AGE-DIFFERENTIATED MINIMUM WAGES IN DEVELOPING COUNTRIES*

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Abstract

The fact that minimum wages seem especially binding for young workers has led some countries to adopt age-differentiated minimum wages. We develop a dynamic competitive two-sector labor market model where workers with heterogeneous initial skills gain productivity through experience. We compare two equally binding schemes of single and age-differentiated minimum wages, and find that although differentiated minimum wages result in a more equal distribution of income, such a scheme creates a more unequal distribution of wealth by forcing less skilled workers to remain longer in the uncovered sector. We also show that relaxing minimum wage solely for young workers reduces youth unemployment but harms the less skilled ones.

JEL Classification: D31, J31, J38, J42.

Key Words: Age-differentiated minimum wage, income distribution, wealth distribution, uncovered sector.

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1 Introduction

Recent empirical research has found that increases in minimum wages lead to reduced employment opportunities, particularly among young workers. This consideration has led some economists and policymakers to propose the replacement of the single minimum wage (hereafter SMW) with an age-differentiated minimum wage (hereafter DMW). For example, Gutierrez Hevia and Schwartz (1997) state that:

> "...if the government's priority is to increase employment of young workers it should seriously consider the possibility of ...lowering the minimum wage for this age group..."

To date, several countries have adopted a DMW scheme, other countries are considering adding more age brackets to their present DMW scheme, while still others are evaluating the possibility of adopting differentiated minimum wages. For example, most of the European Union countries' minimum wages are differentiated according to age (see Table 1). Chile has two age brackets and is planning to add more. Finally, a number of developing countries (e.g. Slovakia) is considering adopting a DMW scheme.

[Insert Table I here]

The main argument in favor of a DMW is that it can increase covered sector employment for young workers and improve the income distribution. This paper aims at showing that a DMW has a negative effect (of intertemporal nature) that has been overlooked: DMW forces less skilled workers to remain longer in the uncovered salaried sector, leading to a worsening in the wealth distribution.

Developing countries labor markets are characterized by large uncovered sectors. For example, recent studies reveal that the uncovered sector employs about half of Latin America's workforce and accounts for about a third of total urban income (see Thomas 1995).

The traditional literature represents the uncovered sector as the disadvantaged sector in a dualistic labor market (see, e.g., Harris and Todaro, 1970). However, at least since Yamada (1996), this view has been questioned. Yamada's critique relied on the disaggregation of the uncovered sector in its two greatest components: self-employment and uncovered salaried employment. After dividing Peru's uncovered sector in these two components, Yamada found that wages in the uncovered salaried sector were lower than wages in both the self-employment and covered sector, suggesting that uncovered wage earners were in a disadvantageous situation. According to the author, the uncovered sector taken as a whole would represent a "tale of two tails": those who voluntarily chose uncovered self-employment earn competitive incomes; while uncovered salaried workers make significantly less than those in other labor options, and probably sought to move out of that sector.

While the view that the uncovered sector should not be taken as a whole has been widely accepted, the use of segmentation tests based on sectoral earnings differentials-comparisons has been criticized. In particular, Maloney (1999) affirmed that earnings differentials cannot be used to prove or disprove labor market segmentation, since the value of unobserved job-related characteristics is not observed. In order to address this problem, Maloney proposed an alternative approach consisting in examining the patterns of worker mobility. Based on this dynamic overview of movements through the labor market, several papers have analyzed labor-market segmentation in both uncovered salaried employment and self employment for different developing countries.

Regarding the uncovered salaried sector, Maloney (1999) found very large and symmetrical worker flows both from and to uncovered salaried work and covered work in Mexico, suggesting that markets were not segmented along the traditional covered/uncovered division. He also found that uncovered salaried workers are on average younger than other-sector workers and that from every sector, workers entering uncovered salaried employment have less experience than other workers. According to the author, this seems to be consistent with the fact that, as noted by Balán et al. (1973), the first working years are meant for learning skills and exploring alternatives. Although this evidence might represent some queuing to enter the covered sector, it may also be closer to internships and apprenticeships that young workers take before settling into their final trajectory, and therefore imply no segmentation. On the other hand, Saavedra and Chong (1999) found that in Peru the probability of a person working in the covered sector with respect to the probability of the same person working in the uncovered sector increases significantly with age. Similarly, Pisani and Pagán (2003) found that uncovered workers in Nicaragua do indeed queue for covered sector employment. Lastly, Veras Soares (2004) found that in Peru the probability of being chosen from the queue and entering covered employment increases significantly with experience.

With respect to self-employment, Maloney (1999) found that it is not an entry occupation from school for young workers. The mean age of self-employed workers is eight years higher than in the next closest sector. Furthermore, the probability of moving into self-employment from every paid sector is associated with greater experience. According to Cunningham and Maloney (2002), this pattern is supportive of the findings of Balán et al. (1973) and increasingly elsewhere, for a "life cycle" model where workers enter into salaried work, accumulate human and financial capital, and then quit to open their own informal businesses.

In sum, the overall evidence seems to broadly coincide with the life-cycle story and the role human capital accumulation plays, but there is some disagreement with respect to the degree of segmentation between the uncovered salaried and covered sectors. However, it is likely that a component of this sector is queuing, and we believe this evidence is enough to support the assumptions made below.

In light of the previous empirical literature, we develop a model that has two sectors: the covered sector, where the minimum wage is enforced, and the uncovered sector, where it is not. There are two kind of workers in the uncovered sector: uncovered salaried and self-employed. Young workers start their working life in the uncovered salaried sector. Once they have gained enough experience, they pass to the covered sector. In the covered sector, they work until they acquire enough human capital that allows them to become self-employed.

As Pettengill (1981) and Heckman and Sedlacek (1981), our model introduces heterogeneous productivity in the labor force. We extend both studies by adding an intertemporal dimension to the problem: we assume that productivity depends on both initial skills and experience. In this framework, the role of the minimum wage is to prevent some low-productivity (mostly young) salaried workers from being employed in the covered sector, forcing them to stay as uncovered salaried workers until they gain enough experience to earn the minimum wage.

Our main conclusions are that moving from a SMW to an equally binding DMW, improves income distribution, since young relatively unexperienced workers have access to the covered sector. However, the wealth distribution worsens, since less skilled workers are forced to remain longer in the uncovered salaried sector.

We also show that relaxing the minimum wage solely for young people may reduce youth unemployment, although it is detrimental for the less skilled workers.

The paper is organized as follows: in section 2 we lay out the model, and in section 3 we determine the equilibrium under the SMW and DMW regimes. In section 4 the outcomes are compared assuming equally restrictive minimum wages. In section 5 the outcomes are compared assuming that the minimum wage is relaxed only for the young workers. In section 6 we add unemployment to the model, and in section 7 we conclude.

2 The Model

2.1 Setup

The economy is composed of two sectors: covered and uncovered. In the uncovered sector, there are two kinds of workers: salaried and self-employed. Let L_c be effective salaried labor in the covered sector and L_u the effective salaried labor in the uncovered sector. Total effective salaried labor in the economy is given by $L_c + L_u \equiv \overline{L}$.

We use a continuous time overlapping generation model in a closed economy. Each individual is born and works for a period of $\overline{A}$. At any given point in time, different generations from ages 0 to $\overline{A}$ live and coexist. The economy is assumed to be at its steady state with no population growth.

2.2 The Firms

There are two types of firms: those in the covered and uncovered sectors. Firms in the covered sector respect the law, thus if there is a legal minimum wage it will hold in this sector.

The production of firms in both sectors is a function of effective salaried labor and capital, $F(L, K)$. We assume that the function exhibits constant returns to scale and decreasing returns to each factor, and that marginal labor productivity is an increasing function of capital, with lower bound $F_L(L, 0)$. Since the uncovered sector does not respect the law and capital is observable by the authority, we assume that this sector cannot use capital. In addition, we assume that the amount of capital in the economy is fixed and equal to $K \equiv \overline{K}$.

The output and labor markets are competitive so firms hire factors up to the point where:

$$
\begin{aligned}
w_c &= F_L\left(L_c, \overline{K}\right) &\quad (1a)\\
w_u &= F_L\left(L_u, 0\right) &\quad (1b)\\
r_K &= F_K\left(L_c, \overline{K}\right), &\quad (1c)
\end{aligned}
$$

where w_c is the price of effective labor in the covered sector, w_u the price of effective salaried labor in the uncovered sector, and r_K the rental price of capital (in units of the final good). Since $F(L, K)$ has constant returns to scale, marginal productivity of labor depends on the ratio of capital to labor. Because this ratio is constant in the uncovered sector (equal to zero), the price of effective salaried labor in this sector is constant. Given that marginal labor productivity is an increasing function of capital, the price of effective labor in the covered sector is always higher than in the uncovered sector.

In addition, there exists a third production technology, that of self-employment. This technology allows the transformation of labor into units of the final good with constant returns to scale.

2.3 The Government

The government owns the capital in the economy and rents it to the firms for $r_K\overline{K}$ at every instant of time. It provides individuals with public goods g, which do not affect the marginal utility of private consumption. We assume the government has an infinite horizon and its utility is a linear function of the discounted value of the public goods it provides. As the government is indifferent between present and future resources discounted at its subjective discount rate, ρ, we assume it is willing to lend any amount of goods to households at the same rate.

2.4 The Individuals

Individuals are assumed heterogeneous with regard to their productivity as salaried workers. This means that a worker with productivity p is twice as productive as one with productivity $p/2$, so any firm will be indifferent between hiring one of the former or two of the latter (see, e.g., Lucas 1988). Each individual is endowed with one hour of labor at every point in time, which he supplies inelastically. An individual with productivity p generates p units of labor for each hour of work. That is:

$$l_p = p \tag{2}$$

The productivity component p is determined by:

(1) Initial skills, which we shall call j, with $j \in [0, J]$. These skills remain constant throughout the individual's lifetime. We assume skills are distributed among individuals according through the density function $f(j)$.

(2) Experience or age, which we shall call a, with $a \in [0, A]$. The gain in productivity through experience is represented by the function $h(a)$, which is increasing in age. We are assuming that as the individual ages he acquires experience, which causes his productivity to grow.[1] At each point in time there exist $f(a)$ individuals of age a. Since the population is stable through time, $f(a)$ is distributed uniformly $U[0, A]$.

As a result, we may rewrite equation (2) as:

$$l_p = j + h(a) \tag{3}$$

The individual can supply his units of salaried labor to either one of the sectors, or work independently as self-employed. His income depends on the product of the price of labor in the sector in which he works and his endowment of productivity. If he is employed in the covered sector he receives a wage of $I_c = w_c(j + h(a))$, while if he works in the uncovered salaried sector he receives $I_u = w_u(j + h(a))$.

Once the individual acquires the necessary experience, which we shall call A, he can become self-employed in the uncovered sector and earn an income proportional to his human capital of $I_s = w_s h(a)$. In equilibrium, we assume it will always be the case that for $a \geq A$, we have that $w_s h(a) > w_c(j + h(a))$.

Therefore, if $a < A$ the individual will work as a salaried worker, and will always prefer to work in the covered sector, since $w_c > w_u$. If $a \geq A$, the individual will choose to work as self-employed.

[1] Strictly speaking, all we need is that p be a function of age, not necessarily an increasing one. It could seem reasonable to assume there is a threshold age after which productivity declines. Therefore a may be reinterpreted as a variable inversely related to the distance of age from this threshold.

Each individual has a level of wealth at birth, which we shall call W. Thus, if an individual of initial skill j works in the uncovered salaried sector until age a^*, and works in the covered sector until age A when he becomes self-employed, his wealth at $a = 0$ will be:

$$W(j) = \int_0^{a^*} w_u (j + h(a)) e^{-\rho a} da + \int_{a^*}^{A} w_c (j + h(a)) e^{-\rho a} da + \int_A^{\overline{A}} w_s h(a) e^{-\rho a} da, \quad (4)$$

where the interest rate ρ is determined by the government's discount factor. Given that $w_c > w_u$, and the fact that the time the individual spends working as self-employed is an exogenous value (determined by $\overline{A}$), there exists a positive monotonic relationship between wealth and the time the individual works in the covered sector.

We assume that individuals are born without assets, they can lend or borrow from the government in order to smooth their consumption, and they die with no assets. For simplicity, we normalize ρ to zero.

3 Equilibrium

In this section, we first compute the equilibrium of the model with no minimum wage in place, then we explain the rationale of the minimum wage, and finally we study the equilibrium under a SMW and a DMW scheme.

3.1 No Minimum Wage

As mentioned above, the price of effective salaried labor in the covered sector is greater than that in the uncovered sector. Everyone wishes to supply their labor to the covered sector until they become self-employed.

Aggregate supply of labor in the covered sector in any moment is:

$$L_c = \int_0^A \int_0^J f(j,a)(j + h(a)) \, dj da, \quad (5)$$

where $f(j,a)$ denotes the joint density function of initial skills and age. Since the distributions $f(j)$ and $f(a)$ are independent of each other, we have that $f(j,a) = f(j)f(a) = \frac{f(j)}{A}$. We may rewrite equation (5) as:

$$L_c = \int_0^A \int_0^J \frac{f(j)}{A} (j + h(a)) \, dj da = \overline{L} \quad (6)$$

Equaling the supply of labor given by equation (6) with the demand of labor given by equation (1a) we obtain the equilibrium price of labor in the covered sector:

$$w_c = F_L\left(\overline{L}, \overline{K}\right) \tag{7}$$

Income

As all individuals receive an income of $I_c = w_c(j + h(a))$ if $a < A$ and $I_s = w_s h(a)$ if $a \geq A$, the distribution of labor income under this scenario is the union of a linear transformation of the distribution of productivity of population under age A, and a linear transformation of the distribution of age for the rest of the population.

3.1.1 Wealth

Finally, we can calculate the wealth of an individual of initial skill j. The individual is born and works in the covered sector until A, age at which he becomes self-employed. As a result, his wealth will be:

$$W(j) = \int_0^A w_c\left(j + h(a)\right) da + \int_A^{\overline{A}} w_s h(a)\, da \tag{8}$$

From equation (8) it follows that wealth is a monotonically increasing function of initial skills. In fact, the distribution of wealth is a linear transformation of the distribution of such skills.

3.2 The Introduction of a Minimum Wage

We assume the government introduces a minimum wage for redistributive purposes. The usual goal of minimum-wage policy is to increase the income of the working poor. Since in our model aggregate income never rises when a minimum wage is established, raising the income of the poorest workers implies raising their share of total income (for further discussion, see Freeman, 1996). For this reason, in our model the role of the minimum wage is to increase labor's share of income. To achieve this, firms are prevented from hiring those low productive workers who gain less than the minimum wage. This reduces the amount of workers that firms in the covered sector can hire, making labor scarcer in this sector and therefore increasing its price. In a one-sector model, the effect on the share of labor depends on whether the elasticity of substitution is above or below unity.

In our case, the necessary condition to increase labor share is less restrictive, given that individuals not hired in the covered sector will still earn positive income (in the uncovered sector). Thus, even if the elasticity of substitution is unity, the share of labor increases with respect to capital when a minimum wage is introduced.

We will assume that the government will set a minimum wage in order to achieve an optimal share of labor. We assume this optimal share is a result of the optimization of an exogenous welfare function.

At this point, it is important to highlight that all the workers in our model are unskilled or blue-collar workers. These are the workers for whom the minimum wage is most relevant. The rest of the productive factors are aggregated in a single factor, which we call K. We could also think of K as skilled labor, and interpret the role of the minimum wage as an income redistributive tool between skilled and unskilled labor. Several papers in the literature, such as Allen (1987), Guesnerie and Roberts (1987) and, more recently, Gorostiaga and Rubio-Ramírez (2004, 2005), have considered the use of minimum wage legislation as a scheme to redistribute income from high-skilled workers to low-skilled ones.

Finally, note that minimum wages in our paper affect only equilibrium wages in the covered sector. This might be at odds with some of the recent empirical work on the impact of minimum wages on labor markets of developing countries. In particular, some recent papers, such as Maloney and Nuñez (2004), have found that minimum wages in some Latin American countries are more binding in the uncovered sector than in the covered sector. However, other papers in the literature have found the opposite results. For example, Jaramillo (2004) fails to find strong effects of the minimum wage in uncovered-sector labor earnings in Peru. Similarly, Gindling and Terrell (2004) found that minimum wages have no effect on the wages of workers in the uncovered sector in Costa Rica. Also, to our knowledge nobody has offered yet a theoretical explanation of why minimum wages might be binding in the uncovered sector. For example, in a recent paper Neumark et al. (in press) state that "...nothing is known about why the minimum wage is binding in the informal sector nor the process while it adjusts..." Since we neither are able to explain why the minimum wage might bind in the uncovered sector, our model by construction will not incorporate this fact.

3.3 Single Minimum Wage

In this section we assume the government imposes a single minimum wage, which we shall call S. We assume that observing individual productivity is impossible for the government, and therefore the minimum wage is set per hour of work, not per unit of effective labor. It stands that every worker must be paid at least S per hour of work.

Firms in the covered sector will hire labor only if the value of the marginal productivity per hour is greater than S. In equilibrium, the following hiring condition will hold for firms:

$$w_c\left(j + h\left(a\right)\right) \geq S \tag{9}$$

This means that individuals whose productivity in equilibrium is less than S/w_c cannot work in the covered sector. We assume that S is sufficiently low such that $S < w_c h\left(A\right)$ and $S < w_c J$ hold. That is, every individual works in the covered sector at some stage in his life. Solving for a the equation $w_c\left(j + h\left(a\right)\right) = S$ leads to $a = h^{-1}\left(S/w_c - j\right)$. Thus individuals with $j = 0$ work in the covered sector from $a = h^{-1}\left(S/w_c\right)$ and individuals with $j \geq S/w_c$ work in the covered sector from $a = 0$.

Figure I illustrates the situation. We draw in the (j, a) plane iso-productivity curves, i.e. curves where the level of productivity is constant, $\left\{(j, a) \mid j + h\left(a\right) = \text{constant}\right\}$. In the figure we draw the iso-productivity curve correspondent to the productivity of S/w_c. All salaried workers with productivity above this curve are able to work in the covered sector, since the value of their marginal productivity exceeds the minimum wage; all salaried workers with productivity below it work in the uncovered sector.

[Insert Figure I here]

To calculate the supply of individuals that satisfy the hiring condition (9) we subtract the labor units of those workers who do not fulfill the condition from the total salaried labor in the economy:

$$L_c^{smw} = \overline{L} - \int_0^{S/w_c} \int_0^{h^{-1}(S/w_c - j)} \frac{f(j)}{A}\left(j + h\left(a\right)\right) da dj \tag{10}$$

Even when the total supply of salaried labor is inelastic, the supply of those individuals who fulfill condition (9) will have a positive elasticity with respect to w_c. This is because as w_c increases, so does the number of individuals who fulfill the hiring condition (9).

Intersecting equation (10) with labor demand equation (1a) we obtain the equilibrium price of labor in the covered sector under a SMW:

$$w_c = F_L\left(L_c^{smw}, \overline{K}\right) \tag{11}$$

Equation (11) has a single solution, which we call w_c^{smw}. It can be shown that w_c^{smw} is increasing in S, which means that w_c^{smw} is greater than the wage in equation (7). The intuition is that as the minimum wage increases, fewer individuals fulfill the hiring condition (9). As labor becomes scarcer in the covered sector, its price must rise.

Now we proceed to calculate the critical instant, which we call a^{smw}, when an individual of initial skill j moves from the uncovered salaried sector to the covered sector. This occurs when $a \geq h^{-1}\left(\frac{S}{w_c^{smw}} - j\right)$. As a cannot be negative, the lowest plausible value for a^{smw} is zero. Thus a^{smw} is given by:

$$a^{smw}(j, S) = \max\left\{h^{-1}\left(\frac{S}{w_c^{smw}} - j\right), 0\right\} \tag{12}$$

Returning to Figure I, we observe that for any j, a^{smw} corresponds to the coordinate on the horizontal axis where j intersects the iso-productivity curve.

In sum, the introduction of a SMW has has two major effects on the labor market:

(1) It will force less productive workers to move from the covered to the uncovered salaried sector, reducing their income.

(2) As the amount of workers in the covered sector is reduced w_c increases, which benefits high productive salaried workers that remain in the covered sector.

Notice that the minimum wage doesn't affect self-employed workers. Finally, note that when the minimum wage is introduced, income's share of labor increases to its (exogenously determined) optimal level.

Income

The income in this case will depend on the sector in which the individual is employed. If he works as salaried in the uncovered sector his wage income is $I_u = w_u(j + h(a))$, while if he works in the covered sector it is $I_c = w_c(j + h(a))$. Self-employed income is the same as the case with no minimum wage. For an explicit characterization of the distribution see Appendix A. The distribution presents segmentation: the distribution is still a linear transformation of the productivity distribution, but it is segmented at the minimum wage level. The least productive individuals will have to work in the uncovered salaried sector, where the price of labor is low, and thus receive a low income. High productivity individuals work in the covered sector where they receive a high price of labor, and thus a higher income.

Wealth

An individual of initial skill j will work in the uncovered sector until $a^{smw}(j, S)$, age at which he works in the covered sector until age A. Therefore, his wealth, given a SMW, is:

$$W^{smw}(j) = \int_0^{a^{smw}} w_u \left(j + h\left(a\right)\right) da + \int_{a^{smw}}^{A} w_c^{smw} \left(j + h\left(a\right)\right) da + \int_{A}^{\overline{A}} w_s h\left(a\right) da \quad (13)$$

Low-skill workers will have a relatively low level of wealth for two reasons: first, because they have a low endowment of productivity. Secondly, because they take a longer time to pass from the uncovered salaried to the covered sector.

3.4 Age-differentiated Minimum Wage

In this section we assume that the authority imposes an age-differentiated minimum wage according to the following formula:

$$S_a = s\left(a\right), \quad (14)$$

where the function $s\left(a\right)$ is increasing and concave in age. For ease of exposition, we assume a linear functional form for the differentiated minimum wage:

$$S_a = \beta + \gamma a \quad (15)$$

The solution to the model with the general functional form $s\left(a\right)$ is considerably more cumbersome, though completely analogous to the treatment of the solution with the general functional form $h\left(a\right)$. We assume that the marginal value of productivity grows faster than the DMW, that is, $w_c h'\left(a\right) > \gamma$.

In this case, the hiring condition becomes:

$$w_c\left(j+h\left(a\right)\right)\geq\beta+\gamma a \tag{16}$$

Individuals whose productivity in equilibrium is less than $\left(\beta+\gamma a\right)/w_c$ are excluded from the covered sector. We assume that every individual will work in the covered sector at some point and some will do so until they become self-employed. We call $a=g(w_c,j,\beta,\gamma)$ the age that solves the equation $w_c\left(j+h\left(a\right)\right)=\beta+\gamma a$. The function $g\left(\cdot\right)$ has the following properties: $\frac{\partial g}{\partial w_c}<0$, $\frac{\partial g}{\partial j}<0$, $\frac{\partial g}{\partial \beta}>0$, and $\frac{\partial g}{\partial \gamma}>0$. Our assumptions imply that individuals with $j=0$ will work in the covered sector from $a=g(w_c,0,\beta,\gamma)$ and individuals with $j\geq\beta/w_c$ will work in the covered sector from $a=0$. See Figure II for graphical representation.

[Insert Figure II here]

We then calculate the labor supply of the individuals who fulfill condition (16):

$$L_c^{dmw}=\overline{L}-\int_0^{\beta/w_c}\int_0^{g(w_c,j,\beta,\gamma)}\frac{f(j)}{A}\left(j+h\left(a\right)\right)da dj \tag{17}$$

Again we observe a positive supply elasticity with respect to w_c among individuals that satisfy condition (16).

Intersecting equation (17) with labor demand equation (1a) we obtain the equilibrium price of labor in the covered sector under a DMW:

$$w_c=F_L\left(L_c^{dmw},\overline{K}\right) \tag{18}$$

Equation (18) has a single solution which we call w_c^{dmw}. It can be shown that w_c^{dmw} is increasing in both β and γ. Higher values of these parameters imply a more restrictive minimum wage and thus scarcer labor in the covered sector.

We call a^{dmw} the critical instant at which an individual of initial skill j moves from the uncovered salaried to the covered sector with the DMW scheme. The individual will move from the uncovered to the covered sector when $a\geq g(w_c^{dmw},j,\beta,\gamma)$. This implies that:

$$a^{dmw}\left(j,\beta,\gamma\right)=\max\left\{g(w_c^{dmw},j,\beta,\gamma),0\right\} \tag{19}$$

Income

Income density function for salaried workers under a DMW is reported in Appendix A. Once again the distribution presents market segmentation. However, in this case the workers in the uncovered sector are not necessarily the least productive ones. Under a DMW some low productivity individuals will work in the covered sector: young, high-skill workers who confront a low minimum wage because of their youth. On the other hand, we will observe some high productivity individuals working in the uncovered sector: older, low-skill workers who face a high minimum wage because of their age.

Wealth

Finally, the wealth of an individual of initial skill j under a DMW scheme is given by:

$$W^{dmw}(j) = \int_0^{a^{dmw}} w_u \left(j + h\left(a\right)\right) da + \int_{a^{dmw}}^{A} w_c^{dmw} \left(j + h\left(a\right)\right) da + \int_A^{\overline{A}} w_s h\left(a\right) da \quad (20)$$

Note that the third term of the right hand of expression (20) is the same regardless of the existence of a minimum wage.

4 Both Schemes Compared: Case I

Recall that the rationale of the minimum wage was to achieve a certain labor's share of income. In this section, we compare a SMW with a DMW keeping labor's share of income fixed.

To this end we choose a vector (S, β, γ) such that the effective labor supplied in the covered sector under both schemes is equal, i.e. $L_c^{smw} = L_c^{dmw}$. As the labor supplied to the covered sector is the same under both schemes and the demand for labor is constant, the equilibrium in both cases is identical, implying that $w_c^{smw} = w_c^{dmw}$.

As the labor excluded from the covered sector under a DMW increases in both β and γ, if we set $\gamma > 0$ then $\beta < S$ must be true for $L_c^{smw} = L_c^{dmw}$ to hold. On the other hand, if $\beta < S$, then $\beta + \gamma A > S$ or else SMW will be more restrictive that DMW for individuals of all ages. These conditions mean that the minimum wage under DMW is lower at $a = 0$ and higher at $a = A$ than the minimum wage under a SMW regime. As under a DMW the minimum wage is increasing in a, there is a single instant at which the minimum wages are the same under both schemes.

If we intersect the hiring condition given by equation (9) with the hiring condition given by equation (16) we obtain a critical level of initial skill, which we shall call $\widehat{j}$. An individual with this level of initial skill enters the covered sector at the same instant, regardless of the minimum wage scheme:

$$\widehat{j} = \frac{S}{w_c} - h\left(\frac{S-\beta}{\gamma}\right) \tag{21}$$

Figure III illustrates the situation.

[Insert Figure III here]

In sum, the following relation holds between the critical ages at which an individual passes from the uncovered salaried to the covered sector under each scheme:

$$\begin{cases} a^{dmw}\left(j,\beta,\gamma\right) > a^{smw}\left(j,S\right) & \forall j \in \left[0,\widehat{j}\right) \\ a^{dmw}\left(j,\beta,\gamma\right) < a^{smw}\left(j,S\right) & \forall j \in \left(\widehat{j},\frac{S}{w_c}\right) \\ a^{dmw}\left(j,\beta,\gamma\right) = a^{smw}\left(j,S\right) & \forall j \in \left[\frac{S}{w_c},J\right] \end{cases} \tag{22}$$

Individuals with low initial skills take longer to enter the covered sector under a DMW than under a SMW. This is because although their productivity grows with age, the minimum wage they face also does. In short, they confront a more restrictive hiring condition under a DMW. Individuals with relatively high initial skills (j between $\widehat{j}$ and $\frac{S}{w_c}$) enter the covered sector more quickly under a DMW. This is due both to their high initial skill and their youth, because it means they confront a low minimum wage, allowing them a quick transition to the covered sector. Finally, individuals with $j \geq \frac{S}{w_c}$ enter the covered sector at $a = 0$ under both schemes.

From equation (22) and from the fact that there is a positive monotonic relationship between wealth and the time it takes an individual to enter the covered sector, the following relation will hold between wealth in both schemes:

$$\begin{cases} W^{smw}(j) > W^{dmw}(j) & \forall j \in \left[0,\widehat{j}\right) \\ W^{smw}(j) < W^{dmw}(j) & \forall j \in \left(\widehat{j},\frac{S}{w_c}\right) \\ W^{smw}(j) = W^{dmw}(j) & \forall j \in \left[\frac{S}{w_c},J\right] \end{cases} \tag{23}$$

Clearly, passing from a SMW to a DMW is not Pareto efficient. The wealth of individuals with low initial skills is lower under a DMW than under a SMW, as they take longer to enter the covered sector. The wealth of individuals whose level of initial skills lies between $\widehat{j}$ and $\frac{S}{w_c}$ is greater under a DMW as they take less time to enter the covered sector. And there is no change in the wealth of individuals of initial skills $j \geq \frac{S}{w_c}$ as they enter the covered sector at $a = 0$ under both schemes.

Another important point to notice is that when a DMW is introduced, there is an outflow of workers moving from the covered to the uncovered salaried sector (represented by area B of figure III) and an inflow of workers coming from the uncovered salaried to the covered sector (represented by area A of figure III). Since by construction the quantity of labor is the same under both schemes, the inflows and outflows of labor cancel each other out.

Individuals that move from the covered to the uncovered salaried sector are those with initial skills given by $j < \widehat{j}$ and with age given by $a^{smw}(j, S) < a < a^{dmw}(j, \beta, \gamma)$. Individuals that move from the uncovered to the covered sector are those with $a^{dmw}(j, \beta, \gamma) < a < a^{smw}(j, S)$. Notice that although total labor in the covered sector remains unchanged, there is a substitution between old and young workers. The average productivity of workers entering the covered sector is lower than the productivity of workers leaving it. As a result, the average productivity of the covered sector falls when a DMW is introduced. In order to obtain the same level of labor in the covered sector under both schemes, with a DMW more individuals will have to work in the covered sector.

Finally, we formally compare the income distribution among all individuals and the wealth distribution among individuals belonging to the same generation, under both minimum wage schemes. We use the Lorenz function as the metric to compare the different distributions. The following two propositions summarize our results:

Proposition 1. *Under a DMW scheme the income distribution is more equal than under a SMW scheme.*

Proof. See Appendix B. □

The intuition behind this result is as follows: as we have already said, an individual's income depends on the product of the price of labor in the sector in which he works in and his productivity. If we place the least productive individuals in the uncovered salaried sector and the most productive in the covered sector, we maximize income differences. This is precisely the effect of a SMW. Any other allocation of individuals -such as that produced by a DMW- will imply a more equal distribution of income. In particular, under a DMW we encounter low productivity individuals receiving a high price of labor (these are young, high-skill individuals) and also high productivity workers that are paid a low price for their labor (older, low-skill individuals). This naturally implies a more equal income distribution than that obtained under a SMW.

Proposition 2. *Under a DMW scheme the distribution of wealth is more unequal than under a SMW scheme.*

Proof. See Appendix B. □

Intuitively, under a single minimum wage, low-skill individuals start out working in the uncovered salaried sector, but as their productivity grows over time the minimum wage eventually ceases to be a binding restriction and they switch to the covered sector. Under a DMW these same individuals take longer to move to the covered sector since they enter the covered sector when they are already old, and the minimum wage rises for them. That is, although their productivity grows over time the minimum wage they confront catches up. On the other hand, with a SMW, high-skill individuals quickly enter the covered sector, and this transition is even faster under a DMW, because their youth ensures they confront a low minimum wage.

We can observe that the differences in the level of wealth among the two minimum wage regimes depend on the differences in the time an individual works in the uncovered salaried sector, and thus they do not depend on the rate of productivity growth in the covered sector. Therefore, the assumption that the rate of productivity growth in the salaried uncovered sector is the same that in the covered sector can be relaxed and both propositions will still hold.

Finally, notice that allowing for a positive interaction between $h(a)$ and j would make our results even stronger. If we were to assume that workers with high natural skills acquire experience faster than workers with low natural skills, we would exacerbate the differences in the wealth of high and low-skill workers. Under a DMW, high-skill individuals would take even less time to enter the covered sector while low-skill individuals would take even more time.

5 Both Schemes Compared: Case II

In some countries, the idea is to lower the minimum wage for young workers without necessarily increasing the minimum for older workers. In this section, we study the result of such policy experiment. In order to do so, it is useful to develop first a simpler version of the model where the price of labor is exogenous.

Exogenous wages

Suppose, for a moment, that the economy is small and open, and the government buys or sells capital in the international market. Since $F(L, K)$ has constant returns to scale, the price of labor in the covered sector will depend only on the government's discount rate, which is constant.

With an exogenous price of labor in the covered sector, the only determinant of individuals' wealth will be the time they take to enter the covered sector. From equation (19) it follows that under a DMW the individual endowed with initial skill j will move from the uncovered salaried to the covered sector at the critical moment a^{dmw}. If we total differentiate this critical moment with respect to the parameters of the DMW, and focus our attention on strictly positive critical ages, we obtain:

$$da^{dmw} = \frac{\partial a}{\partial \beta} d\beta + \frac{\partial a}{\partial \gamma} d\gamma \tag{24}$$

Equation (24) shows that a parallel reduction in the minimum wage ($d\beta < 0$ and $d\gamma = 0$) is Pareto efficient. Given that $h'(a) = h$ in the neighborhood of the critical age, the expression $\frac{da^{dmw}}{d\beta} = \frac{1}{w_c^{dmw} h - \gamma}$ is positive. Observe that this effect does not depend on the initial skill of the individual, and therefore the effect is symmetric for all the individuals.

On the other hand, an increase in the slope γ (keeping β constant) is detrimental for everybody, since $\frac{da^{dmw}}{d\gamma}$ is also positive, although the effect is stronger for individuals with a higher a^{dmw}, which are the less skilled ones.

Relaxing the minimum wage for the young workers while keeping it fixed for the old ones consists precisely in a combination of this two actions, decreasing β and increasing γ. It turns out that the overall effect of this policy is to benefit everybody, although it benefits relatively less the least skilled workers.

This result is formalized in the following proposition:

Proposition 3. *In an economy that faces an exogenous price of labor, relaxing the minimum wage for the young improves everybody's wealth, but it benefits less the least skilled workers.*

Proof. Relaxing the minimum wage for young workers while keeping it fixed for the older ones corresponds to reduce β and increase γ in order to keep $\beta+\gamma A$ constant. This implies that $d\gamma = \frac{-d\beta}{A}$, which allows us to express the change in the critical moment da^{dmw} as a function of the change in β :

$$\frac{da^{dmw}}{d\beta} = \frac{A - a^{dmw}}{A} \frac{1}{w_c^{dmw} h - \gamma} \tag{25}$$

It is clear from (25) that $\frac{da^{dmw}}{d\beta}$ is not negative, since $A \geq a^{dmw}$. However, given that a lower j implies a higher a^{dmw}, the effect will be smaller for a lower j. □

Intuitively, since less skilled workers enter the covered sector when they are old, and the minimum wage for old people barely changes, the reduction in the minimum wage benefits them relatively little.

Endogenous wages

We now turn to the original case of a closed economy with fixed capital. Here the price of labor is endogenous, given by w_c^{dmw} with $\frac{\partial w_c}{\partial \beta}$ and $\frac{\partial w_c}{\partial \gamma}$ strictly positive.

Note that as the minimum wage becomes less restrictive, more young workers are able to enter the covered sector, and therefore the marginal productivity of labor falls. To see why, suppose it didn't. Then the hiring constraint (16) would become less binding for everybody and thus more individuals would be able to work in the covered sector. However, if L_c increases, then w_c^{dmw} would decrease, which would be a contradiction.

Naturally, this makes it more difficult for individuals to fulfill firms' hiring constraint.

Proposition 4. *In an economy with an endogenous price of labor, the strategy of relaxing the minimum wage solely for young workers delays the moment in which the least skilled workers enter the covered sector.*

Proof. By total differentiating the critical moment a^{dmw}, we obtain:

$$da^{dmw} = \frac{\partial a}{\partial \beta} d\beta + \frac{\partial a}{\partial \gamma} d\gamma + \frac{\partial a}{\partial w_c^{dmw}} dw_c^{dmw}, \qquad \text{where } \frac{\partial a}{\partial w_c^{dmw}} = \frac{\partial w_c^{dmw}}{\partial \beta} d\beta + \frac{\partial w_c^{dmw}}{\partial \gamma} d\gamma \quad (26)$$

Recalling that $d\gamma = \frac{-d\beta}{A}$, we can write expression (26) as:

$$\frac{da^{dmw}}{d\beta} = \frac{A - a^{dmw}}{A} \frac{1}{w_c^{dmw} h - \gamma} - \frac{a + j}{w_c^{dmw} h - \gamma} dw_c^{dmw} \quad (27)$$

If we take an individual of sufficient low natural ability (say $\overline{j}$) that enters the covered sector at an age arbitrarily close to A, the effect for him will be:

$$\frac{da^{dmw}}{d\beta} = -\frac{A + \overline{j}}{w_c^{dmw} h - \gamma} dw_c^{dmw} \quad (28)$$

Since dw_c^{dmw} is strictly negative, the total effect in (28) will be positive, and therefore he will take longer to enter the covered sector. □

Now the reduction in minimum wage is biased against the less skilled not only because when they enter the covered sector the minimum wage would have barely changed, but also because the reduction in the price of labor in the covered sector affects them more. We can understand this asymmetric effect due to w_c^{dmw} if we notice that the individual endowed with initial skill j that faces a differentiated minimum wage S_a needs to wait until moment $a^{dmw} = \max\left\{g(w_c^{dmw}, j, \beta, \gamma), 0\right\}$ to enter the covered sector. The lower the j is, the greater the response of a^{dmw} to w_c^{dmw}, because most of the necessary productivity required entering the covered sector is achieved through experience. Intuitively, since the growth rate of the value of productivity through time is proportional to w_c, a reduction in w_c has a larger effect on those workers who take a long time to enter the covered sector, who are precisely the less skilled ones.

According to proposition 4, for individuals with sufficiently low natural abilities, the second (negative) effect will more than outweigh the first (positive) effect, which will finally lead them to wait longer to enter the covered sector.

However, notice that the distribution of wealth doesn't necessarily worsen. Wealth falls for the least productive workers but it also falls for the most productive ones, since individuals who start their working lives in the covered sector would now confront a lower price of labor too.

The main conclusion of this section is that relaxing the minimum wage exclusively for young workers harms the less skilled workers, who will remain for a longer time in the uncovered salaried sector.

6 Unemployment

For some policy makers, the real concern driving the multiple minimum wage is youth unemployment. In order to account for this fact, this section introduces unemployment to our model. We show that the policy experiment reduces youth unemployment, and is still detrimental for the least skilled workers.

We introduce unemployment to the model by assuming that workers at each level of productivity are heterogeneous with regard to their reservation wage (per unit of effective labor), which we shall call r . Therefore, individuals will now choose to work in the sector that pays more, as long as the wage in that sector pays them at least their reservation wage.

6.1 No Minimum Wage

In this scenario, salaried labor supply will be a function of the wage in the covered sector:

$$L_c = \int_0^A \int_0^J \int_0^{w_c} f(j,a,r)(j+h(a))drdjda, \tag{29}$$

where $f(j,a,r)$ denotes the joint density function of initial skills, age, and reservation wages. Observe that supply is an increasing function of w_c. Equilibrium is achieved at the point where $w_c = F_L\left(L_c, \overline{K}\right)$. There is no unemployment and the salaried labor force is equal to L_c.

6.2 Single Minimum Wage

When a SMW is introduced, the supply of salaried labor satisfying the hiring condition (9) is:

$$L_c^{smw} = L_c - \int_0^{S/w_c} \int_0^{h^{-1}(S/w_c - j)} \int_0^{w_c} f(j,a,r)(j + h(a))drdjda \tag{30}$$

Again, equilibrium in this sector is achieved when $w_c = F_L\left(L_c^{smw}, \overline{K}\right)$. Labor supply in the uncovered salaried sector will be:

$$L_u^{smw} = \int_0^{S/w_c} \int_0^{h^{-1}(S/w_c - j)} \int_0^{w_u} f(j,a,r)(j + h(a))drdjda \tag{31}$$

Note that now, there is a group of individuals that are willing to work at the current wage in the covered sector, but due to the existence of the minimum wage they cannot. However, they neither participate in the uncovered sector. These individuals are not working even though at current wages in the covered sector they are willing to work; we regard them as unemployed. Therefore, unemployed workers will correspond to the intersection of two groups of individuals: those who do not satisfy the hiring constraint (9), and those whose reservation wage is in between w_u and w_c.

The measure of unemployed individuals is given by:[2]

$$\mathcal{U}^{smw} = \int_0^{S/w_c} \int_0^{h^{-1}(S/w_c - j)} \int_{w_u}^{w_c} f(j,a,r)drdjda \tag{32}$$

6.3 Age-differentiated Minimum Wage

When a DMW is introduced, the labor supply of the individuals who fulfill condition (16) is given by:

$$L_c^{dmw} = L_c - \int_0^{\beta/w_c} \int_0^{g(w_c,j,\beta,\gamma)} \int_0^{w_c} f(j,a,r)(j + h(a))drdjda \tag{33}$$

As before, the equilibrium in this sector is achieved when $w_c = F_L\left(L_c^{dmw}, \overline{K}\right)$. Labor supply in the salaried uncovered sector will be:

$$L_u^{dmw} = \int_0^{\beta/w_c} \int_0^{g(w_c,j,\beta,\gamma)} \int_0^{w_u} f(j,a,r)(j + h(a))drdjda, \tag{34}$$

[2]Unemployment is measured as individuals, not as efficiency units.

and the measure of unemployment will be:

$$\mathcal{U}^{dmw} = \int_0^{\beta/w_c} \int_0^{g(w_c,j,\beta,\gamma)} \int_{w_u}^{w_c} f(j,a,r)drdjda \tag{35}$$

The following two propositions summarize the results of adding unemployment to the model:

Proposition 5. *The strategy of relaxing the minimum wage solely for young workers reduces youth unemployment.*

Proof. First note that this policy experiment decreases w_c and hence increases the labor force in the covered sector (If it didn't, the hiring constraint (16) would become less binding and more individuals would be able to work in the covered sector, lowering w_c, which would be a contradiction).

Since S must be equal to $\beta+\gamma A$ in order to maintain unchanged the minimum wage for the old salaried workers, we can interpret this policy experiment as changing the minimum wage from $\beta + \gamma A$ to $\beta + \gamma a$.

Given any age, there exists a threshold level of natural skill (which we call $\widetilde{j}$) above which individuals satisfy the hiring condition (16). For the SMW and DMW regime, this threshold is given by:

$$\widetilde{j}^{smw} = \frac{\beta + \gamma A}{w_c^{smw}} - h(a) \tag{36}$$

and

$$\widetilde{j}^{dmw} = \frac{\beta + \gamma a}{w_c^{dmw}} - h(a), \tag{37}$$

respectively. Since after the adoption of a DMW there is more labor in the covered sector, there exists an age level such that $\widetilde{j}^{dmw} < \widetilde{j}^{smw}$, which in term implies that $\frac{\beta}{w_c^{dmw}} < \frac{\beta+\gamma A}{w_c^{smw}}$.

Moreover, since $\widetilde{j}^{dmw}$ and $\widetilde{j}^{smw}$ are continuous functions of a, there must exist a higher age such that $\widetilde{j}^{dmw} \leq \widetilde{j}^{smw}$. Call this age $\widetilde{a}$. Clearly if $a \leq \widetilde{a}$, then $\widetilde{j}^{dmw} < \widetilde{j}^{smw}$, and therefore for $a < \widetilde{a}$, the hiring constraint is strictly less binding for individuals of age below $\widetilde{a}$.

Lets define youth as those individuals for which $a < \widetilde{a}$. Next notice that for this group the hiring condition becomes less binding. Finally notice that w_c decreases, reducing individuals with reservation wages between w_u and w_c. Since youth unemployment consists in those young individuals who fulfill these two conditions, and the two conditions become less binding, youth unemployment will be reduced.

□

Proposition 6. *The strategy of relaxing the minimum wage solely for young workers delays the moment in which the least skilled workers enter the covered sector.*

Proof. Consider an individual with natural skill $\bar{j}$ low enough such that $w_c(j + h(a))$ is arbitrarily close to $\beta + \gamma A$. There are three possible cases for his reservation wage:

(1) $r < w_u$. In this case, the worker will always prefer to work, therefore his case is analogous to the second comparison in section 5. The effect of this policy on the time it will take him to enter the covered sector will be:

$$\frac{da}{d\beta} = -\frac{A + \bar{j}}{w_c^{dmw} h - \gamma} dw_c^{dmw}, \tag{38}$$

which is strictly positive, since dw_c^{dmw} is negative.

(2) $w_u < r < w_c^{dmw}$. This individual will be unemployed regardless of the minimum wage scheme.

(3) $w_c^{dmw} < r < w_c^{smw}$. In this case, the individual was willing to work in the covered sector with a SMW, but couldn't because of the minimum wage (he was unemployed). With a DMW he will simply move out of the labor force.

(4) $r > w_c^{smw}$. In this case the individual was out of the labor force with a SMW and remains to be there after the new policy is implemented.

In sum, in no case is this policy experiment beneficial for the individual, and in the first case it is strictly harmful. □

7 Concluding Remarks

This article has shown that –at the same level of efficiency– an age-differentiated minimum wage results in a more equal income distribution than a single minimum wage. However, low-skill individuals stay longer in the uncovered salaried sector under a DMW, leading to a more unequal distribution of wealth than under a SMW. We have also shown that relaxing the minimum wage solely for young workers reduces youth unemployment, however it is harmful for the less skilled workers, since they will take longer to fulfill the hiring condition of the covered sector.

The problem with the DMW arises from the fact that the authority cannot observe actual productivity and thus sets a minimum wage on a variable that is imperfectly correlated with productivity (i.e. age). Given that productivity also depends on initial skills, this harms particularly those individuals with relatively low productivity for their age.

Finally, we think the analysis should be extended in a number of useful directions. First, we are assuming that individuals gain productivity exogenously through experience. This accumulation of experience might be endogenous to the minimum wage scheme, since the minimum wage might affect, for example, the process of on-the-job training. Secondly, by construction our model does not consider schooling decisions. Since schooling decisions are endogenous to the minimum wage scheme, it could also be interesting to study their possible interaction with our main results.

A Distribution of Income of Salaried Workers

In the model, salaried workers' distribution of skills is $f(j)$ with $j \in [0, J]$ and the distribution of age is $U[0, A]$. We wish to obtain the distribution of $f(I)$, where $I = w\left(j + h\left(a\right)\right)$. To this end, we define the auxiliary variable $x \equiv j$. From probability theory it holds that $f(I, x) = f(j, a) \mid J(I, x) \mid$, where $J(I, x) = \begin{bmatrix} \partial j/\partial I & \partial j/\partial x \\ \partial a/\partial I & \partial a/\partial x \end{bmatrix}$. In this case $\mid J(I, x) \mid = \frac{h^{-1\prime}(I/w - x)}{w}$, where $h^{-1\prime}\left(I/w - x\right) \equiv \frac{\partial h^{-1}(I/w - x)}{\partial x}$. So $f(I, x) = f(x)f(a)\frac{h^{-1\prime}(\cdot)}{w} = \frac{f(x)}{Aw}h^{-1\prime}\left(\cdot\right)$. To obtain the marginal density function $f(I)$, we compute $f(I) = \int_x f(I, x)dx$. We assume that $J < h\left(A\right)$, although assuming the contrary wouldn't alter our results.

Single Minimum Wage

(i) Covered Sector

Individuals work in the covered sector if $w_c\left(x + h\left(a\right)\right) \geq S$, which we may rewrite as $I_c \geq S$, where $I_c = w_c\left(j + h\left(a\right)\right)$. We compute $f^{smw}(I_c) = \int_x f(I_c, x)dx$ where integration must fulfill the following limits: $0 \leq x \leq J$, $0 \leq h^{-1}\left(I_c/w_c - x\right) \leq A$, and $I_c \geq S$. Since by assumption $S/w_c < J$, we obtain the following distribution:

$$f^{smw}(I_c) = \begin{cases} \int_0^{\frac{I_c}{w_c}} \frac{f(x)}{Aw_c}h^{-1\prime}\left(\cdot\right)dx & \forall I_c \in \quad [S, w_c J] \\ \int_0^J \frac{f(x)}{Aw_c}h^{-1\prime}\left(\cdot\right)dx & \forall I_c \in \quad [w_c J, w_c h\left(A\right)] \\ \int_{\frac{I_c}{w_c} - h(A)}^J \frac{f(x)}{Aw_c}h^{-1\prime}\left(\cdot\right)dx & \forall I_c \in \quad [w_c h\left(A\right), w_c\left(J + h\left(A\right)\right)] \end{cases} \tag{A.1}$$

(ii) Uncovered Salaried Sector

Individuals work in the uncovered sector if $w_c\left(x + h\left(a\right)\right) < S$, which we may rewrite as $I_u < \frac{Sw_u}{w_c}$, where $I_u = w_u\left(x + h\left(a\right)\right)$. We compute $f^{smw}(I_u) = \int_x f(I_u, x)dx$ with the following integration limits: $0 \leq x \leq J$, $0 \leq h^{-1}\left(I_u/w_u - x\right) \leq A$, and $I_u < \frac{Sw_u}{w_c}$. This results in the following distribution:

$$f^{smw}(I_u) = \left\{ \int_0^{\frac{I_u}{w_u}} \frac{f(x)}{Aw_u}h^{-1\prime}\left(\cdot\right)dx \qquad \forall I_u \in \quad \left[0, \frac{Sw_u}{w_c}\right] \right. \tag{A.2}$$

(iii) The Whole Economy

The income distribution for the whole economy is given by: $f^{smw}(I) = f^{smw}(I_u) + f^{smw}(I_c)$. Since $w_u < w_c$ it follows that $\frac{Sw_u}{wc} < S$, so the density function of income under a SMW is:

$$f^{smw}(I) = \begin{cases} \int_0^{\frac{I}{w_u}} \frac{f(x)}{Aw_u} h^{-1\prime}(\cdot)\, dx & \forall I \in \left[0, \frac{Sw_u}{w_c}\right] \\ 0 & \forall I \in \left[\frac{Sw_u}{w_c}, S\right] \\ \int_0^{\frac{I}{w_c}} \frac{f(x)}{Aw_c} h^{-1\prime}(\cdot)\, dx & \forall I \in [S, w_c J] \\ \int_0^J \frac{f(x)}{Aw_c} h^{-1\prime}(\cdot)\, dx & \forall I \in [w_c J, w_c h(A)] \\ \int_{\frac{I}{w_c}-h(A)}^J \frac{f(x)}{Aw_c} h^{-1\prime}(\cdot)\, dx & \forall I \in [w_c h(A), w_c (J + h(A))] \end{cases} \tag{A.3}$$

Age-Differentiated Minimum Wage

(i) Covered Sector

Individuals work in the covered sector if $w_c (x + h(a)) \geq \beta + \gamma a$. We shall call $I_c = v_c(x)$ the level of income that solves the equation $I_c = \beta + \gamma h^{-1}(I_c/w_c - x)$, where $v_c(x)$ is a decreasing function in x. We compute $f^{dmw}(I_c)$ with the integration limits: $0 \leq x \leq J$, $0 \leq h^{-1}(I_c/w_c - x) \leq A$, and $I_c \geq v_c(x)$. We assume that $v_c(0) < w_c J$, and we shall call x_c the value of x that solves the equation $v_c(x) = w_c x$. Since $v_c(x)$ is decreasing in x, it follows that $w_c x_c < v_c(0)$, and thus we obtain the following distribution:

$$f^{dmw}(I_c) = \begin{cases} \int_{v_c^{-1}(I_c)}^{\frac{I_c}{w_c}} \frac{f(x)}{Aw_c} h^{-1\prime}(\cdot)\, dx & \forall I_c \in [w_c x_c, v_c(0)] \\ \int_0^{\frac{I_c}{w_c}} \frac{f(x)}{Aw_c} h^{-1\prime}(\cdot)\, dx & \forall I_c \in [v_c(0), w_c J] \\ \int_0^J \frac{f(x)}{Aw_c} h^{-1\prime}(\cdot)\, dx & \forall I_c \in [w_c J, w_c h(A)] \\ \int_{\frac{I_c}{w_c}-h(A)}^J \frac{f(x)}{Aw_c} h^{-1\prime}(\cdot)\, dx & \forall I_c \in [w_c h(A), w_c (J + h(A))] \end{cases} \tag{A.4}$$

(ii) Uncovered Salaried Sector

Individuals work in the uncovered sector if $w_c (x + h(a)) < \beta + \gamma a$. We call $I_u = v_u(x)$ the level of income that solves the equation $I_u = \frac{w_u}{w_c}\left[\beta + \gamma h^{-1}(I_u/w_u - x)\right]$, where $v_u(x)$

is decreasing in x. We assume that $v_u(0) < w_u J$, and we call x_u the value of x that solves the equation $v_u(x) = w_u x$. Since $v_u(x)$ is a decreasing function of x, then $w_u x_u < v_u(0)$, and hence we obtain the following distribution:

$$f^{dmw}(I_u) = \begin{cases} \int_0^{\frac{I_u}{wu}} \frac{f(x)}{Aw_u} h^{-1\prime}(\cdot)\, dx & \forall I_u \in \ [0, w_u x_u] \\ \int_0^{v_u^{-1}(I_u)} \frac{f(x)}{Aw_u} h^{-1\prime}(\cdot)\, dx & \forall I_u \in \ [w_u x_u, v_u(0)] \end{cases} \tag{A.5}$$

(iii) The Whole Economy

The income distribution for the whole economy is $f^{dmw}(I) = f^{dmw}(I_u) + f^{dmw}(I_c)$. For simplicity, we assume that w_u is sufficiently low such that the distributions of both sectors do not overlap, that is $v_u(0) < w_c x_c$. Assuming the contrary wouldn't alter our results. Therefore the distribution of income under a DMW is:

$$f^{dmw}(I) = \begin{cases} \int_0^{\frac{I}{wu}} \frac{f(x)}{Aw_u} h^{-1\prime}(\cdot)\, dx & \forall I \in \ [0, w_u x_u] \\ \int_0^{v_u^{-1}(I)} \frac{f(x)}{Aw_u} h^{-1\prime}(\cdot)\, dx & \forall I \in \ [w_u x_u, v_u(0)] \\ 0 & \forall I \in \ [v_u(0), w_c x_c] \\ \int_{v_c^{-1}(I)}^{\frac{I}{w_c}} \frac{f(x)}{Aw_c} h^{-1\prime}(\cdot)\, dx & \forall I \in \ [w_c x_c, v_c(0)] \\ \int_0^{\frac{I}{w_c}} \frac{f(x)}{Aw_c} h^{-1\prime}(\cdot)\, dx & \forall I \in \ [v_c(0), w_c J] \\ \int_0^{J} \frac{f(x)}{Aw_c} h^{-1\prime}(\cdot)\, dx & \forall I \in \ [w_c J, w_c h(A)] \\ \int_{\frac{I}{w_c} - h(A)}^{J} \frac{f(x)}{Aw_c} h^{-1\prime}(\cdot)\, dx & \forall I \in \ [w_c h(A), w_c(J + h(A))] \end{cases} \tag{A.6}$$

Both Distributions Compared

In section 5 we showed that since both minimum wage schemes were equally binding, then $h^{-1}(S/w_c) < g(w_c, 0, \beta, \gamma)$. This result implies that $\frac{Sw_u}{w_c} < v_u(0)$ and $S < v_c(0)$. Therefore, the following results holds:

- $f^{smw}(I) = f^{dmw}(I)\ \forall I \in [0, w_u x_u]$, since the integrand under both schemes is the same,
- $f^{smw}(I) > f^{dmw}(I)\ \forall I \in \left[w_u x_u, \frac{Sw_u}{w_c}\right]$, since $f^{smw}(I)$ is increasing in I while $f^{dmw}(I)$ is decreasing in I,

- $f^{smw}(I) < f^{dmw}(I)\ \forall I \in \left[\frac{Sw_u}{w_c}, v_u(0)\right]$, since $f^{smw}(I) = 0$ and $f^{dmw}(I) > 0$,
- $f^{smw}(I) = f^{dmw}(I)\ \forall I \in [v_u(0), w_c x_c]$, since $f^{smw}(I) = f^{dmw}(I) = 0$,
- $f^{smw}(I) < f^{dmw}(I)\ \forall I \in [w_c x_c, S]$, since $f^{smw}(I) = 0$ and $f^{dmw}(I) > 0$,
- $f^{smw}(I) > f^{dmw}(I)\ \forall I \in [S, v_c(0)]$, since the lower limit of the integral associated to $f^{dmw}(I)$ is higher than the one associated with $f^{smw}(I)$, and the upper limit of both integrals is the same, and
- $f^{smw}(I) = f^{dmw}(I)\ \forall I \in [v_c(0), w_c(J + h(A))]$, since the integrand under both schemes is the same.

B Proof of Propositions

Proof of Proposition 1

First, we will prove that the proposition holds for salaried workers. We can summarize the results obtained in Appendix A as:

$$\begin{aligned} f^{smw}(I) &\geq f^{dmw}(I) && \forall I \in \left[0, \tfrac{Sw_u}{w_c}\right] \\ f^{smw}(I) &\leq f^{dmw}(I) && \forall I \in \left[\tfrac{Sw_u}{w_c}, S\right] \\ f^{smw}(I) &\geq f^{dmw}(I) && \forall I \in [S, w_c(J + h(A))] \end{aligned} \tag{B.1}$$

Defining the cumulative distribution function as $F(I) = \int_0^I f(i)\,di$, equation (B.1) implies that:

$$\begin{aligned} F^{smw}(I) &\geq F^{dmw}(I) && \forall I \in \left[0, \tfrac{Sw_u}{w_c}\right] \\ F^{smw}(I) &\leq F^{dmw}(I) && \forall I \in [S, w_c(J + h(A))] \end{aligned} \tag{B.2}$$

Given that by construction both schemes are equally binding, the mean income under both regimes is the same:

$$\int_0^{w_c(J+h(A))} I dF^{smw}(I) = \int_0^{w_c(J+h(A))} I dF^{dmw}(I) \tag{B.3}$$

By integrating expressions in (B.3), we can write mean income as:

$$\int_0^{w_c(J+h(A))} [1 - F^{smw}(I)]\, dI = \int_0^{w_c(J+h(A))} \left[1 - F^{dmw}(I)\right] dI \tag{B.4}$$

Which in turn implies that:

$$\int_0^{w_c(J+h(A))} \left[F^{smw}(I) - F^{dmw}(I)\right] dI = 0 \tag{B.5}$$

Since:

$$\int_0^{K} \left[F^{smw}(I) - F^{dmw}(I)\right] dI \tag{B.6}$$

is an increasing function of K for low values of K, and a decreasing function of K for a sufficiently large K, it follows that:

$$\int_0^{K} \left[F^{smw}(I) - F^{dmw}(I)\right] dI \geq 0 \tag{B.7}$$

for all $K \geq 0$. That is, the distribution $f^{smw}(I)$ dominates the distribution $f^{dmw}(I)$ by second-order stochastic dominance. Since the criterion of second degree stochastic dominance is equivalent to non-intersecting Lorenz curves (Atkinson 1970), the Lorenz curve of income under a SMW never exceeds the Lorenz curve of income under a DMW. The proposition is proved for salaried workers.

In order to extend this proof for all workers, note that at age A, the individual becomes self-employed and gains $w_s h(A) > w_c(J+h(A))$. Given that for $I > w_c(J+h(A))$, we have that $f^{smw}(I) = f^{dmw}(I)$, and that $F^{smw}(I) = \int_0^{w_c(J+h(A))} f^{smw}(I) + \int_{w_c(J+h(A))}^{I} f^{smw}(I)$, so the following equality holds:

$$F^{smw}(I)-F^{dmw}(I) = F^{smw}\left[w_c(J+h(A))\right]-F^{dmw}\left[w_c(J+h(A))\right]+\int_{w_c(J+h(A))}^{I} \left(f^{smw} - f^{dmw}\right) di \tag{B.8}$$

We know that the third term in the right hand of the expression is cero, since $f^{smw}(I) = f^{dmw}(I)$. Hence, for $I \geq w_c(J+h(A))$, $\int_0^{K} \left[F^{smw}(I) - F^{dmw}(I)\right] dI = \int_0^{w_c(J+h(A))} \left[F^{smw}(I) - F^{dmw}(I)\right] dI$ which is non-negative.□

Proof of Proposition 2

Equation (23) states that:

$$\begin{aligned}
W^{smw}(j) &> W^{dmw}(j) && \forall j \in \left[0, \widehat{j}\right) \\
W^{smw}(j) &< W^{dmw}(j) && \forall j \in \left(\widehat{j}, \tfrac{S}{w_c}\right) \\
W^{smw}(j) &= W^{dmw}(j) && \forall j \in \left[\tfrac{S}{w_c}, J\right]
\end{aligned} \tag{B.9}$$

Given that $W(j)$ is a monotonically increasing function of j, we can define the inverse function $j = q(W)$. Results in equation (B.9) mean that:

$$\begin{aligned} q^{dmw}(W) &> q^{smw}(W) && \forall W \in \left[W^{dmw}(0), W\left(\widehat{j}\right)\right) \\ q^{dmw}(W) &< q^{smw}(W) && \forall W \in \left(W\left(\widehat{j}\right), W\left(\tfrac{S}{w_c}\right)\right) \\ q^{dmw}(W) &= q^{smw}(W) && \forall W \in \left[W\left(\tfrac{S}{w_c}\right), W(J)\right] \end{aligned} \tag{B.10}$$

From equation (B.10) and from the fact that the cumulative distribution function of wealth, $F(W) = \int_0^W f(i)\,di$, is given by $F(W) = \Pr(W(j) \leq W) = \Pr(j \leq q(W))$, it follows that:

$$\begin{aligned} F^{dmw}(W) &> F^{smw}(W) && \forall W \in \left[W^{dmw}(0), W\left(\widehat{j}\right)\right) \\ F^{dmw}(W) &< F^{smw}(W) && \forall W \in \left(W\left(\widehat{j}\right), W\left(\tfrac{S}{w_c}\right)\right) \\ F^{dmw}(W) &= F^{smw}(W) && \forall W \in \left[W\left(\tfrac{S}{w_c}\right), W(J)\right] \end{aligned} \tag{B.11}$$

Due to the fact that both minimum wage schemes are equally binding, the mean wealth under both regimes is the same:

$$\int_{W^{dmw}(0)}^{W(J)} \left[1 - F^{dmw}(W)\right] dW = \int_{W^{smw}(0)}^{W(J)} \left[1 - F^{smw}(W)\right] dW \tag{B.12}$$

Since $W^{dmw}(0) < W^{smw}(0)$, it follows that $f^{smw}(W) = 0\ \forall W \in \left[W^{dmw}(0), W^{smw}(0)\right]$, so we can write $\int_{W^{smw}(0)}^{W(J)} \left[1 - F^{smw}(W)\right] dW$ as $\int_{W^{dmw}(0)}^{W(J)} \left[1 - F^{smw}(W)\right] dW$. Therefore, equation (B.12) means that:

$$\int_{W^{dmw}(0)}^{W(J)} \left[F^{dmw}(W) - F^{smw}(W)\right] dW = 0 \tag{B.13}$$

Given that:

$$\int_{W^{dmw}(0)}^{K} \left[F^{dmw}(W) - F^{smw}(W)\right] dW \tag{B.14}$$

is decreasing in K for a sufficiently large K, it follows that:

$$\int_{W^{dmw}(0)}^{K} \left[F^{dmw}(W) - F^{smw}(W)\right] dW \geq 0 \tag{B.15}$$

for all $K \geq 0$. Since the distribution $f^{dmw}(W)$ dominates the distribution $f_{smw}(W)$ by second-order stochastic dominance, the Lorenz curve of wealth under a DMW never exceeds the Lorenz curve of wealth under a SMW.□

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C Tables and Figures

Table I: Young Workers' Minimum Wage Differentiation in Selected Countries

Country	Rates for younger employees (age and % of adult minimum)
Australia	Under 18
Belgium	20 (94%) 19 (88%) 18 (82%) 17 (76%) and under 17 (70%)
Chile	Under 18 (80%)
Denmark	Under 18
France	17 and 18 (90%) and under 17 (80%)
Ireland	Under 18
Luxembourg	17 (80%) 16 (70%) and 15 (60%)
Netherlands	22 (85%) 21 (72.5%) 20 (61.5%) 19 (52.5%) 18 (45.5%) 17 (39.5%)
New Zealand	Under 20 (60%)
Portugal	Under 18 (75%)
Spain	Under 18 (89%)
Sweden	Under 25
Turkey	Under 16
UK	Under 22 (85%)

Source: Neumark and Wascher (2003) and OECD Submission to the Irish Minimum Wage Commission (1997)