COMPETITION AMONG EXCHANGES*

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Does competition among financial intermediaries lead to excessively low standards? To examine this question, we construct a model where intermediaries design contracts to attract trading volume, taking into consideration that traders differ in credit quality and may default. When credit quality is observable, intermediaries demand the "right" amount of guarantees. A monopolist would demand fewer guarantees. Private information about credit quality has an ambiguous effect in a competitive environment. When the cost of default is large (small), private information leads to higher (lower) standards. We exhibit examples where private information is present and competition produces higher standards than monopoly does.

I. INTRODUCTION

In spite of the general acceptance of the competitive mechanism as an allocation scheme, many observers consider financial intermediation an exception. Competition is thought to lead to a "race to the bottom" in which financial intermediaries settle for excessively low levels of contractual guarantees in an attempt to increase volume. Evaluating futures exchanges, a joint report from the Treasury and the Federal Reserve Board, written in the wake of the 1987 crash, states that "[T]here is sufficient possibility that at some point SROs [Self Regulatory Organizations] might establish margins that were inconsistent enough to present market problems or set them at levels that might present potential costs to other parties that regulatory approval should be established in all markets."[1]

In this paper we examine the characteristics of the competitive equilibrium in a model of financial intermediation, where traders may default and differ in their credit quality. In particular, we investigate whether competition among exchanges leads to excessively low standards.

In our model, investors' endowments have observable and

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1. This quote is taken from Gammill [1988].

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unobservable risks. The observable risk averages out across agents, and hence can potentially be hedged. In addition, each agent has a small probability of an unobservable negative idiosyncratic shock. We call traders who have a lower (higher) probability of an adverse idiosyncratic shock high (respectively, low) quality traders. These different probabilities may result from other risks already assumed by the traders, which are left outside the model. Traders' quality may be observable or hidden private information. When traders default, society imposes a nonpecuniary punishment proportional to the amount defaulted.

Potential exchanges compete by setting collateral requirements and asset payoffs. An exchange's profit depends on the payoffs promised, and on the defaults that occur. To simplify the analysis, we postulate that each exchange issues a single contract, and each agent must trade in a single exchange. However, investors are allowed to take any long or short position, and exchanges demand margins that are proportional to the absolute size of positions.

When quality is observable, competition among financial intermediaries leads to a (constrained) optimal amount of contractual guarantees. Here, our results formalize the intuition of Telser [1981] and Miller [1988], who argue that futures exchanges choose margin requirements to balance the reduced risk from default against the higher costs brought by the extra degree of protection. In contrast, a monopolist would demand fewer contractual guarantees. This follows because a monopolist's pricing policy induces traders to take smaller positions. As a result, the ex post incentives to default are lower, and this allows the monopolist to economize collateral.

We study the effect of the unobservability of a trader's quality on the competitive equilibrium, and show that this effect depends on the level of the exogenous bankruptcy penalties. When these penalties are high, if quality is observable, low quality traders prefer the terms offered to the high quality traders. As a result, when qualities are not observable, the higher quality traders face a higher collateral level than they would if quality was observable, while the low quality traders face exactly the same terms that they would face if quality was observable. Here collateral is being used as a screening device, and private information about quality generates standards that are too high relative to the observable quality case. On the other hand, if the exogenous penalties are low, the high quality agents prefer the terms of trade offered to the low quality traders in the equilibrium with observable qualities. As a result, in the equilibrium with private information about qualities, the lower quality traders face a lower collateral level than they would in the competitive allocation under observable qualities, while the high quality traders face exactly the same terms that they would face if qualities were observable. In this case, the presence of private information results in a lower standard. For intermediate levels of penalties, the equilibrium with observable qualities survives the presence of private information, and there is no departure from the constrained optimality.

When credit quality is not observable, it is more difficult to compare the behavior of a monopolist with a competitive equilibrium, because a monopolist must take into account the effect that a change in a contract's terms has on the demand for any other contract that he is simultaneously offering. Nonetheless, as we argue in Section IV, there are many cases where it is possible to establish that a monopolist will demand fewer contractual guarantees than would prevail under competition.

To discuss the impact of liquidity in a simple framework, we postulate that there is a fixed cost to operate an exchange. These fixed costs assure that, ceteris paribus, exchanges with more participants can offer better terms. These better terms can be thought of as a measure of liquidity. We show that when fixed costs are present and competition prevails, agents of one quality may benefit from the establishment of minimum collateral requirements by a regulator, at the expense of agents of the other quality, even if these minimum collateral requirements are binding for all qualities. The agents who profit from the establishment of these tougher standards are exactly those who, in the absence of regulation, choose to trade in the exchange with higher collateral. Hence, this model provides a candidate positive theory to explain the frequent demands by more established exchanges for common standards in financial markets.

In our model, collateral is deposited as a performance bond against contractual obligations. The margin requirement specifies the collateral required. This is motivated by the role that margins play in futures markets. Although many of our modeling choices are inspired by the characteristics of futures markets, we believe that our approach can be used to understand the behavior of other financial intermediaries. While our model is necessarily special, we argue that many of the conclusions would survive different specifications and highlight, throughout the paper, the economic forces that drive the conclusions.

We also have a more abstract motivation for this paper—incorporating questions of credit quality into the literature on security design.[2] This literature examines the characteristics of securities when market imperfections prevent the existence of a complete set of securities. Frequently, costs of issuing securities are assumed exogenously. If asymmetric information is treated, it is typically assumed that the designer of the security has private information. Our contribution is to analyze a situation in which securities are competitively designed by intermediaries, and where these intermediaries must consider the characteristics of the agents that they attract.

The remainder of the paper is structured as follows: in the next section we present an overview of some of the evidence on competition among exchanges. Section III contains the model and a more detailed defense of the assumptions. In Section IV we present results for the case of no fixed costs. The fifth section looks at the problem of liquidity, and Section VI contains the conclusions. All proofs are found in the Appendix.

II. Some Examples

In this section we discuss some of the evidence that motivates our modeling choices.

A. Collateral Is Costly

When an exchange chooses higher margins, it lowers expected losses from defaults, but increases the costs of trading. Faced with higher collateral requirements, some traders will opt to trade elsewhere, others will simply trade less. As a consequence, the exchange becomes less liquid and loses revenues. Hardouvelis and Kim [1995] studied the impact of margin changes on volume. They examined eight metal futures contracts from the early 1970s to October 1990—a sample that contains over 500 margin changes. For each margin change, they constructed a benchmark group consisting of the metal contracts that do not change margins in a four-month period around the

2. Some relevant references are Allen and Gale [1988, 1991], Bisin [1998], Duffie and Jackson [1989], and the February 1995 issue of the *Journal of Economic Theory*. Santos [1997] studied the impact of financial innovations on the credit quality of a preexisting bond market.

date of the margin change. They estimated that a 10 percent increase in margins decreases the volume of trade by 1.4 percent and increases the volume of the benchmark group by 1.8 percent. Another example is reported in Figlewski [1984]. In late 1965, at the request of the government, COMEX more than tripled margin requirements on copper futures. As a result, the daily average number of traded contracts went from 832 to 260.

B. Traders Care about the Risk of Nonperformance

In the United States the Commodity Exchange Act expressly precludes the CFTC from imposing minimum margin requirements for futures transactions. Nonetheless, futures exchanges have always demanded positive margins. As a counterparty to every trade, an exchange assumes a risk when it allows a trader to post less than 100 percent collateral.[3] The speculation in silver by Bunker and Herbert Hunt, and associates, from September 1979 to April 1980, provided an experiment that shows that traders are willing to pay to do business in an exchange with less performance risk. In the late 1970s the Hunt brothers established large long positions in silver futures at the Chicago Board of Trade (CBOT) and New York's COMEX. Starting in October 1979, the CBOT took a series of actions, including position limits, that caused the Hunts to move most of their trades to New York. Sellers of futures in New York had reason to worry about the possibility of a default by the Hunt brothers, while those selling in Chicago were insulated from this risk. Bailey and Ng [1991] showed that spreads between the COMEX and the CBOT increased toward the end of the Hunt episode, when it was evident that the brothers were facing financial distress. They also argued that evidence of a deterioration in the Hunts' financial conditions can be matched to positive changes in the COMEX-CBOT spread, and that this spread fell with positive announcements about the Hunts.

3. In reality, it is the clearinghouses that guarantee deliveries. Clearinghouses collect margins, set capital requirements, and other financial standards. They may also establish rules to assess members to cover defaults. All trades that are guaranteed by the clearinghouse must go through a clearing member—other brokers must trade through a member. Traders in turn go through a broker. Clearinghouses and exchanges also set minimum margins that brokers must charge. In this paper we ignore the distinction between clearinghouses and exchanges, and omit the brokers from the picture.

C. Exchanges Compete on Contractual Guarantees

Competition among financial intermediaries involves many dimensions of product differentiation. In some cases exchanges introduce nearly identical instruments. Two examples are the one-month Deutschmark futures contracts which started trading on both Frankfurt's DTB and London's LIFFE in late 1996, and the competition between Singapore's SIMEX and Osaka's OSE in the Nikkei futures. In other cases, exchanges create new contracts that they hope will better fit the needs of potential traders. To make the problem tractable, we concentrate on the guarantees demanded from traders to insure contract performance, even though, in reality, product differentiation occurs simultaneously along several dimensions, only one of which is these guarantees. Exchanges use several devices to guarantee contract performance, including marking to market, trading limits, and capital requirements. However, in this paper we use margin requirements as a proxy for all of these instruments.

The most notorious recent case of competition among exchanges produced the demise of Barings PLC, the oldest merchant bank in Great Britain. In the fall of 1988, the Osaka Stock Exchange (OSE) launched a Nikkei 225 futures contract which mimicked a contract that the Singapore International Money Exchange (SIMEX) had been trading for two years. Trading quickly moved to Japan, and by the end of 1990, the OSE had a market share that exceeded 90 percent. From January 1990 until August 1993, the OSE increased margins on four occasions, while the SIMEX decreased margins five times and increased margins twice. Ito and Lin [1996] documented that each margin increase led to lost volume in favor of the competing exchange. In December of 1993 margin requirements in the SIMEX were 15 percent of a contract's face value, versus 30 percent in Japan, and SIMEX had regained 30 percent of the Nikkei contracts. Among those moving their business to Singapore was Barings. Nicholas Leeson, who was simultaneously Barings' proprietary trader and head of settlements in Singapore, accumulated large losses in futures and on options on futures on the Nikkei index, and hid these losses from the London office. Like most futures exchanges, SIMEX had a policy of limiting "speculative" positions to prevent large losses, but Barings had assured the Singapore exchange that their position was hedged by an opposite position in Osaka. Nonetheless, SIMEX failed to take measures to verify Barings' position at the OSE, as it would normally do. Gerard Pfaudwadel, chairman of MATIF, the French futures exchange, said that, "It's clear that a big part of the problem with Barings was due to the battle for new business between Singapore and Osaka . . . exchanges shouldn't compete by reducing the amount of security they require, because as soon as they do, it increases the risk for everyone involved."[4]

In some instances, competition among exchanges is actually competition among regulators. Commenting on the competition between the Singapore and Hong Kong exchanges, *Euromoney* [February, 1997] wrote: " . . . The regulators in Hong Kong and Singapore . . . want to ensure standards high enough to attract mainstream international investors and yet to encourage less sophisticated regional players too . . . . The joint distinction of Hong Kong and Singapore is that they are tier one, or nearly tier one, regimes. To maintain that distinction they must not be tempted into a race for regulatory leniency." This remark also illustrates that exchanges must bear in mind distinct potential clienteles when choosing contractual guarantees. In our model, we consider two sets of agents, distinguished by their "credit quality." Exchanges take into account the impact of collateral requirements on the demand by each of these qualities, for the exchange's products.

D. Intermediaries' Assessment of Counterparty Risk Is Imperfect

An important aspect of our model is that we do not allow exchanges to use nonlinear prices. We assume that exchanges cannot price discriminate as a function of the size of trades because we believe that, in actual financial markets, intermediaries have difficulties in assessing the total risk of counterparties. The Barings case, and the LTCM debacle that involved the over-the-counter market, illustrate this inability when traders are simultaneously dealing with several counterparties. However, there are also examples involving a single market. In 1977 the CFTC had a limit on positions in the soybean futures markets of three million bushels, approximately 5 percent of the annual U. S. soybean output. Nonetheless, Bunker and Herbert Hunt succeeded in acquiring a long position of 26.9 million bushels at the CBOT, using the name of family members. Among these, Houston Bunker Hunt, a nineteen-year old freshman, "borrowed" 2 million

4. Quoted by Suzanne McGee in *The Wall Street Journal,* March 17, 1995.

dollars from his father to pay for the margins on a position of three million bushels that he bought by placing his orders from a public phone booth at the ΦKA fraternity house at the University of Tulsa.[5] Pirrong [1996] relates a cruder incident at the CBOT, in which two traders sold thousands of T-bond future contracts and bought hundred of puts on these futures. They were eventually caught, but not before imposing severe losses on their clearing firm.

III. The Model

A. Description of the Economy

Consider an economy with two periods. In the first period only asset trading occurs; all endowment realizations and consumption take place in the second period. In this second period there are two equally probable observable states of the world, s_1 and s_2. There are a large number of agents. In each state, exactly half of the agents receive an endowment of x units of the consumption good. The other half "typically" receives a larger endowment y. However, each of these agents has a small probability of receiving $z < x$ instead of the larger endowment y. The occurrence of this bad draw is private information and independent across agents. We call the probability of the bad draw, z, the "quality" of the agent. Formally, each agent is characterized by membership in a group $i \in \{1,2\}$, and a quality $\pi \in \{p,q\}$. Within each group a fraction α_π is of quality π. The table below describes the distribution of endowments of an agent of quality $\pi \in \{p,q\}$.

group	*endowment in* s_1	*endowment in* s_2
1	x	y with probability π z with probability $1 - \pi$
2	y with probability π z with probability $1 - \pi$	x

Since each group is the mirror image of the other, aggregate endowment is constant. Agents maximize the expected utility of consumption, and have a common utility function $u(\cdot)$ with $u'' <$

5. See Fay [1982]. The episode did not cost the brothers much. The Hunts settled out of court in 1981, paying $500,000 and agreeing to stay out of soybean futures for two years.

0, and $u'(0) = \infty$. Since we want to introduce the possibility of default, we fix parameters so that, in equilibrium, an agent would buy assets that transfer consumption to the state where they receive the certain endowment x. That is, we assume

ASSUMPTION 1. $u'(x) > (1 - \pi)u'(z) + \pi u'(y)$.

Notice that this assumption is only plausible if π is close to 1. For definiteness we assume that $p > q$, and this means that the "unlucky" draw (z) is relatively more likely for quality q agents. We refer to agents of quality $p(q)$ as high (low) quality agents. Each agent knows his own quality.

Exchanges compete for customers by issuing contracts.[6] Since first-period endowments are zero, exchanges are limited to issuing forwards. To focus on the ability of the marketed contracts to spread risks among investors, we assume that exchanges must break even in each state of the world.[7] In this paper we assume that investors are precluded from participating in more than one exchange. In addition, we identify each contract with an exchange.[8] Each exchange incurs a fixed cost $\sigma \geq 0$.

We concentrate our analysis on contracts that treat groups (but not qualities) symmetrically. The forward is normalized so that the party that is long must deliver one unit of the consumption good in state s_2. Agents may potentially default. To finance the costs of operation and defaults, the exchange pays less than one unit for each unit of the claim; it delivers to an agent that is long one unit of the contract, $K < 1$ units of the good in state s_1. The exchange is also allowed to require collateral from traders. When a trader deposits collateral C, this amount is taken from his second-period endowment and delivered to the exchange with probability one to satisfy any claims the exchange has on the

6. We model exchanges as competitive firms, even though until recently many exchanges were organized as "mutual nonprofit organizations." Here "mutual" refers to the fact that the financial intermediaries who trade in the exchange also control it. The term "nonprofit" should not be equated to "charitable." Presumably the objective function of these exchanges was to maximize the surplus of its members, but there could be some divergence between this objective and profit maximization. We choose here to assume profit maximization directly because it avoids the explicit modeling of the member firms, and because there is a clear trend among exchanges to organize as for-profit firms.

7. We may assume that any agent can set up an exchange. Since each agent is an infinitesimal portion of the economy, he is not able to sustain finite losses, and this justifies the assumption that exchanges must break even in each state of the world.

8. Hahn [1978] discusses the role of this assumption in the Rothschild-Stiglitz model.

agent. This requires that C does not exceed the minimum endowment that the trader receives in a state where he is required to deliver to the exchange. There is no cost of setting up collateral. Consistent with our search for a symmetric outcome, and to make the model robust to splitting and pooling orders, we assume that each exchange chooses a margin rate $\Phi \geq 0$ and demands that a trader with a position of θ contracts secures at least $\Phi|\theta|$ as collateral. Investors are allowed to take any long or short position in the marketed asset. However, an agent's default triggers an examination of that agent's trades, and his positions are netted before settlement. Since $K < 1$, agents would never simultaneously take a long and a short position. The two assumptions we make—that traders use a single exchange and that exchanges cannot price discriminate as a function of the size of positions—partially cancel each other. Still left in the model is a moral-hazard problem that, as we argued in Section II, affects financial intermediation, and it is valuable to examine the effect of the inability of exchanges to control the risk of their counterparties in this particularly simple case.[9]

Since we assume a large number of traders, the default risk that each trader represents is averaged out and an asset's quality is represented by the scalar K. Hence, in the model, there is no risk associated with trading in a particular exchange—an agent simply faces distinct terms of trade in different exchanges. One possible reinterpretation of the model that allows for differences in risk across exchanges is to assume that there are "risky" times in which the idiosyncratic risk (z instead of y) is present and "quiet" times in which this risk is absent. Agents have to choose an exchange, before it is known whether they are in a risky or quiet period. However, the portfolio of each agent is chosen after it is known whether the idiosyncratic risk is present. Exchanges that pay a lower K in risky times can be rightly regarded as riskier. For simplicity, we do not pursue this interpretation here.

As we already discussed, our assumptions guarantee that, in equilibrium, agents take positions that are out of the money in the state where they face a lottery. The presence of individual uninsurable shocks (receiving z instead of y) makes it undesir-

9. In contrast, in Bisin and Guaitoli [1998] agents can simultaneously buy contracts from several insurance companies, and each insurer can monitor the position of an agent in their own contract. Bisin and Gottardi [1999] discuss existence of equilibrium, in a model where agents can purchase several different contracts and face linear prices.

able to require that agents deliver fully on their commitments, since in this case they would not be able to transfer much to the state where they receive the endowment x. Allowing for default is welfare improving. Default is essentially a method to make payoffs covary, albeit imperfectly, with the idiosyncratic shock. We find it particularly convenient to borrow the default technology of Shubik and Wilson [1977], which was also used by Diamond [1984] and Dubey, Geanakoplos, and Shubik [1996]. This amounts to introducing a penalty that is imposed directly on the utility of the individual and measured in utility units. If in period 2 an agent is committed to deliver an amount $w \geq 0$ in the observed state, and delivers instead $D \geq 0$, he suffers a penalty, $\lambda \max(w - D, 0)$, in utility units. That is, if an agent's deliveries meet his commitments, he suffers no penalties, otherwise the penalty is proportional to the amount that he fails to deliver. Notice that, in conformity with our earlier assumption concerning the robustness to the splitting and pooling of trades, penalties are linear functions of the amount an agent comes short.

The parameter λ should be understood as the economywide bankruptcy code, and exchanges cannot alter it. Since default is introduced to allow for some relief when z is obtained, it is natural to assume that $u'(z) > \lambda$. We will also assume that the penalty is not too small. More exactly, we assume

ASSUMPTION 2. $u'(z) > \lambda > \pi u'(y) + (1 - \pi)u'(z)$.

In addition, to eliminate one multiplier, we assume that z is sufficiently small, so that agents that obtain y will always deliver in excess of collateral.

ASSUMPTION 3. $u'(y - z) < \lambda$.

We are ready now to describe the investor's problem. Suppose that the default penalty parameter is λ, and that the agent is trading in an exchange that demands collateral of Φ (per unit) and pays in the money contracts K (per unit). An agent who trades in this exchange has to decide how many contracts to buy and how many units to deliver when he is out of the money. In the Appendix we write down the decision problem of the agent precisely and show that the agent's maximization problem has a unique solution. Furthermore, the symmetry present in our model guarantees that the position chosen by an agent of group 2 is exactly equal to the negative of the position taken by an agent of group 1. Lemma 1 in the Appendix confirms that group 1 agents take long positions in the forward contract.

Each agent also has to decide how much to deliver. Since $u'(z) > \lambda$, an agent who receives z will make no deliveries in excess of collateral. When endowed with y, the investor will equate the marginal utility of consumption to the marginal penalty, λ. Let θ^* solve $u'(y - \theta^*) = \lambda$. A group 1 agent who is long θ_π contracts, and receives endowment y, will fully deliver if $\theta_\pi \leq \theta^*$. Otherwise, he will deliver $D = \max\{\theta^*, \Phi\theta_\pi\} = \theta^*$, since $\Phi\theta_\pi < z$, and from Assumption 3, $z \leq \theta^*$. The delivery decision depends on the investor's quality only through the position. Hence, given the parameters λ, Φ, and K, a solution to the optimization problem faced by an agent of group 1 and quality π can be parameterized by two positive numbers (θ_π, D_π). The first element is the position he takes, and the second the amount he chooses to deliver when his endowment realization is y.

In the Appendix we show that the first-order conditions for the agents problem define a function $\theta_\pi(\Phi,K)$ that is smooth except perhaps for the (measure zero) set of values of the parameters for which $\theta_\pi = \theta^*$.

B. Equilibrium

In view of the results in the previous subsection, it suffices to make reference to group 1 agents when discussing equilibria. Write $V_\pi(\Phi,K)$ for the expected utility achieved by group 1 agents of quality π trading in an exchange that demands collateral of Φ (per unit) and pays in the money contracts K (per unit). Since we assume that the parameter λ is fixed, we do not make any explicit reference to λ in the definition of V or in the other quantities that follow. We will concentrate on either pooling or separating equilibria. There is no loss in generality in assuming that all agents of a given quality trade in the same exchange. Hence, in a pooling equilibrium there will be a single exchange, while in a separating equilibrium there will be two exchanges.

If (θ_π, D_π) solves the maximization problem for group 1 when facing the contract (Φ,K), we write

$$\Pi_\pi(\Phi,K) \equiv \tfrac{1}{2}\,\alpha_\pi[\pi D_\pi + (1 - \pi)\Phi_\pi\theta_\pi - K\theta_\pi] - \sigma, \tag{1}$$

for the per capita profit generated by a contract (Φ,K), if it attracts all agents of quality π and no agents of the other quality. We will also write $\Pi_{p,q}(\Phi,K) \equiv \Pi_p(\Phi,K) + \Pi_q(\Phi,K) + \sigma$ for the profit generated by a contract (Φ,K), if it attracts all agents of both qualities.

A pooling equilibrium is

(a) a contract (Φ,K) such that $\Pi_{p,q}(\Phi,K) \geq 0$;

(b) there is no other contract (Φ',K') that when offered in addition to the contract (Φ,K) makes positive profits.

(a) guarantees that, in state s_1, the sum of the fixed costs of the exchange plus the amount it delivers to the parties that are long the asset (group 1) does not exceed the amount delivered by the out-of-the-money positions (held by members of group 2). By symmetry we do not need to write the equivalent equation for state s_2. (b) reflects free entry. We also do not need to include market clearing among our equilibrium conditions because group 2 always takes the negative of the position taken by group 1.

A separating equilibrium, when qualities are observable, is

(a) for each $\pi \in \{p,q\}$ a contract (Φ_π,K_π) such that $\Pi_\pi(\Phi_\pi,K_\pi) \geq 0$;

(b) there is no other contract (Φ',K') that when offered in addition to the contracts $(\Phi_\pi,K_\pi)_{\pi\in\{p,q\}}$ makes positive profits.

When qualities are not observable, we must add to the definition of a separating equilibrium:

(c) For each $\pi \in \{p,q\}$, $V_\pi(\Phi_\pi,K_\pi) \geq V_\pi(\Phi_{\pi'},K_{\pi'})$ for any $\pi' \neq \pi$.

Condition (c) states that agents of each quality prefer the contract assigned to that quality to the other contract.

IV. Competition with Costless Exchanges

In this section we study the equilibrium when $\sigma = 0$. In our setup, because agents can vary the size of their positions, the zero-profit combinations do not form a straight line as in the original Rothschild and Stiglitz [1976] model. Proposition 1 describes these zero-profit lines. Proposition 2 characterizes equilibria when qualities are observable. In this case, an exchange is created for each quality. Typically, but not always, the exchange that requires a higher collateral serves the high quality agents. We then discuss the optimality properties of this equilibrium and compare the equilibrium allocation with the monopoly outcome.

The remainder of the section deals with the case in which an agent's quality is private information. When an equilibrium exists, we show that, unlike the Rothschild-Stiglitz model, the incentive constraints may bind for any of the two qualities and as a result the default rate may be higher, lower, or exactly the same as in the case of observable qualities.

We will assume that collateral is expensive when compared with penalties. More precisely, Assumption 4 states that it never pays to extract collateral from the unlucky agents to distribute to the agents who receive x, even after accounting for the reduced penalties. This assumption is used to simplify proofs and to characterize in a more explicit manner the equilibrium when qualities are observable.

ASSUMPTION 4. $u'(x) - [u'(z) - \lambda] \leq 0$.

The properties of equilibria will depend on the level of the exogenous penalties, λ. Suppose that λ_π^* and θ_π^* solve the following system of equations:

$$\pi u'(x + \theta_\pi^* \pi) = u'(y - \theta_\pi^*), \tag{2}$$

$$\lambda_\pi^* = u'(y - \theta_\pi^*). \tag{3}$$

That is, λ_π^* is the minimum level of penalties such that, when there are no collateral requirements and exchanges set $K = \pi$, agents of quality π take a position θ_π^* which leads them to fully deliver, when they obtain the high endowment y. Since there is no collateral posted, agents deliver nothing if they obtain z and the realized default rate from quality π is exactly $1 - \pi$.

The next proposition describes the zero-profit lines for an exchange that only attracts agents of quality π.

PROPOSITION 1. (i) For any $\Phi \in [0,1]$, there exists exactly one $K(\Phi,\pi) > 0$ such that $\Pi_\pi(\Phi, K(\Phi,\pi)) = 0$ and $\theta_\pi > 0$. The function K is increasing in Φ.

(ii) Suppose that $\lambda \leq \lambda_\pi^*$ and θ^* solves $u'(y - \theta^*) = \lambda$. Let $[\Phi_\pi^*, K_\pi^*]$ be the unique solution to

$$u'(x + \theta^* K_\pi^*) K_\pi^* - (1 - \pi)\Phi_\pi^*[u'(z - \theta^* \Phi_\pi^*) - \lambda] - \lambda = 0 \tag{4}$$

$$K_\pi^* - (1 - \pi)\Phi_\pi^* - \pi = 0 \tag{5}$$

if a solution exists with $0 \leq \Phi_\pi^* \leq 1$. Otherwise set $\Phi_\pi^* = K_\pi^* = 1$. Then,

$$K(\Phi,\pi) < (1 - \pi)\Phi + \pi, \quad \text{for all } 0 \leq \Phi, \Phi < \Phi_\pi^*$$

$$K(\Phi,\pi) = (1 - \pi)\Phi + \pi, \quad \text{for all } \Phi \geq \Phi_\pi^*.$$

Proposition 1 (i) establishes that for any collateral requirement there exists precisely one value of $K = K(\Phi,\pi)$ such that if an exchange is the only one serving quality π and uses parameters (Φ,K) then (a) agents of quality π take a nonzero position in that exchange, and (b) the exchange makes zero net revenue from the agents of quality π. Note that $(1 - \pi)\Phi + \pi$ is the expected delivery rate of traders of quality π, conditional on full delivery by those who obtain y (and no deliveries above collateral for those that obtain z). Part (ii) states that there is a collateral level Φ^*_π with the following property: for any Φ, the corresponding zero-profit $K(\Phi,\pi)$ is less than this expected delivery rate if and only if $\Phi \le \Phi^*_\pi$. The equations in part (ii) of the proposition also have a solution if $\lambda > \lambda^*_\pi$. However, in this case $\Phi^*_\pi < 0$. That is, exchanges could demand negative collateral and still obtain full delivery from those who obtain y. In this case, exogenous penalties are high enough, and collateral is not needed to guarantee delivery. In fact, exchanges could deliver a positive amount to agents who claimed to have received z. Since we want to stress the role of collateral, we assume

ASSUMPTION 5. $\lambda \le \lambda^*_\pi$, for each $\pi \in \{p,q\}$.

When qualities are observable, costless entry guarantees that, in equilibrium, each exchange attracts a single quality and the terms offered by the exchange maximizes the utility of that quality subject to the constraint that individuals can choose to default and suffer the penalties. If collateral is set above Φ^*_π, lowering the collateral requirement by an infinitesimal amount would not lead to any defaults by agents that receive endowment y. The marginal gain in the expected utility of a group 1 agent would be

$$(1 - \pi)\theta[u'(z - \Phi\theta) - \lambda]. \tag{6}$$

On the other hand, to maintain profits constant, the exchange would have to lower deliveries proportionately. The marginal loss in expected utility for an agent of group 1, conditional on being long, would be $(1 - \pi)\theta u'(x + \theta K)$. Assumption 4 guarantees that this amount is smaller than the expression in (6). Hence, in equilibrium, the collateral charged by the exchange serving quality π cannot exceed Φ^*_π.

Increasing collateral requirements when $\Phi < \Phi^*_\pi$ penalizes the traders who obtain endowment z, but allows exchanges to increase K. The increase in K is financed by two sources. The first, exactly as in the case where $\Phi > \Phi^*_\pi$, is the increase in deliveries from the agents who receive endowment z, and is proportional to $1 - \pi$. As we just argued above, Assumption 4 guarantees that this increase in collateral and deliveries has a negative net effect on expected utilities. Here, however, there is also a second source to finance the increase in K. To economize on collateral, agents respond by lowering the absolute value of their positions. When $\Phi > \Phi^*_\pi$, this has no effect on the share of each agent's obligations that is fulfilled. However, when $\Phi < \Phi^*_\pi$, lowering θ reduces the default rate and allows exchanges, holding profits constant, to increase K. In the Appendix we show that, when $\Phi < \Phi^*_\pi$, the elasticity of the trading position θ with respect to the collateral requirement Φ is large enough to guarantee an increase in expected utility whenever collateral is raised. The next proposition formally states these results.

PROPOSITION 2. If qualities are observable, there exists a unique equilibrium. In this equilibrium there is one traded contract for each quality $\pi \in \{p,q\}$, $[\Phi^*_\pi, (1 - \pi)\Phi^*_\pi + \pi]$, and agents who obtain endowment y deliver fully.

In this equilibrium, if penalties are low, positive collateral is used as a substitute for penalties.[10] A social planner who could decide on the amount transferred to each player as a function of a message sent by the player concerning their endowment could do better. The social planner would typically make positive transfers to agents who claimed to have received z. However, if such a scheme were in place, individual agents would have an incentive to claim more than one "name," just as in any rationing scheme. In other words, to decentralize such an allocation would require placing limits on traders' positions, something our exchanges cannot do.[11] Suppose that the central planner is restricted to linear schemes, and that he cannot avoid the imposition of penalties on traders who fail to deliver. A standard argument shows that the central planner would choose the competitive allocation described in Proposition 2. In this sense, the equilibrium under symmetric information is a constrained optimum. We also show, in the Appendix, that a monopolist would demand zero collateral. Hence the default rate that a monopolist would face from traders

10. Indeed, it is possible to prove that the equilibrium margin requirement Φ^*_π is a decreasing function of the penalties in place. See Santos and Scheinkman [1998].

11. See the story on the Hunts' speculation on soybeans described in Section II above.

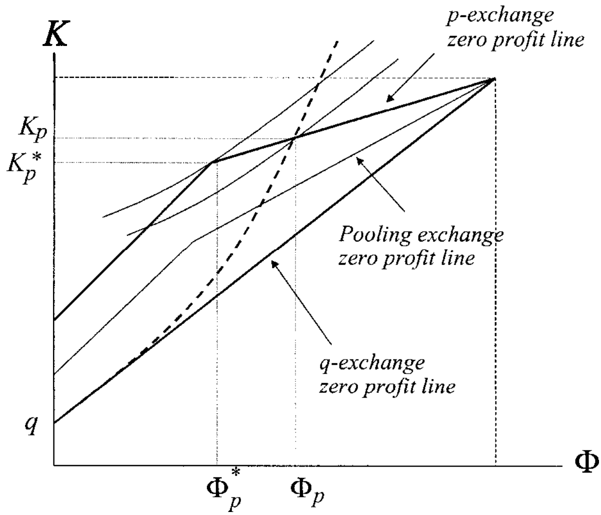

FIGURE I
Overcollateralization
The q agents are indifferent between trading contract $(0,q)$ and trading contract (Φ_p^*,K_p^*). The dashed line is the indifference curve of q agents.

of quality π is at least $1 - \pi$, while, in the competitive equilibrium, the default rate from traders of quality π is $(1 - \pi)(1 - \Phi_\pi^*) \leq 1 - \pi$.

We turn next to the case where an agent's quality is private information. In this case, the equilibrium constructed in Proposition 2 is not necessarily sustainable. In the example displayed in Figure I, $\lambda = \lambda_q^* < \lambda_p^*$. When qualities are observable, the q agents would trade the contract $(\Phi_q^*,K_q^*) = (0,q)$, whereas the p agents would trade the contract (Φ_p^*,K_p^*). Both qualities prefer the contract offered to the p agents. In this example the single crossing property holds. Since the net revenue contributed by agents is a continuous function of the contract terms, (Φ,K), when quality is not observable, any equilibrium must be separating. In such an equilibrium, the q agents must get at least as much utility as they would obtain in the equilibrium with observable qualities. Otherwise, an entrant would offer a contract close to $(0,q)$, attract all the q agents, and make a profit from the q agents. An application of the implicit function theorem shows that at zero collateral p agents deliver at a higher rate than q agents. Hence the entrant gains from any p agents the contract attracts. On the other hand, since $(0,q)$ is the best separating contract for the q agents that breaks even, in equilibrium, the q agents cannot get more utility than in the case of observable qualities. To determine the contract offered to the p agents, first notice that the single-crossing property implies that, along any q indifference curve, the utility of the p agents increases with Φ. Hence, if a separating equilibrium exists, the contract offered the p agents must be at the intersection of the p zero-profit line and the q agents' indifference curve that passes through $(0,q)$. Such a point (Φ_p,K_p) must necessarily satisfy $\Phi_p > \Phi_p^*$; i.e., the p exchange is "overcollateralized." In this equilibrium, collateral is being used as a screening device.[12]

Lemma 2 in the Appendix, establishes that the p quality agents take a larger position than the q quality agents in the preferred contract, (Φ_p^*,K_p^*). Hence a minimum contract size would serve to ameliorate the screening problem. Frequently, contract sizes in exchange are considerable, although this may simply reflect fixed costs of trading.

In the Rothschild-Stiglitz model, the high quality agents never prefer the allocation proposed to the low quality agents. The presence of moral hazard changes this outcome. In our model, when qualities are observable, it is possible that, in equilibrium, the p agents prefer the contract offered on the q exchange.[13] If the margin differential, $\Phi_p^* - \Phi_q^*$, is large enough, although the delivery rate in the q exchange is lower, the p agents may prefer to trade in that exchange because the lower margins would allow them to take a larger position. In this case, the equilibrium, when qualities cannot be observed, involves the p quality agents getting the same allocation as in the case where qualities are observable, and the q quality agents trading in an exchange with sufficiently low collateral (and consequently low delivery rate) to deter the p agents from trading. Here, since the p quality agents would take a large position in the q exchange, position limits can help solve the adverse selection problem.

In the next example we fix all parameters except for the penalty rate λ and examine the equilibria when an agent's quality is private information. We show that if λ is high enough, in equilibrium, the q agents get the same contract and p agents face a higher collateral than they would if quality was observable. On

12. Bester [1985, 1987] uses collateral as a screening device in a different context.

13. Results with a similar flavor can be found in Chassagnon and Chiappori [1997]. However, their model and ours cannot be nested.

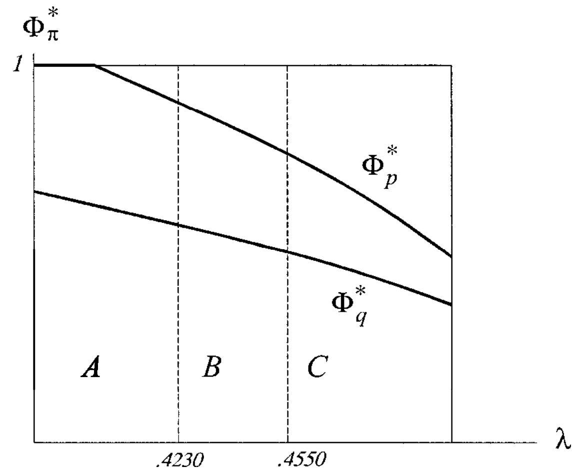

FIGURE II

The plot shows the optimal collateral rate, under full information, as a function of the default penalty. There are three regions. In region A the full information equilibrium is not incentive compatible: agents of quality p would migrate to the q exchange if the full information contracts were offered. In region B the full information equilibrium is incentive compatible. In region C the full information equilibrium is again not incentive compatible, but now the q agents would migrate to the p exchange if the full information contracts were offered.

the other hand, if λ is low, we obtain "undercollateralization"—the p agents face exactly the same contract as they would if quality was observable while the q agents trade in an exchange with a lower collateral than the collateral they deposit in the equilibrium with observable qualities. For intermediate penalty values the equilibrium with observable qualities is incentive compatible. Intuitively, when penalties are low, moral hazard is important, and when quality is observable, the collateral requirement that is imposed on agents of quality p, Φ^*_p, is much larger than the collateral imposed on the quality q agents, Φ^*_q. Hence the p quality agents would prefer the exchange that serves the q quality. On the other hand, when penalties are high, Φ^*_p is not much bigger than Φ^*_q. Hence the q agents prefer to trade on the p exchange, to benefit from the higher delivery rate. For intermediate values the collateral requirements differential is enough to deter the q agents from migrating to the p exchange, but not high enough to encourage the p agents to move to the q exchange. Figure II shows these equilibrium margin requirements as a function of the penalty rates.

Example. In this example $u(c) = -c^{-1}$, and endowments are $\{x,y,z\} = \{.75, 2.2, .625\}$. Each quality represents half of the population, $p = .985$, and $q = .916$. These values imply that $\lambda_p^* = .4596$ and $\lambda_q^* = .4588$. In this example, there are no fixed costs; that is, $\sigma = 0$. Assumptions 1–5 are met provided that $.4043 < \lambda \leq .4588$. If the default penalty $\lambda \geq .4550$, the incentive compatibility constraint that is binding is that of the low quality agents, as in standard adverse selection models. On the other hand, if $\lambda \in (.4230,.4550)$, the equilibrium when quality is observable is also an equilibrium for the private information case. Finally, if $.4043 < \lambda \leq .4230$, the incentive compatibility constraint of the high quality agent is the one that binds—the high quality agents prefer to trade in the low quality exchange.

To illustrate undercollateralization, we compute the equilibrium for $\lambda = .4051$. If quality is observable, the equilibrium contracts are $(\Phi_p^*,K_p^*) = (.57,.9936)$ and $(\Phi_q^*,K_q^*) = (.29,.94)$. The utility the p agents obtain in equilibrium is lower than the one they would obtain by migrating to the q exchange. The p agents would take a larger position in the q exchange than they would take in their own exchange ($\theta_p(\Phi_q^*,K_q^*) = .774$ and $\theta_p(\Phi_p^*,K_p^*) = .6288$).

Figure III shows the equilibrium when quality is private information. Since the single-crossing property holds, the equilibrium is necessarily separating. Compared with the case where quality is observable, the q agents face a lower collateral requirement ($\Phi_q = .26$) and consequently a lower delivery rate ($K_q = .9013$), while the contract that the p agents trade does not change.[14] This lower quality contract deters the p agents from migrating to the q exchange. This undercollateralization is typical when the moral-hazard problem is substantial; i.e., where low penalties require high collateral rates to deter traders from taking excessively large positions. In these cases the q agents, since they are more likely to obtain the low endowment z, are more affected by collateral requirements. As a consequence, when quality is observable, they face a much lower collateral requirement than that faced by the p agents.

14. This separating equilibrium would involve (partial) default by the q agents when they obtain endowment y. This is a consequence of the fact that we have only two possible endowments in the state where endowment is uncertain. If we had more than two possible endowments, the decrease in quality in the asymmetric information equilibrium would be associated with an increase in the fraction of endowments where default occurs.

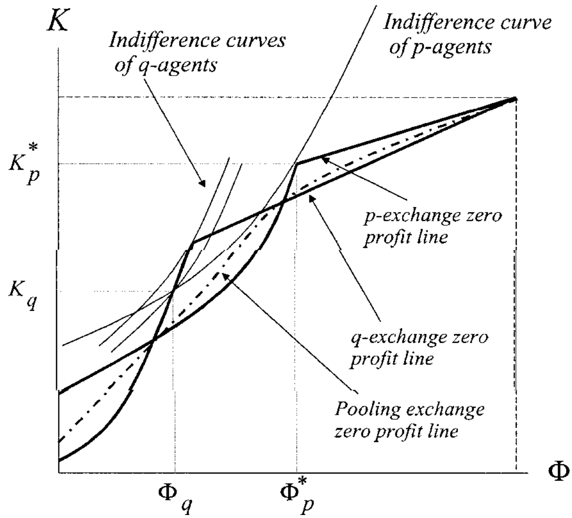

FIGURE III
Undercollateralization
The zero profit lines cross several times. In equilibrium the p agents are indifferent between trading contract (Φ_p^*, K_p^*) and trading contract (Φ_q, K_q).

In contrast with the situation where qualities are observable, it is not easy to compare the contracts offered by a monopolist with the ones that would prevail under competition, when a trader's quality is private information. Nonetheless, there are several situations where one can establish a comparison. The first is when a large fraction of traders is of a given quality π. In this case, since a monopolist suffers a first-order loss when he raises collateral above zero, he would always choose zero collateral for the contract chosen by quality π. The second is when it is optimal for the monopolist to offer a single contract. Here, the reasoning in Lemma 5 in the Appendix can be used to establish that this contract would require zero collateral.

V. LIQUIDITY

Liquidity is a complex phenomenon that can only be properly discussed in a dynamic model. However, in this paper we are only interested in one aspect of liquidity—that traders prefer, ceteris paribus, to trade in a market with higher volume. We postulate fixed costs because in the presence of fixed costs trading is

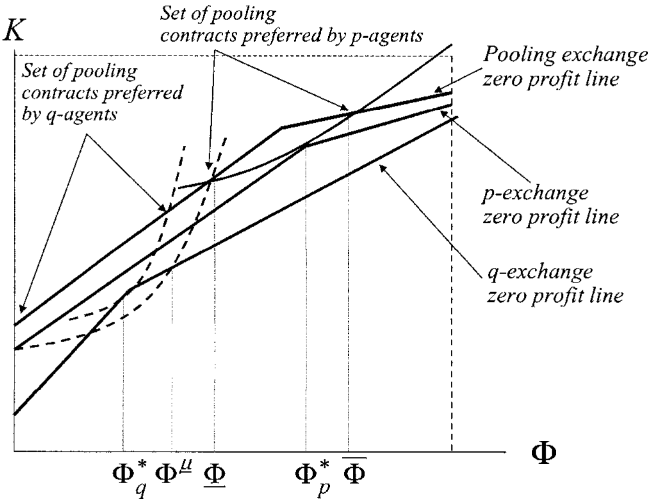

FIGURE IV
Impact of Minimum Margin Requirements

cheaper in exchanges with more volume. As in the case of no fixed costs, it is straightforward to show that, when qualities are observable, the equilibria are constrained efficient. It is also easy to see that, in the presence of these economies of scale, pooling equilibria may exist, whether quality is observable or not. Holding other parameter values constant, pooling equilibria occur for higher values of σ.

In this section we show that, by imposing minimum margin requirements, regulators may cause the equilibrium to shift toward a pooling equilibrium that benefits traders of one quality at the expense of traders of the other quality. In the example illustrated in Figure IV quality is observable, and in the absence of minimum margin requirements, quality p trades the contract (Φ^*_p, K^*_p), and quality q agents the contract (Φ^*_q, K^*_q). In this example $\Phi^*_q < \Phi^*_p$.[15] To each Φ we associate a $K^{\text{pool}}(\Phi)$ such that $(\Phi, K^{\text{pool}}(\Phi))$ is on the zero-profit pooling line. Let F^0_p denote the set of all margin requirements Φ such that the utility of a p quality agent facing the terms $(\Phi, K^{\text{pool}}(\Phi))$ is greater than the utility the p agents obtain when faced with (Φ^*_p, K^*_p). These are

15. The parameters used to generate this example are the same as in the example of Section IV with the exception that $\sigma = .017.$ and $\lambda = .435$.

the candidate pooling equilibria that would deliver to the p quality agents an allocation that they prefer to the allocation they get in the separating equilibrium. Similarly, let F_q^0 denote the set of all margin requirements Φ such that a q quality agent prefers to face the terms $(\Phi, K^{\text{pool}}(\Phi))$ to facing (Φ_q^*, K_q^*). Since the equilibrium is separating, $F_p^0 \cap F_q^0$ is empty. For if Φ is a point in the intersection, an entrant offering terms slightly worse than $(\Phi, K^{\text{pool}}(\Phi))$ would attract everyone and make a profit. In fact, the margin levels that belong to F_p^0 are larger than the margin levels that belong to F_q^0.

Now suppose that the regulator sets a minimum margin requirement $\Phi^\mu = (1 - \mu)\Phi_q^* + \mu\Phi_p^*$, $\mu > 0$. These minimum margin requirements are never binding on the p quality exchange. The q quality exchange, on the other hand, must set a higher margin requirement. Using reasoning similar to the proof of Proposition 2, we can show that, if there is a separating equilibrium, the q exchange will set its margin requirement at the minimum level allowed. To check whether this is actually an equilibrium, we must examine whether there are points on the zero-profit pooling line that are preferred by both qualities. Let F_q^μ denote the set of all margin requirements Φ such that a q quality agent prefers to face the terms $(\Phi, K^{\text{pool}}(\Phi))$ instead of facing the terms prescribed in the candidate equilibrium. Since the utility that the q quality agents can obtain by themselves, when the minimum margin requirements are imposed, is lower than the utility they obtain when these requirements are absent, there are points in F_q^μ that are not in F_q^0. In particular, the maximum margin in F_q^μ is larger than the maximum Φ in F_q^0. For the parameter values used here, we first reach a $\underline{\mu} < 1$ for which there is a common point between F_q^μ and F_p^μ. At this point there are two equilibria—one separating, another pooling. The pooling equilibrium involves a margin requirement $\Phi > \Phi^{\underline{\mu}}$. If the regulator increases the margin requirements beyond $\Phi^{\underline{\mu}}$, there exists a continuum of pooling equilibria. The most favorable equilibrium for the q agents has a margin rate of exactly Φ. All other equilibria treat the p agents better than they are treated in the absence of margin requirements, at the expense of the q agents. Once the regulator sets a minimum margin requirement greater than Φ, all equilibria treat the p agents better than they are treated in the absence of margin requirements, again at the expense of the q agents. The p agents strictly prefer any minimum margin requirement in the interval (Φ, Φ) to no margin requirements. This interval contains margin requirements above Φ_p^*, the margin rate that the p agents would face in the absence of regulation.

In this example the p quality agents benefit from the establishment of minimum standards, at the expense of the q agents. The roles are reversed when $\Phi_q^* > \Phi_p^*$. In any case, the quality that faces higher margin requirements in the absence of regulation may benefit from minimum margin requirements.

VI. Conclusions

In this paper we constructed a simple model where exchanges design contracts to attract trading volume, while simultaneously taking into consideration that potential customers differ in credit quality and may choose to default.

When quality is observable, we showed that competition among exchanges leads to a constrained efficient outcome, and a monopolist would demand smaller guarantees than competitors demand in equilibrium.

In this model, adverse selection interacts in subtle ways with moral hazard. Adverse selection may result in higher, lower, or exactly the same amount of guarantees as in the pure moral hazard case, depending on the intensity of the moral hazard problem. That moral hazard may serve to mitigate adverse selection problems is an interesting and novel aspect of this model.

The often heard assertion that competition among exchanges leads necessarily to low standards is not verified in this model, and does not seem to have empirical support. There is no recorded major incident of default by an American futures exchange. The Barings episode, that has generated much of the more recent criticisms of exchange competition, did not result in losses for anyone but the creditors, stockholders, and employees of the merchant bank. The margins deposited by Barings provided SIMEX with ample funds to meet the costs incurred in liquidating the positions.[16] Nonetheless, exchanges continue to use the threat of a "race to the bottom" as a justification for common margin policies.[17]

16. Szala, Ginger, D. Nusbaum, and J. Reerink, "Barings Abyss," *Futures,* XXIV (May 1995), 68–74.

17. Ecofex, a group of 27 European futures and options exchanges, coordinates policy on margin and its members agree not to compete with each other on margins. (See Suzanne McGee in *The Wall Street Journal,* March 17, 1995.)

We showed that minimum margin requirements can be used to benefit a group of traders at the expense of another. Since we postulated free entry, and an infinite supply of identical potential exchanges, the zero-profit condition would still hold in equilibrium. However, if an exchange has a natural advantage in serving the group of traders that gains from the regulation, it would profit from the introduction of minimum margin requirements.

Although we limit the capacity of exchanges to control positions, we do not allow agents to trade in more than one exchange, or exchanges to issue more than one contract. In the literature on incentives, it is well established that nonexclusive contracts affect the nature of equilibria.[18] Similarly, in the model in this paper, if exchanges cannot enforce exclusivity, the equilibrium in Proposition 2 is no longer necessarily a Nash equilibrium. However, in Santos and Scheinkman [2001] we show that, when qualities are observable, exchanges are restricted to issue a single contract, and agents can trade simultaneously in several exchanges, the equilibrium in Proposition 2 is an "anticipatory equilibrium" in the sense of Wilson [1977]. If an exchange offers the contract described in Proposition 2, $[\Phi^*_\pi, (1 - \pi)\Phi^*_\pi + \pi]$, to agents of quality π, any entrant who makes a profit will cause the incumbent exchange to make losses. In addition, once the incumbent exchange exits, the entrant will lose money. That is, the constrained optimum can be implemented as an anticipatory equilibrium. In this sense, allowing agents to trade simultaneously in more than one exchange does not lead to excessively low standards.

In the model in this paper there are no shocks to the aggregate endowment. It is easy to accommodate such shocks, provided that exchanges can make the contract terms contingent on the aggregate states. Bernanke [1990] argues that in practice it may be difficult for an exchange to make terms contingent in all possible aggregate states, and that the government has a role in protecting the clearing and settlement systems when rare, large, aggregate shocks occur. Bernanke contends that in October 1987 the Fed intervened to redistribute risks from the exchange clearinghouses to the money center banks, and avoided a systemic crisis. If the monetary authorities need to intervene in extreme events, it may be desirable to set standards for exchanges. How-

18. E.g., Bizer and deMarzo [1992], Arnott and Stiglitz [1993], or Kahn and Mookherjee [1998].

ever, these standards would have to be dictated by the nature of the contingencies that lead to the monetary authorities' intervention.

APPENDIX

Investor's Problem

Suppose that an agent of group 1 is trading in an exchange with parameters (Φ,K); that is, the exchange demands collateral of Φ (per unit) and pays in the money contracts K (per unit). If $D(w)$ denotes the amount delivered to the exchange, by an agent who receives endowment $w \in \{x,y,z\}$, we may write the optimization problem of a group 1 agent as[19]

Problem $_{\pi}(\Phi,K)$

$$\max_{\{\theta,D\}} \left\{ \begin{array}{l} \frac{1}{2}[u(c(x)) - \mu \max\{(-\theta - D(x)),0\}] \\ \quad + \frac{\pi}{2}[u(c(y)) - \lambda \max\{(\theta - D(y)),0\}] \\ \quad + \frac{1-\pi}{2}[u(c(z)) - \lambda \max\{(\theta - D(z)),0\}] \end{array} \right\},$$

subject to

$$c(x) = x - D(x) + \max\{\theta,0\}K$$
$$c(y) = y - D(y) + \max\{-\theta,0\}K$$
$$c(z) = z - D(z) + \max\{-\theta,0\}K$$

$$z \geq \max\{\theta,0\}\Phi \text{ and } x \geq \max\{-\theta,0\}\Phi$$

$$D(y),\ D(z) \geq \max\{\Phi\theta,0\} \text{ and } D(x) \geq \max\{-\Phi\theta,0\}.$$

The first three constraints define the income of the agent who buys θ units of the asset and chooses to deliver $D(w)$ in state $w \in \{x,y,z\}$. The fourth constraint expresses the fact that, if an agent goes long (short), the collateral must not exceed z (respectively, x). The last constraint states that deliveries are nonnegative, and that if an agent has an obligation to deliver to the exchange, he will have to deliver at least his collateral.

Problem $_{\pi}(\Phi,K)$ has a unique solution. Write $(\theta^1_{\pi},$

19. Implicit in this formulation is the fact that agents never post excess collateral, and that in each state agents spend all their income.

$[D_\pi(w)]_{w \in \{x,y,z\}})$ for the solution to ${}_{\pi}(\Phi,K)$. Since the optimization problem for group 2 is the mirror image of ${}_{\pi}(\Phi,K)$, $\theta_\pi^2 = -\theta_\pi^1$. Hence, the market clearing condition in the market for the asset is automatically satisfied. Equilibrium will depend only on the supply of contracts by exchanges. Further, $D_\pi^2(w) = D_\pi(w)$, justifying the missing superscript. The next lemma establishes formally that agents of group 1 want to go long.

LEMMA 1. Assumptions 1 and 2 imply that, θ_π^1, the optimal position for an agent of group 1 and quality π, is nonnegative.

Proof of Lemma 1. Suppose in fact that $\theta_\pi^1 < 0$. Let D^* solve $u'(x - D^*) = \lambda$. Assume first that $0 > \theta_\pi^1 \leq -D^*$. In this case, $D_\pi(x) = -\theta_\pi^1$ and using the first-order condition that θ_π^1 must satisfy, we obtain

$$\begin{aligned} u'(x) &< u'(x + \theta_\pi^1) \\ &= K[\pi u'(y - \theta_\pi^1 K) + (1 - \pi)u'(z - \theta_\pi^1 K)] \\ &< \pi u'(y) + (1 - \pi)u'(z), \end{aligned}$$

a contradiction to Assumption 1.

Next, assume that $\theta_\pi^1 < -D^* < \theta_\pi^1 \Phi$. In this case $D_\pi(x) = D^*$, and again using the first-order condition that θ_π^1 must satisfy, we obtain

$$\begin{aligned} u'(x) < u'(x - D^*) &= \lambda \\ &= K[\pi u'(y - \theta_\pi^1 K) + (1 - \pi)u'(z - \theta_\pi^1 K)] \\ &< \pi u'(y) + (1 - \pi)u'(z), \end{aligned}$$

again a contradiction to Assumption 1.

Finally consider the case $D^* \leq -\theta_\pi^1 \Phi$. In this case, $D_\pi(x) = -\theta_\pi^1 \Phi$, and once more using the first-order condition that θ_π^1 must satisfy, we obtain

$$\begin{aligned} \lambda &= u'(x - D^*)\Phi + \lambda(1 - \Phi) \leq u'(x + \theta_\pi^1 \Phi)\Phi + \lambda(1 - \Phi) \\ &= K[\pi u'(y - \theta_\pi^1 K) + (1 - \pi)u'(z - \theta_\pi^1 K)] \\ &< \pi u'(y) + (1 - \pi)u'(z), \end{aligned}$$

a contradiction to Assumption 2. □

Notice that since $\theta_\pi^1 \geq 0$, $D_\pi(x) = 0$. Assumption 1 also guarantees that $D_\pi(z) = \Phi|\theta_\pi|$; that is, an agent who receives the endowment z will deliver only the collateral. This justifies writing the solution to problem ${}_{\pi}(\Phi,K)$ as a pair (θ_π, D_π), where the first element is the position of an agent of group 1 quality π, and the second one is the amount delivered in state y.

Let θ^* be the solution to $u'(y - \theta^*) = \lambda$. Assumption 3, and the fact that $u'(0) = \infty$, guarantee that $\theta^* > z > \Phi\theta$. When receiving endowment y, if $\theta_\pi \leq \theta^*$, the agent fully delivers; while if $\theta_\pi > \theta^*$, the agent delivers only θ^*. By Assumption 2, $\theta^* > 0$. Let $1_{[\theta_\pi > \theta^*]}$ be the indicator function taking the value 1 if $\theta_\pi > \theta^*$ and 0 otherwise, and similarly for $1_{[\theta_\pi \leq \theta^*]}$. Then Lemma 1 allows us to write the consumption of an agent of group 1 and quality π as

$$c_\pi(x) = x + \theta_\pi K \tag{7}$$

$$c_\pi(y) = y - D_\pi = y - (1_{[\theta_\pi \leq \theta^*]}\theta_\pi + 1_{[\theta_\pi > \theta^*]}\theta^*) \tag{8}$$

$$c_\pi(z) = z - \theta_\pi \Phi, \tag{9}$$

where $[\theta_\pi, D_\pi]$ is the unique solution to problem $_\pi(\Phi,K)$.

For each $\theta \geq 0$, let $D = 1_{[\theta \leq \theta^*]}\theta + 1_{[\theta > \theta^*]}\theta^*$ and

$$(10)\quad g_\pi(\Phi,K,\theta) \equiv \left[\begin{array}{c} u'(x + \theta K)K - \pi 1_{[\theta \leq \theta^*]} u'(y - D) \\ -(1 - \pi)\Phi u'(z - \theta\Phi) - \lambda[\pi 1_{[\theta > \theta^*]} + (1 - \pi)(1 - \Phi)] \end{array}\right].$$

The function g defines the first-order conditions for the optimal choice $\theta_\pi > 0$. This choice must satisfy

$$g_\pi(\Phi,K,\theta) \leq 0, \tag{11}$$

with equality if $\theta_\pi > 0$. These first-order conditions define the functions $\theta_\pi(\Phi,K)$ for $\pi \in \{p,q\}$. Since we assumed that $u'' < 0$, and for θ large, $g_\pi(\Phi,K,\theta) < 0$, the functions θ_π are well defined. Further, these functions are smooth whenever they are positive, except for the (measure zero) set of values of the parameters for which $\theta_\pi = \theta^*$, where the right-hand-side derivatives may differ from the left-hand-side derivatives.

The next lemma establishes that, when both types are trading in the same contract, p agents always take larger positions than q agents.

LEMMA 2. For any contract (Φ,K), $\theta_p \geq \theta_q$, with strict inequality if $\Phi > 0$ and $\theta_q > 0$.

Proof of Lemma 2. This follows from equation (11), the concavity of u, and Assumption 2. □

Exchange's Profit Function

The next lemma establishes that the zero-profit line is well defined. Recall that the profit function $\Pi_\pi(\Phi,K)$ was defined in equation (1) above.

LEMMA 3. For each $0 \leq \Phi \leq 1$, there exists at most one K such that $\theta_\pi(\Phi,K) > 0$ and $\Pi_\pi(\Phi,K) = 0$.

Proof of Lemma 3. Fix a $\Phi \in [0,1]$. The derivative of g with respect to θ is defined for every $\theta > 0$, except at θ^*, and this derivative is strictly negative. Hence the function $\theta_\pi(\Phi,\cdot)$ is Lipschitz (see, e.g., Mas-Colell [1985], page 32) at any K where it is strictly positive. The derivative $\partial\theta_\pi/\partial K$ (that exists unless $\theta_\pi = \theta^*$, where $\partial\theta_\pi/\partial K_+ \neq \partial\theta_\pi/\partial K_-$) satisfies for each $K > 0$:

$$\frac{\partial\theta_\pi}{\partial K} > -\frac{\theta_\pi}{K}. \tag{12}$$

Also,

$$\Pi_\pi(\Phi,K) = \tfrac{1}{2}\,\alpha_\pi[\pi(\theta^* 1_{[\theta_\pi(\Phi,K)>\theta^*]} + \theta_\pi(\Phi,K)1_{[\theta_\pi(\Phi,K)\leq\theta^*]}) + (1-\pi)\Phi\theta_\pi(\Phi,K) - \theta_\pi(\Phi,K)K].$$

Hence, $\Pi_\pi(\Phi,\cdot)$ is a Lipschitz function at any K such that $\theta_\pi(\Phi,K) > 0$. Suppose now that K satisfies $\Pi_\pi(\Phi,K) = 0$ and $\theta_\pi(\Phi,K) > 0$. Clearly, $0 < K \leq 1$. If $\theta_\pi(K) \neq \theta^*$ equation (12), implies that $\partial\Pi_\pi/\partial K < 0$. On the other hand, if $\theta_\pi(\Phi,K) = \theta^*$, then we may calculate the derivative of $\Pi_\pi(\Phi,K)$ with respect to K using either $\partial\theta_\pi/\partial K_+$ or, $\partial\theta_\pi/\partial K_-$. In any case, the derivative is negative. Since Π_π is Lipschitz, and has a negative derivative whenever it is zero, it can only cross zero once. □

Lemma 3 allows us to define the zero-profit line as the set of (Φ,K) such that $\theta_\pi(\Phi,K) > 0$ and $\Pi_\pi(\Phi,K) = 0$. Notice that, in principle, for some Φ there may exist no K with (Φ,K) in the zero-profit line, but such K, if it exists, is unique.

Equilibrium

In order to characterize the equilibrium for a given margin requirement we set

$$h_\pi(\Phi,K,\theta) \equiv K\theta - [\pi(1_{[\theta>\theta^*]}\theta^* + 1_{[\theta\leq\theta^*]}\theta) + (1-\pi)\theta\Phi]. \tag{13}$$

For a given Φ, if (K,θ_π) are such that $g_\pi(\Phi,K,\theta_\pi) = 0$ and $h_\pi(\Phi,K,\theta_\pi) = 0$, then agents of group 1 and quality π trading in an exchange with terms (Φ,K) will buy θ_π contracts, and make delivery choices that will produce zero profits for the exchange.

LEMMA 4. If Assumption 4 holds, there exists exactly one $K(\Phi)$ such that $\Pi_\pi(\Phi,K(\Phi)) = 0$ and $\theta_\pi(\Phi,K(\Phi)) > 0$. Further, $K(\Phi)$ is increasing, and $\theta_\pi(\Phi,K(\Phi))$ is decreasing.

Proof of Lemma 4. Suppose that $\bar{\Phi} \in (0,1)$ is such that $(\bar{\Phi},\bar{K})$ is in the zero-profit line, and $\underline{\theta}_\pi > 0$ is the associated position. A simple computation shows that the Lipschitz map $G = (g,h) : (0,1) \times (0,1) \times (0,\infty) \to R^2$ satisfies the conditions of the implicit function theorem (see, e.g., Mas-Colell [1985], page 32), and hence, there is a neighborhood W of $\bar{\Phi}$ and U of $(\bar{K},\underline{\theta}_\pi)$, and a Lipschitz function $H : W \to U$ such that $G(\Phi,K,\theta) = 0$ with $(\Phi,K,\theta) \in W \times U$ if and only if $K = H_1(\Phi)$ and $\theta = H_2(\Phi)$. Further, for $\underline{\theta}_\pi \neq \theta^*$, the functions H_1 and H_2 are smooth, and one can show, using Assumption 4, that $H_2' < 0$. We will prove the result starting from values of Φ near 1 and apply the implicit function theorem to extend the construction of H_2 to the whole interval. Let $K(\Phi) = \pi + (1-\pi)\Phi$, and notice that $K' > 0$. If Φ is close enough to 1, Assumption 1 guarantees that $\theta_\pi(\Phi,K(\Phi)) > 0$, and Assumption 3 insures that $\theta_\pi(\Phi,K(\Phi)) < \theta^*$. Hence, $(\Phi,K(\Phi))$ is in the zero-profit line and, by the uniqueness established in Lemma 3, $K(\Phi) = H_1(\Phi)$, and $\theta_\pi(\Phi,K(\Phi)) = H_2(\Phi)$. Using the implicit function theorem, and the fact that $H_2' < 0$, we can extend this construction until either we reach $\Phi = 0$ or we reach a Φ^* such that $\theta_\pi(\Phi^*,\pi + (1-\pi)\Phi^*) = \theta^*$. If the latter occurs, the implicit function theorem guarantees that we can define H_2 in a neighborhood of Φ^*. In this neighborhood, if $\Phi < \Phi^*$, the monotonicity of H_2 insures that $H_2(\Phi) \geq H_2(\Phi^*) = \theta^*$. If $H_2(\Phi) = \theta^*$, $H_1(\Phi) = \pi + (1-\pi)\Phi$, and, from the first-order condition, we obtain

$$u'(x + \theta^* H_1(\Phi))\, H_1(\Phi) - (1-\pi)\Phi u'(z - \Phi\theta^*) - \lambda(1-\pi)(1-\Phi) = 0. \tag{14}$$

Note that equation (14) also holds for $\Phi = \Phi^*$. However, using Assumption 4, we can show that the left-hand side of equation (14) is strictly decreasing in Φ, a contradiction. Hence, $H_2(\Phi) > \theta^*$, and by uniqueness $H_1(\Phi) = \pi\theta^*/H_2(\Phi) + (1-\pi)\Phi$, an increasing function of Φ. Since $\lambda > 0$, we can establish a global bound for $\theta_\pi(\Phi,K)$ and, consequently for H_2 and H_1. We can use the monotonicity of H_2 to extend the construction for any $\Phi \geq 0$. □

Proof of Proposition 1

(i) This is established in Lemma 4.

(ii) By solving for K_π^* in equation (5), we may rewrite (4) in the form, $M(\Phi_\pi^*) = 0$. Assumption 4 implies that $M' < 0$, and hence uniqueness follows.

Suppose first that a solution exists to (4) and (5) with $0 \leq \Phi_\pi^* \leq 1$. If $0 < \Phi < \Phi_\pi^*$, from equations (4) and (5) we know that the contract $[\Phi_\pi^*, \pi + (1-\pi)\Phi_\pi^*]$ is on the zero-profit line. By Lemma 4 we know that the position θ associated with the contract $(\Phi, K(\Phi,\pi))$ is greater than θ^*. Hence,

$$K(\Phi,\pi) = \frac{\pi\theta^* + (1-\pi)\theta\Phi}{\theta} < \pi + (1-\pi)\Phi.$$

On the other hand, if $\Phi \geq \Phi_\pi^*$, Lemma 4 guarantees that the position taken by agents along the zero-profit line is less than θ^* and agents who receive y will not default.

Finally suppose that no solution exists to equations (4) and (5) in [0,1]. Since, $\lambda \leq \lambda_\pi^*$, $\theta_\pi^* \geq \theta^*$, where θ_π^* is defined by equation (2). Hence,

$$M(0) = u'(x + \theta^*\pi)\pi - \lambda \geq u'(x + \theta_\pi^*\pi)\pi - \lambda_\pi^* = 0,$$

and, since no solution to (4) and (5) with $\Phi_\pi^* \leq 1$ exists, $M(\Phi) > 0$ for each $0 \leq \Phi \leq 1$. Since $K(\Phi,\pi) \leq (1-\pi)\Phi + \pi$, and $M(\Phi) > 0$, $K(\Phi,\pi) < (1-\pi)\Phi + \pi$ for all $\Phi < 1$. □

Proof of Proposition 2. Taking the derivative of the utility function of an agent of quality π with respect to Φ along the zero-profit line we obtain

$$\begin{aligned}\frac{dV}{d\Phi} &= \theta\left\{u'(c_\pi(x))\,\frac{dK}{d\Phi} - (1-\pi)[u'(c_\pi(z)) - \lambda]\right\}\\ &= \theta(1-\pi)(u'(c_\pi(x)) - [u'(c_\pi(z)) - \lambda])\\ &\quad - u'(c_\pi(x))\left(\frac{\pi\theta^*}{\theta}\right)\frac{d\theta}{d\Phi}\,1_{[\theta > \theta^*]}.\end{aligned}$$

First consider $\Phi > \Phi_\pi^* \geq 0$. By Lemma 4, $1_{[\theta > \theta^*]} = 0$, and hence using Assumption 4 we obtain $dV/d\Phi < 0$. Therefore, it is optimal to lower the collateral requirement.

Second consider $0 < \Phi < \Phi_\pi^*$. Then,

$$\begin{aligned}\frac{dV}{d\Phi} &= u'(c_\pi(x))\pi\theta^*\left\{\frac{(1-\pi)\theta(u'(c_\pi(x)) - [u'(c_\pi(z)) - \lambda])}{u'(c_\pi(x))\pi\theta^*} - \frac{1}{\theta}\frac{d\theta}{d\Phi}\right\}\\ &= \frac{u'(c_\pi(x))\pi\theta^*}{\Phi}\,\mathrm{B}\end{aligned}$$

where

$$\text{(15)} \quad \mathrm{B} = \left\{ e_{\Phi}(\theta) - \frac{(1-\pi)\Phi}{K-(1-\pi)\Phi} \left(\frac{u'(c_{\pi}(z)) - \lambda - u'(c_{\pi}(x))}{u'(c_{\pi}(x))} \right) \right\}$$

and

$$e_{\Phi}(\theta) = -\frac{\Phi}{\theta}\frac{d\theta}{d\Phi} > 0.$$

$e_{\Phi}(\theta)$ is the elasticity of the position with respect to the margin requirement. We next show that $\mathrm{B} > 0$. Let

$$A = (1-\pi)\Phi\theta[u''(c_{\pi}(z))\Phi + u''(c_{\pi}(x))K] < 0.$$

A direct application of the implicit function theorem yields

$$e_{\Phi}(\theta) = \frac{(1-\pi)\Phi[u'(c_{\pi}(x)) + \lambda - u'(c_{\pi}(z))] + A}{-u'(c_{\pi}(x))(K-(1-\pi)\Phi) + A}.$$

Substituting the formula for the elasticity in equation (15) and manipulating the resulting expression, we obtain

$$\mathrm{B} = \frac{A[u'(c_{\pi}(x))K - (1-\pi)\Phi(u'(c_{\pi}(z)) - \lambda)]}{[-u'(c_{\pi}(x))(K-(1-\pi)\Phi) + A](u'(c_{\pi}(x))(K-(1-\pi)\Phi))}.$$

Recalling the first-order condition for $\theta \geq \theta^*$,

$$u'(c_{\pi}(x))K - (1-\pi)\Phi(u'(c_{\pi}(z)) - \lambda) - \lambda = 0,$$

we can write

$$\mathrm{B} = \frac{A\lambda}{[-u'(c_{\pi}(x))(K-(1-\pi)\Phi) + A](u'(c_{\pi}(x))(K-(1-\pi)\Phi))} > 0.$$

□

Monopoly

Assume that qualities are observable and $\sigma = 0$. A monopolist will maximize the profits it can extract from each quality:

$$\Pi_{\pi}(\Phi,K) = \alpha_{\pi} \times [\pi(1_{[\theta_{\pi}\leq\theta^*]}\theta_{\pi} + 1_{[\theta_{\pi}>\theta^*]}\theta^*) + (1-\pi)\theta_{\pi}\Phi - \theta_{\pi}K].$$

The next lemma shows that the monopolist sets margins equal to zero.

LEMMA 5. The optimal solution for the monopolist's problem is $\Phi^*_{\pi} = 0$, for all $\pi \in \{p,q\}$.

Proof of Lemma 5. The continuity of the function $\theta_{\pi}(\Phi, K)$ guarantees that the monopolist has an optimal choice. If $\theta_{\pi} \neq \theta^*$, let $\underline{\mu}$ and $\bar{\mu}$ be the positive, possibly infinite, multipliers associated with the restrictions that $\Phi \geq 0$ and $\Phi \leq 1$, respectively. The first-order conditions of the monopolist are given by

$$\frac{d\Pi_\pi}{dK}\} \; [\pi 1_{[\theta_\pi \leq \theta^*]} + (1-\pi)\Phi - K]\frac{d\theta_\pi}{dK} - \theta_\pi = 0 \tag{16}$$

$$\frac{d\Pi_\pi}{d\Phi}\} \; [\pi 1_{[\theta_\pi \leq \theta^*]} + (1-\pi)\Phi - K]\frac{d\theta_\pi}{d\Phi} + (1-\pi)\theta_\pi + (\underline{\mu} - \bar{\mu}) = 0. \tag{17}$$

We first prove that, under the optimal contract, the agent takes a position $\theta_\pi \leq \theta^*$. Suppose that $\theta_\pi > \theta^*$. By combining (16) and (17), we obtain

$$[(1-\pi)\Phi - K] \times \left\{\frac{u''(c_z(\pi))\theta_\pi\Phi + u''(c_x(\pi))\theta_\pi K + [u'(c_x(\pi)) - (u'(c_z(\pi)) - \lambda)]}{u''(c_z(\pi))\Phi^2 + u''(c_x(\pi))K^2}\right\} = \frac{\underline{\mu} - \bar{\mu}}{1-\pi}. \tag{18}$$

If $(1-\pi)\Phi - K < 0$, then the left-hand side of (18) is negative, which necessarily implies that $\bar{\mu} > 0$ and $\underline{\mu} = 0$; that is $\Phi = 1$. But by Assumption 3,

$$u'(y - \theta_\pi) < u'(y - z) < \lambda = u'(y - \theta^*), \tag{19}$$

and hence θ_π must be smaller than θ^*, a contradiction. On the other hand, if $(1-\pi)\Phi - K \geq 0$, then the first-order condition of the agent and Assumption 4 show that $\theta_\pi > \theta^*$ can never be optimal. It follows that, under the optimal monopoly contract, $\theta_\pi \leq \theta^*$. Next, once again combining (16) and (17), we obtain a relation between K and Φ that can only be met if either $\underline{\mu} \neq 0$ or $\bar{\mu} \neq 0$:

$$\left(\frac{\underline{\mu} - \bar{\mu}}{1-\pi}\right) \times \left[\frac{u''(c_x(\pi))K^2 + \pi u''(c_y(\pi)) + (1-\pi)u''(c_z(\pi))\Phi^2}{\pi + (1-\pi)\Phi - K}\right]$$
$$= u''(c_z(\pi))\theta_\pi\Phi + u''(c_x(\pi))\theta_\pi K + [u'(c_x(\pi)) - (u'(c_z(\pi)) - \lambda)].$$

Given Assumption 4, it is immediate that this equation can only hold if $\underline{\mu} > 0$; that is, if $\Phi = 0$. If $\theta_\pi = \theta^*$, consider the function $\Phi_o(K)$ defined by $\theta_\pi(\Phi_o(K), K) = \theta^*$. The implicit function theorem guarantees that this function is well defined in an open set U. Using Assumption 4, it is immediate that $(1-\pi)d\Phi_o/dK < 1$, and hence that the monopolist's profits are maximized at $k = \inf U$. Obviously $k = 0$ is not a candidate. Thus, it must be true that $\Phi_o(k) \in \{0,1\}$. But as equation (19) above shows, if $\Phi = 1$, $\theta_\pi < \theta^*$. □

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