FINANCIAL RISK CAPACITY

Saki Bigio[1]

Financial crises seem particularly lengthy when banks fail to recapitalize after large losses. I explain this through a model where banks provide intermediation in markets with informational asymmetries. Large equity losses reduce a bank's capacity to bear further losses. Losing this capacity leads to reductions in intermediation and exacerbates adverse selection. Adverse selection, in turn, lowers profits from intermediation which explains the failure to attract equity injections or retain earnings quickly. Financial crises are infrequent events characterized by low economic growth which is overcome only as banks slowly recover by retaining earnings. Several policy interventions are explored.

Keywords: Financial Crisis, Adverse Selection, Capacity Constraints.

1. INTRODUCTION

Often, financial crises originate from episodes of extreme bank losses. Prolonged declines in economic activity follow if banks persistently retract lending after their equity is lost. This suggests that the impact and duration of crises could be mitigated if equity were reallocated into the financial industry. Not surprisingly, the slow recovery of bank equity was a major concern for policy makers, academics, and practitioners during the financial crisis of 2008-2009.[1] In fact, during his only television interview, the Chairman of the Federal Reserve, Ben Bernanke, was asked when he would consider the crisis to be over. He answered, "When banks start raising capital on their own."[2] Why is it then that banks cannot attract capital during crises and prolong their duration?

This paper studies this question through a new model of financial intermediation. I model banks as intermediaries in financial markets that feature asymmetric information.[3] By dealing with a large number of parties, banks dilute transaction risks that follow from asymmetric information. However, banks cannot entirely dilute risk and financial intermediation is risky. The capacity to tolerate

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Columbia Business School, 3022 Broadway, Uris Hall 814, New York, NY 10027, `sbigio@columbia.edu`

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[1] For example, the slow recovery of intermediary capital is the subject of Darrell Duffie's 2010 Presidential Address to the American Finance Association (see Duffie, 2010).

[2] Quote from "The Chairman", *60 Minutes*, CBS News, March 15, 2009.

[3] This is a common view in banking theory (see Freixas and Rochet, 2008). A concrete example is Gorton (2010) who argues that "The essential function of banking is to create a special kind of debt, debt that is immune to adverse selection."

financial intermediation risk, *i.e.,* their *financial risk capacity*, is tied to the banking system's net worth. These features imply that large intermediation losses reduce net worth. If the financial sector can recompose its equity quickly, these initial losses should not have macroeconomic consequences. However, a severe financial crises may occur in the presence of strong adverse selection effects. The intuition is that, after banks lose net worth, they must cut back on lending to decrease their exposure to future potential losses. In response, borrowers use assets of lower quality as collateral. This reduction in credit quality leads to an adverse selection cycle that, ultimately, reduces bank profitability. Without profitable opportunities, the financial system can neither attract fresh equity nor accumulate profits quickly. This represents a new mechanism where the credit quality deteriorates when banks provide little intermediation. The paper studies this mechanism and shows that it operates even if the information structure or the production possibility frontier are invariant to aggregate shocks.

Why is asymmetric information a key friction to explain the slow recovery of the financial sector? Without this friction, competition arguments suggest that banks should attract fresh equity precisely when the economy most needs to rebuild its intermediation capacity. After all, as with any other service, marginal profits from intermediation should be high when the supply of this service is low. High expected profits should either attract equity or at least translate into a rapid recovery of bank equity. Consequently, a theory that links financial intermediation to bank net worth must also explain why banks are not quickly recapitalized after these losses. Asymmetric information breaks this stabilizing force. This distinguishes this model from other models with financial intermediaries.[4] Moreover, the paper shows that in presence of asymmetric information, the economy responds very differently to bank losses of different magnitudes: equity injections and retained earnings are effective stabilizers for moderate equity losses, but fail to occur after large losses.

The model has four main ingredients. (1) The reallocation of resources accross sectors fuels growth. (2) Financial intermediation facilitates the reallocation because of asymmetric information. (3) Net worth evolves over time because intermediation is risky. (4) Financial intermediation is tied to net worth because banks face limited liability. These ingredients open the possibility of infrequent but persistent financial crises. The mechanics of these crises work as follows: A sequence of shocks leads to systematic equity losses in the financial industry. These losses are financed by liquidating a substantial portion of bank net worth. As net worth is depleted, banks can no longer sustain the same magnitude of losses in the future. Thus, banks respond by scaling down their operations, but doing so exacerbates adverse selection. Adverse selection decreases expected bank profits and precludes external recapitalization. Instead, the financial system is only recapitalized through retained earnings. However, the recovery is very slow because volumes of intermediation and marginal profits are too low. The implied lack of intermediation dampens growth.

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[4] In most macroeconomic models with intermediaries, banks cannot raise equity because bankers are fully invested specialists and/or face other agency costs. Those models deliver increases in profits from intermediation after large bank losses. This leads to a fast recovery of the financial sector.

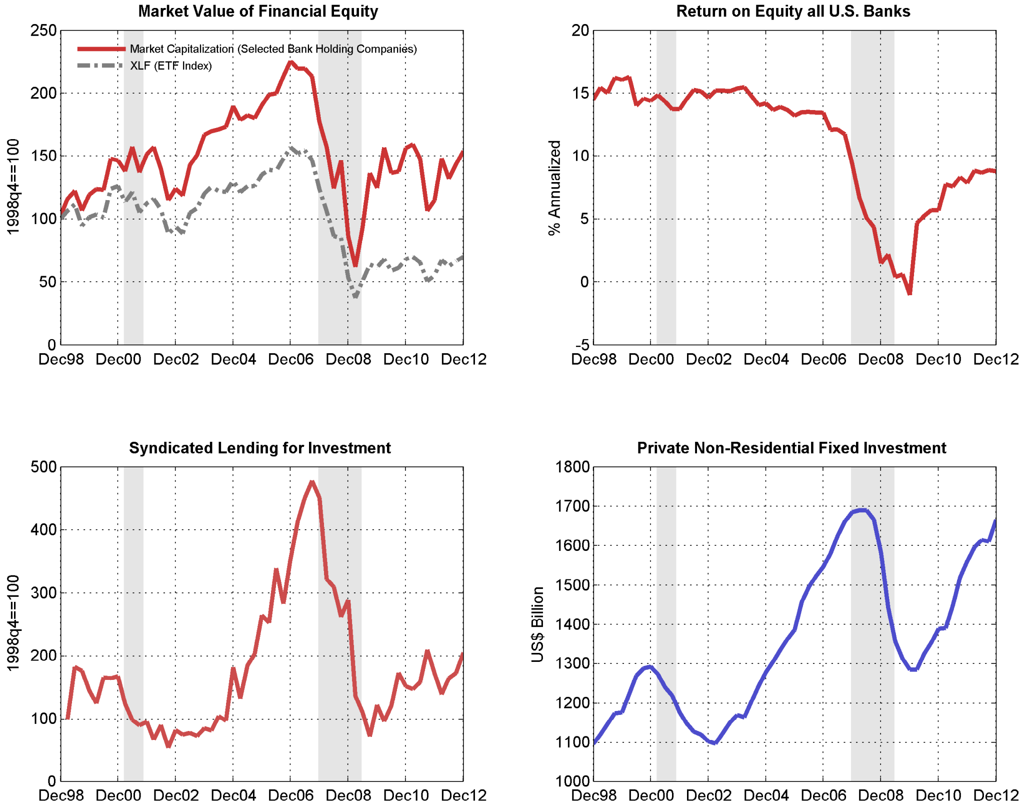

FIGURE 1.— **US Banking Indicators and Investment.**

Figure 1 suggests how these mechanics fit the pattern of the 2008-2009 financial crisis. The top-left panel plots the evolution of the market capitalization of a group of selected US bank holding companies and the value of the XLF index of financial companies during the last decade. The figure shows that these variables began showing a sharp decline two quarters prior to the Great Recession —second gray shaded area. The bottom-left panel presents a measure of financial intermediation, the total flow of syndicated loans with an investment purpose. The pattern closely tracks the behavior of the bank equity capital measures. The top-right panel shows that the bank return on equity (ROE) was stable throughout the decade prior to the crisis. However, bank ROE fell during the recession and remains persistently below its historical average for more than five years. The bottom-right panel describes the decline in nominal private non-residential investment, a symptom of the deceleration in economic activity which echoes the behavior of the other financial time series.[5]

On the technical side, the model features asymmetric information, limited-liability constraints, and aggregate shocks. These ingredients imply that equilibrium objects may feature jumps in the endogenous states. The paper describes a new solution method to obtain global solutions in this class of models.

The next section relates the paper with the literature. Section 3 introduces the model. Section 4 characterizes equilibria. Section 5 presents two analytic examples that underscore the role of asymmetric information. Section 6 presents some numerical examples. Section 7 describes the effects

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[5] See the online data appendix for details.

of several policy experiments. Section 8 concludes. The computational algorithm and proofs are found in the appendix.

2. RELATIONSHIP WITH LITERATURE

The paper is closely related with two areas if research. The first area examines financial intermediation with a focus on bank net worth; the second studies asymmetric information in financial markets. As portrayed in Figure 1, losses of financial net worth seem a potential driving factor of the Great Recession. While the literature explains this connection through several angles, a detailed description would require a more lengthy discussion. However, the common theme in this papers are agency frictions on the side of intermediaries. Agency frictions on the side of non-financial firms were first incorporated into business cycle models by Bernanke and Gertler (1989) and Bernanke et al. (1996). Holmstrom and Tirole (1997) were the first to introduce similar frictions into financial firms but abstracted from business cycle analysis. After the Great Recession, several papers have introduced intermediaries facing these frictions in state-of-the-art business cycle models. Notably, Gertler and Karadi (2011) or Gertler and Kiyotaki (2010) use these models to analyze the benefits of government interventions.

This paper is closely related to Brunnermeier and Sannikov (2011) and He and Krishnamurthy (2009) because intermediary losses here have a non-linear propagation mechanism beyond those found in standard models with financial frictions. The propagation mechanism in those papers follows from fire-sale spirals. Another related paper is Martinez-Miera and Suarez (2011) who incorporate a closer mechanism where contractions in lending also affect the riskiness of loans. As that paper, I also discuss the effects of capital requirements. Another recent paper is Boissay et al. (2013) who study asymmetric information in the interbank market to explain market freezes and consequent declines in lending to non-financial firms.

The novelty here is that financial risk is exacerbated by asymmetric information. This exacerbation formalizes the popular notion that credit quality can deteriorate when banks provide little intermediation. In turn, the deterioration of credit quality reduces the incentives to recapitalize banks during a crises. This differs from other mechanisms prevalent in the literature where agency costs increase as net worth is lost. This feature implies that the value of an additional unit of bank equity is higher during crises because the shadow value of relaxing those constraints is higher. Whenever outside equity injections are possible, those models predict a quick recapitalization of banks and therefore feature a stabilizing force. This distinction is relevant because the return on bank equity fell and remained low after the crisis. Here, in contrast, bankers choose not to inject equity despite free entry because of adverse selection.

Early work by Stiglitz and Weiss (1981) and Myers and Majluf (1984) stressed that asymmetric information in financial markets can cause credit rationing. My paper incorporates their insights into a general-equilibrium framework. For example, Carlstrom and Fuerst (1997) use private information in returns-to-investment to explain credit rationing during the cycle. Eisfeldt (2004) studies a model where assets are sold under asymmetric information for self-insurance. The model is closer to Bigio (2011) because assets here are sold under asymmetric information to relax financial constraints. That paper explains how shocks that exacerbate adverse selection can generate recessions. However, that model does not deliver an internal persistence in adverse selection. Here, in contrast, low bank equity leads to the persistent deterioration of credit markets.

The main contribution of this paper is to present a formal model of the interaction between financial intermediation and asymmetric information. A similar feedback between equity and asymmetric information has been described in the context of liability and catastrophe insurance. Winter (1991) and Gron (1994b,a) survey crises in insurance markets where recurrent and large swings in insurance premia and volumes follow after equity losses in this industry. The connection between this mechanism in insurance markets and credit markets is pointed out by Duffie (2010).

Several modeling choices are taken from different papers. Banks here resemble those in the seminal work of O'Hara (1983). The real side follows directly from Kiyotaki and Moore (2008) and asymmetric information is introduced similarly to Bigio (2011).

3. MODEL

3.1. *Environment*

Time is discrete and the horizon infinite. Every period is divided into two stages, $s \in \{1, 2\}$. There are two goods: consumption goods (the *numeraire*) and capital goods. Consumption is perishable. There are two aggregate shocks: (1) a total-factor productivity (TFP) shock, $A_t \in \{A_1, A_2, ..., A_M\}$, and (2) a shock, $\phi_t \in \Phi \equiv \{\phi_1, \phi_2, ..., \phi_N\}$, that affects the depreciation of capital. (A_t, ϕ_t) form a joint Markov process that evolves according to a transition probability $\chi : (\mathbb{A} \times \Phi) \times (\mathbb{A} \times \Phi) \to [0, 1]$ with the standard assumptions. A_t is realized during the first stage and ϕ_t during the second stage. ϕ_t is the source of intermediation risk.

Notation. I use $y_{t,s}$ to refer to the value of a variable y in period t stage s if the variable changes value between stages. Otherwise, I only use the period subscript.

Demography. There are two populations of agents: producers, and bankers. Each population has a unit mass.

Producers. Producers are identified by some $z \in [0, 1]$ and carry a capital stock $k_t(z)$ as an individual state variable. Producers have log preferences over consumption streams given by:

$$\mathbb{E}\left[\sum_{t \geq 0} \beta^t \log(c_t)\right],$$

where c_t is consumption and β their discount factor.

Production Activities and Technologies. At the beginning of the first stage, producers are randomly segmented into two groups: capital-goods producers (k-producers) and consumption-goods producers (c-producers). Producers become k-producers with probability π independent of time and

z.[6]

C-producers operate a linear technology during the first stage that produces $A_t k_t(z)$ units of consumption. C-producers lack the possibility of augmenting their capital stock directly. In contrast, k-producers have access to an investment technology that transforms one unit of new capital with one unit of consumption. This investment technology is operated during the second stage. Capital held by k-producers cannot be operated so they do not have any inputs for their production. However, their capital can be sold to obtain consumption goods.

Economic Problem. Activity segmentation induces a need for trade. On one hand, k-producers need consumption goods to operate their investment technologies. C-producers, on the other hand, produce those resources —consumption goods— but lack access to the investment technology. The fundamental economic problem here is that consumption goods must be transferred from c-producers to k-producers and capital must be reallocated in the opposite direction. This reallocation fuels growth —this structure is the first main ingredient of the model.

Capital Units. Capital is homogeneous at the beginning of the period. During the first stage, the capital of every producer is divided into a continuum of units. Each unit is identified by some $\omega \in [0,1]$ and can be reallocated individually. The composition of ω-units is the same for all entrepreneurs.

The depreciation rate of each unit is determined by ω and the realization of ϕ_t through the function $\lambda : [0,1] \times \Phi \to \mathbb{R}_+$. In particular, $\lambda(\omega, \phi)$ is the capital that remains out of an ω-unit given the realization of ϕ. Once an $\omega-$unit is scaled by $\lambda(\omega, \phi)$, it becomes homogeneous *t+1* capital by being merged with other pieces. The following period, the capital stock held by producers is divided again into different $\omega's$ that depreciate depending on the t+1 realization of ϕ. The process is repeated indefinitely.

By the end of the second stage, the capital that remains in the economy for t+1 production out of an original t-period stock $k_t(z)$ held by a given producer is $k_t(z) \int_0^1 \lambda(\omega, \phi_t)\, d\omega$. However, this will not equal the producer's capital stock tomorrow because producers may sell an $\omega-$unit individually. This decision is captured by the indicator function $\mathbb{I}(\omega) : [0,1] \to \{0,1\}$. $\mathbb{I}(\omega)$ takes a value of 1 when ω is sold. Thus, when choosing $\mathbb{I}(\omega)$, the producer sells $k(z) \int_0^1 \mathbb{I}(\omega)\, d\omega$. Consequently, the $t+1$ capital that remains with the producer is $k(z) \int_0^1 [1 - \mathbb{I}(\omega)] \lambda(\omega, \phi_t)\, d\omega$. By assumption, the sales take place before the realization of ϕ.

Taking into consideration investments and possible purchases, the producer's capital stock evolves according to:

$$k_{t+1}(z) = i + k^b + k_t(z) \int_0^1 [1 - \mathbb{I}(\omega)] \lambda(\omega, \phi_t)\, d\omega. \tag{1}$$

In this expression, i is t+1 capital created by investing —when possible— and k^b are purchases of *t+1* capital.

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[6] Randomization across activities is convenient to reduce the dimension of the state space of the model.

There is an assumption that provides an interpretation to ω and ϕ. Given ϕ, the average quality under a certain quality ω^* is

$$\mathbb{E}\left[\lambda\left(\omega,\phi\right)|\omega\leq\omega^*\right]\equiv\frac{\int_0^{\omega^*}\lambda\left(\omega,\phi\right)d\omega}{\omega^*}.$$

Assumption 1 *$\lambda\left(\omega,\phi\right)$ satisfies the following: (i) $\lambda(\omega,\phi)$ is increasing in ω. (ii) $\mathbb{E}\left[\lambda\left(\omega,\phi\right)|\omega<\omega^*\right]$ is weakly decreasing in ϕ for any ω^*.*

The first condition implies that lower ω depreciate faster —and are therefore worse qualities. The second condition states that the average quality under some cutoff ω^* is lower for a larger ϕ.[7] I denote by $\bar{\lambda}\left(\phi\right)\equiv\mathbb{E}[\lambda(\omega,\phi)|\omega\leq1]$ the unconditional quality average for any ϕ.

Private Information. A quality ω is only known by its owner. Thus, buyers can only observe the quantity of a pool of sold units, $k\int_0^1\mathbb{I}\left(\omega\right)d\omega$, but cannot discern the composition of ω's within that pool. After ϕ is realized, the *t+1* capital that remains from that pool is $k\int_0^1\mathbb{I}\left(\omega\right)\lambda\left(\omega,\phi_t\right)d\omega$. With this, the effective depreciation of that pool is unknown during the first stage, both because of the composition of ω and the realization of ϕ. Private information about ω is the source of asymmetric information in the model. The uncertainty about ϕ is the source of risk for the buyer. These features are the second and third main ingredients of the model.

Bankers. The reallocation of resources is intermediated through bankers. Bankers are identified by some $j\in[0,1]$. They have linear preferences given by:

$$\mathbb{E}\left[\sum_{t\geq0}(\beta^f)^t c_t\right],$$

where c_t is their consumption and β^f their discount factor. Bankers own legal institutions called banks. Bankers have two sources of wealth. They enter the period with $n_{t,1}(j)$ consumption goods held as equity in their banks. In addition, they have access to an exogenous endowment of consumption goods $\bar{e}_t(j)$ every period as their personal wealth. Although $\bar{e}$ and n are both consumption goods, they have an important legal distinction: the banker's personal endowment is protected by limited liability, whereas his equity is liable to intermediation losses. However, bankers can alter the composition of their wealth through equity injections that add to their banks' equity.

In particular, a banker can choose an amount from his endowment, $e\in\left[0,\bar{e}\right]$, to be his equity injections to his bank. These injections come at the expense of decreasing consumption. He can do the opposite, reducing his bank's net worth by transferring $d\in[0,n]$ consumption units as dividends to be consumed after dividend taxes τ. One can replace this convexity by introducing smooth-convex equity injection costs.[8] Thus, the banker's consumption is $c=\left(\bar{e}-e\right)+\left(1-\tau\right)d$.

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[7] The assumption applies the definition of first-order-dominance to the conditional expectation function.

[8] A distinction between costly equity injections and dividend taxes is common in the dynamic corporate finance literature. See examples, Hennessy and Whited (2005) and Palazzo (2010), among others. In this environment, only the ratio of the cost of equity and dividend taxes matters, so I normalize the tax rate to account for this differences.

His bank's net worth is transformed instantaneously according to $n_{t,1} = n_{t-1,2} + e_{t,1} - d_{t,1}$. Bankers also have access to storage.

Bankers face an exogenous constant probability of exit, ρ. When, $\rho = 0$, they face infinite-horizon problems. When $\rho = 1$, bankers live one period and the model can be solved analytically. Upon exit, bankers sell their bank to an entrant banker. Most of the analysis is carried out with long-lived bankers, but I set $\rho = 1$ when I discuss some analytic examples.

Financial Intermediation. Bankers provide intermediation via the purchase of capital from k-producers. This purchase occurs during the first stage. Then, capital is sold to c-producers during the second stage. Banks buy capital from k-producers issuing them tradeable and riskless IOUs.[9] Those IOUs entitle the holder to a unit of consumption in the second stage. IOUs are redeemed by the end of the period and therefore bear no interest. K-producers use these IOUs to buy consumption goods from c-producers.

Bankers buy capital units from k-producers under asymmetric information. After ϕ is realized, units are depreciated accordingly. The remaining pool of capital is resold as homogeneous *t+1* capital. Since ϕ_t is realized between stages, depreciation is random and therefore a source of intermediation risk. Note that bankers fund themselves with riskless liabilities and thus bear all the financial risk. Bankers experience equity losses when their initial issuance of IOUs exceeds the market value of the capital pool.

When a banker experiences losses, he is forced to draw funds from his bank's net worth. In principle, financial losses could be financed with the banker's personal wealth, but limited liability protects his personal wealth. If IOUs are risk free, this imposes a limited-liability constraint (LLC) on the amount of intermediation. In particular, losses from financial intermediation cannot exceed the bank's net worth under any contingency.[10] Consequently, the bank's net worth restrains the amount of capital that bankers can buy. The greater the amount of capital bought, the greater the risk and the greater the need for net worth as a loss cushion. Since bankers can choose to inject equity from their personal endowment, they can relax that constraint at will. In equilibrium, however, they must have the incentives to do so.

Aggregate States. There are two endogenous aggregate states: the aggregate capital stock, $K_t = \int_0^1 k_t(z)\, dz$, and, $N_t = \int_0^1 n_t(j)\, dj$, the equity of the financial system. I will show that it is only necessary to keep track of the ratio $\kappa_{t,s} \equiv N_{t,s}/K_t$. Since κ determines the economy's capacity to bear losses relative to the wealth of producers, I refer to it as the *financial-risk capacity*. The aggregate state of this economy is summarized by $X_{t,1} = \{A_t, \phi_{t-1}, \kappa_{t,1}\} \in \mathbb{X} \equiv \mathbb{A} \times \Phi \times \mathbb{K}$ and $X_{t,2} = \{A_t, \phi_t, \kappa_{t,2}\} \in \mathbb{X} \equiv \mathbb{A} \times \Phi \times \mathbb{K}$.

Public Information. At every point, $X_{t,s}$ is common knowledge. For tractability, I assume that the producer's type is known. This ensures that, in equilibrium, c-producers are excluded from selling capital. In addition, bankers are informed about their exit at the beginning of the second

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[9] Thus, IOUs are like credit lines (or deposits) used as a medium of exchange.

[10] Equivalently, this a *solvency* or *non-default* constraint.

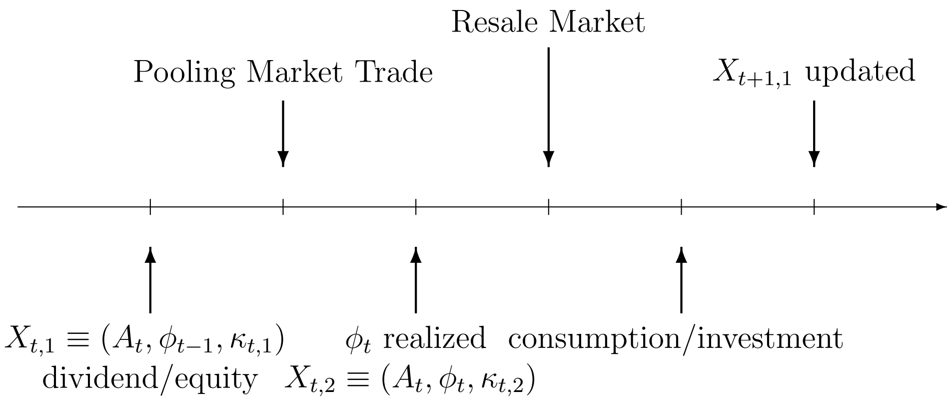

FIGURE 2.— **Timing**

stage.

Markets. There are only two possible markets for capital. The first market is where capital units are sold by k-producers and bought by banks under asymmetric information. This market opens during the first stage and satisfies the following assumption:

Assumption 2 *Capital markets are* ***anonymous*** *and* ***non-exclusive****. Banks are* ***competitive.***

Assumption 2 implies that the first market is a pooling market with a unique pooling price p_t.[11] I refer to this market as the pooling market. The assumption that banks are competitive means that they take prices and qualities as given.[12] The second market opens during the second stage. In this market, bankers sell back all the units purchased during the first stage. This market clears at a price q_t. I refer to it as the resale market.

Timing. The timing summary is as follows: At the beginning of the period, A_t is realized and observed. Then, consumption goods are produced by c-producers. Bankers decide between equity injections and dividend payouts. Bankers then buy capital from k-producers who select which qualities to sell. During the second stage, ϕ_t is realized and $X_{t,2}$ is updated. Bankers learn the average quality of the capital bought and resell the pool as homogeneous *t+1* capital. By the end of the period, bankers settle their claims and realize profits or losses. Producers choose consumption and savings.

The timing of the model is summarized by Figure 2. This economy has a recursive representation, so from now on, I drop time subscripts. I use x' to denote the value of a variable x in the subsequent

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[11] Without anonymity bankers could offer different prices depending on the volume of capital sold. With exclusivity, bankers could use dynamic incentives to screen.

[12] In principle, if intermediaries could commit to random purchase, they could alleviate the lemons problem. Commitment issues aside, this solution is not possible in presence of transaction costs for example. In a related environment, Guerrieri and Shimer (2011) allows agents to trade in multiple markets for which capital is exchanged probabilistically. Differences in probability and prices allow for separation but do not resolve asymmetric information entirely. Bigio (2011) allows for repurchase contracts. Repurchase contracts are another example of contracts that improve allocations but do not alter the essence of the results.

stage. I denote by n the bank's equity at the beginning of the first stage, by n' the equity after equity injections and dividends, and by n'' the equity after profits/losses. The same notation is used for κ.

3.2. *First-Stage Problems*

K-producer's First-Stage Problem. During the first stage, a *k-producer* enters the period with a capital k. At this stage, he decides which qualities to sell:

Problem 1 (*k-producer's s=1 problem*) *The k-producer's first stage problem is:*

$$V_1^k(k,X)=\max_{\mathbb{I}(\omega)\in\{0,1\}}\mathbb{E}\left[V_2^k\left(k'\left(\phi'\right),x,X'\right)|X\right]$$

$$\text{subject to } x=pk\int_0^1\mathbb{I}(\omega)\,d\omega \text{ and } k'\left(\phi'\right)=k\int_0^1\left[1-\mathbb{I}(\omega)\right]\lambda\left(\omega,\phi'\right)d\omega.$$

The first equation is the k-producer's budget constraint. x are the IOUs —consumption goods— carried to the second stage obtained selling $k\int_0^1\mathbb{I}(\omega)\,d\omega$ at a price p. The second equation subtracts the sold units from the original stock.

C-producer's First-Stage Problem. Since c-producers are excluded from the pooling market, they take no actions in this stage. Their value function is the expected value of their second stage's value function.

Problem 2 (*c-producer's s=1 problem*) *The c-producer's first stage value function:*

$$V_1^c(k,X)=\mathbb{E}\left[V_2^c\left(k'\left(\phi'\right),x,X'\right)|X\right]$$

$$\text{where } x=Ak \text{ and } k'\left(\phi'\right)=k\int_0^1\lambda\left(\omega,\phi'\right)d\omega.$$

Banker's First-Stage Problem. The choices of bankers during the first stage are equity injections and dividends. These choices , bankers alter their banks' equity from n to n'. Simultaneously, they also choose, Q, the volume of capital purchased in the pooling market. Thus, he issues IOUs for pQ.

During the second stage, the value capital purchased becomes $q\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|X\right]Q$. The resale market trades at a resale price q and the Q have an average depreciation of $\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|X\right]$. Under LLC, issued IOUs cannot exceed the bank's net worth plus the value of this capital for any realization of ϕ':

$$pQ\leq q\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|X\right]Q+n' \text{ for any } \left(X,X'\right)\in\mathbb{X}\times\mathbb{X}.$$

Let $\Pi\left(X,X'\right)\equiv q\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|X\right]-p$ be the banker's marginal profit. The banker's problem is

Problem 3 *The banker's first-stage problem is*

$$V_1^f(n,X) = \max_{Q, e\in[0,\bar{e}], d\in[0,n]} c + \mathbb{E}\left[V_2^f\left(n' + \Pi(X,X')Q, X'\right)|X\right]$$

$$\begin{aligned} \text{subject to } -\Pi(X,X')Q &\leq n',\ \forall X' \\ c &= (\bar{e}-e) + (1-\tau)d \\ n' &= n+e-d. \end{aligned} \tag{2}$$

The first constraint is the LLC. The second and third constraints are the banker's budget constraints and the law of motion of n.

3.3. *Second-Stage Problems*

K-producer's Second-Stage Problem. During the first stage, k-producers have sold part of their capital for x in IOUs. They bring x and their remaining capital stock to the second stage. They solve:

Problem 4 (*k-producer's s=2 problem*) *The k-producer's problem in the second stage is:*

$$V_2^k(k,x,X) = \max_{c\geq 0, i, k^b\geq 0} \log(c) + \beta\mathbb{E}\left[V_1^k(k',X')|X\right],\ j\in\{i,p\}$$

$$\text{subject to } c + i + qk^b = x \text{ and } k' = k^b + i + k.$$

This budget constraint says that the k-producer uses x to consume c, invest i, or purchase k^b. The capital accumulation equation is consistent with equation (1) since depreciation is accounted the previous stage.

C-producer's Second-Stage Problem. The c-producer's problem is identical to the k-producer's, except when the c-producer is restricted to set $i \leq 0$ because he lacks the investment technology.

Problem 5 (*c-producer's s=2 problem*) *The c-producer's problem at the second stage is:*

$$V_2^c(k,x,X) = \max_{c\geq 0, i\leq 0, k^b\geq 0} \log(c) + \beta\mathbb{E}\left[V_1^j(k',X')|X\right],\ j\in\{i,p\}$$

$$\text{subject to } c + i + qk^b = x \text{ and } k' = k^b + i + k$$

Banker's Second-Stage Problem. Bankers don't take actions during the second stage, but rather, only realize profits or losses. Thus, their value is $V_2^f(n'',X) = \beta^F\mathbb{E}\left[V_1^f(R^b n'', X')|X\right]$ if they remain in the industry or $V_2^f = (1-\tau)\beta^F R^b n''$ if they exit. Here, R^b is the return on storage.

3.4. *Market-Clearing Conditions and Equilibrium*

Notation. I append terms like ${}^j(k,X)$ to variables that indicate the policy function of a producer of type j in state (k,X). I use $\mathbb{I}(\omega,k,X)$ to refer to a k-producer's decision to sell an $\omega-$quality when his state is (k, X).

Aggregation. In every period and stage, there are measures over capital holdings across the population of k- and c-producers. I denote these measures by Γ^k and Γ^c respectively. By independence, these satisfy:

$$\int_0^\infty \Gamma^k(dk) = \pi K \quad \text{and} \int_0^\infty \Gamma^c(dk) = (1-\pi) K. \tag{3}$$

Their evolution is consistent with individual decisions and the segmentation of activities. In addition, there is also a measure Λ over the bankers' net worth.

First Stage. Market clearing during the first stage requires the demand for capital by bankers to equal the supply of capital by k-producers. This condition is:

$$\int_0^\infty Q(n, X)\Lambda(dn) = \int_0^\infty k \int_0^1 \mathbb{I}(\omega, k, X)\, d\omega \Gamma^k(dk)$$

Second Stage. The demands for following-period capital by c- and k-producers are:

$$D^c(X, X') \equiv \int_0^\infty k^{b,c}\left(x(k, X), k\int_0^1 \lambda(\omega, \phi')\, d\omega, X'\right)\Gamma^c(dk)$$

and

$$D^k(X, X') \equiv \int_0^\infty k^{b,k}\left(x(k, X), k\int_0^1 [1-\mathbb{I}(\omega, k, X)]\lambda(\omega, \phi')\, d\omega, X'\right)\Gamma^k(dk).$$

Bankers sell all the units bought. Their supply of capital is:

$$S(X, X') \equiv \mathbb{E}[\lambda(\omega, \phi')|X]\int_0^\infty Q(n, X)\Lambda(dn).$$

The market-clearing condition in the second stage is $S(X, X') = D^c(X, X') + D^k(X, X')$. A recursive competitive equilibrium is:

Definition 1 (Recursive Competitive Equilibrium) *A recursive competitive equilibrium (RCE) is (1), a set of price functions,* $\{q(X, X'), p(X)\}$*; (2) a set of policy functions for c-producers* $c^c(x, k, X)$, $k^{b,c}(x, k, X)$, $i^c(x, k, X)$, *a set of policy functions for k-producers* $c^k(x, k, X)$, $k^{b,k}(x, k, X)$, $\mathbb{I}^k(\omega, k, X)$, *a set of policy functions for bankers* $Q(n, X)$, $e(n, X)$, $d(n, X)$*; (3) sets of value functions,* $\{V_1^l(k, X), V_2^l(x, k, X)\}_{l=c,k}$, $\{V_s^f(n, X)\}_{s=1,2}$*; and (4), a law of motion for the aggregate state X, such that for any measures* Γ^c, Γ^k *and* Λ *satisfying the consistency condition (3) the following hold: (I) The producers' policy functions are solutions to their problems taking* $q(X, X')$, $p(X)$ *and the law of motion for X as given. (II)* $Q(n, X)$, $e(n.X)$, $d(n.X)$ *are solutions to the banker's problem, taking* $q(X, X')$, $p(X)$, *and the law of motion for X as given. (III) Capital markets clear during the first and second stages. (IV) The law of motion X is consistent with policy* *functions and the transition function χ. All expectations are consistent with the law of motion for X and agent policies.*

This definition does not depend on the measures of asset holdings because this economy admits aggregation. This is shown in the following section.

4. CHARACTERIZATION

4.1. *Policy Functions*

Producers' Second-Stage Policies. As a result of log-preferences, the c-producer's policy functions are linear in his wealth $W^c(k,x,X) \equiv \left(A + q\bar{\lambda}(\phi')\right)k$. His wealth is the sum of his output and the value of his capital.

Proposition 1 *In any RCE, the c-producer's policy functions are* $k^{c,\prime}(k,x,X) = \beta\frac{W^c(k,x,X)}{q(X,X')}$ *and* $c^c(k,x,X) = (1-\beta)W^c(k,x,X)$. *His value function is* $V_2^c(k,x,X) = \psi^c(X) + \log(W^c(k,x,X))$, *where* $\psi^c(X)$ *is a function of the aggregate state.*

Policy functions for k-producers are also linear in their wealth. Their wealth is $W^k(k,x,X) \equiv (x + q^i\mathbb{E}[\lambda(\omega,\phi')|\omega > \omega^*(X)])k$. The k-producer's wealth is the sum of the x IOUs —consumption goods— obtained by selling capital and the replacement value of his unsold capital. The average depreciation of unsold units is $\mathbb{E}[\lambda(\omega,\phi')|\omega > \omega^*(X)]$. This is the average depreciation under a cutoff quality $\omega^*(X)$. The following section shows that his sales decision is characterized by a threshold quality. The replacement cost of capital for k-producers is $q^i(X,X') = \min\{1, q(X,X')\}$, the minimum between the market value of capital and the production cost. Their policy functions are:

Proposition 2 *In any RCE, the k-producer's policy functions are* $k^{k,\prime}(k,x,X) = \beta\frac{W^k(k,x,X)}{q^i(X,X')}$ *and* $c^k(k,x,X) = (1-\beta)W^k(k,x,X)$. *His value function is of the form* $V_2^k(k,x,X) = \psi^k(X) + \log\left(W^k(k,x,X)\right)$, *where* $\psi^k(X)$ *is a function of the aggregate state.*

Producers' First-Stage Policies. Proposition 2 leads to a closed-form expression for the k-producer's value function during the first stage. Replacing the definitions of x and $\mathbb{E}[\lambda(\omega,\phi')|\omega < \omega^*(X)]$, their value function is:

$$\begin{aligned} V_1^k(k,X) &= \max_{\mathbb{I}(\omega)\in\{0,1\}} \mathbb{E}\left[\log\left(p(X)\int_0^1 \mathbb{I}(\omega)\,d\omega \; + q^i(X,X')\int_0^1 \lambda(\omega,\phi')\left[1-\mathbb{I}(\omega)\right]d\omega\right)|X\right] \\ &\quad + \psi^k(X) + \log(k). \end{aligned}$$

It is from this expression that quality-sales decisions have to be the same across entrepreneurs regardless of their wealth. These decisions are characterized by a portfolio problem:

Proposition 3 *In any RCE, the k-producer's policy function in the first stage is given by,* $\mathbb{I}^*(\omega,k,X) = 1$ *if* $\omega < \omega^*$ *and 0 otherwise. The cutoff quality is:*

$$\omega^* = \arg\max_{\tilde{\omega}} \mathbb{E}\left[\log\left(p\tilde{\omega} \; + q^i(X,X')\int_{\tilde{\omega}}^1 \lambda(\omega,\phi')\,d\omega\right)|X\right]. \tag{4}$$

Moreover, ω^* *is increasing in* p.

Proposition 3 shows that the solution to the producer's problem during the first stage is given by a unique cutoff quality. This outcome resembles the solution to the lemons problem of Akerlof (1970), but there is a distinction; here, ω^* is chosen by solving a portfolio problem rather than obtained by a deterministic condition. The reason is that the producer does not know the outcome of ϕ.[13] Since ω^* is increasing in $p(X)$, the supply of capital is upward sloping. Consequently, the cutoff quality, $\omega^*(X)$, indicates both the highest quality of capital traded and the volume of intermediation. From now on, I use threshold quality and volume of intermediation to refer to ω^* interchangeably.

Bankers' policies. At the beginning of every period, bankers choose e, d and Q to maximize expected profits. The following Proposition shows that their value function and policies are linear in net worth:

Proposition 4 *The banker's value functions are* $V_1^f(n,X) = v_1^f(X)\,n$ *and* $V_2^f(n,X) = v_2^f(X)\,n$, *where* $v_1^f(X)$ *and* $v_2^f(X)$ *are the marginal value of financial equity in stages 1 and 2, respectively.* $v_1^f(X)$ *solves the following Bellman equation:*

$$v_1^f(X) = \max_{Q\geq 0, e\in[0,\bar{e}], d\in[0,1]} (1-\tau)\,d - e + \mathbb{E}\left[v_2^f(X')\left(\Pi\left(X,X'\right)Q + n'\right)|X\right] \tag{5}$$

subject to,

$$\begin{aligned} -\Pi\left(X,X'\right)Q &\leq n', \forall X' \\ n' &= 1+e-d. \end{aligned}$$

The Bellman equation (5) is solved $e(X)$, $d(X)$ *and* $Q(X)$ *such that:*

$$e(n,X) = e^*(X)\,n,\ \ d(n,X) = d^*(X)\,n,\ \ Q(n,X) = Q^*(X)\left(1+e^*(X)-d^*(X)\right)n.$$

In addition, $v_2^f(X) = \beta^F R^b$ *if the banker exits, and* $v_2^f(X) = \beta^F R^b\mathbb{E}\left[v_1^f(X)\right]$ *otherwise.*

The value function of the banker is linear in his net worth because of risk-neutrality and the linearity of the LLC. In equilibrium, there may be multiple solutions to $d(n,X)$ and $e(n,X)$. Without loss of generality, I restrict attention to linear policies.

Proposition 5 $Q^*(X)$ *is given by,*

$$Q^*(X) = \arg\max_{\tilde{Q}} \mathbb{E}\left[v_2^f(X')\,\Pi(X,X')\,|X\right]\tilde{Q} \text{ subject to } \Pi\left(X,X'\right)\tilde{Q} \leq 1, \forall X'.$$

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[13] This portfolio problem has an intuitive interpretation: ω^* is the fraction of the producer's capital stock sold to banks. Once he exchanges these units, the producer loads the depreciation risk to the bank. In doing so, ω^* becomes the risk-less portion of his portfolio. The remaining fraction, $(1-\omega^*)$, is risky because ϕ' is realized after ω^* is chosen.

In equilibrium, $\min_{\tilde{X}'}\Pi\left(X,\tilde{X}'\right)<0$, *and* $\{e^{*}(X),d^{*}(X)\}$, *satisfy:*

$$\begin{aligned}
e^{*}(X) &> 0 \text{ only if } \left[\mathbb{E}[v_{2}^{f}(X')]+\max\left\{\frac{\mathbb{E}[v_{2}^{f}(X')\Pi(X,X')]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)},0\right\}\right]\geq 1\\
d^{*}(X) &> 0 \text{ only if } \left[\mathbb{E}[v_{2}^{f}(X')]+\max\left\{\frac{\mathbb{E}[v_{2}^{f}(X')\Pi(X,X')]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)},0\right\}\right]\leq(1-\tau).
\end{aligned}$$

When the inequalities are strict, $e=\bar{e}$ *and* $d=1$. $e^{*}(X)$ *and* $d^{*}(X)$ *are indeterminate at the individual level when the relations hold with equality.* $e^{*}(X)$ *and* $d^{*}(X)$ *equal 0 when the inequalities are violated.*

This proposition states that the LLC is binding whenever the expected discounted value of additional intermediation is positive, i.e., when $\mathbb{E}\left[v_{2}^{f}(X')\Pi(X,X')|X\right]>0$. This term is the product of marginal profits, Π, and the marginal value of bank equity v_{2}^{f}. When, $\mathbb{E}\left[v_{2}^{f}(X')\Pi(X,X')|X\right]=0$, Q is indeterminate and 0 when $\mathbb{E}\left[v_{2}^{f}(X')\Pi(X,X')|X\right]<0$.

The proposition also states conditions for capital injections and dividend payoffs. These policies depend on the marginal value of keeping equity within the bank:

$$\tilde{v}(X)\equiv\beta^{F}\left[\mathbb{E}[v_{2}^{f}(X')]+\max\left\{\frac{\mathbb{E}[v_{2}^{f}(X')\Pi(X,X')]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)},0\right\}\right]. \tag{6}$$

$\tilde{v}(X)$ has an intuitive interpretation. β^{F} is the banker's discount factor. The first term inside the bracket, $\mathbb{E}[v_{2}^{f}(X')]$, is the future marginal value of one unit of net worth —multiplied by β^{F} yields the bank's stochastic discount factor. The second term represents the shadow value of relaxing the LLC. Recall that the inverse of the worst-case scenario losses, $-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)$, is the bank's marginal leverage. Increasing a unit of equity allows the bank to issue $-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)$ additional IOUs —and purchase more capital. These additional units of capital yield an expected marginal value of $\mathbb{E}[v_{2}^{f}(X')\Pi(X,X')]$. The *max* operator sets this value to 0 when there are no gains from additional intermediation.

When $\tilde{v}(X)<(1-\tau)$, the banker prefers to pay dividends: the marginal value of equity is $\tilde{v}(X)$, but the after-tax marginal benefit of dividends is greater. In contrast, the banker injects equity into his bank when the value of holding equity exceeds one —the opportunity cost of equity in terms of foregone consumption. This implies that bankers have (S,s)-bands for their dividend policies.

The following section shows that in absence of asymmetric information, $\tilde{v}(X)$ is, in fact, monotone decreasing in κ. However, if asymmetric information is sufficiently severe, $\tilde{v}(X)$ becomes non-monotonic and multiple inaction regions emerge. This explains periods where the financial sector is not recapitalized or grows slowly when adverse selection is severe.

4.2. *Market Prices and Bank Profits*

Resale Market Price Function. The linearity of policy functions allows aggregation. Integrating across c-producers' capital stock yields their aggregate demand for following-period capital:

$$D^c(X,X') = \left[\frac{\beta\left(A + q(X,X')\bar{\lambda}(\phi')\right)}{q(X,X')} - \bar{\lambda}(\phi')\right](1-\pi)K.$$

The demand for capital by k-producers' is obtained similarly. In this case, their aggregate demand is broken up into three tranches depending on q —which determines whether they buy capital or produce it:[14]

$$D^k(X,X') = \begin{cases} \beta\frac{p(X)\omega^*(X)}{q(X,X')} & \text{if} \quad q(X,X') < 1 \\ [0, \beta p(X)\omega^*(X)\pi K] & \text{if} \quad q(X,X') = 1 \\ 0 & \text{if} \quad q(X,X') > 1. \end{cases}$$

Aggregate investment is the difference between the k-producer's desired capital holdings and their demand for units sold by banks:[15]

$$I(X,X') = \beta\frac{p(X)\omega^*(X)}{q(X,X')}\pi K - D^k(X,X').$$

Capital supplied by bankers during the second stage, $S(X,X')$, are those units sold by k-producers during the first stage scaled by their average quality:

$$S(X,X') = \mathbb{E}\left[\lambda(\omega,\phi')\,|\omega < \omega^*(X)\right]\omega^*(X)\pi K.$$

$q(X,X')$ is obtained by clearing out the price from the second-stage, market-clearing equation:[16]

Proposition 6 *In equilibrium $q(X,X')$ is given by,*

$$q(X,X') = \max\left\{\left[\frac{\beta A}{\pi\omega^*(X)\mathbb{E}\left[\lambda(\omega,\phi')\,|\omega<\omega^*(X)\right] + (1-\pi)(1-\beta)\bar{\lambda}(\phi')}\right], g(p(X), X')\right\} \tag{7}$$

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[14] The first tranche is downwards sloping when $q < 1$. This is a region for values of q in which k-producers find it cheaper to buy capital than to produce it. The second region is a flat demand at $q = 1$ since k-producers are indifferent between investing or buying capital. Otherwise, k-producers do not participate in the market when $q > 1$ because it is less costly to produce capital directly.

[15] From the expression, it is clear that investment is 0 when $q < 1$. When $q = 1$, $D^k(X,X')$ is obtained as the residual of the difference between the supply of used units minus purchases by k-producers.

[16] $q(X,X')$ depends on X because the supply of capital is a function of $\omega^*(X)$ which is decided in the first stage. It also depends on X' because the realization of ϕ' determines the effective supply of capital.

where

$$g\left(p\left(X\right),X'\right)\equiv\min\left\{1,\frac{\beta\left(\pi p\left(X\right)\omega^{*}\left(X\right)+A\left(1-\pi\right)\right)}{\left(\beta\omega^{*}\left(X\right)\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega<\omega^{*}\left(X\right)\right]\pi+\left(1-\beta\right)\bar{\lambda}\left(\phi'\right)\right)}\right\}.$$

There are two things worth mentioning about $q\left(X,X'\right)$. First, it is immediate to show that it is decreasing in $\omega^{*}\left(X\right)$. This is natural since larger cutoffs lead to more capital supplied by banks —and capital is a normal good. Second, the price is increasing in ϕ : the shock lowers the average quality of capital for any possible cutoff which is equivalent to lowering the fixed supply of an asset.

Pooling Market Price Function. In equilibrium, market clearing in the first stage requires:

$$\underbrace{Q^{*}\left(X\right)\int_{0}^{\infty}n'\left(n,X\right)d\Lambda\left(n\right)}_{\text{Demand by Bankers}}=\underbrace{\omega^{*}\left(X\right)\int_{0}^{\infty}k\Gamma^{k}\left(dk\right)}_{\text{Supply by k-producers}}\Longleftrightarrow Q^{*}\left(X\right)\kappa=\omega^{*}\left(X\right).$$

Note that $p(X)$ determines $\omega^{*}\left(X\right)$ through the k-producer's portfolio problem independent of the capital stock. This is why the only endogenous state is κ.

Multiplicity. As commonly found in models that feature asymmetric information, there is a potential multiplicity of equilibrium prices given a financial risk capacity κ. During the first stage, there could be two or more equilibrium triplets $\left(\omega,Q,\,p\left(X\right),q\left(X,X'\right)\right)$ that satisfy market clearing and limited liability for the same value of κ. As prices increase, both the average quality and the quantity of capital traded increase. As a consequence, bank profits are possibly non-monotone in ω. Hence, worst-case profits for two different prices may be the same supporting both equilibria. Although multiplicity is an interesting phenomenon, it is not the focus of this paper. For the rest of the paper, I restrict attention to highest-price equilibria. That is, if there is more than one price $p\left(X\right)$ that can be supported by the same κ, I select the highest-price p. This selection implies that the pooling price is increasing as the financial risk capacity is higher. I describe an algorithm to compute highest-price equilibria below.

Bank Profits. By definition, equilibrium marginal profits are:

$$\Pi\left(X,X'\right)=\begin{cases}\frac{\beta(\pi p(X)\omega^{*}(X)+A(1-\pi))}{\beta\omega^{*}(X)\pi+(1-\beta)\frac{\bar{\lambda}(\phi')}{\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega<\omega^{*}(X)\right]}}-p\left(\omega^{*}\left(X\right)\right) & \text{if}\quad q\left(X,X'\right)<1\\ \mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega<\omega^{*}\left(X\right)\right]\ -p\left(\omega^{*}\left(X\right)\right) & \text{if}\quad q\left(X,X'\right)=1\\ \frac{\beta A}{\pi\omega^{*}(X)+(1-\pi)(1-\beta)\frac{\bar{\lambda}(\phi')}{\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega<\omega^{*}(X)\right]}}-p\left(\omega^{*}\left(X\right)\right) & \text{if}\quad q\left(X,X'\right)>1\end{cases}.\tag{8}$$

The behavior of Π is the heart of the model. Recall that $p\left(X\right)$ is determined prior to the realization of ϕ'. Hence, this price does not respond to ϕ'. One can observe that $\Pi\left(X,X'\right)$ is decreasing in ϕ' because $\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega<\omega^{*}\left(X\right)\right]$ is decreasing in ϕ'. Thus, ϕ' affects marginal profits along two margins. On the quantity margin, the implied depreciation implies that ϕ' lowers the supplied quantity of capital. On the price margin, $q\left(X,X'\right)$ increases due to the reduction in supply. The expressions above show that the first effect always dominates.[17] Thus, profits are decreasing in ϕ'.

In turn, $\Pi(X, X')$ is not necessarily monotone in κ. This is the main feature that determines the evolution of κ and, consequently, the dynamics of this economy.

Why are marginal intermediation profits non-monotone in κ? As noted above, $p(X)$ is increasing in κ. Hence, non-monotonicity in profits must follow from the value of capital. Two effects oppose each other. The first effect is a substitution effect: since capital is a normal good, the greater the volume of capital supplied, the lower $q(X, X')$. The second is a composition effect: as the quantity $\omega^*(X)$ increases with κ, so does the average quality $\mathbb{E}[\lambda(\omega, \phi') \,|\omega < \omega^*(X)]$ of capital traded. These two effects interact in a way that makes marginal profits non-monotonic if $\lambda(\omega, \phi)$ is sufficiently sensitive to ω. The non-monotonicity of profits in κ also makes $\tilde{v}(X)$ non-monotone. This is ultimately the critical factor that prevents the recapitalization of the financial system when κ is low. In summary:

Proposition 7 *Let ρ be sufficiently close to one. Then, without asymmetric information —if $\lambda(\omega, \phi') = \bar{\lambda}(\phi')$ for all ω— marginal profits are decreasing in κ. For sufficiently smooth $\lambda(\omega, \phi')$, $\Pi(X, X')$ is non-monotone.*

4.3. *Evolution of Financial Risk Capacity*

First-Stage Evolution of Financial Risk Capacity. At the beginning of the first stage, κ evolves according to:

$$\kappa' = (1 + e^*(X) - d^*(X))\,\kappa. \tag{9}$$

Equity Injections and Dividends. In equilibrium, $\tilde{v}(X) \in [(1-\tau), 1]$ for any X. If it were the case that $\tilde{v}(X) > 1$ for a given X, equity would be injected until this value is one because bankers have the incentives to recapitalize their banks. If the value where above one, this would imply that κ is not in equilibrium. Thus, states where $\tilde{v}(X) > 1$ are instantaneously reflected into a new state where $\tilde{v}(A \times \phi \times \kappa') = 1$ for some $\kappa' > \kappa$.

The opposite occurs when $\tilde{v}(X) < (1-\tau)$. In such states, dividends are paid out until $\tilde{v}(X) = (1-\tau)$. This means that there is an inaction region, where κ is not altered by the financial policies, characterized by states where $\tilde{v}(X) \in [1-\tau, 1]$. This is similar to the (s,S) inventory models.

Multiplicity of Financial Policies. Equation (9) is implicit in κ'. Proposition 5 imposes constraints on the set of equilibrium κ'. However, since a banker's decisions are functions of other banker's decisions, there are potentially multiple equity injection policies consistent with an equilibrium.

Equilibrium Selection in Financial Policies. For the rest of the paper, I use an equilibrium refinement. I refer to this equilibrium simply as *the equilibrium.* In particular, I select an equilibrium that depends on the current realization of κ. Otherwise, note that κ is an inessential state because

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[17] This can be shown by direct inspection of Π.

it can be altered instantaneously.

This implies that if $\tilde{v}(A \times \phi \times \kappa) \in [(1-\tau), 1]$, then $e^*(X) = d^*(X) = 0$. If $\tilde{v}(A \times \phi \times \kappa) > 1$, I select equilibria for which $e^*(X)$ takes κ' to the closest value of κ such that $\tilde{v}(A \times \phi \times \kappa) = 1$. Analogously, if $\tilde{v}(A \times \phi \times \kappa) < (1-\tau)$, then $d^*(X)$ takes κ' to the closest value below κ such that $\tilde{v}(A \times \phi \times \kappa) = 1$.

I have several reasons to select this equilibrium: First, I assume that $\bar{e}$ is never binding to simplify the solution of the model. This assumption makes the point that banks may fail to attract equity in spite having resources are available. However, if $\bar{e}$ is binding, it could be the case that it is insufficient to take κ' to a point where $\tilde{v}(X) \geq 1$. Finally, this would also be the equilibrium in presence of convex adjustment costs.

Thus, this equilibrium selection, by minimizing the change in financial risk capacity, approximates an economy with limited resources. A second reason is that κ is an observed variable that can be used to coordinate strategies. Finally, this equilibrium selection delivers persistent declines in output, intermediation, and traded capital quality, which is what I try to explain with the model. This selection corresponds to the minimal-state equilibrium studied in other models.[18]

Second-Stage Evolution of Financial Risk Capacity. Between the first and the second stages, κ evolves depending on realized profits and the growth rate of the capital stock:

$$\kappa' = R^b \left[1 + \Pi(X, X') Q^*(X)\right] \frac{\kappa}{\gamma(X, X')}.$$

In this expression, $\gamma(X, X')$ is the growth rate of the capital stock:

$$\gamma(X, X') = \pi\beta \left[\frac{p(X)\omega^*(X)}{q^c(X, X')} + \mathbb{E}\left[\lambda(\omega, \phi') \,|\omega > \omega^*(X)\right]\omega^*(X)\right] + (1-\pi)\beta \left[\frac{A}{q(X, X')} + \bar{\lambda}(\phi)\right].$$

Equity Value. We can obtain a recursive expression for $\tilde{v}(X)$ and $v_1^f(X)$ that depends on the transition function from X to X'' by evaluating the Bellman equation at the optimal policies. The marginal value of bank equity at any state X is:

$$v_1^f(X) = \min\left\{\max\left\{\beta^f R^b \mathbb{E}\left[\tilde{v}(X)\,|X\right], (1-\tau)\right\}, 1\right\}. \tag{10}$$

The definitions of $\tilde{v}(X)$ and $v_2^f(X)$ define a self-map for $v_1^f(X)$.[19]

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[18] A similar form of coordination failure has a tradition in macroeconomics. For example, these emerge in the context of strategic complementarities in physical investment. This makes the model similar to early papers by Cooper and John (1988) and Kiyotaki (1988). The difference is the I focus on equity investment.

[19] Blackwell's conditions cannot be checked immediately because it may fail to satisfy the discounting property. The operator is monotone, though. All numerical iterations lead to the same outcome. When $\rho = 1$, analytical expressions can be obtained.

4.4. *States of the Financial Industry*

In any RCE, the state space may be divided into several regions. These regions are determined by the incentives to inject equity or pay dividends and the volume of intermediation. For a given exogenous state, (A, ϕ), incentives are summarized by the value of equity $v(X)$, which in turn, are key to the dynamics of κ.

Dividend-Payoff Reflecting Barrier. When $\tilde{v}(X) < (1-\tau)$, dividend payments instantaneously reduce κ to its closest reflecting barrier. That is, to the closest κ satisfying $\tilde{v}(X) = (1-\tau)$. In turn, a dividend-payoff region associated with a sufficiently large κ always exists.[20] However, there may be dividend-payoff regions for intermediate values of κ when $\Pi(X, X')$ is non-monotone.

Equity-Injection Reflecting Barrier. When $v(X) > 1$, equity injections are attractive. These injections reflect κ toward the closest value satisfying $v(X) = 1$. Similarly, when $\Pi(X, X')$ is non-monotone, there may also be multiple equity-injection barriers.

Competitive Inaction Region. The other two possible regions correspond to inaction regions. The first is a competitive inaction region defined as follows:

Definition 2 (Competitive Inaction Region) *A state X is in a competitive inaction region if (a) $\tilde{v}(X) \in [(1-\tau), 1]$, and (b) when $v_{\kappa}(X)$ exists, $v_{\tilde{\kappa}}(X) \leq 0$ for any $\tilde{\kappa} > \kappa$.*

Condition (a) states that this is an inaction region. Condition (b) states that the expected discounted marginal profits are decreasing in κ. This means that the incentives to recapitalize banks decrease as κ increases, as happens in the frictionless version of the model.[21]

Financial Crisis Inaction Region. The remaining regions are financial crisis regions. In a financial crisis, κ is low and therefore triggers adverse selection. With low intermediation, market clearing requires a low $p(X)$. However, this brings down the quality of capital traded. Moreover, expected profits are low, and this discourages equity injections. By definition, higher κ increase the value of equity —the opposite of a competition effect. Nevertheless, bankers choose not to recapitalize because they lack the incentives to do so given the low level of expected marginal profits. As discussed earlier, bankers face a coordination problem in a financial crisis region.

4.5. *Solving Equilibria*

This section outlines the strategy to compute equilibria. The solution involves two steps. The first is to compute first- and second-stage prices and expected profits for any —possibly off-equilibrium— volume of intermediation given exogenous states (A, ϕ, ϕ'). The second step uses these calculations to find equilibrium cutoffs. With this, one obtains $\tilde{v}(X)$, and the inaction regions X.

Notation. Equilibrium objects are functions of the aggregate state. To compute equilibria, one

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[20] This is because expected profits must be decreasing for large enough ω^*. This occurs because the quantity effect always dominates the quality effect as ω^* approaches 1.

[21] In equilibrium $\tilde{v}(X)$ can potentially feature jumps as a consequence of the highest-price equilibrium refinement. For this reason, condition (b) is not equivalent to $v(X)$ being locally decreasing. Instead, the condition is equivalent to having $v(X)$ being decreasing above κ, except at the finitely many points. This definition captures the idea that the quantity effect dominates the quality effect in a competitive inaction region.

needs to obtain prices and profits for any off-equilibrium ω. I use bold letters to distinguish equilibrium from off-equilibrium objects: I use $\mathbf{p}(\omega,\phi)$ to indicate the first-stage supply schedule given ϕ and a value of ω that is off equilibrium. Also, $\mathbf{q}(\omega,\mathbf{p},A,\phi')$ denotes the price consistent with second-stage market clearing for $(\omega,\mathbf{p},A,\phi')$. Finally, $\mathbf{\Pi}(\omega,\mathbf{p},A,\phi')$ are the corresponding profits given arbitrary prices, volumes, and exogenous states.

Step 1: Off-equilibrium Cutoffs, Prices, and Profits. Through Proposition 3, we can find a first-stage price $\mathbf{p}(\omega,\phi)$ associated with ω by inverting the solution to the k-producer's portfolio problem. We can also use Proposition 6 to obtain $\mathbf{q}(\omega,\mathbf{p},A,\phi')$ for any $(\omega,\mathbf{p},A,\phi')$. With this, we can compute $\mathbf{\Pi}(\omega,\mathbf{p},A,\phi')$. This computation is done once.

Step 2.1: Equilibrium Volumes, Prices, and Profits. Using the calculations in Step 1, we find the equilibrium volume of intermediation given this guess. For each X, we look for the volumes yielding non-negative expected discounted profits and the largest ω such that the worst-case losses are at most κ :

$$\omega^*(X)=\max\left(\omega:\kappa\leq\min_{\phi}\mathbf{\Pi}(\omega,\mathbf{p},A,\phi')\,\omega\text{ and }\mathbb{E}\left[\tilde{v}(X')\,\mathbf{\Pi}(\omega,\mathbf{p},A,\phi')\,|X\right]\geq 0\right).$$

Since $\mathbf{\Pi}(\omega,\mathbf{p},A,\phi')$ is continuous and $\omega\in[0,1]$, this quantity is well defined. $\omega^*(X)$ is the largest volume of intermediation yielding non-negative expected profits such that there is enough capacity to sustain losses for the worst ϕ'.

Step 2.2: Equilibrium $\tilde{v}(X)$. Given this $\omega^*(X)$, one can compute

$$\Pi(X,X')=\mathbf{\Pi}(\omega^*(X),\mathbf{p}(\omega^*(X),\phi),A,\phi').$$

We use the functional equation (10) and the definition of $v_1^f(X)$ to update $\tilde{v}(X)$. Steps 2.1 and 2.2 are iterated until convergence. When $\rho=1$, $v_2^f(X)=(1-\tau)$, so steps 2.1 and 2.2 are performed only once. Appendix A provides details for the implementation of this strategy on a computer.

5. ANALYTIC EXAMPLES

This section provides two examples. This illustrates how equity injections and dividends are stabilizing forces in an economy where intermediation is essential for growth. The first example describes a version of the model under symmetric information. The second introduces asymmetric information. The examples show that in the presence of severe asymmetric information, the recapitalization of banks no longer stabilizes financial intermediation in response to large shocks. For the rest of this section, I assume $\rho=1$ to use analytic expressions.

5.1. *Example 1 - Risky intermediation without asymmetric information.*

The first example is an economy where financial intermediation is risky, but asymmetric information is not present. Assume that $\lambda(\omega,\phi)=\lambda^*(\phi)$, —so all units are of same quality, but decreasing in ϕ. In addition, ϕ takes only two values, $\phi_B > \phi_G$. Draws are i.i.d. and A is constant. We have the following:

Proposition 8 *In any economy without asymmetric information, κ' fluctuates within a unique equilibrium interval $[\underline{\kappa},\overline{\kappa}]$. If $\kappa \leq \underline{\kappa}$, then $e^*(X)$ is such that $\kappa' = \underline{\kappa}$. If $\kappa \geq \overline{\kappa}$, $d^*(X)$ is such that $\kappa' = \overline{\kappa}$. $v(X)$ is decreasing and $\omega(X)$ is increasing in κ.*

Proof: From Proposition 6, we know that $\mathbf{\Pi}(\omega,\mathbf{p},\phi)$ is decreasing in ω since quality effects are not present without asymmetric information. Also, as noted earlier, $\rho = 1$, $v_2^f(X) = (1-\tau)$. We can use $\mathbf{\Pi}(\omega,\mathbf{p},\phi)$ and equation (6) to obtain an expression for the marginal value of equity in terms of any arbitrary ω. Call that value $\tilde{\mathbf{v}}(\omega,\mathbf{p})$. Without asymmetric information, $\tilde{\mathbf{v}}(\omega,\mathbf{p},A,\phi)$ is decreasing in ω. Consequently, by Propositions 5, there is a unique interval for ω such that $\tilde{\mathbf{v}}(\omega,\mathbf{p}) \in [(1-\tau),1]$. Correspondingly, since $\mathbf{\Pi}$ is decreasing in ω, there is a unique equilibrium interval for κ that determines a unique competitive inaction region. *Q.E.D.*

Figure 3 shows a graphic construction of equilibria. The upper-left panel depicts four curves associated with an arbitrary ω. These correspond to the capital-supply schedule, $\mathbf{p}(\omega,\phi)$, the marginal value of bank assets in good and bad states, $\mathbf{q}(\omega,\mathbf{p},\phi_H)\lambda^*(\phi_H)$ and $\mathbf{q}(\omega,\mathbf{p},\phi_L)\lambda^*(\phi_L)$, and their expected value $\mathbb{E}[\mathbf{q}(\omega,\mathbf{p},\phi)\lambda^*(\phi)]$. The difference between $\mathbb{E}[\mathbf{q}(\omega,\mathbf{p},\phi)\lambda^*(\phi)]$ and $\mathbf{p}(\omega)$ are the expected marginal profits of banks. Multiplying this amount by ω yields the total expected bank profits $\mathbb{E}[\mathbf{q}(\omega,\mathbf{p},\phi)\lambda^*(\phi) - \mathbf{p}(\omega)]\omega$ normalized by the capital stock. Total expected bank profits are plotted in the bottom-left panel. The bottom-right panel plots the worst-case profits, $[\mathbf{q}(\omega,\mathbf{p},\phi_L)\lambda^*(\phi_L)] - \mathbf{p}(\omega)]\,\omega$. In equilibrium, κ must be sufficient to sustain the losses induced for the corresponding volumes of intermediation. The top-right panel plots the expected value of bank equity $\tilde{\mathbf{v}}(\omega,\mathbf{p})$ as a function of ω. The horizontal lines in the top-right panel are the marginal costs of injecting equity, 1, and the marginal benefit of dividend pay-offs, $(1-\tau)$. In equilibrium, if a given ω is indeed an equilibrium volume of intermediation, bankers must not alter their net worth for those levels of expected profits. Thus, the set of possible equilibrium ω is characterized by volumes for which the value of equity falls within the marginal cost of injections and the benefit of dividend payments. The shaded areas of the graphs correspond to this set. Since, $\tilde{\mathbf{v}}(\omega,\mathbf{p})$ is decreasing in ω, the equilibrium set is a unique interval. For each ω in that interval, there is an equilibrium κ corresponding to it. We obtain this equilibrium set by computing the maximal losses given each ω in the equilibrium set. The bottom-right panel shows this interval for κ is obtained as the image of worst-case losses for the equilibrium ω-set.

Figure 4 plots four equilibrium objects. The top-left panel plots ω^* as a function of κ —that is, κ before equity injections or dividends. The top-right panel depicts $\tilde{v}$. In equilibrium, κ' must be within the inaction region where $\tilde{v}(\kappa) \in [(1-\tau),1]$. The bottom panel depicts equity injections, dividends, and κ' as functions of κ. In equilibrium, e and d adjust to bring κ' to the equilibrium set depicted in Figure 3. The shaded area of the figure is the competitive inaction region. The regions to the right and left of the shaded area are the dividend payoff and equity injection regions,

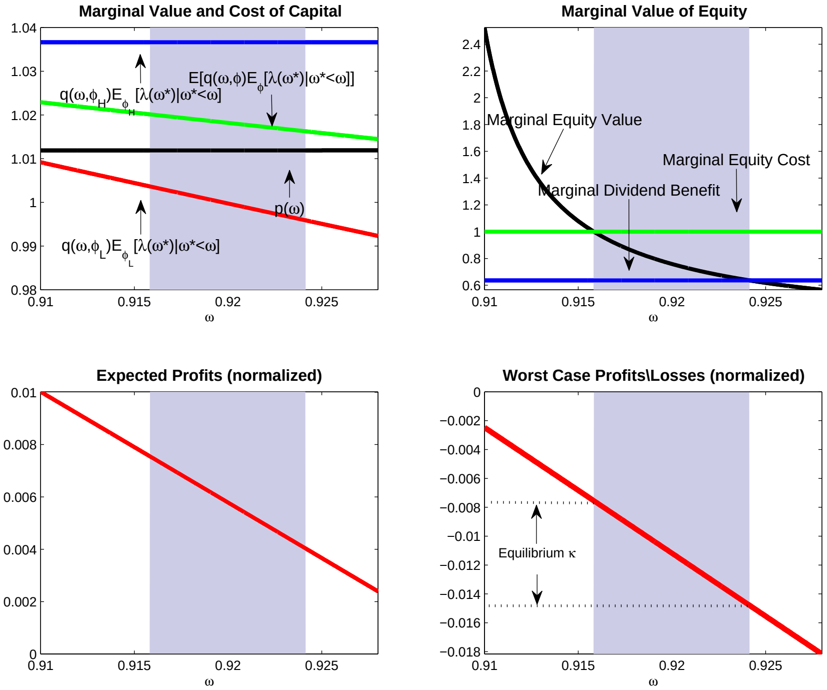

FIGURE 3.— **Model without asymmetric information: Equilibrium objects as functions of ω.**

respectively.

Dynamics. Proposition 8 is useful to understand the dynamics of this economy. Recall that, in equilibrium, worst-case losses are always negative. In contrast, expected profits must be non-negative or, otherwise, no intermediation would be provided. When ϕ_B is realized, profits are negative and drag down κ. Below $\underline{\kappa}$, high expected profits attract equity injections that recapitalize banks and increase the financial risk capacity back to $\underline{\kappa}$. Thus, injections stabilize a financial system with low financial risk capacity. When ϕ_G is realized, positive profits increase κ. When κ increases beyond $\bar{\kappa}$, dividend payoffs reflect the financial risk capacity downwards. Hence, without asymmetric information, κ fluctuates within a unique interval. The next section shows how asymmetric information precludes this stabilizing force.

5.2. *Example 2 - Risky intermediation with asymmetric information.*

I modify $\lambda(\omega,\phi)$ to introduce asymmetric information. I fix values for the lower and upper bounds of $\lambda(\omega,,\phi)$, λ_L and λ_H. I use the following functional form $\mathbb{E}[\lambda(\omega,\phi)|\omega<\omega^*] \equiv \lambda_L + (\lambda_H-\lambda_L)F_\phi(\omega^*)$. I assume F_ϕ is the CDF of a Beta distribution. Hence, ϕ indexes the parameters of that CDF to satisfy Assumption 1. The rest of the calibration is the same as in the previous example. When $\lambda(\omega,\phi)$ varies sufficiently with ω, this introduces asymmetric information.

Figure 5 is the asymmetric-information analogue of Figure 3. The upper-left panel shows four curves that correspond to $\mathbf{p}(\omega,\phi)$, $\mathbf{q}(\omega,\mathbf{p},\phi_H)$, $\mathbf{q}(\omega,\mathbf{p},\phi_L)\lambda(\phi_L)$, and $\mathbb{E}[\mathbf{q}(\omega,\mathbf{p},\phi)\lambda(\phi)]$. Note

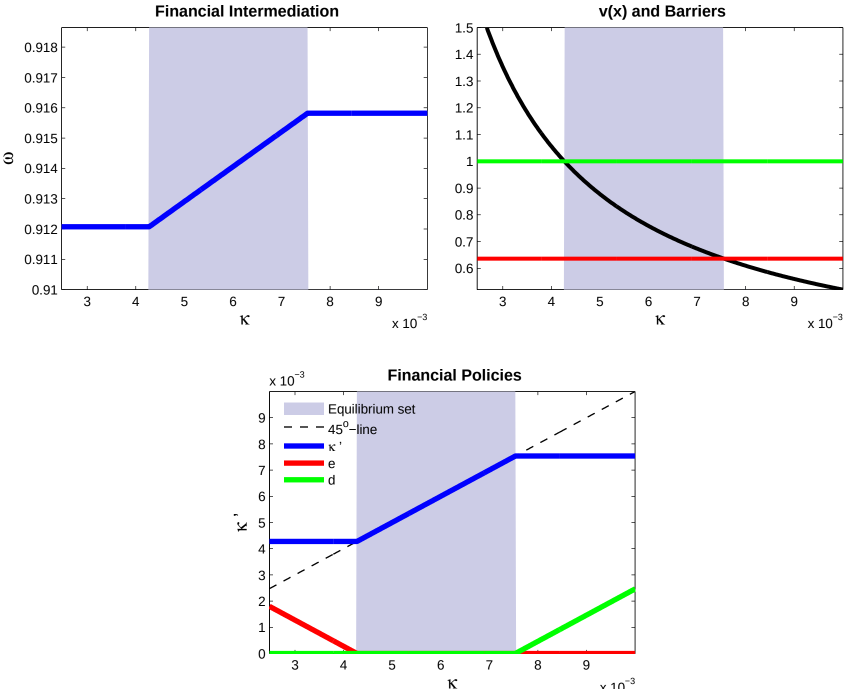

FIGURE 4.— **Model without asymmetric information: Equilibrium objects as functions of κ.**

that $\mathbf{q}(\omega, \mathbf{p}, \phi_H)\,\lambda(\phi_H)$ and $\mathbf{q}(\omega, \mathbf{p}, \phi_L)\,\lambda(\phi_L)$ are no longer decreasing in ω. As volumes increase, the price of capital falls, but the quality improves. The relative strength of either effect governs the shape of the value of the bank's asset position. These forces cause total expected and worst-case profits to be non-monotonic —see the bottom-left and right panels. The same levels of worst-case losses can result in multiple values of ω. This implies that a given κ can possibly sustain multiple levels of intermediation. The highest-price refinement implies that, in equilibrium, ω is the highest amount of intermediation consistent with bank optimality conditions and the capacity constraint. The top-right panel plots the marginal value of bank equity. There is a difference in the shape of the value of bank equity when asymmetric information is present than without it. With asymmetric information, the marginal value of equity inherits the non-monotonic behavior of profits. Once again, the horizontal lines correspond to the marginal cost of equity and the marginal benefit of dividends. Thus, the non-monotonic profits function leads to multiple inaction regions. In particular, for low levels of intermediation, expected profits are low and equity injections are no longer profitable.

The dynamics in the example are much richer than before. Here, there are three equilibrium inaction intervals identified by the shaded areas. The equilibrium intervals for the financial risk capacity are obtained as the image of the worst-case losses for each equilibrium ω-interval. The upper bound of interval II in the figure has a distinctive property: If κ increases slightly at that point, the financial risk capacity can support a much larger volume of intermediation. This happens because there is sharp increase in intermediation that increases worst-case losses only infinitesimally.

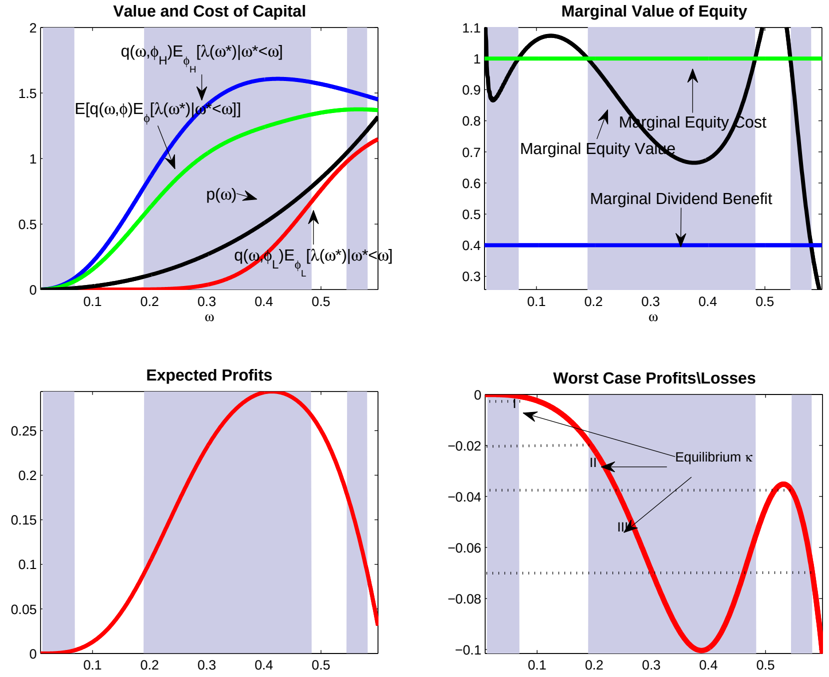

FIGURE 5.— **Model with asymmetric information: Equilibrium objects as functions of ω. Parameters are set to:** $\pi = 0.1,\ \beta = 0.98,\ \beta^f = 0.6,\ A = 1.45,\ \tau = 0.6364,\ \rho = 1$.

Interval III is a financial crisis regime. It is associated with low levels of financial intermediation. This region is an inaction region since bankers do not inject equity here. Note that for larger values of ω, equity injections are profitable since $\tilde{\mathbf{v}}$ is above 1. This underscores the nature of the coordination failure faced by banks. Banks choose to maintain their net worth at current levels and engage in less intermediation.

Figure 6 plots the equilibrium objects as functions of κ. The upper-left panel plots the equilibrium financial intermediation. We can observe a discrete jump in the volume of intermediation between the second and the first region. This jump occurs because for a slightly larger κ can support a much larger volume of intermediation and the highest-price refinement selects the equilibrium with largest volumes. Equity injections between regions I and II are very small: regions I and II are close to each other. Note that to the left of the second region, the marginal value of equity $\tilde{v}$ is increasing in κ. The financial crisis regime is very small because the volume of intermediation and losses associated with it are also very small. Through this example we have shown:

Proposition 9 *For sufficiently severe asymmetric information —when $F_\phi(\cdot)$ varies sufficiently— the return to financial intermediation is non-monotone in κ and there exists a financial crisis regime.*

Dynamics. The immediate effects ϕ on κ are the same as without asymmetric information. However, the dynamics of the economy are very different. With asymmetric information, a realization of $\phi = \phi_B$ can drive the financial system to a financial crisis regime. As adverse selection is aggravated, profitability no longer provides the incentives for capital injections into the banking

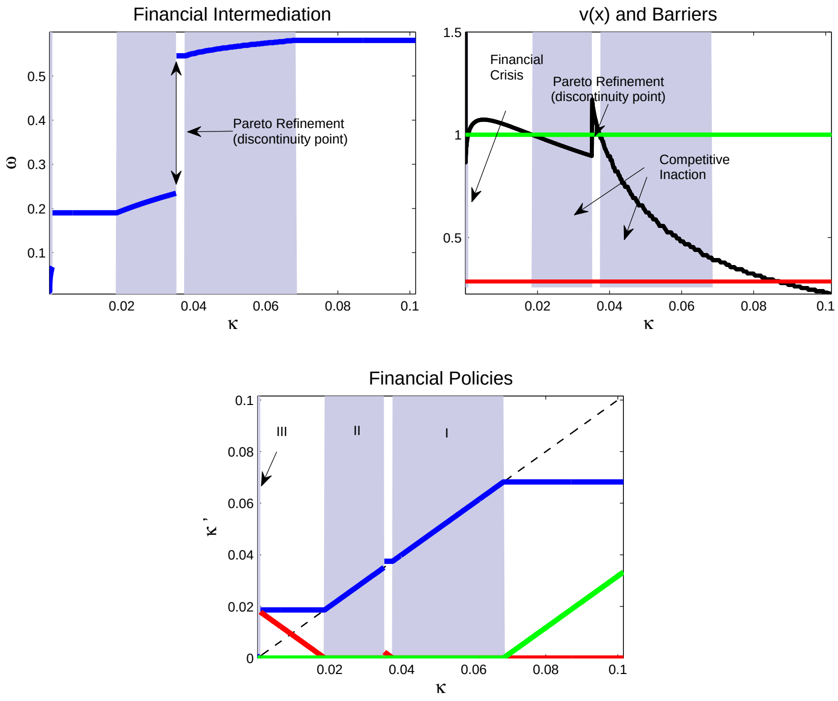

FIGURE 6.— **Model with asymmetric information: Equilibrium objects the as functions of κ.**

system. In crises regimes, the economy may take long to recover. The economy eventually recovers as banks slowly rebuild their equity through retained earnings. Once κ reaches the equity injection regions between intervals III and II, the banking system attracts new equity injections. Eventually, the economy recovers to the reach the competitive inaction region II. The following section develops some numerical examples.

6. NUMERICAL EXAMPLES

6.1. *Additional Features*

In this section, I incorporate three additional features into the model:

Financial-Management Costs. I assume bankers pay a constant amount of their equity every period and a constant bonus if profits are positive. This parameter corresponds to ψ in Table I.

Capital Requirements. I introduce capital requirements as a wedge into the LLC. With capital requirements the LLC reads:

$$-\Pi\left(X, X'\right) Q \leq (1-\theta)\, n'.$$

Physical Capital Cost. I introduce an additional parameter to change the price of capital to a value different than one.

6.2. *Parameters*

Calibrated Parameters. I set β and β^f so that the risk-free rate in the deterministic frictionless version of the model is *3.0*% in annual terms. The return to equity in stage two, R^b, is set to 1. I assume that the average depreciation rate is constant, 0.9756. This gives a lower bound to the growth rate of the economy given its AK structure. The fraction of capital goods producers π is set to 0.1. I set the banker's exit rate, ρ, to 0 —so bankers face an infinite horizon. The tax τ yields a marginal cost of equity of 10% and dividends taxes of 30%. These numbers are obtained from Hennessy and Whited (2005). I set, θ, to 8% to be consistent with Basel-II requirements.

Estimated Parameters. I estimate an $AR(1)$ process for $log\left(A_t\right)$. The auto-correlation coefficient is estimated to be 0.993. Its mean is -0.885, and the standard deviation is 0.0083. I assume that ϕ follows a four-state Markov chain. The transition matrix for this Markov chain is calibrated using data on bank profits.

Matched Parameters. To calibrate the quality distributions, I use the same parametric form as in the analytic examples. Under this form, λ_L is the lowest depreciation and $F_\phi\left(\omega^*\right)$ is any CDF with support in $[0,1]$. I set $\lambda_L = 0$ to have the interpretation that some capital is worthless. For each of the four values of ϕ, there are parameters denoted by (A_ϕ, B_ϕ) that summarize the quality distribution associated with that state. A_ϕ and B_ϕ are uniformly spaced between $[A_L, A_H]$ and $[B_L, B_H]$, respectively. Thus, there are four parameters that characterize four distributions.

These parameters are set to obtain four moments. I target historical and crisis —using observations from the Great Recession— levels for bank leverage and bank ROA. There is mapping from these moments to F_ϕ. In the model, the difference between the expected value of capital and the cost of capital is the bank's return-on-assets (ROA). The cost of capital is given by the supply schedule obtained by solving the k-producer's portfolio problem. This supply schedule is only a function of $[A_L, A_H, B_L, B_H]$ and the transition matrix of ϕ. The demand for capital is also a function of these parameters and the real-side parameters. In turn, the bank's leverage is the inverse of worst-case losses, also a function of $[A_L, A_H, B_L, B_H]$. Using this property, I search for values that lead to leverage and ROA as close as possible to the targets. There are four parameters for four moments —two unconditional moments, and two moments for crisis episodes. However, since these parameters affect the outcome of other moments, I am forced to make some compromises.

Finally, I set the non-interest expense parameter ψ to 8%. This matches the non-interest expenditures to equity ratios for the selected sample of Bank Holding Companies (BHCs). Table I summarizes the parameter values.

6.3. *Results*

The results from this and the following sections illustrate how the model works. The spirit is similar to the numerical examples as in Brunnermeier and Sannikov (2011), He and Krishnamurthy (2009) and Adrian and Boyarchenko (2013). I include financial crises facts for comparison.

Parameters	Value	Note
Calibrated		
β	0.987259	3% annual time discount rate
β^f	0.945742	3% annual time discount rate
R^b	1	0 interest on reserves
π	0.097342	Cooper et al. (1999)
τ	0.08	Hennessy and Whited (2005)
ψ	0.08	Bank non-interest expense per net-worth
θ	0.08	Basel-II capital requirements
Estimated		
μ_A	-0.885	Estimated TFP process
ρ_A	0.993	Estimated TFP process
σ_A	0.0083	Estimated TFP process
Matched		
A_L	3.9	To match bank ROA and leverage
A_h	4	To match bank ROA and leverage
B_L	6.2	To match bank ROA and leverage
B_H	5.2	To match bank ROA and leverage
Φ	$[0.95\ 0.95;\ 0.05\ 0.05]\bigotimes[0.95\ 0.95;\ 0.05\ 0.05]$	To match profit transition of banks

TABLE I

Parameter Values

6.3.1. *Invariant Distribution and Historical Histograms*

Figure 7 reports four histograms. These correspond to the invariant distributions for κ and the growth rate of K, both in the model and for the US data. Bars represent occupation frequencies. The top-left panel depicts the marginal invariant distribution of κ. The top-right is the data analog. In the model, κ takes high values most of time. However, some probability mass concentrated at lower values. A similar concentration mass is found for the empirical counterpart. Both histograms show an empty region in the middle. In the model, occupations at the left of the distribution occur for sufficiently bad combinations of shocks. The red bars correspond to financial crisis regimes in the model. Crises have high occupation times due to their long exit times. Intermediate intervals, for which κ has no occurrence, correspond to equity injection regions. The two panels at the bottom show the corresponding growth rates of K.

6.3.2. *Model and Historical Moments*

Table II reports model and data moments. These moments are computed from the invariant distribution and the distribution conditional on crisis states. The directions of change in the model's moments from historical to crisis statistics follow a similar pattern to the movements from historical averages to Great Recession statistics. However, the model represents an exaggerated version of reality.

The occupation time on crisis regimes is 32.6% in the model. The Great Recession represents 14.6% of the time in the sample, clearly an overstatement of recent US history since the financial system

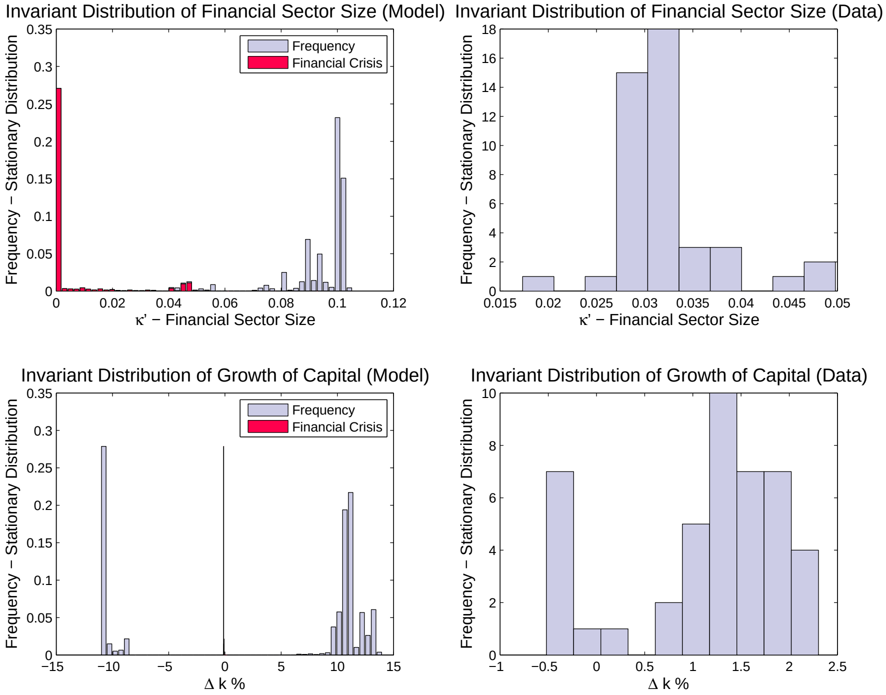

Figure 7.— US Banking Indicators and Capital Accumulation.

was very stable prior to this episode. However, Reinhart and Rogoff (2009) calculate that during the national banking era, crises occurred during 13% of years in their sample —crises defined according to an arbitrary rule. The exit time in the model is 10 quarters, whereas the Great Recession lasted six quarters, twice the length of the average recession.

The average growth rate of the economy in the model is close to the historical growth rate of the US. During a financial crisis, output growth falls to 10.0%, a dramatic description of reality when compared to the Great Recession. However, Cerra and Saxena (2008) report a reduction in the growth rate of about 8% for a cross-country sample.

Financial crises are associated with a strong reductions in intermediation volumes. One can notice this from the reduction in the loans-to-output ratio. Intermediation and qualities also fall during these episodes. This explains the fall in investment to GDP. In the model, this ratio is high in comparison with the data which follows from the AK-technology structure of the model.

A second block of moments reports some financial intermediation indicators. The most important are the values of κ. During a financial crises, κ can fall as much as 92%. This is an exaggeration of the equity losses in reality. Clearly, the dynamics of the model are more extreme than those in the data because the model allows the entire equity of the banking system to disappear in a single period. The unconditional financing premia in the model are also magnified.

Bank ROE is higher within crisis episodes in the model. The direction of the drop depends on parameters. The equilibrium values of bank ROE belong to an interval determined by the inaction regions. Nothing precludes the distribution bank ROE to be higher than their unconditional average within financial crisis regime. The model is successful, however, in that it does not deliver counterfactual increases in bank ROE, as many other models in the literature.

Growth in the model is fully explained by capital accumulation. A variance decomposition of the model shows that in normal times, most of the volatility of output follows from movements in lending. During financial crises, intermediation is responsible for a smaller share of volatility because crises occur more frequently during low TFP states which are likely to revert to trend.

Variable	**Unconditional**	**Crisis**	**Historical**	**Great Recession**
Occupation Times				
Occupation Time	100%	32.6%	100%	14.5%
Duration (quarters)	-	10.26	20.8	6
Economic Activity				
Average Growth Rate	4.3%	-10.3%	2.98%	-2.36%
Investment/Output	39.9%	-0.935%	8.51%	5.64%
Investment/Capital	3.58%	-0.0644%	6.09%	3.92%
Financial Intermediation Indicators				
Average κ	0.0659	0.0048	0.0162	0.0148
Financial Leverage	6.56	1.99	9.87	11.3
Loans Output	6.68%	0.0832%	NaN	NaN
Return on Assets (ROA)	6.94%	16.9%	1.18%	-0.0839%
Return on Equity (ROE)	31.3%	48.1%	16.4%	-1.07%
Financing Premia	39.5%	106%	6.25%	5.89%
Financial Equity Indicators				
Average Dividend Rate	0.643%	0.0193%	1.12%	-18.8%
Financial Stocks Index	100%	8.31%	100%	42.9%

TABLE II

Model moments and reference statistics.

6.3.3. *Response to Dispersion Shocks*

Figure 10 presents the response of the economy to an increase in ϕ. The figures plot expected responses of equilibrium objects after ϕ takes its largest value. The system is initiated from states randomly drawn from the invariant distribution. The top-left panel shows the immediate effect on bank losses. The solid line corresponds to the banks' liabilities relative to their unconditional mean. The dashed line is the corresponding series for bank assets. When the shock is realized, the value of assets falls under the value of liabilities because ϕ affects the value of assets, not liabilities. This discrepancy induces losses —the dashed line in the top-middle panel— on impact. Whereas expected profits —solid line in the top-middle panel— are constant, realized profits fall below expectations. This reduction in profits causes a reduction κ —top-right panel. As a consequence, during subsequent periods, the equilibrium ω^* —middle-left panel— falls. The drop in ω^* represents both a decrease in volumes of intermediation and an exacerbation of adverse selection. After the impact of the shock, one can observe that overall intermediation declines.

On average, there are some equity injections. However, these injections are small in relation to the collapse in bank equity, far from replenishing bank equity immediately. The value of bank net worth also declines, almost entirely driven by the reduction in κ —middle-right panel.

The effects on real economic activity are depicted in the bottom panels. The collapse in financial intermediation after the shock causes a drop in investment —bottom-left panel. Since this economy requires investment to grow, the growth rate declines —bottom-middle panel. The panel on the bottom right shows the level of output with and without the shock. The reductions in growth rates have permanent effects due to the AK structure of the model.

The effects of ϕ share the hallmarks financial crises described as periods of strong adverse selection as argued by, for example, Calomiris and Gorton (1991) or Mishkin (2011). The impact of this large shock is very persistent although the shock itself has no memory. This persistence follows from the internal behavior of the model. Several macroeconometric studies obtain highly persistent filtered time series after financial shocks —e.g., Christiano et al. (2009), Justiniano et al. (2010) or Jermann and Quadrini (2011). Through the lens of the model, these long recoveries are explained by the length of time it takes banks to rebuild their net worth. However, this persistence only occurs in response to large shocks, but not to small shocks as explained below.

Inherent Non-linearities. Consider the response to a smaller shock to ϕ. Figure 11 presents a superimposed impulse response to a smaller shock on the impulse response in Figure 10. It is clear that the responses to smaller shocks are much less persistent. In particular, the growth rate of the economy recovers almost immediately after the small shock. Naturally, bank losses are smaller for the smaller shock, and except for expected bank profits in the period after the shock, the directions of the responses are virtually the same for all of the variables. So what explains these sharp differences in persistence? In contrast to what happens after large shocks, adverse-selection effects are not strong when small shocks occur. Hence, after small shocks, competition effects lead to higher marginal profits with less intermediation. This explains the differences in the directions of expected profits. With higher expected profits, equity injections are attracted. Notice that these are massive, almost twice the size of the average dividend payment. These equity injections mitigate the amplification of the shock. Higher profits imply that by retaining earnings, κ will increase rapidly in the subsequent period after the small shock is realized, something that fails to occur after large shocks. This feature explains the inherent non linearity that this paper attempts to underscore.

7. FINANCIAL STABILITY POLICIES

This section discusses the effects of capital requirements in the context of the model. First, I describe two externalities that emerge in this environment and then the effects of this policy intervention.

7.1. *The Externalities*

Hanson et al. (2011) argue that macroprudential policy frameworks must identify why banks may provide excessive intermediation. There are two externalities here.

Intermediation Externality. In the model, when banks purchase capital they face potential net worth losses. Banks choose their leverage rationally, taking into consideration risks and benefits. However, because they are price takers they fail to internalize that, in the aggregate, they impact the evolution of the financial risk capacity. Although this is also true in a neoclassical growth model, here the presence of limited liability induces an inefficiency. A planner facing the same LLC as banks would take into account the law of motion of κ in his decision to lend. In particular, in a competitive inaction region, a planner would choose to provide less intermediation to reduce large bank losses. In turn, banks may be better off as less intermediation in competitive inaction regions leads to greater profits. Producers may also be better off because the economy may become more stable with less intermediation.[22]

Equity-Injection Externality. In financial crisis regions, there is a coordination failure: when κ is low, bankers expect low future discounted profits. Given this projection, an individual banker will not inject equity because he cannot affect κ. However, if all bankers were to inject capital simultaneously, the value of bank equity would increase. This would benefit both bankers and producers.

7.2. *Capital requirements*

Effects of Capital Requirements. The impact of tighter capital-requirements can be studied through increases θ. This capital requirement has two effects. The first effect is clear from the constraint $\Pi\left(X,X'\right)Q \leq (1-\theta)\,n', \forall X'$: given a level of κ, this parameter increase will lower the volume of intermediation in a given period. As discussed in the previous section, this policy may be desirable if banks are intermediating excessively; although, in general, a reduction in their intermediation will have adverse effects on growth. Depending on the region of the state space, the reduction in Q^* may decrease or increase bank profits —depending on whether competition or adverse selection effects dominate, and the same is true about leverage.

The second effect is embedded in the dynamics of κ. Holding prices fixed, an increase in θ reduces bank ROE. This happens because it forces bank leverage down. From the solution to Q^*, one can show that the shadow value of relaxing the bank's LLC is $(1-\theta)\max\left\{\frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)},0\right\}$. This expression is a linear negative function of θ if $\Pi(X,X')$ is fixed. This effects shows up in the conditions that determine the marginal value of bank equity:

$$\tilde{v}(X) = \beta^F\left[\mathbb{E}[v_2^f(X')] + (1-\theta)\max\left\{\frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)},0\right\}\right]$$

---

[22] A similar externality is found in Lorenzoni (2008).

Holding profits fixed, the value of bank equity is decreasing in θ.

These two effects operate in different directions in determining the equilibrium intervals for κ. Reductions in volumes may increase or decrease the value of bank equity. The reduction in bank leverage reduces bank ROE —by lowering leverage— and this may also reduce the incentive to inject equity.

As argued earlier, a social planner will face a trade off between reducing probability of a financial crises at the expense of a reduction in financial intermediation. Capital requirements may be a useful tool in inducing more favorable outcomes. An optimal government policy potentially involves a state dependent θ. However, the effects on κ will be hard to pin down without specific values of parameters. The following numerical exercises attempt to shed light on how these effects balance out in the current calibration.

Invariant Distribution and Moments under Basel-II and III. This section describes the effects of an increase in θ from 0.08 (Basel-II scenario) to 0.18 (Basel-III scenario).[23] Figure 8 presents the invariant distribution of κ under both parameter values. Both distributions are similar in that they show a bimodal shape with concentrated probability masses at the extremes. However, there are important differences. First, the distribution under Basel-III has less concentration for lower values of κ. This means that the unconditional probability of falling in a financial crisis regime is lower. Second, toward the right of these distributions, the probability mass under Basel-III parameters is higher. Higher average levels of κ can result from higher profitability from intermediation that compensate a lower degree of intermediation.

Table III reports moments under both policy regimes. The duration of a crisis is shorter under Basel-III. Why? Crises last shorter during Basel-III because this regime prevents large falls in κ. Since the financial risk capacity is higher on average, κ can reach an equity injection region faster. In addition, because declines in κ are less extreme, financial crises are also less frequent. It takes less likely combinations of ϕ_t to cause a financial crisis. This figure is also consistent with higher levels of κ observed both unconditionally and for crisis regimes.

Variable	**Basel-II**		**Basel-III**	
	Unconditional	**Crisis**	**Unconditional**	**Crisis**
Occupation Time	100%	32.6%	100%	19.77%
Duration (quarters)	-	10.26	-	3.89
Average Growth Rate	4.3%	-10.3%	9.3%	-9.61%
Average κ	0.0659	0.0048	0.100	0.01

TABLE III

Comparison of Moments Under Basel-II and III in the Model. The table plots some moments corresponding to values of θ equal to 0.08 (Basel-II) and 0.18 (Basel-III).

Timing of Basel-III. This section studies the effects of increasing capital requirements unexpectedly and permanently once the economy falls within a financial crisis. The purpose of this

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[23] A similar exercise is performed by Bianchi (2011) in the context of international capital flows.

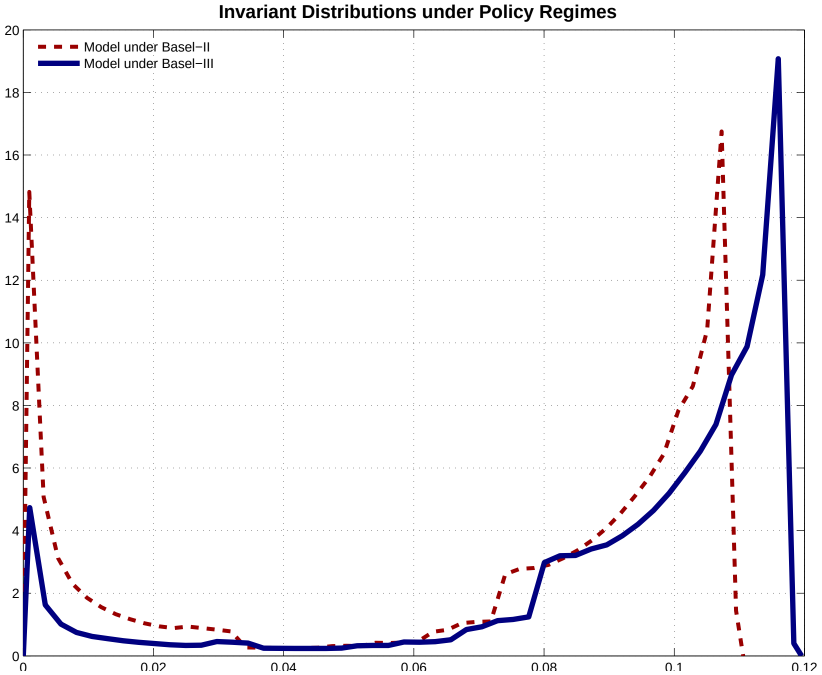

FIGURE 8.— **Invariant Distributions under Basel-II and III.**

exercise is to study the recovery from a financial crises region under Basel-II and Basel-III values of θ once the economy falls on a financial crisis under Basel-II. To do so, I initialize the economy from random states drawn from the Basel-II invariant distribution conditional on being in a financial crisis state. Then, I compare the growth rates when the economy remains under Basel-II with when Basel-III parameters are introduced permanently. Figure 9 presents average paths for the growth rates of output under both policy regimes.

One can observe that the recovery under Basel-II is faster. The intuition behind this numerical experiment is that once in a crisis, capital requirements may actually prolong the decline —although, the same policy may be desirable when implemented starting at a random point of the unconditional distribution. Conditional on being in a crisis, higher capital requirements depress intermediation and exacerbate adverse selection. This reduces profitability beyond what it would be under Basel-II. As a consequence, bank recapitalization is slower.

Comparing this with the previous exercise, the model warns policy makers not to do in bad times what they should have done in good times. There is an ongoing debate between bank managers and policy makers (see Admati et al., 2011). Bank managers argue that increasing capital requirements immediately after a crisis may be too stringent on financial intermediation. In the model, higher capital requirements during a financial crisis will carry two effects: (1) they will aggravate adverse selection and (2) they will reduce the incentives to recapitalize banks, thereby prolonging the crisis. The first effect is also shared by other models with fire-sale externalities —see, for example, Bianchi (2011) or Hanson et al. (2011). However, if marginal profits from intermediation are decreasing, the additional intermediation constraints will be offset by higher marginal profits —the effect on bank

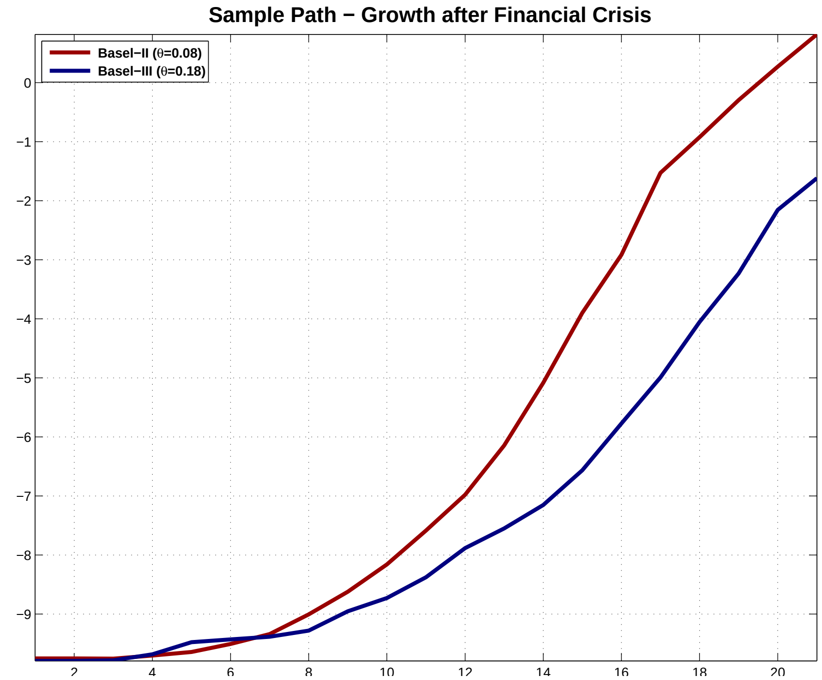

FIGURE 9.— **Expected Growth Recovery Path Under Basel-II and -III.**

ROE could be indeterminate.

Pro-Cyclical Capital Requirements. The results in the previous sections suggest that pro-cyclical requirements are perhaps the optimal policy. In competitive inaction regions, higher requirements may prevent large swings in κ. However, during crisis regimes, by increasing bank profitability at times when adverse selection is severe, lower requirements may shorten the length of a crisis. Kashyap and Stein (2004) argue in favor of pro-cyclical requirements because they argue that lending should be promoted during crises. In a related model Bianchi (2011) performs an exercise along these lines.

8. CONCLUSIONS

This paper provides a theory about risky financial intermediation under asymmetric information. The main message is that financial markets where asymmetric information is a severe friction are likely to be more unstable. The source of this instability is the non-monotonicity in bank profitability as a function of the volume of intermediation. This non-monotonicity implies that banks fail to recapitalize after large losses lead to low intermediation despite the availability of resources. This phenomenon prolongs a financial crisis.

The nature of asymmetric information and financial contracting in this paper is deliberately stark. I believe the model could be improved to introduce more realistic credit markets without altering the essence of the model. This improvement could enhance the mapping from the model to the data.

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APPENDIX A: ALGORITHM

The following algorithm computes equilibria.

1. Build a grid for the state space values of $A \times \phi$ and the transition function χ.
2. Build a grid on the unit interval for ω.
3. For all possible values of $A \times \phi$ and ω on the grid, solve for $\{\mathbf{p}(\omega,A,\phi), \mathbf{q}(\omega,\mathbf{p},A,\phi), \mathbf{\Pi}(\omega,A,\phi)\}$. This step is performed once.
    - Discretize a set of points between $p_L = \min_{\phi} \lambda(0,\phi)$ and $p_H = \max_{\phi} \lambda(1,\phi)$. Solve the optimal portfolio problem assuming $\mathbf{q}^c(\omega,\mathbf{A},\phi) = 1$. To do this, find a value for ω^* in (4) for each possible value of $A \times \phi$ and $\mathbf{p}$ on the grid**.** Interpolate over the grid for ω to obtain $\mathbf{p}(\omega,A,\phi)$.
    - Using this $(\mathbf{p}(\omega,A,\phi), \omega)$, find $\mathbf{q}(\omega,\mathbf{p},A,\phi)$, $\mathbf{\Pi}(\omega,\mathbf{p},A,\phi)$ using the expressions for these variables, equations (7) and (8). Then, check if $\mathbf{q}(\omega,\mathbf{p},A,\phi) \geq 1$.
    - For values where the condition fails, solve $\mathbf{p}(\omega,A,\phi)$, $\mathbf{q}(\omega,A,\phi)$ jointly using (4) and (7). Finally, find $\mathbf{\Pi}(\omega,A,\phi)$ using (8).
4. Guess a candidate function for $\tilde{v}$.
5. Compute the set ω^o using the candidate function $\tilde{v}$.
    - Compute, for each ω on the grid, the value of $\mathbb{E}[\tilde{v}(X')\mathbf{\Pi}(\omega,\mathbf{p}(\omega,A,\phi),\phi')|X]$, where $\mathbf{p}(\omega,A,\phi)$ is found in step 3. ω^o is the set of values of ω that yield a zero for $\mathbb{E}[\tilde{v}(X')\mathbf{\Pi}(\omega,\mathbf{p}(\omega,A,\phi),\phi')|X]$.
6. Compute the set ω^{κ}.
    - For each ω in the grid, compute $\kappa = \min_{\phi} \mathbf{\Pi}(\omega,\mathbf{p}(\omega,A,\phi),\phi)\,\omega$.
7. Compute $\omega^*(X)$.
8. Define $p(X) = \mathbf{p}(\omega^*(X),\mathbf{A},\phi)$, $\Pi(X) = \mathbf{\Pi}(\omega^*(X),\mathbf{A},\phi)$ and $q(X) = \mathbf{q}(\omega^*(X),\ \mathbf{p}(\omega^*(X),\mathbf{A},\phi),\mathbf{A},\phi)$.
9. Compute the transition function for X.
10. Update the $\tilde{v}(X)$ iterating the Bellman equation for $\tilde{v}(X)$ until convergence.
11. Iterate steps 4-10 until convergence.
12. Compute $v(X), d(X), e(X)$.

APPENDIX B: PROOFS

B.1. *Proof of Propositions 1, 2, and 3*

PROOF: The proof of propositions 1, 2, and 3 is presented jointly. The idea of the proof is to transform the producer's problem into a consumption-savings problem with log-preferences and linear constraints. For this, one has to solve the quality sales decisions under asymmetric information first. This is done showing the problem is homogeneous. Once this is done, one can use the dynamic programming arguments for homogenous objectives in Alvarez and Stokey (1998) to argue that all Bellman equations have unique solutions. I proceed by guess and verify.

Define $W^p \equiv w^p k \equiv \left(A + q\bar{\lambda}(\phi')\right)k$ and $W^i \equiv w^i k \equiv (p\omega^* + q^c\mathbb{E}[\lambda(\omega,\phi')|\omega < \omega^*])k$ as in the main text. The guess for the c-producer's policy function is $k^{p,\prime} = \beta\frac{W^p}{q}$ and $c^p = (1-\beta)W^p$ and that his value function is of the form $V_2^p = \psi^p(X) + \frac{1}{(1-\beta)}\log W^p$ where $\psi^p(X)$ is a function of the aggregate state. For k-producers the guess is that $k^{i,\prime} = \beta\frac{W^i}{q^c}$ and $c^i = (1-\beta)W^i$ and that their value function is of the form $V_2^i = \psi^i(X) + \frac{1}{(1-\beta)}\log W^i$ where $\psi^i(X)$ is, again, a function of the aggregate state.

Consider the k-producer's problem during the first stage. Substituting the guess for V_2^i and his constraints yields:

$$V_1^i = \max_{\mathbb{I}(\omega)\in\{0,1\}} \mathbb{E}\left[\psi^i(X) + \log\left(\left(p\int_0^1 \mathbb{I}(\omega)\,d\omega \;\; + q^c\mathbb{E}[\lambda(\omega,\phi')|\mathbb{I}(\omega)=1]\right)k\right)|X\right].$$

From this expression, we show that choosing $\mathbb{I}(\omega)$ is identical to choosing a cutoff ω^* under which all units of quality lower than this cutoff are sold. We show this by arguing that an optimal $\mathbb{I}^*(\omega)$ must be monotone decreasing. Suppose not and assume the optimal plan is given by some $\mathbb{I}'(\omega)$ whose value cannot be attained by any monotone decreasing policy. It is enough to show that the producer can find another candidate $\mathbb{I}(\omega)$ that integrates to the same number, that is monotone decreasing and that makes his value weakly greater. Thus, let's compare $\mathbb{I}'(\omega)$ with some $\mathbb{I}(\omega)$ that integrates to the same number in $[0,1]$.

Since $\mathbb{I}'(\omega)$ and $\mathbb{I}(\omega)$ integrate to the same number, the amount of IOUs obtained by the k-producer during the first stage is the same: $p\int_0^1 \mathbb{I}(\omega)\,d\omega = p\int_0^1 \mathbb{I}'(\omega)\,d\omega$. Now, since $\mathbb{I}(\omega)$ is monotone decreasing and $\lambda(\omega,\phi)$ is monotone increasing in ω,

$$\int [1-\mathbb{I}'(\omega)]\,\lambda(\omega,\phi')\,d\omega \le \int [1-\mathbb{I}(\omega)]\,\lambda(\omega,\phi')\,d\omega$$

implying that any optimal can be attained by some $\mathbb{I}^*(\omega)$ monotone decreasing. This shows that V_1^i is attained by some monotone decreasing function.

Since $\mathbb{I}^*(\omega)$ is monotone decreasing, it is also equivalent to choosing a threshold ω^*. Substituting this threshold into the objective yields an expression for the optimal cutoff rule:

$$\omega^*(X) = \arg\max_{\tilde{\omega}} \mathbb{E}\left[\log\left[p\tilde{\omega} \;\; + q^c(X,X')\int_{\tilde{\omega}}^1 \lambda(\omega,\phi')\,d\omega\right]|X\right]. \tag{11}$$

This proves Proposition 3 provided that V_1^i takes the shape of our guess. I now return to the second stage problems. Taking the solution to (11) as given, we know that the optimal plan for a k-producer sets $x = p(X)\,\omega^*(X)\,k$. Using the optimal policy for ω^* and these definitions, one can write the second stage Bellman equation without reference to the first stage Bellman equation. To do this, one can substitute for x and k in the second stage Bellman equation to rewrite the k-producer's problem as

$$\max_{c\ge 0,i,k^b\ge 0} \log(c) + \beta\mathbb{E}\left[\pi V_1^i + (1-\pi)V_1^p|X\right]$$

$$c + i + qk^b \;\; = \;\; p\omega^*(X)\,k \text{ and } k' = k^b + i + k\int_{\omega^*(X)}^1 \lambda(\omega,\phi')\,d\omega.$$

Now, since $\left(V_1^i, V_1^p\right)$ are increasing in k', and k^b and i are perfect substitutes, an optimal solution will set $i>0$ only if $q\ge 1$ and $k^b>0$ only if $q\le 1$. This implies that substituting the k-producer's capital accumulation equation into his budget constraint simplifies his problem to:

$$\max_{c\ge 0,k'} \log(c) + \beta\mathbb{E}\left[\pi V_1^i(k',X') + (1-\pi)V_1^p(k',X')|X\right] \text{ s.t. } c + q^c k' = w^i k.$$

The same steps allow one to write the c-producer's problem as,

$$\max_{c\ge 0,k'} \log(c) + \beta\mathbb{E}\left[\pi V_1^i + (1-\pi)V_1^p|X\right] \text{ s.t. } c + qk' = w^p k.$$

Replacing the definitions of V_1^i and V_1^p into the objective above, and substituting our guess yields V_2^i and V_2^p, we obtain:

$$\max_{c\geq 0,k'} \log c + \frac{\beta}{(1-\beta)} \log k' + \tilde{\psi}^i (X) \text{ s.t. } c + q^c k' = w^i k$$

and

$$\max_{c\geq 0,k'} \log c + \frac{\beta}{(1-\beta)} \log k' + \tilde{\psi}^p (X) \text{ s.t. } c + q k' = w^p k.$$

respectively. In this expressions $\tilde{\psi}^i (X)$ and $\tilde{\psi}^p (X)$ are functions of X which don't depend on the policy decisions. Taking first-order conditions for (k', c) in both problems leads to:

$$\begin{aligned} c^i &= (1-\beta) w^i (\omega^*, X) k \text{ and } k^{i,\prime} = \frac{\beta}{q^c} w^i (\omega^*, X) k \\ c^p &= (1-\beta) w^p (X) k \text{ and } k^{p,\prime} = \frac{\beta}{q} w^p (X) k. \end{aligned}$$

These solutions are consistent with the statement of Propositions 1 and 2. To verify that the guess for our value functions is the correct one, we substitute the optimal policies:

$$\begin{aligned} & \log (1-\beta) w^i (\omega^*, X) k + \frac{\beta}{(1-\beta)} \log \frac{\beta}{q^c} w^i (\omega^*, X) k + \tilde{\psi}^i (X) \\ = & \frac{\log w^i (\omega^*, X) k}{(1-\beta)} + \psi^i (X) = \frac{\log W^i (k, \omega^*, X)}{(1-\beta)} + \psi^i (X) \end{aligned}$$

for some function $\psi^i (X)$. The same steps lead to a similar expression for c-producers. This verifies the initial guess.

The last claim in Proposition 3 is shown using the implicit function theorem. Define,

$$\Omega (\tilde{\omega}, p; X) \equiv \mathbb{E} \left[\log \left(p\tilde{\omega} + q^i (X, X') \int_{\tilde{\omega}}^1 \lambda (\omega, \phi') \, d\omega \right) |X \right].$$

The optimal cutoff is a maximum of $\Omega (\tilde{\omega}, p; X)$. The Theorem of the Maximum asserts that $\tilde{\omega}$ is continuous in p since $\Omega (\tilde{\omega}, p; X)$ is a continuous function. I use the Implicit Function Theorem to show that $\omega^* (p)$ is increasing in p. Since the objective is continuous and differentiable, the first order condition is necessary for an interior solution:

$$\mathbb{E} \left[\left(p - q^i (X, X') \lambda (\tilde{\omega}, \phi') \right) \left(p\tilde{\omega} + q^i (X, X') \int_{\tilde{\omega}}^1 \lambda (\omega, \phi') \, d\omega \right)^{-1} |X \right] = 0.$$

The second derivative is,

$$\begin{aligned} \frac{\partial^2 \Omega (\tilde{\omega}, p; X)}{\partial \tilde{\omega}^2} &= \mathbb{E} \left[-q^i (X, X') \lambda_{\tilde{\omega}} (\tilde{\omega}, \phi') \left(p\tilde{\omega} + q^i (X, X') \int_{\tilde{\omega}}^1 \lambda (\omega, \phi') \, d\omega \right)^{-1} |X \right] - \\ & \mathbb{E} \left[\left(p - q^i (X, X') \lambda (\tilde{\omega}, \phi') \right)^2 \left(p\tilde{\omega} + q^i (X, X') \int_{\tilde{\omega}}^1 \lambda (\omega, \phi') \, d\omega \right)^{-2} |X \right] \end{aligned}$$

because $\lambda_{\tilde{\omega}} (\tilde{\omega}, \phi')$ is positive for any ϕ'. This shows the objective is concave and therefore, there is a unique maximum. Let's assume the maximum is interior for some p. Then, it solves $\partial \Omega (\tilde{\omega}, p; X) / \partial \tilde{\omega} = 0$. By the implicit function theorem, we have that

$$\frac{\partial \tilde{\omega}}{\partial p} = -\frac{\partial \Omega (\tilde{\omega}, p; X) / \partial p}{\partial^2 \Omega (\tilde{\omega}, p; X) / \partial \tilde{\omega}^2}.$$

It suffices to show $\partial\Omega\left(\tilde{\omega},p;X\right)/\partial p>0$. This expression is:

$$\mathbb{E}\left[\frac{1}{\left(p\tilde{\omega}+q^{i}\left(X,X'\right)\int_{\tilde{\omega}}^{1}\lambda\left(\omega,\phi'\right)d\omega\right)}-\frac{p\left(\tilde{\omega}-q^{i}\left(X,X'\right)\lambda_{\tilde{\omega}}\left(\tilde{\omega},\phi'\right)\right)}{\left(p\tilde{\omega}+q^{i}\left(X,X'\right)\int_{\tilde{\omega}}^{1}\lambda\left(\omega,\phi'\right)d\omega\right)^{2}}|X\right]$$

and arranging terms yields:

$$\mathbb{E}\left[\frac{\left(p\tilde{\omega}+q^{i}\left(X,X'\right)\int_{\tilde{\omega}}^{1}\lambda\left(\omega,\phi'\right)d\omega\right)-p\left(\tilde{\omega}-q^{i}\left(X,X'\right)\lambda_{\tilde{\omega}}\left(\tilde{\omega},\phi'\right)\right)}{\left(p\tilde{\omega}+q^{i}\left(X,X'\right)\int_{\tilde{\omega}}^{1}\lambda\left(\omega,\phi'\right)d\omega\right)}|X\right]>0$$

which is enough to guarantee a positive $\frac{\partial\tilde{\omega}}{\partial p}$. If there is any p for which $\omega^{*}=1$, then by continuity of ω^{*}, ω^{*} must also be increasing at that point also. This proves Proposition 3.

Q.E.D.

B.2. *Proof of Lemma 4 and Proposition 5*

PROOF: Lemma 4 and Proposition 5 are proven jointly here. We begin by guessing that $V_{1}^{f}\left(n,X\right)=v_{1}^{f}\left(X\right)n$, and $V_{2}^{f}\left(n,X\right)=v_{2}^{f}\left(X\right)n$, where $v_{2}^{f}\left(X\right)=\beta^{F}\mathbb{E}\left[v_{1}^{f}\left(X\right)R^{b}\right]$ if the banker remains alive, and $v_{2}^{f}\left(X\right)=\beta^{F}R^{b}n$ if he dies.

Plugging this guess into the bankers problem yields:

$$\max_{Q\geq0,e\in[0,\bar{e}],d\in[0,1]}\left(1-\tau\right)d-e+\beta^{F}R^{b}\mathbb{E}\left[v_{1}^{f}\left(X\right)\left(\Pi\left(X,X^{'}\right)Q+n'\right)|X\right]$$

subject to

$$\begin{aligned}-\min_{X'}\Pi\left(X,X^{'}\right)Q &\leq n'\\ n' &= n+e-d.\end{aligned}$$

Assume that the optimal solution to this problem is characterized by some $e^{*}\left(n,X\right)$ and $d^{*}\left(n,X\right)$ yet to be determined. In equilibrium, $\Pi\left(X,X^{'}\right)$ is finite. Hence, $\mathbb{E}\left[v_{2}^{f}\left(X'\right)\Pi\left(X,X^{'}\right)\right]$ is also finite, provided that the problem has a solution. If $\mathbb{E}\left[v_{2}^{f}\left(X'\right)\Pi\left(X,X^{'}\right)\right]>0$ and $-\min_{X'}\Pi\left(X,X^{'}\right)\leq0$, the banker would set $Q^{*}=\infty$. But this would imply that in equilibrium $\Pi\left(X,X^{'}\right)\leq0$ for any X' because there cannot be a future state where firms provide infinite intermediation and there are positive profits. Hence, it is the case that if $\mathbb{E}\left[v_{2}^{f}\left(X'\right)\Pi\left(X,X^{'}\right)\right]>0\rightarrow-\min_{X'}\Pi\left(X,X^{'}\right)>0$. Now if this is the case,

$$Q^{*}=\frac{n'}{-\min_{X'}\Pi\left(X,X'\right)}>0. \tag{12}$$

If $\mathbb{E}\left[v_{2}^{f}\left(X'\right)\Pi\left(X,X^{'}\right)\right]<0$, the producer optimally sets Q^{*}. If $\mathbb{E}\left[v_{2}^{f}\left(X'\right)\Pi\left(X,X^{'}\right)\right]=0$, Q^{*} is indeterminate (but finite). Thus, in either case, $\mathbb{E}\left[v_{2}^{f}\left(X'\right)\Pi\left(X,X^{'}\right)Q^{*}\right]=0$.

This implies that for any optimal policy,

$$\mathbb{E}\left[v_{2}^{f}\left(X'\right)\Pi\left(X,X^{'}\right)Q^{*}\right]=\max\left\{\frac{v_{2}^{f}\left(X'\right)\Pi\left(X,X^{'}\right)n'}{-\min_{X'}\Pi\left(X,X'\right)},0\right\}.$$

Thus, one can substitute this expression into the objective of the firm and express it without reference to Q :

$$\max_{e\in[0,\bar{e}],d\in[0,n]}(1-\tau)\,d-e+(n+e-d)\beta^{F}R^{b}\mathbb{E}\left[v_{2}^{f}\left(X'\right)+\max\left\{\frac{v_{2}^{f}\left(X'\right)\Pi\left(X,X^{'}\right)}{-\min_{X'}\Pi\left(X,X'\right)},0\right\}|X\right]$$

where I also used the definition of n'. Now, it is clear from this expression that (e,d) are the choices of a linear program. Consequently, any optimal financial policy satisfies

$$\begin{aligned} e &> 0 \text{ only if } \beta^{F}\left[\mathbb{E}[v_{2}^{f}(X')]+\max\left\{\frac{\mathbb{E}[v_{2}^{f}(X')\Pi\left(X,X'\right)]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)},0\right\}\right]\geq 1 \text{ and}\\ d &> 0 \text{ only if } \beta^{F}\left[\mathbb{E}[v_{2}^{f}(X')]+\max\left\{\frac{\mathbb{E}[v_{2}^{f}(X')\Pi\left(X,X'\right)]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)},0\right\}\right]\leq (1-\tau).\end{aligned}$$

If the inequalities are strict, it is clear that $e=\bar{e}$ and $d=n$. By linearity, e and d are indeterminate when the relations hold with equality, and equal 0 if these are not satisfied. This implies that $e\left(n,X\right)=e^{*}\left(X\right)n$, $d\left(n,X\right)=d^{*}\left(X\right)n$ are solutions to the banker's problem without loss in generality.

We now use these results to show that the value function is linear in n. Substituting the optimal policies into the objective we obtain:

$$\left[(1-\tau)\,d^{*}\left(X\right)-e^{*}\left(X\right)+\left(1+e^{*}\left(X\right)-d^{*}\left(X\right)\right)\beta^{F}R^{b}\mathbb{E}\left[v_{2}^{f}\left(X'\right)+\max\left\{\frac{v_{2}^{f}\left(X'\right)\Pi\left(X,X^{'}\right)}{-\min_{X'}\Pi\left(X,X'\right)},0\right\}|X\right]\right]n,$$

which is a linear function of n.

Returning to the optimal quantity decision, it is clear that Q can be written as,

$$Q^{*}\left(X\right)n=\left\{\frac{1+e^{*}\left(X\right)-d^{*}\left(X\right)}{-\min_{X'}\Pi\left(X,X'\right)}\right\}n.$$

and clearly $Q^{*}\left(X\right)=\arg\max_{\tilde{Q}}\mathbb{E}\left[v_{2}^{f}\left(X'\right)\Pi\left(X,X'\right)|X\right]\tilde{Q}$ subject to $\Pi\left(X,X^{'}\right)\tilde{Q}\leq n'$. This proves Proposition 6.

We are ready to show that $v_{1}^{f}\left(X\right)$ solves the functional equation described in the body of the text. Define

$$\tilde{v}\left(X\right)=\beta^{F}R^{b}\mathbb{E}\left[v_{2}^{f}\left(X'\right)+v_{2}^{f}\left(X'\right)\max\left\{\frac{\Pi\left(X,X^{'}\right)}{-\min_{X'}\Pi\left(X,X'\right)},0\right\}|X\right]$$

as the marginal value of equity in the bank, and note that

$$v_{1}^{f}\left(X\right)=\max_{d^{*}(X)\in[0,1],e\geq 0}(1-\tau)\,d^{*}\left(X\right)-e^{*}\left(X\right)+\left(1+e^{*}\left(X\right)-d^{*}\left(X\right)\right)\tilde{v}\left(X\right).$$

If $\tilde{v}\left(X\right)\in\left(\left(1-\tau\right),1\right)$, then $v_{1}^{f}\left(X\right)=\tilde{v}\left(X\right)$ because $\left(d^{*}\left(X\right),e^{*}\left(X\right)\right)=0$. If $\tilde{v}\left(X\right)\leq\left(1-\tau\right)$, then $\mathrm{e}^{*}\left(X\right)=0$ and we have that

$$(1-\tau)\,d^{*}\left(X\right)+\left(1-d^{*}\left(X\right)\right)\tilde{v}\left(X\right)=\left(1-\tau\right).$$

Finally, if $\tilde{v}\left(X\right)=1$, then, $v_{1}^{f}\left(X\right)=1$. This information is summarized in the following functional equation for

$v_1^f(X)$:

$$v_1^f(X) = \min\left\{\max\left\{\beta^F R^b \mathbb{E}\left[v_2^f(X')\left\{1+\max\left\{\frac{\Pi\left(X,X'\right)}{-\min_{X'}\Pi(X,X')},0\right\}\right\}|X\right],(1-\tau)\right\},1\right\}$$

which equals

$$\min\left\{\max\left\{\mathbb{E}\left[\left(\rho+(1-\rho)\,v_1^f(X'')\right)\beta^F R^b\left\{1+\max\left\{\frac{\Pi\left(X,X'\right)}{-\min_{X'}\Pi(X,X')},0\right\}\right\}|X\right],(1-\tau)\right\},1\right\}.$$

This functional equation determines the slope of the banker's value function, $v_1^f(X)$. It can be shown that the solution to this functional equation is unique. Assumptions 9.18-9.20 of Stokey et al. (1989) are satisfied by this problem. It remains to be shown that Assumption 9.5 (part a) is also satisfied. By assumption, X is compact, so the only piece left is that X is countable. Because the transition function for the state is an endogenous object, it depends on an aggregate state, κ. It will be shown that although (d,e) are not uniquely defined, there is unique mapping from ϕ to κ'. By exercise 9.10 in Stokey et al. (1989), together these assumptions ensure that there is a unique solution to the this functional equation. *Q.E.D.*

B.3. *Proof of Proposition 6*

PROOF: To obtain an expression for q, fix any sequence of states (X,X') and call $\bar{\omega}\equiv\omega(X)$. Assume $q>1$ so that $D^i=0$. Market clearing in stage 2 requires $D^p(X,X')=S(X)=\mathbb{E}[\lambda(\omega,\phi')\,|\omega\le\bar{\omega}]\,\bar{\omega}\pi K$. By Proposition 1, we can integrate across the c-producer's policy functions to obtain an expression for $D^p(X,X')$ as a function of q:

$$\beta\int\left[\frac{W^p(k,x,X)}{q}-\bar{\lambda}(\phi')\,k\right]\Gamma^c(dk)=\beta\frac{A+q\bar{\lambda}(\phi')}{q}(1-\pi)\,K.$$

By market clearing, q is such that:

$$\left[\beta\frac{A+q\bar{\lambda}(\phi')}{q}-\bar{\lambda}(\phi')\right](1-\pi)\,K=\mathbb{E}[\lambda(\omega,\phi')\,|\omega\le\bar{\omega}]\,\bar{\omega}\pi K.$$

Manipulating this expression leads to the value of q that satisfies market clearing:

$$q=\frac{\beta A(1-\pi)}{\mathbb{E}[\lambda(\omega,\phi')\,|\omega\le\bar{\omega}]\,\bar{\omega}\pi+(1-\pi)(1-\beta)\,\bar{\lambda}(\phi')}.$$

Recall now that this expression is valid only when $q>1$, because capital good producers are not participating in the market. Thus, the expression is only true for values of

$$\beta A\left[\frac{\pi}{(1-\pi)}\mathbb{E}[\lambda(\omega,\phi')\,|\omega\le\bar{\omega}]\,\bar{\omega}+(1-\beta)\,\lambda(\phi')\right]^{-1}>1. \tag{13}$$

If $q=1$, then it must be the case that the total demand for capital must be larger than the supply provided by financial firms. In this case, $D^i(X,X')$ is obtained again by integrating across the demand for k-producers' capital given in Proposition 1. Thus, for a stage one price p, this demand is given by

$$D^i+I=\beta p\bar{\omega}\pi K-(1-\beta)\,\mathbb{E}[\lambda(\omega,\phi')\,|\omega\le\bar{\omega}]\,(1-\bar{\omega})\,\pi K \text{ for } q=1.$$

The corresponding condition is

$$\beta p\bar{\omega}-(1-\beta)\,\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega\leq\bar{\omega}\right](1-\bar{\omega})\,\pi+\left[\beta A-(1-\beta)\,\bar{\lambda}\left(\phi'\right)\right](1-\pi)\geq\pi\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega\leq\bar{\omega}\right]\bar{\omega} \tag{14}$$

where the aggregate capital stock has been canceled from both sides. If the condition is satisfied, then $q=1$, and

$$D^{i}\left(q,p\right)=\pi\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega\leq\bar{\omega}\right]\bar{\omega}-\left[\beta R-(1-\beta)\,\bar{\lambda}\left(\phi'\right)\right](1-\pi)$$

and

$$I=\left[\beta p\bar{\omega}-(1-\beta)\,\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega>\bar{\omega}\right](1-\bar{\omega})\right]\pi-D^{i}\left(q,p\right).$$

If (13) and (14) are violated, this implies $q<1$ and $I=0$. The corresponding market clearing-condition is obtained by solving q from

$$\begin{aligned}&\left[\frac{\beta p\bar{\omega}}{q}-(1-\beta)\,\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega>\bar{\omega}\right](1-\bar{\omega})\right]\pi+\left[\frac{\beta A}{q}-(1-\beta)\,\bar{\lambda}\left(\phi'\right)\right](1-\pi)\\ \geq\quad&\pi\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega<\bar{\omega}\right]\bar{\omega}.\end{aligned}$$

We can collect the terms where q shows in the denominator to obtain,

$$\frac{\beta\left(p\bar{\omega}\pi+A\right)}{q}=\pi\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega<\bar{\omega}\right]\bar{\omega}+(1-\beta)\left[-\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega<\bar{\omega}\right]\bar{\omega}\pi+\bar{\lambda}\left(\phi'\right)\right].$$

The solution is given by

$$q=\frac{\beta\left(\pi p\bar{\omega}+A\right)}{\left(\beta\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\bar{\omega}<\bar{\omega}\right]\bar{\omega}\pi+(1-\beta)\,\bar{\lambda}\left(\phi'\right)\right)}$$

The formula in Proposition 6 corresponds to this expression. Moreover, the demand function is weakly decreasing so for each p,X there will be a unique q satisfying the market clearing condition. We can express the profit function in the following way:

$$\Pi\left(X,X'\right)=\max\left\{(1-\pi)\,\beta A\frac{\pi\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\bar{\omega}<\bar{\omega}\right]\bar{\omega}}{\pi\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\bar{\omega}<\bar{\omega}\right]\bar{\omega}+(1-\beta)\,\bar{\lambda}\left(\phi'\right)},\tilde{\pi}(X,X')\right\}$$

where

$$\tilde{\Pi}(X,X')=\min\left\{1,\left(\pi p^{i}+A\,(1-\pi)\right)\frac{\pi\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\bar{\omega}<\bar{\omega}\right]\bar{\omega}}{\left(\frac{(1-\beta)\lambda}{\beta}+\pi\bar{\omega}\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\bar{\omega}<\bar{\omega}\right]\right)}\right\}.$$

Since both functions are increasing in $\mathbb{E}\left[\lambda\left(\omega,\phi'\right)|\omega<\bar{\omega}\right]$, the conditional expectation, we know by Assumption **A1,** that these functions are decreasing in the shock ϕ'. Thus, $\Pi\left(X,X'\right)$ is decreasing in ϕ'. *Q.E.D.*

B.4. *Proof of Proposition 7*

PROOF: Take two values of financial risk capacity, $\kappa^{L}<\kappa^{H}$. Fix any exogenous state $A\times\phi$ and denote by X^{L} and X^{H} the corresponding aggregate states for these two levels of financial risk capacity. By the last claim in Proposition 3, it is enough to show that $Q^{*}\left(X^{L}\right)\leq Q^{*}\left(X^{H}\right)$ in any RCE to argue that ω^{*} and p are increasing in κ. This follows from the market-clearing condition in the first stage, $Q^{*}\kappa=\omega^{*}$. We show by contradiction that $Q^{*}\left(X^{H}\right)>Q^{*}\left(X^{L}\right)$

cannot be part of a RCE equilibrium as $\rho \to 1$. Optimality of the banker's problem requires to solve:

$$Q^*\left(X\right) = \arg\max_{\tilde{Q}} \mathbb{E}\left[v_2^f\left(X'\right)\Pi\left(X, X'\right)|X\right]\tilde{Q} \text{ subject to } \Pi_{\min\phi'}\left(X, X^{'}\right)\tilde{Q} \leq 1, \forall X'$$

so either the constraint binds or $\mathbb{E}\left[v_2^f\left(X'\right)\Pi\left(X, X'\right)|X\right] = 0$. Assume the constraint binds when the financial risk capacity is low, κ^L. This implies that:

$$\begin{aligned} \mathbb{E}\left[v_2^f\left(X^{L\prime}\right)\Pi\left(X^L, X^{L\prime}\right)|X^L\right] &> 0 \text{ and} \\ \Pi_{\min\phi'}\left(X^L, X^{L'}\right)\tilde{Q}\left(X^L\right) &= 1. \end{aligned}$$

In turn, market clearing implies that $\min_{\phi'}\Pi\left(X^L, X^{L'}\right)\tilde{Q}\left(X^L\right)N = \min_{\phi'}\Pi\left(X^L, X^{L'}\right)\omega^*\left(X^L\right)K$, which in turn implies that $\min_{\phi'}\Pi\left(X^L, X^{L'}\right)\omega^*\left(X^L\right) = \kappa$. Thus, $\omega^*\left(X^L\right)$ is feasible for κ^H since $\min_{\phi'}\mathbf{\Pi}\left(\omega^*\left(X^L\right), A, \phi\right)\omega^*\left(X^L\right) < \kappa^o$ implies $\mathbf{\Pi}\left(\omega^*\left(X^L\right), A, \phi\right)\tilde{Q}\left(X^L\right) < 1$. Therefore, if a RCE features $Q^*\left(X^L\right) > Q^*\left(X^H\right)$, it must be that $\mathbb{E}\left[v_2^f\left(A, \phi, \kappa^H + \mathbf{\Pi}\left(\omega^*\left(X^L\right), A, \phi\right)\right)\mathbf{\Pi}\left(\omega^*\left(X^L\right), A, \phi\right)|X^L\right] < 0$. However, for $\rho \to 1$, $v_2^f \longrightarrow \beta^F R^b$ pointwise, there always exists some ρ sufficiently close to 1, such that $\mathbb{E}\left[v_2^f\left(X^{L\prime}\right)\Pi\left(X^L, X^{L\prime}\right)|X^L\right] > 0$ also implies:

$$\mathbb{E}\left[v_2^f\left(A, \phi, \kappa^H + \mathbf{\Pi}\left(\omega^*\left(X^L\right), A, \phi\right)\right)\mathbf{\Pi}\left(\omega^*\left(X^L\right), A, \phi\right)|X^L\right] \geq 0. \tag{15}$$

In this case, $Q^*\left(X^H\right) < Q^*\left(X^L\right)$ cannot hold in a highest price equilibrium.

Assume that given X^L, $\mathbb{E}\left[v_2^f\left(X^{L\prime}\right)\Pi\left(X^L, X^{L\prime}\right)|X^L\right] = 0$ so bankers are not constrained. Hence, $Q^*\left(X^L\right)$ is not binding for financial risk capacity κ^L. Then, if a RCE features $Q^*\left(X^H\right) < Q^*\left(X^L\right)$, it must be that $\mathbb{E}\left[v_2^f\left(A, \phi, \kappa^H + \mathbf{\Pi}\left(\omega^*\left(X^L\right), A, \phi\right)\right)\mathbf{\Pi}\left(\omega^*\left(X^L\right), A, \phi\right)|X\right] < 0$. However for $\rho = 1$, $v_2^f \longrightarrow \beta^F R^b$, the two conditions cannot hold at the same time.

Q.E.D.

APPENDIX C: ADDITIONAL FIGURES

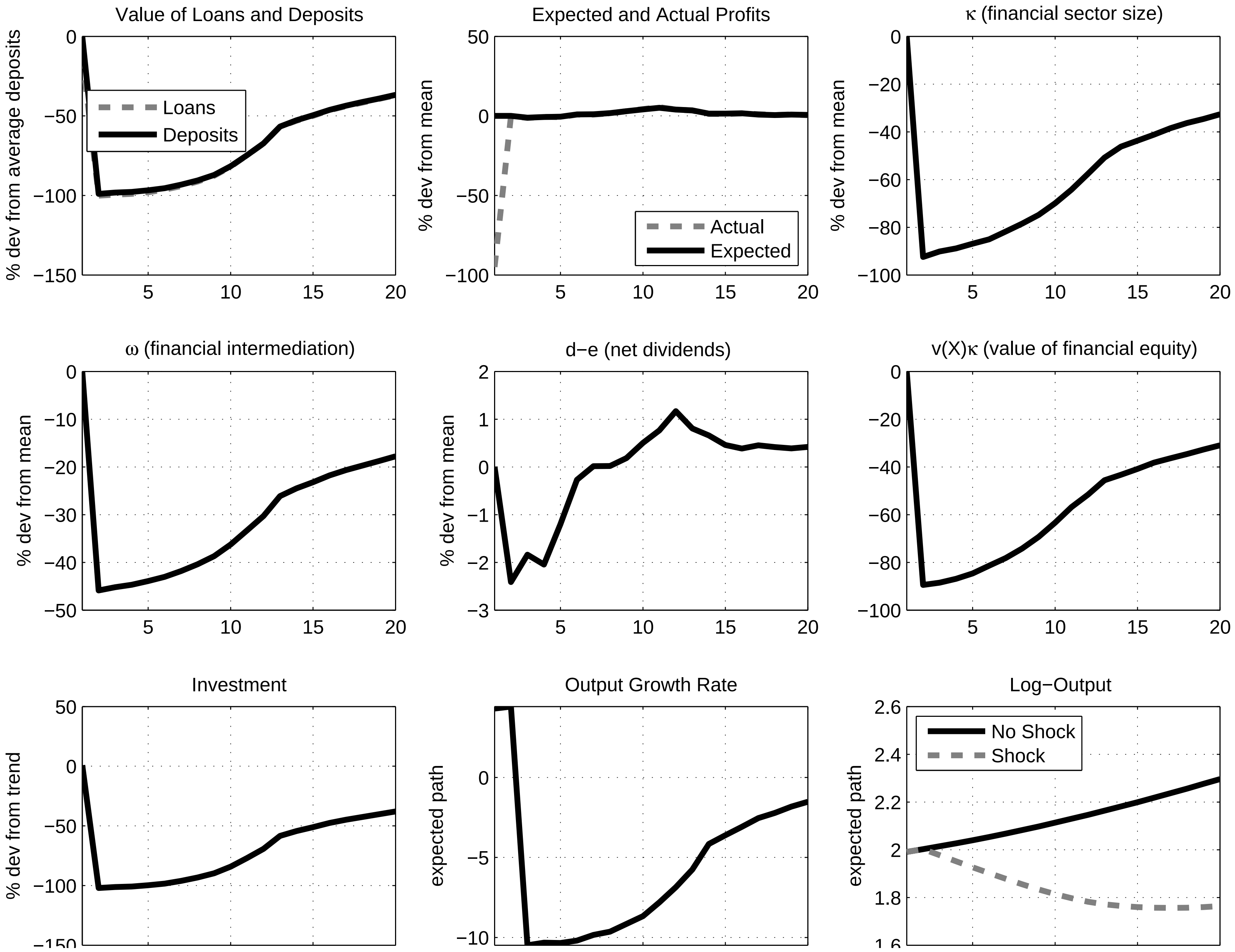

FIGURE 10.— **Impulse response function to small ϕ.** The figure plots the response of the system to a realization of the largest value of ϕ.

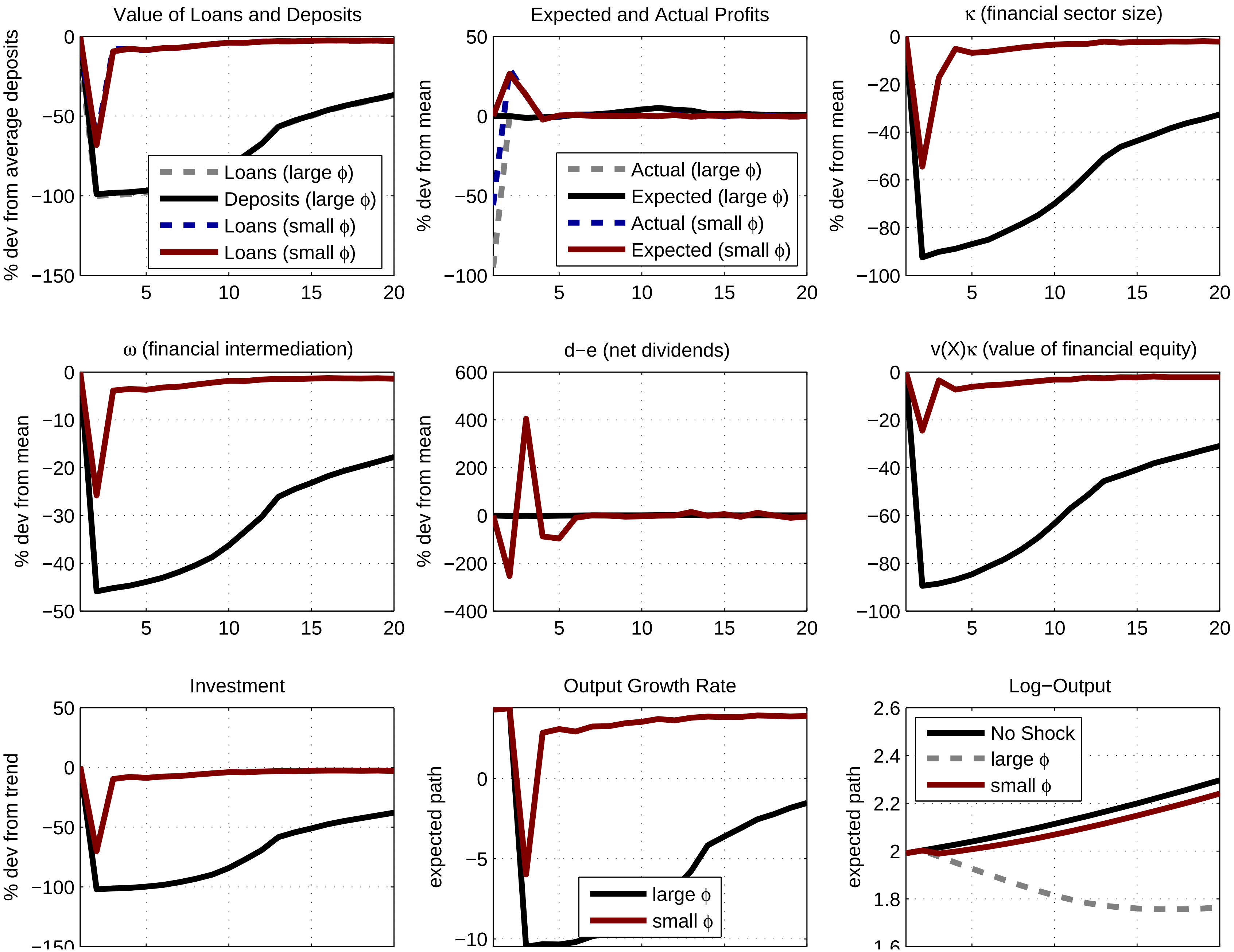

FIGURE 11.— **Impulse response function to large and small ϕ.**