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Corporate Reorganizations and Non-Cash Auctions

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ABSTRACT

This paper extends the theory of non-cash auctions by considering the revenue and efficiency of using different securities. Research on bankruptcy and privatization suggests using non-cash auctions to increase cash-constrained bidder participation. We examine this proposal and demonstrate that securities may lead to higher revenue. However, bidders pool unless bids include debt, which results in possible repossession by the seller. This suggests all-equity outcomes are unlikely and explains the high debt of reorganized firms. Securities also inefficiently determine bidders' incentive contracts and the firm's capital structure. Therefore, we recommend a new cash auction for an incentive contract.

TWO IMPORTANT PAPERS in the legal literature, Baird (1986) and Jackson (1986), suggest that a bankrupt firm's assets should be auctioned off. Their rationale is that a cash auction is an efficient procedure for selling assets and the role of bankruptcy procedures is not to reallocate claims but rather to allow for efficient liquidation.[1]

Subsequently, in an influential paper, Aghion, Hart, and Moore (1992) (henceforth AHM) call for the replacement of the reorganization process (Chapters 7 and 11 in the United States Bankruptcy Code) by a two-stage procedure that separates the valuation decision (price of the firm, who should manage it, and with what capital structure) from the allocation decision for old claimants (which security gets what allocation).[2]

In particular, for the allocation decision, AHM propose a version of a scheme due to Bebchuk (1988). For the valuation decision, AHM advocate a mixture of cash and non-cash auctions.

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[1] This approach is also advocated by Jensen (1991). See Bebchuk (1998) for a historical discussion.

[2] Hart (1995) presents a very readable discussion of the AHM proposals. Similar ideas are proposed in Bradley and Rozensweig (1992) and Adler (1993).

In arguing for non-cash auctions, AHM counsel that the view advanced in Baird (1986), Jackson (1986), and Jensen (1991) is extreme. As they state: "Auctions work well if raising cash for bids is easy and there is plenty of competition among bidders. However, even in the most advanced Western economies, these conditions are unlikely to be met, and they are even less likely to be satisfied in Eastern Europe" (527).[3] Hart (1995) reiterates that "a combination of transactions costs, asymmetric information, and moral hazard makes it difficult for bidders to raise sufficient cash to maintain a company as a going concern (i.e., capital markets are not perfect)" (162). Che and Gale (1998) formalize the claim that cash auctions with credit-constrained bidders lead to inefficient outcomes.

AHM seem to suggest (533, discussion of Task A) that non-cash and cash auctions are not very different. Thus, the use of an auction, they conjecture, will assign the assets of a bankrupt firm to their best use. However, AHM provide no formal analysis of non-cash auctions, focusing instead on the second part of their procedure, the allocation of rights to the old claimants.

The literature on privatization also suggests non-cash auctions as a necessary alternative to the cash auction. For example, Bolton and Roland (1992) state that "because of the low initial level of private wealth [in Eastern Europe], it is important to let potential buyers borrow from the government or issue claims on future revenues (obtained with the privatized assets)" (275). However, again in this literature no formal analysis of the implications of non-cash auctions has been completed.

Our main contribution is to analyze the theory of auctions when different types of non-cash bids are allowed and explore the use of non-cash auctions in bankruptcy proceedings and other situations. We investigate whether non-cash auctions can distinguish between managers, thus insuring an optimal allocation of assets. We also examine the implications of non-cash auctions for revenue, bidder incentives, and ex post capital structure. We find that although non-cash auctions may increase seller revenue, they also introduce inefficiencies into the capital structure and incentive contract and may select the wrong manager. We show that only debt, preferred stock, or convertible debt bids can ensure optimal asset allocation in a non-cash auction. Thus, the auctioned firm's ex post capital structure must contain a type of debt. This explains Gilson's (1997) counterintuitive empirical finding that reorganized firms have high and even increased levels of debt. To gather these results, we first consider a general non-cash auction setup, then auctions with bids of debt, then preferred stock, then equity, then debt and equity, and finally convertibles. In summary, we present an alternative cash auction for an optimally designed incentive contract that addresses the problems that we find.

To clarify the definition of a non-cash auction, Hart (1995) provides four examples of non-cash bids that allow cash-constrained management teams (without access to efficient capital markets) to compete for the bankrupt

[3] Shleifer and Vishny (1992) argue that financial distress is correlated within an industry and hence likely potential buyers face liquidity problems.

company: "(1) The old managers propose to keep their jobs, and offer claimants a share in the post-bankruptcy company; (2) The same financial arrangement might be offered by a new management team; (3) The managers of another company might propose to buy the bankrupt company, offering shares in their company as payment; (4) Management (old or new) might induce some debt in the company's capital structure. One way to do this would be to arrange for a bank to lend money to the post-bankruptcy company (the loan conditional on the bid succeeding), and offer claimants a combination of cash and equity in the (levered) company. Another way would be to offer claimants a combination of shares and bonds in the post-bankruptcy company" (171).

Although the theory of cash auctions is very extensive (see Vickrey (1961), Myerson (1981), Riley and Samuelson (1981), Harris and Raviv (1981), Milgrom and Weber (1982), and the vast literature that follows), the literature on securities or contingent auctions is less developed. Hansen (1985) first demonstrated the seller's expected revenue benefit of requiring equity bids from firms with assets. Comments by Crémer (1987) and Samuelson (1987) extended this work, pointing out the possibility of near full extraction by the seller (Crémer (1987)) and how this may lead to inefficient selection in the auction (Samuelson (1987)). Laffont and Tirole (1987), McAfee and McMillan (1986, 1987), and Samuelson (1986) all examine the bidding for incentive contracts. Their work is relevant because linear incentive contracts are similar to cash and equity bids, although in their work payments are made to the bidder. Each paper explores one or more of the trade-offs between moral hazard, efficient selection, adverse selection, and competition. Riley (1988) studies royalty bidding (cash and equity payments) in a common values setting. We extend all of this work by considering multiple new types of securities, and we explore their implications for non-cash bankruptcy auctions.

Auctions with non-cash (securities) bids create a valuation problem for the seller because the seller has to value the bids to rank which is the highest. With incomplete information, the seller has to infer the bidder's valuation from the security that is bid. In a cash auction the value of a given payment does not depend on the identity of the winner, whereas in a securities auction it does. Thus, the auction has attached to it a signaling aspect: the bidder wants the seller to believe he is a higher type.[4] This increases the bids and may increase the seller's expected revenue. We begin our analysis by presenting a general theorem to rank the revenue from auctions involving different kinds of securities or cash. Under the assumption that the auc-

[4] Our auction model is a screening model, in that the seller designs the auction and thus sets the strategy space (cash, first price, etc.). The bidder's ability to "reveal" or "prove" who he is depends on the strategy space. We show that with securities bids, the bidders desire the seller to think they are a better type independent of what they bid (10 percent of the equity is more valuable from a higher type). Our model is not a signaling model in the traditional sense, because the bidders always bid after the seller announces the allowable strategy space. Our use of the word "signal" or "signaling" in this paper refers to the fact that with securities bids, bidders want to signal that they are a better type. This is similar to signaling models in finance where the securities picked by a firm signal its unknown valuation.

tion yields an incentive-compatible and individually rational separating outcome, we show that any securities auction generates higher expected revenue to the seller than a cash auction. This is because in a securities auction a low type who imitates a higher type does not expect to pay as much as the higher type. Thus, every type must bid more than in a cash auction to separate themselves from the lower types. We further demonstrate that the more dependent the securities bid's value is upon the type of the bidder, the easier it is for low types to imitate higher types and, hence, the greater the seller's expected revenue.

This theorem implies that bids of equity or options could generate higher expected seller revenue than debt and cash. However, the theorem is predicated on the assumption of a separating outcome. The most critical drawback of a non-cash auction is that some securities will not separate the bidders. With equity bids we prove that there is no incentive compatible separating equilibrium. In some situations we can restore separability by requiring debt bids or at least a minimum debt requirement. For other examples, even the addition of debt (or debt only) does not restore separability. In these cases, significant cash bids or large nonpecuniary bankruptcy costs (borne by bidders) are required to attain separation and thus efficient allocation of assets. Therefore, even if the non-cash auction efficiently allocates the assets among the bidders, it may not increase the number of bidders as much as hoped because bidders without enough cash or significant reputation costs cannot participate in the auction.[5] These are the same bidders who find it difficult to obtain financing for a cash auction.

Furthermore, if the bankruptcy costs are small relative to the bidders' highest possible payoff, then the auction may fail the seller in a unique way. When bankruptcy costs are relatively small, debt bids will separate the bidders; however, the bids will be so high that ex post bankruptcy will be a virtual certainty and the seller may repossess the assets sold. This is exactly what occurred in the FCC C block bandwidth auctions (see Landler (1997)). The C block was open only to small firms (low reputation, low assets), and bidders were allowed to put only 10 percent down. Thus, the bids were very similar to debt bids. These bidders bid far higher than expected, and the FCC hailed it as a success. However, as we would predict, ex post most bidders have failed to make future payments, and the FCC has had to renegotiate and reclaim licences.

The use of debt bids also causes the auction to determine the ex post capital structure of the firm, which may affect the firm's value. Because bidders are concerned first about winning and second about the size of the pie, it is unlikely that the winning bid will also be the optimal capital structure. Thus the non-cash auction adds an inefficiency into the ex post capital structure of the firm. Furthermore, ex post restructuring cannot restore optimality because ex ante bidders will incorporate the coming renegotiation and distort their bids, which may lead to pooled, inefficient bids.

[5] However, even in auctions with these requirements, the possibility that bidders have different amounts of cash or that bidders have different bankruptcy costs increases the possibility that the auction is inefficient.

The analysis of an auction when bidders are allowed to bid with both debt and equity or cash and equity results in multiple equilibria. We find, however, that the seller (who typically designs the auction) can set the level of equity required in any bid arbitrarily close to one and still determine the highest type from the debt portion of the bids. Similar to the finding by Crémer (1987), this results in almost full extraction by the seller. Samuelson's (1987) point, that even small noise would destroy the equilibrium in a full extraction cash and equity auction, does not hold in a debt and equity auction. Because the bidders pay the debt only if the firm does well, the debt bids are not reduced even as the amount of equity required in the bids approaches one. Thus, debt bids would still be significantly separated. Therefore, high levels of extraction may be better accomplished by debt and equity.

Full extraction by the seller depends on the assumption that the value of the firm does not depend on the effort level of the buyer. More reasonably, the winner's effort affects the firm's profits, and effort depends on the benefit expected from working, which is a function of the securities that they bid. Consequently, any securities auction introduces a trade-off between inducing the buyer to take the best action and extracting surplus. Although a cash auction does lead to the first-best effort, a mixture of cash and securities raises higher revenue for the seller. Laffont and Tirole (1987) find that the seller's optimal contract is linear (cash and equity). However, if cash is restricted and the choice is debt and equity, the trade-off is more complex because both securities distort the effort of the bidder. Because debt bids distort the effort of low types more than high types and equity distorts both types equally, we find that under a general assumption the optimal rule is to rank bids of high debt but low equity higher.

The final auction we consider allows convertible debt bids. Sellers will prefer convertible debt over straight debt because convertibles allow the seller to affect postauction conditions, such as capital structure, effort, revenue, and so on, in ways that straight debt cannot. We also compare convertibles with debt that is ex post renegotiated and exchanged for equity. We show that the seller prefers convertibles to ex post negotiation because ex post negotiations occur without competition from other bidders. In general, convertible auctions can be designed to improve seller revenue, total welfare, or both.

Our results have interesting implications for corporate reorganizations. The use of the securities auction was suggested as beneficial by AHM because it allowed an increase in the number of possible bidders. Our research argues that a significant cash component or significant bankruptcy costs are necessary to ensure separability and thus efficiency. Thus the auction will not increase the number of bidders as much as hoped. If there is a significant cash component or significant bankruptcy costs, an equity or other security component in the bids is revenue improving for the seller and may improve efficiency if it allows partially cash-constrained bidders to enter. However, because securities bids decrease the buyer's future effort relative to cash bids, and because bankruptcy costs are a deadweight loss, securities bids also add inefficiencies.

The primary disadvantage of a non-cash auction is that it tries to do too much. Even a non-cash auction that separates and determines the best manager is also designing the incentive contract for the new management team and picking the capital structure for the new firm. Because cash is restricted, the non-cash auction will not yield the first-best or even second-best incentive contract, and the capital structure is unlikely to be optimal. Thus, after bidding, the utility of both the seller and the bidder can be improved by renegotiating and removing the securities inefficiency.[6]

In a cash auction, a renegotiation contract binds because the seller receives cash and has no subsequent relationship with the firm. However, in a securities auction, both the seller and the bidder may want to alter the capital structure ex post to improve incentives or lower bankruptcy costs. If this negotiation is a swap of cash for securities (because the bidder solved his cash problems some time after the auction), then the auction still efficiently allocates assets. However, the possibility of a cash payment removes the contingent nature of the auction and lowers the seller's expected revenue (even if he receives all of the eliminated bankruptcy costs). If the negotiation is a swap of preferred stock for debt, or equity for debt, then the negotiation will destroy the efficiency of the auction. Thus, to employ the non-cash auction, the court must restrict subsequent changes to the capital structure for some period of time after the reorganization.

Our research suggests a solution to the financing problem and lack of competition of the cash auction and also to the inefficiencies of the non-cash auction. We suggest that after the claims in the new firm are allocated, the new claim holders first vote on or determine a new incentive contract and capital structure. A great deal of literature focuses on each of these problems. Then, allow management teams to bid with cash for the incentive contract or allow anyone to bid with cash for the whole firm.[7]

The claim holders could then vote on which bid to accept. This would reduce both the financing problem and the lack of competition problem because a bid for the incentive contract would be much smaller than a bid for the whole firm. Moreover, the cash in the auction would separate the management teams and efficiently allocate the assets of the firm.

Our analysis also provides an explanation of the capital structure of firms that emerge from Chapter 11 reorganizations. We argue that the choice of reorganization plans in Chapter 11 is similar to a non-cash auction. Current management often submits a non-cash reorganization plan, and if the exclusivity period for management proposals in Chapter 11 is over, any bidder

[6] Hotchkiss (1995) finds that 32 percent and Gilson (1997) finds that 25 percent of firms that leave Chapter 11 reenter Chapter 11 or privately change their capital structure.

[7] This suggested procedure is similar to what occurs in most of the train privatizations in the United Kingdom, which are auctions of leases to manage the lines. At the end of the period the line reverts to the state, which reauctions the right to manage the lines.

This suggestion is also similar to one put forth in an early paper in the law literature. Roe (1983) suggests the cash sale of 10 percent of the reorganized firm. Our proposal is similar except that any derivative of the firm's future payoffs might be sold, allowing any possible incentive contract.

can present a plan. Hotchkiss and Mooradian (1998) discuss the example of Public Service Company of New Hampshire in which three different bidders presented reorganization plans.[8] The same signaling issues discussed above apply to this procedure, because management teams need to convince the judge that they are the best team. Even within the exclusivity period, the current management faces potential competition from other possible managers (who can enter when the exclusivity period is over). Therefore, current management must "bid" to convince the judge that they are a good choice. Our paper thus provides an explanation of the counterintuitive empirical finding that debt or convertibles are always a part of reorganization plans: the debt helps the judge distinguish among the plans or determine a good management team in a way that equity cannot (debt bids separate while equity bids do not). Furthermore, our work can also determine when the reorganization will involve debt and when it will involve preferred stock. If bankruptcy costs are needed to separate the bidders (higher type means higher probability of good outcome), debt must be used. If this is not necessary (higher type means larger possible outcome), then the seller can improve revenue with preferred stock bids. The need for some type of debt casts doubt on Roe's (1983) view that postreorganization capital structures will be all equity. More important, it provides an explanation consistent with Gilson's (1997) finding that "firms end up more highly leveraged than they were before becoming financially distressed" (162), often leveraged above the industry median (also see Thorburn (1998)). Finally, our work may also explain why after the reorganization and the signaling are over, firms often "renegotiate" (and change their capital structure), as Hotchkiss (1995) and Gilson (1997) find.

The paper is organized as follows. Section I contains a general model for examining non-cash auctions. Section II analyzes debt and preferred stock auctions and under what conditions they succeed and fail. Section III adds nonpecuniary costs to bankruptcy. Section IV examines the effect of renegotiation on the auction. Section V shows the failure of equity auctions. Section VI assesses mixtures of debt and equity. Section VII allows for disutility of effort. Section VIII investigates convertible bids. Section IX discusses the implications of non-cash auctions and a solution for the dissolution of bankrupt firms. Section X concludes.

I. The Model

The model is a two-stage game with private information. In the first stage the current risk-neutral owners of a bankrupt firm sell the firm using a sealed bid first price auction. N potential risk-neutral buyers bid for the object, with $N = \{1, \ldots, n\}$ representing the set of n bidders. In the second stage the winner of the auction realizes the firm's value.

[8] Baird (1993) discusses the example of Financial News Network (FNN), which received two bids and was sold to NBC.

The bidders have private information about their type t, which is their ability to manage the firm. Each bidder's type determines the distribution of possible second stage outcomes. All participants in the auction believe that the types are independently and identically distributed (IID) and drawn from the distribution $F(t)$ with $F(\underline{t}) = 0, F(\bar{t}) = 1$. The cumulative distribution function $F(t)$ is strictly increasing and differentiable over the interval $[\underline{t}, \bar{t}]$. The bidder that wins the auction will earn an unknown amount $\tilde{v}$. The corresponding probability density for bidder i with type t_i is $g(v|t_i) \geq 0$. The corresponding cumulative distribution for v is $G(v|t_i)$ with $G(\underline{v}|t_i) = 0, G(\bar{v}|t_i) = 1$.[9]

A bidder who is a better manager has a higher t. This implies that, if $t_i > t_j$ then

$$\int_{\underline{v}}^{\bar{v}} vg(v|t_i)\,dv > \int_{\underline{v}}^{\bar{v}} vg(v|t_j)\,dv. \tag{1}$$

A higher type has a larger expected value.[10] This may appear as a situation in which bidders draw their values from different distributions. If this were the case, the bidder heterogeneity would eliminate a closed form solution. Our setup, however, is identical to bidders drawing a value k_i where

$$k_i = \int_{\underline{v}}^{\bar{v}} vg(v|t_i)\,dv. \tag{2}$$

Each k_i corresponds to only one t_i and is a monotone increasing function of t_i.

Because each bidder's type is drawn IID, the values are also independent. Therefore, all bidders and the seller have the same information about all other variables that may affect the second period value of the firm. This does not preclude a common component to the bidders' values; we assume only that every bidder has the same information about the value of any common component. We discuss the implications of bidder values that include an unknown common component in Section V.

We now add an unusual twist to the auction. Instead of allowing only cash bids, we will let bidders bid in securities whose value is derived from the future revenue of the firm for which they are bidding. A bid is now a function $s(\cdot)$ that depends on the resulting realization of the firm. For example, if the seller accepts a bid in the form of a \$10 million bond and 20 percent equity, then if the second stage revenue of the firm is greater than \$10 million, the seller gets \$10 million plus 20 percent of the remainder and the winner gets the rest. If the payoff is less than \$10 million, then the seller

[9] The bounds $\underline{v}$ and $\bar{v}$ could depend on the bidder; that is, a bidder with a different type, t_i, could have both a different range of possible outcomes and a different probability of each possible outcome occurring. However, because $g(v|t_i)$ could be zero at any point, the range of values can be written as $[\underline{v} = \min[\underline{v}_1, \ldots, \underline{v}_n], \bar{v} = \max[\bar{v}_1, \ldots, \bar{v}_n]]$.

[10] This requires that $g(v|t)$ be an integrable function of t.

gets all of the second stage revenue. In the general setup, the strategy space S is any form of contingent payment. In subsequent sections, we consider the effects of using specific securities.

In the Bayesian–Nash equilibrium, bids will depend on the type, t_i, and the value of the bid will depend on the ex post realization of the firm's value, v. Thus, the equilibrium bid of bidder i will be written as a value that depends on t_i and v, $s_i(t_i,v)$. For example, if the bid is 20 percent equity, $s_i = 0.2v$, not 0.2.

To rank the equilibrium bids we need two conjectures about the form of equilibrium $s_i(t_i,\tilde{v})$. The seller must rank bid $s_i > s_j$ if, $\forall t_i > t_j$, $E_v[s_i(t_i,v)|t_i] > E_v[s_j(t_j,v)|t_j]$ (the seller expects more from higher types in equilibrium) and if, $\forall t < t_j$ and $\forall t_j < t_i$, $E_v[s_i(t,v)|t] > E_v[s_j(t,v)|t]$ (lower types must expect to pay more if they imitate a higher type).[11] We do not, however, need either inequality to hold for $\forall t_j > t_i$ because a higher type gains nothing if he imitates a lower type with a bid that makes him pay more than his equilibrium bid. Given these assumptions, first price auction bids can be ranked in equilibrium, and the probability of winning the auction is

$$\text{Prob}\{E_v[s_i(t_i,v)|s^{-1}(s_i(t_i,v))] > E_v[s(t_j,v)|t_j]\forall j\}, \tag{3}$$

where $s^{-1}(s_i(t_i,v)) = t_i$ in equilibrium (because $s_i = s$). Thus, we must remember to check that in equilibrium the expected payment is strictly increasing and that a low type would not pay less by imitating a high type. This Bayesian–Nash equilibrium requires that the bid with the highest expected value wins. However, with multidimensional bids (e.g., debt and equity) multiple bids may have the same expected value. In this case there exist multiple equilibria, and we will need to give each bid a score to rank them. How to map each two-dimensional bid into a one-dimensional score is the seller's choice. This choice will determine the equilibrium of the auction. In later sections we will consider the effects of different scoring rules and the optimal choice.

In general, bidders choose their bid $s(\cdot)$ to maximize their profit.

$$\begin{aligned}\max_{s_i(\cdot)} \int_{\underline{v}}^{\bar{v}} (v - s_i(t_i,v))g(v|t_i)\,dv \\ \times \text{Prob}\{E_v[s_i(t_i,v)|s^{-1}(s_i(t_i,v))] > E_v[s(t_j,v)|t_j]\forall j\}.\end{aligned} \tag{4}$$

A bid in this scenario is a function $s(\cdot)$, and a bid function tells bidders which $s(\cdot)$ to use given their type. To simplify the problem we will switch to a direct revelation mechanism. In a first price auction, the high bidder wins.

[11] Usually an auction analysis begins with the conjecture that equilibrium bids are increasing in type (higher types pay more). This conjecture is later verified. With contingent payments, more is needed. If, for example, a high type bids such that 60 percent of the equity goes to the seller, and a low type bids such that 80 percent goes to the seller, then if the high type is high enough his expected payment is higher than the low type. However, a 60 percent bid cannot be considered better than an 80 percent bid, or the low type would bid 60 percent. The second conjecture on the form of equilibrium s_i removes this problem.

Therefore, given that all other bidders use the equilibrium bid function $r_j = r(t_j, \tilde{v})$ where $r: [\underline{t}, \bar{t}] \rightarrow S$, buyer i must want to choose a bid r_i such that $E[r_i(t_i, \tilde{v})|t_i]$ is over the range $\left[E_v[r(\underline{t}, v)|\underline{t}], E_v[r(\bar{t}, v)|\bar{t}]\right]$.[12] Thus, $E_v[r_i(t_i, v)|t_i]$ can be written as $E_v[r(x, v)|x]$, where x is some function that maps to $[\underline{t}, \bar{t}]$. Choosing $x = \eta$ is like choosing to bid like a player with type η. Thus, a bidder's choice of x is a bidder's choice of what to bid.

Note that even in equilibrium, r_i is not necessarily identical to $r(t_i, \tilde{v})$. This is because many bids ($s(\cdot)$ functions) may yield the same expected value. Because the seller is indifferent between two functions s and s' if $E_v[s(t, v)|t] = E_v[s'(t, v)|t]$, bidder i may choose to use a bid function that is different at every point from the conjectured equilibrium but that yields the same expected value as the conjectured equilibrium (as long as the above conjectures still hold for s').[13] Thus, the solution to equation (4) can be solved only down to the expectation of the optimal bid function. The exact format of the optimal bid function cannot be determined. This is the situation in which a scoring function is needed to select an equilibrium.

The bidder's problem in a first price auction can be rewritten

$$\max_x \int_{\underline{v}}^{\bar{v}} (v - r(x, v)) g(v|t_i)\, dv \, \text{Prob}\{E_v[r(x, v)|x] > E_v[r(t_j, v)|t_j] \forall j\}. \tag{5}$$

Due to the two equilibrium conjectures above, $E_v[r(x, v)|x]$ is an invertible function of x. Therefore, the probability of winning can be rewritten

$$\text{Prob}[x > t_j, \ \forall j] = F^{n-1}(x). \tag{6}$$

Because the highest type wins, each bidder's probability of winning is the probability he is the highest of n bidders, where F is the distribution of types. Thus, bidder i's problem reduces to

$$\max_x \int_{\underline{v}}^{\bar{v}} (v - r(x, v)) g(v|t_i)\, dv F^{n-1}(x). \tag{7}$$

The appendix discusses local and global incentive compatibility.

This equation looks remarkably like the standard auction problem with the addition of uncertainty except the bid now depends on the realization of $\tilde{v}$, which depends on the type of bidder. One might expect that the seller's expected revenue from any auction with securities bidding is independent of the particular securities that are allowed. However, examples by Reece (1979) and Hansen (1985) have pointed out that this is not true. The following

[12] In a first price auction, a bidder gains no probability of winning by bidding a function with an expected payoff greater than the highest possible competitor's expectation, and he must pay more than if he just bid the highest expectation. A bidder will lose with certainty if he bids a function with an expectation below the lowest expectation.

[13] Because the seller and the bidders are risk neutral they care only about the first moment.

theorem generalizes this idea and allows the revenue from different types of contingent payments to be compared by examining how the payments change with the type of the bidder. Initially we assume efficiency (i.e., the auction separates, the highest type is sure to win, and the assets are allocated to their best use) and consider revenue; later we consider efficiency.

THEOREM 1: *Let the expected payments of the bidders, $P(\cdot)$, depend on both their bid, x, and their type, t, $P(x,t)$, and let the expected payments increase in the bidder's type, holding their bid constant, ($P_2(x,t) > 0$).*[14] *If, under two alternative sets of securities bids (cash, debt, equity, options, etc.), solutions to the bidder's problem exist that are incentive compatible, individually rational, and invertible, then the securities bid choice where $\int_{\underline{t}}^{t} P_2(x,x)\,dx$ is greatest yields the greatest expected revenue to the seller.*

Proof: Rewriting the bidder's problem (equation (7)) yields

$$\max_x \left\{\int_{\underline{v}}^{\bar{v}} vg(v|t)\,dv\right\} F^{n-1}(x) - \left\{\int_{\underline{v}}^{\bar{v}} r(x,v)g(v|t)\,dv\right\} F^{n-1}(x). \tag{8}$$

The second term is the bidder's expected payment, $P(x,t)$. The derivative of equation (8) with respect to x is

$$\frac{d}{dx}\int_{\underline{v}}^{\bar{v}} vg(v|t)\,dv\, F^{n-1}(x) = P_1(x,t). \tag{9}$$

Setting $x = t$, the integral of both sides from $\underline{t}$ to t is

$$P(t,t) = \left\{\int_{\underline{v}}^{\bar{v}} vg(v|t)\,dv\right\} F^{n-1}(t) - \int_{\underline{t}}^{t}\left\{\int_{\underline{v}}^{\bar{v}} v\,\frac{d}{dy}g(v|y)\,dv\right\} F^{n-1}(y)\,dy + \int_{\underline{t}}^{t} P_2(y,y)\,dy. \tag{10}$$

Because equilibrium bids are assumed to be invertible, the probability of winning cannot change under alternative sets of securities bids. Therefore, the first term and the second term in equation (10) cannot change with the type of securities bid. Thus, the only term that can differ is the third integral. Therefore, if $\hat{P}$ is the expected payment with one type of security bid and P is the expected payment with another type of security bid then

$$\int_{\underline{t}}^{t} \hat{P}_2(y,y)\,dy > \int_{\underline{t}}^{t} P_2(y,y)\,dy \Rightarrow \hat{P}(t,t) > P(t,t). \tag{11}$$

Q.E.D.

[14] This is the definition of a contingent auction.

COROLLARY 1: *Theorem 1 is independent of the form of the auction: first price, second price, English, and so on.*

Proof: Equation (8) was written for a first price auction. However, this equation can be replaced with

$$\max_{x}\left\{\int_{\underline{v}}^{\bar{v}} vg(v|t_i)\,dv\right\}F^{n-1}(x) - P(x,t_i,t_{-i}), \tag{12}$$

and the proof still holds, where $P(x,t_i,t_{-i})$ is bidder i's expected payment given the auction rules and t_{-i} denotes the types of all the other bidders (for some auction rules the bidder's expected payment depends on the types of the other bidders). The only difference in the proof is that $P(x,t)$ is replaced with $P(x,t_i,t_{-i})$ and $P_1(x,t)$, $P_2(x,t)$ are replaced with $P_1(x,t_i,t_{-i})$, $P_2(x,t_i,t_{-i})$. Q.E.D.

Theorem 1 and Corollary 1 allow us to compare auctions that use two alternative types of securities simply by comparing how sensitive the value of the securities is to a change in the type of bidder. If the conditions of the theorem are met, we learn that the seller's revenue can always be increased by allowing securities bids. In a cash auction, the payment does not depend on the winner's true type—it depends only on the winner's bid. In a securities auction, the payment depends on the winner's ability to manage. A high type who imitates a lower type's bid would expect to pay more than the lower type, because the high type is a better manager. However, the low type and the high type would have an equal probability of winning the auction. Thus, the incentive-compatible downward constraint that ensures higher types will not imitate lower types' bids is less difficult to secure. On the opposite side, lower types find it more beneficial to imitate higher types' bids. Therefore, high types must bid higher to keep low types from imitating them, and expected revenue is increased. Thus, the ability to select the medium of exchange in the auction is valuable.[15]

II. Debt and Preferred Stock

The first security auction we consider is a debt auction. All of the previous work on contingent payments (cited above) has examined only cash and equity (linear contracts). We extend the work by restricting the auction (no cash) and considering debt and preferred stock bids. Without additional detail, the debt is identical to preferred stock. In the next section, we introduce nonpecuniary bankruptcy costs and distinguish between debt and preferred stock.

[15] Recent work by Bhattacharyya and Singh (1999) shows the value to different claimants of choosing the method of sale (first price or English). Our work shows that the right to select the medium (equity, cash, etc.) is also valuable.

Theorem 1 leads directly to the following corollary, which compares cash and debt (or preferred stock) auctions.

COROLLARY 2: *The incentive-compatible, individually rational, invertible solution to the bidder's problem, in a first price cash auction, yields less expected seller revenue than the equivalent solution to an auction involving debt or preferred stock.*

Proof: In a cash auction the bidder's problem can be written

$$\max_x \left[\int_{\underline{v}}^{\bar{v}} vg(v|t)\,dv\, F^{n-1}(x) - cash(x)\, F^{n-1}(x) \right], \tag{13}$$

where $cash(x)$ is the cash bid. Let $cash(x)\, F^{n-1}(x) = P(x)$, the bidder's expected payment function. Note that it is not a function of the bidder's true type. Differentiating we find

$$\frac{dP(x)}{dx} = \int_{\underline{v}}^{\bar{v}} vg(v|t)\,dv\, \frac{d}{dx} F^{n-1}(x). \tag{14}$$

Setting $x = t$ and integrating both sides from $\underline{t}$ to t yields the expected cash payment

$$P(t) = \int_{\underline{v}}^{\bar{v}} vg(v|t)\,dv\, F^{n-1}(t) - \int_{\underline{t}}^{t} \int_{\underline{v}}^{\bar{v}} v \frac{d}{dy} g(v|y)\,dv\, F^{n-1}(y)\,dy. \tag{15}$$

In a debt auction the bidder's problem can be written

$$\begin{aligned} \operatorname*{Max}_x \Bigg[& \int_{\underline{v}}^{\bar{v}} vg(v|t)\,dv\, F^{n-1}(x) - b(x) \int_{b(x)}^{\bar{v}} g(v|t)\,dv\, F^{n-1}(x) \\ & - \int_{\underline{v}}^{b(x)} vg(v|t)\,dv\, F^{n-1}(x) \Bigg], \end{aligned} \tag{16}$$

where $b(x)$ is the debt bid; that is, the bidder will pay $b(x)$ next period if he is able and v otherwise (thus, the bid includes principal plus one period of interest). Let $P(x,t)$ equal the bidder's expected payment,

$$P(x,t) = b(x) \int_{b(x)}^{\bar{v}} g(v|t)\,dv\, F^{n-1}(x) + \int_{\underline{v}}^{b(x)} vg(v|t)\,dv\, F^{n-1}(x). \tag{17}$$

Note that the expected payment is a function of the bidder's type, t. Differentiating the bidder's problem, equation (16), we find

$$P_1(x,t) = \int_{\underline{v}}^{\bar{v}} vg(v|t)\,dv\,\frac{d}{dx}F^{n-1}(x). \tag{18}$$

Setting $x = t$ and integrating both sides from $\underline{t}$ to t yields the expected debt payment

$$P(t,t) = \int_{\underline{v}}^{\bar{v}} vg(v|t)\,dv\,F^{n-1}(t) - \int_{\underline{t}}^{t}\int_{\underline{v}}^{\bar{v}} v\,\frac{d}{dy}g(v|y)\,dv\,F^{n-1}(y)\,dy$$
$$+ \int_{\underline{t}}^{t} P_2(y,y)\,dy. \tag{19}$$

Comparing the expected payments in each auction, equation (15) and equation (19), we see that the debt expected payment has an extra term, $\int_{\underline{t}}^{t} P_2(y,y)\,dy$. If this term is positive, then the expected payment in the debt auction is higher than the expected payment in the cash auction. Differentiating equation (17) with respect to t yields

$$b(x)\int_{b(x)}^{\bar{v}} \frac{d}{dt}g(v|t)\,dv\,F^{n-1}(x) + \int_{\underline{v}}^{b(x)} v\,\frac{d}{dt}g(v|t)\,dv\,F^{n-1}(x). \tag{20}$$

This is positive by the definition of a higher type: a higher t ensures that the bidder has a greater chance of earning larger payoffs. A better type will be more likely to pay the debt or pay a larger amount when he cannot pay the full debt. Thus, for all t, $P_2(x,t)$ is positive. Q.E.D.

Consider the following example. Let $t \sim U[0,1]$ and let each player have a probability t of earning t in the next period and a probability $(1 - t)$ of earning zero. The bidder's problem in the cash auction is thus

$$\max_x[[t^2 - cash(x)]x^{n-1}]. \tag{21}$$

Solving, the bidder's expected payment is

$$P(x) = cash(x)x^{n-1} = x^{n+1}\left(\frac{n-1}{n+1}\right). \tag{22}$$

The corresponding bid is

$$cash(x) = x^2\left(\frac{n-1}{n+1}\right), \tag{23}$$

where $x = t$ in equilibrium.

The bidder's problem in the debt auction is

$$\max_x [[t^2 - b(x)t]x^{n-1}]. \tag{24}$$

Solving, the bidder's expected payment is

$$P(x,x) = b(x)x^n = x^{n+1}\frac{n-1}{n+1} + \int_0^x b(y)y^{n-1}dy. \tag{25}$$

The corresponding bid is[16]

$$b(x) = x\left(\frac{n-1}{n}\right), \tag{26}$$

where $x = t$ in equilibrium.

The bid in the debt auction is higher than the bid in the cash auction, as would be expected because the bidder may not pay his debt bid. What is interesting is that given the bidder with type t wins, his equilibrium expected payment in the debt auction, $b(t)t^{n-1}t = t^{n+1}(n-1)/n$, is greater than his expected payment in a cash auction, $cash(t)t^{n-1} = t^{n+1}(n-1)/(n+1)$. If $P_2(x,t)$, the added integral in equation (25), were equal to zero, the bid in the debt auction would be $t(n-1)/(n+1)$ and would still be greater than the bid in the cash auction. However, in this case the equilibrium expected payment in the debt auction, $t(n-1)/(n+1)t^{n-1}t$, would equal the expected payment in the cash auction, $cash(t)t^{n-1}$. Thus, it is clear that it is the addition of term $P_2(x,t)$ that increases the bid even more and results in greater expected revenue in a debt auction than a cash auction.

In general, if the bidders' expected payments increase when their ability increases, even though they do not change their bids, then the seller can extract greater surplus from the bidders. A low bidder who imitates a high type will increase the probability that he will win, but he will not expect to pay as much as the higher type. A high bidder who imitates a low type will decrease his probability of winning, but he will expect to pay more than the lower type. Thus, the low bidders have a greater incentive to imitate higher types; high bidders have a lower incentive to imitate lower types; and the seller can extract more from the bidders.

[16] Global incentive compatibility requires

$$t > x \Rightarrow (2t - b(t))t^{n-1} > (2x - b(x))x^{n-1}.$$

Substituting for the bid function

$$t > x \Rightarrow \left(2 - \frac{n-1}{n}\right)t^n > \left(2 - \frac{n-1}{n}\right)x^n. \quad \text{Q.E.D.}$$

In the previous example, debt bids improved the expected revenue; however, this improvement relied on a particular feature of the example. As noted in Theorem 1, contingent payments are guaranteed to improve revenue only if the solution to the bidder's problem yields invertible bids: debt must separate the bidders. Consider the expected payment function, equation (10), in the proof of Theorem 1, evaluated at the lowest type

$$P(\underline{t},\underline{t}) = \int_{\underline{v}}^{\bar{v}} vg(v|\underline{t})\,dv\,F^{n-1}(\underline{t}). \tag{27}$$

The lowest type's expected payment is his expected value. However, the proof of Theorem 1 did not specify that the theorem was true only if the lowest type expected to earn nothing. Theorem 1 did require, however, a separating equilibrium. As is well known, the lowest type paying their value is a necessary condition for a separating equilibrium.

CONDITION 1: *A necessary condition for an incentive-compatible, individually rational, separating equilibrium in an auction with securities or cash is that the lowest type must expect to earn zero profit, given that he wins the auction.*

Proof: Let the lowest type expect positive profits if he wins the auction. In a separating equilibrium the lowest type has a zero probability of winning. Thus, he expects to make nothing. However, he can raise his bid, improve his probability of winning, and change his expected payoff from zero to a positive number. In a separating equilibrium, the lowest type always has this incentive. Therefore, he raises his bid until he would expect to earn zero profit if he won the auction. Q.E.D.

This condition is easy to satisfy in a cash auction: the lowest type bids his expected value. In a securities auction, the only way to satisfy this condition may be for the lowest bidder to bid the highest possible bid! If the low bidder bids the high bid then the bid function cannot be increasing and invertibility (separation) is lost. In the above debt auction example, the low bidder bid $\underline{t}$, his highest possible payoff in the second stage. Because other bidders had possible payoffs above the highest payoff of type $\underline{t}$, the equilibrium bid function was still invertible.

Consider, instead, the following debt auction example. The lowest type in this auction expects to make H half the time and L the other half of the time. Higher types have a greater chance of earning H and a smaller chance of earning L. In a first price cash auction the lowest type will bid his expected value $H/2 + L/2$ and expect zero if he wins the auction. In a debt auction, however, this same bid would lead to the low type paying L half the time (when he earned L he could not afford to pay his entire bond) and $H/2 + L/2$ half the time (when he earned H). Thus, the bidder would have a positive expected payoff but no probability of winning. So, he would raise his bid. In fact, he would raise his bid to H. At this point he would expect to pay H half the time and L half the time. However, bids above H are meaningless because no bidder ever earns an amount greater than H. Therefore, all of the bidders must bid the highest bid, H.

Thus, the debt or preferred stock auction may separate the bidders and improve revenue or it may not. Surprisingly, as the following theorem shows, the high bid pooling equilibrium is the only one.

THEOREM 2: *If the lowest type must bid the highest possible value to win the auction and make zero profits, then the only subgame perfect Nash equilibrium is for all bidders to bid the highest value. Other equilibria require the seller to believe that a bid that is higher than $b(t)$ but off the equilibrium path belongs to a bidder with a type less than t and will provide a lower payment than $b(t)$.*

Proof: See the appendix.

In a debt auction, where every bidder has a possibility of earning the highest payoff, the low bidder will expect positive profits unless he bids the highest value. Thus, Theorem 2 binds for some debt auctions. The only (subgame perfect) equilibrium is that all bidders bid the largest possible value. This may seem like an excellent outcome from the seller's point of view. However, it is not. Because the seller cannot distinguish among the various types of bidders, he is just as likely to choose the lowest type as the highest. Thus, even though the seller captures all of the value of the selected bidder, the chances of choosing a low type may outweigh the benefit of receiving all of the selected bidder's value. With few bidders, it is better to have an equal chance of capturing all of the value of one bidder. As the number of bidders increases, the loss from not choosing the top bidder quickly dominates.[17]

It follows that if a securities auction is used in the reorganization process, it is not the case that the results would be obviously disastrous. In fact, the bids may be so high that the seller might have trouble distinguishing among them. Those who implement the auction might then hail it a success. But as we see here, efficiency is lost, sellers lose, and the reorganization fails to allocate assets to the best use.

III. Nonpecuniary Cost to Bankruptcy

A. Debt

The failure of the debt auction when bidder payoffs come from a fixed support distribution, that is, $[L, H]$, arises because there is no downside to "over-bidding." Typically, managers who bid in reorganizations are in the business of turning firms around. If they attempt a turnaround and fail, they reduce their reputation. Thus, they have a nonpecuniary cost to bankruptcy. This cost will affect the auction.

[17] In an auction with expected values that are uniform [0,1], randomly choosing a bidder yields an expected value of 0.5 (this would be the result of an equity auction). In a first price cash auction the seller's expected revenue is greater than or equal to 0.5 if the number of bidders is greater than or equal to 3.

Reconsider the fixed support auction. The future firm will earn H with probability λ or L with probability $(1-\lambda)$, where higher types have higher λ. Any bidder who bids above L has probability $(1-\lambda)$ of entering bankruptcy and paying c. This is similar to Ross (1977) and Stein (1992), who both allow for nonpecuniary bankruptcy costs in the context of signaling models.

Bidder i's expected profit is

$$[H\lambda_i + L(1-\lambda_i) - c(1-\lambda_i) - b(x)\lambda_i - L(1-\lambda_i)]F^{n-1}(x). \tag{28}$$

As above, choosing x is like choosing to bid like the bidder of type x. Because the maximum of the bidder's profit is the same when divided by λ_i we can divide through and simplify the mathematics. Taking the derivative with respect to x and dropping the i subscript yields

$$\left[H - c\,\frac{(1-\lambda)}{\lambda}\right]\frac{d}{dx}F^{n-1}(x) = \frac{d}{dx}b(x)F^{n-1}(x). \tag{29}$$

Integrating from λ_0 to λ, we find that the bid is

$$b(\lambda) = H - c\,\frac{(1-\lambda)}{\lambda} - \int_{\lambda_0}^{\lambda}\frac{cF^{n-1}(x)\,dx}{\lambda^2F^{n-1}(\lambda)}. \tag{30}$$

Note the low type bids less than H as long as c is greater than zero. Invertibility requires $b'(\lambda) > 0$, or

$$b'(\lambda) = \int_{\lambda_0}^{\lambda} cF^{n-1}(x)\,dx\left[\frac{2}{\lambda^3F^{n-1}(\lambda)} + \frac{(n-1)f(\lambda)}{\lambda^2F^{n}(\lambda)}\right] > 0, \tag{31}$$

which is true for any positive c; thus the bid function is invertible. As long as c is positive, the low bidder is not willing to bid H. If he did, then he would expect to lose c. Because higher types have less and less chance of paying c, they are willing to bid higher and higher. Thus, the bidders separate over the range of possible bids $[H - c(1-\lambda)/\lambda, H]$.

Now consider what happens if bidders have a possibility of a very large payoff, H, and a small cost to bankruptcy, c (c is small relative to H). The result is a separating auction with very large bids. The seller may then hail the auction a success. However, ex post bankruptcy may be a virtual certainty, and the seller may repossess the assets sold. As mentioned in the introduction, this is exactly what occurred in the FCC C block bandwidth auctions (see Landler (1997) for a discussion). The C block was open only to small firms with low reputation and low assets, and bidders were allowed to put only 10 percent down. Bidders were required to make scheduled future payments on any winning bid. Thus, the bids were very similar to debt bids with low bankruptcy costs. Also, the value of a license was unknown but possibly very large. The bidders bid far higher than expected, and the FCC hailed the auction a success. However, as we would predict, ex post most bidders have failed to make future payments, and the FCC has had to renegotiate and reclaim licenses. Thus, even when a non-cash auction allocates the asset to the best use, it may not leave the asset in the best use. This is another deadweight loss that is not internalized by the auction.

Any time bidders have very small bankruptcy costs and the same highest possible value, the bids will be high and very close together. If the system has any noise in it then very close bids are likely to lead to inefficient asset allocation and lower seller revenue. In general, bidders without a reputation may have a difficult time getting a seller to accept their debt bids (because a small or zero c would allow them to bid high relative to a better type with high costs of reputation loss). Thus, bidders without a reputation are excluded from the auction. These reputationless bidders are the same bidders who will have difficulty raising money for a cash auction in an inefficient capital market with information asymmetries. Therefore, the use of securities may not allow the entry of as many cash-constrained bidders as might be thought.

B. Preferred Stock versus Debt

With a cost to bankruptcy, preferred stock bids are not the same as debt bids because preferred stock bidders suffer no bankruptcy costs. Thus, in an auction where all bidders have the same value supports (H and L), the use of preferred stock will not separate bidders, whereas debt bids (with bankruptcy costs) will. Therefore, if it is possible for any bidder to generate the highest ex post value realization, then the seller will accept only debt bids. If, however, the supports increase with the bidder type, then preferred stock will separate the bidders. This understanding will help explain the use of preferred stock versus debt in reorganization bids.

It is trivial to show that the bidders will bid higher (the seller will make more) if there are no costs to bankruptcy, because increasing their bid is less costly. Using preferred stock bids eliminates the costs of bankruptcy. Thus, if the preferred stock will separate the bidders, the seller will desire, and thus require, bidders to bid in preferred stock. It is interesting to note, however, that the bidders will not necessarily prefer an environment without bankruptcy costs. This is the standard signaling result (though not typical in an auction environment) that a more expensive signal may or may not require the signalers to pay (bid plus bankruptcy costs) more. In this case the cost of signaling is raising the bid. This is more expensive in an environment with bankruptcy costs, but with a more expensive signal bidders can signal without raising their bid as much. The following example shows how bankruptcy costs reduce the seller's revenue but (in this example) improve the bidder's revenue.

Let $t \sim U[0,1]$, let c equal the nonpecuniary bankruptcy costs, and let each player have a probability t of earning t in the next period and a probability $(1 - t)$ of earning zero. The bidder's problem in the debt auction is thus

$$\max_x [[t^2 - b(x)t - (1-t)c]x^{n-1}]. \tag{32}$$

Solving, the bidder's expected payment to the seller is

$$P(x,x) = b(x)x^n = x^{n+1}\frac{n-1}{n+1} - \frac{1-x}{x}c - \frac{c}{n} + \int_0^x b(y)y^{n-1}dy. \tag{33}$$

The corresponding bid is[18]

$$b(x) = \max\left[x\left(\frac{n-1}{n}\right) + c - \frac{(n-1)c}{(n-2)x}, 0\right], \tag{34}$$

where $x = t$ in equilibrium.[19]

The bid of every bidder is obviously below his corresponding preferred stock bid (c = 0); thus the seller's revenue is lower. The expected total payment (to the seller and bankruptcy costs) of every bidder if he wins is $b(t)t + (1 - t)c$. Substituting for the bid function, each bidder's total payment is

$$\max\left[t^2\frac{n-1}{n} - \frac{c}{n-2}, 0\right]. \tag{35}$$

Thus, each bidder's total expected payment is less than if he uses preferred stock. Therefore, although sellers desire preferred stock auctions if these auctions select the best manager, bidders may or may not agree.

[18] Global incentive compatibility requires

$$t > x \Rightarrow (2t - b(t) + c)t^{n-1} > (2x - b(x) + c)x^{n-1}.$$

Substituting for the bid function

$$t > x \Rightarrow \left(2 - \frac{n-1}{n}\right)t^n + \frac{(n-1)c}{(n-2)}t^{n-2} > \left(2 - \frac{n-1}{n}\right)x^n + \frac{(n-1)c}{(n-1)}x^{n-2}. \quad \text{Q.E.D.}$$

[19] The "maximum" expression is required because the costs of bankruptcy are constant across bidders. Therefore, those bidders with low values cannot afford to bid. If they win, they expect to pay more than their value in bankruptcy cost.

Generally, we find that a substantial nonpecuniary cost to bankruptcy can restore separation in the debt auction. However, if the debt auction separates without bankruptcy costs then the seller will prefer bids of preferred stock (which have no bankruptcy costs so the bidders bid higher), whereas the bidders may prefer debt bids (as the above example shows).

IV. Renegotiation

The use of nonpecuniary bankruptcy costs to provide separation presents a problem. We have long known that commitment by the seller is required for a separating auction. If the seller does not commit then he uses the information contained in the invertible bids to extract all of the buyer's surplus: the buyer, seeing this coming, pools. An interesting twist results when securities auctions are considered. The standard restriction is that the seller must commit not to use his ex post information to require the bidder to give more than his winning bid. The seller could, however, give some securities back to the bidder or exchange one type of security for another security of equal or lesser value. If the winning bidder were amenable, no ex ante contract would prevent this. This revenue return destroys separation just as the revenue extraction does.

In some situations noted above, debt bids with bankruptcy costs are required for separation. However, ex post both parties could restructure the capital in a way that results in a Pareto improvement. For example, the debt could be exchanged for preferred stock. The seller would get the same payment in all states of the world, and the bidders would be strictly better off. However, the bidders in the auction would see this renegotiation coming and bid higher, knowing that if they win they can exchange their debt for preferred stock. The lowest type would be willing to bid the largest debt bid, and separation would be lost.

Therefore, to ensure that securities bids efficiently allocate resources, the court must prevent some changes to the capital structure for some period after reorganization. Note that only some types of changes must be prevented. A reorganization that exchanges a security that has a less costly signal for one that is more costly will retain separation in the auction. That is, preferred stock may be exchanged for debt and debt may be exchanged for cash (and equity may be exchanged for preferred stock, debt, or cash).

Preferred stock will never be exchanged for debt because the auction is a zero sum game and debt has a deadweight loss. The more interesting idea is that the winning bidder may buy down the debt in some interim stage before the value of the firm is realized. If the seller required debt bids because the bidders were cash constrained but the constraint was (with some probability) temporary, ex post it will be in everyone's best interest to buy down the debt and remove the possibility of bankruptcy costs.

The seller, with bargaining power, would require the debt bidder to pay, in cash, the expected value of his debt bid plus his expected bankruptcy costs. It would seem that with certainty this would yield more or at least as much revenue as the debt auction. However, it does not. Allowing a middle stage in which the bidders receive cash and buy down the debt actually converts the debt auction into a cash auction. An auction where the debt is converted to cash before the ex post value realization is not a contingent auction: the payment does not depend on the ex post outcome. Note that in equation (10) from Theorem 1, if the payment $P(x,t)$ does not depend on both what the bidder reveals, x, and the truth, t, then $P_2(x,t) = 0$ and the expected payment only depends on the expected value and the probability of winning. Thus, allowing a cash for debt swap removes the contingent nature of the auction.

Thus, the seller faces a trade-off. Theorem 1 ensures that the bidders will make a lower total expected payment in a cash auction than a debt auction. However, in the debt auction with bankruptcy costs, some of their "payment" is in the form of bankruptcy costs that the seller does not receive. Therefore, although the bidders expect to pay less in a cash auction, all of their payment goes to the seller. Thus, the seller may receive more in a cash auction than a debt auction, depending on the size of the bankruptcy costs. If the bankruptcy costs are large, a cash for debt swap is beneficial to everyone; the bidders pay less in total, and the seller gains some of the eliminated bankruptcy costs. As the bankruptcy costs decrease, the seller will desire an ex ante restriction on cash buybacks because the contingent nature of the debt auction improves the seller's revenue. However, this will be difficult to enforce because ex post the seller is always better off selling back the debt and extracting some of the bankruptcy costs from the winning bidder.

Thus, renegotiation is a problem in the non-cash auction. However, the problem can be alleviated by the court if bankruptcy regulation requires that the firm not change its capital structure except through cash payments for some period after the reorganization. Future cash payments, however, eliminate contingencies and may lower the seller's expected revenue. Thus, we might see sellers attempt to restrict all types of restructures.

V. Equity

In this section we investigate the efficiency of using equity bids in a securities auction and the seller's resulting expected revenue. We will show that equity has the greatest potential for revenue enhancement but on its own will never efficiently allocate assets.

An equity-based bid would, like a debt bid, cause the bidder's expected payments to increase with their ability, $P_2(x,t) > 0$, whereas for a cash auction $P_2(x,t) = 0$. Also, given a debt bid and an equity bid with equal expected value, the change in the equity bid if the type of the bidder increases is greater than the debt bid. Thus, Theorem 1 tells us that the expected revenue from a cash auction is less than a debt auction, which is less than an equity auction. We must, however, be careful. Both Theorem 1 and Corollary 1 are predicated upon the existence of an incentive-compatible, individually rational, invertible solution to the bidder's problem. The next theorem demonstrates that an auction that uses only equity bids cannot separate between types.

THEOREM 3: *There exists no incentive-compatible, separating equilibrium, for any auction form, when bidders use only equity bids.*

Proof: An equity auction consists of bidders paying a fraction α_i of their payoff, v. Let $t_i > t_j$. In a separating equilibrium Prob$[i\,wins] >$ Prob-$[j\,wins]$. Incentive compatibilityrequires bidder i to expect to make more playing strategy i than strategy j, that is,

$$\text{Prob}[i\,wins](1-\alpha_i)E[v|t_i] \geq \text{Prob}[j\,wins](1-\alpha_j)E[v|t_i]. \tag{36}$$

Incentive compatibility also requires bidder j to expect to make more playing strategy j than strategy i, that is,

$$\text{Prob}[j\,wins](1-\alpha_j)E[v|t_j] \geq \text{Prob}[i\,wins](1-\alpha_i)E[v|t_j]. \tag{37}$$

These two equations can hold simultaneously only if they are both equalities. Therefore, Prob$[i\,wins](1-\alpha_i) =$ Prob$[j\,wins](1-\alpha_j)$ $\forall t_{i,j} \epsilon [\underline{t}, \bar{t}]$, and every bidder must be indifferent between all of the possible bids. However, the player with the lowest type $\underline{t}$ has no probability of winning in a separating equilibrium. Therefore, Prob$[i\,wins](1-\alpha_i) = 0$ $\forall t_i \epsilon [\underline{t}, \bar{t}]$. This is only possible if $\alpha = 1$ for all bidders, which eliminates separation. Q.E.D.

This theorem shows that any bid mechanism in which bidders give only a fraction of their expected value is not incentive compatible and separating. Because the lowest type has nothing to lose, he is willing to make the highest bid. Thus, all other bidders must do likewise if they hope to win. Therefore, just as in the debt auction with the same value supports for all types, an all-equity auction with any type of value supports does not efficiently allocate assets.

The efficiency of the equity auction, however, can be restored with partial cash bids. The equity bids then separate over the amount of the cash component. Thus, Theorem 3 looks like a knife-edge result: any amount of cash will restore the separability of the equity auction. However, the idea that a single dollar of cash could restore separation in an equity auction for a million-dollar company is predicated on another knife-edge: perfect separation. In reality, bids must be sufficiently far apart to ensure separability: one-hundred-million-dollar equity bids that differ in true value by one hundred dollars would be indistinguishable. Thus, a *significant* cash component to each bid would be necessary if we expect to separate the managers. How significant this component is will depend on the bid increment in the auction. For example, if a bid must contain one percent more equity to be considered larger, and the cash component is only one dollar, then low types may rush to, 99 percent. Higher types are then prevented from separating. If the type space is continuous, then with set bid increments, some bidders will always pool. A larger cash component will increase separation, but, from Theorem 1, decrease expected revenue.

However, the problem we were trying to solve is a lack of cash. Furthermore, if different bidders have access to different amounts of cash, separability will be lost because the seller cannot determine if a bid with low cash is from a low type or a type without access to cash (see Rhodes-Kropf and Viswanathan (2000a)). Thus, the equity plus cash auction will always have some inefficiencies both because types without enough cash cannot participate and because, to some extent, low types will imitate higher types.

The equity auction also fails if the highest type is the type with the greatest outside opportunity. Consider the following English auction example. Three bidders desire the company. If the worst of the three runs the company, then it makes \$20 to be divided between the seller and buyer, but the worst type also has an outside opportunity that would earn him \$10. If the middle type runs the firm, then it makes \$35, but he has an outside opportunity that would earn him \$20. And the best type would generate \$55 if she ran the firm, and she has a \$30 outside opportunity. If the auction were in cash or debt (which is the same as cash if the future payoff is known with certainty) then the low type would be willing to bid up to \$10, the middle type to \$15, and the high type to \$25. Thus the high type would win. In an equity auction, however, the high type would only bid until the seller kept 45.45 percent of the stock; the middle type would bid until the seller kept 42.8 percent of the stock; and the low type would be willing to keep bidding until the seller had 50 percent of the stock. Thus, the low type would win. The bids are not even monotonic. What is surprising is that the inefficiencies cannot be corrected in a secondary equity market in which bidders are allowed to trade the management of the firm. The low type does not end up with enough stock to allow a transaction in which the high type profits by taking over.

Hansen (1985) also considered equity bids. However, he considered their use only in conjunction with verifiable assets. Rhodes-Kropf and Viswanathan (2000b) extend Hansen's and our work by considering equity bids when firms with under- or overvalued assets bid in a takeover setting. In the above discussion it is the use of equity unsupported by assets that causes the auction to fail to separate.

Riley (1988) states that in a common values setting, revenue from an auction can be improved by conditioning the payment of bidders on events after the auction: an equity bid is an example of this. Therefore, we can say that with pure common values, an equity auction is excellent because all bidders bid 100 percent (see Theorem 2). This is efficient because bidders are identical. However, as we move away from pure common values to part common and part independent private values and toward pure independent private values, the seller faces the equity bid problem that we show in Theorem 3. The benefit from the reduction in uncertainty about the common value diminishes, and the probability that the auction does not allocate the assets to the highest type increases. Thus, the equity auction becomes more likely to be socially inefficient and revenue inferior.[20]

The problem with the independent private values equity auction is that the payment to the seller depends on the types of bidders and not just on their bids. Thus, low bidders can bid higher without expecting to pay as much as a high type with the same bid. Even with outside opportunities, there is a minimal downside to "overbidding." Bidding higher causes a bidder to pay more, but until a bidder bids 100 percent, any bidder will make something if he wins.

VI. Debt and Equity

When bids contain both a debt portion and an equity portion, the auction determines the capital structure of the firm, which may affect the value of the firm. Because the optimal capital structure is unlikely to be obtained and restructuring must be restricted, we find non-cash auctions lead to an additional inefficiency.

First we continue to assume that the capital structure does not affect the firm's value. The bidder's problem in this case is

$$\max_x \int_{b(x)}^{\bar{v}} [v - b(x)](1 - \alpha(x)) g(v|t)\, dv\, F^{n-1}(x), \tag{38}$$

where $b(x)$ is the amount of debt offered and $\alpha(x)$ is the fraction of equity offered. Solving (see the appendix), the debt bid as a function of the equity bid is

$$b(t) = \frac{\displaystyle\int_{b(t)}^{\bar{v}} v g(v|t)\, dv}{\displaystyle\int_{b(t)}^{\bar{v}} g(v|t)\, dv} - \frac{\displaystyle\int_{\underline{t}}^{t} \int_{b(x)}^{\bar{v}} [v - b(x)] \frac{d}{dx} [g(v|x)]\, dv\, (1 - \alpha(x)) F^{n-1}(x)\, dx}{\displaystyle\int_{b(t)}^{\bar{v}} g(v|t)\, dv\, (1 - \alpha(t)) F^{n-1}(t)}. \tag{39}$$

[20] With common values, bids that are royalty payments per amount of oil, wood, ore, and so on, extracted reduce the effect of the uncertainty about the quantity in the location and improve the expected revenue of the seller. Riley (1988) assumes that the sale price of oil is known and the average cost schedule is the same for everyone (pure common values). Thus, if the marginal costs are constant, then every bidder bids Price-MC (this is identical to every bidder bidding 100 percent equity). What Riley does not consider is that one extraction process may be more costly than another but able to extract more barrels of oil out of a given reserve. In this case, the seller may increase revenue by accepting a bid with a lower royalty rate per barrel because more barrels will be extracted. However, the seller cannot tell the difference between a low royalty rate bidder who will not produce more and one who will.

Rearranging this equation we can find the equity fraction as a function of the debt bid:

$$\alpha(t) = 1 - \frac{\displaystyle\int_{\underline{t}}^{t}\int_{b(x)}^{\bar{v}} [v - b(x)]\frac{d}{dx}[g(v|x)]\,dv\,(1-\alpha(x))F^{n-1}(x)\,dx}{\displaystyle\int_{b(t)}^{\bar{v}} [v - b(t)]g(v|t)\,dv\,F^{n-1}(t)}. \tag{40}$$

With multidimensional bids, because the seller is indifferent between two bids that yield the same expected payment, there exist multiple equilibria. We can only solve the bidder's problem down to an expected payment. How then can we tell if a bid comes from a better type? In setting up the model we said that in equilibrium bid i would be considered better than bid j if

$$E_v[s(t_i,v)|s^{-1}(s(t_i,v))] > E_v[s(t_j,v)|t_j], \tag{41}$$

where $s(\cdot)$ is the equilibrium bid function. We saw in Section V that in equilibrium we would not be able to invert the equity bid to determine the type of the bidder. Therefore, $s^{-1}(s(t_i,\tilde{v}))$ would not yield the type of the bidder, and we would be unable to tell if higher equity came from a better or worse bidder. The same problem arises here, and so the equity portion of the bid cannot function as a signal. The debt portion, however, can function as a signal (with moving value supports).[21] Thus, the debt is a signal that can be inverted to determine the type of the bidder. Once the type is learned, the type is used to determine the expected value of both the debt and equity portions of the bid. So what we find is that raising the debt portion raises the expected payment and signals a better type, whereas raising the equity portion raises the expected payment (higher bid) but does not signal. Therefore, in equilibrium, higher types must bid higher debt but may bid higher or lower amounts of equity.

To find a particular equilibrium, we must impose some beliefs and find the equilibrium given those beliefs (e.g., α = constant). Because the seller designs the auction, the issues are which equilibrium the seller desires and whether it is incentive compatible. The seller's problem is

$$\max_{b,\alpha} \int_{\underline{t}}^{\bar{t}}\left[\int_{b(x)}^{\bar{v}} [(v - b(x))\alpha(x) + b(x)]g(v|x)\,dv + \int_{\underline{v}}^{b(x)} vg(v|x)\,dv\right]F^{n-1}(x)f(x)\,dx. \tag{42}$$

[21] Theorem 3 showed that an all-equity auction always fails to be efficient. If the debt-only auction also fails to be efficient, then adding an equity choice does not make it efficient. Thus, the only equity and debt combination auctions that are invertible are those that were efficient when just debt was allowed.

Substituting for the debt bid $b(x)$ as a function of α, the seller's problem becomes

$$\max_{\alpha} \int_{\underline{t}}^{\bar{t}} \left[\int_{\underline{v}}^{\bar{v}} vg(v|x)\, dv F^{n-1}(x) - \int_{\underline{t}}^{x} \int_{b(y)}^{\bar{v}} [v - b(y)](1 - \alpha(y)) \frac{d}{dy} g(v|y)\, dv F^{n-1}(y)\, dy \right] f(x)\, dx. \tag{43}$$

Thus, the seller wants to minimize

$$\int_{b(y)}^{\bar{v}} [v - b(y)](1 - \alpha(y)) \frac{d}{dy} g(v|y)\, dv F^{n-1}(y)\, dy \tag{44}$$

for each bidder, subject to incentive compatibility and efficiency (the highest type wins). Equation (44) is minimized by setting α arbitrarily close to one. In this case the debt bid function becomes

$$b(t) = \frac{\int_{b(t)}^{\bar{v}} vg(v|t)\, dv}{\int_{b(t)}^{\bar{v}} g(v|t)\, dv} - \frac{\int_{\underline{t}}^{t} \int_{b(x)}^{\bar{v}} [v - b(x)] \frac{d}{dx} [g(v|x)]\, dv\, F^{n-1}(x)\, dx}{\int_{b(t)}^{\bar{v}} g(v|t)\, dv\, F^{n-1}(t)}, \tag{45}$$

which is the debt bid function when equity is not allowed! This results in almost full extraction by the seller. Crémer (1987) and Samuelson (1987) both show how the seller can almost fully extract the highest bidder's surplus. The debt-equity example and Corollary 1 show that almost full extraction can be obtained in any contingent auction by increasing the contingency. The seller must choose a rule that sets $P_2(x, t_i, t_{-i})$, the change in the bidders' expected payment with respect to their true type, as close as possible to $\int_{\underline{t}}^{t} \{\int_{\underline{v}}^{\bar{v}} v(d/dy) g(v|y)\, dv\}\, F^{n-1}(y)\, dy$, the change in their expected value with respect to a change in their true type. The equity-only auction fails because in equilibrium the change in the bidders' expected payment *equals* the change in their expected value, so better types pay the entire increase in revenue generated by their better skills. The addition of debt reduces the contingency so better types do not pay their full value. Thus bidders separate, and the seller can extract almost everything from the best type.

An interesting result here is that the addition of debt eliminates the knife-edge nature of full extraction. Samuelson (1987) argues that with nearly 100 percent equity bids, bidder "reservation prices are nearly identical (and nearly zero), the bids, though they follow the same ordering as values, must be nearly identical also. But this means that any added factor (even pure white noise) can . . . lead to inefficient selection in the auction" (742). This is not the case in the debt-equity auction, because changing the equity fraction required in the bids does not change the debt bids at all. Thus, the debt bids are just as far apart as in an auction without equity.

Therefore, if the distribution of possible values is such that the equilibrium in a debt-only auction is separating and incentive compatible, then the almost full extraction auction equilibrium is also. The bidders are now bidding for almost none of the bankrupt firm. In a cash auction this would cause the cash bids to decrease drastically. However, debt bids are only paid if the firm does well. Thus, the reduction in equity does not reduce the debt bids. Therefore, revenue extraction may be better accomplished with debt and equity, rather than with cash and equity.

Thus far, we have assumed that the capital structure does not affect the value of the firm ex post. However, it may be the case that the capital structure alters the distribution of the ex post payoffs: $g(v|t,b)$ (where b is the total amount of debt). The bidder's problem then becomes

$$\operatorname*{Max}_{x} \int_{b(x)}^{\bar{v}} [v - b(x)](1 - \alpha(x)) g(v|t, b(x))\, dv\, F^{n-1}(x). \tag{46}$$

It is immediately clear that in general this problem yields no closed form solution. However, it is unlikely that to win the auction, the high type must bid the optimal capital structure, particularly because the optimal capital structure will depend on the type of the winner. Even if the seller can require each type of bidder to bid a particular, optimal amount of debt (and this happens to correspond to an equilibrium), the seller may increase revenue with another equilibrium. Implementing the revenue-maximizing equilibrium then allows ex post gains from trade. However, as Section IV discussed, separability may be lost, and ex ante the seller cannot gain from holding an auction when the bids will be ex post renegotiated. Thus, the non-cash auction may allow increased revenue extraction, but it introduces inefficiencies into the ex post capital structure.

If the signaling in the auction or in Chapter 11 (as discussed in the introduction) leaves the firm with an out-of-equilibrium capital structure, then we would expect firms to renegotiate. Hotchkiss (1995) and Gilson (1997) both find that over 25 percent of firms that leave Chapter 11 reenter Chapter 11 or privately restructure.

VII. Disutility of Effort

In the last section the seller was able to use a combination of debt and equity that would ensure separation and almost full extraction. This was true because the bidders were just as interested in bidding for a small portion of the firm as they were in a large portion of the firm. Presumably, there are control issues, as in Cornelli and Felli (1996), or the bidders are going to have to work to make the bankrupt firm profitable.[22] If effort reduces the bidder's utility, then the seller must be more careful in determining the optimal fraction of the firm for sale.

After a bidder wins the auction, he chooses an effort level, e. This level determines the expected value of the firm. For simplicity we assume the payoff after the winner chooses an effort level is $\tilde{v} + e$. This level of effort, however, reduces the utility of the winner by $\Psi(e)$, where $\Psi'(e) > 0$ and $\Psi''(e) > 0$. More effort is more costly, and the cost of working harder is increasing with effort. Thus, the bidder's problem is

$$\max_{x,e} \int_{b(x)-e}^{\bar{v}} [[v + e - b(x)](1 - \alpha(x))g(v|t)\,dv - \Psi(e)]F^{n-1}(x), \tag{47}$$

assuming that the equilibrium debt bid is above the effort choice.

Because the bidder cannot commit to a choice of effort, the effort decision is made after the winner is determined. Therefore,

$$\int_{b(x)-e}^{\bar{v}} g(v|t)\,dv(1 - \alpha(x)) = \Psi'(e). \tag{48}$$

From the bidder's point of view, the effort decision is a function of both t and x. Because the bidder understands how he will choose e after he wins, the bidder's problem becomes

$$\max_{x} \int_{b(x)-e(x,t)}^{\bar{v}} [[v + e(x,t) - b(x)](1 - \alpha(x))g(v|t)\,dv - \Psi(e(x,t))]F^{n-1}(x). \tag{49}$$

The appendix solves for the debt bid as a function of the equity choice:

$$b(t) = \frac{\int_{b(t)-e(t,t)}^{\bar{v}} vg(v|t)\,dv}{\int_{b(t)-e(t,t)}^{\bar{v}} g(v|t)\,dv} + e(t,t) - \frac{\Psi(e(t,t))}{\int_{b(t)-e(t,t)}^{\bar{v}} g(v|t)\,dv\,(1 - \alpha(t))}$$
$$- \frac{\int_{\underline{t}}^{t} \int_{b(x)-e(x,x)}^{\bar{v}} [v - b(x)] \frac{d}{dx}[g(v|x)]\,dv\,(1 - \alpha(x))F^{n-1}(x)\,dx}{\int_{b(t)-e(t,t)}^{\bar{v}} g(v|t)\,dv\,(1 - \alpha(t))F^{n-1}(t)}. \tag{50}$$

[22] Cornelli and Felli (1996) suggest that when selling a firm the current owners can maximize revenue by selling only a controlling stake, or 51 percent.

Note that it is still an incentive-compatible, invertible equilibrium for the seller to set α arbitrarily close to one. In this case every player chooses an effort level close to zero, and the equilibrium is the same as above. This equilibrium is full extraction with *no* effort. However, it may be possible for the seller to improve his expected revenue by decreasing α. This will increase the effort of the resulting winner and improve the firm's expected value. The possibility of a win-win result emerges. The seller's problem is

$$\max_{b,\alpha} \int_{\underline{t}}^{\bar{t}} \left[\int_{b(x)-e(x,x)}^{\bar{v}} [(v + e(x,x) - b(x))\alpha(x) + b(x)] g(v|x)\, dv + \int_{\underline{v}}^{b(x)-e(x,x)} v g(v|x)\, dv \right] F^{n-1}(x) f(x)\, dx. \tag{51}$$

Substituting for the bid function, equation (50) (which is a function of α), the seller's problem reduces to

$$\max_{\alpha} \int_{\underline{t}}^{\bar{t}} \left[\int_{\underline{v}}^{\bar{v}} (v + e(x,x)) g(v|x)\, dv - \Psi(e(x,x)) F^{n-1}(x) - \int_{\underline{t}}^{x} \int_{b(y)-e(y,y)}^{\bar{v}} [v + e(y,y) - b(y)](1 - \alpha(y)) \frac{d}{dy} g(v|y)\, dv F^{n-1}(y)\, dy \right] \times f(x)\, dx. \tag{52}$$

The seller now has conflicting desires. The seller would like to maximize the sum of the first two terms inside the integral. This would occur if the bidders chose the socially efficient level of effort.[23]

However, the winner would only choose this level of effort if he were going to receive all of the payoff from the firm. In this case the seller would make nothing. On the other hand, the seller would like to minimize the third term. This occurs when the seller raises the required level of α (or when he causes an increase in b). However, higher α and higher b both reduce the effort chosen by the winner. Thus, the general trade-off is clear: the smaller the fraction of the expected surplus the seller takes, the more the winner works, causing the entire pie to increase. If the seller is better off with a smaller fraction of a larger pie, then the seller increases the fraction of the firm for sale. The decision to raise the fraction of the firm for sale depends entirely on the disutility of effort. If the disutility is very small and increases very slowly, then there is no question that the seller will lower α and encourage a great deal of effort. As the disutility function gets higher and higher, the

[23] $$\operatorname*{Max}_{\varepsilon} \left[\int_{\underline{v}}^{\bar{v}} (v + \varepsilon) g(v|x)\, dv - \Psi(\varepsilon) \right] F^{n-1}(x).$$

This equation yields the socially optimal level of effort.

benefit from encouraging the winner to work reduces. This result is similar to that of McAfee and McMillan (1986), who show that in auctions for contracts the fraction of cost overruns the government should pay has a moral-hazard effect, a risk-sharing effect, and a bidding-competition effect. Raising the fraction covered lowers the government's expected payment but decreases cost-reducing effort. The choice of equity, α, in our model has a similar trade-off without the risk-sharing effect.

An all-cash auction would result in the optimal amount of effort. However, this would not be optimal for the seller. Similar to Samuelson (1987) we find that the first-order effect on effort of a decrease in the fraction of the firm for sale from 1 to $1-\epsilon$ is zero. The first-order effect on revenues is positive. Therefore, any seller should hold a cash auction for strictly less than 100 percent of the firm (this is an equity and cash auction with a fixed α). However, what happens if the other option is not cash but debt? If debt and equity are the only alternatives then either choice will distort the effort choice, whereas in a cash and equity auction, moving away from equity to cash unambiguously increases effort. Furthermore, debt and equity will have different effects on different types. A \$10 million debt bid drastically reduces the effort of a bidder who rarely expects to make more than \$10 million but will have little effect on the effort of a bidder who expects to earn \$90 million. Thus, debt may seem to affect lower types more than higher types. However, no general statement can be made.

If we restrict the definition of a high type to be first-order stochastic dominance (FOSD), $G(v|t_{High}) < G(v|t_{Low})\ \forall\, v$, then we can say that for a given bid, a higher type works harder. To see this, consider the bidder's effort choice function, equation (48). Under FOSD,

$$\int_{b-e}^{\bar{v}} g(v|t_{High})\, dv(1-\alpha) > \int_{b-e}^{\bar{v}} g(v|t_{Low})\, dv(1-\alpha). \tag{53}$$

This equation with $\Psi''(e) > 0$ implies that for a given b and α the higher type works harder. Also, if b increases then effort will decrease. Whether this has a greater effect on the effort of the low type or the high type depends on the current b. Under FOSD, as b increases from e, a small increase in the debt bid decreases the low type's effort by more than the high type.[24] This is because the condition

$$-g(b-e|t_{High}) > -g(b-e|t_{Low}) \tag{54}$$

is implied by $G(b-e|t_{High}) < G(b-e|t_{Low})$ for low $b-e$. However, at high enough debt levels the opposite will hold.

[24] If $b = e$ and α is the same for both types, then both types work the same level: $1-\alpha = \Psi'(e)$.

An increase in α will also decrease effort. With FOSD an increase in α will always decrease the effort of the high type by more than the low type because

$$-\int_{b-e}^{\bar{v}} g(v|t_{High})\,dv < -\int_{b-e}^{\bar{v}} g(v|t_{Low})\,dv. \tag{55}$$

Setting a fixed α is easy to implement because the seller just states that only that portion of the firm is for sale. There is nothing, however, that prevents the seller from having the following type of belief: the bidder who bids b_t and α_t is bidder t. Under FOSD, as the seller maximizes the inside of equation (52) for each type, he will find it optimal to require higher types to bid more debt but less equity.[25]

However, there may be an incentive-compatible and individually rational set of beliefs that results in greater (or lower) effort by the higher types. Interestingly, global and local incentive compatibility is helped by an $\alpha(t)$ that is a decreasing function of t. High types do not want to imitate low types with high equity fractions because the high bidders might pay more by imitating a low type than they would with their own equilibrium bid.

McAfee and McMillan (1986) and Laffont and Tirole (1987) consider the choice of equity by the auctioneer in a procurement auction, when bidders bid the amount of cash the procurer should pay them. The trade-off between two types of payments that both reduce effort is not present because cash does not affect effort. McAfee and McMillan (1986) do not allow the seller to choose a different fraction of equity for each player. However, Laffont and Tirole (1987) find a result similar to ours, namely that "the slope of the incentive scheme decreases with the winner's bid" (932). In our setup this maps to higher types bidding lower equity.

In considering the effort level of the buyer, it is clear that a non-cash auction will not only separate the good managers from the bad but will also determine the management wage contract. This results in the seller and buyer using an auction and securities bids to design a contract. The choice of possible securities is endless: convertibles, options, cash (possibly from the seller to the buyer), debt, preferred stock, and so on. Allowing the bidders to bid with securities is akin to requiring the old debtholders of the bankrupt firm to hire a new management team rather than sell the firm for cash. There is an extensive literature on the design of incentive contracts, and McAfee and McMillan (1986,1987), Laffont and Tirole (1987), and Samuelson (1986) have considered the effects of auctioning contracts. Laffont and Tirole (1987) demonstrate that the optimal incentive contract is linear. Because this cannot even be achieved in an auction without cash, the non-cash auction will not result in the optimal contract.

[25] We know from above that separation requires higher types to bid more debt. Because these debt bids will distort the effort decision, it is possibly optimal to set $\alpha = 0$ rather than decreasing with type.

Thus, the same ex post renegotiation that caused the unraveling in the debt auction (as discussed above) will exist here. The auction's failure to design a contract that achieves even the second best effort may result in an ex post renegotiation that eliminates the ex ante invertibility. Firms cannot contract not to renegotiate, because both the bidder and the seller will want to renegotiate once the bidder's type is known. The seller is essentially hiring a management team, but without cash he will be restricted from offering the optimal contract. There may be a contract desired by the seller, for example, the buyer gets 20 percent of the equity. If during the bidding process the buyer ends up with less than 20 percent of the equity, then the seller may want to renegotiate and *give some of the bid back*. Thus, both the seller and buyer would find it in their self-interest to throw out any no-negotiation commitment. However, because all of the bidders would ex ante expect the negotiation, it would eliminate invertibility just as surely as if the seller tried to use the ex post information to extract more from the bidder.

Although the non-cash auction may improve revenue and may determine the best management team, it will not result in the optimal incentive contract. The designers and users of the auction must trade off distortions. Once again, we find that because cash is a necessary part of the optimal incentive contract, it is a necessary part of a reorganization auction.

VIII. Convertibles

Convertibles often play a part in reorganization plans. This section finds the equilibrium when bids are in the form of convertible debt and demonstrates why sellers prefer convertible bids to straight debt bids. The benefit of convertibles is greater revenue for the seller and the ability of the auction designer to affect deadweight losses, effort decisions, or control. This section also compares convertibles with debt that is ex post renegotiated and exchanged for equity. Because convertible debt is just regular debt with ex ante rather than ex post conversion negotiations, what is the benefit of ex ante negotiations? The answer is that ex ante negotiations occur with competition from other bidders, whereas ex post the buyer will renegotiate only if his payoff is improved. Thus, convertible auctions can be designed to improve seller revenue or total welfare or both.

The bidder's problem in a convertible auction with debt component $b(t)$ that can be converted to $\alpha(t)$ shares is

$$\max_x \left[\int_{b(x)/\alpha(x)}^{\bar{v}} v(1-\alpha(x))g(v|t)\,dv + \int_{b(x)}^{b(x)/\alpha(x)} [v - b(x)]g(v|t)\,dv \right]. \tag{56}$$

Using techniques identical to those used in the above sections, the debt bid as a function of α and the conversion ratio as a function of the debt bid can be determined (see the appendix). Note that because there are multiple equilibria, an exact solution to the bid cannot be found. This is the indeterminacy referred to early in the model.

The benefit of convertible debt over regular debt can be demonstrated with Theorem 1. As α approaches 1, $P_2(t,t)$ approaches 1; thus $\int_{\underline{t}}^{\bar{t}}\int_{\underline{t}}^{y} P_2(x,x)\,dxdy$ is maximized and the expected payment is maximized. The seller can accomplish almost full extraction by requiring that the debt convert to almost all of the equity.

Now consider the effect of allowing the seller to negotiate an exchange of debt for equity after a debt-only auction. In reality, regular debtholders can always attempt to renegotiate.[26]

Thus, the difference between convertible and regular debt is the timing of the negotiation of the conversion ratio. As shown above, the ex ante negotiation implicit in the convertibles allows the seller almost full extraction. However, ex post negotiation has no effect. This is because there are no gains from trade. Once the firm value is realized, the size of the pie is fixed.

However, we might think of conversion as occurring between the auction and the final realization. If we add a period between the auction and the firm realization, during which time the seller negotiates an exchange, it is easy to show that the negotiation stage does not affect the total expected payment of the bidders (assuming that the seller/debtholder *exogenously* learns the type of the buyer). This is true even though the seller does expect greater revenue. The revenue increase results because the competition in the auction will cause the benefit of eliminating the costs of bankruptcy to accrue to the seller. However, bidders cannot be induced during negotiations to pay in equity more than their current expected debt payment plus any costs of bankruptcy.[27]

Thus, it is clear that ex post negotiation changes nothing and middle period conversion negotiations eliminate deadweight loss and result in greater seller expected revenue, but no greater total payments by the bidders. However, ex ante negotiations via convertibles remove the deadweight loss of debt and allow full extraction.

However, in the above sections on debt and equity we saw that the goal of the seller may not be full extraction due to control issues or effort decisions. The seller (government) may also care about the total dead-weight loss. The use of debt and equity or debt and the conversion ratio provides the seller with the ability to affect the amount and the kind of bid. Because either debt and equity or convertibles can be used for full extraction, the only difference between them is the shape of the payoff function. Thus, the superiority of one over the other depends on the particulars of the auction.[28]

[26] We showed earlier that renegotiation using information from the auction destroys the efficiency in the auction. Thus, the assumption here is that the seller/debtholder exogenously learns the type of the buyer or observes the firm realization.

[27] This is a result of Theorem 1 because $P_2(t,t)$ does not change with the addition of negotiation.

[28] Interested readers may request a working paper version that includes a few examples that enumerate the results of the paper. These examples show some of the power of the controls of the auction designer. The designer can extract more or less from everyone or less from low types and more from high types, or vice versa, and thereby alter the makeup of the bids. Understanding and using these controls is a powerful auction design tool.

In general, the convertible auction is similar to the debt and equity auction. In either auction the debt must signal the type of the bidder, and the equity (either direct or via conversion) raises the expected payment. As discussed in the introduction, even Chapter 11 has features like a non-cash auction, because management teams submit reorganization plans in an attempt to convince the judge they are the superior team. Therefore, because the higher types in a reorganization process want to signal their private information, we would expect to see firms leaving Chapter 11 with high debt or convertible debt levels. Gilson (1997) finds that firms leave Chapter 11 with debt ratios that are high for their industry and sometimes higher than what they went in with. He correlates high debt with high reorganization transactions costs. We suggest that asymmetric information may be a part of the transaction costs. Thus, the signaling power of debt may account for this result.

IX. Implications and a Potential Solution

The above sections examined non-cash auctions that involved debt, equity, and convertibles. It would be interesting to consider the effects of using a myriad of other securities. However, the fundamental problems with the non-cash auction can already be seen: it is attempting to do too much. If the non-cash auction separates, which in many circumstances it does not, then it determines the best manager. However, at the same time, the auction is mandating the incentive contract that will govern the firm's relationship with the manager. Because the incentive contract will not be first best or even second best, both parties may want to renegotiate. In so doing they eliminate the separability in the auction which was predicated on commitment. If the courts can provide a commitment not to renegotiate, then they are bound away from even the second best incentive contract.

The winning non-cash bid determines not only the incentive contract but also the capital structure of the reorganized firm. This creates another tradeoff. For example, it might be so important to the firm to have a particular capital structure that, in a non-cash auction, the owner simply picks the bid closest to this optimum, essentially choosing a manager at random. Even away from this extreme, it is unlikely that the non-cash auction chooses the best manager and an optimal capital structure.

Our analysis indicates that non-cash auctions are complicated relative to cash auctions. Dybvig (1994) echoes this in his discussion of the AHM proposal when he states that the "comparison of various bids, and especially combinations of partial bids, does not seem like an easy matter. When non-cash or partial bids are allowed, their assigned valuation will affect the number of shares that each existing claimant will get" (877). Hence we suggest the following modification of the AHM proposal, which keeps the spirit of their idea but simplifies it to some extent.

Much of the AHM proposal is predicated on the sale of the firm. We suggest instead consideration of the sale of only the managerial contract, which is just a set of derivatives of the firm's cash flow. The claimants (who were allocated their rights as in AHM's proposal using Bebchuk's scheme) vote both for the capital structure and the managerial incentive scheme they desire (this may involve trade-offs). Cash bids are then accepted for the incentive contract. This reduces the cash requirements required by a full sale but maintains the incentive-compatible separability of cash. Consequently, we do not have to consider the possible misallocation of assets induced by securities bids. We do not prohibit a direct sale of the firm, because there may be operating strategies that require substantial cash infusion. Hence, our suggestion involves a mixture of a standard cash auction and a simultaneous cash auction of the incentive contract (a derivative claim). The claimants then vote on the bids. Each claim turns into X percent of the debt and X percent of the equity and/or X percent of the cash. The debt and equity could then be traded.

This solution addresses AHM's problem because it allows bidders with relatively low cash to enter: a bid for a management contract would be far less than for the firm as a whole. And it separates the managers, insuring that the best manager wins. It also allows for an optimal (second best) managerial contract and the creation of the firm's optimal capital structure.

X. Conclusion

In this paper, we study the strengths and weaknesses of the proposal to use non-cash auctions to sell a bankrupt firm in a corporate reorganization or to privatize a public sector entity. In the process, we survey the small body of literature on contingent auctions. Our results demonstrate that a non-cash auction is revenue superior because the use of securities introduces a valuation problem and hence the securities that are bid must correctly "signal" the expected value of the firm. If the choice of securities makes it easier for a low type to imitate a high type, then bidders must bid higher to signal their type. This is true unless the choice of security allows the lowest type to imitate the highest type, eliminating invertible bids and efficient asset allocation. Bids in an all-equity auction are never invertible; thus cash or debt is a necessary component of any equity bid. With fixed supports bounding the possible firm outcomes and with no nonpecuniary bankruptcy costs, the bids in all securities auctions are not invertible. In contrast, with moving supports or nonpecuniary costs, auctions involving debt or convertible securities do separate. However, separation in the auction does not ensure that the assets will continue to be used efficiently; the seller may repossess the assets if the debt cannot be serviced. If the bankruptcy costs are small relative to the bidder's highest possible payoff, the equilibrium will result in very high bids and thus virtual certainty of bankruptcy. This explains the large number of bankruptcies following the FCC C Block bandwidth auctions, in which small bidders were allowed to use debt. Thus, although separation in a non-cash auction requires the use of securities like debt, convertible debt, preferred stock or cash, the seller must consider more than just separation.

Securities bids also distort the effort choices made by the winning bidders. Furthermore, the signaling in the securities bidding may leave the firm with a less than optimal capital structure. Such distortions imply that there exist ex post incentives to renegotiate the capital structure or managerial contract. Hotchkiss (1995) and Gilson (1997) both find that over 25 percent of firms that leave Chapter 11 reenter Chapter 11 or privately restructure. Hotchkiss (1995) suggests that these future restructurings are evidence of a bias toward the continuation of unprofitable firms. We show that even economically viable firms may want to restructure after leaving Chapter 11 to remove inefficiencies in the capital structure and the incentive contract.

Overall, our analysis suggests that non-cash auctions do involve some loss of efficiency, and hence the idea that securities bidding should be used in corporate organizations is not as attractive as it might seem. As an alternative to the non-cash auction we suggest that after the optimal capital structure and incentive contract are designed, management teams should bid in cash for the given incentive contract. This increases the competition for the bankrupt firm, reduces the financing problem, finds the optimal manager, and does not distort the capital structure.

Our analysis of non-cash auctions has an important positive implication. It provides an explanation for the use of securities other than equity in reorganizations and hence explains the use of securities such as debt and convertibles. More importantly, it provides an answer to the puzzle posed in Gilson (1997) as to why bankrupt firms have so much debt even after a reorganization and why many of those firms later restructure.

Appendix

A. Global Incentive Compatibility

We can rewrite the bidder's general problem, equation (7), more succinctly as

$$t \epsilon \arg\max \Pi(r(x,\tilde{v}),x,t). \tag{A1}$$

This yields the first-order condition

$$\Pi_1(r(x,\tilde{v}),x,t)r'(t) + \Pi_2(r(x,\tilde{v}),x,t) = 0 \tag{A2}$$

and the envelope condition

$$\frac{d\Pi(r(t,\tilde{v}),t,t)}{dt} = \Pi_3(r(t,\tilde{v}),t,t). \tag{A3}$$

For the first-order condition to be sufficient, the inequality

$$\Pi(r(t,\tilde{v}),t,t) \geqq \Pi(r(x,\tilde{v}),x,t)\ \forall x \tag{A4}$$

must be satisfied for all v. We can use the envelope condition to rewrite this inequality:

$$t > x \Rightarrow \int_x^t \Pi_3(r(s,v),s,s)\,ds > \int_x^t \Pi_3(r(x,v),x,s)\,ds. \tag{A5}$$

If $x = v$ then the inequality is an equality. Therefore, a related necessary condition is

$$\Pi_{31}(r(t,v),t,t)r_1(t,v) + \Pi_{32}(r(t,v),t,t) \geqq 0. \tag{A6}$$

Therefore, a sufficient condition that implies equation (A4) is

$$t > x \Rightarrow \Pi_3(r(t,v),t,s) > \Pi_3(r(x,v),x,s)\ \forall s. \tag{A7}$$

The derivative of the bidder's problem, equation (7), with respect to s is

$$\int_{\underline{v}}^{\bar{v}} (v - r(t,v))\,\frac{d}{ds}\,g(v|s)\,dv\,F^{n-1}(t). \tag{A8}$$

Hence, for all $t > x$ we need

$$\int_{\underline{v}}^{\bar{v}} (v - r(t,v))\,\frac{d}{ds}\,g(v|s)\,dv\,F^{n-1}(t) > \int_{\underline{v}}^{\bar{v}} (v - r(x,v))\,\frac{d}{ds}\,g(v|s)\,dv\,F^{n-1}(x). \tag{A9}$$

We know that $F^{n-1}(t) > F^{n-1}(x)$. Thus,

$$\int_{\underline{v}}^{\bar{v}} r(t,v)\,\frac{d}{ds}\,g(v|s)\,dv < \int_{\underline{v}}^{\bar{v}} r(x,v)\,\frac{d}{ds}\,g(v|s)\,dv \tag{A10}$$

is sufficient but not necessary for global incentive compatibility. Or, for $s' > s$

$$\begin{aligned}&\int_{\underline{v}}^{\bar{v}} r(t,v)g(v|s')\,dv - \int_{\underline{v}}^{\bar{v}} r(t,v)g(v|s)\,dv \\ &\quad < \int_{\underline{v}}^{\bar{v}} r(x,v)g(v|s')\,dv - \int_{\underline{v}}^{\bar{v}} r(x,v)g(v|s)\,dv.\end{aligned} \tag{A11}$$

Rewriting, we find the Spence-type sufficiency condition

$$\int_{\underline{v}}^{\bar{v}} (r(t,v) - r(x,v))g(v|s')\,dv < \int_{\underline{v}}^{\bar{v}} (r(t,v) - r(x,v))g(v|s)\,dv. \tag{A12}$$

It is interesting to note that this condition is violated by any debt contract and most other contingent contracts, though we see throughout the paper that global incentive compatibility may still hold. Therefore, global IC cannot be shown in general and will need to be checked in any particular example.

B. Proof of Theorem 2

The lowest type cannot participate in a separating equilibrium if he would expect to earn positive profits if he won. Therefore, the lowest types $\underline{t}$ to $t + \phi$ must pool or mix (ϕ may cover the entire range of t). Consider first their option to pool. For any group that pools, a small increase in the bid provides a large increase in the probability of winning, because they then beat all of the members of the pool. Thus, the pool is not viable. This effect does not occur in two cases: (1) the pool is at the largest possible bid, or (2) the seller believes that a bid just above the pool belongs to the worst type and will yield lower revenue than a bid from within the pool. This is not subgame perfect because the bidder who has the most to gain from bidding above the pool is the highest type in the pool. Thus, the only pooling equilibrium is the equilibrium in which all bidders bid the highest bid.

Any bidder who mixes over a range of bids must be indifferent to choosing any of the bids. Consider the lowest bid, $\underline{b}$, in any of the bidders' ranges. This bid may be part of a continuous set, $[\underline{b}, \beta]$ or it may be the lowest in a discrete set. First consider a continuous choice set. Any bidder who bids over the range that includes the lowest bid has the following problem: if he bids the low bid he has a zero probability of winning the auction. Because any higher bid gives him a positive probability of winning and a positive expected payoff, he will not be indifferent between them. The only possibility left is a discrete choice for the lowest possible bid. If bids are discrete, a bidder who bids the lowest bid still has a positive probability of winning. However, this now raises the same problem as the pooling equilibrium: instead of mixing over a choice set that involves the lowest possible discrete bid, he can choose a bid ϵ larger and increase his probability of winning while raising his expected payment by an insignificant amount. The only way a bidder will not choose to do this is if the seller believes that this higher bid will provide less revenue (subgame imperfect).

C. Debt and Equity Bid

Taking the derivative of equation (38) and rearranging the first-order condition yields

$$\int_{b(x)}^{\bar{v}} vg(v|t)\,dv\,\frac{d}{dx}\left[(1-\alpha(x))F^{n-1}(x)\right]$$
$$= \int_{b(x)}^{\bar{v}} g(v|t)\,dv\,\frac{d}{dx}\left[b(x)(1-\alpha(x))F^{n-1}(x)\right]. \tag{A13}$$

Integrating both sides from $\underline{v}$ to v we find

$$\int_{b(t)}^{\bar{v}} vg(v|t)\,dv\,(1-\alpha(t))F^{n-1}(t) - \int_{\underline{t}}^{t}\int_{b(x)}^{\bar{v}} v\,\frac{d}{dx}\,[g(v|x)]\,dv\,(1-\alpha(x))F^{n-1}(x)\,dx$$
$$= \int_{b(t)}^{\bar{v}} g(v|t)\,dv\,b(t)(1-\alpha(t))F^{n-1}(t) - \int_{\underline{t}}^{t}\int_{b(x)}^{\bar{v}} \frac{d}{dx}\,[g(v|x)]\,dv\,(1-\alpha(x))F^{n-1}(x)\,dx. \tag{A14}$$

D. Debt and Equity Bid with Effort

The first-order condition is

$$\int_{b(x)-e(x,t)}^{\bar{v}} [v+e(x,t)-b(x)]g(v|t)\,dv\,\frac{d}{dx}\,[(1-\alpha(x))F^{n-1}(x)] + \int_{b(x)-e(x,t)}^{\bar{v}} [e_1(x,t)-b'(x)]g(v|t)\,dv[(1-\alpha(x))F^{n-1}(x)] - \frac{d}{dx}\,[\Psi(e(x,t))F^{n-1}(x)] = 0. \tag{A15}$$

Integrating from $\underline{t}$ to t we find

$$\int_{b(t)-e(t,t)}^{\bar{v}} [v+e(t,t)-b(t)](1-\alpha(t))g(v|t)\,dvF^{n-1}(t) - \int_{\underline{t}}^{t}\int_{b(x)-e(x,x)}^{\bar{v}} [v+e(x,x)-b(x)]\,\frac{d}{dx}\,[g(v|x)]\,dv\,(1-\alpha(x))F^{n-1}(x)\,dx - \int_{\underline{t}}^{t}\int_{b(x)-e(x,x)}^{\bar{v}} \epsilon_2(x,x)g(v|x)\,dv\,(1-\alpha(x))F^{n-1}(x)\,dx$$
$$= \Psi(e(t,t))F^{n-1}(t) - \int_{\underline{t}}^{t} \Psi'(e(x,x))e_2(x,x)F^{n-1}(x). \tag{A16}$$

Substituting for $\Psi'(e(x,x))$ from equation (48), equation (A16) reduces to

$$\int_{b(t)-e(t,t)}^{\bar{v}} [v+e(t,t)-b(t)](1-\alpha(t))g(v|t)\,dvF^{n-1}(t) - \int_{\underline{t}}^{t}\int_{b(x)-e(x,x)}^{\bar{v}} [v+e(x,x)-b(x)]\,\frac{d}{dx}\,[g(v|x)]\,dv\,(1-\alpha(x))F^{n-1}(x)\,dx$$
$$= \Psi(e(t,t))F^{n-1}(t). \tag{A17}$$

E. Convertible Bids

Taking the derivative of the bidder's problem in the convertible auction, equation (56), with respect to x and integrating back up we find

$$\begin{aligned}&\int_{b(t)/\alpha(t)}^{\bar{v}} v(1-\alpha(t))g(v|t)\,dvF^{n-1}(t)\\&\quad-\int_{\underline{t}}^{t}\int_{b(x)/\alpha(x)}^{\bar{v}} v(1-\alpha(x))\frac{d}{dx}[g(v|x)]\,dvF^{n-1}(x)\,dx\\&\quad+\int_{b(t)}^{b(t)/\alpha(t)}[v-b(t)]g(v|t)\,dvF^{n-1}(t)\\&\quad-\int_{\underline{t}}^{t}\int_{b(x)}^{b(x)/\alpha(x)}[v-b(x)]\frac{d}{dx}[g(v|x)]\,dvF^{n-1}(x)\,dx\\&\quad-\int_{\underline{v}}^{b(t)} cg(v|t)\,dvF^{n-1}(t)+\int_{\underline{t}}^{t}\int_{\underline{v}}^{b(x)} c\frac{d}{dx}[g(v|x)]\,dvF^{n-1}(x)\,dx=0.\end{aligned} \tag{A18}$$

Solving for the optimal debt bid as a function of the conversion ratio, we get

$$\begin{aligned}b(t)=&\frac{\int_{b(t)/\alpha(t)}^{\bar{v}} v(1-\alpha(t))g(v|t)\,dv}{\int_{b(t)}^{b(t)/\alpha(t)} g(v|t)\,dv}\\&-\frac{\int_{\underline{t}}^{t}\int_{b(x)/\alpha(x)}^{\bar{v}} v(1-\alpha(x))\frac{d}{dx}[g(v|x)]\,dvF^{n-1}(x)\,dx}{\int_{b(t)}^{b(t)/\alpha(t)} g(v|t)\,dvF^{n-1}(t)}\\&+\frac{\int_{b(t)}^{b(t)/\alpha(t)} vg(v|t)\,dv}{\int_{b(t)}^{b(t)/\alpha(t)} g(v|t)\,dv}-\frac{\int_{\underline{t}}^{t}\int_{b(x)}^{b(x)/\alpha(x)}[v-b(x)]\frac{d}{dx}[g(v|x)]\,dvF^{n-1}(x)\,dx}{\int_{b(t)}^{b(t)/\alpha(t)} g(v|t)\,dvF^{n-1}(t)}\\&-\frac{\int_{\underline{v}}^{b(t)} cg(v|t)\,dv}{\int_{b(t)}^{b(t)/\alpha(t)} g(v|t)\,dv}+\frac{\int_{\underline{t}}^{t}\int_{\underline{v}}^{b(x)} c\frac{d}{dx}[g(v|x)]\,dvF^{n-1}(x)\,dx}{\int_{b(t)}^{b(t)/\alpha(t)} g(v|t)\,dvF^{n-1}(t)}.\end{aligned} \tag{A19}$$

Once again, the lowest type $\underline{t}$ has an expected debt payment equal to his expected value from winning the auction.[29] Thus he expects to make nothing.

The simultaneous conversion ratio choice of the bidder is

$$\alpha(t) = 1 - \frac{\int_{\underline{t}}^{t}\int_{b(x)/\alpha(x)}^{\bar{v}} v(1-\alpha(x))\,\frac{d}{dx}\,[g(v|x)]\,dv F^{n-1}(x)\,dx}{\int_{b(t)/\alpha(t)}^{\bar{v}} vg(v|t)\,dv F^{n-1}(t)}$$
$$+ \frac{\int_{b(t)}^{b(t)/\alpha(t)} [v-b(t)]g(v|t)\,dv}{\int_{b(t)/\alpha(t)}^{\bar{v}} vg(v|t)\,dv}$$
$$- \frac{\int_{\underline{t}}^{t}\int_{b(x)}^{b(x)/\alpha(x)} [v-b(x)]\,\frac{d}{dx}\,[g(v|x)]\,dv F^{n-1}(x)\,dx}{\int_{b(t)/\alpha(t)}^{\bar{v}} vg(v|t)\,dv F^{n-1}(t)}$$
$$- \frac{\int_{\underline{v}}^{b(t)} cg(v|t)\,dv}{\int_{b(t)/\alpha(t)}^{\bar{v}} vg(v|t)\,dv} + \frac{\int_{\underline{t}}^{t}\int_{\underline{v}}^{b(x)} c\,\frac{d}{dx}\,[g(v|x)]\,dv F^{n-1}(x)\,dx}{\int_{b(t)/\alpha(t)}^{\bar{v}} vg(v|t)\,dv F^{n-1}(t)}. \tag{A20}$$

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[29] Expected debt payment of the lowest type is

$$b(\underline{t})\int_{b(\underline{t})}^{b(\underline{t})/\alpha(\underline{t})} g(v|\underline{t})\,dv = \int_{b(\underline{t})/\alpha(\underline{t})}^{\bar{v}} v(1-\alpha(\underline{t}))g(v|\underline{t})\,dv + \int_{b(\underline{t})}^{b(\underline{t})/\alpha(\underline{t})} vg(v|\underline{t})\,dv - \int_{\underline{v}}^{b(\underline{t})} cg(v|\underline{t})\,dv.$$

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