper we explored this link between the type of decisions that a firm makes and the extent to which its managers share information with each other. For this purpose we developed a simple model of a multi-divisional firm in which division managers communicate with headquarters about the demand conditions in their markets. The main result in the paper is that the nature and quality of communication between the division managers and headquarters depends crucially on whether headquarters decides on the price that each division can charge or on the quantities they need to produce. This is the case even though, once communication has take place, expected profits do not depend on whether headquarters sets prices or quantities.

Our main result is robust to various changes in our model. It continues to hold, for instance, for different values of the externality parameter $g$ and when the firm faces both demand and cost externalities. Additional results $-$ such as the relative quality of communication under price and quantity setting $-$ are likely to be sensitive to changes in the specific model we looked at. We leave a general analysis of the interaction between managerial communication and the decisions that firms make for future research.

# 7 Appendix

**Proof of Proposition 1:** The proof that all equilibria are interval equilibria (parts *i*, *ii* and *iv.*) follows the same rationale as in Alonso, et al. 2008. Let $\overline{m}_j$ be headquarter's posterior belief of the expected value of $\alpha_j$ after receiving message $m_j$ For any communication rule of the other Manager, in state $\alpha_{j,i}$ Manager $j$ must be indifferent between sending a message that induces a posterior $\overline{m}_{j,i}$ and a posterior $\overline{m}_{j,i+1}$ so that $\mathrm{E}_{\alpha_{-j}}\left[U_j \mid a_{j,i}, \overline{m}_{j,i}\right] - \mathrm{E}_{\alpha_{-j}}\left[U_j \mid a_{j,i}, \overline{m}_{j,i+1}\right] = 0$ which given headquarter's equilibrium posterior belief $\overline{m}_{j,i} = \left(a_{j,i-1} + a_{j,i}\right)/2$ implies that $a_{j,i+1} - a_{j,i} = a_{j,i} - a_{j,i-1} + 4b_P$. This condition is formally equivalent to the constant bias example in Crawford and Sobel (1982). ■

**Proof of Proposition 2:** See Crawford and Sobel (1982), pp. 1440-1442. ■

**Proof of Proposition 3:** The proof that all equilibria are interval equilibria (parts *i*, *ii* and *iv*) follows the same rationale as in Alonso et al. (2008). Let $\overline{m}_j$ for $j = 1, 2$ be headquarter's posterior belief of the expected value of $\alpha_j$ after receiving message $m_j$ For any communication rule of the other manager, in state $\alpha_{j,i}$ manager $j$ must be indifferent between sending a message that induces a posterior $\overline{m}_{j,i}$ and a posterior $\overline{m}_{j,i+1}$ so that $\mathrm{E}_{\alpha_{-j}}\left[U_j \mid a_{j,i}, \overline{m}_{j,i}\right] - \mathrm{E}_{\alpha_{-j}}\left[U_j \mid a_{j,i}, \overline{m}_{j,i+1}\right] = 0$ which given headquarter's equilibrium posterior belief $\overline{m}_{j,i} = \left(a_{j,i-1} + a_{j,i}\right)/2$ implies that $a_{j,i+1} - a_{j,i} = a_{j,i} - a_{j,i-1} + 4b_P\left(a_{j,i} - \mu\right)$ for $i = 1, ..., N_j - 1$ and $a_{j,-(i+1)} - a_{j,-i} = a_{j,-i} - a_{j,-(i-1)} + 4b_P\left(\mu - a_{j,-i}\right)$ *for* $i = 1, ..., N_j - 1$ This condition is formally equivalent to the communication equilibria analyzed in Proposition 1 in Alonso et al. (2008). ■

**Proof of Proposition 4:** Following a similar derivation as in Lemma 1 in Alonso et al. (2008), we have that the residual variance for the most informative equilibrium is given by $V_Q = \left[\left|b_P\right| / \left(3 + 4\left|b_P\right|\right)\right]\sigma^2$. Substituting $\left|b_P\right| = \left(2\lambda - 1\right)/2$ we obtain $V_P = \left[\left(2\lambda - 1\right) / \left(8\lambda + 2\right)\right]\sigma^2$. ■

**Proof of Proposition 5:** First we establish a lower bound on $V_Q$. The function $V(N) = s^2/3N^2 + b^2\left(N^2 - 1\right)/3$ for $N, b, s \geq 0$ achieves its minimum at $N = \sqrt{s/b}$ with $V\left(\sqrt{s/b}\right) = 2bs/3 - b^2/3$ which is increasing in $b$ whenever $b \leq s$. From the non-negativity restrictions we have that $\mu - c \geq 3s$ which implies that $b_Q \geq 3s\left(2\lambda - 1\right)/4\lambda$. Therefore

$$V_Q \geq \frac{2b_Q s}{3} - \frac{b_Q^2}{3} \geq \frac{2\left[3s\left(2\lambda - 1\right)/4\lambda\right]s}{3} - \frac{\left[3s\left(2\lambda - 1\right)/4\lambda\right]^2}{3} = \frac{3}{16}\frac{\left(2\lambda + 3\right)\left(2\lambda - 1\right)}{\lambda^2}\sigma^2$$

Since $3\left(2\lambda+3\right)/\left(16\lambda^{2}\right)>1/\left(2\left(4\lambda+1\right)\right)$ for $\lambda\in(1/2,1]$ we have that $V_Q\geq V_P$. ■

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