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ESTIMATING BOND LIQUIDITY

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Abstract

Liquidity significantly affects bond prices, yet this variable remains little understood. This paper gives a definition and measure of bond liquidity, and shows why it tends to decay over time. A model of bid-ask spreads as a function of term, price and liquidity is then developed. It is used to estimate the liquidity of individual U.S. Treasury bond issues outstanding during 1987-1990. The pattern of liquidity decay is illuminated by regressing these estimates on age and quantity outstanding.

I. **Introduction**

The prices of bonds are determined primarily, but not exclusively, by the bonds' cash flows. Also important is the bonds' *liquidity*: some bonds can be traded more readily, and traded in larger quantities, than others. In many cases investors will sacrifice some cash value to acquire securities which can be traded more readily and in larger quantities at a later time.

The observation that bond liquidity is positively priced was made at least as early as Fisher (1959) who correlated excess corporate bond yields with, among other things, the total quantity of each bond outstanding.[1] Only quite recently, however, has bond liquidity received detailed academic attention.

Amihud and Mendelson (1991) observed that U.S. Treasury bills display greater liquidity than otherwise comparable U.S. Treasury notes near their maturity dates.

In consequence, the notes carry higher yields and higher bid-ask spreads than bills. They documented such differences for 37 trading days in 1987. They found a mean yield difference of 0.43% and a standard deviation of 0.47%. Similarly Kamara (1990) documented positive yield differences between Treasury notes and comparable Treasury bills during 1977-1984; these yield differences have a mean of 0.34% and a standard deviation of 0.39%.

Sarig and Warga (1989) noted that the age of a U.S. Treasury issue is a proxy for its illiquidity; they correlated age with the stale-price problem which can occur when bonds trade infrequently. Warga (1991) then studied excess returns to constant duration U.S. Treasury bond portfolios. Since 1982, bond portfolios provided excess returns to (more liquid) Treasury bills in virtually all cases, and whole-market bond portfolios provided higher returns than the most liquid ("on the run") bond portfolios.

This research demonstrates that liquidity significantly affects Treasury bond prices. To understand liquidity pricing in detail, however, one needs a more precise definition and measure of liquidity. This not only has inherent interest, it is critically important to estimating interest rates.

The term structure of interest rates is typically estimated from the prices of U.S. Treasury bonds and bills. But if these prices contain significant liquidity values, as indicated by the above studies, all term structure estimates which do not deal with the liquidity factor are biased. If liquidity is a positive value, conventional interest rate estimates are biased low. To put it simply, we do not know what interest rates are until we have taken liquidity into account.

But what exactly is liquidity? How do we measure it? Is it meaningful to say that one bond is twice as liquid as another? The literature offers surprisingly little guidance to these questions. Lippman and McCall (1986) define liquidity as (the inverse of) the expected time until an asset can be exchanged for money, when following an optimal pricing and search policy. They show the consistency of this definition with concepts of liquidity advanced by numerous others.[2] While this is an important contribution to our understanding, we have no hope of directly measuring this expected time, and so the concept remains somewhat elusive.

Section II of this paper offers a definition and measure of bond liquidity, and shows why it tends to decline over time. This discussion leads to an expected relationship between bid-ask spreads and liquidity. Section III examines bid-ask spreads in the CRSP bond file and models their dependence on term, price and liquidity. Section IV then uses this model to estimate the liquidity of individual Treasury bonds during the period 1987-1990. Section V analyzes the results, in which the tendency of bond liquidity to decay over time is clearly evident. Section VI offers some concluding thoughts.

II. Defining and Measuring Liquidity

What is meant by bond[3] liquidity, and why should it decay over time? Amihud and Mendelson (1991) and Sarig and Warga (1989) maintain that liquidity declines because over time bonds are "locked away" or "absorbed" into untraded portfolios. To make this concept precise, let us assume that the universe of bond holders is divided into two groups: traders and nontraders. Traders are prepared to buy and sell bonds at or near current market prices, while nontraders will only buy. Note that this definition of "traders" is much broader than dealers (market-makers).

It may seem strange to assume that any group of bondholders should not trade their portfolios, since the benefits of trading are so widely accepted. The general case for securities trading as set out, for example, in Duffie (1988), turns on the ability of trading strategies to replicate a diversity of (generally unavailable) contingent claims. This diversity increases with the number of uncorrelated securities and the frequency of trading opportunities. The prototype portfolio is a set of common stocks.

Treasury bonds, however, are relatively few in number (about 200 are outstanding at any given time); more importantly, their prices are highly correlated with each other. Any information or economic variable affecting one bond is likely to affect all in a similar way. The benefit of trading such bonds against each other is not at all clear.

In practice, Treasury bond price anomalies occur because of supply or demand imbalances in particular issues. Market-makers bring to the attention of bond holders opportunities to sell at relatively high prices or buy at relatively low prices based primarily on such anomalies. But there are two reasons why institutions may not want to avail themselves of such offers. The two reasons are risk and cost.

As to risk, bond buyers are typically institutions, many of which have relatively fixed liabilities. Such institutions often seek to match sets of liabilities with portfolios of fixed-rate assets such as Treasury bonds. When this has been done, the institution is relatively immune to future changes in the term structure. Trades typically swap a bond of one term for a bond of a different term, based on price anomalies. But accepting such trades will generally alter the risk profile of the investor, and the investor may not wish to do this.

As to cost, trading involves not only transaction costs but also the compensation of one or more skilled persons to conduct the activity. Such persons not only need to be hired, they need to stay in continuing contact with the market in order to remain current. Institutions should and will hire such a person only if they intend to trade on an ongoing basis. This tends to force a decision whether, as a matter of policy, the portfolio should be traded or not. Many smaller and more conservative institutions decide to be nontraders.

The trader / nontrader distinction leads quickly to a measure of liquidity and an expectation that it should decline over time. Assume that bond issues are indexed by i. Define the liquidity of each bond i as *the total quantity of bond i owned by traders at time t*, and let $\ell_i(t)$ represent this quantity. The aggregate liquidity of the market, i.e. the total quantity of bonds owned by traders at time t is then $\mathcal{L}(t) = \sum \ell_i(t)$.

Assume further that trades are initiated when a trader offers to sell a randomly-selected bond; the probability that a particular sell offer involves bond i is $\ell_i(t)/\mathcal{L}(t)$. The mean rate of bond trading is proportional to $\mathcal{L}(t)$. The mean rate of trading bond i is proportional to $\ell_i(t)$, as is the expected quantity of bond i sold to nontraders. Thus $\ell_i(t)$ declines at a rate proportional to its size, i.e. it decays exponentially.

There is no reason to assume that this rate is constant over time or the same for all issues. For example, issues may lose liquidity most rapidly during their initial distribution, when they are said to be "on the run". The pace of liquidity decay may decrease thereafter. But when the issue becomes sufficiently illiquid to command a premium return, it will be preferred by nontraders; the decay rate may well accelerate beyond this point.

Furthermore, each issue has its own idiosyncrasies. Some issues are callable, which may affect their appeal to traders. Some issues are in the Treasury's "strips" program; this may reduce their liquidity, by taking a significant portion of such issues out of circulation, or it may increase their liquidity by making such issues favored trading vehicles. In other cases, vagaries of supply and demand may cause some issues to fall into inactive portfolios to a greater degree than others.

The trading of each issue is thus a separate process in which nontraders represent an absorbing barrier. Even without knowing the details of the process, we can see that the expected interarrival time between trades is simply the inverse of the trading rate.

This fact enables us to link the concept of liquidity given here with the liquidity definition in Lippman and McCall (1986), for the expected interarrival time between trades of bond i is proportional to $1/\ell_i(t)$. The advantage of the present formulation is that $\ell_i(t)$ is in principle a measurable quantity.

Let us now turn to the problem of estimating bond liquidity. The best measure would be a direct one: since trading rates are proportional to $\ell_i(t)$, we could sum the daily trading volumes in each issue across the major Treasury dealers. Unfortunately, dealers do not report or retain their issue-by-issue trading volumes. Thus a direct measure is not generally available. We can achieve an indirect estimate, however, from the relationship between liquidity and bid-ask spreads.

Dealers' profits and losses have three components: (i) change in market value of positions, (ii) cost of carry, and (iii) bid-ask spreads. These are separate elements, with little if any correlation. The rate at which (iii) is earned will be the same for all issues if the spread on each issue is proportional to the expected waiting time between trades. Small spreads on each trade are acceptable if trades are frequent; larger spreads are required for less active issues, where the dealer's expected holding period is longer. In the formalism used here, the bid-ask spread should be inversely proportional to $\ell_1(t)$.

It is widely assumed that this relation holds. Amihud and Mendelson (1986) assert that the bid-ask spread is "a natural measure of illiquidity", citing studies of stock spreads. They then correlate bid-ask spreads with asset returns. The strategy followed here will be to extract an estimate of bond liquidity from spread quotation data.

III. A Model of Bid-Ask Spreads

Before modeling bid-ask spreads, it is useful to look at the character of the available data, to see whether they contain enough information to enable us to estimate bond liquidity.

The data studied here are from the CRSP bond tape for 48 month-ends during 1987-1990. The bid and ask prices on the CRSP tape are derived from the Federal Reserve Bank of New York's daily survey of Treasury dealers, and do not represent actual transactions. Treasury bills are not studied. The total number of observations available is 9530, about 200 at each of the 48 time points.

Close observers of Treasury markets have long noted that publicly quoted spreads are wider than the "inside" spread available at the moment of trading.[4] The latter will vary frequently during the day, reflecting changing conditions and the dealer's desire to increase or reduce positions at any particular moment. If we could observe such inside spreads, we would need to average them over some period of time to estimate a mean or "normal" level which should be inversely proportional to liquidity.

Although the CRSP spread data represent public quotations only, they vary significantly from month to month and from bond to bond, and appear to contain significant information. It is reasonable to assume that such public quotations bear some orderly relationship, bond by bond, to the mean level of inside spreads.

If they were exactly proportional to the mean level of inside spreads, they would convey the full effect of liquidity differences. I shall assume that they are essentially proportional, but are censored by rounding.

The CRSP spread quotations almost always equal 1/16, 1/8 or 1/4 % of the bonds' face value, with an occasional spread of 1/2 %. The six "flower bonds"[5] have spread quotations of 1% of face value, but only one observation shows a 1% spread for a non-flower bond. Ten observations are excluded because they do not fall into these five size categories, so the data set involves 9520 observations.

The observed spreads can be conveniently represented as $2^k/16 = 2^{k-4}$, where k = 0, 1, 2, 3 or 4. The upper two categories are quite rare. As noted, the k=4 case is almost entirely restricted to six issues, while the k=3 case occurs in just 40 observations involving only four issues.[6] Thus, except for ten issues, U.S. Treasuries in this period carried spreads characterized by k = 0, 1 or 2.

Figure 1 illustrates this pattern by displaying spread data against term for eight of the 48 cross-sections. As is apparent from this figure, the spread is primarily a step function of the bond's term. In each cross-section there are two or three primary steps separated by one or two truncation points.

In the first cross-section, representing March 31, 1987, the dealers' primary rule seems to be that bonds with terms of about eight years or less are quoted with spreads of 1/8 % (k=1) while longer bonds are set at 1/4 % (k=2), though in both cases there are exceptions. By September 30, 1987, in the second cross-section, bonds under 5 years term are generally attracting spreads of 1/16 % (k=0): the curve has shifted downward.[7] By the time we reach the last cross-section, representing September 28, 1990, bonds over two years are quoted generally at 1/8 % (k=1) and the k=2 case has become relatively rare. But in all cases, spreads seem to be predominantly a function of term.

A considerable literature has developed in the past few years which attempts to model the spreads on common stocks in terms of possible adverse information costs. George, Kaul and Nimalendran (1991) both summarize and extend this literature. However, spread differences among Treasury bonds cannot be attributed to asymmetric information since any information which affects one Treasury bond price will likely affect all. While dealers will have different and changing positions in various bonds at various times, the spread effects of such differences will be fleeting: there are no "lemons" in this market. The spreads on Treasury bonds thus present a substantially simpler problem than stock spreads.

Let us assume that there is an underlying, continuous spread function which we shall call $F(\tau)$, where τ is the term of the bond. Let p_a represent the ask price and p_b the bid price. Then a model of bid-ask spreads, dropping the index i, is

$$\frac{p_a - p_b}{p_b} = \frac{F(\tau)}{\ell} \tag{1}$$

That is, the spread as a percentage of price is some unknown function of term divided by the bond's liquidity. It is understood that both ℓ and $F(\tau)$ change over time, although this functional dependence is suppressed for simplicity of notation.

Taking logarithms,

$$\ln (p_a - p_b) = \ln p_b + \ln F(\tau) - \ln \ell$$

As noted above, $\ln (p_a - p_b)$ can be represented by the variable k; in fact, $\ln (p_a - p_b) = (k-4) \ln 2$. It will be assumed that k is a censored version of an underlying spread variable $\hat{s}(\tau)$ which is observed with error:

$$\hat{s}(\tau) = 4 + [\ln p_b + \ln F(\tau) - \ln \ell] / \ln 2 \tag{2}$$

$$k = \text{round}\,[\hat{s}(\tau) + \varepsilon]$$

where ε is a normally-distributed random error of mean zero.

This model resembles ordered probit, except that the categories of k have a precise quantitative relationship to each other, so that truncation points in the model of k need not be estimated; based on the nature of the observed spreads, the truncation points are precisely 0.5, 1.5, 2.5 and 3.5. The model can be viewed as a constrained ordered probit.

The estimation procedure consists of two phases. In the first phase, a variation on (2) will be estimated month by month assuming that all bonds have the same liquidity ℓ_0. That is, we set

$$\begin{aligned} s(\tau) &= 4 + [\ln p_b + \ln F(\tau) - \ln \ell_0] \,/\, \ln 2 \qquad (3) \\ &= \hat{s}(\tau) + \delta \\ \delta &= (\ln \ell - \ln \ell_0) \,/\, \ln 2 = \frac{\ln(\ell/\ell_0)}{\ln 2} \\ k &= \text{round } [s(\tau) - \delta + \varepsilon] \end{aligned}$$

This transfers the relative liquidity measure δ into the disturbance term. It is assumed that δ is also normally distributed with mean zero over the data set, but that a subset consisting of a single bond observed in different months could show residuals which were not mean zero, due to a persistent liquidity difference of ℓ from ℓ_0. The goal of the second phase is to recover the liquidity measure δ from the residuals of the first phase.

To estimate (3) against actual data, we need to specify the functional form of $F(\tau)$. In general, spread increases with term because longer-term bonds carry greater price volatility than shorter-term bonds, and risk-averse market-makers will require a higher return to carry them in inventory. To model this adequately would require, *inter alia,* (i) a term structure model, (ii) an estimate of the volatility of bonds at each term, (iii) an assumed utility function for market-makers.

All of this would carry us far afield from the subject of liquidity, and would involve numerous arbitrary assumptions. Furthermore, the end result is observed in such a severely censored form that, in a sense, the sophisticated approach to $F(\tau)$ would be wasted. A simpler, atheoretical approach is therefore taken here.

Based solely on the observed data, it will be assumed that on the range $0 \le \tau \le \infty$, $\ln F(\tau) \,/\, \ln 2$ rises asymptotically to a value higher by 2 than its starting point. Such a function can without loss of generality be represented in the form $\hat{a} - 2e^{-f(\tau)}$, where $f(0)=0$, $f(\infty)=\infty$, $f(\tau)>0 \;\; \forall \; \tau>0$. Thus we shall rewrite (3) as:

$$s(\tau) = \ln p_b \,/\, \ln 2 + a - 2\, e^{-f(\tau)} \qquad (4)$$

where a collects the various constant terms.

To summarize, s as a function of τ is assumed confined to a range of approximately 2 (the presence of $\ln p_b / \ln 2$ in (4) makes the range slightly greater than 2). In contrast, k is observed to take values of 0, 1, 2, 3, or 4.

A general function such as $f(\tau)$ can be represented to any desired degree of accuracy by a sufficiently high-order polynomial or by a set of spline functions. If we had finely-tuned spread information, it might be meaningful to utilize such a representation. Given the severe censoring, however, we need not go beyond a linear form: $f(\tau) = b\tau$; the two parameters a and b are then sufficient to specify the two truncation points. Since $s(\tau)$ is censored by rounding, the truncation points are the solutions of $s(\tau) = 0.5$ and $s(\tau) = 1.5$.

Thus we have, finally,

$$\begin{aligned} s(\tau) &= \ln p_b / \ln 2 + a - 2\, e^{-b\tau} \\ k &= \text{round}\,[s(\tau) - \delta + \varepsilon] \end{aligned} \tag{5}$$

This is the model which is estimated in the next section.

IV. The Estimation Procedure

Phase 1. In Phase 1 the function $s(\tau)$ is estimated according to equations (5) above using the method of maximum likelihood. Since the conception of $s(\tau)$ is based on at most three steps with two truncation points, the appropriate probability structure is:

$$\begin{aligned} \text{Prob}[k=0] &= \Phi[(0.5-s)/\sigma_1] \\ \text{Prob}[k=1] &= \Phi[(1.5-s)/\sigma_1] - \Phi[(0.5-s)/\sigma_1] \\ \text{Prob}[k\geq 2] &= 1 - \Phi[(1.5-s)/\sigma_1] \end{aligned}$$

where $\Phi[\cdot]$ is the unit normal cdf and σ_1 is the standard deviation of $-\delta+\varepsilon$.

The likelihood function $L(a,b,\sigma_1)$ of each observation is formed by substituting $s(\tau)$ from (5) into one of the above probabilities based on its k-value. Maximum likelihood estimates of the parameters a, b and σ_1 are then found for each of the 48 month-ends in 1987-90.

These values are reported in Table 1. Note that a falls more or less steadily over the 48 months, while b rises. As noted above, bid-ask spreads generally fell throughout the period. The substantial drop in months 6-9 (July-September, 1987) is visible in the parameters. Model curves of $s(\tau)$ over time for τ = 2, 5, 10 and 30 years based on the estimated parameters are displayed in Figure 2; the patterns are perhaps more easily seen in the graphs. Note, for example, that spreads not only have fallen but also that the upper three term categories have tended to converge.

The estimated $s(\tau)$ functions correctly predict about three-quarters of the spreads in sample, in the sense that the observed $k = \text{round}[s(\tau)]$. Their lowest success rate is 60.0% in January, 1987 and their highest is 83.7% in February, 1988. The liquidity information, of course, comes from the failures of $s(\tau)$ to predict k.

Phase 2. The residuals k-s from phase 1 are studied in phase 2. Our goal is to extract estimates of δ from these residuals. We can do this by comparing all the residuals for the same bond observed in different months. A tendency for the residuals from one bond to be persistently and significantly non-zero is taken as evidence of a liquidity effect.

The data set covers 324 separate bond issues, some of which matured and some of which were newly issued during the 48-month period. Of these, spreads on 79 are perfectly predicted by $s(\tau)$; no deviations from the k-value found by rounding $s(\tau)$ are observed. This set of bonds forms the "base group" whose liquidity is deemed equal to ℓ_0. Alternatively, δ for these bonds is sufficiently close to zero that it cannot be measured given the noise created by ε and by rounding.

The mean residual for each bond is a biased estimator of the relative liquidity, given censored observations over a limited number of data points. We need a maximum likelihood estimator of δ for each bond. This may be obtained from the probability structure:

$$\text{Prob}[k=0] = \Phi[(0.5-s)/\sigma_2)]$$
$$\text{Prob}[k=1] = \Phi[(1.5-s)/\sigma_2)] - \Phi[(0.5-s)/\sigma_2)]$$
$$\text{Prob}[k=2] = \Phi[(2.5-s)/\sigma_2)] - \Phi[(1.5-s)/\sigma_2)]$$
$$\text{Prob}[k=3] = \Phi[(3.5-s)/\sigma_2)] - \Phi[(2.5-s)/\sigma_2)]$$
$$\text{Prob}[k=4] = 1 - \Phi[(3.5-s)/\sigma_2)]$$

where, since δ is now being estimated, σ_2 is the standard deviation of ε alone. The right-hand sides of these relations may be summarized as:

$$\Phi[(k-s+0.5)/\sigma_2] - \Phi[(k-s-0.5)/\sigma_2]$$

except that the second cdf is 0 when k=0 and the first cdf is 1 when k=4. But k-s is the residual from phase 1, which we have modeled as $-\delta$ plus a random disturbance ε. Thus the likelihood function for phase 2 is:

$$L(\delta,\sigma_2) = \Phi[(-\delta+0.5)/\sigma_2] - \Phi[(-\delta-0.5)/\sigma_2] \quad (7)$$

except that the second cdf is 0 when k=0 and the first cdf is 1 when k=4. Since σ_2 is the standard deviation of ε alone, so we should find that $\sigma_2 < \sigma_1$.

The major problem posed by estimation of (7) is that, when abs(δ) is large compared to 0.5, $L(\delta,\sigma_2)$ becomes highly collinear in the two parameters. That is, pairs of parameter values with δ/σ_2 = constant become nearly interchangeable with each other. The likelihood function has a long, flat ridge associated with such pairs, along which a maximum is often difficult to locate. Even when it can be found, the covariance matrix of the parameters usually fails to invert, and the Hessian cannot be estimated.

This near-multicollinearity greatly increases the error with which δ is estimated, particularly extreme values of δ. Yet it is precisely the extreme values which are of greatest interest in studying the range of liquidity variation among bonds. The best solution is to estimate the two parameters separately rather than together.

The base group of bonds provides an excellent opportunity to do this. Because δ is zero or near-zero in this subset, σ_1 (the variance of $-\delta+\varepsilon$) should be very close to σ_2 (the variance of ε alone). The base group contains 1342 observations. The residuals in this group have a sample variance of 0.0522, i.e. a standard deviation of 0.2284. A 99% confidence interval for σ_1 in this subset is $0.2170 < \sigma_1 < 0.2398$.

This reasoning leads to the estimate $\sigma_2 = 0.2284$. The likely presence of a small amount of δ-variance in the base group may bias this estimate slightly upward. However, any such bias would lower estimates of δ based upon its use. Thus $\sigma_2 = 0.2284$ is a conservative value to use in estimating δ, and is used herein.

The reader may wonder whether, since liquidity decays exponentially over time, we could model δ as $\alpha - \lambda t$ in (7), thereby achieving estimates of the individual decay rate λ as well as the log liquidity α. While an attractive idea, such estimation is distorted by the significant time trends in $s(\tau)$. As noted, $s(\tau)$ tends to decline over time; in addition, a given bond's term τ shortens with each month, further reducing $s(\tau)$. Since k is often constant, this time trend frequently passes directly into the residual k-s. Yet since k is not always constant, it does not pass perfectly and cannot easily be filtered out. The result is indistinguishable from rising δ, so λ is biased quite high and α is misestimated as well. On balance, it seems sounder to settle for point estimates of mean liquidity for each bond during the 4-year period.

V. The Results

The procedure described above results in estimates of relative log liquidity $\ln(\ell/\ell_0) = \delta \ln 2$ for all 324 bonds in the sample, including estimates of 0 (i.e. $\ell=\ell_0$) for the 79 bonds in the base group. The highest value is 1.4 and the lowest is -3.5. This implies that the most liquid bond has an ℓ value about four times that of the base group, and that the least liquid bond has an ℓ value of about three percent of the base group's liquidity. As might be expected, the six flower bonds show relative log liquidity well below all the others, with a mean of -2.8.

Figure 3 displays a histogram of the results, aside from the flower bonds. The distribution is skewed somewhat to the right, presumably because the large federal deficits of the late 1980's caused an historically unusual quantity of new bonds to be issued. This departure from normality in the distribution of δ may affect the accuracy of the estimates.

We can test the hypothesis H_0: $\delta=0$ against H_1: $\delta\neq0$ by computing a likelihood ratio for each bond. Because of the model's structure, this ratio is massive for the extreme values of δ. Highly liquid or highly illiquid bonds are quoted with spreads persistently different than those of the base group. Given the small value of σ_2, the probability of this happening by chance for 48 times is virtually zero.

The likelihood ratios for the outlying bonds therefore tend to infinity. In all, 58 highly liquid bonds and 24 highly illiquid bonds show likelihood ratios in excess of 10^6. The null hypothesis thus can be rejected for an important segment of the market.

Many bonds, of course, have more normal likelihood ratios. No effort is made to reject any estimate based on this statistic, for indeed we expect that some of the relative log liquidity values are at or near zero. These are simply best estimates based on available data.

The 21 bonds whose relative log liquidity exceeds 1.0 all have a mean age during the period of less than one year. This means that all of them were outstanding for less than half the full 48 months, i.e. they are being observed for a limited period of time immediately after issuance. All but four are Treasury notes with an original maturity of 2-5 years.

Conversely, the 11 bonds whose relative log liquidity is less than -0.5 are all older issues. The six flower bonds have mean ages of 25-34 years during the period, and original maturities of 30-40 years. The five non-flower bonds in this group have mean ages of 11-16 years during the period, and original maturities of 15-30 years.

Figure 4 displays the relative log liquidity estimates plotted against the mean age of each issue during the period. The six flower bonds appear in the lower right hand corner. The line of symbols at zero log liquidity represents the base group. The overall pattern is one of rapid decline during the first two years, followed by a leveling out and eventually a further decline.

Given the discussion of liquidity in Section II, we expect relative log liquidity to be correlated with quantity of the issue outstanding and with mean age of the issue during the period studied. If a bond's liquidity equaled its total issue amount on the first day of its life (say Q), and declined exponentially thereafter at a fixed rate (say λ), we would have $\ell = Qe^{-\lambda g}$, where g is the age of the bond, or $\ln \ell = \ln Q - \lambda g$.

To generalize this, the following model was estimated by OLS:

$$\ln \ell \,/\, \ln \ell_0 \;=\; \beta_0 \ln Q + \beta_1\, g_1 + \beta_2\, g_2 + \beta_3\, g_3 \qquad (8)$$

where g_m is the mean age of the bond times an indicator variable which is one if the mean age is in range m and zero otherwise. The ranges used, based primarily on the patterns visible in Figure 4, are:

m = 1:	$0 < g \leq 2$ years
m = 2:	$2 < g \leq 12$ years
m = 3:	$12 < g \leq 30$ years

The results are reported in Table 2.

The mean age is a significant determinant of liquidity in all three ranges, and is of the expected (negative) sign. But β_1 is 3 times greater than β_2, suggesting that liquidity decays more rapidly during the first two years of issuance than later. Since $e^{-2(0.2283)} = 0.63$, it appears that bonds lose about a third of their liquidity during their first two years of life. Since $\beta_3 < \beta_2$, it appears that liquidity decay does accelerate in the late stages of a bond's life.

The logarithm of the quantity issued is also significant and of the expected (positive) sign, but the magnitude of the coefficient is substantially lower than one. The demand of traders appears related to the issue's size in a significant way, but not on a one-for-one basis.

The impact of the government's strips program on bond liquidity was also studied. Surprisingly, an indicator variable equal to one if the bond were in the government's strips program and zero otherwise turned out to have no significance in the regressions. As a second check, the actual quantity of bonds stripped, and therefore out of circulation, was obtained for each month-end and each issue from the Treasury.

The quantity of each issue held by non-governmental entities, and therefore in circulation, is supplied in the CRSP file, but it does not take the Treasury's strips program into account. A variable was therefore prepared which reduced the CRSP number by the total amount of the bond which had been stripped at the end of each month. Substituting this variable for ln Q in the regressions, however, produces results almost identical to those reported in Table 2. The explanatory power of this new variable is, perhaps surprisingly, neither greater nor less than that of ln Q. This confirms the insignificance of the strips indicator variable.

VI. Conclusion

Bond traders have long understood the importance of liquidity. They feel its immediacy and importance in the rhythm of their daily business. But because records of trading volumes by issue are not usually kept, liquidity leaves no direct record to be studied at a later time.

Bid-ask spread quotations, however, constitute an indirect record. A careful analysis of such spreads can produce bond-by-bond liquidity estimates, as demonstrated in this paper. Such estimates give us a first look at how liquidity decays with a bond's age, and how liquidity is affected by the quantity of the issue outstanding.

The estimates developed in this paper suggest that bonds lose more than a third of their liquidity during their first two years after issuance. Liquidity decay slows after the second year, but increases slightly in later years, when higher returns may make illiquid bonds preferred investments for nontraders.

Liquidity is significantly related to the quantity of the issue outstanding, but not on a directly proportional basis as might have been expected. The Treasury's strips program has no measurable impact on the liquidity estimates developed here.

The greatest significance of these results may lie in their capacity to explain bond pricing. Term structure estimates which assume that the prices of bonds are strictly equal to the present value of their cash flows are biased, because liquidity is positively priced. Only by studying liquidity carefully, and taking it into account when estimating interest rates, can unbiased term structure estimates be achieved.

Notes

[1]"The smaller the amount of bonds a firm has outstanding, the less frequently we should expect its bonds to change hands. The less often its bonds change hands, the thinner the market; and the thinner the market, the more uncertain is the market price. Hence, other things being equal, the larger the market value of publicly traded bonds a firm has outstanding, the smaller is the expected risk premium on those bonds." (p. 225)

[2]The lack of academic attention to liquidity is indicated by the fact that Lippman and McCall's 21 references date back as far as 1921; only three were more recent than 1977; and one of these three was also by Lippman and McCall.

[3]For the purpose of this discussion, a Treasury "bond" means either a bond or a note. The liquidity estimates of this paper have no statistically significant correlation with whether the security in question is formally designated "bond" or "note".

[4]See, for example, Scott (1965), chapters 3 and 4.

[5]Flower bonds can be delivered at par plus accrued interest in settlement of estate taxes upon the death of the holder. They are held by investors at a late stage of life, and go permanently out of circulation on the holder's death. See Mayers and Smith (1987). The flower bonds in this sample are these: 3 1/2 % due 2/15/90, 4 1/4 % due 8/15/92, 4 % due 2/15/93, 4 1/8 % due 5/15/94, 3 % due 2/15/95, and 3 1/2 % due 11/15/98.

[6]These are the 6 3/4 % due 2/15/93, the 7 7/8 % due 2/15/93, the 7 % due 5/15/98, and the 8 1/2 % due 5/15/00. The third of these accounts for the sole non-flower bond occurrence of a 1 % spread.

[7]The number of bonds with spreads of 1/16 % in June, 1987 was 9; this became 26 in July, 49 in August and 85 in September.

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Table 1
Estimated parameters of $s(\tau)$

$$s(\tau) = \ln p_b / \ln 2 + a - 2 e^{-b\tau}$$

$$k = \text{round}\ [s(\tau) - \delta + \varepsilon]$$

The observed spread variable k is modeled as a censored version of an underlying continuous function s of bid price p_b, term τ, a liquidity measure δ and a random error ε. The parameters a, b and σ_1, the standard deviation of $-\delta+\varepsilon$, are estimated by the method of maximum likelihood.

Month	a	b	σ ~~Sigma~~
1	-4.087	0.100	0.733
2	-3.822	0.050	0.746
3	-4.043	0.092	0.756
4	-3.997	0.096	0.691
5	-4.081	0.114	0.740
6	-4.085	0.108	0.760
7	-4.372	0.151	0.657
8	-4.697	0.203	0.676
9	-4.942	0.181	0.881
10	-5.020	0.188	0.891
11	-5.030	0.188	0.929
12	-5.068	0.175	0.934
13	-5.096	0.184	0.900
14	-5.128	0.187	0.931
15	-5.098	0.181	0.936
16	-5.067	0.183	0.915
17	-5.073	0.187	0.943
18	-5.097	0.188	0.947
19	-4.876	0.196	0.907
20	-4.876	0.197	0.921
21	-4.941	0.219	0.941
22	-4.934	0.226	0.948
23	-4.926	0.232	0.971
24	-4.912	0.242	0.899
25	-5.045	0.271	0.874
26	-5.057	0.310	0.854
27	-5.072	0.292	0.885
28	-5.139	0.288	0.908
29	-5.185	0.287	0.935
30	-5.114	0.292	0.869
31	-5.043	0.297	0.877
32	-5.055	0.311	0.863
33	-5.013	0.314	0.849
34	-4.889	0.261	0.883
35	-4.878	0.263	0.859
36	-4.858	0.261	0.848
37	-4.963	0.302	0.862
38	-5.104	0.321	0.898
39	-5.092	0.325	0.910
40	-5.046	0.317	0.785
41	-5.141	0.314	0.795
42	-5.487	0.533	0.752
43	-5.509	0.543	0.767
44	-5.503	0.564	0.763
45	-5.518	0.594	0.766
46	-5.579	0.636	0.778
47	-5.614	0.528	0.835
48	-5.609	0.514	0.839

Table 2
Results of regressing relative log liquidity on log quantity and term variables

$$\ln \ell \,/\, \ln \ell_0 \;=\; \beta_0 \ln Q + \beta_1 g_1 + \beta_2 g_2 + \beta_3 g_3$$

Coefficient	Est. Value	Std. Error	t-value
β_0	0.0601	0.0054	11.14
β_1	-0.2283	0.0389	- 5.86
β_2	-0.0743	0.0075	- 9.94
β_3	-0.0984	0.0068	-14.49

($N = 324$, $R^2 = 0.67$)

The relative log liquidity estimates developed in this paper are here regressed on the logarithm of the quantity of each issue outstanding, and on three variables g_m equal to the mean term of the issue during the period studied times an indicator variable equal to 1 if the mean term is 0-2 years (g_1), 2-12 years (g_2) and 12-30 years (g_3), zero otherwise. The standard errors are estimated on a heteroscedasticity-consistent basis.

Figure 1

Bond spreads as a function of term

Bond spread quotations for eight of the 48 month-ends in 1987-90 are displayed as a function of term. Spreads of 1/16, 1/8, 1/4, 1/2 and 1% are displayed as k = 0, 1, 2, 3 and 4. An assumed underlying curve $s(\tau)$ for par bonds is computed for each cross-section as described in the text.

BOND SPREADS AS A FUNCTION OF TERM

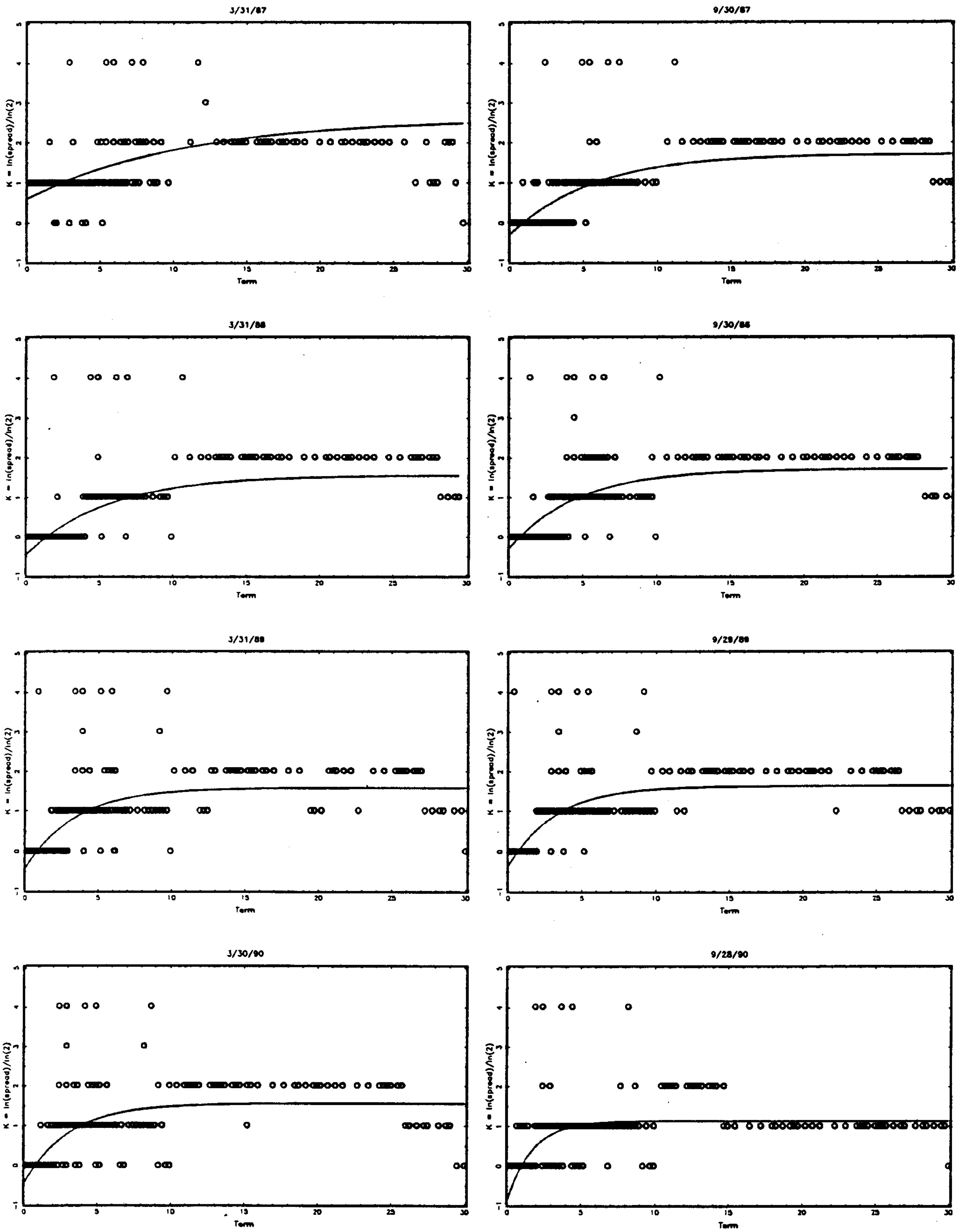

Figure 2

Changing values of the spread function

The parameters of the underlying spread function $s(\tau)$ estimated for each of the 48 month-ends are used to compute four representative curves, namely τ = 2, 5, 10 and 30 years. These curves show the decline of spreads during the period studied and the convergence of certain term categories.

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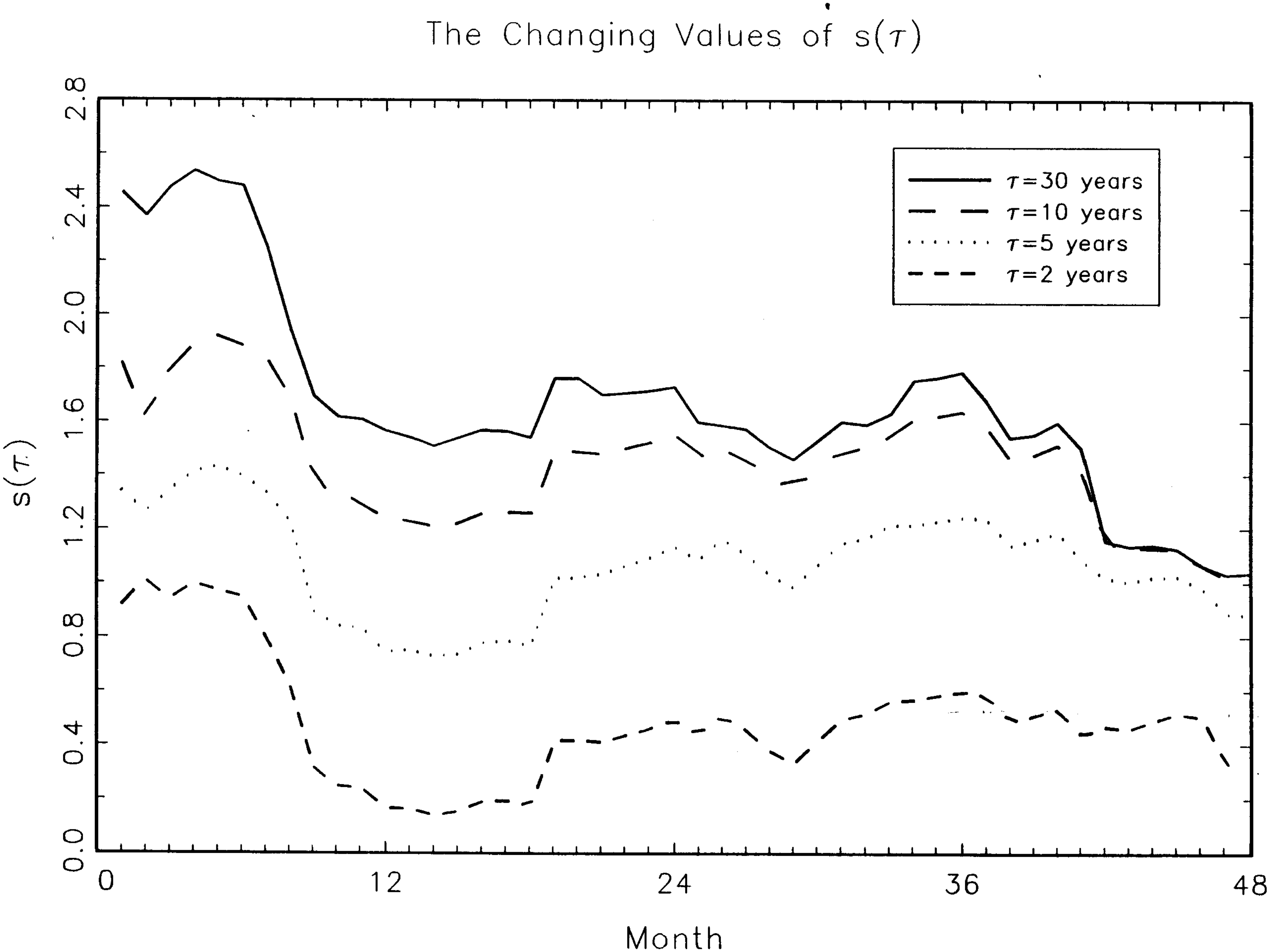

Figure 3

Frequency distribution of log liquidity estimates

Log liquidity estimates for U.S. Treasury bonds are grouped into sets centered on the values shown. The base group of 79 bonds with zero relative log liquidity are added to the central set. The six flower bonds are not shown.

Frequency Distribution of Relative Log Liquidity Estimates

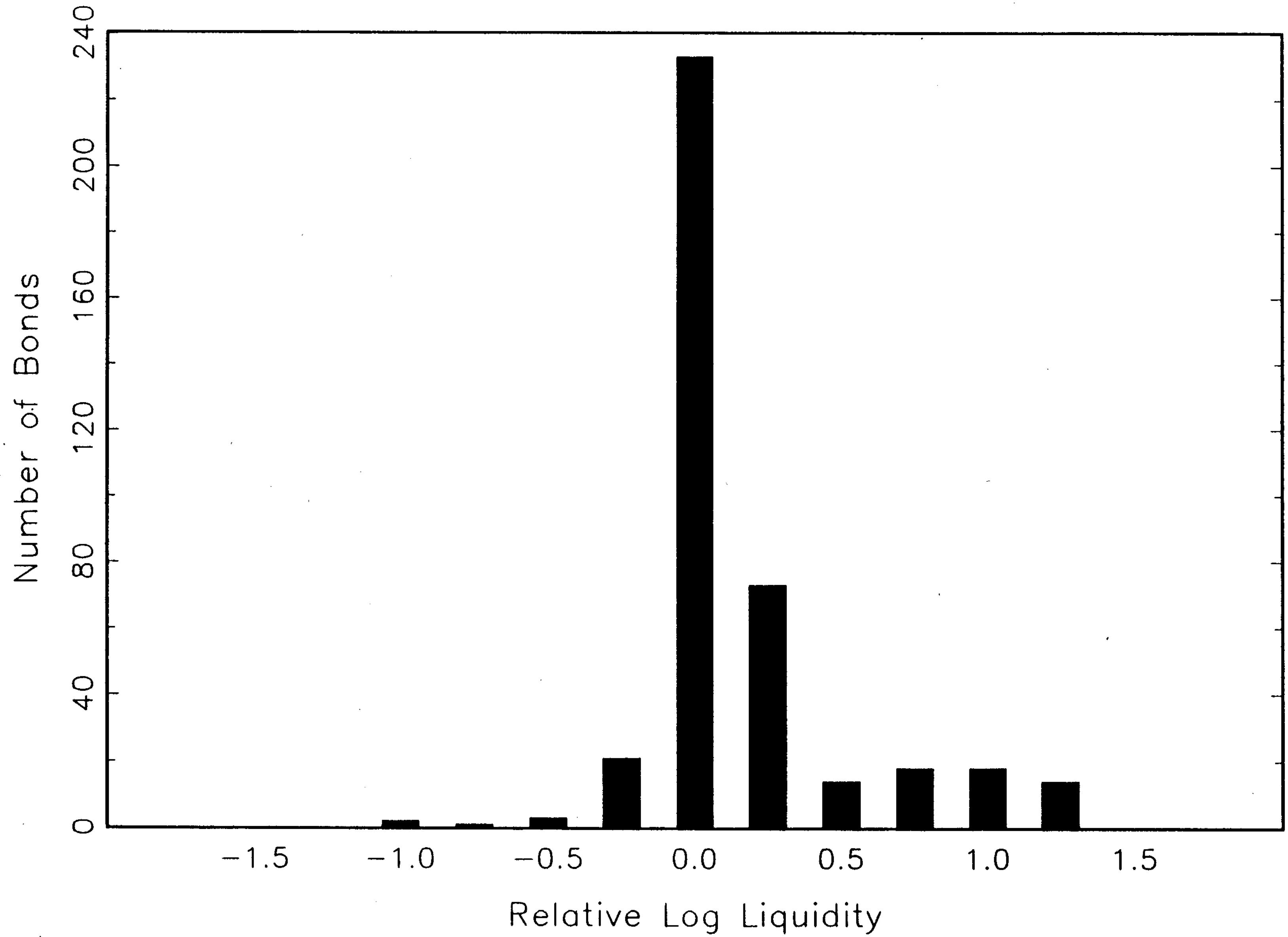

Figure 4

The decay of relative log liquidity

Each bond is plotted according to its relative log liquidity estimate and the mean age of the issue during the period studied. The tendency of bond liquidity to decay with time can be observed.

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The Decay of Relative Log Liquidity