# Capacity Expansion and Replacement in Growing Markets with Uncertain Technological Breakthroughs

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The accelerated pace of technological change has led to rapid obsolescence of productive capacity in electronics and other industries. Managers must consider the impact of future technologies while making acquisition and replacement decisions in such environments. We consider a problem where a sequence of technological breakthroughs are anticipated but their magnitude and timing are uncertain. A firm, operating in such an environment, must decide how much capacity of the current technology to acquire to meet future demand growth. It must also determine whether to upgrade any of the older vintages. We formulate this problem and present some structural results. Using these results, we then develop a highly efficient regeneration point-based dynamic programming algorithm. The effectiveness of the proposed algorithm is illustrated through a computational study. The sensitivity of the first period decision to various parameters is also explored.
(*Capacity Expansion*; *Equipment Replacement*; *Technology Adoption Models*; *Stochastic Dynamic Programming*; *Management of Technology*)

## 1. Introduction

Consider a firm in year 1992, with a large installed base of 8086-, 286-, and 386-based personal computers (PCs). To keep up with the ever increasing demand for computing and information sharing, management is contemplating purchases of several 486-based machines, the current best technology available. To take advantage of quantity discounts offered by PC manufacturers and economies of scale in the operation and maintenance of these equipment, the firm is proposing volume purchases to satisfy the current computing needs as well as those for the next few quarters. The firm is also planning to replace some of the older technologies currently in use. But many in the firm believe that this may not be a good idea, considering the impending release of 586- or Pentium-based PCs, a more attractive product that is likely to render the older technologies obsolete. Also, prices of 486-based PCs are likely to decrease substantially soon and may drop further with the release of Pentium-based PCs. Unfortunately, there is uncertainty about when 586-based PCs will actually become available and about their relative benefits. In fact, several other potentially superior technologies such as 686-, 786-, PowerPC, and RISC processor-based PCs are also on the horizon. Management faces several difficult choices in such a dynamic technological environment. Should the firm upgrade or replace the older PCs now or should it wait for an even better product? When a new innovation appears, should the firm adopt it immediately or wait for its price to come down? Among older models (8086, 286, and 386), which ones should be upgraded? Is it advisable to replace the older equipment with only the best available technology? Should the firm take advantage of economies of scale by buying

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excess PCs for the future? Or, is the risk of obsolescence too high to render the exploitation of scale economies unattractive? Do the answers to these questions change as more time goes by without a new innovation?

The managerial dilemma described above, albeit dated in terms of model names, is typical in industries such as banking, insurance, transportation, etc., where billions of dollars are invested in computing equipment. As an illustration, consider the following quote from a leading computer trade journal (Ubois 1993):

> Facing a fast-moving market, managers must make difficult choices to keep their systems current. With processor speed doubling every year and a half, planning for obsolescence is critical but difficult for most managers. Rapid change presents managers with a dilemma. New products are introduced at premium prices and then depreciate quickly, so managers who buy the latest products with the longest useful life will probably see the same item advertised at half the price a few months later. But choosing slightly older, less expensive systems whose prices have stabilized also can seem foolish a few months later when yet another generation of products is introduced.

When the technology is evolving rapidly, the firm needs to carefully consider the erosion in value of its existing equipment each time a new, more powerful equipment is released. The loss in value of existing equipment due to the downward pricing pressure of new technology can be staggering. For example, consider the following report by Hastings (1994):

> The purchase price of all microcomputers is approaching $200 billion. The market value today of all these computers is approximately $50 billion. Although productivity gains have been enormous, companies and individuals have lost almost $150 billion in investment. While losses cannot be totally avoided, they can be minimized by planning the purchase and sale of equipment. Most of the loss in value occurs immediately after the announcement of a more powerful system.

The problems described above are not confined to the computer industry. The growth of electronics has led to a rapid pace of technological change in industries such as medical equipment, telecommunications, machine tools, and image processing equipment. For instance, consider the technological changes in the medical imaging equipment industry over the past two decades. Successive generations of technologies have provided better diagnostic information that has helped physicians in eliminating expensive surgeries and in selecting more appropriate medical therapies. X-ray was the dominant imaging modality until the mid-1970s. It was gradually replaced by computed tomography (CT). The CT equipment themselves have undergone revolutionary changes, with successive generations of equipment resulting in higher patient throughput and better image quality. CT technology matured in the early 1980s, but equipment based on magnetic resonance (MR) imaging technologies appeared in 1984 (Trajtenberg 1990). Lately, not only have the MR equipment been undergoing significant improvements, but new equipment based on positron emission tomography (PET), a promising new imaging technology, have emerged. Hospitals and firms leasing such imaging equipment have to consider future demand growth as well as the forthcoming technological changes while making equipment acquisition and replacement decisions.

The uncertain path of technological evolution and its importance to a firm's adoption decision has long been recognized in the economics literature. For example, Rosenberg (1982), in an illuminating essay, notes that ''the technological future is inevitably shrouded in uncertainty'' and ''the optimal (adoption) timing of an innovation becomes heavily influenced by expectations concerning the timing and the significance of future improvements.'' Empirical studies by Karlson (1986), Antonelli (1989), and Cainarca et al. (1989) similarly underscore the profound influence of technological expectations on firms' acquisition and replacement timing in industries as diverse as steel making, cotton spinning, and flexible automation. Despite this widespread recognition, there is a paucity of normative models in the literature that can help managers choose appropriate technologies (the timing and size of investment) in a rapidly changing and uncertain technological environment. This paper is an attempt to bridge this gap. Specifically, we present a capacity expansion and replacement model that recognizes the possibility of a sequence of technological breakthroughs. The model allows acceleration and saturation of breakthroughs and uncertainty in the technological life cycle. By letting the purchase price and salvage value of equipment be dependent on the current best vintage available, we model the downward pricing pressure and investment losses due to new technological breakthroughs. The purchase price and salvage value functions are chosen so as to allow economies of scale. We differentiate among the vintages

of technologies that a firm possesses in terms of both operating cost and the value loss for different technologies. Most importantly, the choice of technologies and their capacity sizes are considered simultaneously, for both acquisition and replacement decisions. The acquisition choice is not restricted to the newest technology; vintages that were once uneconomical may be purchased later due to price drops. Similarly, one or more vintages in use may be replaced partially or completely.

The rest of the paper is organized as follows. In the next section, we briefly review the relevant literature. In §3, we present a model for situations where the technological evolution is predictable. The analysis of this model provides important insights for §4, where a model with stochastic evolution of technology is presented. The optimal solution is shown to possess several structural properties that are exploited in §5 in an efficient stochastic dynamic programming procedure. Section 6 presents results of a computational study using this solution procedure. We conclude in §7 with a few final observations.

## 2. Review of the Literature

We briefly review related work in the literature on machine replacement, technology adoption, and capacity expansion wherein some of the aspects described earlier have been addressed. The early equipment replacement models (refer to Pierskella and Voelker 1976 for a survey) simply consider the issue of optimal replacement timing of a single machine with a new machine of the same or better technology. Chand and Sethi (1982) consider such replacement decisions in an improving technological environment, with better machines *available with certainty* in successive time periods. Jones et al. (1991) present solution procedures for a more general deterministic replacement model with multiple identical machines, and fixed and variable costs associated with replacing the machines. Cohen and Halperin (1986) present a model with a known choice of technologies available at different times, where capacity of a technology is fully replaced by another technology for some fixed cost. There are a few interesting papers in the replacement literature that model stochastic technological breakthroughs. Goldstein et al. (1988) present a solution procedure for a single machine replacement problem with one anticipated technological breakthrough characterized by a constant hazard rate. Nair and Hopp (1992) and Nair (1995), respectively, use forecast horizon-based approaches to solve more general models with one and several anticipated breakthroughs. Balcer and Lippman (1984) present a rich model of technological innovation and replacement with uncertainty in the time between discoveries, the size of each discovery, and the future pace of discovery. They provide valuable insights on the impact of future technological expectations on replacement policies. In general, equipment replacement under stochastic evolution of technology has remained underexplored. Models that do allow technological uncertainty restrict themselves to two alternatives: Either replace all existing capacity or none of it. An important exception is Monahan and Smunt (1989) who consider a problem where fixed capacity of an old labor-intensive production process can be converted incrementally to a new flexible automation process, which lowers inventory-related costs. Uncertainty in both interest rates and technological improvements complicates this decision. Technological improvements in Monahan and Smunt lower the acquisition cost of flexible automation. In contrast, each technological improvement in our model yields a new vintage of technology available for adoption, which lowers the acquisition cost and salvage value of *all* earlier vintages. While the choice in Monahan and Smunt is the rate of conversion from ''old-to-new,'' our model considers incremental replacement of any of the existing equipment vintages with more recent vintages, as well as capacity expansion to meet future demand growth.

Capacity expansion models, unlike machine replacement models, permit consideration of scale economies and incremental acquisition of capacity. While there is a vast literature in this area (Freidenfelds 1981, Luss 1982), we are not aware of any paper that considers capacity expansion issues simultaneously with replacement in the context of a sequence of random technological developments. Gaimon (1989) presents a dynamic game analysis to understand the impact of competitive forces on the acquisition of new flexible technology capacity and disposal of old technology. Gaimon mentions at the outset (p. 410) that ''. . . firms also run the risk of making an enormous investment in technology

that may soon become obsolete.'' Klincewicz and Luss (1985) present solution procedures for determining when to install facilities of fixed capacity using current technology under different demand growth conditions, given spare capacity of the old technology. However, none of these papers models the stochastic evolution of technology and obsolescence effects.

Overall, there is a rich literature in machine replacement, capacity expansion, and technology adoption. However, there appears to be no prior work that can help a manager in making *detailed capacity and technology acquisition and replacement decisions* in the type of dynamic and uncertain technological environment described earlier. In fact, to our knowledge, there is no such work even in a scenario with a *predictable* technology evolution. We believe that the comprehensive nature of our model makes it valuable to a manager making these decisions.

## 3. Predictable Technological Evolution

Consider the situation where the sequence of technological advances over the problem horizon is predictable, i.e., the time of appearance of future technologies is known. The analysis of this model is important for the understanding of the general model because any sample realization of stochastic technology evolution gives rise to the deterministic model. Moreover, the analysis presented here is of value since the deterministic model has not been considered in the literature. In each period, there is demand for *additional* capacity that must be satisfied. Let $d_t \geq 0$ denote the *increase* in demand for capacity in period $t$. This model is applicable to dynamic environments where the appearance of new technologies is accompanied by sustained demand growth. Most firms and organizations have seen demand for microcomputers increase steadily over time (*Economist* 1994). Similarly, the medical imaging needs of hospitals have grown continually in the last three decades.

Let the potential technological innovations be indexed $1, 2, \ldots, M$, where $M$ corresponds to the highest technology level achievable within the problem horizon. Technology level $(m + 1)$ represents a clear improvement over technology $m$. For instance, successive generations of medical imaging equipment have higher patient throughput rate or better image quality, and successive PCs have higher processing speeds. However, this does not imply that a firm would only purchase the latest technology at any time. This is consistent with what we observe in practice. For example, if the price of a 486-based PC drops significantly when a Pentium-based PC is introduced, many firms may buy the 486-based PC even though Pentium-based PCs are available. Also, the actual costs and benefits of a technology may be specific to an adopting firm (Rosenberg 1982). For instance, while the scan time of an MR unit may be an important attribute for one user, the variety of body parts that can be imaged may be more important for another user. Thus, a strength of the model is that the best technology to acquire at any time is an endogenous variable.

We now define the decision variables for this model. Let $x_{ptj}$ denote the amount of vintage $p$ capacity acquired in period $t$ and disposed in period $j(>t)$. This definition allows all possibilities of capacity purchases and disposals. The index $j$ in variable $x_{ptj}$ ensures that capacity disposed in a period must have been purchased earlier. Replacement of capacity is modeled by disposal followed by an immediate acquisition. Let $Y_{pt}$ be the total vintage $p$ capacity acquired in period $t$, i.e.,

$$Y_{pt} = \sum_{j>t} x_{ptj} \quad \forall p, t. \tag{1}$$

Let $Z_{pj}$ be the total vintage $p$ capacity disposed in period $j$, i.e.,

$$Z_{pj} = \sum_{t<j} x_{ptj} \quad \forall p, j. \tag{2}$$

At any time, the firm may have unused capacity on hand that was purchased to take advantage of the scale economy in acquisition. Let the unused capacity of vintage $p$ at the beginning of period $t$ be denoted by $I_{pt}$. Then the total vintage $p$ capacity *utilized* in period $t$ is

$$= I_{p1} + \sum_{\tau \leq t} \sum_{j>t} x_{p\tau j} - I_{p,t+1}.$$

Let $m_t$ denote the highest technology level available in period $t$. The equation balancing the demand and supply of capacity in each period is

$$\sum_{p=1}^{m_t} (I_{pt} + Y_{pt} - Z_{pt} - I_{p,t+1}) = d_t \quad \forall t. \tag{3}$$

We distinguish between unused and used (or utilized) capacity since their salvage costs may be quite different. For example, replacement of used capacity may involve disruption of production activities, resulting in a higher salvage cost for used capacity relative to unused capacity. As another example, consider a medical equipment leasing firm that places orders for *future delivery* of MR units to take advantage of quantity discounts. However, it may not take delivery later and instead pay a penalty. This situation is equivalent to disposing unused capacity. The penalty or equivalent salvage cost in this scenario will be much lower than the salvage cost for disposing a unit of used capacity of the same vintage. Let $Z^e_{pt}$ denote the amount of unused capacity of vintage $p$ disposed in period $t$. The amount of unused capacity disposed in a period cannot exceed the amount of unused capacity available,

$$Z^e_{pj} \leq I_{pj} \quad \forall p, j. \tag{4}$$

Finally, the nonnegativity conditions on the variables are

$$x_{ptj}, I_{pt}, Y_{pt}, Z_{pt}, Z^e_{pt} \geq 0 \quad \forall p, t, j. \tag{5}$$

We now specify the various cost functions in the model. Let $f_{pmt}(\cdot)$ be the cost of acquiring capacity of technology $p$ in period $t$, given that the latest available technology is $m(\geq p)$. Let the cost of carrying capacity of technology $p$ in period $t$ be represented by $h_{pt}(\cdot)$. The carrying cost includes the cost of maintenance, space, and insurance, and is incurred on total capacity, used and unused. We assume that the cost functions $f_{pmt}(\cdot)$ and $h_{pt}(\cdot)$ are concave to reflect potential economies of scale; this is a common assumption in the capacity expansion literature (Luss 1982, Monahan and Smunt 1989) and has been validated in a variety of industries (Liebermann 1987). For example, MR manufacturers generally provide quantity discounts of about 10% for purchasing more than 5 units, about 20% for more than 10 units, etc. However, deliveries of the units can be taken over a two-year period, resulting in lower carrying costs. Also, maintenance costs often exhibit scale economies due to learning effects and the ability to obtain quantity discounts in maintenance contracts. The operating cost per unit capacity for vintage $p$, when used in period $t$, is $c_{pt}$. All costs are discounted to the present (i.e., beginning of period 1) and the appropriate discount factors are assumed to be incorporated in the cost parameters, as in Luss (1982). Therefore, the carrying cost does not include the cost of capital.

When a new technology becomes available, there may be both unused and used capacity of older technologies, both of which may be disposed fully or partially. Let $g_{pmt}(\cdot)$ denote the net salvage cost (may be negative) from disposal of *unused* capacity of technology $p$ in period $t$, given that the latest technology available is $m(\geq p)$. The function $\bar{g}_{pmt}(\cdot)$ is similarly defined for *used* capacity. We assume the following functional form for the salvage cost function $g_{pmt}(\cdot)$:

$$g_{pmt}(z) = s_{pmt}\delta(z) - r_{pmt}z,$$

where $\delta(z) = 1$ if $z > 0$, and $\delta(z) = 0$ if $z = 0$. The nonnegative parameters $s_{pmt}$ and $r_{pmt}$ represent, respectively, the fixed cost and unit revenue from the disposal of technology type $p$, when $m$ is the latest technology. The cost of disposing used capacity, $\bar{g}_{pmt}(\cdot)$, is expressed similarly. Observe that all the parameters, $f_{pmt}(\cdot)$, $h_{pt}(\cdot)$, $c_{pt}$, $g_{pmt}(\cdot)$, and $\bar{g}_{pmt}(\cdot)$, are functions of time and of the vintage ($p$). This can be used to model features such as declining purchase costs with time (due to wider acceptance of a technology), declining salvage values for older vintages, and lower operating costs for newer vintages. Further, the acquisition cost and salvage values are also functions of the latest vintage, $m$. This enables us to model the decline in prices and salvage values of older vintages discussed earlier. A mathematical programming formulation of this deterministic problem, denoted as ($\mathcal{D}$), is given by:

$$\begin{aligned}\underset{x_{p\tau j}, Z^e_{pt}}{\text{Min}} \sum_{t=1}^{T}\sum_{p=1}^{m_t}\Bigg( & f_{pm_tt}(Y_{pt}) + h_{pt}\Bigg(I_{p1} + \sum_{\tau\leq t}\sum_{j>t} x_{p\tau j}\Bigg) \\ & + c_{pt}\Bigg(I_{p1} + \sum_{\tau\leq t}\sum_{j>t} x_{p\tau j} - I_{p,t+1}\Bigg) \\ & + g_{pm_tt}(Z^e_{pt}) + \bar{g}_{pm_tt}(Z_{pt} - Z^e_{pt})\Bigg),\end{aligned}$$

subject to: (1–5),

where $m_t$ is the best technology level available in period $t$. The first term within the summation signs is the acquisition cost for capacity purchases of technology type $p$ made in period $t$. The second term is the carrying cost and the third term is the operating cost in period $t$ for

technology $p$. The fourth and fifth terms, respectively, are the salvage cost of disposing unused and used capacity in period $t$. In any period $t$, the only technologies that can be acquired, operated, and disposed are those that have appeared by period $t$, i.e., $p = 1, \ldots, m_t$.

We now present an important result for this model. The proof is given in the appendix.

THEOREM 1. *There exists an optimal solution to problem* ($\mathcal{D}$) *that has the following properties*:

(i) *One would never purchase capacity of a vintage in a period when there is unused capacity of the same or different vintage on hand, i.e.,*

$$I_{pt}Y_{mt} = 0 \quad \textit{for all } p, m, t. \tag{6}$$

(ii) *One would never purchase capacity of more than one vintage in any period, i.e.,*

$$Y_{pt}Y_{mt} = 0 \quad \textit{for all } p, m, t. \tag{7}$$

(iii) *At any time, there would never be unused capacity of more than one vintage on hand, i.e.,*

$$I_{pt}I_{mt} = 0 \quad \textit{for all } p, m, t. \tag{8}$$

We now state a number of key implications of Theorem 1. These corollaries follow largely from the conditions (6)–(8); detailed proofs can be found in Rajagopalan et al. (1993).

COROLLARY 1. *There exists an optimal solution to problem* ($\mathcal{D}$) *in which capacity purchases to meet future demand and unused capacity disposed correspond to demand increments for an integral number of periods.*

COROLLARY 2. *There exists an optimal solution to problem* ($\mathcal{D}$) *in which, whenever a unit of used capacity is replaced, all units corresponding to that vintage are replaced*.

COROLLARY 3. *There exists an optimal solution to problem* ($\mathcal{D}$) *in which all capacity purchases and disposals correspond to an integral number of periods*.

COROLLARY 4. *There exists an optimal solution to problem* ($\mathcal{D}$) *in which capacity meant to satisfy demand increments for earlier periods is purchased before purchasing capacity for later periods*.

These results significantly reduce the choices to be considered in deciding which type of technology and how much capacity to purchase, dispose, and replace. There is another important consequence of these corollaries. Together, they imply that there exists an optimal solution to problem ($\mathcal{D}$) such that the amount of unused or used capacity of any vintage in any period equals the sum of demand increments for an integral number of periods. In the next section, we extend these results to the general case with stochastic technological evolution.

# 4. Uncertain Technological Evolution

We model the stochastic evolution of technology as a semi-Markov process. Specifically, the technology evolution is modeled as a function of two factors: (i) the number of periods between two consecutive innovations, and (ii) the new level of technology achieved with an innovation. Given that vintage $m$ has just become available, let $Q_m(\cdot)$ and $q_m(\cdot)$ denote the cumulative distribution function (cdf) and probability distribution function (pdf) of the amount of time until the next vintage becomes available. That is, the number of periods between the appearance of successive vintages depends on the vintage $m$ last achieved, but is independent otherwise. We assume $q_m(0) = 0$. The level of technology available, once a breakthrough occurs, changes according to a Markov process with a one-step transition matrix $\mathbf{P} = [P_{mn}]$, where $P_{mn}$ is the probability that vintage $n$ appears, given that $m$ is the last vintage that appeared. This is similar to the approach used by Monahan and Smunt (1989) and by Muth (1986) in the context of learning effects.

The model of technological evolution proposed above can accommodate a variety of realistic situations by proper choice of time-to-discovery distributions $Q_m(\cdot)$ and transition probabilities $P_{mn}$. For instance, in many industries, there is an initial phase with rapid technological breakthroughs, followed by a second phase with mostly small but a few major improvements, and a final phase wherein the rate of innovation diminishes gradually to zero. This has been observed in memory chips (Methe 1992), medical imaging equipment (Tratjenberg 1990), machine tools (Rosenberg 1982), and many other industries. CT equipment was first introduced in 1973 and there were a number of significant advances in the first few years followed by a gradual maturation period, with no significant advances since 1983. Such

acceleration and saturation in technological breakthroughs can be modeled as follows: The $Q_m(\cdot)$ values are chosen such that the mean time-to-discovery is small for small $m$ values and increases for large values of $m$. The transition matrix $P_{mn}$ will have probability mass centered relatively far from the diagonal in the initial few rows, while the mass will be centered closer to the diagonal in the last few rows.

As time passes without any discovery, this may imply an increase in the likelihood of a discovery in situations where competing manufacturers are making final design modifications or fixing software bugs for a new vintage. Alternatively, a delay in discovery may imply a reduction in the likelihood of an impending discovery due to the failure of a line of research or temporary abandonment of development efforts by the equipment manufacturers. Both situations can be accommodated in the model as it allows for an arbitrary distribution of the time between innovations.

The semi-Markov process for the evolution of technology can be analyzed like a Markov process by letting the state at time $t$ be the pair $(m_t, k_t)$, with $m_t$ being the most recent technology and $k_t$ the period in which it was introduced. The dynamics of technological evolution can then be represented by the Markov state transition process

$$(m_{t+1}, k_{t+1}) = \Psi_t(m_t, k_t), \tag{9}$$

where the transition function $\Psi_t$: $(m_t, k_t) \to (m_{t+1}, k_{t+1})$ depends on the one-step transition matrix $\mathbf{P}$ and the time-to-discovery distributions $Q_m(\cdot)$. Let vectors $\mathbf{X}_t$ and $\mathbf{I}_t$, respectively, represent the levels of used and unused capacities at the beginning of period $t$. To keep the exposition uncluttered, we assume without loss of generality that initially (at the beginning of period 1) there is capacity of only one technology type, say type 1. This consists of some used capacity and, possibly, unused capacity to satisfy the demand increments for an integer number of future periods. The state of technology becomes known at the beginning of a period. Based on this information, one may choose to dispose capacities already on hand and purchase additional capacities. Let vectors $\mathbf{Z}_t$ and $\mathbf{Z}_t^e$ represent, respectively, the total capacity disposed and unused capacity disposed in period $t$. Let the capacity purchases in period $t$ be represented by vector $\mathbf{Y}_t$. All vectors have $M$ columns and each column corresponds to a technology. Let $\hat{\mathbf{e}}$ be a unit row vector with $M$ elements. Then the capacity balance equation relating the state and decision vectors can be written as

$$\mathbf{I}_{t+1}\hat{\mathbf{e}} = (\mathbf{I}_t + \mathbf{Y}_t - \mathbf{Z}_t)\hat{\mathbf{e}} - d_t, \tag{10}$$

$$\mathbf{X}_{t+1} = \mathbf{X}_t - (\mathbf{Z}_t - \mathbf{Z}_t^e) + (\mathbf{I}_t - \mathbf{I}_{t+1})^+. \tag{11}$$

It is important to note that, while the technological evolution, characterized by Equation (9), is stochastic, the evolution of capacity vectors, given by Equations (10) and (11), is deterministic. That is, the capacities of various technologies on hand for the next period become known as soon as this period's purchase and disposal decisions are made. Another important aspect of this model is that the technological evolution (9) is independent of both current capacities, $(\mathbf{X}_t, \mathbf{I}_t)$, as well as purchase and disposal decisions, $(\mathbf{Y}_t, \mathbf{Z}_t, \mathbf{Z}_t^e)$. To exploit these important properties of the model, we partition the state into two sets, $(m_t, k_t)$ and $(\mathbf{X}_t, \mathbf{I}_t)$.

Given the current state of technology $(m_t, k_t)$ and capacity $(\mathbf{X}_t, \mathbf{I}_t)$, the problem is to find purchase and disposal vectors $(\mathbf{Y}_t, \mathbf{Z}_t, \mathbf{Z}_t^e) \in U_t$, where $U_t$ is the set of all *admissible* alternatives. In particular, $U_t$ takes into account that (i) the amount of capacity purchased and disposed is nonnegative, (ii) the purchase and disposal decisions are limited to the set of technologies available in period $t$, (iii) for any technology, the capacity disposed does not exceed the capacity currently on hand, and (iv) the amount of capacity purchased in period $t$ does not exceed the sum of capacity disposed plus future capacity requirements. Let the possible values of $(m_{t+1}, k_{t+1})$ in (9) be indexed by $i$ such that the technology next period will be in state $(m_{t+1}, k_{t+1})_i$ with probability $\phi_i$, where $\phi_i$ is a function of $(m_t, k_t)$. Probabilities $\phi_i(m_t, k_t)$ are obtained from the one-step transition matrix $\mathbf{P}$ and the time-to-discovery distributions $Q_m(\cdot)$. Let $C_t((m_t, k_t), (\mathbf{X}_t, \mathbf{I}_t), (\mathbf{Y}_t, \mathbf{Z}_t, \mathbf{Z}_t^e))$ be the expected total cost associated with decision $(\mathbf{Y}_t, \mathbf{Z}_t, \mathbf{Z}_t^e)$, assuming that all future decisions are taken optimally. The terminal cost is assumed to be zero, i.e., $C_{T+1}(\cdot) \equiv 0$. The stochastic dynamic programming formulation ($\mathcal{P}$) of the problem is given by the set of recursive equations

$$C_t((m_t, k_t), (\mathbf{X}_t, \mathbf{I}_t), (\mathbf{Y}_t, \mathbf{Z}_t, \mathbf{Z}_t^e))$$

$$= \Big( L_t(m_t, \mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t, \mathbf{Z}_t, \mathbf{Z}_t^e) + \sum_i \phi_i(m_t, k_t) C_{t+1}^*((m_{t+1}, k_{t+1})_i, (\mathbf{X}_{t+1}, \mathbf{I}_{t+1})) \Big),$$

$$C_t^*((m_t, k_t), (\mathbf{X}_t, \mathbf{I}_t)) = \min_{(\mathbf{Y}_t, \mathbf{Z}_t, \mathbf{Z}_t^e) \in U_t} C_t((m_t, k_t), (\mathbf{X}_t, \mathbf{I}_t), (\mathbf{Y}_t, \mathbf{Z}_t, \mathbf{Z}_t^e)),$$

subject to: State Equations (10) and (11),

where the function $L_t(\cdot)$ is the sum of acquisition, carrying, operating, and salvage costs incurred in period $t$, and is given by

$$L_t(m_t, \mathbf{X}_t, \mathbf{I}_t, \mathbf{Y}_t, \mathbf{Z}_t, \mathbf{Z}_t^e) = \sum_{p=1}^{m_t} (f_{pm_t t}(Y_{pt}) + h_{pt}(I_{p,t+1} + X_{p,t+1}) + c_{pt}(X_{p,t+1}) + g_{pm_t t}(Z_{pt}^e) + \bar{g}_{pm_t t}(Z_{pt} - Z_{pt}^e)).$$

If the sequence of technological discoveries were known, formulation ($\mathcal{P}$) would be equivalent to formulation ($\mathcal{D}$). That is, for any sample path realization of the stochastic technological evolution, the optimal solution to ($\mathcal{P}$) can be obtained by solving ($\mathcal{D}$). Recall that an optimal solution to ($\mathcal{D}$) can be found by restricting all capacity purchases and disposals to be equal to demand increments for an integral number of periods. Let $A_t$ be a subset of admissible alternatives, $A_t \subset U_t$, such that capacity purchases and disposals are restricted to an integral number of periods. The following theorem (proved in the Appendix) restricts an optimal solution to problem ($\mathcal{P}$) to a policy that allows capacity purchases and disposals only for an integral number of periods:

THEOREM 2. *There exists an optimal solution,* $(\mathbf{Y}_t^*, \mathbf{Z}_t^*, \mathbf{Z}_t^{e*})$, *to formulation* ($\mathcal{P}$) *such that* $(\mathbf{Y}_t^*, \mathbf{Z}_t^*, \mathbf{Z}_t^{e*}) \in A_t \quad \forall t$.

This theorem, in effect, generalizes the results in §3 to the model with stochastic technology evolution. This leads to an enormous reduction in the state space and the decision alternatives, which is exploited in the next section when we develop a dynamic programming solution procedure. It should be noted that, while the optimal time of disposal is determined at the time of purchase in the deterministic model, this can clearly not be the case in the stochastic model. However, the results obtained in §3 still hold here for the following reason: The only effect of the stochastic technological evolution is on the number and types of technologies available in a period, which impacts the capacity units and types that may be disposed and the capacity types that can be purchased. Since the deterministic results are independent of the number and types of technologies available in a period, they hold in the stochastic case too. However, the disposal period would depend on the sample path of technological evolution, which is uncertain in the stochastic case but known in the deterministic case.

Before presenting the procedure, we introduce some terminology and present additional results that would further reduce the state space and the set of decision alternatives. Let a period ending with zero unused capacity on hand be referred to as a *regeneration period*. A period in which capacity is acquired to meet future demand increments is referred to as an *acquisition period*. From Corollaries 1 through 4, it is clear that capacity of exactly one vintage is acquired in an *acquisition period* to meet the demand increments for all the periods until the *next acquisition period*. Also, all the unused capacity of an earlier vintage must be completely disposed before purchasing capacity of a later vintage.

If technological breakthroughs were predictable, i.e., if the sequence of discoveries and their timing were all known, it would never be optimal to acquire unused capacity and then dispose of it unused at a future time. It is the uncertainty in the nature and timing of breakthroughs that makes disposal of unused capacity possible. However, not all unused capacity need be disposed and the new vintage adopted immediately. For instance, if further breakthroughs are anticipated soon with high probability, the firm may be unwilling to dispose of all the unused capacity and adopt the new vintage immediately. The firm may dispose part of the unused capacity and wait for even better vintages. The following result provides us some guidance in restricting the choices to be considered in deciding what part of the unused capacity to dispose.

COROLLARY 5. *There exists an optimal solution to problem* ($\mathcal{P}$) *in which unused capacity meant to satisfy demand increments for later periods is disposed before disposing unused capacity meant for earlier periods*.

The next result restricts the periods in which used capacity may be replaced.

COROLLARY 6. *There exists an optimal solution to problem ($\mathcal{P}$) such that used capacity is never replaced while unused capacity is still on hand. All unused capacity must either be disposed or put into use before used capacity can be replaced.*

Corollary 6 implies that we need to consider replacing used capacity only in periods where capacity will, in any case, be acquired to meet future demand. That is, replacement of used capacity can occur *only in acquisition periods* (note, however, that every acquisition need not be accompanied by replacement), thus reducing the computational effort significantly. Next, we assume that

$$g_{pmt}(x) \leq h_{pt}(x) + g_{pm,t+1}(x) \quad \forall x, t \text{ and } m \geq p, \quad (12)$$

which stipulates that the cost of salvaging an amount of capacity in a period shall not exceed the cost of carrying it to the next period and then salvaging it. This condition appears quite reasonable in high technology environments where revenue from salvaging capacity typically decreases over time. Suppose the firm has some unused capacity on hand when a new technology appears. The next proposition restricts the periods in which part of this unused capacity may be disposed.

PROPOSITION 1. *It is optimal to restrict disposal of unused capacity to periods when a new technology becomes available.*

From condition (12), it is clear that postponing the actual disposal of the unused capacity to a period later than the period of appearance of the new technology can only result in higher costs. When a technology appears, (i) the characteristics of the new technology that has appeared relative to the vintage of the unused capacity become known, (ii) the probabilistic information about the next innovation becomes available. Neither of these pieces of information is going to change until the next innovation occurs. Based on this information, the amount of unused capacity to be disposed can be determined and this decision is not going to be revised until the next innovation occurs.

If we denote a period in which unused capacity is disposed as a *disposal period*, it follows from Proposition 1 that the disposal period always coincides with the appearance of a new vintage. A similar result holds for replacement of used capacity.

PROPOSITION 2. *Any capacity acquired for the sake of replacement is put into use immediately.*

The proof for all corollaries and propositions can be found in Rajagopalan et al. (1993). From the above results, it is clear that acquisition and disposal periods constitute the only two decision epochs in the model, with replacement of used capacity restricted to acquisition periods. Of course, disposal and acquisition periods may coincide. This will happen if a new technology becomes available in an acquisition period. In the next section, we use these observations to develop regeneration point-based dynamic programming recursions in terms of the two decision epochs.

## 5. Development of an Efficient Solution Procedure

In this section, we develop an efficient regeneration point-based dynamic programming recursion procedure using the results of the last two sections. The problem can be regarded as a sequence of acquisition, replacement, and disposal decisions. Disposal of unused capacity is considered only when a new technology appears (Proposition 1). Acquisition and replacement are considered only when the firm has no unused capacity (Theorem 2 and Corollary 6). A formulation is developed in terms of these two decision epochs.

### Optimal Acquisition and Replacement Decision

Let $\langle u \rangle_{\xi=1}^{p} = \{u(1), u(2), \ldots, u(p)\}$ be the acquisition history, where $u(\xi)$ is the period in which the $\xi$th acquisition was made, and $p$ is the number of acquisitions made so far. Let $\langle w \rangle_{\xi=1}^{p} = \{w(1), w(2), \ldots, w(p)\}$ be the technology-mix sequence such that $w(\xi)$ represents the technology type acquired in period $u(\xi)$. As time progresses and more capacity is acquired, both acquisition history and technology-mix sequences are updated by appending the acquisition period and technology type to the respective sequences. It is assumed, for ease of exposition, that the newest technology is acquired to satisfy future demand increments or to replace existing capacity.

The initial capacity consists of $D_0$ units already in use and, possibly, unused capacity to satisfy the demand increments for an integer number of future periods. Let vintage 1 represent this initial capacity, i.e., $u(1) = 1$ and

$w(1) = 1$. More general initial conditions can be incorporated by simply redefining $u(1)$ and $w(1)$. Let $d(i, j)$ represent the cumulative demand increment over periods $i$ through $j$, i.e.,

$$d(i, j) = \sum_{t=i}^{j} d_t,$$

with the convention that $d(i, j) = 0$ whenever $i > j$. Let $\bar{d}_\xi$ be the sum of demand increments in periods $u(\xi)$ through $u(\xi + 1) - 1$, i.e., $\bar{d}_\xi = d(u(\xi), u(\xi + 1) - 1)$, except for $\bar{d}_1$, which includes the capacity initially in use, i.e., $\bar{d}_1 = d(u(1), u(2) - 1) + D_0$. Clearly, the cumulative demand, $\bar{d}_\xi$, is satisfied by technology type $w(\xi)$.

Let $C(m, k, i, \langle u\rangle_{\xi=1}^{p}, \langle w\rangle_{\xi=1}^{p})$ be the minimum expected total cost from period $i$ onwards, given that the firm has no unused capacity on hand, the newest technology is $m$, which was first introduced in period $k$, and the acquisition history and technology-mix sequences until this point in time are $\langle u\rangle_{\xi=1}^{p}$ and $\langle w\rangle_{\xi=1}^{p}$, respectively. Then

$$\begin{aligned} C(m, k, i, \langle u\rangle_{\xi=1}^{p}, \langle w\rangle_{\xi=1}^{p}) \\ = \min_{\substack{j, \langle \bar{w}\rangle \\ i<j\le T+1 \\ \bar{w}(\xi)=w(\xi) \text{ or } m;\ \forall \xi}} \Bigg\{ & f_{mmi}\left(d(i, j-1) + \sum_{\xi=1}^{p} \bar{d}_\xi \bar{\delta}_\xi\right) + \sum_{\xi=1}^{p} g_{w(\xi),m,i}(\bar{d}_\xi \bar{\delta}_\xi) + \left(\frac{1 - Q_m(j-k)}{1 - Q_m(i-k)}\right) \\ & \times (G(m, i, j, j, \langle u\rangle_{\xi=1}^{p}, \langle \bar{w}\rangle_{\xi=1}^{p}) + \mathrm{C}(m, k, j, \{\langle u\rangle_{\xi=1}^{p}, i\}, \{\langle \bar{w}\rangle_{\xi=1}^{p}, m\})) \\ & + \sum_{\nu=i+1}^{j} \left(\frac{q_m(\nu - k)}{1 - Q_m(i-k)}\right) \Bigg( G(m, i, \nu, j, \langle u\rangle_{\xi=1}^{p}, \langle \bar{w}\rangle_{\xi=1}^{p}) \\ & \qquad + \sum_{n(>m)} P_{mn} D(n, \nu, j, \{\langle u\rangle_{\xi=1}^{p}, i\}, \{\langle \bar{w}\rangle_{\xi=1}^{p}, m\}) \Bigg) . \end{aligned} \tag{13}$$

This minimization determines the number of future periods for which to acquire the newest technology using a decision variable $j$, which represents the next scheduled acquisition period. The firm acquires capacity of size $d(i, j - 1)$ to satisfy the demand increments for periods $i$ through $(j - 1)$. Replacement of used capacity is considered through decision variables $\langle \bar{w}\rangle_{\xi=1}^{p}$, the technology mix after the replacement. For each technology currently in use, $\bar{w}(\xi)$ takes the value $m$ or $w(\xi)$, based upon whether it is replaced by the newest technology or not, respectively. We now explain each of the terms in the minimand.

The first term of the minimand in (13) represents the total acquisition cost of capacity purchased to satisfy the future demand increments, and to replace capacity currently in use. The replacement indicator, $\bar{\delta}_\xi = \delta(\bar{w}(\xi) - w(\xi))$, takes the value 1 or 0, depending upon whether technology $w(\xi)$ is replaced or not, where $\delta(\cdot)$ is the Kronecker delta function. Since technology $w(\xi)$ satisfies cumulative demand $\bar{d}_\xi$, the amount of capacity replaced is $\bar{d}_\xi \bar{\delta}_\xi$. The total capacity acquired for replacement is $\Sigma_{\xi=1}^{p}\ \bar{d}_\xi \bar{\delta}_\xi$. The second term of the minimand represents the cost of disposing used capacity.

The carrying costs and operating expenses incurred in periods $i$ through $(\nu - 1)$ by starting in period $i$ with acquisition history $\langle u\rangle_{\xi=1}^{p}$, technology mix $\langle \bar{w}\rangle_{\xi=1}^{p}$, and an amount of capacity $m$ sufficient to exactly satisfy the demand increments in periods $i$ through $(j - 1)$ is,

$$\begin{aligned} G(m, i, \nu, j, \langle u\rangle_{\xi=1}^{p}, \langle \bar{w}\rangle_{\xi=1}^{p}) &= \sum_{l=i}^{\nu-1} h_{ml} d(l+1, j-1) \\ &+ \sum_{\xi=i}^{\nu-1} d_\xi \sum_{l=\xi}^{\nu-1} c_{ml} + \sum_{\xi=1}^{p} \bar{d}_\xi \sum_{l=i}^{\nu-1} c_{\bar{w}(\xi),l} \quad i < \nu \le j. \end{aligned} \tag{14}$$

The third term of the minimand in (13) represents the expected carrying charges, operating expenses, and cost-to-go for the scenario when no new technological innovation appears until period $j$. The next acquisition

and replacement decisions take place, as scheduled, in period $j$ while technology $m$ is still the newest technology. The probability of this event, given that technology $m$ has been available for the last $(i - k)$ periods, is

$$\frac{1 - Q_m(j - k)}{1 - Q_m(i - k)}.$$

The last term of the minimand in (13) represents the expected carrying charges, operating expenses, and cost-to-go for the scenario when a newer technology does appear before the next scheduled acquisition decision is taken. The terms in the summation enumerate the mutually exclusive possibilities for $\nu$, the period in which a newer technology appears; the weighting factors, $[q_m(\nu - k)] / [1 - Q_m(i - k)]$, represent their probabilities. The cost-to-go in this scenario is computed by conditioning on the next technological breakthrough. Suppose $n$ is the new technology that appears in period $\nu$ when the firm still has unused capacity of technology $m$. Let

$$D(n, \nu, j, \{\langle u\rangle_{\xi=1}^{p}, i\}, \{\langle \bar{w}\rangle_{\xi=1}^{p}, m\})$$

represent the minimum expected total cost from period $\nu$ onwards, given that a new technology $n$ was introduced this period, the acquisition history and technology mix sequences until this point in time are $\{\langle u\rangle_{\xi=1}^{p}, i\}$ and $\{\langle \bar{w}\rangle_{\xi=1}^{p}, m\}$, respectively, and unused capacity from the last acquisition of technology $m$ can satisfy demand increments until period $(j - 1)$. Then the cost-to-go for this scenario is given by

$$\sum_{n(>m)} P_{mn} D(n, \nu, j, \{\langle u\rangle_{\xi=1}^{p}, i\}, \{\langle \bar{w}\rangle_{\xi=1}^{p}, m\}).$$

The computation of $D(\cdot)$ involves the optimal disposal decision, which we consider next.

**Optimal Disposal Decision**

Disposal is triggered by the advent of a new technology $\nu$, in period $n$. The firm still has enough unused capacity of the last acquired technology, $m$, to satisfy demand increments until period $(j - 1)$. Suppose, after disposal, there is just enough capacity to satisfy demand increments until period $(\tau - 1)$. The next acquisition and replacement decisions are scheduled for period $\tau$, when the firm will consider the choice of acquiring technology $n$. The cost-to-go,

$$D(n, \nu, j, \langle u\rangle_{\xi=1}^{p+1}, \{\langle w\rangle_{\xi=1}^{p}, m\})$$

$$= \min_{\substack{\tau \\ \nu \le \tau \le j}} \left\{ \begin{array}{l} g_{m,n,\nu}(d(\tau, j-1)) + (1 - Q_n(\tau - \nu)) \\ \quad \times \left( G(m, \nu, \tau, \tau, \langle u\rangle_{\xi=1}^{p+1}, \langle w\rangle_{\xi=1}^{p+1}) \right. \\ \quad \left. + C(n, \nu, \tau, \langle u\rangle_{\xi=1}^{p+1}, \langle w\rangle_{\xi=1}^{p+1}) \right) \\ \quad + \sum_{\sigma=\nu+1}^{\tau} q_n(\sigma - \nu) \\ \quad \times \left( G(m, \nu, \sigma, \tau, \langle u\rangle_{\xi=1}^{p+1}, \langle w\rangle_{\xi=1}^{p+1}) \right. \\ \quad \left. + \sum_{s(>n)} P_{ns} D(s, \sigma, \tau, \langle u\rangle_{\xi=1}^{p+1}, \{\langle w\rangle_{\xi=1}^{p}, m\}) \right). \end{array} \right. \tag{15}$$

This minimization determines the amount of surplus capacity to dispose through the choice of decision variable $\tau$. Choosing $\tau = \nu$ corresponds to disposal of all surplus capacity, while $\tau = j$ implies no disposal.

The first term of the minimand in (15) represents the cost (possibly negative) of disposing $d(\tau, j - 1)$ units of unused capacity of technology $m$ in period $\nu$, when the newest technology is $n$. The second term represents the expected carrying charges, operating expenses, and cost-to-go for the scenario when no new technological innovation appears until period $\tau$. The next acquisition decision takes place, as scheduled, in period $\tau$ with technology $n$ still the latest technology. The probability of this event, given that technology $n$ has just appeared, is $(1 - Q_n(\tau - \nu))$.

The last term of the minimand in (15) represents the expected carrying charges, operating expenses, and cost-to-go for the scenario when an even newer technology appears in period $\sigma$, before the scheduled acquisition decision could be implemented in period $\tau$. The firm needs to reconsider its future strategy by possibly making yet another disposal decision in period $\sigma$. The terms in the summation represent the mutually exclusive possibilities for period $\sigma$, when a newer technology appears; the weighting factors, $q_n(\sigma - \nu)$, represent their probabilities. The cost-to-go for this scenario is obtained again by conditioning on the next technological breakthrough following $n$. Suppose the new technology that appears in period $\sigma$ is of type $s$ (the probability of this event is $P_{ns}$). The firm will find itself in period $\sigma$ with enough unused capacity of the last

acquired technology, $m$, to satisfy demand until period $(\tau - 1)$. The acquisition history and technology mix sequences will still be $\langle u \rangle_{\xi=1}^{p+1}$ and $\{\langle w \rangle_{\xi=1}^{p}, m\}$, respectively. The cost-to-go is then given by

$$\sum_{s(>n)} P_{ns} D(s, \sigma, \tau, \langle u \rangle_{\xi=1}^{p+1}, \{\langle w \rangle_{\xi=1}^{p}, m\}).$$

Equations (13) and (15) together define the recursive algorithm used to solve the problem in a sequential backward fashion.

## An Illustrative Example

We now demonstrate the interplay between the technology evolution and technology choice using a simple example with a horizon length of eight periods. This example also illustrates how the recursion algorithm presented in this section exploits the properties of the optimal solution developed in §§3 and 4. Demand for capacity increases by 10 units in each period. Initially, the firm has no installed base of old technologies. At the beginning of period 1, technology 1 has just appeared and three more technologies—2, 3, and 4—are anticipated in the future. Each vintage has an identical lifetime distribution such that the next vintage in the sequence appears after either two or three periods, both equally likely. Acquiring capacity $m$ incurs a fixed cost, $K_m$, and a variable cost, $v_m$, per unit. The values of cost parameters $K_m$, $v_m$ and the operating cost per unit $c_m$ are shown in Table 1. The impending arrival of newer technologies, offering lower operating cost, is the prime factor discouraging larger capacity purchases. To emphasize this, carrying charges are assumed to be zero in this example. The revenue from the disposal of a unit of used capacity, $\bar{r}_{mn}$, decreases with the arrival of newer technologies, as shown in Table 2. Disposal of unused capacity incurs a minor loss over the variable purchasing cost paid, as shown by the per unit revenue, $r_{mn}$, in Table 2. There is a fixed cost of 300 for the disposal of used capacity. There is no fixed cost incurred for the disposal of unused capacity.

**Table 1** **Costs Parameters for the Example**

| | Technology Level, $m$ | | | |
|---|---|---|---|---|
| Costs | *1* | *2* | *3* | *4* |
| Fixed Cost for Acquisition, $K_m$ | 200 | 200 | 100 | 100 |
| Variable Cost for Acquiring a Unit of Capacity, $v_m$ | 6 | 9 | 11 | 13 |
| Unit Operating Cost, $c_m$ | 12 | 8 | 4 | 1 |

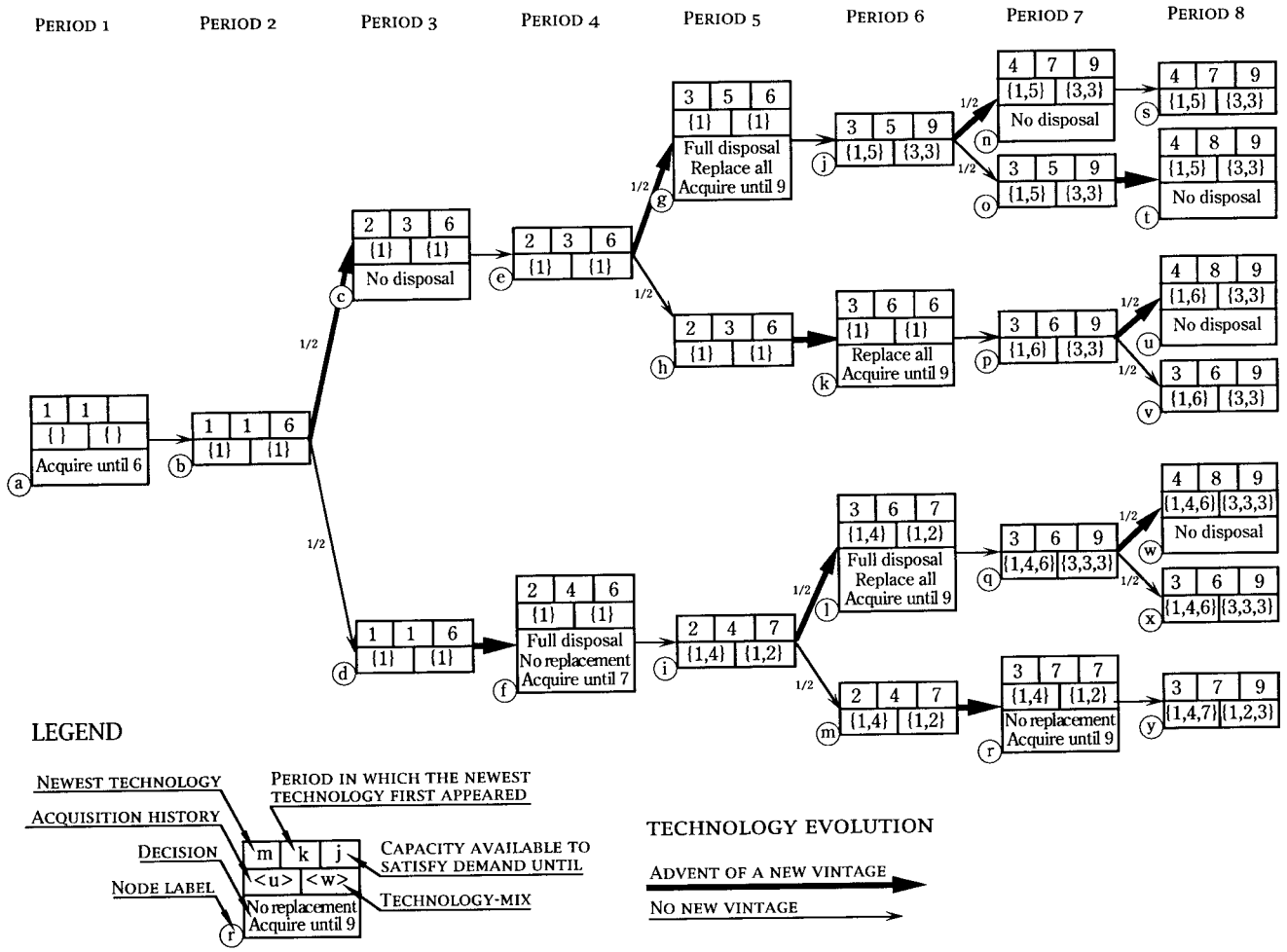
**Table 2** **Revenue from Disposal of a Unit of Used ($\bar{r}_{mn}$) and Unused ($r_{mn}$) Vintage $m$ When $n$ Is the Newest Vintage Available**

| | $\bar{r}_{mn}$ | | | $r_{mn}$ |
|---|---|---|---|---|
| $m$ | $n = 2$ | *3* | *4* | $\forall n > m$ |
| *1* | 2 | 1 | 0 | 5 |
| *2* | | 4 | 2 | 8 |
| *3* | | | 6 | 10 |

The sequence of optimal decisions for different technological evolutions are shown in Figure 1. To facilitate exposition, nodes have been assigned alphabetical labels—*a*, *b*, *c*, etc. For each node, the state at the beginning of the period is shown at the top and the optimal decision corresponding to that state is shown below it. Consider period 1, for example, where $m = 1$ and $k = 1$, because technology 1 has just become available for purchase in period 1. No installed base of capacity exists at this point; both acquisition history and technology-mix sequences are null. The optimal first period decision at node *a* is to acquire enough capacity of technology 1 to satisfy demand until the beginning of period 6. No new technology can appear in period 2. This leads to a deterministic transition to node *b*, with state $m = 1$, $k = 1$, $j = 6$, $\langle u \rangle = \{1\}$, and $\langle v \rangle = \{1\}$. Since decisions are triggered either by the advent of a new technology or by the lack of unused capacity, period 2 requires no decision.

Several alternative technology evolution paths exist beyond period 2. We consider one such path $c \rightarrow e \rightarrow g \rightarrow j \rightarrow n \rightarrow s$ for discussion. The next innovation, technology 2, becomes available at the beginning of period 3 (there is only a 50% chance that this will really happen). At this point, the firm still has enough capacity of technology 1 to satisfy demand until period 6. The firm has to decide whether to dispose—partially or fully—of the excess capacity of technology 1 and whether to replace technology 1 capacity put into use during the last two periods. The optimal decision is not to dispose

**Figure 1 Optimal Decisions for Different Technological Evolution Paths in the Illustrative Example**




any of the technology 1 capacity at this time. Node *e*, like node *b*, requires no decision. Technology 3 becomes available at the beginning of period 5 (node *g*), while the firm still has capacity to satisfy period 5 demand. The optimal decision is to dispose of all excess capacity. The firm must now decide how much capacity of technology 3 to acquire and whether to replace any of the technology 1 capacity already in use. The optimal decision is to acquire enough capacity of technology 3 to satisfy demand until the beginning of period 9 (the end of the horizon) and to replace all technology 1 capacity in use. As a result of these decisions, the acquisition history and technology-mix sequences at node *j* are given by {1, 5} and {3, 3}, respectively. Since no new innovation has appeared ($m = 3$, $k = 5$) and enough capacity exists to satisfy demand until beginning of period 9, no decision is required at node *j*. Technology 4 appears at the beginning of period 7 (node *n*), but the optimal decision is not to adopt the new technology at this point. Like nodes *b*, *e*, and *j*, no decision is required at node *s*.

This technology evolution path illustrates several interesting aspects of the problem. First, the advent of a superior technology does not imply its immediate adoption if the firm has unused capacity of an older vintage. For example, the firm continued to use technology 1, as planned, despite the availability of technology 2 in period 3. On the other hand, the firm immediately disposed all technology 1 capacity to embrace technology 3 when it first appeared in period 5. The choice of

whether to dispose all the unused capacity and adopt the new technology immediately or to continue using the unused capacity of the older vintage and thus delay the adoption depends on a number of factors including salvage cost of unused capacity, relative merit of the newest technology compared to the old one, and the likelihood of an even better forthcoming technology. An optimal technology path may not follow the chain of technological development link by link. For example, a decision to delay the adoption of technology 2 led to its exclusion altogether due to the advent of an even better innovation—technology 3.

Other technology evolution paths can be interpreted similarly. A comparison of different technological evolution paths illustrates many additional features of this problem. First, the timing of an innovation critically affects the likelihood of its adoption by a firm. This does not imply that a technology that appears early is more likely to be adopted. For example, if technology 2 appears in period 3 (node $c$), it is never adopted by the firm, but if it appears in period 4 (node $f$), it is immediately adopted. Second, the timing of an innovation also affects the extent to which a technology is accepted by a firm. For example, if technology 3 appears in period 6 (node $l$), the firm adopts it to the fullest extent by disposing all excess capacity, acquiring technology 3 for future periods and replacing all existing capacities by technology 3. In contrast, under the same circumstances (i.e., technology mix and acquisition history), if technology 3 appears in period 7 (node $r$), the firm adopts it only to a limited extent—capacities currently in use are not replaced by the new technology. Whether an early appearance of a technology promotes or discourages its likelihood and extent of adoption depends upon the possibility of future innovations and their relative attractiveness, current technology mix, and acquisition history and acquisition and salvage costs of various technologies.

The optimal installed base of technologies that a firm has in the future depends not only on the available technologies, but also on their evolution path. For example, consider nodes $x$ and $y$. Starting with the same initial conditions in period 1, the firm finds itself with the same choice of available technologies—1, 2, and 3—in period 8. Yet their technology-mix sequences are vastly different: Node $x$ has only technology 3, while node $y$ has all the three technologies.

Recall that in an optimal policy, capacity is never acquired with the sole intention of disposing it at a future point. At the beginning of period 1, if it is optimal to acquire capacity to last until period 6, then there must exist a technology evolution path such that it is optimal to put all this technology into use and take an acquisition decision in period 6, as planned. Path $a \rightarrow b \rightarrow c \rightarrow e \rightarrow h \rightarrow k$ satisfies this condition. Though it is optimal to acquire capacity of technology 1 to last until period 6, by the time one reaches period 6, it is possible that none of this capacity is actually in use (see node $j$). Revision of a decision in the face of innovation does not imply suboptimality; neither does lack of revision of the original decision imply optimality.

## 6. Sensitivity to Parameters

We now report the results of a computational study performed to explore the impact of key problem parameters on the acquisition decision. This study also illustrates the feasibility and effectiveness of the proposed recursive procedure in solving realistic size problems. For the purpose of this study we assumed that capacity, once put into use, is not replaced. For example, many organizations let the older personal computers trickle down the organizational hierarchy as newer models are acquired. Similar phenomena occur in many other industries where alternative usage for older vintages is identified instead of discarding them. Therefore, the acquisition decision could be based solely on the characteristics of future innovations since replacement of older technology was not an issue. This allowed us to illustrate the sensitivity of the acquisition decision to factors such as number of forthcoming innovations, elapsed time since the last innovation, the variability in time between successive innovations, economies of scale in acquisition, and length of the problem horizon. We have chosen to report the impact of these parameters only on the first period decision since, as illustrated in the last section, decisions for future periods are contingent upon the technological evolution path.

### Problem Details

We considered problems with up to four possible forthcoming technologies and horizon length ($T$) varying

from 6 to 20 periods. The demand increments were set equal to 10 units in each period. All costs and revenues were assumed to be constant over time. The purchase cost function used was $f_m(x) = K_m x^\alpha$, where $x$ is the amount of capacity purchased. The parameter $K_m$ was higher for newer technologies, but the value of $\alpha$ was assumed to be the same for all technologies. To explore the sensitivity of the decision to economies of scale in acquisition, the parameter $\alpha$ was varied from 0.8 to 0.975. The carrying costs, $h_m$, were assumed to be proportional to the investment costs and were chosen such that the ratio $(K_m / h_m)$ was the same for all technologies. This ensured that the tradeoff between scale economy in acquisition and carrying costs was identical for all technologies. The operating costs, $c_m$, were taken to be lower for newer technologies. The purchase costs, carrying costs, and the operating costs for the successive technologies were chosen such that it was always more attractive to acquire a newer technology. These cost parameters are given in Table 3.

In the salvage value function, the *fixed* salvage cost, $s_{mn}$, was taken to be 10 for all technologies. The unit salvage revenue, $r_{mn}$, was lower for older technologies and decreased with the appearance of a new technology. The salvage revenues were chosen carefully so that it would not be profitable to acquire capacity in a period with the sole intention of disposing it later. The specific values of salvage revenues, $r_{mn}$, are listed in Table 4. Technologies were assumed to follow the evolutionary chain $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5$. The time between successive innovations was assumed to be identically and uniformly distributed with a mean value of five periods. To explore the impact of technological uncertainty on the first period decision, we considered distributions with supports ranging from three periods to nine periods (centered at period 5). For comparison, we also considered predictable technological evolution where successive innovations appear exactly five periods apart. The sensitivity to elapsed time without innovation was explored by varying the number of periods, $p$, that have passed since technology 1 appeared. The following values were used for $p$: 0, 2, 4, and 6 (where allowed by the spread of the lifetime distribution).

**Table 3** **Costs Used in Computational Experiments for Sensitivity Analysis**

| | Technology Level, $m$ | | | | |
|---|---|---|---|---|---|
| Costs | *1* | *2* | *3* | *4* | *5* |
| Purchase Cost Parameter, $K_m$ | 20 | 30 | 40 | 50 | 60 |
| Unit Carrying Cost, $h_m$ | 0.6 | 0.9 | 1.2 | 1.5 | 1.8 |
| Unit Operating Cost, $c_m$ | 4.0 | 3.1 | 2.1 | 1.25 | 0.5 |

**Table 4** **Unit Revenue ($r_{mn}$) from Disposal of Used Capacity of Vintage $m$ When $n$ ($>m$) Is the Newest Vintage**

| | $r_{mn}$ | | | |
|---|---|---|---|---|
| $m$ | $n = 2$ | *3* | *4* | *5* |
| *1* | 2.5 | 2.0 | 1.5 | 1.0 |
| *2* | | 3.5 | 3.0 | 2.25 |
| *3* | | | 5.0 | 4.0 |
| *4* | | | | 6.0 |

Given a horizon length of $T = 20$, we solved 100 problems with five values of $\alpha$ (scale economies parameter), five values of the variance of interarrival time, and four values of $p$ to study the impact of these parameters on the optimal first period decision. Further, to investigate the impact of the horizon length, we solved 80 problems with five values of the variance in interarrival time and eight values of $T$, ranging from 6 to 20.

**Computation Time.** The recursive procedure was implemented without any special data structures or coding schemes. The study was performed on an IBM-compatible personal computer based on a 486 chip running at 33 MHz clock speed (almost the *current best technology* in personal computers at the time of this study) and the problems were solved within 1–8 seconds for horizon lengths of 12–20 periods.

## Results

Tables 5 through 8 report the number of periods for which capacity is acquired in the first period. The cost and demand parameters were kept invariant, except for $\alpha$. The values of the parameters $\alpha$, $T$, $m$, $p$, and variance used in the experiments are reported in each table. The sensitivity results are discussed below.

**Sensitivity to the Concavity in Purchase Costs.** Table 5 shows that, as $\alpha$ is decreased from 0.975 to 0.8

**Table 5** **Sensitivity of the First Period Decision to the Economy of Scale in Acquisition and the Variance of the Time Between Successive Innovations** ($T = 20$, $m = 1$, $p = 0$)

| Cumulative Distribution for Time Between Successive Innovations | Mean | Variance | The Optimal First Period Decision (Number of Periods for Which to Buy the Capacity) | | | | |
|---|---|---|---|---|---|---|---|
| | | | $\alpha = 0.8$ | 0.9 | 0.925 | 0.95 | 0.975 |
| (0, 0, 0, 0, 1, 1, 1, 1, 1) | 5 | 0 | 5 | 5 | 5 | 2 | 2 |
| (0, 0, 0, $\frac{1}{3}$, $\frac{2}{3}$, 1, 1, 1, 1) | 5 | $\frac{2}{3}$ | 4 | 4 | 4 | 2 | 2 |
| (0, 0, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, 1, 1, 1) | 5 | 2 | 4 | 3 | 3 | 3 | 2 |
| (0, $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$, 1, 1) | 5 | 4 | 3 | 3 | 2 | 2 | 2 |
| ($\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{6}{9}$, $\frac{7}{9}$, $\frac{8}{9}$, 1) | 5 | $6\frac{2}{3}$ | 3 | 2 | 2 | 1 | 1 |

(increasing concavity in purchase costs), capacity is acquired for more periods. This is consistent with the observations in Monahan and Smunt. However, there is an interesting interplay between $\alpha$ and the time between successive innovations. We first discuss this interaction for the deterministic case (first row in Table 5), where the time between successive innovations is exactly five periods. When $\alpha = 0.8$, it is optimal to buy capacity for six periods based on the tradeoff between acquisition and carrying costs. However, if a new technology is forthcoming after five periods, we would buy the current technology only for five periods, since the sixth period is bound to be an acquisition period. On the other hand, when $\alpha = 0.925$, it would be optimal to buy capacity for four periods based on the tradeoff between acquisition and carrying costs. This tradeoff, however, does not consider the end of horizon effect, which becomes important since the horizon is again five periods due to advent of a new technology. For the five-period subproblem with the current technology, it is more economical to buy capacity for five periods (and pay the extra carrying cost) rather than to acquire capacity twice. Finally, for $\alpha = 0.9$, it is optimal to buy capacity for five periods, which coincides with the horizon length for the subproblem. In summary, if *no* new innovation was forthcoming, it would have been optimal to buy capacity for six, five, and four periods, respectively, for $\alpha = 0.8$, 0.9 and 0.925. But, in all cases, the first period decision was to buy capacity for five periods. The time between successive innovations thus has a smoothing effect on the number of periods for which capacity is acquired. The same effect holds when the time between successive innovations is uncertain, though the horizon length of the subproblems is uncertain in this case.

**Sensitivity to the Horizon Length.** Table 6 provides the optimal first period decision (expressed as the number of periods for which capacity is purchased) for problems with horizon lengths ranging from 6 to 20 periods. It is clear from Table 6 that the first period decision is relatively insensitive to the horizon length beyond $T = 12$. This result is very useful in practice because it implies that long-term forecasts, which are likely to be inaccurate, are not necessary. In contrast, in deterministic capacity acquisition models with a single technology (Luss 1982, Rajagopalan 1992), the first period decision is typically very sensitive to the horizon length. The insensitivity of the first period decision to horizon length can be attributed to two factors. First, even in the deterministic case (first row in Table 6), the advent of

**Table 6** **Sensitivity of the First Period Decision to the Horizon Length and the Variance of the Time Between Successive Innovations** ($\alpha = 0.8$, $m = 1$, $p = 0$)

| Variance | The Optimal First Period Decision | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $T = 6$ | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 0 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| $\frac{2}{3}$ | 6 | 6 | 6 | 5 | 5 | 4 | 4 | 4 |
| 2 | 6 | 7 | 5 | 4 | 4 | 4 | 4 | 4 |
| 4 | 6 | 8 | 4 | 4 | 4 | 4 | 3 | 3 |
| $6\frac{2}{3}$ | 6 | 8 | 3 | 3 | 3 | 3 | 3 | 3 |

each new technology defines an acquisition period that, in effect, decomposes the problem into a sequence of smaller horizon subproblems, each with a single technology. Any change in horizon length typically affects the last subproblem. Second, and more significantly, the effect of future events on the first period decision diminishes as uncertainty about the future increases. This diminishing influence and the optimality of myopic decisions have been observed in other problems (Hopp et al. 1987) due to the relative insensitivity of the probability distribution of future states to the first decision.

**Sensitivity to the Number of Forthcoming Innovations.** Since a maximum of five technologies can be forthcoming, by changing the technology type available in period 1, we were able to vary the number of technologies that can appear in the future. Table 7 provides the results for the optimal first period decision as a function of the newest vintage, $m$, available in period 1. As the best technology type initially available increases from 1 to 4, one tends to acquire capacity for more periods. The larger capacity purchases can be attributed to the lower risk of obsolescence, since fewer technologies are forthcoming.

**Sensitivity to the Elapsed Time Without Innovation.** Consider the deterministic technology evolution first. Recall that for $\alpha = 0.8$, one would have liked to buy capacity for six periods, based on the tradeoff between acquisition and carrying costs. If the next innovation is bound to appear sooner, one buys just enough capacity to last until then. As $p$ increases, the initial capacity purchase decreases proportionately, as shown in the first row of Table 8. A similar argument holds for uncertain technological evolution, though one tends to purchase even less in this case due to an increased obsolescence risk. The longer the elapsed time since the last innovation, the greater the probability that a better vintage is forthcoming. This will be true for all lifetime distributions that have increasing failure rate (IFR).

**Table 7** **Sensitivity of the First Period Decision to the Number of Forthcoming Innovations and the Variance of the Time Between Successive Innovations** ($\alpha = 0.8$, $T = 20$, $p = 0$)

| | The Optimal First Period Decision When Initial Vintage is $m$ | | | | |
|---|---|---|---|---|---|
| Variance | $m = 1$ | 2 | 3 | 4 | 5 |
| 0 | 5 | 5 | 5 | 5 | 6 |
| $\frac{2}{3}$ | 4 | 4 | 5 | 5 | 6 |
| 2 | 4 | 4 | 4 | 5 | 6 |
| 4 | 3 | 3 | 4 | 4 | 6 |
| $6\frac{2}{3}$ | 3 | 3 | 4 | 4 | 6 |

**Table 8** **Sensitivity of the First Period Decision to Elapsed Time Without Innovation and the Variance of the Time Between Successive Innovations** ($\alpha = 0.8$, $T = 20$, $m = 1$)

| | The Optimal First Period Decision When $p$ Periods Have Elapsed Without Innovation | | | |
|---|---|---|---|---|
| Variance | $p = 0$ | 2 | 4 | 6 |
| 0 | 5 | 3 | 1 | – |
| $\frac{2}{3}$ | 4 | 2 | 1 | – |
| 2 | 4 | 2 | 1 | 1 |
| 4 | 3 | 1 | 1 | 1 |
| $6\frac{2}{3}$ | 3 | 1 | 1 | 1 |

**Sensitivity to the Variance in Interarrival Times.** Going down each column in any of the Tables 5 through 8, it is clear that the capacity acquisition decision is quite sensitive to the variance in interarrival times. The amount of capacity purchased in period 1 decreases with increasing variance. In general, less capacity is acquired as the uncertainty in the timing of future innovations increases or the next innovation becomes more imminent. This is consistent with the conclusions in Monahan and Smunt, where increased uncertainty in interest rate results in a wait-and-see policy.

These results on the impact of uncertainty in the number and time of appearance of future innovations on current replacement decisions are consistent with a number of recent empirical studies. Antonelli (1989) found that firms delayed or slowed the adoption of open-end spinning rotors when the pace of technical progress was rapid and uncertain. The rate of adoption increased substantially when technical progress slowed and few future innovations were anticipated. Karlson (1986) observed that expectations about the pace of technical progress in basic oxygen and large electric fur-

nace technologies played a significant role on the rate and time of replacement of open hearth furnaces by the U.S. steel industry.

# 7. Conclusion

In this paper we have studied a fairly comprehensive model of capacity and technology acquisition and replacement in environments where successive technological breakthroughs take place stochastically, leading to frequent and uncertain obsolescence of equipment. We modeled this problem and presented some key properties of the model that are used to reduce the state space and computational effort to solve this otherwise intractable problem. In particular, we showed that it is optimal to purchase, dispose, and replace capacity in amounts equal to the demand increments for an integral number of periods. Further, we showed that it is optimal to: (i) dispose excess capacity only in periods when a new technology appears, and (ii) replace used capacity only in *acquisition periods*, i.e., periods when capacity is in any case going to be acquired to meet future demand increments. Using these results, we developed regeneration point-based dynamic programming recursions for the models *with* and *without* replacement of used capacity. Finally, we showed that it is possible to solve moderate size problems on a personal computer in a reasonable amount of time. Future research needs to address extensions of this model to scenarios where there may be multiple demand types, revenue impacts of new technologies, and uncertainty in demand.[1]

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## Appendix

### Proof of Theorem 1

First, note that the objective function is concave since the functions $f(\cdot)$ and $h(\cdot)$ are concave, $g_{pmt}(\cdot)$ and $\bar{g}_{pmt}(\cdot)$ are fixed charge functions, and operation costs are linear. Second, we have a set of linear constraints. Given these two observations, *the optimal solution is at an extreme point*. Note that in Equations (1)–(3), all the variables—$x$, $I$, $Y$, $Z$, and $Z^e$—appear exactly once with a positive (+1) coefficient; in all other occurrences they have a negative (−1) coefficient. This can be verified simply by transferring the right-hand side terms to the left in constraints (2)–(3) and transferring the left-hand-side terms to the right in Equation (1). Also, all the variables are nonnegative from (5). As a result, the constraint matrix determined by Equations (1)–(3) is *Leontief* (Veinott 1969). From the characterization of extreme points in Leontief matrices (Veinott 1969, also Theorem 6 in Veinott 1968), it follows that if more than one variable appears with a positive coefficient in the same constraint, then only one of these variables can be positive in the optimal solution. Therefore, conditions (6)–(8) follow directly from constraint (3). □

### Proof of Theorem 2

The proof is by induction. Let $S$ be a set of $1 \times M$ vectors whose elements can be either 0 or any combination of the sums of the demands $d_t$, for all $t$. Consider the last period $T$. Suppose $\mathbf{X}_T, \mathbf{I}_T \in S$. Then, from Corollary 3, a policy $(\mathbf{Y}_T, \mathbf{Z}_T, \mathbf{Z}^e_T) \in A_T$ minimizes

$$C_T((m_T, k_T), (\mathbf{X}_T, \mathbf{I}_T), (\mathbf{Y}_T, \mathbf{Z}_T, \mathbf{Z}^e_T)) = L_T(m_T, \mathbf{X}_T, \mathbf{I}_T, \mathbf{Y}_T, \mathbf{Z}_T, \mathbf{Z}^e_T),$$

for any value of $(m_T, k_T)$.

Now consider period $(T-1)$. Again suppose $\mathbf{X}_{T-1}, \mathbf{I}_{T-1} \in S$. Then choosing the policy $(\mathbf{Y}_{T-1}, \mathbf{Z}_{T-1}, \mathbf{Z}^e_{T-1}) \in A_{T-1}$ ensures that $\mathbf{X}_T, \mathbf{I}_T \in S$, irrespective of the state of technology $(m_{T-1}, k_{T-1})$. This follows directly from the definitions of $S$ and $A_t$ and the deterministic evolution of states $(\mathbf{X}_t, \mathbf{I}_t)$ in Equations (10) and (11). The fact that $(\mathbf{X}_T, \mathbf{I}_T) \in S$ ensures that $C_T(\cdot)$ will be minimized by a policy $(\mathbf{Y}_T, \mathbf{Z}_T, \mathbf{Z}^e_T) \in A_T$, as noted earlier. Given current state $((m_{T-1}, k_{T-1}), (\mathbf{X}_{T-1}, \mathbf{I}_{T-1}))$, the state in period $T$ will be $((m_T, k_T)_i, (\mathbf{X}_T, \mathbf{I}_T))$ with probability $\phi_i(m_{T-1}, k_{T-1})$. But for any realization $(m_T, k_T)_i$ of the technology, a policy $(\mathbf{Y}_{T-1}, \mathbf{Z}_{T-1}, \mathbf{Z}^e_{T-1}) \in A_{T-1}$ minimizes

$$L_{T-1}(m_{T-1}, \mathbf{X}_{T-1}, \mathbf{I}_{T-1}, \mathbf{Y}_{T-1}, \mathbf{Z}_{T-1}, \mathbf{Z}^e_{T-1}) + \phi_i(m_{T-1}, k_{T-1}) C^*_T((m_T, k_T)_i, (\mathbf{X}_T, \mathbf{I}_T)). \quad \text{(A.1)}$$

This is true because all the results in §3 are independent of the number of types of technology choices available in a period. Since transition probabilities, $\phi_i(m_{T-1}, k_{T-1})$, are independent of decisions $(\mathbf{Y}_{T-1}, \mathbf{Z}_{T-1}, \mathbf{Z}^e_{T-1})$, expression

$$L_{T-1}(m_{T-1}, \mathbf{X}_{T-1}, \mathbf{I}_{T-1}, \mathbf{Y}_{T-1}, \mathbf{Z}_{T-1}, \mathbf{Z}^e_{T-1}) + \sum_i \phi_i(m_{T-1}), K_{T-1} C^*_T((m_T, k_T)_i, (\mathbf{X}_T, \mathbf{I}_T)),$$

is a weighted linear combination of the cost expression (A.1), and is also minimized by a policy $(\mathbf{Y}_{T-1}, \mathbf{Z}_{T-1}, \mathbf{Z}^e_{T-1}) \in A_{T-1}$. That is, $C_{T-1}((m_{T-1,}, k_{T-1}), (\mathbf{X}_{T-1}, \mathbf{I}_{T-1}), (\mathbf{Y}_{T-1}, \mathbf{Z}_{T-1}, Z^e_{T-1}))$ is minimized by a policy $(\mathbf{Y}_{T-1}, \mathbf{Z}_{T-1}, \mathbf{Z}^e_{T-1}) \in A_{T-1}$, provided that $\mathbf{X}_{T-1}, \mathbf{I}_{T-1} \in S$. Repeating this argument for periods $(T-2)$, $(T-3)$, . . . , 1, it follows that there exists an optimal policy, $(\mathbf{Y}_t, \mathbf{Z}_t, \mathbf{Z}^e_t) \in A_t$, $\forall t$ since $\mathbf{X}_1, \mathbf{I}_1 \in S$. □

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