Moral Hazard and Debt Maturity

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Abstract

We present a model of the maturity of a bank's uninsured debt. The bank borrows funds and chooses afterwards the riskiness of its assets. This moral hazard problem leads to an excessive level of risk. Short-term debt may have a disciplining effect on the bank's risk-shifting incentives, but it may lead to inefficient liquidation. We characterize the conditions under which short-term and long-term debt are feasible, and show circumstances under which only short-term debt is feasible and under which short-term debt dominates long-term debt when both are feasible. Thus, short-term debt may have the salutary effect of mitigating the moral hazard problem and inducing lower risk-taking. The results are consistent with key features of the common narrative of the period preceding the 2007-2009 financial crisis.

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"It is difficult to establish any archetype for failure. Banks with high capital ratios imploded while those with lower ratios survived. Plain-vanilla retail banks blew up while some black-box trading shops prospered. Both small and big firms collapsed. Yet there was a common ingredient in most failures: an over-reliance on (short-term) wholesale borrowing." *The Economist*, September 5, 2009.

"Borrowers design their financial structures to their own benefit, and one cannot presuppose that dangerous forms of debt constitute suboptimal liability structures." *Jean Tirole*, 2003.

1   Introduction

Funding long-term investments with short-term debt risks failure to roll over the debt. Such failure can happen if adverse news about the investments' final payoff arrive at the rollover date, or if short-term lenders have better or more urgent uses for their funds. In that case the investments are liquidated, even when liquidation might be inefficient. *Why then fund long-term investments with short-term rather than long-term debt?*

Following Diamond and Dybvig (1983), a voluminous literature focuses on the lenders' demand for liquidity. This paper is different. In our model the lenders have no demand for liquidity, but they observe some relevant information on the prospects of the investment that may lead them to withdraw their funds. But if early liquidation is inefficient, the question about using short-term debt remains. Here is where moral hazard enters the picture. Suppose that the borrowers can choose the risk of their investments after the borrowing is done. In such situation, they will have an incentive to take excessive risks. We argue that *using short-term may be justified as a way to ameliorate the borrowers' risk-shifting incentives.*

In our model, the borrowing firm has three attributes of a bank. First, it funds itself mostly (in the model only) by issuing debt. Second, it can easily modify the risk profile of its investments. Third, it invests in financial assets, not real assets that can be redeployed to other sectors of the economy, which means that their liquidation value is related to (in the model a fraction of) their expected continuation value.

A comparison between short- and long-term debt entails the analysis of the optimal decision at the outset of the bank's shareholders. At that point they know that if short-term debt is used, they will have to refinance it. We argue that when there is a moral hazard problem in the choice of risk, the anticipation of the refinancing needs acts as a disciplining device that may render short-term debt superior to long-term debt. So the trade-off is between the disciplining benefits of short-term debt and the risk of inefficient liquidation.

The model has three dates: an initial date where the financing of the investment is arranged and its risk is privately chosen by the bank, an interim date where noisy public information about the eventual investment payoff is revealed, and depending on this information the bank may be liquidated, and a final date where investment returns are realized, if the bank was not liquidated before.

We start analyzing the simpler case of long-term debt finance, where the information revealed at the interim date is irrelevant. Next we examine short-term debt finance. In this case, if there is no liquidation at the interim date the bank has to repay the initial lenders by issuing new short-term debt, which matures at the final date, and if there is liquidation at the interim date the liquidation proceeds go to the initial lenders. Finally, we compare the bank's payoff in the optimal contract with long- and short-term debt, to derive the determinants of the optimal debt maturity structure.

The main results may be summarized as follows. First, we show that the positive incentive effects of short-term debt only obtain when it is risky, that is, when it implies a positive probability of early liquidation. Second, we show that there are circumstances in which short-term debt may be the only way to secure funding and in which short-term debt may dominate long-term debt when both are feasible. Third, we show that using short-term debt may involve paying an up-front dividend to the bank shareholders.

To explain the intuition for these results is it useful to refer to the seminal paper on credit rationing by Stiglitz and Weiss (1981). They present two models, one based on adverse selection and the other one on moral hazard. In the latter, they show how "higher interest rates induce firms to undertake projects with lower probabilities of success but higher payoffs when successful." We apply this argument to banks instead of firms.

In our model higher borrowing costs induce banks to undertake investments with lower probabilities of success but higher payoffs when successful. From this perspective, the difference in risk-shifting incentives between long- and short-term debt lies in the relevant cost of the bank's borrowing. With long-term debt the relevant cost reflects the unconditional probability of success, whereas with short-term debt it reflects the probability of success conditional on observing the good signal that leads to the rollover of the debt at the interim date, because in the other state the bank is liquidated and the shareholders get nothing. Since the unconditional probability of success is, ceteris paribus, lower than the probability of success conditional on observing the good signal, it follows that the relevant cost of borrowing will tend to be higher with long-term debt, so the bank will have an incentive to choose riskier investments.[1]

There are two caveats to this argument. First, the information that arrives at the interim date may be noisy, which reduces the probability of success conditional on the good signal and consequently increases the relevant cost of borrowing. Second, liquidating the investment may be costly, in which case the lenders should be compensated with a higher payoff when the short-term debt is rolled over, which increases the relevant cost of borrowing. Thus, the two key parameters that determine the optimal maturity structure of the bank's debt are the quality of the lenders' information (which in the model will be denoted by q) and the proportional liquidation costs, or the complementary recovery rate (which in the model will be denoted by λ). Short-term debt will dominate long-term debt when both q and λ are sufficiently high.

The intuition for the result that short-term debt only makes a difference when it is risky should now be clear. If the initial short-term debt is safe and therefore is always rolled over, the cost of the long-term debt will be the same as the average cost of the short-term debt, and so long-term debt will be equivalent to (safe) short-term debt.

The key role of liquidation at the interim date also explains the result that using short-

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[1]A popular rationale for the use of short-term debt is that it is generally cheaper, because the yield curve tends to have a positive slope. In our setup, where the yield curve is flat, short-term debt is (endogenously) cheaper through its positive incentive effects.

term debt may involve paying an up-front dividend to the bank shareholders. Such dividend does not make sense in the case of long-term debt, since it increases the amount due to the lenders and consequently worsens the moral hazard problem. But it may be useful in the case of short-term debt in order to guarantee that early liquidation obtains with positive probability. Paying an up-front dividend raises the hurdle for the rollover of short-term debt, increasing the conditional probability of success and reducing the relevant cost of borrowing for the bank, which explains the positive incentive effect.

According to Bernanke (2010), "Leading up to the crisis, the shadow banking system, as well as some of the largest global banks, had become dependent on various forms of short-term wholesale funding. Over the past 50 years or so, a number of forms of such funding have emerged, including commercial paper, repurchase agreements (repos), certain kinds of interbank loans, contingent funding commitments, and others. In the years immediately before the crisis, some of these forms of funding grew especially rapidly; for example, repo liabilities of U.S. broker dealers increased by 2-1/2 times in the four years before the crisis." Brunnermeier (2009), Shin (2010), and Tirole (2010), among others, share the view that the run-up to the financial crisis of 2007-2009 was associated with banks increasingly financing their asset holdings with shorter maturity instruments. Thus, although the empirical evidence is still scant, maturity shortening is central to the common narrative of the period preceding the crisis.

Other aspects of the common narrative of the pre-crisis period include a decline in profitability which led to widespread search for yield, increased liquidity in global financial markets (captured by our parameter λ), and increased opacity of the financial sector's balance sheets (captured by our parameter q).[2] The results presented here suggests that these these changes are consistent with a shift from long- to short-term debt financing. Thus, the model is useful in organizing a narrative for the crisis and in providing a framework for further work in this area.

[2] See Rajan (2005), Brunnermeier (2009), and Gorton (2010), among others.

Literature review Liquidity risk plays a major role in most papers that analyze short-term debt finance. The model presented here is an exception in that the lenders have no demand for liquidity. We focus on the possibility that adverse news about the bank's investments could lead to early liquidation, which happens when the conditional expected payoff is lower than the amount due to the lenders. The reason why short-term debt may good is that, aware of the possibility of failure to refinance in the future, the bank chooses safer investments.

Liquidity risk is the focus of the seminal paper by Diamond and Dybvig (1983). They show how banks may efficiently insure this risk, but may be subject to runs by demand depositors suspecting that other depositors may want to withdraw their funds, and therefore render the bank illiquid. Our model is closer to the work of Jacklin and Bhattacharya (1988) on informationally-based bank runs. But their focus is very different from ours.

Theoretical research on the maturity structure of firms' debt includes the seminal work of Diamond (1991). He considers an adverse selection model of a firm's choice of debt maturity in which firms with high credit ratings issue short-term debt and firms with lower credit ratings issue long-term debt. The optimal maturity structure trades off a borrower's preference for short-term debt (due to private information about the future credit rating) against liquidity risk. Rajan (1992) studies a moral hazard model of a firm's choice between a bank and an arm's-length lender. The bank monitors the firm and can lend either short-term or long-term, whereas the arm's-length lender must lend long-term. The choice of financing mode depends crucially on the relative bargaining power of the firm and the bank after they acquire information on the future payoff of the investment.

Calomiris and Kahn (1991) provide a rationale for the issuance of demandable debt by banks. In their model, bank shareholders can abscond with bank assets, which they will have an incentive to do when they learn that investment returns will be low. Thus, their focus is on the possibility that shareholders might choose value-destroying actions that will benefit them at the expense of the lenders. In this context, it is optimal to use short-term demandable debt, because it gives the lenders the option to force liquidation before the absconding is done. The related work of Diamond and Rajan (2001) shows how short-term

demandable debt allows bank shareholders to commit to paying depositors ex-post the full return of their loans. Huang and Ratnovski (2011) use a variation of the model of Calomiris and Kahn (1991) to examine the trade-off between the "bright side" (efficient liquidation) and the "dark side" (inefficient liquidation) of banks' short-term wholesale financing, showing that in the presence of a noisy public signal on asset quality the dark side may dominate. In contrast with these papers, the focus of our paper is on the role of short-term debt as a disciplinary device on ex-ante risk-shifting incentives.

Flannery (1994) points out that banks can easily modify the risk profile of their assets, and that contracts preventing such modifications are difficult to write and enforce, so a reasonable alternative for the intermediary is to issue short-term debt. The need to roll over the debt will act as a disciplinary device that may restrain the bank's risk-shifting incentives. The formal model in the present paper shows circumstances under which Flannery's intuition is confirmed and suggests further implications of that intuition.

Cheng and Milbradt (2012) construct a dynamic model of a firm's optimal debt maturity in the presence of rollover externalities (because of the use of staggered short-term debt as in He and Xiong, 2012) and risk-shifting incentives. It is shown that although short-term debt can lead to freezes, it mitigates the risk-shifting problem by imposing a punishment in the form of costly liquidation. Eisenbach (2013) presents a general equilibrium model of the choice of a combination of short-term and long-term debt by a continuum of banks that invest in long-term projects. The paper shows that using rollover risk as a disciplining device is effective when banks face purely idiosyncratic risks, but with correlated risks it leads to excessive risk-taking in good times and excessive fire sales in bad times.

As an alternative to the theories of debt maturity based on the disciplinary role of short-term debt, Diamond and He (2013) consider the relationship between debt maturity and debt overhang (the reduced incentive of borrowers to invest because some value accrues to the existing lenders). They examine the idea of Myers (1977) that short-term debt should reduce overhang, and show circumstances under which this is not necessarily the case. Brunnermeier and Oehmke (2013) consider a model in which borrowers cannot commit to a maturity structure, showing that in this case a maturity rat race may lead to extreme reliance on

short-term financing.

Finally, we note the connection of our paper with the literature on sovereign debt maturity. For example, Jeanne (2009) shows how a short-term maturity structure helps to ensure that the debtor country implements investor-friendly policies, but it also makes the country vulnerable to crises caused by bad shocks. Thus, the benefits of short-term debt in terms of incentives are traded off against the costs in terms of unwarranted crises.

Structure of paper Section 2 presents the model. Section 3 characterizes the optimal contract with long-term debt. Section 4 introduces an interim date where some public information about the final return of the bank's investment is revealed, and characterizes the optimal contracts with safe and risky short-term debt. Section 5 analyzes under what conditions risky short-term debt dominates long-term debt, when both are feasible, and uses the results to discuss the shortening of the maturity of banks' debt in the run-up to the 2007-2009 financial crisis. Section 6 examines the possible mixing of short-term and long-term debt, the consequences of regulating liquidity risk, and the effects of monetary policy. Section 7 concludes. The proofs of the analytical results are in the Appendix.

2   The Model

Consider an economy with three dates ($t = 0, 1, 2$), a risk-neutral *bank*, and a large number of risk-neutral (wholesale) *lenders*. Both the lenders and the bank have a discount rate that is normalized to zero.

At $t = 0$ the bank wants to fund a unit investment that at $t = 2$ yields a random payoff

$$
R = \begin{cases} R_0 & \text{with probability } 1 - p, \\ R_1 & \text{with probability } p, \end{cases} \tag{1}
$$

where $R_0 = 0$ and $R_1 > 1$. The lenders may fund the bank using *long-term debt*, that matures at the terminal date $t = 2$, or *short-term debt*, that matures and has to be rolled over at the interim date $t = 1$. If the short-term debt is not rolled over the asset is liquidated, which yields a *liquidation value L*.

In principle, the bank could raise more than one unit of funds at $t = 0$ and pay out the excess as an *up-front dividend* D to the bank shareholders. This possibility turns out to be useful in some circumstances discussed below.

To introduce a moral hazard problem, we assume that the probability p of the high payoff R_1 is chosen by the shareholders at $t = 0$ after raising $1 + D$ from the lenders, and that this payoff is a decreasing function of p, that is

$$R_1 = R(p), \tag{2}$$

with $R'(p) < 0$. Therefore, higher risk (lower p) is associated with a higher success payoff.[3]

The expected payoff of the bank's investment $pR(p)$ is maximized at the *first-best probability of success*

$$p^* = \arg\max_p (pR(p)).$$

Since the first derivative $(pR(p))' = R(p) + pR'(p)$ equals $R(0) > 0$ for $p = 0$, we have $p^* > 0$. We further assume that $R(1) + R'(1) \leq 0$, so $p^* \leq 1$. Finally, we assume that $(pR(p))'' < 0$, which implies that p^* is characterized by the first-order condition

$$(p^* R(p^*))' = 0. \tag{3}$$

Note that we have $p^* R(p^*) \geq R(1) > 1$, so in the absence of informational problems the bank's investment has a positive net present value.

An example The linear payoff function

$$R(p) = a(2 - p), \tag{4}$$

with $a > 1$, satisfies the required properties and will be used to derive the numerical results of the paper. Parameter a characterizes the profitability of the bank's investment. For this function we have $(pR(p))' = 2a(1 - p)$, which implies $p^* = 1$. Thus, the first-best would be a safe investment with $R(p^*) = a$. Assuming $a > 1$ ensures a positive NPV.

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[3]This setup is borrowed from Allen and Gale (2000, Chapter 8) and is essentially the moral hazard model in Stiglitz and Weiss (1981). A possible alternative would be to follow the approach in Holmström and Tirole (1997), where the success payoff is fixed at $\overline{R}_1$ and the bank gets private benefits $\Pi(p)$. The two approaches yield similar results, and may be related by assuming that $\Pi(p) = p[R(p) - \overline{R}_1]$.

3   Long-term Debt

Suppose that the bank is funded with long-term debt, and let B denote the face value of the debt maturing at $t = 2$ that the lenders receive in exchange for $1 + D$ funds provided at $t = 0$, where $D \geq 0$ is the dividend paid up-front to the shareholders.

A *contract with long-term* debt specifies the initial dividend D paid to the shareholders at $t = 0$ and the face value B of the debt payable to the lenders at $t = 2$. Such contract determines the probability of success p chosen by the bank at $t = 0$.

An *optimal contract with long-term* debt is a triple (D_L, B_L, p_L) that solves the problem

$$\max_{(D,B,p)} \left[D + p \left(R(p) - B \right) \right]$$

subject to the *bank's incentive compatibility constraint*

$$p_L = \arg\max_p \left[p \left(R(p) - B_L \right) \right], \tag{5}$$

and the *lenders' participation constraint*

$$p_L B_L \geq 1 + D_L. \tag{6}$$

The incentive compatibility constraint (5) characterizes the bank's choice of p given the promised repayment B_L, and the participation constraint (6) ensures that the lenders get the required expected return on their investment.

The solution to (5) is characterized by the first-order condition

$$(pR(p))' = B. \tag{7}$$

Since $pR(p)$ is concave, the left-hand side of (7) is decreasing in p, which implies that higher face values of the long-term debt B are associated with lower values of the probability of success p, that is $dp/dB < 0$. This is the standard risk-shifting effect that obtains under debt finance. Moreover, using the characterization (3) of the first-best probability of success p^*, it follows that $p_L < p^*$, that is the bank will take on more risk than in the first-best.

The following result shows that raising more than one unit of funds and paying out the excess as an up-front dividend D worsens the moral hazard problem, and hence it will not

be optimal when funding the bank with long-term debt. For this reason, a contract with long-term debt will simply be written as (B_L, p_L).

Lemma 1 *The optimal contract with long-term debt satisfies $D_L = 0$.*

By Lemma 1 the participation constraint (6) may be written as $pB = 1$. Solving for B in this expression and substituting it into the first-order condition (7) gives the condition

$$H(p) = 1, \tag{8}$$

where

$$H(p) = p \, (pR(p))'. \tag{9}$$

Since $(pR(p))'$ is positive for $0 \leq p < p^*$, with $(p^* R(p^*))' = 0$, it follows that the function $H(p)$ is positive for $0 < p < p^*$, and satisfies $H(0) = H(p^*) = 0$.

The equation $H(p) = 1$ may have no solution, a single solution, or multiple solutions. In the first case, financing the bank with long-term debt will not be feasible: the bank's risk-shifting incentives are so strong that the lenders' participation constraint cannot be satisfied. In the second case, the single solution characterizes the optimal contract with long-term debt. And in the third case, the following result shows that the optimal contract is characterized by the solution with the highest probability of success.

Proposition 1 *Financing the bank with long-term debt is feasible if the equation $H(p) = 1$ has a solution, in which case $B_L = 1/p_L$ and*

$$p_L = \max\{p \in (0, p^*) \mid H(p) = 1\}, \tag{10}$$

is the optimal contract with long-term debt.

Summing up, it will be possible to fund the bank with long-term debt if the function $H(p)$ takes values greater than or equal to 1 somewhere in the interval $(0, p^*)$. In this case, the bank's payoff will be

$$\pi_L = p_L(R(p_L) - B_L) = p_L R(p_L) - 1, \tag{11}$$

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where p_L is the probability of success in the optimal contract with long-term debt. Notice that $\pi_L > 0$ by the incentive compatibility constraint (5) and the result $p_L > 0$. This payoff will be compared with the one corresponding to the optimal contract with short-term debt.

An example (continued)   For the payoff function $R(p) = a(2 - p)$ we have

$$H(p) = 2ap(1 - p), \tag{12}$$

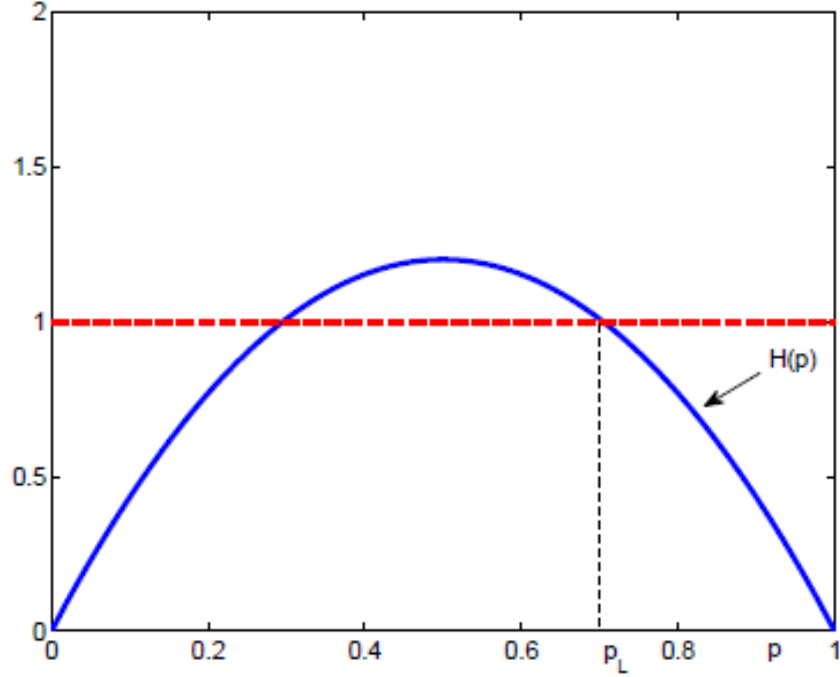so solving for the optimal contract with long-term debt gives

$$p_L = \frac{1}{2}\left(1 + \sqrt{\frac{a - 2}{a}}\right). \tag{13}$$

The term inside the square root will be non-negative if $a \geq 2$. Hence, financing the bank with long-term debt requires that the profitability of the bank's investment be sufficiently high. The probability of success p_L in (13) is increasing in the profitability parameter a, with $\lim_{a \to \infty} p_L = p^* = 1$, and the face value of the debt $B_L = 1/p_L$ is decreasing in a, with $\lim_{a \to \infty} B_L = 1$. Figure 1 represents the function $H(p)$ in (12) and the determination of p_L for $a = 2.4$.

4   Short-term Debt

To introduce some meaningful difference between short- and long-term debt some information about the prospects of the bank's investment must be revealed at the interim date $t = 1$ in which the initial short-term debt has to be rolled over until the terminal date $t = 2$.

Specifically, assume that at $t = 1$ the lenders observe a *public signal* $s \in \{s_0, s_1\}$ on the future payoff of the bank's investment, and based on this signal they decide whether to refinance the bank. If they do, final payoffs will be obtained at $t = 2$. If they do not, the bank will be liquidated at $t = 1$ and the initial lenders will receive the liquidation value L.

Figure 1. Optimal contract with long-term debt

This figure shows the determination of the probability of success in the optimal contract
with long-term debt as the highest value of p for which $H(p) = 1$.

Following Repullo (2005), we assume that the signal s observed by the lenders at the
interim date $t = 1$ satisfies

$$\Pr(s_0 \mid R_0) = \Pr(s_1 \mid R_1) = q,$$

where parameter $q \in [1/2, 1]$ describes the quality of the lenders' information.[4] Notice that
this information is only about whether the final payoff R of the bank's investment will be
high (R_1) or low (R_0), and not about the particular value $R(p)$ taken by the high payoff.[5]

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[4] More generally, we could have $\Pr(s_0 \mid R_0) \neq \Pr(s_1 \mid R_1)$, but then we would have two parameters to
describe the quality of the lenders' information.

[5] Thus, s is not a signal of the bank's action at $t = 0$ (the choice of p) but of the consequences of such
action at $t = 2$ (the final payoff R). See Prat (2005) for a discussion of the distinction between signals on
actions and signals on consequences of actions.

By Bayes' law we have

$$\Pr(R_1 \mid s_0) = \frac{\Pr(s_0 \mid R_1)\Pr(R_1)}{\Pr(s_0)} = \frac{(1-q)p}{p+q-2qp}, \tag{14}$$

and

$$\Pr(R_1 \mid s_1) = \frac{\Pr(s_1 \mid R_1)\Pr(R_1)}{\Pr(s_1)} = \frac{qp}{1-p-q+2qp}. \tag{15}$$

When $q = 1/2$ the posterior probabilities satisfy $\Pr(R_1 \mid s_0) = \Pr(R_1 \mid s_1) = p$, so the signal is uninformative. When $q = 1$ we have $\Pr(R_1 \mid s_0) = 0$ and $\Pr(R_1 \mid s_1) = 1$, so the signal completely reveals the final payoff. In general, when $1/2 < q < 1$ (and $0 < p < 1$) we have $\Pr(R_1 \mid s_0) < p < \Pr(R_1 \mid s_1)$.[6] For this reason, the states corresponding to observing signals s_0 and s_1 will be called, respectively, the bad and the good state.

Finally, we assume that the *liquidation value* L of the bank's investment at the interim date $t = 1$ is a fraction $\lambda \in [0, 1]$ of its conditional expected payoff, that is

$$L = \lambda E(R \mid s).$$

Parameter λ describes the *recovery rate* of the value of the investment, so $(1 - \lambda)E(R \mid s)$ are the liquidation costs. Notice that for any $\lambda < 1$ liquidating the bank at $t = 1$ will be inefficient.

Compared to the case of long-term debt, the model of short-term debt involves two additional parameters, namely the quality q of the lenders' interim information and the recovery rate λ of the bank's investment when it is liquidated early.

Suppose now that the bank is funded with short-term debt that matures at $t = 1$, and let M denote the face value of the debt that the lenders receive in exchange for $1 + D$ funds provided at $t = 0$, where as before $D \geq 0$ is the dividend paid up-front to the shareholders.

At $t = 1$ the bank will try to issue new debt, payable at $t = 2$, in order to repay the initial lenders. The face value of this debt will naturally depend on the signal s observed at the interim date. Let N_s denote the face value of the debt that the interim lenders receive in exchange for funding the repayment of the initial debt, if it is rolled over in state s.

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[6]Both inequalities are satisfied if $p(1-p)(2q-1) > 0$.

The decision to roll over the initial debt depends on the corresponding posterior probabilities of success of the investment, $\Pr(R_1 \mid s_0)$ or $\Pr(R_1 \mid s_1)$. As stated in (14) and (15), these probabilities depend on the quality of the signal q, which is known, and the prior probability p, which is not. Hence, the interim lenders will have to decide on the basis of the value $\widehat{p}$ that they conjecture the bank chose at $t = 0$.[7] Let $\widehat{\Pr}(R_1 \mid s_0)$ and $\widehat{\Pr}(R_1 \mid s_1)$ denote the corresponding posterior probabilities.

At the interim date $t = 1$, the lenders will roll over the bank's initial debt in state s if it satisfies

$$\widehat{E}(R \mid s) = \widehat{\Pr}(R_1 \mid s)R(\widehat{p}) \geq M,$$

that is, if the conjectured expected value of the bank's investment is greater than or equal to the face value M of the debt to be refinanced. In this case, there exists a face value $N_s(\widehat{p}) \leq R(\widehat{p})$ of the new debt that satisfies the interim lenders' participation constraint

$$\widehat{\Pr}(R_1 \mid s)N_s(\widehat{p}) = M.$$

From here it follows that if the initial debt is rolled over in state s, the face value of the debt issued at the interim date will be

$$N_s(\widehat{p}) = \frac{M}{\widehat{\Pr}(R_1 \mid s)}. \tag{16}$$

When $\widehat{E}(R \mid s) < M$ the initial debt is not rolled over, in which case the initial lenders get $\widehat{L} = \lambda\widehat{E}(R \mid s) < \widehat{E}(R \mid s)$. The expected loss of $(1 - \lambda)\widehat{E}(R \mid s)$ could be avoided if they could renegotiate down their claim M, so we are implicitly assuming that such renegotiation is impossible—for example, because the lenders are dispersed and cannot coordinate such renegotiation.

We are implicitly assuming that no dividend is paid to the bank shareholders nor they can inject equity at the interim date $t = 1$. The first assumption is made without loss of generality: as we will see below, paying a dividend at the initial date $t = 0$ may have a positive effect on the bank's choice of p, but paying a dividend at the interim date $t = 1$ entails no

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[7]Under rational expectations, the conjectured $\widehat{p}$ must be equal to the value of p chosen by the bank.

incentive effect, since at this point p has already been chosen. The second assumption is restrictive, since the equity could come from saving (part of) a positive initial dividend D, but it is made for simplicity.

To describe the refinancing decision at $t = 1$ it is convenient to introduce the indicator function

$$I(x) = \begin{cases} 1, & \text{if } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then, for each state s we let

$$x_s = \widehat{E}(R \mid s) - M,$$

that is, the difference between the conjectured expected value of the bank's investment and the face value of the debt to be refinanced. The initial debt M will be rolled over in state s if $x_s \geq 0$, so $I(x_s) = 1$. Otherwise, $I(x_s) = 0$ and the bank will be liquidated.

Using this notation, the *initial lenders' participation constraint* may be written as

$$\varphi(\widehat{p}, M) = 1 + D, \tag{17}$$

where

$$\varphi(\widehat{p}, M) = \sum_{s=s_0,s_1} \widehat{\Pr}(s) \left[I(x_s)M + (1 - I(x_s))\lambda\widehat{E}(R \mid s) \right]. \tag{18}$$

Note that when $I(x_s) = 1$, that is when $\widehat{E}(R \mid s) \geq M$, the initial debt is rolled over and the initial lenders are repaid M, and when $I(x_s) = 0$, that is when $\widehat{E}(R \mid s) < M$, the bank is liquidated and the initial lenders get the liquidation value $L = \lambda\widehat{E}(R \mid s)$. The participation constraint (17) is written as an equality, because otherwise the dividend D could be increased without changing the bank's incentives, improving its payoff.

Similarly, the *bank's payoff* for given values of the initial dividend D and the face value of the initial debt M may be written as

$$\pi(D, M, p, \widehat{p}) = D + \sum_{s=s_0,s_1} \Pr(s)I(x_s)\Pr(R_1 \mid s) \max\left\{R(p) - N_s(\widehat{p}), 0\right\}, \tag{19}$$

where $N_s(\widehat{p})$ is the face value of the debt issued at the interim date, if the initial debt is rolled over in state s. Note that when $I(x_s) = 1$ the bank's payoff equals the dividend D

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plus the expected continuation payoff $\Pr(R_1 \mid s) \max\{R(p) - N_s(\widehat{p}), 0\}$, and when $I(x_s) = 0$ the bank is liquidated and the shareholders only get the dividend D.

A *contract with short-term debt* specifies the initial dividend D paid to the shareholders at $t = 0$ and the face value M of the initial debt payable to the lenders at $t = 1$. Such contract determines the probability of success p chosen by the bank at $t = 0$, the contingent rollover decision $I(x_s)$ at $t = 1$, and the face value of the interim debt $N_s(p)$, if the initial debt is rolled over in state s.

An *optimal contract with short-term debt* is a triple (D_S, M_S, p_S) that solves the problem

$$\max_{(D,M,p,\widehat{p})} \pi(D, M, p, \widehat{p})$$

subject to the *bank's incentive compatibility constraint*

$$p_S = \arg\max_p \pi(D_S, M_S, p, \widehat{p}), \tag{20}$$

the *initial lenders' participation constraint*

$$\varphi(\widehat{p}, M_S) = 1 + D_S, \tag{21}$$

and the *rational expectations constraint*

$$\widehat{p} = p_S. \tag{22}$$

The incentive compatibility constraint (20) characterizes the bank's choice of p given the promised interim repayment M and the rollover decision implied by the lenders' conjecture $\widehat{p}$ of the value of p chosen by the bank. The participation constraint (21) ensures that the initial lenders get the required expected return on their investment. Finally, the rational expectations constraint (22) requires that the conjectured probability of success $\widehat{p}$ equals the value p_S chosen by the bank in the optimal contract.

There are two possible types of contracts with short-term debt: one in which the initial debt is safe, in the sense that the initial lenders are always fully repaid, and another one in which the initial debt is risky, in the sense that the initial lenders are fully repaid in the good

state s_1 and the bank is liquidated in the bad state s_0.[8] We next characterize the optimal contracts with safe and risky short-term debt.

4.1   Safe short-term debt

The easier case to analyze is that of safe short-term debt. This case is also less interesting because, as will be shown below, to every optimal contract with safe short-term debt there corresponds an optimal contract with long-term debt that is characterized by the same success probability p_L and the same payoff for the bank π_L (but not vice-versa).

With safe short-term debt, we have $I(x_s) = 1$, so the initial lenders' participation constraint (17) reduces to $M = 1 + D$. Using the definition (16) of $N_s(\widehat{p})$ and the expressions (14) and (15) of $\Pr(R_1 \mid s_0)$ and $\Pr(R_1 \mid s_1)$ the bank's payoff (19) becomes

$$\pi(D, M, p, \widehat{p}) = D + p \left[R(p) - \frac{M}{\widehat{p}} \right].$$

From here it follows that the first-order condition that characterizes the bank's choice of p is

$$(pR(p))' = \frac{M}{\widehat{p}}.$$

Substituting the participation constraint $M = 1 + D$ and the rational expectations constraint $\widehat{p} = p$ into the first-order condition, and using the definition (9) of $H(p)$ gives

$$H(p) = 1 + D.$$

For $D = 0$ this is identical to the condition (8) that characterizes the optimal contract with long-term debt. And for the same incentive reasons as before, there should be no up-front dividend D. Therefore, the candidate optimal contract with safe short-term debt is $(M_S, p_S) = (1, p_L)$, where p_L is the probability of success in the optimal contract with long-term debt defined in (10).

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[8] We ignore contracts in which there is liquidation in both states, since one can show that they are either not feasible or dominated by one of the other two possible types of contracts.

However, for $(1, p_L)$ to be an optimal contract with safe short-term debt it must be the case that the initial debt is rolled over in the the bad state s_0,[9] which requires

$$E(R \mid s_0) = \frac{(1-q)p_L}{p_L + q - 2qp_L} R(p_L) \geq 1. \tag{23}$$

For $q = 1/2$ (uninformative signal) the condition reduces to $p_L R(p_L) \geq 1$, which holds if long-term financing is feasible. For $q = 1$ (perfectly informative signal) the condition is never satisfied, because the left-hand side of the inequality is zero. Since $\Pr(R_1 \mid s_0)$ is decreasing in q, there is an intermediate value of q for which the constraint is satisfied with equality. Solving for q in (23), the condition that guarantees that the initial debt is rolled over in the bad state s_0 becomes

$$q \leq q(p) = \frac{p(R(p) - 1)}{1 + p(R(p) - 2)}. \tag{24}$$

Hence, we have the following result.

Proposition 2 *Financing the bank with safe short-term debt is feasible if financing the bank with long term debt if feasible and $q \leq q(p_L)$, where $q(p)$ is defined in (24) and p_L is the probability of success in the optimal contract with long-term debt defined in (10), in which case $(M_S, p_S) = (1, p_L)$ is the optimal contract with safe short-term debt.*

Proposition 2 shows that it will be possible to fund the bank with safe short-term debt only if the quality q of the lenders' information is not too high. The intuition for this result is clear. When q is close to 1, observing the bad state s_0 means that the conditional expected value of the bank's investment is close to zero, so the initial debt will not be rolled over. On the other hand, since the upper bound $q(p_L)$ is strictly greater than $1/2$,[10] when q is sufficiently low funding the bank with safe short-term debt will be feasible (as long as funding it with long-term debt is).

We conclude that using safe short-term debt does not add anything relative to using long-term debt. Thus, the only possible role of short-term debt is when it is risky.

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[9] Since $E(R \mid s_1) > E(R \mid s_0)$, if the initial debt is rolled over in the bad state s_0 it will also be rolled over in the good state s_1.

[10] Note that $q(p_L) > 1/2$ if and only if $p_L R(p_L) > 1$, which holds because $\pi_L = p_L R(p_L) - 1 > 0$.

An example (continued)   For the payoff function $R(p) = a(2 - p)$ the optimal contract with long-term debt is characterized by the probability of success p_L in (13). This will also characterize the optimal contract with safe short-term debt if the quality of the lenders' information satisfies $q \leq q(p_L)$. For example, for $a = 2.4$ we have $p_L = 0.7$ and $q(p_L) = 0.83$. In this case, values of q higher than 0.83 imply that $(1, p_L) = (1, 0.7)$ will not be feasible, because the initial debt will not be rolled over in the bad state s_0.

It should be noted that the short-term debt issued after the rollover of the initial debt is not safe. For the case $a = 2.4$, taking $q = 0.75 < 0.83 = q(p_L)$, and substituting $M = 1$, $p_L = 0.7$, and $q = 0.75$ into (16) we get $N_{s_0} = [\Pr(R_1 \mid s_0)]^{-1} = 2.26$ and $N_{s_1} = [\Pr(R_1 \mid s_1)]^{-1} = 1.14$. Thus, in both states the bank pays a premium over the riskless rate to cover the default risk, which is higher in the bad state s_0.

4.2   Risky short-term debt

When the short-term debt is risky, the initial lenders are only repaid in the good state s_1, and the bank is liquidated in the bad state s_0, in which case they anticipate getting a fraction λ of the conjectured expected value of the bank's investment $\widehat{E}(R \mid s_0)$.

The initial lenders' participation constraint (17) then becomes

$$\varphi(\widehat{p}, M) = \lambda(1 - q)\widehat{p}R(\widehat{p}) + (1 - \widehat{p} - q + 2q\widehat{p})M = 1 + D, \tag{25}$$

where we have used $\widehat{\Pr}(s_0)\widehat{E}(R \mid s_0) = (1 - q)\widehat{p}R(\widehat{p})$ and $\widehat{\Pr}(s_1) = 1 - \widehat{p} - q + 2q\widehat{p}$.

The bank's payoff (19) simplifies to

$$\pi(D, M, p, \widehat{p}) = D + qp\left[R(p) - \frac{1 - \widehat{p} - q + 2q\widehat{p}}{q\widehat{p}}M\right], \tag{26}$$

where we have used $\Pr(s_1)\Pr(R_1 \mid s_1) = qp$, the definition (16) of $N_s(\widehat{p})$, and the expression (15) of $\Pr(R_1 \mid s_1)$. From here it follows that the first-order condition that characterizes the bank's choice of p is

$$(pR(p))' = \frac{1 - \widehat{p} - q + 2q\widehat{p}}{q\widehat{p}}M. \tag{27}$$

Solving for M in the participation constraint (25) and substituting it together with the rational expectations constraint $\widehat{p} = p$ into the first-order condition (27), and using the

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definition (9) of $H(p)$ gives the condition

$$H(p) = F(p, q, \lambda, D), \tag{28}$$

where

$$F(p, q, \lambda, D) = \frac{1 + D - \lambda(1 - q)pR(p)}{q}. \tag{29}$$

Since $pR(p)$ is increasing and concave for $p < p^*$, the function $F(p, q, \lambda, D)$ is decreasing and convex in p over the same range (except for $\lambda = 0$ or $q = 1$, when it is constant).

To determine the initial dividend D in the optimal contract with risky short-term debt we have to introduce the constraint that the initial debt is not rolled over in the bad state s_0, which requires

$$E(R \mid s_0) = \frac{(1 - q)p}{p + q - 2qp} R(p) \leq M. \tag{30}$$

For $q = 1$ (perfectly informative signal) the condition is always satisfied, because the left-hand side of the inequality is zero. For $q = 1/2$ (uninformative signal) the condition implies $E(R \mid s_1) = E(R \mid s_0) \leq M$, so the bank would also be liquidated in the good state s_1.[11] Solving for M in the participation constraint (25), substituting it into condition (30), and solving for D gives

$$D \geq G(p, q, \lambda), \tag{31}$$

where

$$G(p, q, \lambda) = \left[\frac{1}{p + q - 2qp} - (1 - \lambda) \right] (1 - q)pR(p) - 1. \tag{32}$$

Summing up, condition (28) characterizes the values of p and D that satisfy the bank's incentive compatibility constraint and the initial lenders' participation constraint. Condition (31) characterizes the values of p and D for which the initial debt will not be rolled over in the bad state s_0. The set of feasible contracts with risky short-term debt are those that satisfy both conditions.

To determine the optimal contract with risky short-term debt, note that substituting M from (25) into (26), and taking into account the rational expectations constraint $\widehat{p} = p$, we

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[11]It should be noted that condition (30) may be written with a weak inequality, because when $E(R \mid s_0) = M$ the face value $N_{s_0}(p)$ of the new debt issued at $t = 1$ equals $R(p)$, in which case the shareholders' stake is the same as in the case of liquidation (that is, zero).

can write the bank's payoff as

$$[q + \lambda(1-q)]\, pR(p) - 1. \tag{33}$$

This expression is easy to explain. Since the initial lenders' participation constraint is satisfied with equality, the bank gets the expected payoff of the investment minus the unit cost that, in expectation, is repaid to the lenders. With probability $\Pr(s_1)$ the bank is not liquidated at $t = 1$ and the conditional expected payoff is $\Pr(R_1 \mid s_1)R(p) = qpR(p)/\Pr(s_1)$, and with probability $\Pr(s_0)$ the bank is liquidated at $t = 1$ and the conditional expected payoff is $\lambda \Pr(R_1 \mid s_0)R(p) = \lambda(1-q)pR(p)/\Pr(s_0)$, which gives

$$\Pr(s_1)\Pr(R_1 \mid s_1)R(p) + \Pr(s_0)\lambda\Pr(R_1 \mid s_0)R(p) = [q + \lambda(1-q)]\, pR(p).$$

Since $pR(p)$ is increasing for $p < p^*$, it follows that the optimal contract will be the feasible contract with the highest probability of success. Hence, we can state the following result.

Proposition 3 *Financing the bank with risky short-term debt is feasible if the equation* $H(p) = F(p, q, \lambda, D)$ *has a solution for some* $D \geq \max\{G(p,q,\lambda), 0\}$, *in which case*

$$
\begin{aligned}
D_S &= \max\{G(p_S, q, \lambda), 0\}, & \text{(34)}\\
M_S &= \frac{1 + D_S - \lambda(1-q)p_S R(p_S)}{1 - p_S - q + 2qp_S}, \quad \text{and} & \text{(35)}\\
p_S &= \max\left\{p \in (0, p^*) \mid H(p) = F(p, q, \lambda, D) \text{ and } D \geq \max\{G(p,q,\lambda), 0\}\right\} & \text{(36)}
\end{aligned}
$$

is the optimal contract with risky short-term debt.

Proposition 3 shows that the feasibility of funding the bank with risky short-term debt depends in a somewhat complex manner on the quality of the lenders' information q and the recovery rate λ of the value of the investment when the bank is liquidated at $t = 1$. Interestingly, the optimal contract may involve raising more than the unit cost of the investment at $t = 0$ and paying the difference as an up-front dividend $D > 0$.

To explain the characterization of the optimal contract with risky short-term debt, and to derive some analytical results on the effect of changes in parameters q and λ, it is useful

to define the following functions

$$D_{HF}(p,q,\lambda) = qH(p) + \lambda(1-q)pR(p) - 1, \tag{37}$$

$$D_G(p,q,\lambda) = \max\{G(p,q,\lambda), 0\}. \tag{38}$$

The function $D_{HF}(p,q,\lambda)$ is obtained by solving for D in the equation $H(p) = F(p,q,\lambda,D)$, and the function $D_G(p,q,\lambda)$ characterizes the initial dividend in the optimal contract. Increases in p have an ambiguous effect on $D_{HF}(p,q,\lambda)$ (since $H(p)$ is not monotonic in p), and increase the value of $D_G(p,q,\lambda)$ in the range for which $D_G(p,q,\lambda) > 0$ (since the derivative of the function $G(p,q,\lambda)$ in (32) with respect to p is positive in the relevant range). The results in Proposition 3 can now be restated as follows: The optimal contract with risky short-term debt is characterized by the highest value of p that satisfies $D_{HF}(p,q,\lambda) = D_G(p,q,\lambda)$.

An example (continued) For the payoff function $R(p) = a(2-p)$ the function $D_{HF}(p,q,\lambda)$ is the concave parabola

$$D_{HF}(p,q,\lambda) = -a[2q + \lambda(1-q)]p^2 + 2a[q + \lambda(1-q)]p - 1, \tag{39}$$

and the function $D_G(p,q,\lambda)$ becomes

$$D_G(p,q,\lambda) = \max\left\{\left[\frac{1}{p+q-2qp} - (1-\lambda)\right](1-q)ap(2-p) - 1, 0\right\}. \tag{40}$$

The lower panel of Figure 2 plots these functions for $a = 2.4$, $q = 0.9$, and $\lambda = 0.8$, in which case the rightmost intersection between $D_{HF}(p,q,\lambda)$ and $D_G(p,q,\lambda)$ happens for $p_S = 0.75$ and $D_S = 0$, so the optimal contract has no up-front dividend. The upper panel of Figure 2 shows the corresponding functions $H(p)$ and $F(p,q,\lambda,0)$.

The lower panel of Figure 3 plots these functions for $a = 2.4$, $q = 0.75$, and $\lambda = 0.8$, in which case the rightmost intersection between $D_{HF}(p,q,\lambda)$ and $D_G(p,q,\lambda)$ happens for $p_S = 0.68$ and $D_S = 0.21$, so the optimal contract is characterized by a positive up-front dividend. The upper panel of Figure 3 shows the functions $H(p)$, $F(p,q,\lambda,0)$, and $F(p,q,\lambda,D_S)$. Notice that in this case the no refinancing in the bad state condition is violated for $D = 0$, which explains the need to increase the value of D (shifting $F(p,q,\lambda,D)$ upwards) until the condition is satisfied.

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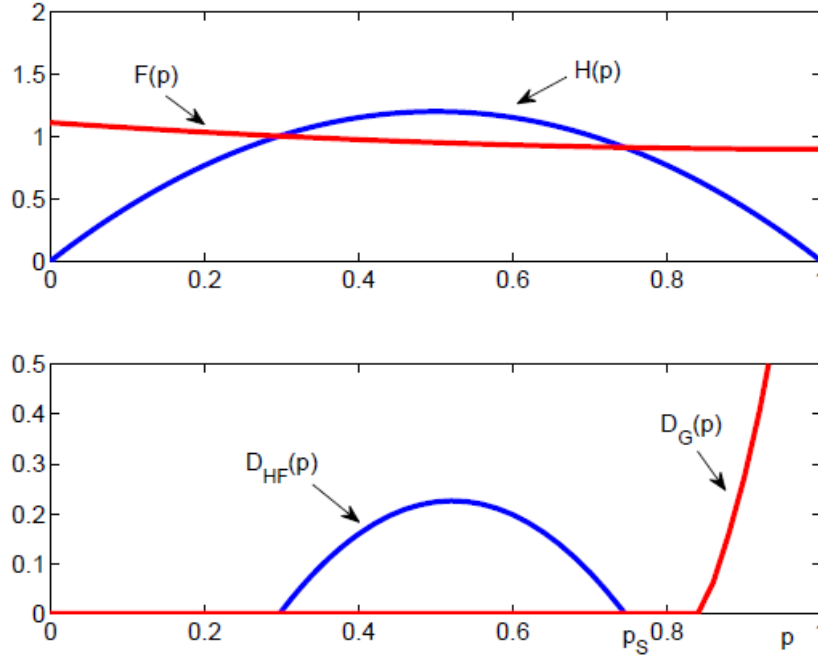
**Figure 2. Optimal contract with risky short-term debt
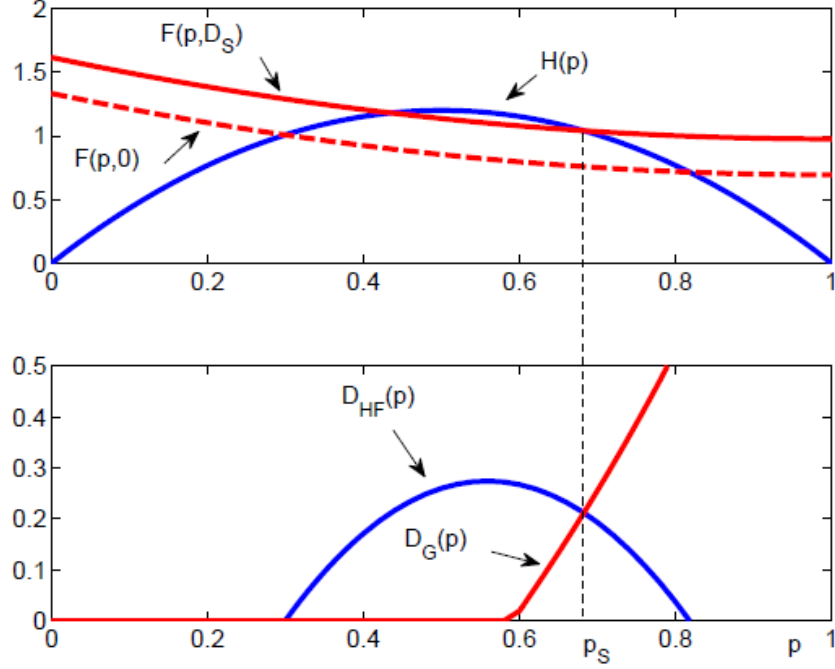The case of a zero initial dividend**

The lower panel of this figure shows the determination of the probability of success
in the optimal contract with risky short-term debt when there is no initial dividend
as the highest value of p for which $DHF(p) = DG(p)$. The upper panel shows the
corresponding functions $H(p)$ and $F(p)$.

Comparing the results in Figures 2 and 3, it appears that a reduction in the quality of
the lenders' information q from 0.9 to 0.75 worsens the moral hazard problem, leading to
a reduction in the probability of success p_S chosen by the bank and to the payment of a
positive up-front dividend D_S. The intuition for the latter result is that when q is low it
may be difficult to ensure that the initial debt is only refinanced in the good state, in which
case paying an up-front dividend and consequently worsening the moral hazard problem may
allow to separate the two states.

4.3 Comparative statics of risky short-term debt

We next discuss the effects of changes in the quality of the lenders' information q and in the
recovery rate λ on the optimal contract with risky short-term debt.

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Figure 3. Optimal contract with risky short-term debt
The case of a positive initial dividend

The lower panel of this figure shows the determination of the probability of success in the optimal contract with risky short-term debt when there is a positive initial dividend as the highest value of p for which $DHF(p) = DG(p)$. The upper panel shows the corresponding functions $H(p)$, $F(p,0)$, and $F(p,D)$.

Doing the comparative statics of parameter q is difficult because $\partial D_{HF}(p,q,\lambda)/\partial q$ and $\partial D_G(p,q,\lambda)/\partial q$ cannot in general be signed. One exception is when the optimal contract has no initial dividend and the recovery rate λ is close to 1. By Proposition 3, when $D_S = 0$ the optimal contract is characterized by the highest solution p_S to the equation $D_{HF}(p,q,\lambda) = 0$, in which case we have

$$\frac{\partial D_{HF}(p,q,\lambda)}{\partial q} = H(p) - \lambda p R(p) = \frac{1 - \lambda p R(p)}{q}.$$

If funding the bank with risky short-term debt is feasible, the bank's payoff (33) will be positive, so $pR(p) > 1$. Thus, when λ close to 1 this derivative will be negative, which implies

$$\frac{\partial p_S}{\partial q} = -\left[\frac{\partial D_{HF}(p,q,\lambda)}{\partial p}\right]^{-1} \frac{1 - \lambda p R(p)}{q} < 0,$$

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where we have used the fact that the slope of $D_{HF}(p, q, \lambda)$ evaluated at p_S must be negative. Therefore, in this case a reduction in the quality q of the lenders' information has a positive incentive effect, increasing the probability of success p_S chosen by the bank in the optimal contract.

Explaining the intuition for this result is important, since it is key to understanding the potential benefits of short-term debt. In principle, the result seems surprising. *Why would noise improve incentives?* Standard arguments would predict the opposite.[12] To explain why this is not the case in our model, let us write the bank's objective function (26) as

$$qp[R(p) - N_{s_1}],$$

and the lenders' participation constraint (25) as

$$\lambda(1 - q)pR(p) + qpN_{s_1} = 1.$$

The bank gets a positive payoff, equal to the difference between the success return $R(p)$ and the final amount due to the lenders N_{s_1}, when the initial short-term debt is rolled over and the investment succeeds, that is with probability $\Pr(s_1, R_1) = qp$. Increases in the amount due N_{s_1} worsen the bank's moral hazard problem, and consequently reduce the probability of success p chosen by the bank. Therefore, the only way in which adding noise (reducing q) could increase p is by reducing the amount due to the lenders N_{s_1}.

To see why this may be the case notice that the lenders' payoff has two components: They get N_{s_1} when the initial debt is rolled over and the investment succeeds, that is with probability qp, which gives the term qpN_{s_1}, and they get the liquidation value of the investment $\lambda E(R \mid s_0)$ when the initial debt is not rolled over, that is with probability $\Pr(s_0)$, which gives the term $\lambda(1 - q)pR(p)$. Reductions in q increase this component of the lenders' payoff. If this effect is sufficiently strong, which happens when λ is large, there will be a reduction in N_{s_1}, which explains the result $\partial p_S/\partial q < 0$. In other words, the noise in the lenders' information leads to a type I error (liquidating the bank when the investment would

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[12]For example, in a principal-agent context, if effort is not fully rewarded because of the existence of noise we would expect the agent to exert less effort.

succeed) that increases the payoff of the lenders in the liquidation state and leads them to reduce what they require when the initial debt is rolled over.

In contrast with the comparative statics of parameter q, the comparative statics of the recovery rate λ is fairly easy. Consider a pair (q, λ) for which risky short-term debt is feasible, and suppose first that the corresponding optimal contract is such that there is no initial dividend. Then, according to Proposition 3, the optimal contract is characterized by the highest solution p_S to the equation $D_{HF}(p, q, \lambda) = 0$. Differentiating this equation gives

$$\frac{\partial p_S}{\partial \lambda} = - \left[\frac{\partial D_{HF}(p, q, \lambda)}{\partial p} \right]^{-1} (1 - q)pR(p) > 0,$$

where we have used the fact that the slope of $D_{HF}(p, q, \lambda)$ evaluated at p_S must be negative. Thus, the higher the recovery rate λ the higher the probability of success p_S in the optimal contract with risky short-term debt. But by the definition (32) of $G(p, q, \lambda)$ we have

$$\frac{\partial G(p, q, \lambda)}{\partial \lambda} = (1 - q)pR(p) > 0,$$

so there might be a $\lambda_1(q) \leq 1$ for which $G(p_S, q, \lambda_1(q)) = 0$. Hence, for further increases in λ the optimal contract will be characterized by a positive up-front dividend. In terms of Figure 2, the increase in the recovery rate λ shifts both $D_{HF}(p, q, \lambda)$ and $D_G(p, q, \lambda)$ upwards, so at some point they may intersect at some $D_S > 0$.

Suppose next that the optimal contract corresponding to the pair (q, λ) is such that there is a positive initial dividend. Then, according to Proposition 3, the optimal contract is characterized by the highest solution p_S to the equation

$$D_{HF}(p, q, \lambda) - G(p, q, \lambda) = qH(p) - \left[\frac{1}{p + q - 2pq} - 1 \right] (1 - q)pR(p) = 0,$$

where we have used the definitions (37) and (32) of $D_{HF}(p, q, \lambda)$ and $G(p, q, \lambda)$. Differentiating this equation gives $\partial p_S / \partial \lambda = 0$. Thus, changing the recovery rate λ when $D_S > 0$ does not have any effect on the probability of success p_S in the optimal contract with risky short-term debt. And since

$$\frac{\partial D_{HF}(p, q, \lambda)}{\partial \lambda} = \frac{\partial G(p, q, \lambda)}{\partial \lambda} = (1 - q)pR(p) > 0,$$

increasing λ has a positive effect on the up-front dividend D_S.

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The preceding results show that increasing the recovery rate λ either makes it more likely that the optimal contract with risky short-term debt is characterized by a positive initial dividend, or increases the dividend when it is positive.

Finally, in the limit case of a perfectly informative signal ($q = 1$) we have $D_{HF}(p, 1, \lambda) = H(p) - 1$ and $G(p, 1, \lambda) = -1$ (for $p < 1$), so by Proposition 3 the optimal contract with risky short-term debt is characterized by the highest solution to the equation $H(p) = 1$, which is the probability of success p_L in the optimal contract with long-term debt, and by a zero initial dividend. The intuition for this result is that when the signal is perfect, liquidation at $t = 1$ (with short-term debt) takes place exactly in those states in which the bank would fail at $t = 2$ (with long-term debt). Thus, the optimal contract with risky short-term debt is isomorphic to the optimal contract with long-term debt. Moreover, in this situation there is no point in paying an initial dividend, which would only worsen the bank's moral hazard problem. Note also that for values of q close to 1 we have $G(p, 1, \lambda) < 0$, which means that if funding the bank with risky short-term debt is feasible, the optimal initial dividend will be zero. In other words, paying a positive dividend only makes sense when the signal is sufficiently noisy.

We can summarize these results as follows.

Proposition 4 *The set of pairs* (q, λ) *for which risky short-term debt is feasible is such that there exists* $\lambda_0(q)$ *and* $\lambda_1(q)$, *with* $0 \le \lambda_0(q) \le \lambda_1(q) \le 1$, *such that:*

1. *For* $\lambda \in [\lambda_0(q), \lambda_1(q))$ *and* $\lambda = \lambda_0(q) = \lambda_1(q) > 0$ *the optimal contract is characterized by* $D_S = 0$, *and* $\partial p_S / \partial \lambda > 0$.

2. *For* $\lambda \in (\lambda_1(q), 1]$ *and* $\lambda = \lambda_0(q) = \lambda_1(q) = 0$ *the optimal contract is characterized by* $D_S > 0$, $\partial D_S / \partial \lambda > 0$, *and* $\partial p_S / \partial \lambda = 0$.

3. *For* q *close to 1 we have* $\lambda_1(q) = 1$, *so the optimal contract is characterized by* $D_S = 0$. *In this case, for* λ *close to 1 we have* $\partial p_S / \partial q < 0$.

Proposition 4 shows that for any given value of q the range of values of λ for which financing the bank with risky short-term debt is feasible is either empty or a closed interval

$[\lambda_0(q), 1]$. Moreover, we can have either (i) $\lambda_0(q) < \lambda_1(q) = 1$ in which case paying an initial dividend will not be optimal, or (ii) $\lambda_0(q) < \lambda_1(q) < 1$ in which case paying an initial dividend will (will not) be optimal for high (low) values of λ, or (iii) $0 < \lambda_0(q) = \lambda_1(q) < 1$ in which case paying an initial dividend will be optimal except at the boundary $\lambda_0(q) = \lambda_1(q)$ where $D_S = 0$, or (iv) $\lambda_0(q) = \lambda_1(q) = 0$ in which case paying an initial dividend will always be optimal.

To illustrate these results, Figures 4 and 5 show the combinations of the recovery rate λ (in the horizontal axis) and the quality of the lenders' information q (in the vertical axis) for which (i) risky short-term debt is feasible and the optimal contract has no initial dividend (Region I), (ii) risky short-term debt is feasible and the optimal contract is characterized by a positive initial dividend (Region II), and (iii) risky short-term debt is not feasible (Region III). The frontiers between these regions depict the functions $\lambda_0(q)$ and $\lambda_1(q)$ in Proposition 4. The figures are drawn for the payoff function $R(p) = a(2 - p)$.

In Figure 4 the parameter that characterizes the profitability of the bank's investment takes the value $a = 1.9$. Recall that financing the bank with long-term debt (or safe short-term debt) requires $a \geq 2$, so this is a case in which risky short-term debt allows to expand the range of values of the profitability parameter a for which financing the bank is feasible. Figure 5 corresponds to $a = 2.4$.[13] The results in Figures 4 and 5 indicate that risky short-term debt is feasible for a set of values of λ and q that is increasing in the profitability parameter a. The intuition for this result is that increases in a ameliorate the bank's moral hazard problem, so it is easier to fund it with risky short-term debt.

Summing up, we have characterized the conditions under which it is possible to fund the bank with risky short-term debt. By (33) the corresponding payoff is

$$\pi_S(q, \lambda) = [q + \lambda(1 - q)] \, p_S R(p_S) - 1, \tag{41}$$

where p_S is the probability of success in the optimal contract with risky short-term debt.[14]

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[13]In this figure Regions I and II are divided into two subregions, denoted with subindices S and L, which correspond to parameter values for which risky short-term debt dominates long-term debt (subindex S) or vice versa (subindex L); see the discussion in Section 5.

[14]Using the same argument as in the case of long-term debt one can show that $\pi_S(q, \lambda) > 0$.
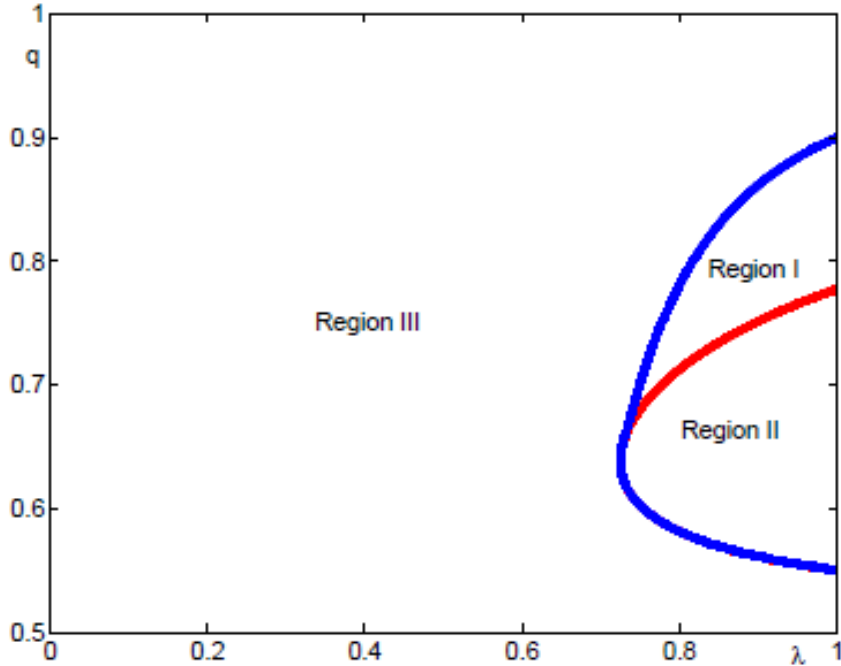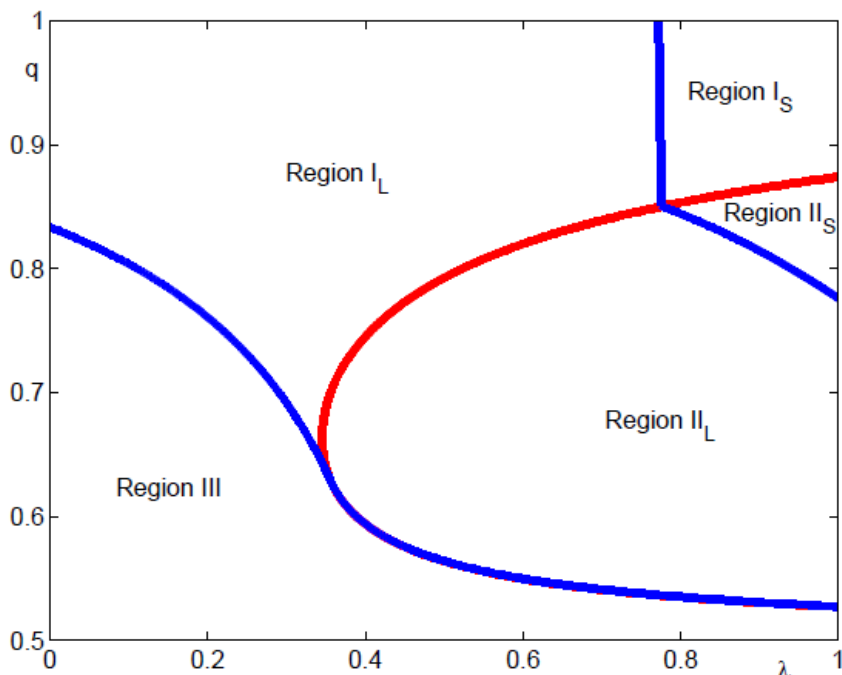
Figure 4. Optimal contract with risky short-term debt
The case of low profitability

This figure shows the combinations of the recovery rate l and the quality of the lenders' information q for which risky short-term debt is feasible when the profitability parameter takes the low value $a = 1.9$ (for which long-term debt is not feasible). In Region I risky short-term debt is feasible and the optimal contract has no initial dividend, in Region II risky short-term debt is feasible and the optimal contract is characterized by a positive initial dividend, and in Region III risky short-term debt is not feasible.

In contrast with the case of safe short-term debt, risky short-term debt is essentially different from long-term debt. In the next section we compare the payoffs $\pi_S(q, \lambda)$ and π_L corresponding to these alternative forms of bank financing.

5 Optimal Bank Financing

In previous sections we have characterized the optimal contracts with long- and short-term debt, and we have shown that safe short-term debt is equivalent to long-term debt, but this is not the case for risky short-term debt. In particular, there are situations in which (i) long-

**Figure 5. Optimal contracts with risky short-term debt
The case of high profitability**

This figure shows the combinations of the recovery rate l and the quality of the lenders'
information q for which long-term or risky short-term debt are optimal when the prof-
itability parameter takes the high value $a = 2.4$. In Regions IL and IIL risky short-term
debt is feasible (and characterized, respectively, by a zero or a positive initial dividend),
but it is dominated by long-term debt. In Regions IS and IIS risky short-term debt is
feasible (and characterized, respectively, by a zero or a positive initial dividend), and
it dominates long-term debt. In Region III risky short-term debt is not feasible and
hence the bank can only be funded with long-term debt.

term debt is the only feasible financing instrument, (ii) risky short-term debt is the only
feasible financing instrument, and (iii) both long-term and risky short-term debt are feasible.
An example of (i) is Region III of Figure 5 in which risky short-term debt is not feasible,
but long-term debt is, an example of (ii) is Regions I and II of Figure 4 in which long-term
debt is not feasible, but risky short-term debt is, and an example of (iii) is Regions I and
II of Figure 5 in which both long-term and risky short-term debt are feasible. This section
considers the conditions under which risky short-term debt dominates or is dominated by
long-term debt, when both are feasible. It then relates the results to features of the common

narrative of the 2007-2009 financial crisis.

The comparative static results on risky short-term debt obtained in the previous section suggest that we could derive some analytical results on the comparison between long-term and risky short-term debt. However, for the sake of clarity and brevity, we will conduct our discussion in terms of our simple parametric example, based on the payoff function $R(p) = a(2 - p)$.

Suppose that the parameter that characterizes the profitability of the bank's investment takes the value $a = 2.4$. Since $a \geq 2$, long-term debt if feasible, the optimal contract is characterized by the probability of success $p_L = 0.7$ (derived from (13)), and the corresponding bank's profits are $\pi_L = 1.19$ (derived from (11)). Figure 5 shows the combinations of the recovery rate λ and the quality of the lenders' information q for which risky short-term debt is feasible. According to Proposition 3, for each feasible pair (q, λ) the optimal contract with risky short-term debt is characterized by a probability of success $p_S(q, \lambda)$ equal to the highest value of p that satisfies $D_{HF}(p, q, \lambda) = D_G(p, q, \lambda)$, where the functions $D_{HF}(p, q, \lambda)$ and $D_G(p, q, \lambda)$ are given by (39) and (40). Substituting $p_S(q, \lambda)$ into (41) gives the corresponding bank's profits $\pi_S(q, \lambda)$. The question is: For which feasible pairs (q, λ) it is the case that risky short-term debt dominates long-term debt (that is, we have $\pi_S(q, \lambda) \geq \pi_L$)?

Figure 5 shows the answer, with Regions I and II divided into two subregions, denoted with subindices S and L. The former (latter) indicates that risky short-term (long-term) debt dominates long-term (risky short-term) debt. Thus, we have five regions. In Regions I_L and II_L risky short-term debt is feasible, but it is dominated by long-term debt. The difference between the two regions is that in Region I_L the optimal contract with risky short-term debt is characterized by a zero initial dividend, while in Region II_L the initial dividend is positive. In Regions I_S and II_S risky short-term debt is feasible, and it dominates long-term debt. As before, the difference between the two regions is that in Region I_S the optimal contract is characterized by a zero initial dividend, while in Region II_L the initial dividend is positive. Finally, in Region III risky short-term debt is not feasible and hence the bank can only be funded with long-term debt.

It is interesting to note that when risky short-term debt dominates long-term debt, by

(11) and (41) we have

$$\pi_S(q, \lambda) = [q + \lambda(1 - q)] \, p_S R(p_S) - 1 > p_L R(p_L) - 1 = \pi_L.$$

For $\lambda < 1$ or $q < 1$ this implies $p_S R(p_S) > p_L R(p_L)$. But since $(pR(p))'$ is positive for $p < p^*$, it follows that $p_S > p_L$. In other words, when risky short-term is the optimal financing instrument, the bank will choose safer investments. In this sense we may conclude that *risky short-term debt has a disciplining effect on the bank's risk-shifting incentives.*

Inspired by Rajan (2005), we next consider the implications of a reduction in the profitability of investments, which leads to the search for yield phenomenon, together with an increase in the liquidity of global asset markets. In terms of our parametric example, these changes correspond to a reduction in the profitability parameter a and an increase in the recovery rate λ. Figure 6 shows the regions depicted in Figure 5 for different combinations of the recovery rate λ (in the horizontal axis) and the profitability parameter a (in the vertical axis), and for a value of the quality of the lenders' information $q = 0.75$. In Regions I_L and II_L risky short-term debt is feasible, but it is dominated by long-term debt. In Regions I_S and II_S risky short-term debt is feasible, and it dominates long-term debt.[15] Finally, in Region III risky short-term debt is not feasible and hence the bank can only be funded with long-term debt, when $a \geq 2$, or not at all, when $a < 2$.

Suppose, for example, that parameter a goes down from 2.4 to 2.1 and parameter λ goes up from 0.75 to 0.9. This means that we would move from point A in Region II_L to point B in Region II_S, that is, the bank would find it optimal to change its funding strategy from long-term debt to risky short-term debt with a positive up-front dividend. Figure 6 shows that there is a wide range of changes in parameters a and λ that yield the same outcome. Hence, our model is consistent with banks shifting to "dangerous forms of debt" (to use the terminology of Tirole, 2003), as well as making high payments to bank shareholders (the empirical counterpart of the positive up-front dividend),[16] as an optimal response to the

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[15] As before, in Regions I_L and I_S (Regions II_L and II_S) the optimal contract with risky short-term debt is characterized by a zero (positive) initial dividend D_S.

[16] The interpretation could also apply to the high bonuses to bank executives, since it can be argued that such payments are (at least in part) earnings distributions. We are grateful to Hyun Shin for pointing this out.
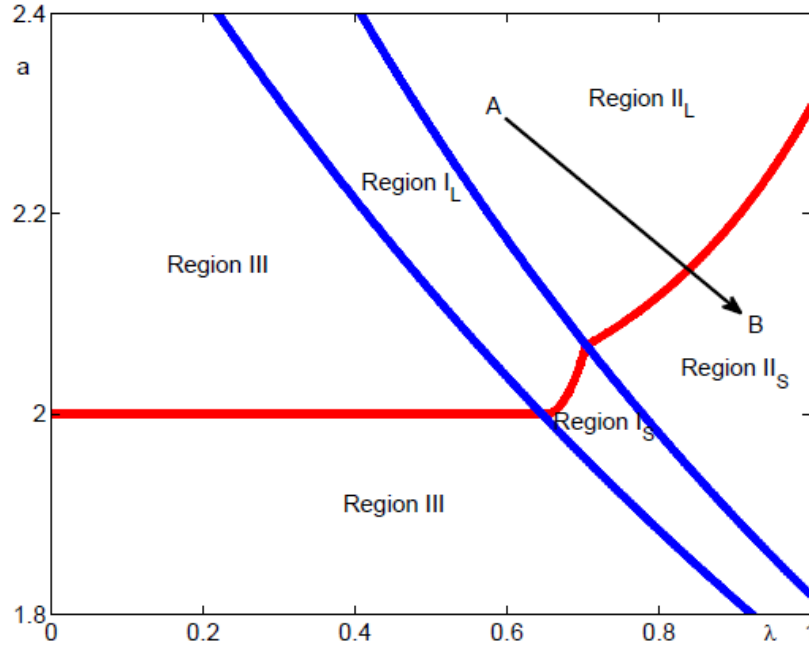
Figure 6. Optimal contracts with long-term or risky short-term debt

This figure shows the combinations of the recovery rate l and the profitability parameter a for which long-term or risky short-term debt are optimal when the quality of the lenders' information $q = 0.75$. In Regions IL and IIL risky short-term debt is feasible (and characterized, respectively, by a zero or a positive initial dividend), but it is dominated by long-term debt. In Regions IS and IIS risky short-term debt is feasible (and characterized, respectively, by a zero or a positive initial dividend), and it dominates long-term debt. In Region III risky short-term debt is not feasible and hence the bank can only be funded with long-term debt, when $a > 2$, or not at all, when $a < 2$. The move from point A to point B shows the effect on the optimal contract of a reduction in the profitability parameter a and an increase in the recovery rate l.

change in the economic and financial environment.

Finally, as noted by Brunnermeier (2009), a novel feature of the recent crisis was the extent of securitization, which led to "an opaque web of interconnected obligations." In terms of our model, an increase in opacity may be captured by a reduction in the quality q of the lender's information, which according to our results makes it more likely that the optimal contract with risky short-term debt is characterized by a positive up-front dividend.

Summing up, we have shown that the positive incentive effects of risky short-term debt may compensate the negative effects associated with using it when information is noisy

$(q < 1)$ and liquidation of bank assets is inefficient $(\lambda < 1)$, so that in some circumstances it may either dominate long-term debt of even become the only feasible form of finance. We have also pointed out important parallels between the model predictions and features of the commonly accepted narrative of the 2007-2009 financial crisis.

6   Extensions

6.1   Mixed debt finance

We have assumed so far that the bank may be funded with either short- or long-term debt. We now consider the possibility that the bank raises a fraction γ of its funding at $t = 0$ by issuing short-term debt, and the remaining $1 - \gamma$ by issuing long-term debt. As before, there are two possible cases, namely the case where the short-term debt is rolled over in both states and the case where the short-term debt is only rolled over in the good state and the bank is liquidated in the bad state. If the bank is liquidated at $t = 1$ or fails at $t = 2$, we will assume that short-term debt is senior to long-term debt.

Let γM denote the face value of the debt that matures at $t = 1$ that the lenders receive in exchange for $\gamma(1 + D)$ funds provided at $t = 0$, and let $(1 - \gamma)B$ denote the face value of the debt that matures at $t = 2$ that the lenders receive in exchange for $(1 - \gamma)(1 + D)$ funds provided at $t = 0$.

When the short-term debt is safe, the initial short-term lenders participation constraint reduces to $M = 1 + D$, and the face value of the debt issued in state s becomes $N_s(\widehat{p}) = (1 + D)/\widehat{\Pr}(R_1 \mid s)$. Hence, using the same derivation as in Section 4.1, the bank's payoff becomes

$$\pi(D, M, B, p, \widehat{p}, \gamma) = D + p\left[R(p) - \frac{\gamma(1 + D)}{\widehat{p}} - (1 - \gamma)B\right].$$

From here it follows that the first-order condition that characterizes the bank's choice of p is

$$(pR(p))' = \frac{\gamma(1 + D)}{\widehat{p}} - (1 - \gamma)B.$$

Substituting the participation constraint of the long-term lenders $pB = 1 + D$ and the rational expectations constraint $\widehat{p} = p$ into the first-order condition, and using the definition

(9) of $H(p)$ gives $H(p) = 1 + D$. Since, as before, it is optimal to set $D = 0$, we get the same condition (8) that characterizes the optimal contract with long-term and safe short-term debt. Thus, in this case mixed debt does not add anything relative to using long-term debt.

When the short-term debt is risky, the initial short-term lenders participation constraint becomes

$$\widehat{\Pr}(s_0) \min\{\lambda \widehat{E}(R \mid s_0), \gamma M\} + \widehat{\Pr}(s_1)\gamma M = \gamma(1 + D),$$

and the long-term lenders participation constraint becomes

$$\widehat{\Pr}(s_0) \max\{\lambda \widehat{E}(R \mid s_0) - \gamma M, 0\} + \widehat{\Pr}(s_1)\widehat{\Pr}(R_1 \mid s_1)(1 - \gamma)B = (1 - \gamma)(1 + D),$$

where we have used the assumption that short-term debt is senior to long-term debt. Adding up the two constraints gives

$$\lambda \widehat{\Pr}(s_0)\widehat{E}(R \mid s_0) + \widehat{\Pr}(s_1)\gamma M + \widehat{\Pr}(s_1)\widehat{\Pr}(R_1 \mid s_1)(1 - \gamma)B$$
$$= \lambda(1 - q)\widehat{p}R(\widehat{p}) + q\widehat{p}[\gamma N_{s_1}(\widehat{p}) + (1 - \gamma)B] = 1 + D.$$

where we have used that $N_{s_1}(\widehat{p}) = M/\widehat{\Pr}(R_1 \mid s_1)$. The bank's payoff may be written as

$$\pi(D, M, B, p, \widehat{p}, \gamma) = D + qp\left[R(p) - \gamma N_{s_1}(\widehat{p}) + (1 - \gamma)B\right].$$

From here it follows that the first-order condition that characterizes the bank's choice of p is

$$(pR(p))' = \gamma N_{s_1}(\widehat{p}) + (1 - \gamma)B.$$

Substituting the joint participation constraint derived above and the rational expectations constraint $\widehat{p} = p$ into the first-order condition, and using the definition (9) of $H(p)$ gives $H(p) = F(p, q, \lambda, D)$, which is the same condition (28) that characterizes the optimal contract with risky short-term debt.

As before, to determine the initial dividend D we have to introduce the constraint that the initial short-term debt is not rolled over in the bad state s_0, which requires $E(R \mid s_0) \leq \gamma M$. Clearly, this constraint becomes tighter as the proportion γ of short-term funding goes down. Hence, in this case mixed debt may be worse than risky short-term debt (which corresponds to $\gamma = 1$), when the latter is feasible.

The conclusion from this discussion is that using a mixture of short- and long-term debt is at best equivalent to using only long-term debt or only risky short-term debt, so it does not add anything relative to the cases analyzed above.

6.2 Liquidity regulation

One of the elements of the new regulation proposed by the Basel Committee on Banking Supervision in 2010, known as Basel III, is a pair of liquidity standards, called the *Liquidity Coverage Ratio* and the *Net Stable Funding Ratio* (see Basel Committee on Banking Supervision, 2010 and 2013). The former requires banks to have "an adequate stock of unencumbered high-quality liquid assets that can be converted into cash easily and immediately in private markets to meet its liquidity needs for a 30 calendar day liquidity stress scenario."

We next consider the effect of introducing in our model a regulation that requires banks to match the amount of short-term borrowing with liquid assets. To do this we need to introduce a liquid asset, which we assume yields a zero return—the same as the expected return required by investors.

Let γM denote the face value of the debt that matures at $t = 1$ that the lenders receive in exchange for $\gamma(1 + C + D)$ funds provided at $t = 0$, and let $(1 - \gamma)B$ denote the face value of the debt that matures at $t = 2$ that the lenders receive in exchange for $(1 - \gamma)(1 + C + D)$ funds provided at $t = 0$, where C denotes the bank's investment in the liquid asset. The liquidity requirement may then be written as $C \geq \gamma M$.

The first thing to note in this setup is that if short-term debt is senior to long-term debt, we have $\gamma M = \gamma(1 + C + D)$ because, whatever the signal observed at $t = 1$, the conditional expected payoff at $t = 2$ of the bank's investment (including the payoff of the liquid asset) will always be greater than the amount due to the short-term lenders. The bank's payoff will then be

$$\pi(D, B, C, p, \gamma) = D + p\left[R(p) + C - \gamma(1 + C + D) - (1 - \gamma)B\right].$$

From here it follows that the first-order condition that characterizes the bank's choice of p

is

$$(pR(p))' = \gamma(1 + C + D) + (1 - \gamma)B - C.$$

Substituting into this expression the participation constraint of the long-term lenders

$$p(1 - \gamma)B + (1 - p)[C - \gamma(1 + C + D)] = (1 - \gamma)(1 + C + D),$$

and using the definition (9) of $H(p)$ gives $H(p) = 1 + D$. Since, as before, it is optimal to set $D = 0$, we get the same condition (8) that characterizes the optimal contract with long-term debt. Hence, imposing a liquidity requirement effectively eliminates the possibility of using risky short-term debt. By our previous results, this may imply riskier bank investments.

As an alternative to quantity-based liquidity regulation, it has been suggested (by Perotti and Suarez, 2009, among others) the possibility of using levies on uninsured short-term liabilities, which would operate like Pigouvian taxes. To discuss the effects of such regulation, suppose that we introduce a proportional levy τ on using short-term debt, payable ex ante (so that to fund a unit investment with short-term debt you have to raise $1 + \tau$).[17] In this setup, safe short-term debt will be clearly dominated by long-term debt, which in the absence of the levy is payoff-equivalent to safe short-term debt. In the case of risky short-term debt the initial lenders' participation constraint (25) becomes

$$\lambda(1 - q)\widehat{p}R(\widehat{p}) + (1 - \widehat{p} - q + 2q\widehat{p})M = 1 + \tau + D.$$

Following the same steps as in Section 4.2, the condition that characterizes the bank's choice of p is

$$H(p) = \frac{1 + \tau + D - \lambda(1 - q)pR(p)}{q}.$$

Since for any $\tau > 0$ the right-hand side of this expression is greater than $F(p, q, \lambda, D)$ in (29), we conclude that the levy will lower the bank's choice of p. Hence, either the levy will have no effect (when long-term debt is optimal), or it will shift the optimal choice of financing from risky short-term debt to long-term debt, or it will not change the bank's choice of risky short-term debt. However, in the last two cases the bank will choose riskier investments.

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[17]As noted by Stein (2012), such levy could be implemented by means of a reserve requirement remunerated below market rates.

It should be stressed that the preceding results on the effects of liquidity regulation are not welfare statements, because they only consider the impact on the bank's choice of risk, without any reference to the possible positive effects of the regulation—such as, for example, reducing or eliminating costly bank failures at the interim date.

6.3   Monetary policy

We have up to now normalized to zero the expected return required by investors. We next consider what happens when this return is a variable r, which is interpreted as the one-period policy rate set by the central bank.

To characterize the optimal contract with long-term debt it suffices to note that the lenders' participation constraint now becomes $pB = (1+r)^2$. Substituting this expression into the bank's first-order condition (7), and using the definition (9) of $H(p)$ gives

$$H(p) = (1+r)^2.$$

From here it follows that an increase in r will lower the bank's choice of p.

To characterize the optimal contract with risky short-term debt we first note that the initial lenders' participation constraint (25) becomes

$$\lambda(1-q)\widehat{p}R(\widehat{p}) + (1 - \widehat{p} - q + 2q\widehat{p})M = (1+r)(1+D).$$

Similarly, the interim lenders' participation constraint (16) is now

$$N_{s_1}(\widehat{p}) = (1+r)\frac{M}{\widehat{\Pr}(R_1 \mid s_1)}.$$

Substituting these expressions into the bank's payoff function (26), differentiating with respect to p, and taking into account the rational expectations constraint $\widehat{p} = p$ and the definition (9) of $H(p)$ gives the condition

$$H(p) = \frac{(1+r)^2(1+D) - (1+r)\lambda(1-q)pR(p)}{q}.$$

Since the derivative with respect to r of the right-hand side of this expression is positive, it follows that an increase in r will lower the bank's choice of p.

We conclude that a tightening (loosening) of monetary policy will always increase (decrease) risk-taking.[18] Thus, in contrast with most discussions on search for yield, which focus on the low interest rates driven by either capital inflows from Asian countries or the policy of the Federal Reserve, our results indicate that what really matters is the relationship between banks' funding costs and the profitability of their investments.[19] This may be illustrated with reference to our parametric example, where $H(p) = 2ap(1-p)$. The choice of p in the optimal contract with long-term debt depends on the ratio $(1+r)^2/a$ (and a similar result obtains in the case of the optimal contract with risky short-term debt), so changes in the profitability parameter a have the opposite effect as changes in the interest rate r. This suggests that *the search for yield phenomenon should not be related to low interest rates, but to low spreads between the (risk-adjusted) returns of banks' assets and liabilities.*

7   Concluding Remarks

This paper presents a model of the maturity of a bank's uninsured debt. We consider a single bank that borrows funds and chooses afterwards the riskiness of its assets. This moral hazard problem causes excessive risk-taking. Short-term debt may act as a disciplinary device when the lenders observe some interim signal of the assets' risk, but it may lead to inefficient liquidation. We characterize the conditions under which short- and long-term debt are feasible, and show circumstances under which only short-term debt is feasible and under which short-term debt dominates long-term debt when both are feasible. These cases obtain when the profitability of the bank's investments is sufficiently low, and when the liquidity of the markets for the resale of bank assets and the quality of the lenders' information are sufficiently high. We also show that to ensure the credibility of the liquidation threat the optimal contract with short-term debt may involve raising more than the cost of the investment and paying the difference as an up-front dividend.

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[18]The positive relationship between banks' funding costs and their portfolio risk should not be surprising, since as noted above it is a straightforward implication of the analysis in Stiglitz and Weiss (1981).

[19]One notable exception is Shin (2010): "The phenomenon of search for yield often appears in the late stages of a boom as investors migrate down the asset quality curve as *risk spreads* (my italics) are compressed."

These results provide a rationale for the reported widespread use of wholesale short-term debt before the 2007-2009 financial crisis. Our model suggests that a reduction in the profitability of banks' investments and an increase in the liquidity of global asset markets and in the opacity of banks' balance sheets are consistent with a change in the optimal funding strategy from long- to short-term debt, possibly with an increase in payouts.

The paper entirely focuses on debt finance, abstracting from the possibility of funding the bank with equity. Since equity finance would ameliorate the banks' risk-shifting incentives, adding this possibility would require the introduction of a differential cost of equity; otherwise we would end up with 100% equity. This might require modeling the possible conflicts of interest between managers and shareholders, and distinguishing between inside and outside equity, which would merit a separate paper.

It should also be noted that the model focuses on the optimal behavior of a single bank, ignoring general equilibrium effects, which could be relevant for a welfare analysis. For example, as examined by Stein (2012), short-term debt together with correlated shocks may generate fire-sale externalities at the rollover date that may justify liquidity regulation or other public policies. In the context of our model, this means that the liquidation costs would depend on the maturity structure of banks' debt: in particular, more short-term debt would lead to more liquidations and hence lower liquidation values.[20] Thus, the equilibrium would be characterized by an endogenous liquidation cost. Another possibility, following the neglected risk idea of Gennaioli, Shleifer, and Vishny (2012), would be that banks underestimate the true liquidation costs, which would lead them to choose too much short-term debt.[21] Modeling these general equilibrium effects, with or without behavioral biases, is a topic for future research.

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[20]We may conjecture that the equilibrium value of the recovery rate λ would be at the boundary between the regions in which long-term debt and short-term debt are optimal, with some banks choosing the former and others the latter.

[21]For example, banks could choose their debt maturity structure on the basis of current values of the recovery rate λ, which may be higher than those following the realization of a tail event.

Appendix

Proof of Lemma 1 The participation constraint (6) in the optimal contract must hold with equality, for otherwise the dividend D could be increased without changing B and p, improving the bank's payoff. Then, suppose that $D = pB - 1 > 0$ and consider the effect on the bank's payoff of a change in the face value B, which is

$$\frac{d}{dB} \left[(pB - 1) + p \left(R(p) - B \right) \right] = \frac{d}{dB} \left[pR(p) - 1 \right] = B \frac{dp}{dB} < 0,$$

where we have used the first-order condition (7) and the result $dp/dB < 0$. This means that whenever the constraint $D \geq 0$ is not binding, reducing the face value B would increase the bank's payoff, which proves the result. $\square$

Proof of Proposition 1 Suppose that there exist p_1 and p_2, with $p_1 < p_2$, such that $H(p_1) = H(p_2) = 1$. Since the function $pR(p)$ is increasing in p in the interval $(0, p^*)$ we have

$$p_1 \left(R(p_1) - B_1 \right) = p_1 R(p_1) - 1 < p_2 R(p_2) - 1 = p_2 \left(R(p_2) - B_2 \right),$$

so the contract with $B_1 = 1/p_1$ is dominated by the contract with $B_2 = 1/p_2$. Hence, if equation (8) has multiple solutions, the optimal contract with long-term debt is characterized by the one with the highest probability of success. $\square$

Proof of Proposition 2 By Proposition 1, if financing the bank with long-term debt is feasible, then the optimal contract with long-term debt is characterized by the highest solution p_L to the equation $H(p) = 1$. This solution also characterizes the optimal contract with safe short-term debt if the initial debt is rolled over in the bad state s_0, that is if $q \leq q(p_L)$. We next show that if this condition is violated, no other solution to the equation $H(p) = 1$ will satisfy it. Let $p_1 < p_L$ be one such solution. To prove that $q(p_1) < q(p_L)$ it suffices to show that for $p_1 \leq p \leq p_L$ we have

$$\frac{dq(p)}{dp} = \frac{2(1 - p) \left(pR(p) \right)' + pR(p) - 1}{\left[1 + p(R(p) - 2) \right]^2} > 0.$$

But the first term in the numerator of this expression is positive because $(pR(p))' > 0$ for $p < p^*$ (and, by Proposition 1, $p_L < p^*$), and the second term is greater than 1 because $pR(p) \geq p_1 R(p_1) > 1$ for $p_1 \leq p \leq p_L$. $\square$

Proof of Proposition 3 Since the bank's payoff (33) is increasing in p, the optimal contract is characterized by the highest value of p that satisfies conditions (28) and (31) for some $D \geq 0$, which gives (36). Since $H(p)$ in (9) is positive for $0 < p < p^*$ and satisfies $H(0) = H(p^*) = 0$, and $F(p, q, \lambda, D)$ in (29) is decreasing and convex in p for $0 < p < p^*$ and is increasing in D, it follows that reducing the initial dividend D will increase the highest value of $p \in [0, p^*]$ that satisfies condition (28), except when such value is p^*. But in this case $F(p^*, q, \lambda, D) = H(p^*) = 0$ implies $1 + D - \lambda(1 - q)p^* R(p^*) = 0$, which in turn implies that the no refinancing in the bad state condition (31) is violated. Hence the optimal contract is characterized by the lowest value of $D \geq 0$ that satisfies $D \geq G(p, q, \lambda)$, which gives (34). Finally, the face value M_S of the initial debt in the optimal contract is obtained by solving for M in the participation constraint (25), taking into account the rational expectations constraint $\widehat{p} = p$, which gives (35). $\square$

Proof of Proposition 4 It only remains to characterize the lower boundary $\lambda_0(q)$. Suppose first that financing the bank with risky short-term debt is feasible for the pair (q, λ), and that the corresponding optimal contract is such that $D_S = 0$. By (37) we have $\partial D_{HF}(p, q, \lambda)/\partial \lambda > 0$, which implies that for some low value of the recovery rate λ, denoted $\lambda_0(q)$, the equation $D_{HF}(p, q, \lambda) = 0$ may only have a (tangency) solution, so for lower values of λ it will have no solution. And if $D_{HF}(p, q, 0) > 0$ for some p we set $\lambda_0(q) = 0$.

Suppose next that financing the bank with risky short-term debt is feasible for the pair (q, λ), and that the corresponding optimal contract is such that $D_S > 0$. By our previous results, reducing the recovery rate λ shifts down the functions $D_{HF}(p, q, \lambda)$ and $G(p, q, \lambda)$ without changing the value p_S at which they intersect, which implies that for some low value of λ we may have $D_{HF}(p_S, q, \lambda) = G(p_S, q, \lambda) = 0$. Then, if there is no $p < p_S$ such that $D_{HF}(p, q, \lambda) > 0$, this value of λ defines the boundary of the feasible region, with $\lambda_0(q) = \lambda_1(q)$. Otherwise, the boundary will be characterized by a lower value of λ, with $\lambda_0(q) < \lambda_1(q)$. Finally, if $D_{HF}(p_S, q, 0) = G(p_S, q, 0) > 0$ we set $\lambda_0(q) = \lambda_1(q) = 0$. $\square$

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