Analysts' sale and distribution of non-fundamental information

Edwige Cheynel • Carolyn B. Levine

Abstract

We examine an analyst's sale and distribution of information related to short-term price movements, but unrelated to underlying firm value. By selling non-fundamental information, the analyst increases competition on the signal, but prices become more sensitive to net order flow, creating an offsetting increase in the non-fundamental signal's value. More precise non-fundamental information is more widely distributed. In the limit, a perfect non-fundamental signal will be publicly disclosed for an arbitrarily small fee, and the analyst earns profits as if he possessed fundamental information. Consistent with empirical findings, analysts' recommendations can be profitable, even when widely distributed or seemingly inconsistent with detailed forecasts. Analysis based on non-fundamental information does *not* contribute to greater price efficiency, but reduces liquidity costs. In a multi-period setting, traders with non-fundamental information do not front run, preferring to transact only in the period in which uninformed demand is executed.

Keywords Non-fundamental information · information sales · securities regulation

JEL Classification D82 · G24 · G28

E. Cheynel
Graduate School of Business, Columbia University, New York, NY 10025, USA
email: ec2694@columbia.edu, phone: 212-851-5863, fax: 212-316-9219.

C. Levine
Rutgers Business School, Rutgers University, Newark, NJ 07102, USA
email: clevine@business.rutgers.edu

1 Introduction

This paper examines the role of non-fundamental information, defined as information on the demand or supply of *uninformed* traders, in analysts recommendations and distribution of reports. Recommendations based on non-fundamental information are contrary to uninformed demand yet unrelated to fundamentals. In contrast to the results on fundamental information (e.g., (Admati and Pfleiderer, 1988)), analysts will always directly sell non-fundamental information; when an analyst's information is very precise, his reports will be distributed widely. Despite wide distribution, following analysts' recommendations can be profitable. Our model helps explain several empirical puzzles and discredit some common trading myths.

Analysts gather and disseminate information about liquidity and investors react to it. Craig (2009) reports that Goldman Sachs supplies trading ideas to top clients based on market color gleaned in trading huddles, and that analysts comment on events with short-term price impact, even if they are in a different direction from their overall forecasts. Schaeffer's Investment research maintains a team of analysts who track 'anecdotal sentiment.'[1] Additionally, empirical evidence highlighting the use of non-fundamental information in analysts' research includes Bagnoli et al. (2009), which finds that recommendations include a component of investor sentiment, which is price-relevant, but not value-relevant. Therefore, improving our understanding of the dissemination and pricing of such information, as well as its effect on adverse selection costs for uninformed traders is of interest.

In our model, a risky asset is exchanged among four types of traders: a single proprietary trader with private fundamental information; uninformed traders who trade randomly; demand-based traders who have private non-fundamental information; and a competitive market maker. Demand-based traders become informed by purchasing a signal from an analyst who has unique knowledge of the demands of uninformed traders.

[1] http://www.schaeffersresearch.com/commentary/dailycontrarian.aspx

An analyst's access to non-fundamental information may include direct observation of customers' orders, estimation of liquidity arising from heavily promoted investment strategies, or analysis of internet message boards. Strategic traders (i.e., the proprietary trader and demand-based traders) submit their orders to maximize their expected trading profits. The market maker cannot observe the individual trading quantities, and sets price equal to the expected value of the asset, based on aggregate order flow.

Standard economic models suggest that sharing information reduces its value and that a monopolist will never directly sell information. Still, research continues to flow out of brokerage houses and following recommendations is profitable, even when widely distributed (Barber et al. (2001)). With non-fundamental information, greater distribution increases competition, leading to more aggressive contrarian trade which lowers profits in the standard way. On the flip side, more intense demand-based trading makes it more likely that order imbalances come from proprietary trading and thus prices are more sensitive to order flow. This externality increases the value of non-fundamental information, leaving analysts to choose the optimal distribution of non-fundamental information to balance these two forces. We show that analysts always choose to sell their non-fundamental information rather than retain monopoly rights over it, and as information becomes more precise and/ or the variance of liquidity shocks in the market becomes large, the number of clients to whom the analyst sells (i.e., distribution) becomes large.

In research integrating earnings forecasts (inputs) with stock recommendations (the final product), a surprising inconsistency has emerged. Analysts' recommendations cannot be tied to the earnings or growth forecasts in analysts' reports using standard valuation methods (Block, 1999; Bradshaw, 2002, 2004).[2] The SEC has interpreted recommendations that differ from analysts' internal (supporting) memos as evidence of

[2]Bradshaw (2002) finds that analysts' opinions are not explained by per-period earnings forecasts despite the fact that earnings forecasts can predict one year ahead stock returns. Barniv et al. (2009) confirms these findings, but finds a dampened negative relation between recommendations and returns following Regulation FD.

the analyst having succumbed to conflicts of interest, prioritizing investment banking revenues over analysis. However, our model justifies a divergence between recommendations and fundamental analysis absent these conflicts of interest. Short term recommendations can appear at odds with long-term prospects if they are based on different (separable) types of information; although uncorrelated with fundamentals, recommendations based on liquidity demand can be profitable. Additionally, our paper suggests a complementarity across departments in an integrated financial services firm. If sell-side analysts gain access to non-fundamental information through relations with the brokerage or investment banking departments, they can generate profits for demand-based traders while simultaneously reducing costs for uninformed clients of the firm.

In contrast to the common belief that sharing confidential customer orders is detrimental, uninformed traders in our model prefer that demand-based traders exploit their trading information. With demand-based trading, proprietary traders strategically scale down their fundamental demand leaving prices no more informative than absent demand-based trading. Although price efficiency is unchanged and demand-based-trading is profitable, demand-based traders are not parasitic. Demand-based trading reduces the transaction costs for uninformed traders, by supplying the offsetting liquidity.[3]

We extend our analysis to include a two period model with long lived fundamental information. By trading in the first period, an analyst can gather a fundamental signal for use in the second period. However, we show that the analyst will not front run the uninformed traders in the first period. Trades in the first period provide liquidity to a fundamentally informed proprietary trader and generate losses for the analyst that cannot be offset by proprietary trading on his noisy fundamental signal in the second period. Moreover, possessing fundamental information, per se, does not eliminate the analyst's desire to sell non-fundamental information, as long as the non-fundamental signal is

[3]Contrast this with Brunnermeier and Pedersen (2005) in which a distressed trader's liquidity needs cannot be immediately satisfied, or Carlin et al. (2007) in which conditions for cooperation are repeated play, and a limited number of investors.

sufficiently precise. He sells fundamental information, as it provides a commitment device for more aggressive trading.[4]

Intense demand-based trading reduces the value of fundamental trading, suggesting that a proprietary trader might be willing to offer an analyst fundamental information to create conflicting revenue incentives for the analyst. Information flows within firms and strategies to prevent such flows have been the source of considerable debate.[5] We demonstrate that there is no mutually acceptable price at which a proprietary trader will exchange fundamental information with the analyst. In fact, when the analyst's information is very precise, the analyst is strictly better off without access to the fundamental information. If it is not optimal for a proprietary trader to voluntarily exchange information with an analyst, it is also not in the interest of a centralized firm to require such exchanges to take place.[6] While many have expressed doubts as to the feasibility of Chinese walls within a firm, our model provides a setting in which they can arise endogenously and benefit the integrated firm.

Our paper integrates two previously distinct literatures. Most of our understanding of information sales in financial markets pertains to value relevant (hereafter, fundamental) information, sold either directly or indirectly (e.g., mutual fund).[7] In a standard strategic trading model, a single risk-neutral insider with private, fundamental information will *never* sell it to any other trader (Admati and Pfleiderer, 1988). Once informed, traders compete with one another for order flow and lower *total* informed trading profits. Risk-averse information monopolists may sell their information, but for distribution to be wide, it must be sold with personalized noise (Garcia and Sangiorgi, 2009). Although

[4]Sabino (1993) and Fishman and Hagerty (1995) similarly discuss this benefit of fundamental information sales.

[5]For a summary, see Demski et al. (1999).

[6]Consistent with this, "The Strategic/Proprietary Trading desk leverages the Firm's capital .... As a buy-side desk within a predominately sell-side Firm, this desk does not directly interact with Lehman Brothers clients. Instead, it relies on other sell-side firms in the market to provide incoming research (Lehman Brothers)."

[7]See, for example, Admati and Pfleiderer (1986, 1990), Brennan and Chordia (1990), and Biais and Germain (2002).

the case of risk-aversion is an interesting one for an individual seller, it is less likely to apply to sell-side analysts employed by large, diversified, investment firms.[8] Moreover, information widely distributed by sell-side analysts tends to be in the form of generic and non-personalized newsletters or press releases. The second literature analyzes private non-fundamental information, and we are the first paper, to our knowledge, to endogenize the sale and distribution of non-fundamental information. Röell (1990) examines the costs and benefits of dual trading, where a single broker observes information about his client's trading motives. Fishman and Longstaff (1992) and Madrigal (1996) focus on the use of non-fundamental information to make inferences about long-lived fundamentals. Finally, papers considering the benefits to a monopolist (fundamental) insider of observing non-fundamental information along with his fundamental information include Back (1992), Rochet and Villa (1994), and Yu (1999). As the precision of non-fundamental information increases, the non-fundamental information comprises a larger fraction of a single trader's profits, but does not eliminate the profitability of the fundamental signal. In contrast, when non-fundamental information is optimally sold, increasing non-fundamental precision drives fundamental profits to zero.

Section 2 describes the basic model, where a single proprietary trader holds fundamental information and a single analyst holds non-fundamental information. Section 3 demonstrates the optimal sale and use of information. In section 4, the analyst's information set is expanded to include both non-fundamental and fundamental information. Additionally, we evaluate information sharing within a firm, as the proprietary trader can be the source of the fundamental signal. In Section 5 we evaluate front running, in which demand-based traders use their non-fundamental information prior to the period in which uninformed demand is executed. Section 6 concludes.

[8] Most large investment firms have proprietary desks that gather information but do not sell it directly. If risk sharing motives were significant at the firm or individual analyst level, we would expect direct sales of information by these buy-side analysts as well.

2 The Model

We analyze a Kyle (1985) security market for a single risky asset. The fundamental value (terminal cash flows) of the asset, v, is a normally distributed random variable with mean zero and variance Σ. Uninformed traders face exogenous liquidity needs, modeled as a random variable u, with mean zero and variance σ_u^2 (hereafter, "liquidity variance").

A proprietary trader (e.g., an insider or fund manager) receives a noisy signal on the fundamental value of the security, $s = v + \eta$, where η is normally distributed with mean zero and variance, σ^2. An information seller (hereafter, the analyst) receives a signal, $\tau = u + \epsilon$, on the order flow originating from uninformed traders, where ϵ represents the noise in the signal, with $\epsilon \sim N(0, \sigma_\epsilon^2)$.[9] Thus, there are two types of information in the model, fundamental (s) and non- fundamental (τ).[10] The expected value of v, given the noisy signal s, is $E(v|s) = \Psi s$; the expected value of u given the noisy signal τ is $E(u|\tau) = \theta\tau$, where $\Psi \equiv \Sigma/(\Sigma + \sigma^2)$ and $\theta \equiv \sigma_u^2/(\sigma_u^2 + \sigma_\epsilon^2)$. We refer to Ψ and θ as the "signal weight" on the fundamental and non-fundamental information, respectively. An increase in the precision of the fundamental signal $(1/\sigma^2)$ or the non-fundamental signal $(1/\sigma_\epsilon^2)$ results in a corresponding increase to the respective signal weight.

The analyst offers to sell his non-fundamental information for a fee, c (publicly known). Let m be the number of demand-based traders that decide to purchase information.[11] We assume that newly informed demand-based traders cannot resell information. To focus on the direct value of non-fundamental information, we initially assume that the analyst does not have fundamental information.

[9]The non-fundamental signal is an abstract representation of the activities of financial analysts that lead to superior knowledge about market sentiment, including road-shows, the analysis of various data feeds, relationships with institutional investors, orders over a squawk box, general market data, etc.

[10]While knowledge of non-fundamental information is valuable in our market order model, it might not necessarily be so under other market structures.

[11]In our setting, observing c is equivalent to observing m, whereas Admati and Pfleiderer (1988) assume direct observability of the number of traders. In practice, the number of subscribers to a database (e.g., I/B/E/S, FirstCall) may effectively function as a commitment to a fixed number of traders.

Traders submit market orders for shares. Uninformed demand is determined by the realization of u, whereas the proprietary and demand-based traders choose their quantities based on their private information. All traders with information (either fundamental or non-fundamental) trade strategically, anticipating the impact of their trades on order flow and consequently on price. A competitive market-maker observes aggregate order flow (i.e., the sum of the orders of the proprietary trader, the uninformed traders and the demand-based traders), but not its individual components, and sets price equal to the expected value of the asset. All agents in our model are risk-neutral, allowing us to focus on non risk-sharing incentives for selling or exchanging information.

We denote proprietary and demand-based trading strategies as $X(s, m)$ and $Z(\tau, m)$, and actual proprietary trader and demand-based trader quantities, given realized information sets as x and z, respectively. Aggregate order flow is the sum of the demands of all players, or $y = x + mz + u$. Trading profits (Π) are computed as the difference between price and intrinsic value, multiplied by the quantity traded. A demand-based trader's *net* profits are equal to his trading profits less the cost of information, c. The analyst's profits are the sum of the fees he collects from each demand-based trader, or $m \cdot c$. A financial market equilibrium is a set of trading rules that maximize traders' expected profits, given their information sets and the other market participants' strategies, and a pricing rule so that the market maker breaks even in expectation. To the standard financial market equilibrium, we add two additional conditions to define an information sales equilibrium.

Definition: *An information sales equilibrium (ISE) is a fee, an information purchase decision, and demand and pricing schedules such that*

(A) *Purchase. Exactly m^* demand-based traders are willing to purchase at price c^*.*

(B) *Optimal Sharing. The analyst's revenues are maximized at (c^*, m^*).*

(C) *Financial Market Equilibrium (FME). For demand-based profits $\Pi^D(X, P, Z) = (v - p)z$ and proprietary trading profits $\Pi^P(X, P, Z) = (v - p)x$, demand and price schedules satisfy:*

(i) *Demand-based trader profit maximization. For all trading strategies Z and any information τ,* $Z^* = \text{argmax } E(\Pi^D(X, P, Z)|\tau, m)$,

(ii) *Proprietary trader profit maximization. For all trading strategies X and any information s,* $X^* = \text{argmax } E(\Pi^P(X, P, Z)|s, m)$ *and,*

(iii) *Market efficiency. Price satisfies $P = E(v|y, m)$.*

Parts (A) and (B) together allow the analyst to extract all the profits from the demand-based traders, leaving them indifferent between purchasing and not purchasing information (i.e., they earn zero expected profits either way).[12] Part (C) involves a pricing schedule and trading rules for informed market participants, both those with fundamental information (the proprietary trader) and those with non-fundamental information (the demand-based traders).

We adopt the convention that the analyst does not trade, consistent with the observed separation between sell-side and buy-side analysts. However, for analytical purposes, this assumption is without loss of generality: if the analyst were to trade, then the information would be sold to $m^* - 1$ demand-based traders, leaving the predictions of the model unchanged. Second, we assume that the price at which the information is sold is public. Since c is in a one-to-one mapping with the number of demand-based traders buying the information (by the zero-profit condition), this is equivalent to assuming the number of demand-based traders m is public, and trading decisions in Part (C) can be conditioned on m. Third, as is common in entry games (here, the purchase decision is analogous to an "entry" decision), we approximate the number of demand-based traders m as a continuous variable. Although in practice, the number of traders is discrete (and the analyst must choose either the integer above or below the optimal m^*), the qualitative conclusions of our model are unchanged.

[12] Mikhail et al. (2004) show that analysts can produce profitable recommendations, but after accounting for transactions costs, returns are no longer reliably positive.

3 Non-Fundamental Information Sales

Initially, we assume that non-fundamental information is transmitted and used by demand-based traders in the same period that the uninformed trades are executed. We solve for the optimal information sales equilibrium by holding fixed the number of demand-based traders at arbitrary m, and solving for a financial market equilibrium (Part (C) of the definition above). Then, we solve for the analyst's profit maximizing combination of intermediation fee and number of demand-based traders, (c^*, m^*) using the FME and the constraint that the net value of the information to a demand-based traders is nonnegative.

3.1 Financial Market Equilibrium

Suppose the market order of each type of informed trader is a linear function of his information. Then, by the normality of all random variables, the market maker's pricing function can be written as $P(y) = \lambda y$, where y is net order flow and λ represents the sensitivity of price to the order flow. Because the demand-based traders are identically informed and prevented from reselling their information, their trading strategies are identical, and thus we can scale up the demand of a single representative trader by the total number of demand-based traders.[13] Then, given the trading strategies of the proprietary trader and demand-based traders, we solve for the market maker's break-even pricing function.

Lemma 1 *With m demand-based traders and a single proprietary trader, there is a unique linear financial market equilibrium. Let $r_m = (m(m+2)(1-\theta)+1)/(m+1)^2$ and define constants β_m, γ_m and λ_m by $\beta_m = (\Psi r_m \sigma_u^2/\Sigma)^{1/2}$, $\gamma_m = -\theta/(m+1)$, and $\lambda_m = (\Sigma\Psi/(4r_m\sigma_u^2))^{1/2}$. Then, equilibrium demand and price schedules are*

[13] Specifically, we solve for a single demand-based trader's optimal response function given the expected demand posted by other demand-based traders and then apply the symmetry of types to get the Cournot solution. Because we assume the traders cannot resell their information, they are prevented from credibly acting more aggressively (i.e., like a Stackelberg leader).

$$X_m^* = \beta_m s \qquad Z_m^* = \gamma_m \tau \qquad P_m^* = \lambda_m(x + mz + u)$$

The demand-based traders trade against uninformed demand as one of an m-trader oligopoly. The sensitivity of their demand to the non-fundamental signal (τ) depends on its precision, $1/\sigma_\epsilon$, and the underlying variance in uninformed trades, σ_u^2. Since u and v are independently distributed, trading strategies on fundamental and non-fundamental information are additively separable. The effect of demand-based trading is to reduce the liquidity available for the proprietary trader to exploit, and leads to a higher price sensitivity parameter, λ_m. To see this, consider the impact of m traders observing τ. Initially, the uninformed traders provide an order imbalance of u. Then, demand-based traders demand $m\gamma\tau$ shares, driving order flow (in expectation) back towards zero. The residual liquidity variance, when there are m demand-based traders, is the variance of the net non-proprietary trades, calculated as

$$Var(\beta_m s) = Var(u + m\gamma_m \tau) = \left(\frac{m(m+2)(1-\theta)+1}{(m+1)^2}\right)\sigma_u^2 \equiv r_m \sigma_u^2 \qquad (1)$$

with $r_m \leq 1$ for all $m \geq 1$. Since the market maker incorporates all sources (proprietary, demand-based, and uninformed traders) of order flow in calculating his conditional expectation, break-even prices are more sensitive to the order imbalance than they would be in the absence of demand-based trading. The proprietary trader chooses his trading intensity β_m so that the variance of his trades is exactly equal to the residual variance he faces.

Price sensitivity is increasing in the precision of both non-fundamental and fundamental information, the underlying liquidity variance, and the number of demand-based traders. The proprietary trading intensity (β_m) has the same relation to the exogenous parameters as in a model without demand-based trading, but original liquidity variance is replaced with *residual* variance (1). A large uninformed trade moves price along the linear pricing function, but demand-based trading reverses the move through trades in the opposite direction of the uninformed demand.

3.2 Optimal Information Sales and Purchases

The larger the number of demand-based traders, the more each trader must moderate his order, reducing individual profits. However, there is a novel offsetting benefit to competition in demand-based trading. Increasing the number of demand-based traders moves net demand (before proprietary trading) closer to zero, making observed order imbalances more likely to come from proprietary trading. Consequently, there is an endogenous increase in price sensitivity that increases per unit demand-based trading profits.

The shaded region in Panel (a) of Figure 1 shows feasible combinations of fees and number of demand-based traders. At each combination (c, m) in the shaded region, m demand-based traders would be willing to purchase the information at fee c, using the financial market equilibrium price sensitivity for the particular m. The solid boundary line is the (c, m)-frontier representing the largest fee the analyst can charge such that m demand-based traders will purchase.[14] The analyst chooses the fee along the frontier that maximizes his revenues. The profit maximizing fee, c^*, is on the analyst's iso-profit curve tangent to the (c, m) frontier (depicted in Panel (b) of Figure 1) and is unique.

[14]Equivalently, the frontier can be interpreted as the maximum number of demand-based traders m willing to purchase information at every given price c.

Proposition 1

i. For $\theta \in (0,1)$, there exists a unique information sales equilibrium (ISE) where

$$\begin{aligned} 0 &= m^* \left(1 - m^* - m^{*2}\right)(1-\theta) + 1 \\ c^* &= \frac{\theta}{2(m^*+1)^2} \left(\frac{\Sigma\Psi\sigma_u^2}{r_{m^*}}\right)^{\frac{1}{2}} \end{aligned}$$

and trading strategies and prices follow Lemma 1 at m^*.

ii. The analyst always sells his information, or $m^* > 1$.

The optimal number of demand-based traders, m^*, is a function of the variance of uninformed trades and the precision of the non-fundamental information alone, whereas the price of non-fundamental information, c^*, is a function of all market parameters. To see this, recall that the normality assumption generates demand on fundamental and non-fundamental information that are additively separable.[15] In turn, the demand-based trading quantities are a function of m and the components of θ, but not the other market fundamentals. Relative to a market absent demand-based trading, this market has lower total market profits (i.e., for all proprietary traders); to see this, the term on the left of (2) is the profits for the insider in a standard Kyle (1985) setting, whereas the argument on the right is the deflator, which we label k where $0 < k < 1$.

$$\overbrace{\frac{1}{2}(\Sigma\Psi\sigma_u^2)^{\frac{1}{2}}}^{\text{constant}} \overbrace{\left(\underbrace{\left(\frac{(m(m+2)(1-\theta)+1)^2}{(m+1)^2(m(m+2)(1-\theta)+1)}\right)^{\frac{1}{2}}}_{\Pi^P} + \underbrace{\left(\frac{m^2\theta^2}{(m+1)^2(m(m+2)(1-\theta)+1)}\right)^{\frac{1}{2}}}_{\Pi^D} \right)}^{k\in(0,1)} \tag{2}$$

The first term in k represents the fraction allocated to proprietary profits and the second term represents demand-based profits. Holding θ fixed, as m initially increases, total

[15]The proprietary trader can be one of the analyst's clients as proprietary and demand-based trading are additively separable, with price sensitivity affected only by the number (not identity) of demand-based traders.

market profits decrease (k decreases), but the *share* of the analyst's profits increase. That is, over the range $(0, m^*)$, the numerator of Π^P is increasing in m less rapidly than the numerator in Π^D. Therefore, the analyst trades off increases in the share of profits with reductions in total market profits, where the choice of m is unaffected by the constant term in (2).[16] The analyst then uses the sales price to extract the full profits from the optimal number of demand-based traders. Another feature of the model is the convexity of total market profits in θ. When either the variance of liquidity gets very large or the error in non-fundamental information gets large (θ approaches zero or one), k in (2) converges to one. Therefore, when there is effectively no non-fundamental information, uninformed traders' losses are the same as if there is perfect non-fundamental information. In the former case, the profits are earned entirely by the proprietary trader, whereas in the latter, by the analyst.

Part (ii) indicates that non-fundamental information is always sold by a (monopolist) analyst.[17] Whether information is fundamental or non-fundamental, more informed traders lead to higher price sensitivity such that smaller quantities of order imbalance have a larger impact on price. Uninformed trades move price away from value, giving demand-based traders a larger gap (between price and value) to exploit when price sensitivity is high. The sale of non-fundamental information occurs without risk-sharing motives and without a reduction in the quality of the seller's information. Our result contrasts with the results on *fundamental* information-sharing whereby a risk-neutral monopolist information owner will not engage in direct information sales (Admati and Pfleiderer, 1988). Corollary 1 provides comparative statics on the optimal number of traders and resulting equilibrium profits.

[16] Mathematically, the constant term disappears when differentiating non-fundamental profits and setting equal to zero.

[17] By allowing the number of demand-based traders to be a continuous variable we find that some degree of information sale is optimal, conceptually. If m is required to be an integer, there may be values of θ such that $m = 1$ is preferred to $m = 2$. Additionally, allowing the proprietary trader to acquire the analyst's signal (i.e., also become a demand based trader) does not alter the characterization of equilibrium prices or the results obtained.

Corollary 1

(i.) The number of demand-based traders is increasing in the underlying liquidity variance (σ_u^2) and the precision of non-fundamental information ($1/\sigma_\epsilon^2$). As $\theta \to 1$, $m^* \to \infty$.

(ii.) The profits of the proprietary trader and the analyst are increasing in the variance of the asset value (Σ) and the precision of fundamental information ($1/\sigma^2$).

(iii.) The analyst's profits are increasing in the precision of the non-fundamental signal and the underlying liquidity variance $1/\sigma_\epsilon^2$ and σ_u^2, respectively; the proprietary trader's profits are decreasing in the precision of non-fundamental information, $1/\sigma_\epsilon^2$, and increasing in liquidity variance, σ_u^2.

Part (i) of Corollary 1 reveals that non-fundamental information is more widely distributed the larger the liquidity variance or more precise the non-fundamental signal. When non-fundamental information is extremely precise, total market profits are large (recall the convexity in θ) and go primarily to the analyst. To see this, note that the non-fundamental factor in (2) converges to 1 and the fundamental factor converges to zero (m^* endogenously gets large, and thus 1/(m^*+1)^2 gets small) as $1/\sigma_\epsilon$ approaches ∞. Therefore, the analyst extracts rents as if he were a fundamental information monopolist when his non-fundamental information is very precise. The larger the market information asymmetry, the larger the total pool of profits for informed (both demand-based and proprietary) traders (the constant multiplier in (2)). Thus the profits for the proprietary trader and demand-based traders are increasing in the underlying market parameters Σ and $1/\sigma$ (Corollary 1, part (ii)).

Greater precision in the analyst's signal increases the value of the signal to the analyst; because greater precision increases the number of demand-based traders, it increases price sensitivity, thus reducing the proprietary trader's profits. Although an increase in the underlying variance of uninformed trades (σ_u^2) increases the analyst's distribution of information, it also increases residual variance. The net effect is positive for both proprietary and demand-based traders.[18] Higher initial liquidity variance implies higher residual liquidity variance, or $r_m \sigma_u$ for proprietary traders, whereas higher initial liquidity variance increases both the fraction of total profits and the constant multiplier in (2).

Non-fundamental information is priced in our model such that traders are *indifferent* between receiving information and not receiving it. The clients willing to pay for the tips are exactly the clients that receive them. Supporting this, "executives who dealt with Galleon said it regularly received 'colour' on market developments" and reportedly paid Stanley and Goldman Sachs \$250m for this information (Sender, 2009).[19] Goldman Sachs asserts that trading tips typically go to top clients (loosely translated as those clients who generate the most revenue for Goldman).[20] Irvine et al. (2007) finding of abnormally high institutional trading volume in the direction of the recommendation in the five days prior to public disclosure, concluding that institutions implicitly pay for the (profitable) tips via commissions. Lang and Lundholm (1996) write "analysts provide different forecasts primarily because of differences in non-firm provided information, rather than differences in interpretation of common information." Although their focus is on (fundamental) earnings forecasts, the statement applies equally well to non-fundamental information. In some ways, it is easier to imagine different analysts having different signals on short-term demand (due to interactions with different sets of uninformed clients) than on fundamentals.

Both price discovery (efficiency) and market liquidity are important considerations in the regulation of securities markets (Madhavan et al., 2005). Corollary 2 reveals that non-fundamental information sales do not improve price efficiency. In addressing market liquidity, we are interested in both the price sensitivity and the expected losses of

[18] To the extent σ_u^2 proxies for volume, our conclusions are consistent with Bhushan (1989) and O'Brien and Bhushan (1990), which document a positive relation between trading volume and analyst following.

[19] The insider trading case against Galleon is unrelated to these transactions.

[20] Goldman's Trading Tips Reward Its Biggest Clients, August 24, 2009, Wall Street Journal, p. A1.

uninformed traders. Corollary 3 demonstrates that uninformed traders benefit from the presence of demand-based traders, but prefer that the demand-based traders get information that is neither too precise, nor too imprecise.

Corollary 2 *Price efficiency is not affected by non-fundamental information sales.*

The sale and use of non-fundamental information does not contribute to price efficiency. The information content of prices $Var(v|P) = \Sigma\left(1 - \frac{\Psi}{2}\right)$ does not depend on the number of demand-based traders, m, the precision of the non-fundamental information, or the underlying liquidity variance. Intuitively, using the residual (rather than the *ex ante*) liquidity variance, the optimal trading strategy is to leave hidden half of the proprietary trader's private information.

Corollary 3 *Uninformed traders' losses are minimized at an intermediate $\theta = \theta^*$.*

Perhaps surprisingly, the uninformed traders' losses are lowest for an interior level of θ. When non-fundamental precision is high, m^* is high; although significant competition is likely to drive net order flow back to zero, small unanticipated shocks can lead to large costs for the uninformed traders, as price sensitivity is strictly increasing in precision. At low levels of precision, m^* is low and the proprietary trader has the greatest opportunity to take advantage of (i.e., hide behind the trades of) uninformed traders. Our model suggests that restricting an analyst's use of non-fundamental information, ($\sigma_\epsilon \Rightarrow \infty$) or promoting transmission of information across markets ($\sigma_\epsilon \Rightarrow 0$) may hurt, rather than help market liquidity.[21] Regulators with the interest of uninformed investor protection should support trading systems that help uninformed traders split their trades and lead to intermediate precisions (i.e., alternative trading systems along with traditional brokers).

[21] Efforts to promote transparency include the Securities Acts Amendments of 1975 and recent regulation against dark pools.

Heretofore, we have taken the information parameters as exogenous, but recognize that other market participants may affect the analyst's information gathering incentives. First, consider corporate managers that trade for diversification (liquidity) reasons only. Through voluntary disclosure, managers can reduce the outstanding information asymmetry on fundamental information. Lower information asymmetry (residual variance of fundamental information) simultaneously reduces the total profits on proprietary and demand-based trading. Therefore, if non-fundamental precision is costly, reducing information asymmetry effectively reduces an analyst's incentives to gather precise non-fundamental information. For certain parameters, the manager would forego increased disclosure to maintain incentives for non-fundamental information gathering. Turning to the information preferences of the proprietary trader, his decision to increase fundamental precision creates an externality, motivating the analyst to gather more precise non-fundamental information. There too, for certain parameters, the proprietary trader prefers less precise fundamental information as it eliminates the analyst's incentive to further increase his precision and distribute more widely.

With costly information gathering, the model predicts that firms with low liquidity, low underlying uncertainty, or low information asymmetry will have less widely distributed recommendations. Additionally, recommendations for low uncertainty firms will be less profitable not only because prices are closer (in expectation) to fundamentals, but also because analysts, understanding the low potential profitability of the market, will not expend resources to improve the quality of their non-fundamental signal.

3.3 Buy Side Analysts

In the previous sections, we define non-fundamental information as information about uninformed demand following Madrigal (1996). Analysts with non-fundamental information have the characteristics of sell-siders; they gather and disseminate private information. In this section, we consider the collection of information related to the noise in the proprietary signal. The component of the proprietary trader's demand arising from the noise in his signal contributes to market mispricing and can be exploited by an analyst in the same way as the demand of uninformed traders. In this section, the analyst gathers information on η, the error in the proprietary trader's fundamental signal, and determines the optimal use of this alternate type of non-fundamental information. We call traders using this information "error-based" traders. As before, the proprietary trader possesses signal $s = v + \eta$ where v and η are normally distributed with mean zero and variances Σ and σ^2, respectively. There is uninformed demand $u \sim N(0, \sigma_u^2)$. The difference in this section is that the analyst observes information η (rather than a signal on u).[22] The insider sets demand to maximize expected profits, given his information $v + \eta$ and the number of error-based traders. Each error-based trader maximizes expected profits given the signal, and the market maker sets price competitively.

Although there are many similarities between this section and the previous ones, the results are quite different. When the analyst has information about u, uninformed traders benefit from the offsetting demand at the expense of the proprietary trader. When the analyst has information about η, the proprietary trader is the beneficiary of the offsetting demand-based trades; uninformed traders have higher expected losses for all $m \geq 1$ and are increasing in m. The proprietary trader reduces his bad trades while error-based traders help to lower net order flow, and thus offset the impact of higher price sensitivity.

Recall from the previous section, the analyst can sell uninformed demand information to any individual willing to purchase it. In contrast, the identity of the client affects the sale and use of η related information. Specifically, if the proprietary trader purchases the analyst's information, he cleans up his fundamental signal, leaving no mispricing to exploit. Therefore, error-based traders are willing to pay a positive price only if the proprietary trader is excluded from information sales.

First, consider the sale of information to error-based traders excluding the propri-

[22]The analyst's information could be imperfect; the results are notationally more cumbersome, but qualitatively similar.

etary trader. Substituting the parameters derived for the financial market equilibrium, the analyst's expected profits with m demand based traders is

$$\frac{m\Sigma\sigma^2\sigma_u}{((1+m)^2\Sigma+(1+m(3+m))\sigma^2)^{\frac{1}{2}}\,(2(1+m)\Sigma+(2+m)\sigma^2)} \tag{3}$$

Differentiating, and setting equal to zero, gives the optimal number of sales. Unlike the previous section, where distribution could be wide, there are severe limits on distribution of η-related information.[23]

Next, consider sales of information to the proprietary trader alone. The proprietary trader's maximum willingness to pay for information is the difference between his profits when the analyst sells η to m^*_η demand-based traders and his profits with perfect fundamental information, given by

$$\left(\Sigma\sigma_u^2\right)^{\frac{1}{2}}\left(\frac{1}{2}-\frac{(1+m)^2(\Sigma+\sigma^2)\Sigma^{\frac{1}{2}}}{((1+m)^2\Sigma+(1+m(3+m))\sigma^2)^{\frac{1}{2}}\,(2(1+m)\Sigma+(2+m)\sigma^2)}\right) \tag{4}$$

If the proprietary trader's maximum willingness to pay exceeds the analyst's profits from selling the information to other clients, there is always a price at which both parties are better off interacting with each other alone. Taking the difference between the analyst's profits at m^*_η and the maximum willingness to pay, we have the following result.

Proposition 2 *The proprietary trader's willingness to pay for η exceeds the profits the analyst can generate from excluding the proprietary trader and selling to error-based traders.*

The analyst always prefers selling directly to the proprietary trader and no others. Essentially, the analyst gathering information on η resembles a *buy side* analyst, hired by the proprietary trader to improve his otherwise noisy fundamental signal. The *buy*

[23] For all values of (positive) variance, the derivative of the analyst's profits is strictly negative for all values of $m > 1.5$.

side analyst does not resell his information to other clients, whereas an analyst gathering information on u resembles a *sell side* analyst, widely distributing his information.

4 Analyst's Access to Fundamental Information

To address the robustness of information sales to the possession of fundamental and non-fundamental information, we consider two scenarios. In 4.1, the analyst is endowed with a fundamental signal (via direct fundamental analysis), while in section 4.2, the proprietary trader is the source of the analyst's fundamental signal.

4.1 Analyst Endowed With Fundamental Signal

Suppose the analyst possesses the fundamental signal s, along with the non-fundamental signal τ. The analyst can sell his fundamental signal to $n-1$ traders such that the total number of proprietary traders (i.e., traders with fundamental information) is n. As before, the analyst can sell the non-fundamental signal to m demand-based traders.

As expected, with sales of fundamental information, price sensitivity is higher and per trader fundamental intensity is lower. That is, competition on fundamental information forces each individual trader to trade less than he would as a monopolist, but total informed trading is higher. Therefore, order flow is more informative and prices are more efficient. In contrast, non-fundamental intensity is unaffected by the resale of fundamental information because it is a function of the quality of the non-fundamental information alone.

The analyst optimally chooses the number of demand-based traders and proprietary traders.[24] Selling fundamental information allows the analyst to effectively act as a Stackelberg leader and benefit from more aggressive trades than if he traded alone on

[24]We also consider the additional restriction where $n = m$, or the analyst can have a single set of clients to which he sells both pieces of information. The results are qualitatively similar. Proof available on request from authors.

the information.[25] Unless the precision of the non-fundamental signal is extremely high, the analyst always prefers some degree of fundamental information sales. The analyst's desire to sell fundamental information can be interpreted as selling non-fundamental information, and allowing the buyers to make inferences about (long-lived) fundamental value from the non-fundamental signal.

Proposition 3

(i.) *For $\theta \in (0,1)$, there exists a unique information sales equilibrium (ISE) where the analyst solves for the optimal number of demand-based and proprietary traders, $\hat{m}$ and $\hat{n}$ respectively, following*

$$\begin{aligned} 0 &= \hat{m}(1-\theta)\left(2+\hat{m}-(\hat{m}+1)^2\hat{n}\right)+1 \\ 0 &= \hat{m}\,\theta\left(\hat{m}+2-(\hat{m}+1)\hat{n}^2+(4\hat{m}+7)\hat{n}\right)+(\hat{m}+1)^2((\hat{n}-4)\hat{n}-1). \end{aligned}$$

The prices for non-fundamental and fundamental information, $\hat{c}$ and $\hat{\xi}$ respectively, are

$$\begin{aligned} \hat{c} &= \frac{\theta}{(\hat{m}+1)^2(\hat{n}+1)}\left(\frac{\hat{n}\Sigma\Psi\sigma_u^2}{r_{\hat{m}}}\right)^{\frac{1}{2}} \\ \hat{\xi} &= \frac{1}{(\hat{m}+1)(\hat{n}+1)}\left(\frac{\Sigma\Psi\sigma_u^2 r_{\hat{m}}}{\hat{n}}\right)^{\frac{1}{2}} \end{aligned}$$

Trading strategies and prices follow Lemma 2 (in Appendix) at $\hat{m}$ and $\hat{n}$.

(ii.) *The optimal number of demand-based traders, $\hat{m}$, is increasing in the non-fundamental precision; the optimal number of proprietary traders, $\hat{n}$, is decreasing in non-fundamental precision.*

(iii.) *If $\theta > \overline{\theta}$, the analyst always sells his information ($\hat{m} > 1$).*

[25] Fishman and Hagerty (1995) solves a model with duopolist information sellers of fundamental information only.

How does the fundamental signal affect the sale of contemporaneous non-fundamental information? Greater competition on the fundamental signal increases price sensitivity, making non-fundamental sales more valuable. Therefore, the analyst has conflicting interests and must carefully trade off the reduction in fundamental profits with the increase in non-fundamental profits associated with distributing non-fundamental information. As long as the non-fundamental information is sufficiently precise, the analyst sells his non-fundamental signal despite participating in proprietary trading. Only at low levels of non-fundamental precision (when the number of demand-based traders would have been low absent fundamental information), will the analyst forgo non-fundamental sales.

In the financial market equilibrium, demand-based traders, who are held to zero *net* profits, are unaffected by the number of fundamentally informed traders. Because prices are more sensitive with increased competition in proprietary trading, per unit trades on non-fundamental information are more profitable.

Corollary 4 *When the analyst has fundamental and non-fundamental information, price efficiency is maximized and uninformed traders' losses are minimized at* $1/\sigma_\epsilon^2 = 0$. *Uninformed traders' losses are everywhere lower than when the analyst does not possess fundamental information.*

Demand-based trading is weakly lower whenever the analyst possesses both types of information. In our single period model, price efficiency is highest when the analyst's non-fundamental information is noisy (or equivalently, if he is restricted from using non-fundamental information). With low precision (or an inability to sell or use non-fundamental information), the analyst sells to the largest number of proprietary traders. Turning to liquidity, uninformed traders benefit both from competitive proprietary trading and demand-based trading. However, the benefits from increased competition on the fundamental signal are greater than the benefits from demand-based trading. Uninformed traders' losses are minimized when the analyst's profits are entirely generated from fundamental information sales. This contrasts with the setting where the analyst does not possess fundamental information and price efficiency is constant across all values of non-fundamental precision.

Based on the discussion above, if the analyst's source of fundamental information is corporate management, Reg FD, which eliminates selective disclosure to analysts, can reduce price efficiency. However, when analysts have access to fundamental information, restricting their use of non-fundamental information will reduce uninformed traders' losses. The worst regulatory outcome, for price efficiency and uninformed traders, then, is the simultaneous elimination of private fundamental information for analysts, and a restriction against using non-fundamental information.

Fundamental information (e.g., earnings forecasts) and non-fundamental information (e.g., recommendations) are often provided jointly in an analyst's report. Because the trading on fundamental and non-fundamental signals is additively separable, the analyst may have overall reports that appear inconsistent or contradictory. Our model predicts that the divergence between recommendations and forecasts will be larger (albeit not more frequent) for firms with large information asymmetry and large liquidity interest (i.e., the underlying variances Σ and σ_u^2 are large). Moreover, if an analyst's fundamental information is noisy (precise), he has greater (lesser) incentives to improve his precision on non-fundamental information if it is costly. This suggests that non-fundamental recommendations are likely to be most (least) profitable when earnings forecasts are least (most) profitable.

4.2 Information Exchange

Wide distribution of non-fundamental information significantly reduces the proprietary trader's profits. Might the proprietary trader sell fundamental information it to an analyst who otherwise has only non-fundamental information to effectively reduce the degree of demand-based trading?[26,27] Would an integrated firm require its proprietary trader to share information with its analysts?

To answer the first question, we determine the costs and benefits to a proprietary trader of exchanging information. Although the distribution of non-fundamental information (i.e., number of demand-based traders) is lower when the analyst possesses fundamental information than when he does not, the increase in residual liquidity is insufficient to offset the increase in competition on the fundamental signal. Consequently, the proprietary trader would always demand a strictly positive price (the solid line in Panel (a), Figure 2). The proprietary trader is most willing to sell fundamental information exactly when the analyst is least willing to buy it. For very precise levels of non-fundamental information, the analyst would prefer not to have fundamental information at all, as he is already extracting most of the total market rents. To see this, note that the analyst's willingness to pay, represented by the dashed line in Panel (a), Figure 2 dips below zero when σ_ϵ^2 approaches zero. A negative "price" occurs when the analyst's equilibrium profits without fundamental information (i.e., when the market maker correctly sets price as if he does not have fundamental information) are higher than equilibrium profits with fundamental (i.e., the market maker correctly sets price as if he has fundamental information).

The minimum fee at which the proprietary trader offers his information is everywhere above the analyst's maximum willingness to pay. Therefore, the proprietary trader and analyst will never voluntarily exchange the fundamental signal. Our analysis presumes equilibrium, such that when the analyst possesses fundamental information,

[26] An alternative way to think about the resource transfer from the analyst to the proprietary trader would be for the analyst to commit to sell to a smaller number of demand-based traders than is optimal in exchange for the information. Whether such commitments are feasible may be debatable, however we can imagine an unmodeled repeated relationship between the proprietary trader and analyst where a failure to stick to the agreed upon distribution would lead to no fundamental sales in all subsequent periods.

[27] In this section, we restrict the analyst from reselling fundamental information. However, the results are stronger when the analyst can resell and chooses the optimal demand-based and fundamental distribution $\hat{m}$ and $\hat{n}$ respectively. Proof available upon request from authors.

the market maker sets prices as if there are two proprietary traders and when the analyst does not, there is a single proprietary trader. Implicitly, our comparison assumes the market maker knows when an integrated firm possesses both types of information and can determine (observe) when information has been shared. We discuss the implications of unobservable sharing and preferences over which information to gather below.

Proposition 4 *The fee a proprietary trader would charge to sell his fundamental information to an analyst with imperfect non-fundamental information is strictly positive. At a price sensitivity consistent with the number of fundamentally informed traders, there is no fee at which both the proprietary trader is willing to sell and the analyst is willing to buy fundamental information.*

Since the analyst and proprietary trader do not share fundamental information voluntarily, does this extend to information sharing within a firm? Suppose there is an integrated financial services firm that provides financial analysis, advisory services, investment banking, brokerage, proprietary trading, etc. A common criticism of providing all of these services under one roof is that it creates a conflict of interest: departments interact (e.g., exchange information) to benefit some at the expense of others. Proposition 5 shows that it would not be in the firm's best interest to facilitate (i.e., force) information exchange.

Proposition 5 *Overall firm profits cannot be enhanced by requiring fundamental information exchange between an analyst and proprietary trader working for separate divisions in a common firm.*

Panel (b) of Figure 2 depicts total firm profits with and without (solid and dashed lines, respectively) fundamental information exchange.[28] The optimal number of demand-based traders is lower when the analyst is fundamentally informed. However, the increase in fundamental profits associated with lower demand-based trading is not enough

[28]Holding σ_u^2 fixed, increases in the signal weight θ are the direct result of increases in precision (i.e., decreases in σ_ϵ^2).

to offset the reduction due to competition on the fundamental signal. To understand why, consider low levels of non-fundamental precision. The potential benefits of exchange are low because the degree of demand-based trading is already low, while the costs are high, as the majority of profits come from proprietary trading. In contrast, when non-fundamental information is very high, the majority of profits are generated by demand-based trading. Reducing the number of demand-based traders to favor proprietary trading, which contributes very little to total profitability, is not beneficial for the firm.

This suggests a natural division among groups in the firm. Even without the requirement that firms erect a Chinese wall, this is a setting where one would arise endogenously. However, the market maker must believe in the efficiency of the Chinese wall (i.e., information flows can genuinely be avoided when it is beneficial for the firm). If the absence of a formal Chinese wall was taken as *prima facie* evidence that information leakage (exchange) would occur, it would be self fulfilling. That is, if the market maker assumes the information is exchanged (i.e., the actual sharing decision is unobservable, but organizational structure is observable) and prices the asset accordingly, the proprietary trader would indeed share its information as long as the non-fundamental information is sufficiently precise or liquidity variance is sufficiently large.[29] If regulation requiring Chinese walls enhances the credibility that divisions are not sharing, it is beneficial for the integrated firms and detrimental to uninformed investors (which is not the intent of the regulation).

As part of the Global Analyst Research Settlement, analysts' compensation is restricted (i.e., cannot be tied to investment banking deals) to avoid potential conflicts of interest. If the connection between analysts' compensation and firm wide profits has diminished, the analyst is more likely to resell fundamental information (if he has it), improving price efficiency and reducing uninformed traders' losses. Although the sale

[29] The threshold for precision is $\left(49\left(10713+491\sqrt{3449}\right)\right)/3856896 \approx 0.5$. Proof available from authors.

of non-fundamental information does not affect price efficiency, it does decrease uninformed traders' losses while reducing overall firm profits. While details of analysts' compensation is difficult to obtain, those firms that pay their analysts based on firm wide, rather than divisional profits, are themselves likely to be more profitable.

To the extent an integrated firm can "direct" the information gathering activities of its analysts, it would encourage analysts to gather fundamental or precise non-fundamental information primarily in those firms in which it *does not* have a proprietary interest. Empirically, the model suggests that recommendations will be more profitable when the analyst's employer does not have a proprietary interest in the firm. Unlike conventional wisdom that would attribute this finding to analysts misrepresenting their information (bias due to conflicts of interest), our model suggests that recommendations for these firms may be based on less precise information. Thus, we would not predict a persistent directional bias, but only reduced profitability, on average.

5 Front Running

Front-running is the (illegal) practice of trading on knowledge of pending orders. Orders submitted to brokers by customers will predictably affect the price of a security in the period in which they are executed. The front-runner hopes to earn profits by trading *before* the expected price movement associated with the pending trades. A natural question to ask is whether an analyst with a non-fundamental signal will trade in advance of the uninformed traders on which the signal is based.

When fundamental information is short lived, allowing the analyst (or, demand-based trader) the opportunity to transact on non-fundamental information provides no advantage to him. The analyst loses money in the first period, increasing overall the proprietary trader's first round profits by adding uninformed variance. Following the public announcement of fundamental value, price adjusts to the "correct" value, or $p_1^{END} = v$.

In period 2, the proprietary trader trades on his superior inference about the analyst's signal compared to the market-maker. In other words, front-running makes the analyst's signal partly public for free and reduces his rent extraction from knowledge of non-fundamental information. Therefore, with short-lived fundamental information, there is no front-running.

To make the problem more interesting, we develop a multi-period trading model where fundamental information v is long lived. Prior to first period trading, the proprietary trader observes v and the analyst learns $\tau_2 = u_2 + \epsilon$.[30] As before, the analyst sells his non-fundamental signal to m demand-based traders.

A strategic trader participating in the first period can decompose order flow better than the market maker by backing out the portion attributed to his own trade. Therefore, a demand-based trader (i.e., one who has purchased the non-fundamental signal from the analyst) generates a superior signal about *fundamental* information, when fundamental information is long lived and the proprietary trader's strategy is a function of his private information. By the same reasoning, the proprietary trader gathers a signal about τ if first period strategies are a function of second period non-fundamental information. Therefore, in the second period, the demand based traders possess both the non-fundamental information, τ_2 along with a noisy signal about fundamental information, $s^A = E(v|p_1, z_1)$, while the proprietary trader has fundamental information v and a noisy signal about non-fundamental information, $\tau^P = E(u_2|p_1, x_1)$. Whether demand-based traders prefer to transact, and choose demands as a function of their information about second period uninformed demand (i.e., front running) is the subject of this section.

Unlike the single period simplification in Section 4, the analyst cannot provide fundamental information to one set of clients and non-fundamental to another since one piece of information is fully revealing of the other. Therefore, the number of

[30]Notation is simplified by making the proprietary trader's information perfect, but it is not necessary, and does not change the qualitative nature of the results.

demand-based clients of the analyst is also the number of fundamentally informed clients in the second period. As well, the original proprietary trader has fundamental (non-fundamental) information that is superior (inferior) to the demand-based traders in the second period.

To solve for the optimal sale and use of information, we begin with the second period. Let first period demand be y_1 and let the demand of the original proprietary trader and demand based traders be x_2 and z_2 respectively, where each party places weights on his information (direct or inferred), or $x_2(v, y_1)$ and $z_2(\tau, y_1)$ The market maker sets price in the second period as a function of total order flow and priors, or $p_2 = p_1 + \lambda_2(x_2 + mz_2 + u_2)$. Rewriting the proprietary trader and demand-based traders' expectations, and using the linear pricing function, we can rewrite demand as a function of private information and prior period price:

$$x_2 = \beta_2 v + b_2 p_1 \qquad z_2 = \gamma_2 \tau_2 + g_2 p_1$$

Each trader chooses demand to maximize second period profits. Then, solving for the first period problem, the proprietary trader (demand-based trader) selects his demand $x_1(z_1)$, with attention to the way in which first period demand affects second period inferences and prices.

For the first period, the proprietary trader solves:

$$\max_{x_1} E((v - p_1)x_1|v) + E((v - p_2)x_2|v, x_1) \tag{5}$$

while the demand-based traders solve:

$$\max_{z_1} E((v - p_1)z_1|\tau) + E((v - p_2)z_2|\tau, z_1) \tag{6}$$

First period actions affect first period price which in turn affects second period price.

The market maker sets price in the first period as a function of total order flow and unconditional expectations (here set to zero) as $p_1 = \lambda_1(x_1 + mz_1 + u_1)$. Rewriting (5) and (6), substituting the solutions from the second period problem and taking expectations allows us to solve for the first period optimal demand.

Proposition 6 *Neither the analyst, nor his clients engage in front running on non-fundamental information. There exists an equilibrium in which zero weight is placed on second period non-fundamental information in the first period.*

To gain some intuition for this result, consider any non-zero, non-random first period trading intensity that is anticipated by the market maker. In the first period, demand-based traders would enhance the proprietary trader's disguise by provide additional liquidity (random orders). Specifically, first period demand-based trades move price in the direction of the net order, and their first period *expected* profits are non-positive. Further increasing the costs of front running is the fact that the market maker and the proprietary trader (by backing out from order flow) can infer the otherwise private non-fundamental information for non-random γ_1. The upside to front running is the ability to extract fundamental information for use in the second period, but second period profitability is relatively low, since the proprietary trader has "used up" much of his fundamental information by trading in the first period.[31] A deviation by the analyst to a zero trade in the first period (when front running is expected) would eliminate the first period losses, but still permit the analyst to better decompose order flow than the market maker. Consequently, the analyst would always deviate from a non-zero, non-random first period trading intensity. This result contrasts with Bernhardt and Taub (2008) where front-running is beneficial to a proprietary trader endowed with both fundamental and second period non-fundamental information. In that setting, front-running allows the proprietary trader to smooth his profits. In the first period he trades in the direction of the

[31] If there is no fundamental information in the first period, front-running is similarly unattractive. First round trading reduces the value of second period profits through an update on u_2 and with zero expected first period profits.

net order flow, adding liquidity to the market and helping him to disguise more of his fundamental information. In period 2, he reverses his trade on non-fundamental information. Our result confirms theirs as front-running is not used by the proprietary trader to benefit from the knowledge of liquidity trade but only on the opportunity to even profits over time.

Therefore, even if an analyst knows non-fundamental information in advance of the execution of uninformed trades, it does not change the sale and use dynamics in our model. That is, the analyst does not forego sales of non-fundamental information to privately front-run and gather fundamental information in the earlier periods. Moreover, he cannot charge a higher price for non-fundamental information by selling it early, as it does not benefit the demand-based traders other than in the period in which the trades are executed. It then follows that even without legal restrictions prohibiting it, front-running future uninformed demand shocks is undesirable. Our conclusion that non-fundamental information cannot be exploited in advance of trades suggests that profitable front-running observed in practice is likely to be based on fundamental signals.

The practice of placing orders immediately *following* executed order flow is called tailgating. If fundamental information is short lived, *ex post* order flow information (i.e., u_1) has no value. Disclosure of fundamental value eliminates any incorrect price movement due to liquidity trading. In addition to learning fundamental value, all traders can back out the individual components of order flow, leaving the analyst with nothing to exploit. With long lived information, tailgating is equivalent to the setting of Madrigal (1996) in which demand-based traders learn first period non-fundamental information after the first trading period. To limit the inferences of the demand-based traders the proprietary trader noises up his first period trades. Tailgating is profitable to the extent that it provides demand-based traders an information advantage over the market maker. Tailgating does not eliminate the analyst's interest in selling second period non-fundamental information if he has it prior to trading (Section 4.1).

6 Conclusion

This paper analyzes the sale of non-fundamental information by a single analyst to demand-based traders who can trade on that information in a Kyle (1985) market. The analysis highlights the differences between fundamental and non-fundamental information. While a single, risk neutral agent with fundamental information always prefers to remain an information monopolist, an analyst with non-fundamental information increases the information's value by selling it. Demand-based traders issue orders in the opposite direction of noise traders, driving expected order flow towards zero and reducing the variance of the disguise for proprietary trading. Order imbalances are more likely to come from the proprietary trader possessing fundamental information, leading to an increase in price sensitivity. With more sensitive prices, demand-based traders can better capitalize on incorrect price swings stemming from uninformed trades.

In our model, the analyst, who possesses non-fundamental information, extracts the total surplus from the demand-based traders. With this feature, the optimal number of demand-based traders is always greater than one. The more precise the non-fundamental information, the more widely it is sold. The analyst has no incentives to add noise or distort his signals, as lower precision reduces demand-based traders' willingness to purchase.

Proprietary traders in our model are strictly worse off with an analyst who possesses and optimally sells non-fundamental information to demand-based traders than in a market absent demand-based trading. Despite the costs of demand-based trading, the proprietary trader will not exchange fundamental information with analysts to limit their willingness to sell non-fundamental information. The reduction in the number of demand-based traders that would result is insufficient to compensate the proprietary trader for competition on the fundamental signal.

Our results suggest complementarities between some services in a financial services firm despite the typically one-sided criticism that they lead to conflicts of interest. By offering brokerage services, analysts gain access to non-fundamental information that can be sold to demand-based traders. This reduces insider trading profits and enhances market liquidity, although it leaves price efficiency unchanged. If a firm has divisions producing (or gaining access to) both fundamental and non-fundamental information, it will not be shared suggesting the possibility of naturally arising limits to intra-firm information flow.

Acknowledgments We appreciate the comments of Stan Baiman, Anne Beyer, Mark Bradshaw, Gus DeFranco, Diego Garcia, Ilan Guttman, Jack Hughes, Paul Irvine, Stephannie Larocque, Roby Lehavy, Beatrice Michaeli, Christine Parlour, and seminar participants at Carnegie Mellon University, Dartmouth College, The University of Chicago, The University of Georgia, Rutgers University, and The University of Wisconsin.

References

Admati, A. R., & Pfleiderer, P. (1986). A monopolistic market for information. *Journal of Economic Theory*, 39(1), 400–438.

Admati, A. R., & Pfleiderer, P. (1988). Selling and trading on information in financial markets. *American Economic Review*, 78(2), 96–103.

Admati, A. R., & Pfleiderer, P. (1990). Direct and indirect sale of information. *Econometrica*, 58(4), 901–928.

Back, K. (1992). Insider trading in continuous time. *Review of Financial Studies*, 5(3), 387–409.

Bagnoli, M., Clement, M., Crawley, M., & Watts, S. (2009). The profitability of analysts stock recommendations: Is estimating intrinsic value all that matters? *Working Paper,* University of Texas at Austin, http://ssrn.com/abstract=1430617.

Barber, B., Lehavy, R., McNichols, M. S., & Trueman, B. (2001). Can investors profit from the prophets? Security analyst recommendations and stock returns. *Journal of Finance*, 56(2), 1083–1124.

Barniv, R., Hope, O.-K., Myring, M., & Thomas, W. (2009). Do analysts practice what they preach and should investors listen? effects of recent regulations. *Accounting Review*, 84(4), 1015–1039.

Bhushan, R. (1989). Firm characteristics and analyst following. *Journal of Accounting and Economics*, 11(2-3), 255–274.

Biais, B., & Germain, L. (2002). Incentive-compatible contracts for the sale of information. *Review of Financial Studies*, 15(4), 987–1003.

Block, S. (1999). A study of financial analysts: practice and theory. *Financial Analysts Journal*, 55(4), 86–95.

Bradshaw, M. T. (2002). The use of target prices to justify sell-side analysts' stock recommendations. *Accounting Horizons*, 16(1), 27–41.

Bradshaw, M. T. (2004). How do analysts use their earnings forecasts in generating stock recommendations? *Accounting Review*, 79(1), 25–50.

Brennan, M. J., & Chordia, T. (1990). Brokerage commission schedules,. *Journal of Finance*, 48(4), 1379–1403.

Brunnermeier, M. K., & Pedersen, L. H. (2005). Predatory trading. *Journal of Finance*, 60(4), 1825–1863.

Carlin, B., Lobo, M., & Viswanathan, S. (2007). Episodic liquidity crises: Cooperative and predatory trading. *Journal of Finance*, 62(5), 2235–2274.

Craig, S. (2009). Goldman's trading tips reward its biggest clients. *Wall Street Journal*, page A1.

Demski, J., Lewis, T., Yao, D., & Yildirim, H. (1999). Practices for managing information flows within organizations. *Journal of Law, Economics, & Organization*, 15(1), 107–131.

Fishman, M. J., & Hagerty, K. (1995). The incentive to sell financial market information. *Journal of Financial Intermediation*, 4(2), 95–115.

Fishman, M. J., & Longstaff, F. A. (1992). Dual trading in futures markets. *Journal of Finance*, 47(2), 643–671.

Garcia, D., & Sangiorgi, F. (2009). Information sales and strategic trading. *Working Paper,* University of North Carolina, http://www.unc.edu/ garciadi/paper19v05.pdf.

Irvine, P., Lipson, M., & Puckett, A. (2007). Tipping. *Review of Financial Studies*, 20(3), 741–768.

Kyle, A. (1985). Continuous auctions and insider trading. *Econometrica*, 53(6), 1315–1335.

Lang, M., & Lundholm, R. (1996). Corporate disclosure policy and analyst behavior. *Accounting Review*, 71(4), 467–492.

Madhavan, A., Porter, D., & Weaver, D. (2005). Should securities markets be transparent? *Journal of Financial Markets*, 8(3), 265–287.

Madrigal, V. (1996). Non-fundamental speculation. *Journal of Finance*, 51(2), 553–578.

Mikhail, M. B., Walther, B. R., & Willis, R. H. (2004). Do security analysts exhibit persistent differences in stock picking ability? *Journal of Financial Economics*, 74(1), 67–91.

O'Brien, P. C., & Bhushan, R. (1990). Analyst following and institutional ownership. *Journal of Accounting Research*, 28(Supplement), 55–76.

Rochet, J.-C., & Villa, J.-L. (1994). Insider trading without normality. *Review of Economic Studies*, 61(1), 131–152.

Röell, A. (1990). Dual-capacity trading and the quality of the market. *Journal of Financial Intermediation*, 1(2), 105–124.

Sabino, J. (1993). Incentives to sell information. *Working Paper, University of Pennsylvania*.

Sender, H. (2009). Galleon paid banks millions for 'edge'. *Financial Times*, October 28, http://www.ft.com/cms/s/0/9b7b329e-c400-11de-8de6-00144feab49a.html#axzz1DyuMfBnM.

Bernhardt, D., & Taub, B. (2008). Front-running dynamics. *Journal of Economic Theory*, 138(1), 288–296.

Yu, F. (1999). What is the value of knowing uninformed trades? *Economics Letters*, 64(1), 87–98.

Proofs

Proof of Lemma 1:
Suppose for constants β_m, γ_m and λ_m, X, Z, and P are given by

$$X_m = \beta_m s \quad Z_m = \gamma_m \tau \quad P_m = \lambda_m y.$$

Given the linear rules, the proprietary trader and a (representative) demand-based trader's profits can be written as

$$\begin{aligned} E\{(v - P_m)x_m|s, m, x_m\} &= E(x_m(v - \lambda_m(x_m + m\gamma_m\tau + u))|s, m, x_m) \\ &= x_m(\Psi s - \lambda_m x_m) \\ E\{(v - P_m)z_m|\tau, m, z_m\} &= E(z_m(v - \lambda_m(\beta s + z_m + (m-1)\bar{\gamma}_m\tau + u))|\tau, m, z_m) \\ &= -z_m\lambda_m(z_m + (m-1)\bar{\gamma}_m\tau + \theta\tau), \end{aligned}$$

where $\bar{\gamma}$ is the expected trading strategy of each of the other $m-1$ demand-based traders. Taking first order conditions and replacing $\bar{\gamma}$ with γ (since all demand-based traders are identical) gives

$$\beta_m = \frac{\Psi}{2\lambda_m} \quad \text{and} \quad \gamma_m = -\frac{\theta}{m+1}. \tag{7}$$

The market efficiency condition $P_m = \lambda_m y = E\{v|y\}$ implies

$$\lambda_m = \frac{\beta_m \Sigma}{\beta_m^2 \Sigma/\Psi + \sigma_u^2 \left(\frac{m^2\gamma_m^2}{\theta} + 2m\gamma_m + 1\right)}. \tag{8}$$

Solving (7) and (8) subject to the second order condition $\lambda_m > 0$ gives the equilibrium.

Proof of Proposition 1:

(*i.*) The *expected* profit of an individual demand-based trader in a market with a total of m demand-based traders, $\Pi_{i,m}^D$ is:

$$\Pi_{i,m}^D = \frac{\gamma_m \left(-\gamma_m\lambda - \left(\frac{(m-1)\gamma_m}{\theta} + 1\right)\theta\lambda\right)\sigma_u^2}{\theta}. \tag{9}$$

Replacing the expression of γ_m and simplifying yields

$$\Pi_{i,m}^D = \frac{\theta\lambda\sigma_u^2}{(m+1)^2}, \tag{10}$$

making a demand-based trader indifferent between buying and not buying information at

$$c = \frac{\theta\lambda\sigma_u^2}{(m+1)^2}. \tag{11}$$

Replacing λ with λ_m, we get

$$c_m = \frac{\theta}{2(m+1)}\left(\frac{\Sigma\Psi\sigma_u^2}{m(m+2)(1-\theta)+1}\right)^{\frac{1}{2}}. \tag{12}$$

The analyst's profits are equal to $\Pi^A = mc_m$. The analyst chooses the optimal number of demand-based traders to maximize profits. Taking the FOC of Π^A in m:

$$\theta\sqrt{\Sigma\Psi\sigma_u^2}\left(\frac{\left(m\left(1-m-m^2\right)(1-\theta)+1\right)}{2(m+1)^2(m(m+2)(1-\theta)+1)^{3/2}}\right) = 0. \tag{13}$$

Therefore m^* and c^* are implicitly defined by the following equations:

$$0 = m^*\left(1-m^*-m^{*2}\right)(1-\theta)+1 \tag{14}$$

$$c^* = \frac{\theta}{2(m^*+1)^2}\left(\frac{\Sigma\Psi\sigma_u^2}{r_{m^*}}\right)^{\frac{1}{2}}. \tag{15}$$

(*ii.*) Write (14) as $\Gamma(\theta, m^*) = 0$ where

$$\Gamma(\theta, m) = m\left(1-m-m^2\right)(1-\theta)+1. \tag{16}$$

$\forall\ m \geq 0$, $\Gamma(\theta, m)$ is a third degree polynomial, increasing in $m \in [0, 1/3]$, taking values in $[1, 1+(5/27)(1-\theta)]$ and decreasing in $m \in (1/3, +\infty)$, taking values in $(-\infty, 1+(5/27)(1-\theta)]$. Therefore, there exists a unique real solution to $\Gamma(\theta, m^*) = 0$, which can be written as $m^* = \Delta(\theta)$. Using the implicit function theorem,

$$\begin{aligned} \Delta'(\theta) &= -\frac{\partial\Gamma}{\partial\theta}(\theta, m)/\frac{\partial\Gamma}{\partial m}(\theta, m) \\ &= \frac{m\left(1-m-m^2\right)}{\left(1-2m-3m^2\right)(1-\theta)}. \end{aligned} \tag{17}$$

By (14), $(1-m-m^2) < 0$ and $(1-2m-3m^2) < 0$. Thus $\Delta'(\theta) > 0$ and m^* is increasing in θ. As $\theta \to 0$, $m^* \to 1$ and therefore $m^* > 1\ \forall\ \theta \in (0, 1)$.

Taking the second order condition, solving (14) for θ, and substituting gives

$$-\frac{\sqrt{\Sigma\Psi\sigma_u^2}(1-m^*)(1-3m^*)}{2m^{*2}(1+m^*)^2\sqrt{-(1-m^*-m^{*2})}} \leq 0$$

for all $m^* \geq 1$. Since m^* is unique, greater than 1, and the second order condition (SOC) is negative at m^*, m^* is a global maximum.

Proof of Corollary 1:

(*i.*) $\lim_{\theta\to 1} \Pi^A = \frac{m}{2(m+1)} \left(\Sigma\Psi\sigma_u^2\right)^{\frac{1}{2}}$ which is increasing in m and thus maximized at $m^* \to \infty$. As m^* is increasing in θ, m^* is increasing in $1/\sigma_\epsilon^2$ and σ_u^2.

(*ii.*) Ψ is increasing in $1/\sigma^2$. Proprietary trading profits are

$$\Pi^{P*} = \sqrt{\Sigma\Psi\sigma_u^2}\left(\frac{\sqrt{(1+m^*(m^*+2)(1-\theta))}}{2(m^*+1)}\right), \tag{18}$$

and the analyst's profits, at m^* are

$$\Pi^{A*} = \sqrt{\Sigma\Psi\sigma_u^2}\left(\frac{m^*\theta}{2(m^*+1)\sqrt{1+m^*(m^*+2)(1-\theta))}}\right). \tag{19}$$

Since m^* is independent of Ψ and Σ, it follows immediately that Π^{P*} and Π^{A*} are increasing in Σ and $1/\sigma^2$.

(*iii.*) Differentiating (18) with respect to θ gives:

$$-\sqrt{\Sigma\Psi\sigma_u^2}\left(\frac{2\theta\Delta'(\theta)+m^*(m^*+1)(m^*+2)}{4(m^*+1)^2\sqrt{1+m^*(m^*+2)(1-\theta)}}\right) \leq 0, \tag{20}$$

and since $\Delta'(\theta) > 0$, and thus Π^{P*} is decreasing in $1/\sigma_\epsilon^2$.

First, replace θ=$(\sigma_u^2/(\sigma_u^2+\sigma_\epsilon^2))$ and differentiate (18) with respect to σ_u^2. Then, replace $m^{*'}(\theta)$ with $\Delta'(\sigma_u^2/(\sigma_u^2+\sigma_\epsilon^2))$ and $\sigma_u^2 = (m^*-1)(1+m^*)^2\sigma_\epsilon^2$ (14) to yield

$$\frac{m^*\left(m^{*2}+m^*-2\right)+2}{4(m^*+1)(3m^*-1)\left(m^{*2}+m^*-1\right)^{3/2}\sqrt{(m^*-1)\sigma_\epsilon^2}} \geq 0 \tag{21}$$

and thus Π^{P*} is increasing in σ_u^2.

Differentiating (19) with respect to θ, substituting for $\Delta^{'}(\theta)$ and θ yields:

$$\sqrt{\Sigma\Psi\sigma_u^2}\left(\frac{m^{*2}\sqrt{-\left(1-m^{*}-m^{*2}\right)}}{4(1+m^{*})}\right) \geq 0. \tag{22}$$

Thus Π^{A*} is increasing in non-fundamental precision, $1/\sigma_\epsilon^2$.

First, replace θ=$(\sigma_u^2/(\sigma_u^2+\sigma_\epsilon^2))$ and differentiate (19) with respect to σ_u^2. Then, replace $m^{*'}(\theta)$ with $\Delta'(\sigma_u^2/(\sigma_u^2+\sigma_\epsilon^2))$ and $\sigma_u^2 = (m^*-1)(1+m^*)^2\sigma_\epsilon^2$ which yields for $m \geq 1$

$$\sqrt{\frac{\Sigma\Psi}{\sigma_\epsilon^2}}\left(\frac{m(m+2)\sqrt{(m^3-2m+1)}}{4(m+1)\left(m^2+m-1\right)^2}\right) \geq 0.$$

Thus Π^{A*} is increasing in σ_u^2.

Proof of Corollary 2:
Price efficiency is computed as:

$$V(v|P_m) = V(v) - cov(v,P_m)^2/V(P_m) = \Sigma - \frac{\beta_m^2\Sigma^2}{m^2\gamma_m^2\sigma_u^2/\theta + 2m\gamma_m\sigma_u^2 + \beta_m^2\Sigma/\Psi + \sigma_u^2}.$$

Using the coefficients given in Lemma 1, we obtain

$$V(v|P^*) = \Sigma - \Sigma\Psi/2.$$

Proof of Corollary 3:
Uninformed traders' losses are the sum of the profits of the proprietary trader and the analyst, or

$$L(\theta) = \Pi^{A*}(\theta) + \Pi^{P*}(\theta).$$

Losses are minimized by setting the first order condition to zero, or

$$0 = \frac{\sqrt{\Sigma\Psi\sigma_u^2}}{4}\left(m^*\left(m^*(2\theta - 1 - (1-\theta)m^*) - 2(1-\theta)\theta m^{*'}(\theta)\right)((1+(1-\theta)m^*(m^*+2))^{\frac{3}{2}}\right).$$

Replacing $m^{*'}$ with 17 and simplifying,

$$\theta^* = \frac{(m^*+1)^2(3m^*-1)}{(3m^*(m^*+2)+1)m^*}.$$

Using 14 and $\theta = \sigma_u^2/(\sigma_u^2+\sigma_\epsilon^2)$, we have $\sigma_u^{2*} = 9\sigma_\epsilon^2$ or $\theta^* = 9/10$ and $m^* = 2$.

Proof of Proposition 2:
Let $K = \left(\frac{\sigma_u^2}{(1+m)^2\Sigma + \sigma^2(1+m(3+m))}\right)^{\frac{1}{2}}$. Solving for FME following Lemma 1 gives

$$\beta_\eta = (1+m)K, \quad \gamma_\eta = -K, \quad \text{and} \quad \lambda_\eta = \frac{\Sigma}{(2(1+m)\Sigma + (2+m)\sigma^2)}\frac{1}{K}.$$

The difference between expressions (3) and (4) yields

$$\left(\Sigma\sigma_u^2\right)^{\frac{1}{2}}\left(\frac{1}{2} - \frac{\Sigma^{\frac{1}{2}}((m+1)^2\Sigma + (m(m+3)+1)\sigma^2)^{\frac{1}{2}}}{2(m+1)\Sigma + (m+2)\sigma^2}\right). \tag{23}$$

Differentiating with respect to m, gives:

$$-\frac{\Sigma^{\frac{1}{2}}\sigma^2\left((m+4)\sigma^2 + 4\Sigma\right)\left(\Sigma\sigma_u^2\right)^{\frac{1}{2}}}{2\left(2(m+1)\Sigma + (m+2)\sigma^2\right)^2\left((m+1)^2\Sigma + (m(m+3)+1)\sigma^2\right)^{\frac{1}{2}}} \leq 0.$$

Further,

- at $m = 0$, (23) is equal to $\frac{1}{2}\left(\Sigma\sigma_u^2\right)^{\frac{1}{2}}\left(1 - \left(\frac{\Sigma}{\Sigma+\sigma^2}\right)^{\frac{1}{2}}\right) > 0$,
- as $m \Rightarrow +\infty$, (23) converges to $\left(\Sigma\sigma_u^2\right)^{\frac{1}{2}}\left(\frac{1}{2} - \frac{(\Sigma(\Sigma+\sigma^2))^{\frac{1}{2}}}{2\Sigma+\sigma^2}\right) > 0$.

Thus $\forall\ m$, (23) is positive.

Lemma 2 *Let* $r_m = (m(m+2)(1-\theta)+1)/(m+1)^2$ *and define constants* $\beta_{m,n}$, $\gamma_{m,n}$ *and* $\lambda_{m,n}$ *by* $\beta_{m,n} = (\Psi r_m \sigma_u^2/(n\Sigma))^{1/2}$, $\gamma_{m,n} = -\theta/(m+1)$, *and* $\lambda_{m,n} = (n\Sigma\Psi/((n+1)^2 r_m \sigma_u^2))^{1/2}$. *Equilibrium demand and price schedules are*

$$X^*_{m,n} = \beta_{m,n}s \qquad Z^*_{m,n} = \gamma_{m,n}\tau \qquad P^*_{m,n} = \lambda_{m,n}(x+mz+u).$$

Proof of Lemma 2:
Given the linear rules, the proprietary trader (or equivalently a representative proprietary trader) and a (representative) demand-based trader choose $X_{m,n}$ and $Z_{m,n}$ to maximize profits, that can be written as

$$\begin{aligned} E\{(v-P_m)x_m|s,m,x_m\} &= x_{m,n}(\Psi s - \lambda_{m,n}(x_{m,n}+(n-1)\bar{\beta}_{m,n}s))) \\ E\{(v-P_m)x_m|\tau,m,z_m\} &= -z_{m,n}\lambda_{m,n}(z_{m,n}+(m-1)\bar{\gamma}_{m,n}\tau+\theta\tau) \end{aligned}$$

where $\bar{\beta}_{m,n}$ and $\bar{\gamma}_{m,n}$ are the trading strategies of the other $n-1$ proprietary traders, and $m-1$ demand-based traders, respectively. Taking first order conditions and substituting $\bar{\gamma}_{m,n}$ with $\gamma_{m,n}$ and $\bar{\beta}_{m,n}$ with $\beta_{m,n}$ gives

$$\beta_{m,n} = \frac{\Psi}{(n+1)\lambda_{m,n}} \quad \text{and} \quad \gamma_{m,n} = -\frac{\theta}{m+1}. \tag{24}$$

The market clearing condition, $P = \lambda_{m,n}y = E\{v|y\}$ implies

$$\lambda_{m,n} = \frac{n\beta_{m,n}\Sigma}{(n^2\beta_{m,n}^2\Sigma)/\Psi + \sigma_u^2\left(\frac{m^2\gamma_{m,n}^2}{\theta} + 2m\gamma_{m,n} + 1\right)}. \tag{25}$$

Solving (24) and (25) subject to the second order condition $\lambda_{m,n} > 0$ gives the equilibrium.

Proof of Proposition 3:

(i.) From Lemma 2, the *expected* profit of an individual demand-based trader in a market with a total of m demand-based traders and n proprietary traders , $\Pi^D_{i,m,n}$ is:

$$\Pi^D_{i,m,n} = \frac{\theta\lambda_{m,n}\sigma_u^2}{(m+1)^2}, \tag{26}$$

leaving a demand-based trader indifferent between buying and not buying information at $c = \frac{\theta\lambda_{m,n}\sigma_u^2}{(m+1)^2}$. Using Lemma 2, substitute for $\lambda_{m,n}$, yielding

$$c_{m,n} = \frac{\theta}{(m+1)(n+1)}\left(\frac{n\Sigma\Psi\sigma_u^2}{m(m+2)(1-\theta)+1}\right)^{\frac{1}{2}}. \tag{27}$$

The *expected* profit of an individual proprietary trader in a market with a total of m demand-based traders and n proprietary traders , $\Pi^P_{i,m,n}$ is:

$$\Pi^P_{i,m,n} = \frac{\Sigma\Psi}{(n+1)^2\lambda_{m,n}}, \tag{28}$$

making a trader indifferent between buying and not buying fundamental information at $\xi = \frac{\Sigma\Psi}{(n+1)^2\lambda_{m,n}}$. Substituting for $\lambda_{m,n}$, we get

$$\xi_{m,n} = \left(\frac{\Sigma\Psi\sigma_u^2}{n}\right)^{\frac{1}{2}} \frac{((1+m(m+2)(1-\theta))^{\frac{1}{2}}}{(n+1)(m+1)}. \tag{29}$$

The analyst's profits are equal to the sum of the proprietary and demand based sales, or $\Pi^A_{m,n} = (n-1)\xi_{m,n} + mc_{m,n}$. The analyst maximizes profits by choosing m and n optimally. Taking the first order condition of $\Pi^A_{m,n}$ in m and n and simplifying yields the implicit expressions for the optimal number of proprietary $(\hat{n})$ and demand-based $(\hat{m})$ traders:

$$\begin{aligned} \hat{m}\theta(-\hat{n}(\hat{m}(\hat{n}-4)+\hat{n}-7)+\hat{m}+2) & \\ +(\hat{m}+1)^2((\hat{n}-4)\hat{n}-1) &= 0 \qquad (30) \\ \hat{m}(1-\theta)\left(2+\hat{m}-(\hat{m}+1)^2\hat{n}\right)+1 &= 0. \qquad (31) \end{aligned}$$

Solving (31) for $\hat{n}$ gives

$$\hat{n} = \frac{\hat{m}(\hat{m}+2)(1-\theta)+1}{\hat{m}(\hat{m}+1)^2(1-\theta)}. \tag{32}$$

Applying 32 into (30) and simplifying gives

$$\frac{(\hat{m}(\hat{m}+2)(1-\theta)+1)^2(1-\hat{m}(\hat{m}(\hat{m}+5)+3)(1-\theta))}{\hat{m}^2(\hat{m}+1)^3(1-\theta)^2} = 0. \tag{33}$$

Denote $\tilde{\Gamma}(\theta,\hat{m}) = 0$ where

$$\tilde{\Gamma}(\theta,m) = (m(m+2)(1-\theta)+1)^2(1-m(m(m+5)+3)(1-\theta)). \tag{34}$$

Differentiating $\tilde{\Gamma}(\theta,m)$ in m gives

$$\begin{gathered} (1-\theta)(m(m+2)(1-\theta)+1)\times \\ (m(m(55\theta-m(7m+40)(1-\theta)-58)-6(4-3\theta))+1). \end{gathered} \tag{35}$$

Solving $\tilde{\Gamma}(\theta,\hat{m}) = 0$ for θ gives the two (unique) roots below, with only 36 feasible.

$$\begin{aligned}\theta &= \frac{(\hat{m}+1)^2}{\hat{m}(\hat{m}+2)} > 1 \text{ and}\\ \theta &= \frac{(\hat{m}+1)(\hat{m}(\hat{m}+4)-1)}{\hat{m}(\hat{m}(\hat{m}+5)+3)}.\end{aligned} \tag{36}$$

Substituting 36 into 35 yields

$$-\frac{(\hat{m}+1)^2(\hat{m}+3)(\hat{m}+5)^2(3\hat{m}+1)}{\hat{m}(\hat{m}(\hat{m}+5)+3)^3} < 0. \tag{37}$$

Thus $\hat{m}$ and $\hat{n}$ are unique.

(ii.) Rewriting (30) and (31) in matrix form:

$$\phi = \begin{bmatrix} \phi_1(m,n,\theta) & \phi_2(m,n,\theta) \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

The Jacobian matrix of ϕ in m and n is

$$J_\phi(m,n) = \begin{bmatrix} \frac{\partial\phi_1}{\partial m}(m,n,\theta) & \frac{\partial\phi_1}{\partial n}(m,n,\theta) \\ \frac{\partial\phi_2}{\partial m}(m,n,\theta) & \frac{\partial\phi_2}{\partial n}(m,n,\theta) \end{bmatrix}$$

where

$$\begin{aligned}\frac{\partial\phi_1}{\partial m}(m,n,\theta) &= 2(m((n-4)n-1)-1)(1-\theta)+n(7\theta-8+n(2-\theta))\\ \frac{\partial\phi_1}{\partial n}(m,n,\theta) &= 2(n-2)(m+1)^2+m\theta(4m-2n(m+1)+7)\\ \frac{\partial\phi_2}{\partial m}(m,n,\theta) &= (m+1)(2-3mn-n)(1-\theta)\\ \frac{\partial\phi_2}{\partial n}(m,n,\theta) &= -m(m+1)^2(1-\theta).\end{aligned}$$

The Jacobian matrix of ϕ in θ is

$$J_\phi(\theta) = \begin{bmatrix} \frac{\partial\phi_1}{\partial\theta}(m,n,\theta) \\ \frac{\partial\phi_2}{\partial\theta}(m,n,\theta) \end{bmatrix} = \begin{bmatrix} m(m-n(m(n-4)+n-7)+2) \\ m\left(n(m+1)^2-m-2\right)(2\theta-1)+1 \end{bmatrix}.$$

The determinant of $J_\phi(\hat{m},\hat{n})$, is

$$\frac{\partial\phi_1}{\partial m}(m,n,\theta)\times\frac{\partial\phi_2}{\partial n}(m,n,\theta)-\frac{\partial\phi_1}{\partial n}(m,n,\theta)\times\frac{\partial\phi_2}{\partial m}(m,n,\theta). \tag{38}$$

Substituting 32 and 36 into 38 and simplifying yields

$$\frac{(m+1)(m+3)(m+5)^2(3m+1)}{m(m(m+5)+3)^2}. \tag{39}$$

Expression 36 positive ($\theta > 0$) implies $m > \sqrt{5}-2$ and the determinant is always different from zero. Applying the implicit function theorem,

$$\eta(\theta) = \begin{bmatrix} \frac{\partial \hat{m}}{\partial \theta}(\theta) \\ \frac{\partial \hat{n}}{\partial \theta}(\theta) \end{bmatrix} = -[J_\phi(m,n)]^{-1}[J_\phi(\theta)].$$

Using the expressions for $\hat{n}$ and θ (32 and 36, respectively) and simplifying in the expressions of $\frac{\partial \hat{m}}{\partial \theta}(\theta)$ and $\frac{\partial \hat{n}}{\partial \theta}(\theta)$ gives

$$\begin{aligned} \frac{\partial \hat{m}}{\partial \theta}(\theta) &= \frac{m^2(m(m+5)+3)^2}{(m+3)(3m+1)} > 0 \\ \frac{\partial \hat{n}}{\partial \theta}(\theta) &= -\frac{4m^2(m(m+5)+3)^2}{(m+1)^2(m+3)(3m+1)} < 0. \end{aligned}$$

Thus $\hat{m}$ is increasing in θ and $\hat{n}$ is decreasing in θ.

Further:

- at $\theta = 0$, from (36) and (30), $\hat{m} = \sqrt{5}-2 \approx 0.236$, and $\hat{n} = 2+\sqrt{5} \approx 4.236$
- $\lim_{\theta\to 1} \pi^A_{m,n} = \frac{m\sigma_u\sqrt{n\Sigma\Psi}}{mn+m+n+1} + \frac{(n-1)\sigma_u\sqrt{\Sigma\Psi}}{(m+1)(n+1)\sqrt{n}}$, which is increasing in m.

Thus profits are maximized at $\hat{m} \to \infty$ and $\hat{n} = 1$.

(iii.) Substituting $m = 1$ into 31 and using 30, we solve for θ and find $\bar{\theta} = 8/9$.

To prove $\hat{m}$ and $\hat{n}$ characterize a global maximum, define the Hessian matrix

$$H(m,n) = \begin{bmatrix} \frac{\partial^2 \Pi^A}{\partial m^2}(m,n,\theta) & \frac{\partial^2 \Pi^A}{\partial m \partial n}(m,n,\theta) \\ \frac{\partial^2 \Pi^A}{\partial n \partial m}(m,n,\theta) & \frac{\partial^2 \Pi^A}{\partial n^2}(m,n,\theta) \end{bmatrix}.$$

Substituting for $\hat{n}$ and θ and simplifying yields

$$H(\hat{m}) = \left(\frac{\Sigma\sigma_u^2\Psi}{(\hat{m}(\hat{m}+5)+3)}\right)^{\frac{1}{2}} \frac{(\hat{m}+1)}{(\hat{m}+3)(\hat{m}+5)^2} \begin{bmatrix} -\frac{(\hat{m}(\hat{m}+4)-1)(\hat{m}(3\hat{m}+14)+3)}{2\hat{m}^2(\hat{m}+1)^2} & -\frac{(\hat{m}(\hat{m}+4)-1)}{2\hat{m}} \\ -\frac{(\hat{m}(\hat{m}+4)-1)}{2\hat{m}} & -\frac{(\hat{m}+1)^2(\hat{m}+7)}{8} \end{bmatrix}.$$

Define $G(\hat{m}) = \begin{bmatrix} -\frac{(\hat{m}(\hat{m}+4)-1)(\hat{m}(3\hat{m}+14)+3)}{2\hat{m}^2(\hat{m}+1)^2} & -\frac{(\hat{m}(\hat{m}+4)-1)}{2\hat{m}} \\ -\frac{(\hat{m}(\hat{m}+4)-1)}{2\hat{m}} & -\frac{(\hat{m}+1)^2(\hat{m}+7)}{8} \end{bmatrix}$.

$\hat{m} > \sqrt{5} - 2 \Rightarrow -\frac{(\hat{m}(\hat{m}+4)-1)(\hat{m}(3\hat{m}+14)+3)}{2\hat{m}^2(\hat{m}+1)^2} < 0$ and the determinant of $G(\hat{m})$ is $\frac{(m+5)^2(3m+1)(m(m+4)-1)}{16m^2} > 0$. Thus the Hessian is negative definite. Given $\hat{m}$ and $\hat{n}$ are unique, Π^A attains a global maximum at $\hat{m}$ and $\hat{n}$.

Proof of Corollary 4:

(i.) Replacing $\hat{n}$ (using 32) and θ (using 36) into the expression of the uninformed traders' losses, $L_{\hat{m},\hat{n}}(\theta)$ (i.e., total profits of demand-based and the proprietary traders) yields

$$\sqrt{\Sigma\Psi\sigma_u^2}\left(\frac{(\hat{m}(\theta)+1)(\hat{m}(\theta)+4)}{2(\hat{m}(\theta)+3)\sqrt{\hat{m}(\theta)(\hat{m}(\theta)+5)+3}}\right). \tag{40}$$

Differentiating the above expression in θ yields

$$\frac{(\hat{m}(\hat{m}(\hat{m}+10)+27)+6)\hat{m}'(\theta)}{4(\hat{m}+3)^2(\hat{m}(\hat{m}+5)+3)^{3/2}} > 0. \tag{41}$$

Thus uninformed traders' losses are minimized at $\theta = 0$.

(ii.) Price efficiency, the inverse of $Var(v|P_{m,n}) = \Sigma - \frac{n\Sigma\Psi}{n+1}$, is increasing in n, n is decreasing in θ, and thus price efficiency is maximized at $\theta = 0$.[32] From expression 41 the uninformed traders' losses are increasing in θ, when there are $\hat{n}$ fundamental traders and $\hat{m}$ demand-based traders. From Corollary 3, the uninformed traders' losses are minimized at $\theta = 9/10$ when the analyst does not have fundamental information.

From 30 and 36,

- At $\theta = 0$, $\hat{m} = \sqrt{5} - 2 \approx 0.236$, and $\hat{n} = 2 + \sqrt{5} \approx 4.236$. The uninformed traders' losses are $L_{\hat{m},\hat{n}}(\theta) = \frac{(\sqrt{5}-1)\sqrt{2+\sqrt{5}}}{3+\sqrt{5}\sqrt{1+\sqrt{5}(\sqrt{5}-2)}} \approx 0.393$.
- At $\theta = 9/10$, $\hat{n} \approx 2.941$ and $\hat{m} \approx 1.061$. The uninformed traders' losses are $L_{\hat{m},\hat{n}}(\theta) \approx 0.418$
- At $\theta \to 1$, $\hat{m} \to \infty$ and $\hat{n} = 1$. The uninformed traders' losses are $L_{\hat{m},\hat{n}}(\theta) = 1/2$.

[32]If restricted to integer values, there will be no non-fundamental sales whenever $\theta < \hat{\theta} \approx 0.478$, and thus price efficiency is constant for $\theta \in (0, \hat{\theta})$.

When the analyst has only non fundamental information, the uninformed traders' losses are:

- $L(\theta) = 1/2$ at $\theta \in \{0, 1\}$ and
- $L(\theta) = 1/\sqrt{5} \approx 0.447$ at $\theta = 9/10$, the level at which losses are minimized.

Therefore the uninformed traders' losses are always lower when the analyst also has fundamental information.

Proof of Proposition 4:
From Lemma 2, the price sensitivity with two proprietary traders and m demand-based traders is equal to

$$\lambda_m = \frac{\sqrt{2}}{3}\left(\frac{\Sigma\Psi}{\sigma_u^2}\right)^{\frac{1}{2}}\left(\frac{(m+1)^2}{(m(m+2)(1-\theta)+1)}\right)^{\frac{1}{2}}. \tag{42}$$

Using 11, and replacing λ with 42

$$c_m = \frac{\sqrt{2}\theta}{3(m+1)}\left(\frac{\Sigma\Psi\sigma_u^2}{m(m+2)(1-\theta)+1}\right)^{\frac{1}{2}}.$$

Taking the FOC of the analyst's (proprietary and demand-based) profits with respect to m:

$$\frac{\sqrt{\Sigma\sigma_u^2\Psi}}{3\sqrt{2}}\left(\frac{\theta\sqrt{1+m(m+2)(1-\theta)}\,(1-m^2(2m+3)(1-\theta))}{((m+1)^3-m(m+1)(m+2)\theta)^2}\right) = 0. \tag{43}$$

Therefore the optimal number of demand-based traders and fees, when there are two proprietary traders are implicitly defined by the following equations:

$$0 = 1-(m^{**})^2(2m^{**}+3)(1-\theta) \tag{44}$$

$$c^{**} = \frac{\sqrt{2}\theta}{3(m^{**}+1)}\left(\frac{\Sigma\Psi\sigma_u^2}{m^{**}(m^{**}+2)(1-\theta)+1}\right)^{\frac{1}{2}}. \tag{45}$$

Taking n as exogenous, we differentiate the uninformed traders' losses in θ. It yields after simplifying:

$$\frac{m^2\left(-\frac{n}{m(m+2)(\theta-1)-1}\right)^{3/2}\left(-\frac{2\theta\left((m+1)^2n-m-2\right)}{(m+1)(3mn+n-2)}+m(\theta-1)+2\theta-1\right)}{2n(n+1)} = 0. \tag{46}$$

Solving (31) and (46), we obtain:

$$\theta = \frac{4n^2+4n+1}{(n+1)(4n+1)}$$
$$m = \frac{n+1}{n}.$$

- When $n = 1$, $\theta^* = 9/10$ and $m^* = 2$. Uninformed traders' losses are equal to $1/\sqrt{5} \approx 0.447$. For $\theta = 0$ and $\theta = 1$, $L(\theta) = 1/2$.
- When $n = 2$, $\theta^{**} = 25/27 \approx 0.926$ and $m^{**} = 3/2$. Uninformed traders' losses are equal to $4/9 \approx 0.444$. For $\theta = 9/10$, uninformed traders' losses are approximately equal to 0.445 and for $\theta = 0$ and $\theta = 1$, uninformed traders' losses are $\sqrt{2}/3 \approx 0.471$.

Thus $\Pi^D(m^*) + \Pi^P(m^*) \geq \Pi^D(m^{**}) + 2\Pi^P(m^{**})$. Given that the maximum fee the proprietary trader can extract from the analyst is

$$\bar{f} = \Pi^D(m^{**}) + \Pi^P(m^{**}) - \Pi^D(m^*)$$

it follows that $\Pi^P(m^*) \geq \Pi^P(m^{**}) + \bar{f}$, and thus the proprietary trader does not sell fundamental information.

Proof of Proposition 5:
Since $\Pi^D(m^*) + \Pi^P(m^*) \geq \Pi^D(m^{**}) + 2\Pi^P(m^{**})$, interdivisional sharing cannot increase overall profits.

Proof of Proposition 6:
Let the linear demand functions of the proprietary trader and demand-based traders be denoted $x_2 = \beta_2 v + b_2 p_1$ and $z_2 = \gamma_2 \tau_2 + g_2 p_1$ with conditional expectations

$$\begin{aligned} E^P(\tau|v,y_1) &= \kappa v + \omega p_1; \quad E^P(u_2|v,y_1) = kv + wp_1 = \theta\kappa v + \theta\omega p_1 \\ E^D(v|\tau,y_1) &= \phi\tau + \zeta p_1; \quad E^M(v|y_1,y_2) = p_2 = d_2 p_1 + \lambda_2 y_2. \end{aligned} \tag{47}$$

Then, writing the proprietary trader's objective function, replacing for the expressions of p_2, y_2, u_2 and taking expectations, we have

$$\begin{aligned} E((v-p_2)x_2|v,p_1) &= E(v - \lambda_2 y_2 - d_2 p_1)x_2 \\ &= (v - \lambda_2(x_2 + m(\gamma_2(\kappa v + \omega p_1) + g_2 p_1) + kv + wp_1) - d_2 p_1)x_2. \end{aligned}$$

Differentiating, setting equal to zero, and solving for β_2 and b_2 gives

$$\beta_2 = \frac{1 - \lambda_2\kappa(\theta + \gamma_2 m)}{2\lambda_2}$$

$$b_2 = -\frac{d_2 + \lambda_2(\theta\omega + m(g_2 + \gamma_2\omega))}{2\lambda_2}.$$

The SOC confirms $\lambda_2 > 0$.
Doing the same for an individual demand-based trader, we have

$$\begin{aligned} E((v - p_2)z_2|\tau, p_1) &= (v - (\lambda_2(x_2 + z_2'(m-1) + z_2 + u_2) + d_2p_1))z_2 \\ &= (\zeta p_1 + \phi\tau - (\lambda_2(\beta_2(\zeta p_1 + \phi\tau) + b_2p_1 + \\ &\quad (\gamma_{2i}\tau + g_{2i}p_1)(m-1) + z_2 + \theta\tau) + d_2p_1))\, z_2. \end{aligned}$$

Differentiating, substituting symmetric demand functions for all demand-based traders, setting equal to zero, and solving for γ_2 and g_2 gives

$$\begin{aligned} \gamma_2 &= \frac{2\theta\lambda_2 - \phi(1 + \theta\kappa\lambda_2)}{\lambda_2(m\kappa\phi - 2(1+m))} \\ g_2 &= \frac{-d_2 + \zeta + \lambda_2(\theta\omega + \gamma_2 m\omega + \zeta\kappa(\theta + \gamma_2 m))}{(2+m)\lambda_2} \end{aligned}$$

The SOC confirms $\lambda_2 > 0$.

Turning to the first period problem, the proprietary trader maximizes total expected profits, or $E\{(v - p_1)x_1 + (v - p_2)x_2\}$. Using the expressions above and appropriate substitutions leads to the following optimization program

$$\begin{aligned} &(v - \lambda_1 x_1)x_1 + (v - (\lambda_2(\beta_2 v + b_2\lambda_1 x_1) + m(\gamma_2(\kappa v + \omega\lambda_1 x_1) + \\ &\qquad g_2\lambda_1 x_1) + (\theta\kappa v + \theta\omega\lambda_1 x_1)) + d_2\lambda_1 x_1)(\beta_2 v + b_2\lambda_1 x_1). \end{aligned}$$

Differentiating and setting the FOC to zero yields

$$\beta_1 = -\frac{(-1 + b_2\lambda_1(-1 + \lambda_2(2\beta_2 + \kappa(\theta + \gamma_2 m)) + \beta_2\lambda_1(d_2 + \lambda_2(\omega(\theta + \gamma_2 m) + g_2 m)))}{2\lambda_1(1 + b_2\lambda_1(d_2 + \lambda_2(b_2 + \theta\omega + m(g_2 + \gamma_2\omega))))}.$$

The SOC confirms $2\lambda_1(-b_2\lambda_1(\lambda_2(b_2 + \gamma_2 m\omega + g_2 m + \theta\omega) + d_2) - 1) < 0$.
Similarly an individual demand based trader optimizes total expected profits which can be written as

$$\begin{aligned} &-\lambda_1((m-1)z_1' + z_1)z_1 + ((\zeta(\lambda_1((m-1)z_1' + z_1)) + \phi\tau) - (\lambda_2((\beta_2(\zeta(\lambda_1((m-1)z_1' + \\ &\quad z_1)) + \phi\tau) + b_2(\lambda_1((m-1)z_1' + z_1))) + (\gamma_2\tau + g_2(\lambda_1((m-1)z_1' + z_1)))m + \theta\tau) + \\ &\qquad d_2(\lambda_1((m-1)z_1' + z_1))))(\gamma_2\tau + g_2(\lambda_1((m-1)z_1' + z_1))). \end{aligned}$$

Differentiating, substituting, setting equal to zero, and recognizing the symmetry in demand-based strategies, yields

$$\gamma_1 = \frac{-\gamma_2(d_2 - \zeta + \lambda_2(b_2 + 2g_2 m + \beta_2\zeta)) + g_2(\phi - \lambda_2(\theta + \beta_2\phi))}{1 + m(1 + 2g_2\lambda_1(d_2 - \zeta + \lambda_2(b_2 + g_2 m + \beta_2\zeta)))}. \tag{48}$$

The SOC confirms $2\lambda_1(-C_2\lambda_1(\lambda_2(\beta_2\zeta + b_2 + g_2 m) + d_2 - \zeta) - 1) < 0$.

Using the projection theorem to calculate the conditional expectations in 47 yields

$$\begin{array}{llllll} \phi = & -\frac{\gamma_1 m}{\lambda_1(\beta_1^2\Sigma+\sigma_u^2)} & \zeta = & \frac{\beta_1\Sigma}{\lambda_1(\beta_1^2\Sigma+\sigma_u^2)} & \kappa = & -\frac{\beta_1\gamma_1 m}{\gamma_1^2 m^2+\theta} \\ \omega = & \frac{\gamma_1 m}{\lambda_1(\gamma_1^2 m^2+\theta)} & \lambda_1 = & \frac{\theta\beta_1\Sigma}{\theta(\beta_1^2\Sigma+\sigma_u^2)+\gamma_1^2 m^2\sigma_u^2}. & & \end{array}$$

$$\begin{aligned} \lambda_2 &= \frac{(\gamma_1 m^2(-\beta_2\gamma_1 + \beta_1\gamma_2) - \theta(\beta_2 - \beta_1\gamma_1 m))\Sigma}{2\beta_1\beta_2\gamma_1 m(\gamma_2 m + \theta)\Sigma - \gamma_2 m(\gamma_2 m + 2\theta)(\beta_1^2\Sigma + \sigma_u^2) - \gamma_1^2 m^2(\sigma_u^2(1-\theta) + \beta_2^2\Sigma) - \theta(\beta_1^2 + \beta_2^2)\Sigma + \sigma_u^2)} \\ d_2 &= \Sigma(-\beta_1\gamma_2 m(\gamma_1 m\lambda_1(b_2 + g_2 m) + 2\theta) - \beta_1\theta(\gamma_1 m\lambda_1(b2 + g_2 m) + 1) \\ &\quad -\beta_1\gamma_2^2 m^2 + \beta_2\gamma_1^2 b_2 m^2\lambda_1 + \beta_2\gamma_1^2 g_2 m^3\lambda_1 + \beta_2\gamma_1 m(\gamma_2 m + \theta) + \beta_2 b_2\theta\lambda_1 + \beta_2 g_2 m\theta\lambda_1)/ \\ &\quad (\lambda_1\left(-\theta\left(\left(\beta_1^2 + \beta_2^2\right)\Sigma + \sigma_u^2\right) - \gamma_2^2 m^2\left(\beta_1^2\Sigma + \sigma_u^2\right) - 2\gamma_2 m\theta\left(\beta_1^2\Sigma + \sigma_u^2\right)\right. \\ &\quad +2\beta_1\beta_2\gamma_1 m\Sigma(\gamma_2 m + \theta) - \gamma_1^2 m^2\left(\sigma_u^2(1-\theta) + \beta_2^2\Sigma\right)). \end{aligned}$$

Using conjectures of $\gamma_1 = 0$ and $d_2 = 1$, substitute, simplify, and solve for equilibrium. Without first period trading by the demand-based traders, the proprietary trader does not update his priors on u_2 and demand-based traders do not update beyond p_1, and

$$\begin{array}{lll} E^P(\tau|v, y_1) & = E(\tau) = 0 & \to \kappa = 0; \omega = 0; k = 0; w = 0 \\ E^D(v|\tau, y_1) & = E(v|y_1) = p_1 & \to \phi = 0; \zeta = 1. \end{array}$$

Substituting, solve for the remaining parameters.

$$\begin{array}{llllll} \beta_1 & = \frac{\lambda_1 - 2\lambda_2}{\lambda_1^2 - 4\lambda_1\lambda_2} & \lambda_1 & = \frac{\beta_1\Sigma}{\beta_1^2\Sigma+\sigma_u^2} & \gamma_2 & = -\frac{\theta}{(1+m)} \\ g_2 & = 0 & \beta_2 & = \frac{1}{2\lambda_2} & b_2 & = -\frac{1}{2\lambda_2} \end{array} \tag{49}$$

$$\lambda_2 = \frac{(1+m)}{2}\left(\frac{\Sigma}{(1 + m(2+m)(1-\theta))(\beta_1^2\Sigma + \sigma_u^2)}\right)^{\frac{1}{2}}.$$

The inequalities to verify are:

$$\frac{\lambda_1(\lambda_1 - 4\lambda_2)}{2\lambda_2} \;<\; 0$$

$$
\begin{aligned}
\lambda_1 &> 0 \\
\lambda_2 &> 0.
\end{aligned}
$$

Replacing the above parameters in expressions 49 and 48 yields $d_2 = 1$ and $\gamma_1 = 0$. Conjecture $\beta_1 = q(\sigma_u^2/\Sigma)^{1/2}$, where $q \geq 0$ (as $\beta_1 > 0$ given $\lambda_1 > 0$). Substituting all the above parameters from 49 in the expression of β_1 and simplifying yields:

$$
q = -\frac{\left(q^2+1\right)\left(q\sqrt{\left(q^2+1\right)\left(m(m+2)(1-\theta)+1\right)} - (m+1)\left(q^2+1\right)\right)}{q\left(2(m+1)\left(q^2+1\right) - q\sqrt{\left(q^2+1\right)\left(m(m+2)(1-\theta)+1\right)}\right)}.
$$

Rearranging the terms implies:

$$
(m+1)q^4 + q\sqrt{\left(-q^2-1\right)\left(m(m+2)(\theta-1)-1\right)} - m - 1 = 0. \tag{50}
$$

Differentiating the left hand side (LHS) yields :

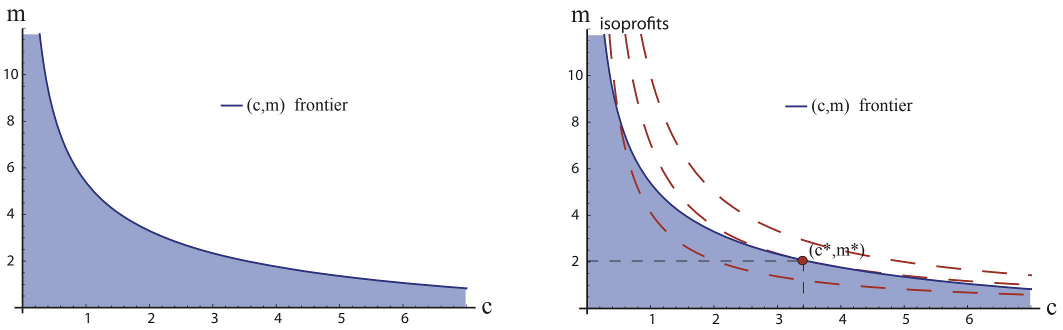
$$
4(m+1)q^3 + \frac{\left(2q^2+1\right)\sqrt{\left(q^2+1\right)\left(1+m(m+2)(1-\theta)\right)}}{q^2+1}. \tag{51}
$$

The derivative 51 is positive for $q \geq 0$. For $q = 0$, the LHS of (50) is equal to $-(1+m)$ and for $q \to +\infty$, the LHS of (50) goes to $m+1$. Thus there exists a unique q satisfying (50).

Figures

Figure 1: Optimal Non-Fundamental Information Sales

Parameter values: $\Sigma = 10; \sigma_u^2 = 10; \Psi = 1; \theta = 0.9$

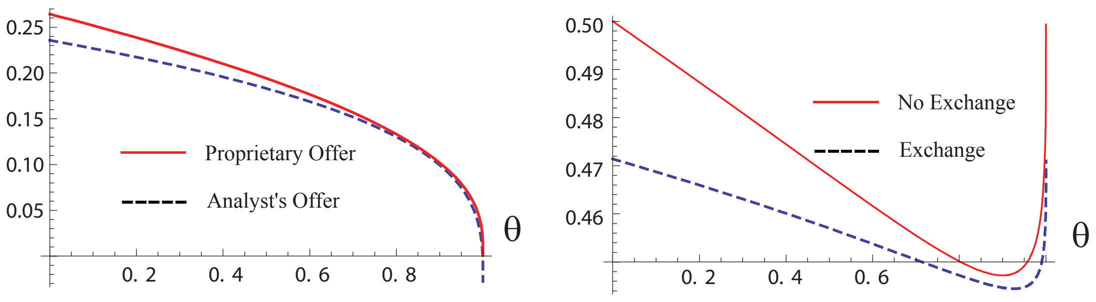


(a) The (c,m) Frontier

(b) The Information Sales Equilibrium

Figure 2: Information Exchange

Parameter values: $\Sigma = 1; \sigma_u^2 = 1; \Psi = 1$



(a) No voluntary exchange

(b) Common firm prefers no exchange