THE EFFECT OF USAGE UNCERTAINTY ON MOBILE PHONE PACKAGE SIZE AND SUBSCRIBERS' CHOICES

by

R. Iyengar
O. Koenigsberg
E. Muller

Working Paper No 16/2010 December 2010

Research No. 04800100

This paper was partially financed by the Henry Crown Institute of Business Research in Israel.

The Effect of Usage Uncertainty on Mobile Phone Package Size and Subscribers' Choices

Raghuram Iyengar
The Wharton School
University of Pennsylvania, Philadelphia
riyengar@wharton.upenn.edu

Oded Koenigsberg
Columbia Business School
Columbia University, New York, NY 10027
ok2018@columbia.edu

Eitan Muller
Stern School of Business
New York University, New York, NY 10012
Leon Recanati Graduate School of Business Administration
Tel Aviv University, Tel Aviv, Israel 69978
emuller@stern.nyu.edu

November, 2009

The Effect of Usage Uncertainty on Mobile Phone Package Size and Subscribers' Choices

Abstract

Wireless services typically charge their subscribers based on a three-part tariff package that consists of a fixed monthly fee, an amount of free minutes per month (bucket size), and an overage price that is charged for all minutes in excess of the stated number of free minutes. For a subscriber, with a constant known consumption rate, the choice of a package is straightforward. However, as in many other markets, mobile phone subscribers confront uncertainty regarding their usage. In this paper, we study the role of such usage uncertainty with respect to minutes on both a service provider's package design and a subscriber's choice of package. Specifically, we study how usage uncertainty affects a firm's decisions about package size when it offers calling plans. In addition, from a subscriber's perspective, we study the effect of uncertainty on whether a subscriber stays with a plan or upgrades to another.Our main results, (analytical and empirical) are that the effect of variation in usage on the subscriber's optimal choice of free minute allotment and service package is conditional on whether the subscriber expects to use less than the entire bucket of minutes purchased. When this happens, the service provider's optimal bucket size decreases as a function of the subscriber's air-time variance. In addition, surprisingly, a subscriber's usage variance, after controlling for mean usage, is actually a *negative* predictor of the subscriber choosing to upgrade.

1. Introduction

At the end of 2008, 4.1 billion people around the world were using mobile phones (International Telecommunications Union, 2008). Mobile phone services were commercially launched in Scandinavia in 1981 and since then have become a part of the everyday life of about half of the world's population in more than 200 countries (Peres, Muller and Mahajan, 2009). Over 90% of all subscribers in the United States (there are more than 270 million subscribers in the U.S. alone (Cellular Telecommunications Industry Association, 2008)) had contractual agreements with their mobile phone providers (McGovern, 2005). While these providers offer different types of programs, the most popular one is a three-part tariff package that consists of *a*) a fixed monthly fee, *b*) an amount of free minutes per month ("bucket size"), and *c*) a variable price, the "overage price," that is charged for all minutes used in excess of the stated number of free minutes.[1]

For a subscriber who knows her consumption rate and which does not change from month to month, the choice of a calling package is straightforward. However, as in many other markets, mobile phone subscribers confront uncertainty regarding their telephone usage time. Table 1 depicts monthly usage during a one-year period for a typical subscriber in our database (to be described shortly):

Table 1: Monthly usage in minutes (MOU) and choice of package – subscriber 1

Month	Oct 2001	Nov 2001	Dec 2001	Jan 2002	Feb 2002	Mar 2002	Apr 2002	May 2002	Jun 2002	Jul 2002	Aug 2002	Sep 2002
Package	1	1	1	1	1	1	1	1	1	1	1	1
MOU	21	40	60	99	200	168	135	93	104	26	30	0

This subscriber's use was highly variable, ranging from none to 200 minutes per month, but the user chose a package and stayed with it during the entire year. Table 2 depicts a

[1] There are two additional components (that we do not address) for each contract—the duration of the contract and the handset subsidy, which are usually related.

subscriber whose usage varies between 88 and 485 minutes and who upgraded her choice of package toward the end of the year (package 2 has more free minutes than package 1). This subscriber has a lower coefficient of variation (0.61) as compared to the subscriber in the earlier example (0.71). As these two examples show, variance in usage is prevalent and large and may have an effect on the choice of package by the subscriber.

Table 2: Monthly usage in minutes (MOU) and choice of package – subscriber 2

Month	Jun 2002	July 2002	Aug 2002	Sep 2002	Oct 2002	Nov 2002	Dec 2002	Jan 2003	Feb 2003	Mar 2003	Apr 2003	May 2003
Package	1	1	1	1	1	1	1	1	1	2	2	2
MOU	188	88	133	126	101	170	97	241	118	176	183	485

The main objective of our research is to study the role of subscriber uncertainty with respect to air-time minutes on both the service provider's package design and a subscriber's package choice. In particular, we investigate how this uncertainty affects a firm's decision regarding package size (in terms of free minutes) when it offers calling plans. In addition, we study the effect of this uncertainty from a subscriber's perspective since it affects whether she stays with a calling plan or switches to another plan with more free minutes (termed as upgrade).

In our model, subscribers maximize their utility while facing random demand (which depends on the price) for mobile phone use. Each subscriber has to take into account the fact that she might either not use all of the free minutes—might leave minutes on the table (termed as underage)—or might consume more than this allotment of free minutes (overage). The latter will force the subscriber to pay more than the fixed fee since overage minutes are costly. Given this framing of the problem, subscribers can be assumed to be facing a newsvendor type problem. They sign up for a package after paying a fixed fee, and can consume either more or less minutes than the package provides: If they consume less they leave some unused minutes and if they consume more they pay overage. For that reason, we incorporated the newsvendor framework (Ferguson and Koenigsberg, 2007) into the subscriber's utility function. A subscriber's utility is a function of both package characteristics - bucket size, the fixed fee, the overage price that the firm charges, and her usage characteristics (mean and variance of consumption rate). A subscriber chooses the package that maximizes her utility. In our model, a service provider has to make decisions about calling packages given uncertainty regarding subscribers' consumption rates. Specifically, the service provider has to set both the price of the overage minutes and the size of the bucket of free minutes (which dictates the fixed fee).

Intuitively, if the average number of minutes that subscribers consume remains constant and only the consumption variance increases, then one could argue that subscribers will respond by purchasing larger packages since overages are costlier than leaving minutes on the table. This is also the conclusion of Lambrecht, Seim, and Skiera (2007), who estimated individual-specific variances in usage uncertainty and, via a what-if analysis, showed that subscribers were more likely to switch to a flat-fee plan (that is, more likely to upgrade) with an increase in usage variance. However, our analysis indicates that the situation is more complex and this intuitive explanation should be reevaluated.

Our main results, both theoretical and empirical, are that the effects of variance in usage on the subscriber's optimal choice of a service package are *conditional* on whether the subscriber leaves minutes on the table. Thus the choice depends on whether subscribers expect to use less than the entire bucket of minutes they bought from the service provider. When this happens, the service provider's optimal bucket size decreases as a function of the subscriber's air-time variance. In addition, a subscriber's usage variance, after controlling for mean usage, is actually a *negative* predictor of a subscriber's choice of a plan upgrade. To understand this result, consider the tradeoff that a subscriber makes in choosing a package. While keeping a smaller package saves on the fixed cost, it loses on any overage that may occur. We show that the overall result of such a tradeoff, in equilibrium, is that the cost of under-consumption actually exceeds that of overconsumption. The subscriber then is worse off upgrading her package, as compared to either staying with the same package or downgrading, when her average consumption remains the same and there is an increase in her consumption variance.

The paper continues as follows: In Section 2 we review the literature on consumers' usage uncertainty under nonlinear pricing schemes. Section 3 reviews the basic model, Section 4 studies the effect of usage variance on choices made by the mobile network provider, and Section 5 discusses the effect of variance on choices made by subscribers. Section 6 describes the two sets of data we use for the empirical analysis, which is presented in Section 7. We conclude with managerial implications in the final section of the paper.

2. Nonlinear pricing schemes and usage uncertainty

A nonlinear pricing schedule refers to any pricing structure in which the total charges payable by customers are not proportional to the quantity of the services consumed. Such pricing schemes fall broadly into two categories: increasing block and decreasing block. In an increasing block pricing scheme, marginal (per unit) prices increase with quantity. In a decreasing block pricing scheme, marginal prices decrease with quantity. The increasing block tariff penalizes excess consumption of units. A recent application of a multi-tier increasing block tariff for conservation is the electricity tariff in California. After the financial crisis in 2001, the California Public Utilities Commission imposed a new five-tier increasing block structure (see Reiss and White

(2005, p. 875) for more details). A typical example of a decreasing block tariff is a quantity discount.

In the wireless industry, the typical pricing scheme is a special form of a two-tier increasing block. This type of pricing scheme, also termed a three-part tariff, consists of a fixed fee (F), an associated number of free minutes (T), and a per-minute rate (p) for any consumption in excess of the free minutes. Thus, this pricing scheme is a two-tier pricing scheme with the marginal price in the first tier set to zero.

With notable exceptions to be discussed shortly, most empirical marketing studies that have investigated nonlinear pricing have been concerned with two-part tariff pricing, e.g., Danaher (2002); Esseghaier, Gupta, and Zhang (2002); and Narayanan, Chintagunta, and Miravete (2007). Lambrecht, Seim, and Skiera (2007) used a simple example to show how such usage uncertainty can affect subscriber choice. They considered symmetric distributions of usage under a two-part and a three-part tariff. Under a two-part tariff, the expected bill is the same regardless of the degree of uncertainty about usage. This is not so under a three-part tariff. Under such pricing schemes, the greater the uncertainty regarding usage given the same level of mean usage, the higher the overall bill. This clearly suggests that, under a three-part tariff and more complex multi-part tariffs, subscribers' usage expectations can influence which service package they choose.

Several researchers have found evidence to support this hypothesis (Nunes, 2000; Lemon, White and Winer, 2002; Lambrecht and Skiera, 2006). For instance, Nunes (2000) explored the cognitive process of how people anticipate service usage and integrate their expectations of usage to choose between a flat-fee package and a measured (pay per use) package. He proposed that subscribers calculate a break-even number and then see whether the break-even point implies a choice of a flat-fee package or a measured package.

Other researchers have empirically investigated the effect of subscribers' usage uncertainty and learning on their package choices and subsequent consumption. For instance, Narayanan, Chintagunta, and Miravete (2007) analyzed data from an experiment conducted by South Central Bell in which customers had a choice between a flat rate and a two-part tariff. They found that subscriber learning is rapid when subscribers are under the two-part tariff scheme but slow while under the flat-fee package.

Iyengar, Ansari, and Gupta (2007) developed a model that captures subscriber learning and uncertainty within the context of three-part tariff pricing schemes. They found that subscriber learning can lead to a win-win situation for both subscribers and the firm—subscribers leave fewer minutes on the table while the firm sees an increase in overall customer lifetime value. More recently, Iyengar, Jedidi, and Kohli (2008) investigated how subscribers choose among three-part tariff packages. With data from a choice-based conjoint task using multi-part tariffs, they built an economics-based model to investigate how changes in the pricing scheme of packages coupled with subscriber usage uncertainty affected the probability of subscriber choices.

Lambrecht, Seim, and Skiera (2007) developed a discrete choice model of subscribers in a three-part tariff environment and estimated it using subscriber-level data on Internet usage in Germany. The empirical work shows that demand uncertainty is a key driver of choice among the packages. The subscribers' overall expenditures increase with their usage variance and thus they tend to choose packages with higher allowances.[2]

In this paper we study a service provider's decisions regarding calling plans to offer given uncertainty about subscriber consumption rates. The service provider has to decide the price of overage minutes and the size of the buckets of free minutes (the fixed fee depends on the number of minutes). Subscribers choose the package that maximizes their overall utility, which is a function of bucket size, the fixed fee, the overage price, and the mean and variance of their consumption rates. In the next section we provide a more formal description of the model.

3. Model

We assume that there is a single service provider (firm) in the market that offers a single package to the market. We later model and analyze a case in which the firm introduces multiple packages. Each package involves a three-part tariff to subscribers. The packages consist of three components: free minutes (or bucket size) represented by T, which indicates the number of free calling minutes in the package; a fixed fee, $f(T)$, that depends on the number of free minutes, T; and price, p, per minute for any usage that exceeds T. We assume that $f(T)$ is an increasing concave function of the number of free minutes, T. That is, $f'(T) > 0$ while $f''(T) < 0$.

We assume a unit mass market where subscribers generate an average per minute utility designated u. We further assume that a subscriber's demand, $z(\alpha, p)$, is a multiplicative function of two components: $z(\alpha, p) = \alpha\hat{g}(p)$ where α is a *uniform* random variable between $[a - v, b + v]$ for which a and b are the deterministic lower and upper bounds, respectively, of

[2] In the final section of the paper, we compare our results with these results.

minutes used by subscribers and the parameter v captures the uncertainty (variation in usage).. Given our specification of usage distribution, an increase in the parameter v will increase the variance without changing the mean of the distribution. Recall that the variance of the uniform distribution on the interval $(a - v, b + v)$ is given by $(b-a+2v)^2/12$ while the mean is given by $(a + b) / 2$, which is independent of v. We, of course, assume that $b > a > v > 0$. Furthermore, we assume $\hat{g}(p)$ is the following function: $\hat{g}(p) = \begin{cases} 1 & when \quad \alpha \le T \\ g(p) & when \quad \alpha > T \end{cases}$, where $g(p)$ is the subscriber demand function of price p, which becomes effective only when consumption exceeds the free minutes, T. We assume that $g(p)$ is a decreasing function of the price and that the following condition has to be satisfied: $2(g')^2 - gg'' > 0$.[3]

Figure 1 demonstrates subscriber demand below and above T. Note that our demand function is uniformly distributed below T. However, above T, the demand is equal to a product of α (still uniformly distributed) and $g(p)$. This creates a demand function that is still uniformly distributed albeit with a mean that is a decreasing function of overage price.

Figure 1: Single package

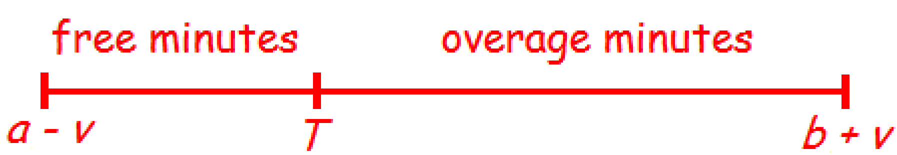


Table 3 summarizes the parameters and decision variables that we use in the paper.

[3] We will discuss this condition in the next section.

Table 3: Summary of parameters and decision variables

free minutes included in the package	*T*
fixed monthly fee in the package	*f(T)*
price per airtime minute for any consumption above the free minutes	*p*
overage minutes demanded by the subscriber when the price per minute is *p*	*g(p)*
per minute utility of the subscriber	*u*
deterministic lower and upper bounds of the uniform distribution of the subscriber's air time	*a, b*
a parameter that correlates with the variance of the subscriber's air time	*v*
cost per airtime minute of the service provider	*c*

Consider a subscriber with demand that is generated from a uniform random variable between $[a - v, b + v]$ and with utility of u per minute. The expected overall utility of such a subscriber from the package $[T, f(T), p]$ is:

$$(1)\ U\left[T, f(T), p\right] = -f(T) + \frac{1}{b-a+2v}\left[\int_{a-v}^{T} u\alpha\, d\alpha + \int_{T}^{b+v}\left[uT + (u-p)(\alpha - T)g(p)\right]d\alpha\right].$$

Note that the first term in Equation 1, $(-f(T))$, represents the fixed fee paid by the subscriber; the second term, $\left[\int_{a-v}^{T}\frac{u\alpha}{b-a+2v}d\alpha\right]$, represents the utility from the free minutes; and the third term, $\int_{T}^{b+v}\frac{\left[uT+(u-p)(\alpha-T)g(p)\right]}{b-a+2v}d\alpha$, represents the utility from the paid minutes. In this case, the subscriber does not pay for the first T minutes while enjoying the per-minute utility u, but he pays for each minute above T and thus enjoys a per-minute utility of $(u - p)$. The overall package utility will play a major role in Section 5, where subscribers have a choice between different packages. In a similar manner, the mobile service provider's profit from a segment of subscribers who register for package $[T, f(T), p]$ is given by:

$$(2)\ \pi = f(T) + \frac{1}{b-a+2v}\left[-\int_{a-v}^{T} c\alpha\, d\alpha + \int_{T}^{b+v}\left[-cT + (p-c)(\alpha - T)g(p)\right]d\alpha\right],$$

where c is the cost per airtime minute of the service provider.

4. Effect of usage variance on choices made by the service provider

As described earlier, the package consists of three components: T, $f(T)$, and p. To derive the optimal price, we differentiate the profit (Equation 2) with respect to p and find the following implicit equation:

$$(3) \qquad g(p)+(p-c)g'(p)=0\,.$$

It is easy to verify that this equation will have a unique optimal price as long as the objective function is quasi-concave with respect to the price. That is, the following condition has to be satisfied:

$$(4) \qquad 2(g')^2-gg''>0\,.$$

We define $\overline{c}=c+(p-c)g(p)$ where $\overline{c}$ measures the cost of an additional minute added to the free time T. To see this, note that if the subscriber does not use all of her minutes then the firm's marginal cost of one minute added to T is zero. If, however, the subscriber usage that month exceeds the free time T, then the cost to the firm of one additional free minute is the cost of producing the minute plus the opportunity cost of not being able to sell this minute but rather give it away.

Next, we consider the optimal free minutes that a service provider should offer. The first-order conditions of the profit function (Equation 2) with respect to T yield the following implicit function for T:

$$(5) \qquad f'(T)(b-a+2v)+\overline{c}(T-b-v)=0\,.$$

The second-order condition with respect to T yields $\partial^2\pi / \partial T^2 = f''(T)(b-a+2v)+\overline{c}$, which is negative if and only if $f''(T) < -\overline{c}/[(b-a+2v)]$. To find the effect of usage variance on the optimal free minutes, we differentiate equation (5) with respect to v, which yields $2f'(T)+(b-a-2v)f''(T)\,\partial T^* / \partial v + \overline{c}\,\partial T^* / \partial v - \overline{c} = 0$. Solving for $\partial T^* / \partial v$, we get the following equation:

(6) $$\partial T^* / \partial v = \frac{\overline{c} - 2f'(T)}{\overline{c} + f''(T)(b-a+2v)}.$$

The denominator is negative for the relevant parameter set (which satisfies the second-order condition as we showed previously) and therefore the derivative in (6) is negative when the numerator is positive; i.e., $f'(T^*) < \overline{c}/(2)$. Since we only need quasi-concavity of the objective function with respect to T, we can substitute the solution for T^* from equation (5) into this condition to derive the following condition for $\partial T^* / \partial v$ to be negative:

(7) $$T^* > \frac{a+b}{2}.$$

We therefore have the following result: $\partial T^* / \partial v < 0$ *if and only if* $T^* > (a+b)/2$; that is:

Result 1: ***When the average subscriber "leaves minutes on the table" (the mean of the subscriber's air time is less than the number of free minutes (T)), then the optimal number of free minutes (T^*) decreases as a function of the subscriber's air-time variance (v).***

To understand the intuition behind the result, one must realize that the trade-off in adding one minute of free time is that the firm gains from the higher fixed fee but loses the overage fee for that minute. Thus the intuition is that when T^* is large, $f'(T^*)$ is small (since $f''(T) < 0$). When the variance of a subscriber's usage decreases but her mean consumption remains fixed, there are less overage minutes and less underage minutes. But when T^* is large, the firm loses more from adding one unit of free time than it gains in the overage fee. This is reversed when T^* is small. The firm gains more from adding one unit of free time than it loses in the overage fee.

5. Effect of usage variance on choices made by the subscriber

In this section we study the effect of usage variance on choices made by the subscriber and thus we relax the assumption of a single package. We assume that there are two subscriber segments ($i = (1,2)$) that can choose between two packages ($j = (1,2)$), each with a subscriber usage time, z_i, that, as in the previous section, is a multiplicative function of two components: $z_i = \alpha_i \hat{g}_i\left(p_j\right)$ where α_i is a uniform random variable between $[a_i - v_i, b_i + v_i]$ and $\hat{g}_i\left(p_j\right)$ is the following function: $\hat{g}_i\left(p_j\right) = \begin{cases} 1 & when \quad \alpha_i \le T_j \\ g_i\left(p_j\right) & when \quad \alpha_i > T_j \end{cases}$. Thus the expected utility, U_i^j, of a subscriber from segment i using package j is given by:

$$(8)\quad U_i^j = -f_j(T_j) + \frac{1}{b_i - a_i + 2v_i}\left[\int_{a_i - v_i}^{T_j} u_i \alpha_i \, d\alpha_i + \int_{T_j}^{b_i + v_i}\left[uT_j + (u - p_j)(\alpha - T_j)g(p_j)\right]d\alpha_i\right]$$

and the mobile phone provider's profit, π_i^j, from segment i when subscribers from this segment choose package j is given by:

$$(9)\quad \pi_i^j = f_j(T_j) + \frac{1}{b_i - a_i + 2v_i}\left[-\int_{a_i - v_i}^{T_j} c\alpha_i \, d\alpha_i + \int_{T_j}^{b_i + v_i}\left[-cT_j + (p_j - c)(\alpha - T_j)g(p_j)\right]d\alpha_i\right].$$

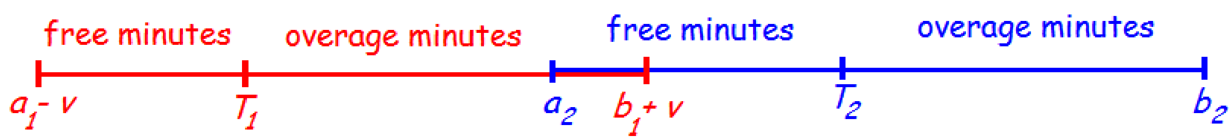
Assuming that subscribers from segment $i=1$ choose package $j=1$ and that subscribers from segment $i=2$ choose package $j=2$, the overall firm's profit is given by:

$$(10)\ \Pi = \sum_{i=1}^{2}\{f_i(T_i) + \frac{1}{b_i - a_i + 2v_i}\left[-\int_{a_i - v_i}^{T_i} c\alpha_i \, d\alpha_i + \int_{T_i}^{b_i + v_i}\left[-cT_i + (p_i - c)(\alpha - T_i)g(p_i)\right]d\alpha_i\right]\}.$$

Recall that Π is the optimal profit if and only if the following individual rationality constraints are satisfied: $U_1^1 > U_1^2$ and $U_2^2 > U_2^1$.

Using Equation (8), we can compute a subscriber's utility from the smaller package and compare it to the utility achieved from the larger package (see Figure 2).

Figure 2: Two packages



Note that in this section we focus on the case in which subscriber segments are identified and therefore we analyze a case in which $v_2 = 0$ and $v_1 = v$. We further assume that v is low enough that $b_1 + v < T_2$. From this figure, we can deduce that a low-use subscriber (with utility of U_1^1) who upgrades to the second larger package (with utility of U_1^2) will derive utility only from the free minutes of the larger package since her usage will still be uniformly distributed between $a_1 - v$ and $b_1 + v$. Thus,

$$(11)\ U_1^2 = -f_2(T_2) + \frac{1}{b_1 - a_1 + 2v}\int_{a_1 - v}^{b_1 + v} u_1 \alpha \, d\alpha = -f_2(T_2) + u_1(b_1 + a_1)/2\,.$$

Since U_1^2 does not depend on v, we can compute only dU_1^1 / dv:

$$(12)\ dU_1^1 / dv = -f_1' \partial T_1 / \partial v + \frac{[(p_1 - u)g(p_1) + u](b_1 + v - T_1)}{b_1 - a_1 + 2v}\left(\partial T / \partial v_1 - \frac{T_1 + v - a_1}{b_1 - a_1 + 2v}\right).$$

Clearly, a subscriber will be less inclined to upgrade her package as a result of an increase in usage variance if the utility for her current package actually increases with the variance. When would this happen? We find that a subscriber's utility increases with the variance when $\partial T^* / \partial v < 0$ [4] and the derivative of the fixed fees with respect to time *T*, f', satisfies the following condition:

$$f_1' > \frac{[(p_1 - u)g(p_1) + u](b_1 + v - T_1)}{(b_1 - a_1 + 2v)^2}\left(b_1 - a_1 + 2v - \frac{T_1 + v - a_1}{\partial T_1 / \partial v}\right) \tag{13}$$

Thus the subscriber is less likely to upgrade with an increase in her usage variance. Similarly, for a package downgrade, if we investigate the symmetrical case in which $v_2 = v$ and $v_1 = 0$ and consider downgrading, then, since the utility increases with variance, the same result holds for downgrading. Since we know from *Result 1* that $\partial T^* / \partial v < 0$ *if and only if* $T^* > (a+b)/2$, we can conclude with the following result.

Result 2: ***The subscriber's variance in usage, after controlling for mean usage, is a negative predictor of a subscriber choosing to upgrade or downgrade a package as long as the optimal number of offered free minutes is larger than the mean of the distribution of air time of subscribers and the derivative of the fixed fees with respect to T is large enough.***

This conditional result indicates that it is not always the case that an increase in usage variance will make a subscriber upgrade their plan. To understand this result, one must consider the tradeoff that a subscriber makes in choosing a package. While keeping a smaller package saves on the fixed cost, the subscriber loses on the overage fee. Note that subscribers' utility is a

[4] We consider the case in which $\partial T^* / \partial v < 0$, which is supported by our empirical analysis. A parallel condition for the case in which $\partial T^* / \partial v > 0$ can be derived in a similar way.

function of the value they generate from consuming minutes (not purchasing minutes) minus the fixed fees they pay.

Consider a subscriber who purchases a package in which the amount of free minutes exceeds her expected usage rate. Such a subscriber can consume more (up to the package limit) and still pay the same price (only the fixed fee). Thus, with an increase in the variance of consumption, the subscriber faces one of two scenarios—either lower or higher consumption than average. If realized consumption is lower, a subscriber is worse off if she upgrades as she pays a higher fixed fee while consuming less. A subscriber who stays with the same package is better off (as compared to the one who upgraded) as, while she generates less value from the consumed minutes, she still pays the same fee. What happens if the realized consumption is higher? In that case, a subscriber who upgrades her package does generate more utility from usage but pays more as well. In contrast, a subscriber who stays with the same package still has some free minutes to use before she is penalized with the overage price. Thus, it is not immediately obvious whether a subscriber will be better off with an upgrade.

As our result shows, we find that the outcome of such tradeoffs is that, in equilibrium, the cost of under-consumption actually exceeds that of overconsumption. A subscriber, therefore, is worse off upgrading, as compared to staying with the same package, when there is an increase in her consumption variance and her average consumption remains the same.

6. Data

We use two data sets in the empirical analysis. We briefly describe the data sets.

Data set 1: The first data set contains information on subscriber-level monthly billing records of 12,500 customers from September 2001 to May 2003 that were sent by a wireless service provider. There is no left truncation in the data—we have monthly billing data for each customer from their first month with the provider. In addition, these customers were not bound by an annual contract and could change their calling plans on a monthly basis.

The monthly billing data provide information on the service package a customer was using and her monthly consumption of minutes. Typically, the included (free) minutes are of two types: off-peak/weekend minutes and peak minutes (bucket size). Usually, off-peak/weekend minutes are free. Our data set contains information on customers' monthly peak minutes of usage.

There are four calling packages in the data set. Table 4 describes the characteristics of the packages, which are numbered in ascending order of included free peak minutes. We refer to a switch from a package with fewer allowed minutes to one with more as an upgrade and a switch from a package with more allowed minutes to one with less as a downgrade.

Table 4: Package Characteristics and Summaries

Package 4	**Package 3**	**Package 2**	**Package 1**	**Variables**
50	40	35	30	Monthly Fee ($)
500	350	300	200	Free Minutes
0.40	0.40	0.40	0.40	Overage Rate ($/minute)
320	215	185	117	Average Consumption
229	155	108	104	Std. Deviation
0.72	0.72	0.58	0.89	Coefficient of Variation
7.25	38.62	3.19	50.94	Share (%)

The data set contains an average of sixteen months of data for each individual. Of these sixteen months, subscribers on average experience thirteen months (81%) of underage and three months (19%) of overage. Thus, there is much greater underage than overage in the data. Table 4 also shows the share of the provider's customers who have each package, average monthly consumption of minutes, usage variance, and the underage and overage under the packages.

Around 20% of customers (2,507) changed their packages during the period covered. In addition, of the total number of package changes, 72% were upgrades (a customer moved from a package with less free minutes to one with more); the rest (28%) were downgrades. The overall churn rate is about 39% with 4,914 customers out of the 12,500 leaving the company's service during the data period. Of these 4,914 customers, 41% left after being on package 1 as their last chosen package and about 57% were on package 3 or package 4 before they decided to leave the company (the remaining were on package 2).

Data set 2: This data set contains the characteristics of all of the packages offered by four major U.S. wireless service providers: Cingular, Verizon, T-Mobile, and Sprint. For each package, we have information on its monthly access fee, the number of free minutes included, and the overage rate. This data set was collected in 2007 by collating all of the package-related information from the websites of these service providers. This data set is available from the authors upon request. In the following section, we will use this dataset to test our assumption that the fixed fee is an increasing concave function of the free minutes.

7. Empirical analysis

We divide the empirical analysis into two sections. The first section shows empirical justification for our main model assumption. The second offers empirical evidence for the model results.

7.1. Concavity assumption

We assume that the fixed fee, $f(T)$, is an increasing concave function of the free minutes, T. We use the information in data set 2 to provide evidence for this assumption. To establish the link between the monthly fee and number of free minutes we use the following equation

$F = \tilde{\delta} T^{\beta}$ and check whether $\beta<1$. We estimate the following regression using the packages offered by each provider (where $\delta = \log(\tilde{\delta})$):

$$\log(F) = \delta + \beta \log(T) + \varepsilon, \qquad \varepsilon \sim N(0, \sigma^2).$$

Here, F refers to the monthly fixed fee, T is free minutes, and δ, β, and σ are parameters to be estimated. To show a concave relationship, the parameter β must be less than 1.0. Table 5 shows the results for this regression by provider. We note that in each case the parameter β is significantly less than 1.0.

Table 5: Concavity Assumption: $\log(F) = \delta + \beta \log(T)$

Verizon	**Sprint**	**T-Mobile**	**Cingular**	**Coefficient**
0.02	0.03	0.54	0.06	Intercept (δ)
(0.15)	(0.12)	(0.23)	(0.13)	
0.65	0.59	0.39	0.59	Slope (β)
(0.05)	(0.04)	(0.07)	(0.04)	

7.2. Model results

Our key result is that a subscriber's variance in usage, after controlling for mean usage, is a negative predictor of the subscriber choosing to upgrade as long as the number of free minutes offered in the plan is *larger* than the mean of the distribution of air time of subscribers.

Our data set is such that more than 80% of the observations in our data set show an underage. On further analysis, we find that there are only 29 subscribers out of 12,500 who never experienced any underage during their tenure with the provider. Given this low percentage of customers who do not experience any underage, we use the entire data set to provide empirical evidence for the key result. In addition, recall that 4,914 customers out of the 12,500 leave the company's service during the data period. The results reported here use the data from both churners and non-churners (robustness checks are reported at the end of this section).

For the main analysis, we construct the mean (termed as "Mean_Usage") and variance (termed as "Var_Usage") based on each subscriber's past usage in two ways. In the first analysis, we construct past average usage and past variance in usage based on a moving window. For instance, if we use a window length of three months, then the usage data from the preceding three months is used to compute average usage and variance in usage. Note that usage in each of the past three months is weighted equally. In the second analysis, we allow for recent usage to be weighted more than past usage. We do so by using weighted usage. Thus, for a month *t*, a past month *t-k* (*k=1...t-1*) and discount factor δ, weighted usage ($q_{it-k}^{weighted}$) is constructed using actual usage for that month (q_{it-k}) as follows:

$$q_{it-k}^{weighted} = \delta^{k-1} q_{it-k}, \quad k = 1,\dots,t\text{-}1.$$

Next, using weighted past usage, we construct its mean and variance based on a moving window. For instance, if we use a window length of three months, then the weighted usage data (based on a specific decay factor) from the preceding three months is used to compute average usage and variance.

Note that the data contains information on which package (i.e., Package 1, 2, 3 or 4) a customer chooses each month and her subsequent monthly usage. We can, therefore, model a customer's monthly package choice as function of her package characteristics and her past usage. Our key result, however, relates a subscriber's usage characteristics to her decision to upgrade a package. Thus, to corroborate the analytical results, we only model a customer's monthly decision to upgrade her package (a binary decision) and not her monthly package choice.

In our data set, there are no contractual agreements and subscribers are free to change their packages on a monthly basis. Thus, we assume that in the beginning of each month, subscribers make a decision about whether they wish to upgrade. Thus, for subscriber *i* and month *t,* we posit the following utility for upgrading her package:

$$U_{it} = \gamma_0 \ + \ \gamma_1(\text{Mean Usage})_{it\text{-}1} \ + \gamma_2 \ (\text{Var Usage})_{it\text{-}1} \ + \varepsilon_{it}.$$

Here, for subscriber *i*, the variable Mean Usage is her past average usage and Var Usage is the variance of her past usage (based on either weighted or unweighted usage). The parameters γ_1 and γ_2 capture the how the mean and variance of usage, respectively, affect the utility of upgrading her package. Note that the parameters are assumed to be common across subscribers. Later, we show that our main results are robust to the inclusion of unobserved heterogeneity. The error ε_{it} is assumed to be logistically distributed and independent across subscribers and time. We use maximum likelihood methods to estimate the model.

We begin by discussing the results from an analysis that uses equally weighted past usage for constructing the mean and variance. First, we investigate which window length provides the best model fit. To do so, we calculate the likelihood for models with past covariates based on various window lengths. Next, we use the Akaike Information Criterion (AIC) to find the window length that provides the best model fit. Table 6 shows the results. A smaller AIC indicates a better model. Note that a model with a moving window of 1 month assumes that, for each subscriber and month, their previous month's usage is their average usage and the usage variance is set to 0. The results suggest that a model with a window length of two months (i.e., past 2 months) gives the best AIC. In addition, the finding that AIC for a model using a window of one month is worse indicates that an inclusion of variance in past usage is important.

Table 6: AIC values for models with varying window lengths

AIC	Window Length
21204.53	1
21084.17	**2**
21119.13	3
21188.51	4
21239.99	5

Table 7 shows the parameter estimates for this model. The estimates show that, after controlling for average past usage, the variance in usage is significant and a negative predictor of package upgrade behavior.

Table 7: Parameter estimates for model with window length of two months

Coefficient	Parameter
–5.21 (0.03)	Intercept
0.003 (0.0001)	Past Mean (γ_1)
–2.93E –06 (8.51 E-07)	Past Variance (γ_2)

*All parameters are significant at $p < 0.001$.

Next, we discuss the results from an analysis that uses weighted usage for constructing the mean and variance of past usage. As before, we investigate which window length and what value of decay factor provides the best model fit. To do so, we calculate the AIC for models with past covariates based on various window lengths and several values of the discount factor. Table 8 shows the results. We again find that a window length of two months provides the best model fit albeit coupled with a discount rate of 0.15. Table 9 shows the parameter estimates for this model. The results corroborate the findings from the first analysis and indicate that the variance in usage is significant and negative predictor of upgrade behavior.

Our result is quite robust to variations in model specification. First, we added unobserved heterogeneity in the model. Note that as many subscribers do not upgrade during the data period, we cannot estimate subscriber-specific fixed effects (e.g., Chamberlain 1980). Thus, we included a normally distributed random effect on the intercept of the utility function. We estimated the model using adaptive Gaussian quadrature in SAS NLMIXED and found variance estimates of 10^{-8}, the boundary value. This documents the absence of detectable effects of unobserved heterogeneity.

Second, we included a dummy for each month. This has two advantages. First, it captures the effect of any system-wide time-varying factors that may affect the upgrade behavior. The second advantage of including monthly dummies is that it provides a nonparametric control for duration dependence. Again, our results were robust.

Third, we checked for the presence of serial correlation within subscribers to investigate whether the results are robust to our assumption of utility errors being uncorrelated across time. We estimated the model using the Generalized Estimating Equations method (GEE) and found an estimated AR(1) coefficient of -0.01. There was no change in the substantive results.

Fourth, we removed all data from subscribers who churned during the data period and only considered the data from subscribers who remained with the service provider. The results were similar to our main analysis.

Finally, to test the stability of our results to the occurrence of overage, we also estimated the model with data from only those customers who experienced at least one month of overage (8,169 customers), at least two months of overage (6,105 customers), or at least three months of overage (4,667 customers). As before, we find that, after controlling for average past usage, the variance in usage is significant and a negative predictor of package upgrade behavior.

Table 8: AIC values for models with varying window lengths and discount rates

AIC	Discount Rate	Window Length
21003.91	0.05	2
20988.96	0.10	2
20987.81	**0.15**	**2**
20994.70	0.20	2
21064.39	0.05	3
21036.32	0.10	3
21019.32	0.15	3
21012.15	0.20	3
21012.41	0.25	3
21017.77	0.30	3
21203.19	0.05	4
21164.93	0.10	4
21130.71	0.15	4
21103.31	0.20	4
21084.39	0.25	4
21074.18	0.30	4
21071.76	0.35	4
21075.61	0.40	4
21320.67	0.05	5
21287.29	0.10	5
21253.01	0.15	5
21219.45	0.20	5
21188.52	0.25	5
21162.35	0.30	5
21142.57	0.35	5
21130.37	0.40	5
21125.98	0.45	5
21128.65	0.50	5

Table 9: Parameter estimates with window length of two months and discount rate of 0.15

Coefficient	Parameter
–5.52 (0.05)	Intercept
0.01 (0.0005)	Past Mean (γ_1)
–2.01E –05 (2.23 E-06)	Past Variance (γ_2)

*All parameters are significant at $p < 0.001$.

8. Conclusions

In this paper we study the role of subscriber uncertainty with respect to air-time minutes. Specifically, we study how this uncertainty affects the firm's decisions regarding the size of packages (in terms of free minutes) to offer in its calling programs. In addition, we study the effect of this uncertainty from the subscriber's perspective, examining how it affects a subscriber's decision to stay with an existing plan or upgrade to another package.

We develop an analytical model that considers the effect of a subscriber's usage variance on both a service provider's decision about the characteristics of a package and a subscriber's decision to upgrade her package. Our key result is that a subscriber's variance in usage, after controlling for mean usage, is a negative predictor of the subscriber upgrading her package as long as the optimal number of free minutes is larger than the mean of the distribution of air time of subscribers. This result relies on the trade-off that subscribers make between keeping a smaller package, which contains the cost of the fixed fee, and switching to a larger package that reduces overage costs. We show that both our results have empirical support.

It is interesting to contrast our results with those in Lambrecht, Seim, and Skiera (2007). They modeled a scenario in which subscribers were choosing between two three-part tariff packages and a flat-rate package with unlimited usage. They estimated their model using data from an Internet service provider that included the package that subscribers chose and their subsequent usage under the chosen package. They also estimated individual-specific variances in usage uncertainty and, via a what-if analysis, showed that as the variance increased, subscribers were more likely to choose a flat-fee package—that is, more likely to upgrade. At first glance, their results seem at odds with our findings. However, there are a few differences between our setting and their context. First, in our context we consider the upgrade decision only between three-part tariff packages. Interestingly, to motivate the structural model, Lambrecht et al. used a reduced-form model and analyzed whether the coefficient of variation, after controlling for mean usage, plays a role in consumers' upgrade decisions. They found no significant effect of the coefficient of variation (with a negative sign) when subscribers downgraded from one three-part tariff to another. In fact, these results support our analytical result that, with an increase in usage variance, subscribers should be less likely to downgrade. Lambrecht et al. also found a positive effect of variance only when they considered an upgrade decision involving a three-part tariff and a flat-fee package, which we do not consider in our paper. In addition, they estimated in their model only a person-specific variance and not a person- and package-specific variance.

While there has been some empirical work in linking subscribers' usage variance to their choice and consumption decisions, relatively little analytical work has been developed. We develop a framework that provides testable predictions and we then test those predictions using a data set from a telecommunication service provider that contains subscriber-level information on choice of packages and consumption. Our model can be extended in several directions. One possibility is to study how competition affects firms' package offerings and subscribers' choices and, in particular, how offers of packages by multiple firms differ from offers of multiple packages by a single firm. In addition, the distribution of the minutes of use of subscribers in other data sets could be generated differently (we currently assume a uniform distribution), which might have an effect on the design of optimal packages and choices of subscribers.

Another interesting avenue would be to analyze data on European phone plans and see whether the different nature of European packages affects our prediction. In particular, one can argue that the U.S. data should have more variance than the European data because Europe uses calling-party-pays regimes that insure that the subscriber pays only for outgoing minutes. In the U.S., where a receiving-party-pays regime is in effect, the subscriber also pays for incoming minutes that inherently have more variance since the subscriber has limited control over them.

References

Chamberlain, Gary (1980), "Analysis of covariance with qualitative data," *Review of Economic Studies,* 47, 225-238.

Cellular Telecommunications Industry Association (2009), Wireless Quick Facts. Available at http://www.ctia.org/media/industry_info/index.cfm/AID/10323.

Danaher, Peter J. (2002), "Optimal Pricing of Subscription Services: Analysis of a Market Experiment," *Marketing Science*, 21, 2, 119–138.

Esseghaier, Skander, Sunil Gupta and John Z. Zhang (2002), "Pricing Access Services," *Marketing Science*, 21, 2, 160–177.

Ferguson, Mark E. and Oded Koenigsberg (2007), "How Should a Firm Manage Deteriorating Inventory?" *Production and Operations Management*, 16, 3, 306–321.

International Telecommunication Union (2008), Telecommunication Indicators. Available at http://www.itu.int/ITU-D/ICTEYE/Indicators/Indicators.aspx#.

Iyengar, Raghuram, Asim M. Ansari and Sunil Gupta (2007), "A Model of Consumer Learning for Service Quality and Usage," *Journal of Marketing Research*, 44, 4, 529–544.

Iyengar, Raghuram, Kamel Jedidi and Rajeev Kohli (2008), "A Conjoint Approach to Multi-Part Pricing," *Journal of Marketing Research*, 45, 2, 195–210.

Lambrecht, Anja and Bernd Skiera (2006), "Paying Too Much and Being Happy about It: Existence, Causes and Consequences of Tairff-Choice Biases," *Journal of Marketing Research*, 43, 2, 212–223.

Lambrecht, Anja, Katja Seim and Bernd Skiera (2007), "Does Uncertainty Matter? Consumer Behavior under Three-Part Tariffs," *Marketing Science*, 26, 5, 698–710.

Lemon, Katherine N., Tiffany B. White and Russell S. Winer (2002), "Dynamic Customer Relationship Management: Incorporating Future Considerations into the Service Retention Decision," *Journal of Marketing*, 66, 1–14.

McGovern, Gail (2005), "Virgin Mobile USA: Pricing for the Very First Time," Harvard Business School Publication, 9-504-028.

Narayanan, Sridhar, Pradeep Chintagunta and Eugenio J. Miravete (2007), "The Role of Self Selection, Usage Uncertainty and Learning in the Demand for Local Telephone Service," *Quantitative Marketing and Economics*, 5, 1–34.

Nunes, Joseph C. (2000), "A Cognitive Model of People's Usage Estimation," *Journal of Marketing Research*, 37, 397–409.

Peres, Renana, Eitan Muller and Vijay Mahajan (2009), "Innovation Diffusion and New Product Growth: A Critical Review and Research Directions," working paper.

Reiss, Peter C. and Matthew W. White (2005), "Household Electricity Demand, Revisited," *Review of Economic Studies*, 72, 3, 853–883.