Why Do Investors Trade?

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Abstract

We propose and estimate a structural model of daily stock market activity to test competing theories of trading volume. The model features informed rational speculators and uninformed agents who trade either to hedge endowment shocks or to speculate on perceived information. To identify the model parameters, we exploit enormous empirical variation in trading volume, market liquidity, and return volatility associated with regular and extended-hours markets as well as news arrival. We find that the model matches market activity well when we allow for overconfidence. At plausible values of overconfidence and risk aversion, overconfidence—not hedging—explains nearly all uninformed trading, while rational informed speculation accounts for most overall trading. Without overconfident investors, over 99% of trading volume disappears even when informed rational traders disagree maximally. These findings illustrate that modest overconfidence can help explain stark patterns in stock market activity.

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Investors in US equities now trade *tens of trillions* of dollars each year—a fivefold increase in less than 20 years (Chordia, Roll, and Subrahmanyam (2011)). Even before this trend, DeBondt and Thaler (1995) argue the high trading volume observed in financial markets "is perhaps the single most embarrassing fact to the standard finance paradigm," and speculate that most volume comes from traders who are overconfident in their abilities to pick stocks. Odean (1998) provides a model in which overconfidence generates excessive trade and argues this activity is costly to society. Others, such as Harris and Raviv (1993) and Kim and Verrecchia (1997), model trading that results from disagreement among agents. More generally, disagreement within Kyle's (1985) informed trading framework can occur either among fully rational agents who receive noisy but informative signals or among biased agents. While Chordia, Roll, and Subrahmanyam (2011) provide some evidence that informed trading by institutions has driven recent volume increases, there are disappointingly few tests of which models of market activity are consistent with the data. Consequently, the causes and social consequences of trading activity are not well understood.

In this paper, we propose and estimate a structural model in the spirit of Kyle (1985) and Spiegel and Subrahmanyam (1992) with agents exhibiting canonical theoretical motives for trading. This framework allows us to evaluate the relative importance of each motive and implications for market activity and welfare. Our model features rational speculators who may collect valuable information, along with two groups of risk-averse uninformed agents: one who rationally trades to hedge endowment shocks, and another who overconfidently speculates on perceived information. All agents optimally choose whether to enter the market and how much to trade conditional on their information and expectations of other traders' behavior. A risk-neutral competitive market maker sets the equilibrium stock price in response to aggregate demand.[1]

[1] The model builds on Spiegel and Subrahmanyam's (1992) analysis of rational uninformed trading by allowing for overconfident speculation and by endogenizing entry into the market.

Trading volume, liquidity, and return variance arise as endogenous outcomes of this market equilibrium. Importantly, the model is flexible enough to allow for disagreement among rational informed agents as well as their overconfident uninformed counterparts. It also nests both rational heterogeneous belief models and behavioral models with no hedgers. To our knowledge, we are the first to use trading volume and liquidity as the basis for distinguishing rational and behavioral models of stock market activity.

Our novel identification strategy exploits *enormous* empirical variation in trading volume, market liquidity, and return volatility associated with the time of day and news arrival in the electronic trading era. Since 2000, revolutions in information and trading technology have enabled a dramatic expansion in the hours of market activity in US stocks. Newswires now arrive continuously throughout the day, and electronic communication networks give any institution or retail trader with a brokerage account the ability to trade outside regular market hours (from 9:30am to 4pm). Despite having round-the-clock information and expanded market access, most still choose to trade in regular market hours. Less than 5% of total trading in our sample takes place in the pre-market (defined as 7am to 9:30am) and after-market (4pm to 6:30pm), and both periods are far less liquid than the regular market. In contrast, stock return volatility during extended hours is more than half of that during the regular market. In fact, in periods when news arrives, extended hours volatility is on par with that in regular hours.

In estimation, our model matches this rich set of stylized facts quite well. The model's prediction errors for variance, volume, and liquidity are small in economic terms, and we cannot statistically reject the model's over-identifying restrictions. Equally important, the parameters' small standard errors indicate that they are well-identified.

The parameter estimates show how plausible cross-period variation in the information environment can explain enormous differences in market activity. The precision of signals observed by informed traders is roughly twice as large in extended hours as it is in regular market periods. The resulting information asymmetry reduces liquidity and overconfident agents' willingness to trade in extended hours, which further reduces liquidity. Illiquidity also deters participation and limits the aggressiveness of informed traders. Consequently, predicted volume is roughly 100 times smaller during extended hours than it is in the regular market. Similar, but less dramatic, comparisons arise for periods without and with news arrival. In addition, estimates of acquirable private information are higher in regular market periods than in extended hours. This results in more informed trading and volatility in the regular market than in other periods. The same is true when comparing periods with and without news arrival.

One especially striking empirical finding is that a modest level of overconfidence is key to explaining market activity. Uninformed agents, whose perceived signals correctly predict the direction of future returns with a 0.50 probability (i.e., they are pure noise), only need to *believe* that they observe a signal that is correct 54% of the time. The strength of this mistaken belief is modest in comparison to survey and experimental evidence on overconfidence in the precision of information. Moreover, it would take hundreds of independent trials for such biased agents to recognize that they are overconfident, even if they accurately remember their successes and failures. Overconfidence is a particularly potent source of trading volume because of its feedback effects. If uninformed agents are more overconfident, they trade more, which increases market liquidity; and this increased liquidity promotes additional trading by all agents.

We estimate several nested models with no overconfidence, and all fail at both fitting the data and generating economically plausible parameter estimates. Intuitively, such a model would require all uninformed trading to come from hedgers who weigh the benefits of hedging endowment shocks against the costs of price impact and market entry. In the estimations, hedging activity is unable to generate sufficiently high trading in the regular market and sufficiently low trading during extended hours, even with large cross-period variation in information asymmetry. The hedging model also predicts that prices incorporate very little private information, which is difficult to reconcile with the empirical evidence. In the unrestricted model that allows for overconfidence, hedgers play a small role in observed trading activity.

Interestingly, while some overconfidence is important, our analyses of market activity suggest its role is subtle. Overconfident agents directly account for only 19% of trading volume. In contrast, informed trading accounts for 71% as these traders' signals are only weakly correlated and they often trade with each other. In addition, our return variance decomposition indicates prices incorporate 84% of information collected by informed traders within the same period.[2] Thus, overconfident agents' direct impact on volume is smaller than their indirect impact: their presence primarily enhances market liquidity and promotes informed trading. Through this indirect channel, overconfident agents provide a societal benefit. In sum, our findings demonstrate that overconfidence can play a critical role in explaining trading volume and market liquidity without distorting market prices.

Our structural approach complements existing theoretical and empirical studies of overconfidence. Other models of overconfidence, such as those by Odean (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Gervais and Odean (2001), have only been calibrated

[2] Because there is competition among rational risk-neutral market makers in our model, market prices are always *semi-strong* efficient, meaning that they fully incorporate public information (Fama (1970)). The 91% estimate above is one way to quantify the degree to which the *strong* form of the efficient market hypothesis holds.

to selected empirical moments.[3] Empirical studies, such as those by Barber and Odean (2000) and Statman, Thorley, and Vorkink (2006), uncover strong patterns in individual investor behavior and market activity that are qualitatively consistent with these models of overconfidence. A complete explanation of the rich evidence in these studies as well as ours likely requires modeling additional behavioral biases.[4] Our study is an important step in this direction as it provides a framework within which such biases could be added.

The paper is organized as follows. Sections 1 and 2 describe our model of market activity and identification strategy. Section 3 describes the data. Sections 4 and 5 summarize and analyze our results. Section 6 concludes.

1. Model of Market Activity

We consider a model of a single risky asset (hereafter "stock"), which features rational risk-neutral informed traders along with two types of risk-averse uninformed traders: overconfident speculators and rational hedgers. The model borrows key elements from the Spiegel and Subrahmanyam (1992), Kyle and Wang (1997), and Scheinkman and Xiong (2003) models. In our empirical work, we apply this model of a single stock to all stocks. We are thus assuming that the model parameters—when scaled as discussed in Section 2.A—are constant across firms. We later provide separate analyses of firms sorted by market capitalization to evaluate this assumption. We also assume market activity is independent across stocks after controlling for systematic factors, such as market returns.

The stock's fundamental value in the distant future trading round T is given by

[3] Alti and Tetlock (2013) provide structural estimates of overconfidence based on asset return predictability, but they use a representative agent framework and thus do not consider trading volume.

[4] For example, Barberis and Xiong (2012) and Ingersoll and Jin (2013) convincingly argue that myriad patterns in trading volume, such as the disposition effect, are consistent with models of realization utility in which traders directly experience pleasure (pain) from realizing investment gains (losses).

$$F_T = \overline{F} + \sum_{t=1}^{T-1} d_t, \tag{1}$$

where $\overline{F}$ is the initial value of the stock at time 0 and the d_t terms are independently normally distributed with means of zero and standard deviations of σ_{dt}. In each trading round t, public information reveals the value of d_{t-1}, traders choose whether to enter the market and submit orders based on private signals, and a competitive market maker sets the equilibrium price (p_t).

Agents of each type are drawn from respective homogeneous populations. Each agent enters the market at most once to maximize expected utility defined over future profits. Optimal entry decisions endogenously determine the numbers of informed traders (m_I), overconfident speculators (m_B), and uninformed hedgers (m_H). We assume that informed agents and speculators have one-period horizons, which matches the assumed useful life of their perceived information, while hedgers have one-year horizons to match endowment shock duration estimates in Heaton and Lucas (2000).[5] We later analyze robustness to our hedger horizon and entry assumptions. All traders are strategic in the sense that each considers his/her price impact.

In trading round t, informed trader i can observe a noisy signal (s_{it}) of d_t:

$$s_{it} = \eta d_t + \sqrt{1-\eta^2}\, z_{it}, \tag{2}$$

$$z_{it} = \rho_z z_t + \sqrt{1-\rho_z^2}\, \delta_{it}, \tag{3}$$

where δ_{it} are independent across traders. The signal noise terms z_t and δ_{it} are independently normally distributed with means of zero and standard deviations of σ_{dt}. The parameter $\eta \in [0,1]$ is the signal's informativeness, while the parameter ρ_z governs disagreement among traders, where $\rho_z = 0$ implies maximum disagreement. The pairwise correlation between signals is

[5] Because informed traders are risk-neutral and their information is short-lived, they behave identically whether they have one-period or infinite horizons and whether they enter the market once or many times.

$$\rho_I^2 = \eta^2 + (1-\eta^2)\rho_z^2. \tag{4}$$

To enter the market and learn the signal, each informed trader *i* must pay a cost c_i, which includes the opportunity cost of investigating the stock and trading. We assume the cost of the marginal trader entering is a quadratic function of the number of informed traders who enter, reflecting differences in traders' opportunity costs.

We assume uninformed yet overconfident speculators (j) have utility functions $u_{jt} = -\exp(-\frac{1}{\tau_B}\pi_{jt})$ with common risk tolerance of τ_B. The combination of constant absolute risk aversion (CARA) utility and normally distributed signals and dividends implies that trader j's expected utility (U_{jt}) depends only on the mean and variance of next period's profits according to

$$U_{jt} = E(u_{jt} \mid I_{jt}) = E_j(\pi_{jt+1}) - \frac{1}{2\tau_B}Var_j(\pi_{jt+1}), \tag{5}$$

where I_{jt} is trader j's information at time t. Such traders believe that they observe an informative signal s_{jt}^B with a structure similar to the s_{it} signals such that

$$s_{jt}^B = \eta_B d_t + \sqrt{1-\eta_B^2}\, z_{jt}^B, \tag{6}$$

where the B sub- or superscript denotes a variable specific to biased agents and $\eta_B \in [0,1]$ is perceived signal informativeness. They believe the error component of their signals follows

$$z_{jt}^B = \rho_{Bu} z_t^B + \sqrt{1-\rho_{Bu}^2}\,\phi_{jt}, \tag{7}$$

where z_t^B is independent of z_t and the ϕ_{jt} are independent across traders. The parameter ρ_{Bu} governs the perceived correlation among uninformed traders' signal errors. Uninformed traders therefore perceive the pairwise correlation between their signals to be

$$\rho_B^2 = \eta_B^2 + (1-\eta_B^2)\rho_{Bu}^2. \tag{8}$$

Analogous to the informed traders, each speculator j must pay a cost c_j to enter the market and observe the signal, where the cost is again quadratic in the number of speculators who enter.

In contrast, the *true* signal process for overconfident agents, which is understood by both informed agents and market makers, is simply

$$s_{jt} = \rho_u z_t^B + \sqrt{1-\rho_u^2}\phi_{jt}. \tag{9}$$

The parameter ρ_u determines the actual correlation among uninformed traders' signals and does not depend on perceived signal informativeness. Uninformed traders' signals in Equation (9) are not actually correlated with dividends, even though traders with $\eta_B > 0$ believe they are. Because overconfidence in our model is based on misplaced faith in the accuracy of a common signal, one can interpret it as trading on common investor sentiment.[6] In this interpretation, overconfident traders follow similar spurious signals, such as investment newsletters, technical trading rules, or flawed valuation methods.

Overconfident agents have correct beliefs about the number of other traders and their strategies and maximize their subjective expected utility accordingly. Thus, aside from incorrectly perceiving their signals, overconfident traders behave rationally. This formulation follows Scheinkman and Xiong (2003) and Alti and Tetlock (2013) and is similar to that in Kyle and Wang (1997), Odean (1998), and Daniel, Hirshleifer, Subrahmanyam (1998). Importantly, our model nests the $\eta_B = 0$ case in which uninformed speculators rationally do not trade and all uninformed trading comes from hedgers.

Hedgers (k) have utility functions $u_{kt} = -\exp(-\frac{1}{\tau}\pi_{kt})$ with common risk tolerance of τ and trade only to hedge endowment shocks h_{kt} that follow

$$h_{kt} = \rho_h h_t + \sqrt{1-\rho_h^2}\psi_{kt}, \tag{10}$$

where h_{kt} are measured in shares and ψ_{kt} are independent across traders. The parameter

[6] This use of the term sentiment is consistent with the model by DeLong, Shleifer, Summers, and Waldmann (1990).

$\rho_h \in [0,1]$ allows for positive correlation among hedgers' endowment shocks, which has two important effects. First, it allows aggregate hedging demand to have a significant impact on prices. Second, insofar as the aggregate liquidity shock (h_t) affects prices, each hedger has an incentive to speculate on information about the aggregate shock conferred by the realization of his own private endowment shock (h_{kt}). Analogous to other traders, each hedger k must pay a cost c_k to enter the market, monitor endowment shocks, and trade. The marginal entry cost is quadratic in the number of hedgers who enter. Entry costs are presumably lower for hedgers and speculators than for informed traders, who actually collect informative signals.

As in Kyle (1985), we focus on symmetric linear equilibriums in which informed traders and uninformed speculators and hedgers submit utility-maximizing market orders (x_{it}, x_{jt}, and x_{kt}, respectively) that are linear in their private information. All traders within each group use the same strategies, and the competitive market maker uses a pricing rule that depends linearly with a slope of λ_t on the aggregate net order flow (Q_t), where

$$Q_t = \sum_{i=1}^{m_I} x_{it} + \sum_{j=1}^{m_B} x_{jt} + \sum_{k=1}^{m_H} x_{kt}. \tag{11}$$

Recall that informed trader i observes $I_{it} = \{s_{it}\}$, while uninformed speculators and hedgers observe $I_{jt} = \{s_{jt}^B\}$ and $I_{kt} = \{h_{kt}\}$, respectively. We denote the aggressiveness of informed trading on s_{it} by β_1 and the aggressiveness of uninformed trading based on s_{jt}^B and h_{kt} by β_2 and β_3.

The market clearing condition for the stock results in a price that is linear in traders' information. We assume that any buy-sell imbalance is accommodated by a rational risk-neutral market maker, which ensures that the market clearing price is an unbiased estimate of firm value ($E(F_T|p_t) = p_t$). In other words, there are no limits to arbitrage in the model, so the price is semi-strong efficient. The absence of mispricing allows us to focus on the model's predictions for variance, volume, and liquidity.[7]

We solve the model by conjecturing an equilibrium and evaluating whether agents have an incentive to deviate using backward induction. In Appendix A, we characterize the symmetric equilibrium in which all traders and market makers follow the linear strategies above and all traders endogenously choose to participate in the market. The key endogenous parameters are the equilibrium sensitivity of prices to order flow (λ_t), informed and uninformed trader aggressiveness (β_1, β_2, and β_3), and the number of informed traders (m_I), overconfident speculators (m_B), and uninformed hedgers (m_H). The Appendix provides expressions for these parameters.

2. Identification

A. Model Predictions

We identify the model parameters by their impacts on the model's predictions of return variance, expected trading volume, and market liquidity. We adopt standard definitions of stock returns (r_t) and share volume (v_t) used in studies such as Admati and Pfleiderer (1988),

$$r_t = p_t - p_{t-1} \tag{12}$$

$$v_t = \max(\sum buys_t, \sum sells_t), \tag{13}$$

and compute return variance and expected trading volume as

$$Var(r_t) = Var(\lambda_t Q_t) + Var(d_{t-1} - \lambda_{t-1} Q_{t-1}) \tag{14}$$

$$E(v_t) = \frac{1}{2}(m_{It} E[|x_{it}|] + m_{Bt} E[|x_{jt}|] + m_{Ht} E[|x_{kt}|]) + \frac{1}{2} E[|Q_t|]. \tag{15}$$

Equations (A72) and (A73) in the Appendix provide detailed expressions for these two moments in terms of model parameters.

[7] Alti and Tetlock (2013) analyze the impact of overconfidence on asset prices and firm investment behavior, but they do not consider trading volume or market liquidity.

Our definition of illiquidity (ILQ_t) captures the idea that an illiquid market is one in which stock prices are highly sensitive to trading volume. To avoid the difficulty of measuring signed (i.e., buyer- or seller-initiated) trading volume, we define illiquidity in the spirit of Amihud (2002) as the regression coefficient of the absolute value of stock returns on unsigned trading volume:

$$ILQ_t = \frac{Cov(|r_t|, v_t)}{Var(v_t)}. \tag{16}$$

The ratio of Equation (A74) to (A75) in the Appendix provides a closed-form expression for this illiquidity coefficient. In simulations, the model's predicted illiquidity coefficient is very close to the model's predicted value for λ, the slope of the market maker's pricing schedule.

To facilitate comparisons of empirical moments and parameter estimates across firms and time, we define scaled versions (denoted by $*$) of several moments and parameters in terms of each firm's shares outstanding (θ) and stock price (p):

$$c_i^* = c_i / (p\theta) \tag{17}$$

$$c_j^* = c_j / (p\theta) \tag{18}$$

$$c_k^* = c_k / (p\theta) \tag{19}$$

$$\tau^* = \tau / (p\theta) \tag{20}$$

$$\sigma_{dt}^* = \sigma_{dt} / p \tag{21}$$

$$\sigma_{ht}^* = \sigma_{ht} / \theta \tag{22}$$

$$\lambda_t^* = \lambda_t \theta / p \tag{23}$$

$$r_t^* = r_t / p \tag{24}$$

$$v_t^* = v_t / \theta \tag{25}$$

$$Var(r_t^*) = Var(r_t / p) \tag{26}$$

$$E(v_t^*) = E(v_t / \theta) \tag{27}$$

$$\frac{Cov(|r_t^*|, v_t^*)}{Var(v_t^*)} = \frac{Cov(|r_t / p|, v_t / \theta)}{Var(v_t / \theta)}. \tag{28}$$

With these definitions, the equations expressing the parameter estimates in terms of the empirical moments remain identical to the original equations in Appendix A, except that all variables in the new equations have star superscripts. For practical reasons, we measure market capitalization and shares outstanding at the end of the previous period.[8] Hereafter, we omit star superscripts, except when explicitly comparing raw and scaled values.

B. Use of Time of Day and News Arrival

Our identification strategy exploits enormous empirical variation in variance, trading volume, and illiquidity moments across two dimensions. First, over the past decade, while virtually any investor can trade stocks at electronic venues both before and after normal market hours, the vast majority of trading occurs between 9:30 am and 4:00 pm. Second, regardless of the intraday period, all three types of moments vary with public news arrival. Consequently, we separately estimate variance, volume, and illiquidity moments in three intraday periods—the pre-market from 7:00am to 9:30am; the regular market from 9:30am to 4:00pm; and the after-market from 4:00pm to 6:30pm—and conditional on public news arrival.

Some model parameters, such as the amount and precision of acquirable information (σ_{dt} and η_t, respectively), are properties of the information environment and thus may vary. We allow both to depend on the intraday period due to variation in normal business activity. Moreover, intuition suggests that public news about the firm increases the amount of available information.

[8] Technically, this timing induces a small approximation error in the parameter estimates, but this is usually negligible because the gross returns for intraday periods are usually very close to 1.0.

At the same time, interpreting newly arriving information is often difficult, implying that the precision of information may decline. We thus allow these parameters to vary with news arrival as well. Other model parameters, such as risk tolerance and overconfidence (τ and η_B, respectively), are properties of the agents in the model and remain fixed.

C. *Parameter Restrictions*

Table I Panel A describes the parameters and parameter restrictions in two versions of the model. In the main version, we estimate both overconfidence η_B and a single risk tolerance parameter $\tau = \tau_B$. We also estimate a nested version of the model to analyze the contribution of overconfident agents. In this "no overconfidence" or "rational" model, we fix overconfidence to be $\eta_B = 0$. In both versions, we estimate the amount and precision of acquirable information (σ_{dt} and η_t, respectively) in the three intraday periods with and without public news arrival.

We determine the seven remaining parameters (ρ_z, ρ_h, ρ_u, σ_h, c_I, c_B, and c_H) as follows. We set the disagreement parameter that governs the correlation in informed traders' signal errors to be $\rho_z = 0$, which maximizes the amount of trading that occurs among rational agents. As we will show, this $\rho_z = 0$ specification is able to fit the data well. For parsimony and symmetry, we assume uninformed traders *perceive* the correlation among their signal errors to be the same as the actual correlation among informed traders' signal errors, so that $\rho_{Bu} = \rho_z = 0$.

We set the magnitude of endowment shocks (σ_h) equal to a high value to give hedging a reasonable chance of explaining the data. In the model, each firm is held by investors whose uncertainty in wealth comes solely from stock holdings in that firm and an endowment shock that is perfectly correlated with the firm's stock. Shocks to individuals' wealth from non-stock sources could serve as empirical counterparts to endowment shocks. Heaton and Lucas (2000) argue that proprietary business wealth is a key consideration for the typical stockholder. They estimate that individual business wealth has the following properties: an annual correlation with aggregate stock returns of 0.14, an annual volatility of 65%, and a value of 18.3% of total wealth.[9] Using these inputs, one can estimate the annual dollar volatility of endowment shocks as a fraction of uninformed investors' dollar wealth (*w*) to be

$$f_h = p\sigma_h / w = 0.14 \cdot 0.65 \cdot 0.183 = 0.0167 \tag{29}$$

We convert f_h to a fraction of the firm's market capitalization (i.e., σ_h^* in Equation (22)) by assuming that each hedger's wealth is a fraction $f_w = 10^{-4}$ of the firm's market capitalization.

Importantly, the assumed f_w parameter has only a small influence on our estimates of *relative* risk aversion (*RRA*). Specifically, the definition of the *RRA* coefficient in terms of uninformed investor wealth and the estimated scaled *ARA* coefficient ($1/\tau^*$) is

$$RRA = \frac{w}{\tau} = \frac{w}{p\theta}\frac{p\theta}{\tau} = \frac{f_w}{\tau^*} \tag{30}$$

The estimated *RRA* coefficient is quite insensitive to f_w for a large range of values. Intuitively, increasing the wealth of hedgers proportionally increases the magnitude of endowment shocks, which decreases the estimate of *ARA* ($1/\tau^*$), but increasing wealth proportionally increases the *RRA* coefficient relative to *ARA*.

We base the actual correlation among uninformed signals (ρ_u^2) on empirical data. Specifically, for two uninformed speculators j' and j'' in our model with signals arising from normal variables with correlation ρ_u^2, the probability of trading in the same direction is

$$\Pr(x_{j'}x_{j''} \geq 0) = 0.5 + \frac{1}{\pi}\arcsin(\rho_u^2) \tag{31}$$

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[9] Heaton and Lucas (1996) calibrate a model in which agents trade based on uninsurable labor income shocks that are uncorrelated with stock returns—shocks that we do not model. They find that such trading amounts to just 15% turnover annually even when there are no trading costs, which is small compared to empirical turnover of 400%.

Dorn, Huberman, and Sengmueller (2008) report that the probability that two retail brokerage customers submit market orders in the same direction (in the same stock) is 0.538. We set Equation (31) equal to this value and solve for ρ_u^2, resulting in an estimate of 0.119. We analogously set the pairwise correlation ρ_h^2 among hedgers' endowment shocks to be 0.119 because the uninformed retail traders in Dorn, Huberman, and Sengmueller (2008) could be hedging rather than speculating on perceived information. In unreported tests, we find that the model's main predictions are not sensitive to changing this correlation by a factor of two.

Lastly, we determine the parameters in the three agents' cost functions described in Equations (A53) to (A55) in the Appendix as follows. A recent *Wall Street Journal* story provides a direct estimate of the cost of informed trading in large firms.[10] Roughly twenty hedge funds paid *up to* \$10,000 each to acquire private information about a December 8th, 2009 health care law that affected four large health care stocks. As in the model, the information was acquired during the regular market, one intraday period in advance of its release during the after-market period. Based on the cumulative market capitalization of the stocks (about \$100B), the implied value of c_i^* is about \$10,000 / \$100B = $1.0 * 10^{-7}$. Setting Equation (A53) with twenty informed traders to this value, we solve for the cost parameter $c_I^* = 2.77 * 10^{-10}$. This estimate is an upper bound for three reasons: 1) some hedge funds paid less than \$10,000; 2) the meeting may have provided more precise information than investors can typically obtain; and 3) it may have provided relevant information about other firms in the health care sector, which has a much larger market capitalization than the four firms. We thus reduce c_I^* by a factor of 10 to $2.77 * 10^{-11}$ in our main estimation, though the results are similar with no reduction or a reduction by a factor of 100.

[10] *Wall Street Journal*, December 20, 2011, "Inside Capital, Investor Access Yields Rich Tips" by Brody Mullins and Susan Pulliam.

We estimate the costs of uninformed agents by considering a maximum plausible value for the number of hedgers (m_{H_max}) who enter the market in an intraday period. Based on flow of funds statistics from the Federal Reserve, the aggregate wealth (W) in the US is roughly four times the US stock market capitalization ($W = 4p\theta$) during the sample period. If hedgers hold all wealth, enter uniformly across days, and concentrate in one intraday period per day ($252 \cdot w \cdot m_{H\_max} = W$), then the number of hedgers entering in that period would satisfy:

$$252 \cdot w \cdot m_{H\_max} = 4p\theta. \tag{32}$$

Rearranging and substituting for f_w, we obtain:

$$m_{H\_max} = \frac{4}{252 f_w} = 158.7. \tag{33}$$

We insert this m_{H_max} value into Equation (A57) to compute the reservation utility of a marginal hedger with an *RRA* of 2 who expects to fully hedge his endowment shock ($\beta_3 = -1$) in a perfectly liquid market ($\lambda = 0$). These substitutions result in a value of $c_H^* = 4.19 * 10^{-13}$. As we will show, these assumptions allows for the aggregate wealth and endowment shocks of hedgers to be very large.[11] To create a level playing field for speculators, we set their cost parameter equal to the hedger cost parameter ($c_B = c_H$). Thus, the relative strengths of the endogenous motives for overconfident speculation and rational hedging determine the relative numbers of the two types of uninformed traders who choose to enter.

[11] If hedgers' estimated *RRA* exceeds 2, far more than m_{H_max} hedgers could enter the market in an intraday period.

3. Data and Empirical Moments

A. Data

Our sample spans 2001 to 2010 and includes NYSE, AMEX, and NASDAQ stocks. We obtain trade-by-trade price and volume data from the NYSE TAQ database. We adjust for non-standard opening and closing times and define the regular trading period as the actual hours in which the market is open (typically 9:30 am to 4:00 pm ET), and the pre-market and after-market as the 2.5 hours prior to the open and following the close, respectively.[12] We then construct stock-specific return, turnover, and illiquidity observations for each day's pre-market, regular market, and after-market periods. Each variable is designed to mimic the corresponding theoretical moment introduced in Section 2.

As discussed in the Appendix, we employ standard microstructure techniques to compute accurate trade-based returns. In addition, we adjust all stock returns for market returns by subtracting the contemporaneous intraday return of the SPDR S&P 500 ETF (SPY). When a stock's return is zero during an extended hours period, we set its market-adjusted return to zero to avoid introducing additional microstructure noise. We compute share turnover as the market value of share volume scaled by market cap. Finally, we compute illiquidity as the regression coefficient of absolute one-minute VWAP returns on contemporaneous turnover.[13]

We measure firm-specific news using the Dow Jones archive to distinguish between periods with and without news arrival. These data include all DJ newswire and *Wall Street Journal* stories from 2001 to 2010. For each story, DJ provides stock codes indicating which firms are meaningfully mentioned and a timestamp indicating when the story became publicly accessible.

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[12] Barclay and Hendershott (2008) argue that extended hours trades and quotes are adequately represented in the TAQ database. See their section 2.1 for a detailed analysis.

[13] In the illiquidity regressions, we drop absolute one-minute returns greater than five percent and only estimate the model for regular (extended hours) periods with at least 50 (20) VWAP returns. We restrict the intercepts for each extended hours regression to equal the average regular period intercept for the same size-group and quarter.

We ensure that we focus on firm-specific news by excluding stories that mention more than two listed US stocks. The variable $News_{it}$ equals one when news stories mention firm i during intraday period t and zero otherwise. For small and mid cap (large cap) firms, we require at least one (two) story mention(s) to constitute news. This convention does not count isolated stories for large cap stocks as news because large firms frequently receive news coverage even when no new public information exists.

Motivated by the modern day 24-hour news cycle, we consider whether news has arrived in the past 24 hours. For example, sequences of related news stories about a firm may unfold during a 24-hour period, bringing the news and the stock to the attention of more traders and altering the information gathering process. In our estimation below, we keep only intraday periods in which no news has occurred since the same intraday period in the prior trading day. Imposing a brief period of non-news prior to measuring market activity mitigates the confounding impacts of recently released news stories and better establishes the release of new public information. Thus, we estimate moments conditional on news using observations where $News_{it} = 1$ *and* no news occurred since the same intraday period from the prior trading day. Likewise, we estimate moments conditional on no-news using only observations where $News_{it} = 0$ *and* no news has occurred since the same intraday period from the prior trading day.

We employ additional sample filters based on size, news coverage, and extended hours trading activity from the prior calendar year. First, we retain only stocks having market capitalizations above $100 million and share prices greater than $1 at the end of the prior year. Second, we require a firm to have a news story in a minimum of four pre-market periods and four after-market periods. Third, firms must also have trading in at least 20 pre-market periods and 20 after-market periods. Finally, we divide the firms into three size subsamples based on market capitalization from the prior year-end. We define “large cap,” “mid cap,” and “small cap” stocks as those with market capitalizations within the intervals [\$10B,¸∞), [\$1B,\$10B), and [\$100M,\$1B), respectively. These size groups contain an average of 95, 245, and 237 firms per year, respectively.

B. Empirical Moments

In each intraday period (pre-market, regular hours, and after-market) and quarter, we pool firm-specific observations across similarly-sized firms. We then estimate pooled variance, volume, and illiquidity moments conditional on the occurrence of either “news” or “no-news”. In all computations, we weight intraday observations such that each firm is given equal representation within a size group and quarter. For our full sample analysis, we average each moment estimate across size groups within each quarter and then average across quarters to obtain a set of eighteen conditional moments. This procedure mitigates measurement error resulting from firm-level estimates and assigns each size group and quarter equal weight. Moreover, we use the joint quarterly time series of moment estimates to estimate the weighting matrix in the efficient generalized method of moments procedure described below.

Figure 1 plots news probabilities for each quarter in the full sample. These values indicate the probability of news arrival conditional on no news releases since the same intraday period of the past trading day. Four general patterns are noteworthy. First, while the regular period has the highest probability of news, the majority of news stories occur during extended hours—the probability of news in a regular period is about 0.10, while the probabilities of news in the pre-market and after-market periods are 0.07 and 0.05, respectively. Furthermore, measured as an hourly arrival rate, the probability of news is actually highest during the pre-market. Second, the probability of news arrival in every intraday period increases substantially around 2003 and plateaus through 2010. Third, there is some seasonality for regular market news in which more stories occur during the first quarter of the year. This is possibly related to annual reports of the disproportionate number of firms with December fiscal year ends. Fourth (not shown), in all periods, news occurs more frequently for large cap stocks than for mid or small caps.

[Insert Figure 1 here.]

The first column in Table II presents full-sample estimates of conditional variance, volume, and illiquidity moments, and Figure 2 highlights how each varies with news. We first discuss variance. To facilitate comparison across periods, we multiply extended hours variances by 6.5/2.5 (the ratio of the lengths of the regular period to an extended hours period) and in all cases take the square root to report volatilities. In each variance calculation, we account for spurious reversal due to transitory noise in the period t price by computing $Var(r_t)^* = Var(r_t) + Cov(r_t, r_{t-1}) + Cov(r_t, r_{t+1})$. Interestingly, this adjustment affects estimates by less than 15% in each intraday period, implying that microstructure noise is not too severe.

[Insert Table II here.]

[Insert Figure 2 here.]

Our estimates reveal that news is consistently associated with higher variance—variance conditional on news always exceeds that conditional on no-news, and the difference is economically staggering. Figure 2 shows that in the regular period, volatility on news days is about 130% of that on no-news days. In the pre-market and after-market periods, this ratio is even higher at 302% and 370%, respectively. Qualitatively similar patterns emerge within each size group (not shown). Second, volatility during extended trading hours is of a similar magnitude as that during regular trading, especially when comparing periods with news. This finding is surprising in light of earlier evidence from French and Roll (1986) that volatility when the regular market is open far exceeds that when the regular market is closed. Our results may differ from theirs because at least some trading now occurs in extended hours markets and because new technologies have changed the nature of trading and news dissemination since their time period.

Table II also presents the conditional volume moments for the pre-, regular, and after-market periods. Extended hours moments are again multiplied by a factor of 6.5/2.5, and numbers in the table are turnover expressed in percent. Similar to the conditional variance results, conditioning on news matters. As illustrated in Figure 2, turnover with news in the regular, pre-, and after-market periods, respectively, is 145%, 723%, and 688% of that without news. However, unlike the variance patterns, almost all trading takes place during regular market hours irrespective of the occurrence of news.

Table II displays conditional illiquidity moments, which are average coefficients from regressing absolute one-minute VWAP returns on turnover. Not surprisingly, extended hours markets are far less liquid than the regular market. Illiquidity in the pre-market and after-market, respectively, is about 30 times and 15 times that of the regular market. The effect of news, however, varies across periods. The regular market and pre-market are more liquid in the presence of news, while the opposite is true for the after-market. Together with those in variance and volume, these stark patterns are key to identifying our model.

4. Structural Estimates of the Model

We use the generalized method of moments (GMM) as in Hansen and Singleton (1982) to estimate the 14 parameters that best match the 18 empirical moments. Formally, GMM minimizes the distance between the model's predicted moments and the empirical moments. We use an efficient GMM procedure that weights this distance using a matrix equal to the inverse of the covariance matrix of the empirical moments, so that moments that are measured with greater precision receive proportionally greater weight. We measure the covariance matrix using quarterly variation in the moments while taking persistence into account. We describe the details of this procedure in Appendix C.

We estimate both versions of the model described in Panel A in Table I. Recall the main model nests a fully rational version in which overconfidence (η_B) is held fixed at zero. Table II shows how the models fit each empirical moment, and Figure 3 reports the *t*-statistics of prediction errors. These *t*-statistics indicate that the main model matches all 18 of the empirical moments within two standard errors and all but two moments within one standard error. One cannot statistically reject this model based on its four overidentifying restrictions—the $\chi^2(4)$ statistic of 5.89 has a *p*-value of 0.208. Table IV below reveals the main model also matches the moments very well in the size subsamples and in both subperiods.[14] The latter is notable because the recent increase in high-frequency trading is likely to affect our empirical moments mainly in the second half of the sample. Moreover, the parameters' small standard errors—shown in parentheses in Table III below—indicate that all 14 of the parameters are reasonably well-identified in the full sample and in subsamples.[15] Thus a lack of statistical power cannot explain why we do not reject the hypothesis that the overconfidence model fits the data.

[Insert Figure 3 here.]

We also stress that the economic magnitudes of the unrestricted model's prediction errors

[14] Although we reject the model in the large firm subsample, this poor fit only applies to the early half of the sample ($\chi^2(4) = 17.59$; *p*-value = 0.001) in which there are sometimes as few as 23 large firms. In the late half of the sample, the main model matches the empirical moments of large firms very well ($\chi^2(4) = 4.07$; *p*-value = 0.397).

[15] The parameters' standard errors come from the GMM delta method formula based on the covariance matrix of the moments and the sensitivity of each moment to each parameter—e.g., see Cochrane (2001). The local standard error for the overconfidence parameter is misleading in some cases; therefore, we estimate a global standard error by comparing the main model's χ^2statistic to that from a "quasi-rational" restricted model that is equidistant from the main and rational models in η_B space. This method does not materially alter the standard errors in the full sample estimation, though it mitigates some outlying standard errors in the subsample results reported in Table III below.

are tiny in comparison to the huge empirical variation in moments across periods—e.g., turnover and illiquidity often vary by orders of magnitude. Most importantly for this study, this model that allows for overconfident traders matches *both* the enormous trading volume during the regular market period and the light trading in extended hours. Likewise, it matches the high regular market liquidity and the high extended hours *illiquidity*. Finally, it replicates large increases in volatility and volume that accompany public news releases, especially during extended hours. In this sense, the model *can* explain the main features of market activity.

In stark contrast to the main model, the version without overconfidence does not fit basic qualitative features of the data. One abject failure is that it predicts regular market turnover to be two orders of magnitude too low, and a second is that it does not capture cross-period variation in turnover. Overall, the prediction error *t*-statistics are greater than 3.0 for five of the six turnover moments and two of the six volatility moments. As a result, one can strongly reject this model's five overidentifying restrictions at the 1% level—the $\chi^2(5)$ statistic of 449.14 has a p-value < 0.001. We analyze the rational model further below.

[Insert Table III here.]

We turn next to the parameter estimates reported in Table III. In the discussion that follows, we emphasize both statistical significance and economic plausibility: if we reject the hypothesis that a model's parameter estimates are plausible, we reject the model as an explanation of market activity even if it fits the data statistically. Because it distinguishes the two models, we discuss the overconfidence parameter first. The full-sample estimates indicate that uninformed speculators believe they observe signal with precision of η_B = 0.122. The estimates in the two subperiods are 0.138 and 0.075; and the full-sample estimates range from 0.125 for large caps to 0.149 for small caps (also in Table IV). Based on the difference in the restricted and unrestricted model fit of $\chi^2(1) >> 100$, we strongly reject the hypothesis of no overconfidence ($\eta_B = 0$).

Estimated overconfidence is economically plausible in two respects. First, it is not high relative to the rational confidence of informed agents. Notably, across six intraday periods, estimates for the parameter η range from 0.205 to 0.680; thus informed agents' signals are uniformly more precise than overconfident speculators' perceived signals.[16] The perceived information asymmetry between these agents is especially large during the extended hours periods, which explains why the model correctly predicts small amounts of trading during these periods. Intuitively, when informed agents observe more precise signals, overconfident speculators become less willing to trade with them.

Second, estimated overconfidence is consistent with experimental estimates. Consider the accuracy of agents' directional forecasts of dividends based on their signals. Because overconfident agents' signals are not informative, their directional forecasts of dividends are correct exactly half of the time (probability 0.5). Our parameter estimates suggest these agents believe that they are right 53.9% of the time.[17] Even in the first half of the sample when overconfidence is highest, uninformed agents only believe they are correct 54.4% of the time. We compare this overconfidence level with Lichtenstein and Fischhoff's (1977) analogous experiment, in which subjects taught how to read stock charts believe they can predict a stock's directional price movement 65.4% of the time, whereas they are correct slightly less than half of the time. Our smaller overconfidence estimates are consistent with the notion that investors' behavioral biases may diminish when material sums of money are at stake. In addition, real-world investors may obtain rapid and repeated feedback on their forecasting performance, which could

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[16] We also estimate a model in which the overconfidence parameter varies across periods as a constant multiple of that period's η parameter. The estimated multiple of 0.58 is statistically less than 1.0, consistent with our conclusion from Table III that the overconfidence of speculators is smaller than the rational confidence of informed traders.

[17] We compute the overconfident agents' perceived probability of being correct using Equation (31), except that we replace $x_{j'}x_{j''}$ with $s_j^B\, d$ and ρ_u^2 with η_B.

mitigate their overconfidence. Nevertheless, an agent exhibiting our estimated level of overconfidence would need hundreds of independent trials to learn that he is in fact overconfident, even if he accurately remembers his successes and failures.[18]

To complement the evidence in Table III, Table IV shows that, with a few exceptions, the values of the other parameter estimates are plausible and stable across two subperiods and three size groups. This suggests our assumption that the model parameters are the same across firms is not too restrictive. The estimated values of the amount of acquirable information (σ_d) closely mimic the patterns in empirical volatility noted in Section 3. The model produces this outcome because market prices incorporate most private information in the period in which it is acquired, as we discuss further below. The model matches the evidence that news increases return volatility because the estimated amount of acquirable information increases with news, especially in extended hours periods.

[Insert Table IV here.]

The estimates of information precision (η) are consistently lower in the regular market than in the two extended hours markets, which is critical for matching the stark volume differences across periods. Trading volume decreases with η because increases in precision exacerbate adverse selection in pricing and deter uninformed traders from entering the market. Increases in adverse selection are an especially powerful deterrent to overconfident traders who enter and trade only because they think they can profit from their information. Remarkably, although the average non-news precision estimate of 0.518 in extended hours periods is only double the non-news precision of 0.243 in the regular market, predicted trading in these extended hours periods is roughly 100 times lower than regular market trading. Finally, we insert these estimates into

[18] An investor with an infallible memory would require on average 628 independent trials to obtain a *t*-statistic of 1.96, rejecting the null hypothesis that the probability the investor is correct is 0.539.

Equation (4) to infer the pairwise correlation between informed traders' signals, which simplifies to η^2. In all periods except the pre-market without news, this value is below 0.13. The weak correlation results in many offsetting informed trades in equilibrium.

We now analyze the rational model more rigorously. Its failures to capture both the level and variation in turnover in Table III are closely connected. If estimated RRA were higher, the model would predict much more hedging activity and could explain much higher levels of trading activity. However, this would dramatically increase trading activity in *all* intraday periods, not just the regular market and news periods, meaning that the model would predict far too much trading during extended hours and non-news periods.

We demonstrate this point in Table V, which shows estimates of the models with and without overconfidence that are based only on the six empirical moments from the regular market—i.e., not using the 12 from extended hours periods.[19] As conjectured, the rational model is able to match the level of trading volume by increasing *RRA* from 1.223 to 1698, a factor of more than 1000. However, it still does not match the variation in volume across news and non-news periods, which leads to its rejection at the 5% level. Intuitively, hedgers are so risk averse and their endowment shocks are so large that their desire to enter the market and fully hedge their shocks is little offset by their expected price impact. As a result, cross-period variation in adverse selection has a minimal influence on hedging activity and overall trading activity. This mechanism underlies the rejection of the rational model in both Tables III and V, though the rejection is more dramatic in Table V where cross-period variation in volume is far greater.

[Insert Table V here.]

In addition to its inability to fit basic facts about trading volume, the fully rational model

[19] We assume RRA = 2 in the unrestricted model, rather than estimating RRA, because RRA is weakly identified when we exclude the extended hours moments. Because the fit of the unrestricted model is very insensitive to our RRA assumption in this estimation, our conclusions from Table V are similar if we assume RRA is 0.2 or 20.

requires several implausible parameter values. From Table III, estimates of the information environment parameters during extended hours periods with news are unrealistic. For example, in pre-market periods with news, the combination of the extremely low information precision (η = 0.009) with high volatility of tractable information (σ_d = 32.5%) has absurd implications. Given that 21 informed traders enter during this period and they each have such imprecise signals, over 99.9% of pre-market news is not revealed through prices. Thus, in the model, nearly all of the enormous amount of discoverable information would be revealed in the regular market period following this news, predicting a return volatility of over 32% according to Equation (A72). Although *post-news* volatility is not an empirical moment in the estimation, we measure this volatility and find it to be 3.78%. This demonstrates that the rational model's parameter estimates are inconsistent with features of the data beyond our empirical moments.

Both models produce reasonable *RRA* estimates in Table III. Consider the implied risk aversion of an investor with constant *RRA* who holds a portfolio fully invested in the US stock market. With an equity premium of EP and a volatility (in log returns) of 20% per year, this investor must have a risk aversion of EP / $(0.20)^2$ in order to hold the market. Fama and French (2002) argue the EP ranges from 2.55% to 4.32%. Combining this EP range with the assumptions above implies *RRA* estimates of 0.638 to 1.080. Our estimates of 0.615 in the main model and 1.223 in the restricted model are similar to these values, though they are smaller than *RRA* values used to justify the high historical average returns of US stocks.

We interpret the risk aversion estimates in light of the fact that the risk faced by agents in the model affects only the returns of a single stock—i.e., it is idiosyncratic, not systematic. If uninformed investors hold well-diversified portfolios, small idiosyncratic risks should be irrelevant for their overall portfolio risk, implying they should exhibit no risk aversion at all. More generally, under the reasonable assumption that investors exhibit lower effective risk aversion in response to idiosyncratic risks, one would expect a structural estimate of *RRA* based on idiosyncratic risks to be lower than an estimate based on systematic risks. This could explain why our estimates are low relative to studies analyzing the historical equity premium.

5. Analysis of the Model

A. Interpreting Volume, Variance, and Welfare

The model also allows one to analyze the relative contributions of each trader type to market activity. We decompose trading volume in Equation (15) by separately considering the quantities of buy and sell orders that transact between traders and the net order flow (Q) where market makers take the other side of the trade. We assign half the volume from a trade to each participating counterparty.

We denote the volume arising from informed traders, uninformed speculators, hedgers, and market makers by v_I, v_B, v_H, and v_M. Expected market maker volume is half of the volume arising from expected net order flow aggregated across other trader groups, which is

$$E(v_M) = \frac{1}{4} E(|Q|). \tag{34}$$

One can easily compute closed-form solutions for this volume expression and all those below by adjusting the terms in Equation (A73) appropriately. Each trader group's volume is half of the sum of its buy and sell orders plus half of its proportion of the net trading with market makers. We apportion the volume arising from net order flow to each trader type based on the fraction of variance in net order flow arising from that type, as measured in Equation (A13). Because the four components of volume equal total volume, we can express each as a fraction of the total using

$$\frac{E(v_I)}{E(v)} + \frac{E(v_B)}{E(v)} + \frac{E(v_H)}{E(v)} + \frac{E(v_M)}{E(v)} = 1. \tag{35}$$

Table VI reports analyses of volume and other market activity based on the parameter estimates shown in Table III. In the main model, informed trading and overconfident speculation together account for the lion's share of volume at 71% and 19% respectively. Hedging, on the other hand, plays a miniscule role. In the no overconfidence model, informed trading still represents 56% of trading, while hedging is far more important in percentage terms and accounts for 36% of volume. Recall, however, that total trading in this latter model is only 1% of trading that is observed in regular market periods.

[Insert Table VI here.]

Following an analogous procedure to the volume decomposition, one can decompose return variance into the two components shown in Equation (14), the first based on trading on private information ($Var_T(r)$) and the second based on public information ($Var_P(r)$)

$$\frac{Var_T(r)}{Var(r)} + \frac{Var_P(r)}{Var(r)} = 1. \tag{36}$$

The first term represents price discovery arising from informed trading on newly acquired private information and serves as a natural measure of market efficiency. The second term measures the revelation of discoverable information that the previous period's price did not fully reveal. This residual information term can be viewed as public information that is revealed through sources such as newswires, social media, television, and radio.

Table VI reveals that market prices are extremely efficient in the main model with 84% of acquirable *private* information revealed in prices in the same period in which it arises. In this sense, the strong-form of the efficient market hypothesis nearly holds. Intuitively, overconfidence increases market liquidity because the rational market maker knows that she sometimes transacts with overconfident agents. The increased liquidity motivates rational informed agents to trade more aggressively on their information, which is then incorporated in prices. Despite their awareness of informed traders, uninformed overconfident agents continue to trade aggressively because they believe that they are taking advantage of excessive liquidity provision by the market maker. The lack of this mechanism results in much lower price informativeness in the model with only rational traders. Equilibrium trading activity is very low in this restricted model, so few informed agents find it worthwhile to collect costly signals.

Finally, we analyze traders' expected trading profits and deadweight costs to provide insights into welfare. The total trading profits of the informed must equal the total trading losses of overconfident speculators and hedgers. We compute expected profits of informed traders each period by multiplying their gross expected profit from Equation (A61) by the number of traders who enter and then summing across all intraday periods in a year. We then compute expected trading losses of overconfident speculators by multiplying their periodic expected loss (under rational expectations) by the number entering and then summing again across all intraday periods. The expected trading loss of hedgers is simply the residual that ensures the market maker makes zero profits. For each trader type, we compute expected deadweight costs related to entry as the area under its cost curve summed across all intraday periods in a year.

In the main model, expected annual trading profits for informed agents are 69 bps of firm value and their deadweight cost of trading is 23 bps, shown near the bottom of Table VI. These numbers are in line with the 67 bps magnitude of the cost of active management estimated by French (2008). Multiplying our 69 bps estimate by the average US equity market capitalization of \$15.3T during our sample, the annual aggregate trading gains for informed traders and losses for uninformed traders are \$105B. This amount is almost entirely a wealth transfer from overconfident speculators; hedger losses are relatively inconsequential. Annual deadweight entry costs summed across all traders are \$15.3T x 25 bps = \$38B. We do not take a stand on whether this is a fair price for society to pay for price discovery and liquid capital markets. We also note that there may be other costs associated with delegated asset management that lie beyond the scope of our model.

A natural concern with behavioral models is that traders exhibiting biases may quickly cease to be relevant if they lose all of their wealth. To evaluate this possibility, we suppose that overconfident investors hold 25% of stocks, which is roughly equal to direct retail ownership of US stocks. In this case, their trading losses of 69 bps would decrease their annual stock returns by 0.0069 / 0.25 = 2.76% per year. Because average annual stock returns easily exceed this amount, this trading loss is not sufficiently large to eradicate the wealth of overconfident investors—even if no new overconfident investors begin participating. This 2.76% estimate of trading losses is realistic in two respects: 1) it is only slightly higher than the expenses and fees charged by many actively managed mutual funds, which represent a natural alternative investment vehicle for uninformed retail investors; and 2) it is somewhat lower than Odean's (1999) estimate that the most active direct retail traders in the 1990s underperform the market by 6.5%.

B. Sensitivity Analysis

Every model necessarily simplifies reality and ours is no different. Here we consider several forces that influence empirical estimates of market activity that lie outside our model, including dynamic hedging, trading on long-lived information, trading in equity derivatives, program trading, and trades between market makers.

Hedgers enter only once per year in our model, but they may trade more frequently in practice. To explore this possibility, we assume they hedge their annual endowment shocks in 50 independent increments. Formally, we reduce hedgers' time horizons (T_H) from 252 (annual) to 5 (weekly). We reduce the endowment shock (f_h) by a factor of $\sqrt{5/252}$ for those who enter and increase the maximum number of entering hedgers (m_{H_max}) by a factor of 252/5.

Untabulated estimation of the resulting model with no overconfidence indicates that it can fit basic volume patterns in the data, though it is still overwhelmingly rejected ($\chi^2(5) = 263$; p-value << 0.001) based on its variance and liquidity predictions. Moreover, several of the parameter estimates in this model suffer from the problems discussed previously. For example, the precision of informed agents' signals is implausibly low (<<0.01), while the amount of acquirable information is implausibly high (over 30% in some periods). As a result, prices incorporate almost no acquirable information and the model fails to fit out-of-sample moments, such as the modest return volatility observed during the regular market following pre-market news. Based on this analysis, it seems unlikely that modeling dynamic hedging would change our main findings.

Trading on long-lived private information is another potentially significant feature that is absent in our model. In models such as Kyle (1985), a trader with long-lived information camouflages his trades by smoothing trading evenly across many periods. Information must be truly private in order for a trader to implement this strategy. However, "it is reasonable to expect that at least a few players will have access to private information and ... face competition," as argued in Holden and Subrahmanyam (1992). In this case, nearly all trading will occur soon after private information is acquired—as it does in our model. In this respect, our model of short-lived information may approximate a model in which trading on long-lived information is competitive.

Although trading in equity derivatives, such as stock options, is not an explicit part of our model, such trading need not lie outside our framework. Consider what happens to stock market volume when a trader in our model transacts in the options market instead. For example, suppose an overconfident trader buys a call option from an options market maker, rather than obtaining the equivalent exposure by buying the underlying stock. If the options market maker engages in delta hedging in the stock market to offset her options inventory, the market maker will buy the equivalent amount of stock as a hedge. With complete delta hedging, the ultimate impact of the overconfident trader on stock market activity is the same whether the trader uses the stock or options market to acts on his beliefs. This logic generalizes to traders with other motives.

Our model does not account for either program trading or trading between market makers, but both could be quantitatively important features of real-world market activity.[20] Accordingly, we consider how our results would change if we adjust our empirical moments to account for the market activity attributable to these unmodeled trading motives. Specifically, we suppose that unmodeled trading motives account for half of all measured volume and increase measured illiquidity by a factor of two. We then reestimate our main model using moments based on the parts of volume and liquidity that are not driven by unmodeled motives. The results of this estimation are very similar to those shown in Table III. As before, we cannot reject the model ($\chi^2(4) = 6.99$; p-value = 0.136) or any of its 18 moment predictions at even the 10% level. Estimated overconfidence changes from 0.122 to 0.110, while estimated *RRA* changes from 0.613 to 0.515. The information environment parameters also change very little. We infer from these analyses that our main conclusions are robust to several types of model misspecification.

6. Conclusions

We propose and estimate a model in which market activity arises endogenously from the interaction between utility-maximizing informed traders, uninformed hedgers, and overconfident

[20] A common form of program trading is arbitrage between stock indexes and the underlying stocks, while market makers often trade with each other to share inventory risk.

speculators. We find that this model fits the data reasonably well when the magnitude of overconfidence is modest. In this ostensibly behavioral model, most trading volume arises from rational informed agents and market prices are remarkably efficient. When we restrict overconfidence to be zero, however, nearly all trading volume dissipates, prices incorporate a negligible amount of information, and the model's predictions do not match the data.

Although we are unable to identify a model without overconfidence that fits the data, our results point to several promising directions for future research. In our estimations, models of uninformed trading based solely on rational hedging have difficulty matching the high level of volume and its large variation across intraday periods. Intuitively, fitting the data would require extremely high effective risk aversion during the regular market period—when most trading occurs—and much lower effective risk aversion during extended hours periods. One way to generate high effective risk aversion during the day is to impose tight constraints on the behavior of uninformed agents. Institutional money managers could act as though their risk aversion is extremely high because they are sometimes forced to liquidate their holdings to meet large and unanticipated client withdrawals. Alternatively, institutional risk management policies such as position size limits could generate high effective risk aversion as well. Modeling agency problems in institutions and endowment shocks to their clients' portfolios could provide valuable new insights into market activity.

Another promising direction for future research is integrating behavioral models designed to explain trading volume and those focused on mispricing. Our study suggests that a modest amount of overconfidence can explain trading volume, while other studies such as Alti and Tetlock (2013) argue that overconfidence can help explain mispricing. To distinguish among behavioral and rational models, we use empirical moments such as liquidity and volume, while Alti and

Tetlock (2013) use empirical patterns in stock returns. Models with heterogeneous agents—rational or behavioral—often make sharp predictions about both volume *and* return predictability. Using both types of empirical moments to identify the parameters in such models would enable researchers to construct more powerful tests of rational and behavioral theories.

Finally, structural models of trading activity, especially if they allow for mispricing, have the potential to inform the longstanding debate over the impact of proposed transaction or “Tobin” taxes. Whereas studies such as Tobin (1978) and Summers and Summers (1989) and current proponents of a Tobin tax argue that it would curb short-term speculation and promote price stability, empirical studies by Roll (1989) and Habermeier and Kirilenko (2003) do not support this claim. Because most empirical studies are subject to endogeneity and measurement concerns, analyzing the impact of Tobin tax in a structural model such as ours would provide complementary evidence. In particular, a structural framework could elucidate the mechanisms and distributional consequences of a Tobin tax, which are often difficult or impossible to measure empirically.

Appendix A – Solving for the Model's Equilibrium Moment Predictions

Here we solve for the symmetric linear equilibrium and the endogenous outcomes in the model introduced in Section 2. We suppress variables' time (t) subscripts in all equations, except those with ambiguous timing.

A. Equilibrium with Exogenous Entry

We impose market clearing to examine the market maker's equilibrium pricing function. The aggregate order flow Q is

$$\begin{aligned} Q &= \sum_{i=1}^{m_I} x_i + \sum_{j=1}^{m_B} x_j + \sum_{k=1}^{m_H} x_k \\ &= \beta_1 S + \beta_2 S_B + \beta_3 H, \end{aligned} \tag{A1}$$

where the aggregate signal and endowment shock realizations S, S_B, and H are

$$S = \sum s_i = m_I[\eta d + \sqrt{1-\eta^2}\rho_z z] + \sqrt{1-\eta^2}\sqrt{1-\rho_z^2}\sum \delta_i \tag{A2}$$

$$S_B = \sum s_j = m_B \rho_u z^B + \sqrt{1-\rho_u^2}\sum \varphi_j \tag{A3}$$

$$H = \sum h_k = m_H \rho_h h + \sqrt{1-\rho_h^2}\sum \psi_k . \tag{A4}$$

In contrast to (A3), overconfident traders' misperception of their aggregate signal realization is

$$S_{BB} = \sum s_j^B = m_B[\eta_B d_t + \rho_{Bu}\sqrt{1-\eta_B^2} z_t^B] + \sqrt{1-\rho_{Bu}^2}\sqrt{1-\eta_B^2}\sum \varphi_{jt} . \tag{A5}$$

The linear pricing function with market depth λ_t is

$$\begin{aligned} p_t - F_t &= \lambda_t Q_t \\ &= \lambda\left[\beta_1 S + \beta_2 S_B + \beta_3 H\right], \end{aligned} \tag{A6}$$

where the market maker's expectation of fundamental value at the beginning of period t is

$$\begin{aligned} F_t &= E(F \mid Q_{t-1}, .., d_{t-1}, d_{t-2}, ...) \\ &= \overline{F} + \sum_{s=1}^{t-1} d_s . \end{aligned} \tag{A7}$$

The zero profit condition for the market maker is:

$$0 = E[Q_t(p_{t+1} - p_t)]. \tag{A8}$$

This requires that the slope of the market maker's linear price schedule is

$$\begin{aligned}\lambda &= \frac{Cov(d,Q)}{Var(Q)} \\ &= \frac{\beta_1 \sigma_{dS}}{\beta_1^2 \sigma_{SS} + \beta_2^2 \sigma_{SBSB} + \beta_3^2 \sigma_{HH}}\end{aligned} \tag{A9}$$

where

$$\sigma_{dS} = Cov(d,S) = m_I \eta \sigma_d^2 \tag{A10}$$

$$\sigma_{SS} = Var(S) = m_I[1+(m_I-1)\rho_I^2]\sigma_d^2 \tag{A11}$$

$$\sigma_{SBSB} = Var(S_B) = m_B[1+(m_B-1)\rho_u^2]\sigma_d^2 \tag{A12}$$

$$\sigma_{HH} = Var(H) = m_H[1+(m_H-1)\rho_h^2]\sigma_h^2. \tag{A13}$$

To simplify expressions throughout, we use the equilibrium liquidity equation

$$\lambda^2 Var(Q) = \lambda^2[\beta_1^2 \sigma_{SS} + \beta_2^2 \sigma_{SBSB} + \beta_3^2 \sigma_{HH}] = \lambda \beta_1 m_I \eta \sigma_d^2. \tag{A14}$$

We characterize traders' beliefs about future dividends. For convenience, we denote the market price that would prevail in the absence of trader *i*'s demand as

$$p_{t(-i)} = p_t - \lambda x_{it}. \tag{A15}$$

We first summarize some properties of traders' beliefs

$$\sigma_{siS} = Cov(s_i, \sum s_{-i}) = (m_I - 1)\rho_I^2 \sigma_d^2 \tag{A16}$$

$$\sigma_{sjS} = Cov_B(s_j^B, S) = m_I \eta_B \eta \sigma_d^2 \tag{A17}$$

$$\sigma_{dSB} = Cov_B(d_t, S_{BB}) = m_B \eta_B \sigma_d^2 \tag{A18}$$

$$\sigma_{sjSB} = Cov_B(s_j^B, \sum s_{-j}^B) = (m_B - 1)\rho_B^2 \sigma_d^2 \tag{A19}$$

$$\sigma_{hkH} = Cov(h_k, \sum h_{-k}) = (m_H - 1)\rho_h^2\sigma_h^2. \qquad \text{(A20)}$$

We also use the following approximation in what follows:

$$\begin{aligned} Var[h_{kt} p_{t+T_H} + y_{kt}(p_{t+T_H} - p_t) \mid h_{kt}] &\approx Var[h_{kt} F_{t+T_H} + y_{kt}(F_{t+T_H} - F_t) \mid h_{kt}] \\ &= (h_{kt} + y_{kt})^2 \sigma_D^2, \end{aligned} \qquad \text{(A21)}$$

where we define

$$\sigma_D^2 \equiv \sum_{l=t}^{t+T_H-1} \sigma_{dl}^2. \qquad \text{(A22)}$$

Rational informed traders choose demand to maximize their expected utility (*i.e.*, profit):

$$\max_{x_{it}} U_{it} = E[x_{it}(p_{t+1} - p_t) \mid s_{it}] = E[x_{it}(d_t - (p_{t(-i)} + \lambda x_{it})) \mid s_{it}]. \qquad \text{(A23)}$$

Using the first-order condition, we can solve for demand

$$x_{it} = \frac{(\eta - \lambda\beta_1\sigma_{siS}\sigma_{dt}^{-2})s_{it}}{2\lambda}, \qquad \text{(A24)}$$

which implies informed trader aggressiveness is

$$\beta_1 = \lambda^{-1}\eta(2 + \sigma_{siS}\sigma_{dt}^{-2})^{-1}. \qquad \text{(A25)}$$

Overconfident but uninformed traders maximize their perceived expected utility from speculation, which is:

$$\max_{x_{jt}} E_B[x_{jt}(p_{t+1} - p_t) \mid s_{jt}^B] - \frac{1}{2\tau_B} Var_B[x_{jt}(p_{t+1} - p_t) \mid s_{jt}^B]. \qquad \text{(A26)}$$

Maximizing this quadratic equation in x_{jt} and solving for x_{jt} gives:

$$x_{jt} = \frac{(\eta_B - \lambda\beta_1\sigma_{sjS}\sigma_{dt}^{-2} - \lambda\beta_2\sigma_{sjSB}\sigma_{dt}^{-2})}{2\lambda + \frac{1}{\tau_B}\sigma_d^2} s_{jt}^B, \qquad \text{(A27)}$$

where we have applied the variance approximation above in the special case in which $h_{jt} = 0$ and $T_H = 1$. Defining $f_k \equiv \lambda\beta_k$, this implies

$$\beta_2(2\lambda+\frac{1}{\tau_B}\sigma_d^2)=(\eta_B-\lambda\beta_1\sigma_{sjS}\sigma_{dt}^{-2}-\lambda\beta_2\sigma_{sjSB}\sigma_{dt}^{-2}) \tag{A28}$$

$$\begin{gathered}f_2=\frac{\eta_B-f_1\sigma_{sjS}\sigma_{dt}^{-2}-\beta_2(\frac{1}{\tau_B}\sigma_d^2)}{2+\sigma_{sjSB}\sigma_{dt}^{-2}}\\(2+\sigma_{sjSB}\sigma_{dt}^{-2})f_2=\eta_B-f_1\sigma_{sjS}\sigma_{dt}^{-2}-\beta_2(\frac{1}{\tau_B}\sigma_d^2).\end{gathered} \tag{A29}$$

Rational uninformed hedgers also maximize their expected utility, which is:

$$\max_{x_{kt}} E[h_{kt}p_{t+T_H}+x_{kt}(p_{t+T_H}-p_t)\,|\,h_{kt}]-\frac{1}{2\tau}Var[h_{kt}p_{t+T_H}+x_{kt}(p_{t+T_H}-p_t)\,|\,h_{kt}]. \tag{A30}$$

Again using the variance approximation in Equation (A21), we write the first-order condition as

$$x_{kt}=-\frac{\lambda\beta_3\sigma_{hkH}\sigma_{ht}^{-2}+\frac{1}{\tau}\sigma_D^2}{2\lambda+\frac{1}{\tau}\sigma_D^2}h_{kt} \tag{A31}$$

which implies hedging aggressiveness is

$$\beta_3=-\frac{f_3\sigma_{hkH}\sigma_{ht}^{-2}+\frac{1}{\tau}\sigma_D^2}{2\lambda+\frac{1}{\tau}\sigma_D^2}=-1-(\frac{1}{\tau}\sigma_D^2)^{-1}(2+\sigma_{hkH}\sigma_{ht}^{-2})f_3 \tag{A32}$$

and

$$f_3=-\frac{(1+\beta_3)(\frac{1}{\tau}\sigma_D^2)}{2+\sigma_{hkH}\sigma_{ht}^{-2}}. \tag{A33}$$

From Equation (A32), one can see that as hedgers' risk aversion becomes infinite, they fully hedge their endowment shocks ($\beta_3=-1$). Hedgers' own price impact and that of the aggregate endowment shock prevent complete hedging in the more general case.

We now solve for the equilibrium endogenous parameters. We first analyze the ratio of the aggressiveness of the two types uninformed traders' as given by Equations (A29) and (A33):

$$\frac{\beta_2}{\beta_3}=\frac{(2+\sigma_{hkH}\sigma_{ht}^{-2})[\eta_B-f_1\sigma_{sjS}\sigma_{dt}^{-2}-\beta_2(\frac{1}{\tau_B}\sigma_d^2)]}{-(2+\sigma_{sjSB}\sigma_{dt}^{-2})(\frac{1}{\tau}\sigma_D^2)(1+\beta_3)}. \tag{A34}$$

Substituting $\beta_2=\beta_3(f_2/f_3)$ and solving for f_2/f_3, we have

$$\frac{f_2}{f_3}=\frac{(2+\sigma_{hkH}\sigma_{ht}^{-2})[\eta_B - f_1\sigma_{sjS}\sigma_{dt}^{-2}]}{-(2+\sigma_{sjSB}\sigma_{dt}^{-2})(\frac{1}{\tau}\sigma_D^2)(1+\beta_3)+(2+\sigma_{hkH}\sigma_{ht}^{-2})(\frac{1}{\tau_B}\sigma_d^2)\beta_3}. \tag{A35}$$

We then substitute Equation (A32) for β_3 and rearrange to write

$$\begin{aligned}\frac{f_2}{f_3}&=\frac{\eta_B - f_1\sigma_{sjS}\sigma_{dt}^{-2}}{-(\frac{1}{\tau_B}\sigma_d^2)+[(2+\sigma_{sjSB}\sigma_{dt}^{-2})-(\frac{1}{\tau_B}\sigma_d^2)(\frac{1}{\tau}\sigma_D^2)^{-1}(2+\sigma_{hkH}\sigma_{ht}^{-2})]f_3}\\ &=\frac{a_0}{a_1+a_2 f_3},\end{aligned} \tag{A36}$$

where

$$a_0=\eta_B - f_1\sigma_{sjS}\sigma_{dt}^{-2} \tag{A37}$$

$$a_1=-\frac{1}{\tau_B}\sigma_d^2 \tag{A38}$$

$$a_2=(2+\sigma_{sjSB}\sigma_{dt}^{-2})-(\frac{1}{\tau_B}\sigma_d^2)(\frac{1}{\tau}\sigma_D^2)^{-1}(2+\sigma_{hkH}\sigma_{ht}^{-2}). \tag{A39}$$

We combine the equilibrium liquidity condition (A14) and the three first-order conditions above:

$$f_1^2\sigma_{SS}+f_2^2\sigma_{SBSB}+f_3^2\sigma_{HH}=f_1\sigma_{dS} \tag{A40}$$

$$f_3^2[(f_2/f_3)^2\sigma_{SBSB}+\sigma_{HH}]=f_1\sigma_{dS}-f_1^2\sigma_{SS}=f_1^2 m_I\sigma_d^2 \tag{A41}$$

$$f_3^2[a_0^2\sigma_{SBSB}+\left(a_1+a_2 f_3\right)^2\sigma_{HH}]=\left(a_1+a_2 f_3\right)^2 f_1^2 m_I\sigma_d^2 \tag{A42}$$

$$\begin{aligned}&(a_0^2\sigma_{SBSB}+a_1^2\sigma_{HH}-a_2^2 f_1^2 m_I\sigma_d^2)f_3^2+2a_1a_2\sigma_{HH}f_3^3+a_2^2\sigma_{HH}f_3^4\\ &=a_1^2 f_1^2 m_I\sigma_d^2+2a_1a_2 f_1^2 m_I\sigma_d^2 f_3\end{aligned} \tag{A43}$$

We solve for f_3 as the negative real root of Equation (A43). Given a value of f_3, we can solve for hedgers' aggressiveness (β_3) according to Equation (A32) and, in turn, equilibrium illiquidity (λ). We then use Equation (A36) and equilibrium liquidity to solve for overconfident traders' aggressiveness (β_2). Finally, we use the equilibrium liquidity condition to solve for informed traders' aggressiveness (β_1).

B. Special Cases

Special cases in which there are no overconfident traders ($\eta_B = 0$) or no hedgers ($\sigma_h = 0$) are easier to solve and provide intuition. In the case without overconfident traders, hedgers' aggressiveness is simply

$$f_1^2 \sigma_{SS} + f_3^2 \sigma_{HH} = f_1 \sigma_{dS} \tag{A44}$$

$$\left[\frac{(1+\beta_3)(\frac{1}{\tau}\sigma_D^2)}{2+\sigma_{hkH}\sigma_{ht}^{-2}} \right]^2 \sigma_{HH} = f_1 \sigma_{dS} - f_1^2 \sigma_{SS} = f_1^2 m_I \sigma_d^2 \tag{A45}$$

$$\beta_3 = -1 + \frac{f_1 \sigma_d \sqrt{m_I}[2+(m_H-1)\rho_h^2]}{\frac{1}{\tau}\sigma_D^2 \sigma_h \sqrt{m_H[1+(m_H-1)\rho_h^2]}} \tag{A46}$$

and equilibrium illiquidity is

$$\begin{aligned} \lambda &= f_3 / \beta_3 \\ &= -\frac{(1+\beta_3^{-1})(\frac{1}{\tau}\sigma_D^2)}{2+\sigma_{hkH}\sigma_{ht}^{-2}} \\ &= \frac{\frac{1}{\tau}\sigma_D^2 f_1 \sigma_d \sqrt{m_I}}{\frac{1}{\tau}\sigma_D^2 \sigma_h \sqrt{m_H[1+(m_H-1)\rho_h^2]} - [2+(m_H-1)\rho_h^2] f_1 \sigma_d \sqrt{m_I}}. \end{aligned} \tag{A47}$$

Liquidity improves as the number of hedgers increases because there is more uninformed trading. Liquidity can actually improve with the number of informed traders, too, because competition among informed traders leads to more aggressive trading from each trader and reveals their private information. There is a countervailing effect on liquidity to the extent that additional informed traders possess novel information that is not already incorporated in prices.

No equilibrium exists if

$$\beta_3 = -1 + \frac{f_1\sigma_d\sqrt{m_I}[2+(m_H-1)\rho_h^2]}{\frac{1}{\tau}\sigma_D^2\sigma_h\sqrt{m_H[1+(m_H-1)\rho_h^2]}} > 0$$

$$\frac{\eta\sqrt{m_I}}{[2+(m_I-1)\rho_I^2]}\sigma_d > \frac{1}{\tau}\sigma_D^2\sigma_h\frac{\sqrt{m_H[1+(m_H-1)\rho_h^2]}}{[2+(m_H-1)\rho_h^2]} \tag{A48}$$

$$\frac{\sigma_d}{\eta\sqrt{m_I}} > \frac{1}{\tau}\sigma_D^2\sigma_h\rho_h^{-1}.$$

One obtains similar intuition from an analysis of the case in which there are no hedgers

$$f_1^2\sigma_{SS} + f_2^2\sigma_{SBSB} = f_1\sigma_{dS} \tag{A49}$$

$$\left[\frac{\eta_B - f_1\sigma_{sjS}\sigma_{dt}^{-2} - \beta_2(\frac{1}{\tau_B}\sigma_d^2)}{2+\sigma_{sjSB}\sigma_{dt}^{-2}}\right]^2\sigma_{SBSB} = f_1\sigma_{dS} - f_1^2\sigma_{SS} = f_1^2 m_I\sigma_d^2 \tag{A50}$$

$$\beta_2 = (\frac{1}{\tau_B}\sigma_d^2)^{-1}\left[\eta_B - f_1\sigma_{sjS}\sigma_{dt}^{-2} - \frac{f_1\sigma_d\sqrt{m_I}(2+\sigma_{sjSB}\sigma_{dt}^{-2})}{\sqrt{\sigma_{SBSB}}}\right] \tag{A51}$$

and equilibrium illiquidity is

$$\begin{aligned}\lambda &= f_2/\beta_2 \\ &= \frac{(\eta_B - f_1\sigma_{sjS}\sigma_d^{-2})\beta_2^{-1} - (\frac{1}{\tau_B}\sigma_d^2)}{2+\sigma_{sjSB}\sigma_{dt}^{-2}} \\ &= \frac{(\frac{1}{\tau_B}\sigma_d^2)f_1\sqrt{m_I}}{\sqrt{m_B[1+(m_B-1)\rho_u^2]}\eta_B(1-f_1m_I\eta) - f_1\sqrt{m_I}[2+(m_B-1)\rho_B^2]}.\end{aligned} \tag{A52}$$

Here, liquidity increases with the number of overconfident traders.

C. Endogenous Entry

The three trader types simultaneously choose whether to enter the market, which endogenously determines m_I, m_B, and m_H in equilibrium. Our solution strategy is to express all equations in terms of the number of hedgers (m_H) and then solve for m_H numerically. In the equations below, we use the following quadratic cost functions:

$$C_I(m_I) \equiv c_I(m_I-1)^2 \text{ for } m_I \geq 1, \tag{A53}$$

$$C_B(m_B) \equiv c_B(m_B - 1)^2 \text{ for } m_B \geq 1, \tag{A54}$$

$$C_H(m_H) \equiv c_H(m_H - 1)^2 \text{ for } m_H \geq 1, \tag{A55}$$

in which the cost of the first trader entering is zero. We choose the quadratic functional form for its tractability and realism. Cross-trader variation in entry costs reflects both differential access to information and different opportunity costs of time.

First, we compute the equilibrium utility of rational hedgers net of participation costs

$$\begin{aligned} E_k U_{kt} &= E[h_{kt} p_{t+T_H} + x_{kt}(p_{t+T_H} - p_t)] - \frac{1}{2\tau} Var[h_{kt} p_{t+T_H} + x_{kt}(p_{t+T_H} - p_t)] - c_H(m_H - 1)^2 \\ &= \frac{1}{2}(2\lambda + \frac{1}{\tau}\sigma_D^2) E[(x_{kt})^2] + E[h_{kt} p_{t+T_H}] - \frac{1}{2\tau} Var[h_{kt} p_{t+T_H}] - c_H(m_H - 1)^2. \end{aligned} \tag{A56}$$

By setting this utility equal to hedgers' reservation utility, we obtain:

$$\frac{1}{2}(2\lambda + \frac{1}{\tau}\sigma_D^2)\beta_3^2\sigma_h^2 = c_H(m_H - 1)^2. \tag{A57}$$

Substituting Equation (A32) for hedger aggressiveness (β_3) and Equation (A33) for f_3 and then rearranging, we obtain a quadratic equation in hedger aggressiveness (β_3):

$$(m_H - 1)\rho_h^2\beta_3^2 - 2\beta_3 - 2\tau\sigma_D^{-2}\sigma_h^{-2}(2 + (m_H - 1)\rho_h^2)c_H(m_H - 1)^2 = 0. \tag{A58}$$

The negative root of this equation yields β_3 as a function of the number of hedgers:

$$\beta_3(m_H) \equiv \frac{1 - \sqrt{1 + 2\tau\sigma_D^{-2}\sigma_h^{-2}c_H(m_H - 1)^3\rho_h^2[2 + (m_H - 1)\rho_h^2]}}{(m_H - 1)\rho_h^2} \text{ if } m_H > 1. \tag{A59}$$

Using $\beta_3(m_H)$ and Equation (A33), we express illiquidity in terms of the number of hedgers

$$\lambda(m_H) \equiv -\frac{(1 + \beta_3(m_H)^{-1})(\frac{1}{\tau}\sigma_D^2)}{2 + \sigma_{hkH}\sigma_{ht}^{-2}}. \tag{A60}$$

This implies that $f_3(m_H) \equiv \lambda(m_H)\beta_3(m_H)$.

Next, to express the number of informed traders as a function of the number of hedgers, we set informed traders' expected profits net of entry costs (c_I) equal to zero

$$\begin{aligned} E_i U_{it} &= E[(p_{t+1} - p_{t(-i)} + F_t)x_{it}] - \lambda E[x_{it}^2] - c_I(m_I - 1)^2 \\ &= \lambda E[x_{it}^2] - c_I(m_I - 1)^2 \\ &= \lambda^{-1} f_1^2 \sigma_{dt}^2 - c_I(m_I - 1)^2 = 0. \end{aligned} \tag{A61}$$

We substitute Equations (A25) and (A16) and simplify, yielding a quadratic equation in $(m_I - 1)$:

$$0 = \rho_I^2 (m_I - 1)^2 + 2(m_I - 1) - \eta \sigma_d [\lambda(m_H) c_I]^{-1/2}, \tag{A62}$$

where we express equilibrium illiquidity ($\lambda(m_H)$) in terms of m_H. The solution to the quadratic allows us to express the number of informed traders as a function of the number of hedgers:

$$m_I(m_H) \equiv 1 + \rho_I^{-2} \left[-1 + \sqrt{1 + \rho_I^2 \eta \sigma_d [\lambda(m_H) c_I]^{-1/2}} \right]. \tag{A63}$$

We can then use Equations (A25) and (A16) to express f_1 as a function of m_H:

$$f_1(m_H) \equiv \eta [2 + (m_I(m_H) - 1)\rho_I^2]^{-1}, \tag{A64}$$

and thus we have $\beta_1(m_H) = f_1(m_H) / \lambda(m_H)$.

Next we set the expected utility of uninformed overconfident traders net of participation costs (c_B) equal to their zero reservation utility

$$E_j^B U_{jt} = \frac{1}{2}(2\lambda + \frac{1}{\tau_B}\sigma_d^2)\beta_2^2 \sigma_d^2 - c_B(m_B - 1)^2 = 0. \tag{A65}$$

This implies that we can express the number of overconfident traders in terms of equilibrium illiquidity and their aggressiveness

$$m_B(m_H, \beta_2) = 1 + \sqrt{c_B^{-1}(\lambda(m_H) + \frac{1}{2\tau_B}\sigma_d^2)}\beta_2 \sigma_d. \tag{A66}$$

We substitute Equations (A59), (A60), and (A64) into Equation (A36) to write overconfident trader aggressiveness as

$$\beta_2 = \frac{\beta_3(m_H)a_0(m_H)}{a_1(m_H)+[2+\sqrt{c_B^{-1}(\lambda(m_H)+\frac{1}{2\tau_B}\sigma_d^2)}\sigma_d\rho_B^2\beta_2 - (\frac{1}{\tau_B}\sigma_d^2)(\frac{1}{\tau}\sigma_D^2)^{-1}(2+(m_H-1)\rho_h^2)]f_3(m_H)}. \quad \text{(A67)}$$

We solve the above quadratic equation in β_2 to obtain $\beta_2(m_H)$, which we substitute into Equation (A66) to express m_B in terms of m_H:

$$m_B(m_H, \beta_2(m_H)) \equiv 1 + \sqrt{c_B^{-1}(\lambda(m_H) + \frac{1}{2\tau_B}\sigma_d^2)}\beta_2(m_H)\sigma_d. \quad \text{(A68)}$$

Knowing $\lambda(m_H)$ and $\beta_2(m_H)$ also defines $f_2(m_H)$. In summary, we have now defined all key endogenous variables (f_1, f_2, f_3, m_I, m_B) in terms of the equilibrium number of hedgers (m_H).We substitute these expressions for (f_1, f_2, f_3, m_I, m_B) into the market maker's zero-profit condition to obtain a polynomial equation in m_H , which we solve using standard numerical methods.

The equilibria with endogenous entry decisions are a subset of the possible trading game equilibria in which the equilibrium utilities are consistent with the number of traders entering the market. Specifically, each trader type must earn a non-negative equilibrium utility if and only if at least one trader of that type enters the market. Formally, an equilibrium outcome must satisfy

$$EU_i(m_I, m_B, m_H) = C_I(m_I) \text{ and } \frac{\partial EU_i(m_I, m_B, m_H)}{\partial m_I} \le 0 \text{ if and only if } m_I > 0 \quad \text{(A69)}$$

$$EU_j(m_I, m_B, m_H) = C_B(m_B) \text{ and } \frac{\partial EU_j(m_I, m_B, m_H)}{\partial m_B} \le 0 \text{ if and only if } m_B > 0 \quad \text{(A70)}$$

$$EU_k(m_I, m_B, m_H) = C_H(m_H) \text{ and } \frac{\partial EU_k(m_I, m_B, m_H)}{\partial m_H} \le 0 \text{ if and only if } m_H > 0. \quad \text{(A71)}$$

The derivative restriction ensures that no trader has an incentive to deviate from equilibrium by unilaterally entering the market. We also require equilibrium illiquidity $\lambda(m_I, m_B, m_H)$ to be non-negative to satisfy each trader's second-order condition. If these restrictions are met, the entry cost parameters (c_I, c_B, c_H) implement the trading game equilibrium with (m_I, m_B, m_H) traders of each type.

D. Implications for Empirical Moments

We now analyze the return variance, trading volume, and liquidity implications of the model using the definitions in Equations (14), (15), and (16). First, variance is

$$\begin{aligned} Var(r_t) &= Var(\lambda_t Q_t) + Var(d_{t-1} - \lambda_{t-1} Q_{t-1}) \\ &= (2 + (m_{It} - 1)\rho_{It}^2)^{-1} m_{It} \eta_t^2 \sigma_{dt}^2 \\ &\quad + (2 + (m_{It-1} - 1)\rho_{It-1}^2)^{-1} (2 + (m_{It-1} - 1)\rho_{It-1}^2 - m_{It-1}\eta^2)\sigma_{dt-1}^2. \end{aligned} \tag{A72}$$

We compute volume as

$$\begin{aligned} E(v_t) &= \frac{1}{2}(m_{It} E[|x_{It}|] + m_{Bt} E[|y_{jt}^B|]) + m_{Ht} E[|y_{kt}|]) + \frac{1}{2} E[|Q_t|] \\ &= \frac{1}{\sqrt{2\pi}}(m_I \sqrt{Var[\beta_{1t} s_{it}]} + m_{Bt}\sqrt{Var[\beta_{2t} s_{jt}^B]} + m_{Ht}\sqrt{Var[\beta_{3t} h_{jt}]}) + \frac{1}{\sqrt{2\pi}}\sqrt{Var[Q_t]} \\ &= \frac{1}{\sqrt{2\pi}}(m_{It}\beta_{1t}\sigma_{dt} + m_{Bt}\beta_{2t}\sigma_{dt} + m_{Ht}\beta_{3t}\sigma_{ht}) + \frac{1}{\sqrt{2\pi}}\sqrt{\beta_{1t}^2\sigma_{SSt} + \beta_{2t}^2\sigma_{SBSBt} + \beta_{3t}^2\sigma_{HHt}}. \end{aligned} \tag{A73}$$

Finally, we compute illiquidity (ILQ_t) as the ratio of the covariance between absolute returns and volume to the variance of volume. The numerator and denominator are

$$\begin{aligned} Cov(|r_t|, v_t) &= Cov(|r_t|, \frac{1}{2}(\sum |x_{it}| + \sum |x_{jt}| + \sum |x_{kt}| + |Q_t|)) \\ &= \frac{\sqrt{Var(r_t)}}{2} \left[\begin{array}{c} m_{It}\beta_1\sigma_d g\left[\frac{\lambda_t\beta_1(1+\sigma_{siS}\sigma_d^{-2})\sigma_d}{\sqrt{Var(r)}}\right] + m_{Bt}\beta_2\sigma_d g\left[\frac{\lambda_t\beta_2(1+\sigma_{RsjSB}\sigma_d^{-2})\sigma_d}{\sqrt{Var(r)}}\right] \\ + m_{Ht}|\beta_3|\sigma_h g\left[\frac{\lambda_t|\beta_3|(1+\sigma_{hkH}\sigma_h^{-2})\sigma_h}{\sqrt{Var(r)}}\right] + g\left[\frac{\lambda_t\sqrt{Var(Q_t)}}{\sqrt{Var(r)}}\right] \end{array} \right] \end{aligned} \tag{A74}$$

and

$$Var(v_t) = Cov(\frac{1}{2}(\sum|x_{it}|+\sum|x_{jt}|+\sum|x_{kt}|+|Q_t|), \frac{1}{2}(\sum|x_{it}|+\sum|x_{jt}|+\sum|x_{kt}|+|Q_t|))$$

$$= \frac{1}{4}\begin{bmatrix} (1-2/\pi)VarQ_t + 2m_{It}\beta_1\sigma_{dt}\sqrt{VarQ}g\left[\frac{\beta_1(1+\sigma_{siS}\sigma_d^{-2})\sigma_d}{\sqrt{VarQ}}\right] \\ 2m_{Bt}\beta_2\sigma_d\sqrt{VarQ}g\left[\frac{\beta_2(1+\sigma_{RsjSB}\sigma_d^{-2})\sigma_d}{\sqrt{VarQ}}\right] \\ +2m_{Ht}|\beta_3|\sigma_h\sqrt{VarQ}g\left[\frac{|\beta_3|(1+\sigma_{hkH}\sigma_h^{-2})\sigma_h}{\sqrt{VarQ}}\right] \\ +m_{It}\beta_1^2\sigma_{dt}^2[(1-2/\pi)+(m_{It}-1)g(\rho_I^2)] \\ +m_{Bt}\beta_2^2\sigma_{dt}^2[(1-2/\pi)+(m_{Bt}-1)g(\rho_u^2)] \\ +m_{Ht}\beta_3^2\sigma_{ht}^2[(1-2/\pi)+(m_{Ht}-1)g(\rho_h^2)] \end{bmatrix}, \tag{A75}$$

where the covariance between the absolute value of two correlated standard normal random variables with correlation ρ is given by the function

$$g(\rho) = \frac{2}{\pi}\left[\sqrt{1-\rho^2}-1\right] + \frac{\rho}{\pi}\left[\arctan\left(\frac{\rho}{\sqrt{1-\rho^2}}\right) - \arctan\left(\frac{\sqrt{1-\rho^2}}{\rho}\right) + \frac{\pi}{2}\right], \tag{A76}$$

and the covariance of each uninformed agent's signal with the aggregate uninformed signal is computed under rational, not biased, expectations

$$\sigma_{RsjSB} = Cov(s_j^B, \sum s_{-j}^B) = (m_B - 1)\rho_u^2\sigma_{dt}^2. \tag{A77}$$

The illiquidity moment equation is thus the ratio of Equation (A74) to Equation (A75). The moment equations above are the key testable implications of the model.

Appendix B – Measuring Returns Using TAQ Data

During regular trading, we only keep trades and quotes meeting standard filters used in the microstructure literature. We drop trades with non-positive price or size and those with correction codes not equal to zero or condition code of M, Q, T, or U. For the pre-market and after-market periods, however, the filters for trades necessarily differ. Importantly, we do not exclude trades with a condition code of T, which explicitly identifies extended hours trades. For extended hours periods, we exclude those that occur at prices probably determined within the trading day (*e.g.*, crosses and block trades), appear out of sequence, or contain non-standard delivery options. This filter eliminates any trades from NYSE, AMEX, or CBOE and trades with "cond" codes B, G, K, M, L, N, O, P, W, U, Z, 4, 5, 6, 8, or 9. We drop trades of at least 10,000 shares or $200,000 regardless of their "cond" codes as these are likely pre-negotiated blocks. Finally, we drop all trades and quotes in the final minute of the pre-market and in the first minute of the after-market period to mitigate effects of bid-ask bounce.

Within each of the pre-market, regular, and after-market periods, we construct a beginning and ending trade price as the volume-weighted average price (VWAP) based on the first and last minute of trades in the dataset and then compute trade-based returns.[22] Using a VWAP instead of a single trade price further mitigates the effects of bid-ask bounce in returns. When there is only one trade observation in an intraday period, its return is computed from the last price from the most recent intraday period. When there is no trading in a period, the return is zero.

Appendix C – Data and Estimation Procedures

We estimate the covariance matrix of the empirical moments using a model that allows each moment to be persistent and to depend on persistent systematic factors. For simplicity, we model persistence as an AR(1) process at the quarterly frequency. We define three factors (f_t) as the sum across moments of each type (i.e., variance, volume, and illiquidity). We estimate an AR(1) model for each factor:

$$f_t = \alpha_f + \rho_f f_{t-1} + e_{ft}, \tag{A78}$$

where α_f and ρ_f are estimated via OLS.

[22] Because dividend and stock split adjustments occur between the final trade on the trading day prior to the ex date and the first trade on the ex date, they are not considered in our pre-market, regular market, and after market returns.

For each moment m_t, we estimate an AR(1) model in which the moment can also depend on the systematic factor:

$$m_t = \alpha_m + \rho_m m_{t-1} + \beta_f f_t + e_{mt}, \tag{A79}$$

where α_m, ρ_m, and β_f are estimated via OLS. Using this equation, one can define an "abnormal moment" that is orthogonal to the factor:

$$\tilde{m}_t = m_t - \beta_f f_t. \tag{A80}$$

Computing expectations of the last three equations leads to the following expressions for the means of the factors, the abnormal moments, and the raw moments:

$$E[f_t] = \frac{\alpha_f}{1-\rho_f} \tag{A81}$$

$$E[\tilde{m}_t] = \frac{\alpha_m}{1-\rho_m} \tag{A82}$$

$$E[m_t] = \frac{\alpha_m + \beta_f E[f_t]}{1-\rho_m} = \frac{\alpha_m}{1-\rho_m} + \frac{\alpha_m + \beta_f \left(\alpha_f \middle/ 1-\rho_f \right)}{1-\rho_m}. \tag{A83}$$

We compute standard errors by applying the delta method to the abnormal and raw moment mean equations. First, we obtain the covariance matrix of $E[\tilde{m}_t]$ using the delta method. Second, we separately apply the delta method to the second term in each $E[m_t]$ equation to add in the uncertainty in each raw moment mean that is caused by uncertainty in the factor means. We treat the raw and abnormal moments as conceptually distinct because our model analyzes firm-specific information, rather than systematic information. Thus, the model's moment predictions do not depend on the realizations of systematic factors. Similarly, our empirical method above eliminates the factors' influence on the off-diagonal terms in the moment covariance matrix by regressing each moment on the appropriate factor (e.g., pre-market non-news volume is regressed on the volume factor) and using the covariances in the resulting residuals e_{mt} in our covariance matrix calculation. The covariance matrix of residual moments has off-diagonal correlations that are usually quite reasonable—e.g., 95% of the elements have correlations with absolute values of less than 0.7.

Bibliography

Admati, Anat R., and Paul Pfleiderer, 1988, A theory of intraday patterns: Volume and price variability, *Review of Financial Studies* 1, 3-40.

Alti, Aydogan, and Paul C. Tetlock, 2013, Biased beliefs, asset prices, and investment: A structural approach, *Journal of Finance*, forthcoming.

Amihud, Yakov, 2002, Illiquidity and stock returns: Cross-section and time-series effects, *Journal of Financial Markets* 5, 31-56.

Barberis, Nicholas, and Wei Xiong, 2012, Realization utility, *Journal of Financial Economics* 104, 251-271.

Barclay, Michael J., and Terrence Hendershott, 2003, Price discovery and trading after hours, *Review of Financial Studies* 16, 1041-1073.

Chordia, Tarun, Richard Roll, and Avanidhar Subrahmanyam, 2011, Recent trends in trading activity and market quality, *Journal of Financial Economics* 101, 243-263.

Cochrane, John H., 2001, *Asset Pricing*, Princeton University Press.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market under- and overreactions, *Journal of Finance* 53, 1839-1885.

DeBondt, Werner F.M., and Richard H. Thaler, 1995, Financial decision-making in markets and firms: A behavioral perspective, In *Handbooks in Operations Research and Management Science: Finance*. Edited by Robert A. Jarrow, Vojislav Maksimovic, and William T. Ziemba. Amsterdam: Elsevier: 385-410.

DeLong, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann, 1990, Noise trader risk in financial markets, *Journal of Political Economy* 98, 703-738.

Dorn, Daniel, Gur Huberman, and Paul Sengmueller, 2008, Correlated trading and returns, *Journal of Finance* 58, 885–920.

Fama, Eugene F., 1970, Efficient capital markets: A review of theory and empirical work, *Journal of Finance* 25, 383-417.

French, Kenneth R., 2008, The cost of active investing, *Journal of Finance* 58, 1537-1573.

French, Kenneth R., and Richard Roll, 1986, Stock return variances: The arrival of information and the reaction of traders, *Journal of Financial Economics* 17, 5-26.

Gervais, Simon, and Terrance Odea, 2001, Learning to be overconfident, *Review of Financial Studies* 14, 1-27.

Habermeier, Karl, and Andrei Kirilenko, 2003, Securities transaction taxes and financial markets, *International Monetary Fund Staff Papers* 50, 165-180.

Hansen, Lars P., and Kenneth J. Singleton, 1982, Generalized instrumental variables estimation of nonlinear rational expectations models, *Econometrica* 50, 1269-1288.

Harris, Milton, and Artur Raviv, 1993, Differences of opinion make a horse race, *Review of Financial Studies* 6, 473-506.

Heaton, John, and Deborah J. Lucas, 1996, Evaluating the effects of incomplete markets on risk sharing and asset prices, *Journal of Political Economy* 104, 443-487.

Heaton, John, and Deborah J. Lucas, 2000, Portfolio choice and asset prices: The importance of entrepreneurial risk, *Journal of Finance* 55, 1163-1198.

Holden, Craig W., and Avanidhar Subrahmanyam, 1992, Long-lived private information and imperfect competition, *Journal of Finance* 47, 247-270.

Ingersoll, Jonathan E., and Lawrence J. Jin, 2013, Realization utility with reference-dependent preferences, *Review of Financial Studies* 26, 723-767.

Kim, Oliver, and Robert E. Verrecchia, 1997, Pre-announcement and event-period private information, *Journal of Accounting and Economics* 24, 395-419.

Kyle, Albert S., 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315-1335.

Kyle, Albert S., and F. Albert Wang, 1997, Speculation duopoly with agreement to disagree: Can overconfidence survive the market test? *Journal of Finance* 52, 2073-2090.

Lichtenstein, Sarah, and Baruch Fischhoff, 1977, Do those who know more also know more about how much they know? *Organizational Behavior and Human Performance* 20, 159-183.

Obizhaeva, Anna, 2009, Portfolio Transitions and Stock Price Dynamics, University of Maryland working paper.

Odean, Terrance, 1998, Volume, volatility, price, and profit when all traders are above average, *Journal of Finance* 53, 1887-1934.

Roll, Richard, 1989, "Price volatility, international market links, and their implications for regulatory policies," *Journal of Financial Services Research* 3, 211–46.

Scheinkman, José A., and Wei Xiong, 2003, Overconfidence and speculative bubbles, *Journal of Political Economy* 111, 1183-1219.

Spiegel, Matthew, and Avanidhar Subrahmanyam, 1992, Informed speculation and hedging in a noncompetitive securities market, *Review of Financial Studies* 5, 307-329.

Meir Statman, Steven Thorley, and Keith Vorkink, 2006, Investor overconfidence and trading volume, *Review of Financial Studies* 19, 1531-1565.

Summers, Lawrence H., and Victoria P. Summers, 1989, "When financial markets work too well: A cautious case for a securities transaction tax," *Journal of Financial Services Research* 3, 261–86.

Tobin, James, 1978, A proposal for international monetary reform, *Eastern Economic Journal* 4, 153-159.

Table I
Comparative Statics and Parameter Identification

This table summarizes the model presented in Section 2. Panel A describes exogenous parameters and restrictions imposed in the estimation of the main (MAIN) and no overconfidence (NO-OC) versions of the model. Panel B provides comparative statics for variance, volume, and illiquidity moments.

Panel A: Parameter Descriptions and Restrictions

		Restriction by Model		Periodic
Parameter	Description	MAIN	NO-OC	Variation
η_B	Overconfidence (uninformed perceived precision)	None	0	No
RRA	Relative risk aversion of uninformed traders	None	None	No
η	Informed signal precision	None	None	Yes
σ_d	Volatility of acquirable information	None	None	Yes
σ_h	Volatility of annual endowment shocks	0.0167	0.0167	No
$\rho_z^2 = \rho_{Bu}^2$	Perceived correlations among signal errors	0	0	No
ρ_u^2	Actual correlation of uninformed signals	0.119	0.119	No
ρ_h^2	Correlation among endowment shocks	0.119	0.119	No
c_I	Informed entry cost parameter	2.77 x 10^{-11}	2.77 x 10^{-11}	No
c_B	Overconfident entry cost parameter	c_H	N/A	No
c_H	Hedger entry cost parameter	4.19 x 10^{-13}	4.19 x 10^{-13}	No

Panel B: Comparative Statics

	Impact on the Model's Prediction		
Parameter	Variance	Volume	Illiquidity
η_B	Inverted U-shape	Inverted U-shape	U-shape
RRA_B	Decrease	Decrease	Increase
RRA_H	Increase	Increase	Decrease
η	Inverted U-shape	Decrease	Increase
σ_d	Increase	Inverted U-shape	Increase

Table II
Empirical and Model Predicted Moments

This table presents empirical moment estimates for return variance, trading volume, and illiquidity moments for the Regular Market (9:30 AM to 4:00 PM), Pre-Market (7:00 AM to 9:30 AM), and After Market (4:00 PM to 6:30 PM) conditional on news or no news. Standard errors computed as in Appendix B appear in parentheses. Variance and volume moments in the Pre-Market and After-Market are scaled by a factor of 6.5/2.5 for comparison with the Regular Market period. Variance moments are expressed as volatility and volume moments are expressed as turnover. The second and third columns contain predicted moments from GMM estimations of the model. The main version (MAIN) estimates the 14 parameters listed in Table I Panel A, while the no overconfidence version (NO-OC) restricts the overconfidence parameter (η_B) to be zero. Table I Panel A lists all additional restrictions. The χ^2 statistics and *p*-values from a test of overidentifying restrictions for each model appear in the bottom two rows. DoF refers to degrees of freedom.

Table II: continued

Regular Market Moments	Empirical	Prediction (MAIN)	Prediction (NO-OC)
News Volatility (%)	3.999	4.074	3.458
	(0.471)		
No-News Volatility (%)	3.040	2.743	3.458
	(0.361)		
News Turnover (%)	2.113	2.094	0.019
	(0.181)		
No-News Turnover (%)	1.455	1.416	0.018
	(0.129)		
News Illiquidity	7.747	7.223	7.265
	(1.488)		
No-News Illiquidity	7.920	6.189	7.248
	(1.519)		
Pre-Market Moments			
News Volatility (%)	3.572	3.606	0.942
	(0.234)		
No-News Volatility (%)	1.182	1.185	1.319
	(0.179)		
News Turnover (%)	0.061	0.056	0.012
	(0.008)		
No-News Turnover (%)	0.008	0.008	0.008
	(0.001)		
News Illiquidity	227.125	179.876	215.316
	(49.760)		
No-News Illiquidity	259.393	252.393	419.176
	(64.241)		
After-Market Moments			
News Volatility (%)	6.044	5.965	1.994
	(0.556)		
No-News Volatility (%)	1.635	1.962	1.944
	(0.247)		
News Turnover (%)	0.148	0.144	0.011
	(0.018)		
No-News Turnover (%)	0.021	0.020	0.012
	(0.003)		
News Illiquidity	112.389	112.136	109.141
	(15.064)		
No-News Illiquidity	90.207	84.045	80.446
	(15.195)		
χ^2 with 4 (MAIN) or 5 (NO-OC) DoF		5.886	449.138
p-value		0.208	0.000

Table III
Parameter Estimates

This table presents GMM parameter estimates for two versions of the model. The main version (MAIN) estimates the 14 parameters listed in Table I Panel A, while the no overconfidence version (NO-OC) restricts the overconfidence parameter (η_B) to be zero. Table I Panel A lists all additional restrictions. Parameters vary across intraday periods as listed, and "News" and "No News" indicate parameters from periods with and without public news, respectively. Standard errors appear in parentheses. When a computed standard error exceeds ten times the parameter estimate, we replace it with the phrase "*wk-id*" to denote the parameter as weakly identified.

Parameters for all periods	MAIN	NO-OC
η_B	0.122	0.000
	(0.027)	N/A
RRA	0.615	1.223
	(0.095)	(0.109)
Parameters for Regular Market		
η(*News*)	0.205	0.133
	(0.019)	*wk-id*
η(*No News*)	0.243	0.084
	(0.017)	(0.013)
σ_d(*News*)	0.047	0.012
	(0.005)	*wk-id*
σ_d(*No News*)	0.031	0.018
	(0.003)	(0.002)
Parameters for Pre-Market		
η(*News*)	0.318	0.009
	(0.033)	*wk-id*
η(*No News*)	0.680	0.120
	(0.237)	(0.014)
σ_d(*News*)	0.044	0.325
	(0.003)	*wk-id*
σ_d(*No News*)	0.013	0.037
	(0.002)	(0.003)
Parameters for After-Market		
η(*News*)	0.294	0.018
	(0.021)	*wk-id*
η(*No News*)	0.356	0.034
	(0.058)	*wk-id*
σ_d(*News*)	0.067	0.170
	(0.006)	*wk-id*
σ_d(*No News*)	0.015	0.082
	(0.002)	*wk-id*

Table IV
Subsample Estimates

This table presents GMM fit statistics and parameter estimates for the main model within two subperiods as well as within three size groups. The "Early" and "Late" subperiods include the years 2001-2005 and 2006-2010, respectively. The size groups "Small Cap," "Mid Cap," and "Large Cap" contain stocks whose prior year-end market capitalizations are within the intervals [\$100M,\$1B), [\$1B,\$10B), and [\$10B,∞), respectively. Table I Panel A lists all parameter restrictions. The χ^2 statistics and *p*-values are from a test of overidentifying restrictions. Parameter estimates vary across intraday periods as listed, and "News" and "No News" indicate parameters from periods with and without public news, respectively. Standard errors appear in parentheses.

Fit Statistics	Early	Late	Small-Cap	Mid-Cap	Large-Cap
$\chi^2(4)$	6.473	1.615	4.476	4.919	17.236
p-value	0.167	0.806	0.345	0.296	0.002
Parameters for all periods					
η_B	0.138	0.075	0.149	0.131	0.125
	(0.036)	(0.006)	(0.036)	(0.025)	(0.004)
RRA	0.624	0.595	0.530	0.536	0.792
	(0.161)	(0.166)	(0.118)	(0.113)	(0.218)
Parameters for Regular Market					
η(*News*)	0.199	0.185	0.177	0.221	0.259
	(0.036)	(0.019)	(0.049)	(0.014)	(0.015)
η(*No News*)	0.245	0.213	0.225	0.255	0.275
	(0.026)	(0.013)	(0.017)	(0.012)	(0.012)
σ_d(*News*)	0.054	0.041	0.063	0.041	0.031
	(0.006)	(0.008)	(0.008)	(0.005)	(0.003)
σ_d(*No News*)	0.032	0.029	0.040	0.027	0.023
	(0.004)	(0.004)	(0.004)	(0.003)	(0.002)
Parameters for Pre-Market					
η(*News*)	0.321	0.263	0.295	0.333	0.360
	(0.081)	(0.001)	(0.067)	(0.030)	(0.018)
η(*No News*)	0.786	0.637	1.000	0.922	0.474
	(0.389)	(0.394)	(0.221)	(0.330)	(0.179)
σ_d(*News*)	0.046	0.044	0.053	0.039	0.032
	(0.005)	(0.004)	(0.004)	(0.003)	(0.003)
σ_d(*No News*)	0.015	0.011	0.012	0.011	0.014
	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)
Parameters for After-Market					
η(*News*)	0.284	0.259	0.273	0.307	0.328
	(0.032)	(0.005)	(0.021)	(0.015)	(0.009)
η(*No News*)	0.369	0.267	0.348	0.361	0.453
	(0.180)	(0.011)	(0.328)	(0.056)	(0.115)
σ_d(*News*)	0.079	0.055	0.081	0.062	0.054
	(0.007)	(0.009)	(0.008)	(0.006)	(0.005)
σ_d(*No News*)	0.014	0.017	0.016	0.015	0.011
	(0.003)	(0.002)	(0.003)	(0.002)	(0.001)

Table V
Estimation Using the Regular Period Only

This table presents GMM predicted moments and parameter estimates for two versions of the model based on six regular market moments. The first model restricts relative risk aversion (RRA) to equal 2.0, while the second model restricts the overconfidence parameter (η_B) to be zero. These restricted parameter values appear in brackets. Table I Panel A lists all other parameter restrictions. Panel A presents empirical and predicted moments. The χ^2 statistics and *p*-values are from a test of overidentifying restrictions. Panel B presents parameter estimates. The labels "News" and "No News" indicate parameters from the Regular Market with and without public news, respectively. Standard errors appear in parentheses.

Panel A: Predicted Moments

	Empirical	Prediction (MAIN)	Prediction (NO-OC)
News Volatility (%)	3.999	3.906	4.182
	(0.471)		
No-News Volatility (%)	3.040	3.007	2.931
	(0.361)		
News Turnover (%)	2.113	2.137	1.750
	(0.181)		
No-News Turnover (%)	1.455	1.462	1.638
	(0.129)		
News Illiquidity	7.747	8.182	7.163
	(1.488)		
No-News Illiquidity	7.920	8.135	7.878
	(1.519)		
$\chi^2(1)$		0.165	6.359
p-value		0.685	0.012

Panel B: Parameter Estimates

Invariant Parameters	MAIN	NO-OC
η_B	0.164	0.000
	(0.168)	(0.000)
RRA	2.000	1698.465
	(0.000)	(404.165)
Parameters that Vary with News		
η(*News*)	0.149	0.290
	(0.034)	(0.089)
η(*No News*)	0.184	0.130
	(0.019)	(0.015)
σ_d(*News*)	0.051	0.046
	(0.006)	(0.004)
σ_d(*No News*)	0.038	0.042
	(0.003)	(0.002)

Table VI
Analysis of the Main and No Overconfidence Models

This table presents analyses based on estimates from the main (MAIN) and no overconfidence (NO-OC) models shown in Tables II and III. The *RRA* and η_B parameters from those tables appear with standard errors in parentheses. The Volume and Variance Decompositions are based on Equations (35) and (36), respectively, with parameter estimates for the two models. The Expected Trading Profit from Informed Trading is the gross expected trading profit of each informed trader from Equation (A61) multiplied by the number of informed traders and summed across all intraday periods in a year. The Wealth Transfer from Overconfident Traders (Hedgers) is the rational expectation of the trading loss of each overconfident trader (hedger) multiplied by the number of overconfident traders (hedgers) and summed across all intraday periods in a year. The Deadweight Cost for a given trader type is the area under that type's cost curve summed across all intraday periods in a year.

Parameter Values	MAIN	NO-OC
Overconfidence (η_B)	0.122	0.000
	(0.044)	-
Relative Risk Aversion (*RRA*)	0.615	1.223
	(0.095)	(0.109)
Volume Decomposition		
Informed Trading	71%	56%
Uninformed - Overconfident	19%	
Uninformed - Hedging	1%	36%
Market Making	9%	8%
Variance Decomposition		
Private Information	84%	15%
Public Information	16%	85%
Aggregated Statistics (% mkt cap, annualized)		
Expected Trading Profit from Informed Trading	0.689%	0.015%
Wealth Transfer from Overconfident Traders	0.685%	
Wealth Transfer from Hedgers	0.004%	0.015%
Deadweight Cost of Informed Trading	0.227%	0.005%
Deadweight Cost of Overconfident Trading	0.019%	
Deadweight Cost of Hedging	0.004%	0.005%

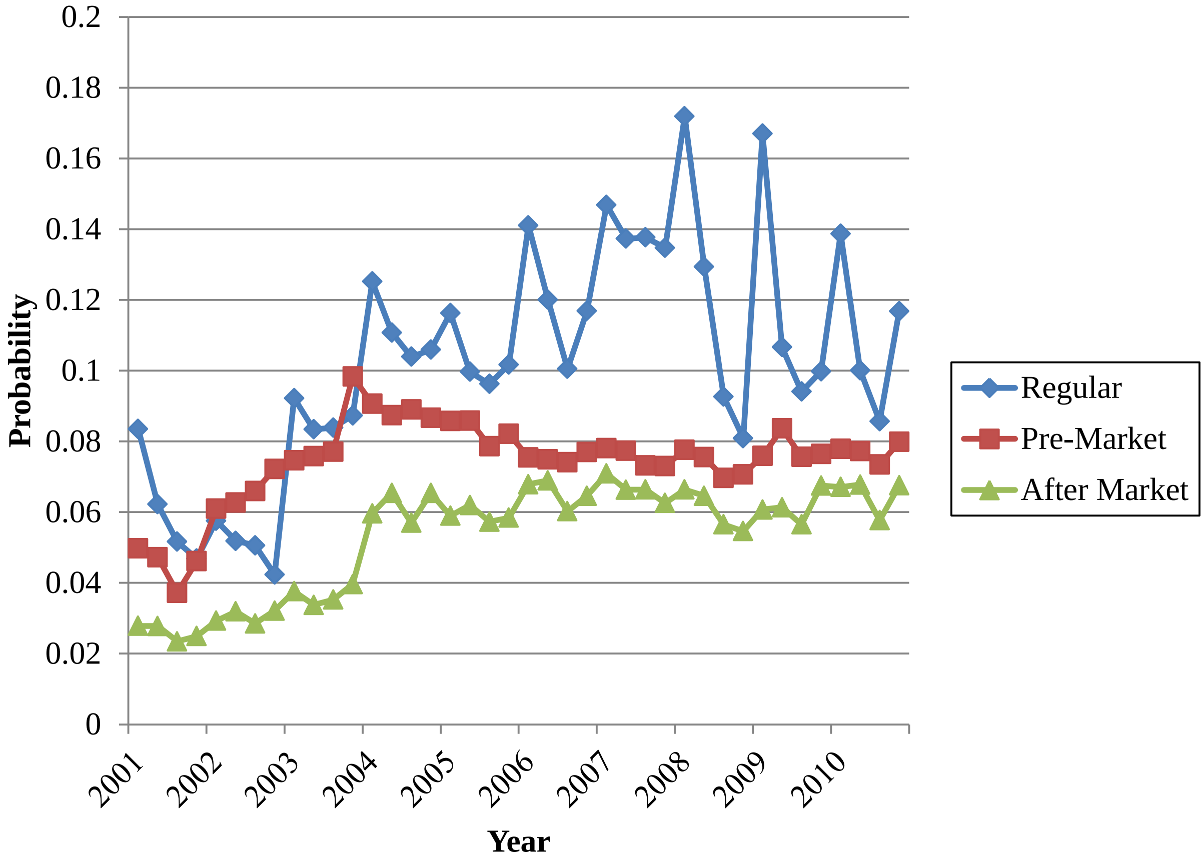

Figure 1: Probability of news arrival.
This figure plots quarterly probabilities of news arrival for each of three intraday periods (the Regular Market, the Pre-Market, and the After Market) during 2001-2010. The indicator variable *News* is 1 if there is at least one story (two stories) in the Dow Jones Newswires mentioning a particular Small or Mid Cap (Large Cap) firm and 0 otherwise. To precisely measure news arrival, only periods for which there are no news stories since the same intraday period on the prior trading day are considered. All probabilities are calculated by pooling observations for all firms within each quarter and size group and then averaging across size groups. Intraday periods are as defined above.

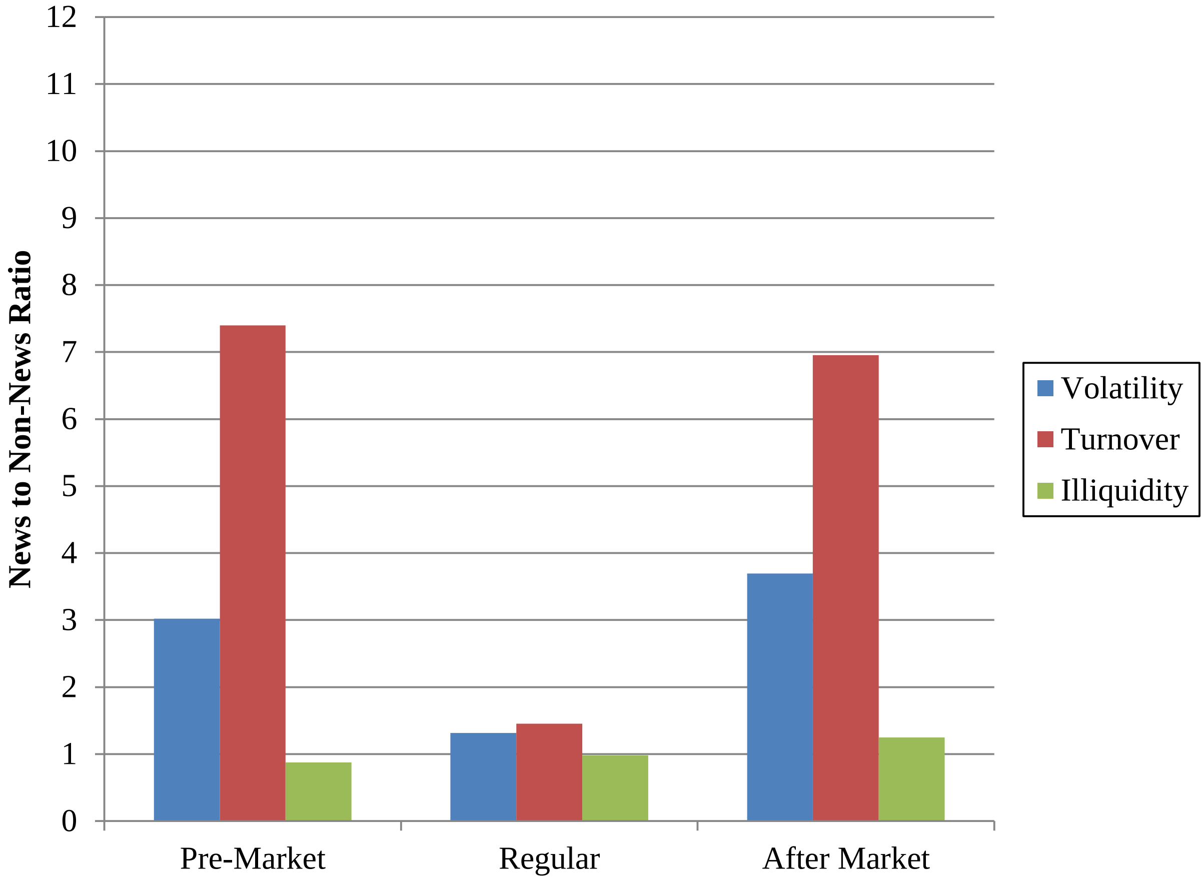

Figure 2: Empirical moments in periods with and without news arrival.
This figure plots ratios of news to no-news moment estimates of variance, volume, and illiquidity within each of three intraday periods (the Regular Market, the Pre-Market, and the After Market) during 2001-2010. Calculations and intraday period definitions are as described in Table II.

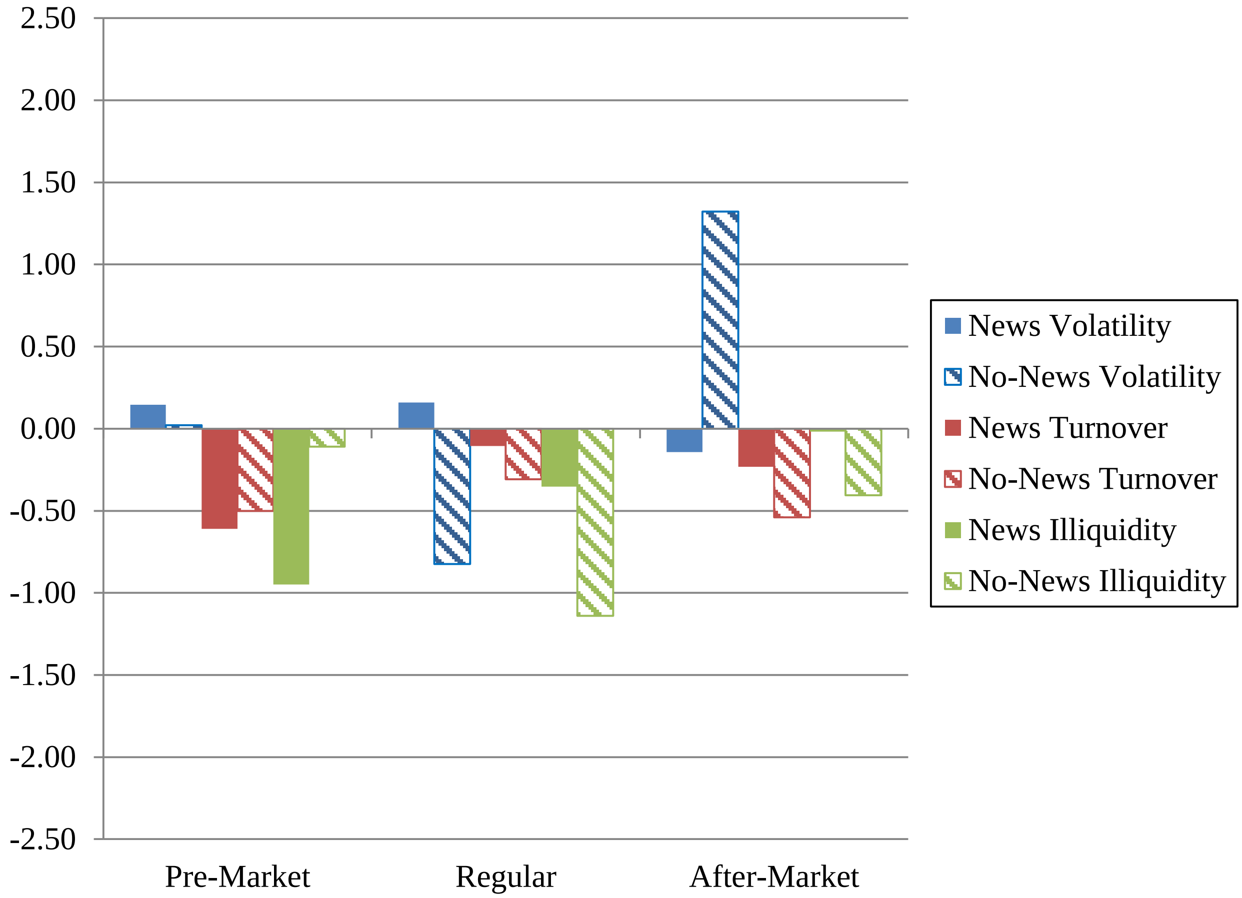

Figure 3, Panel A: Prediction error *t*-statistics for the main model.
This figure shows prediction error *t*-statistics for variance, volume, and illiquidity moments in each of three intraday periods (the Regular Market, the Pre-Market, and the After Market) conditional on news or no-news. The overconfidence version model is estimated using GMM for the 2001-2010 sample period using as in the middle column of Table II.

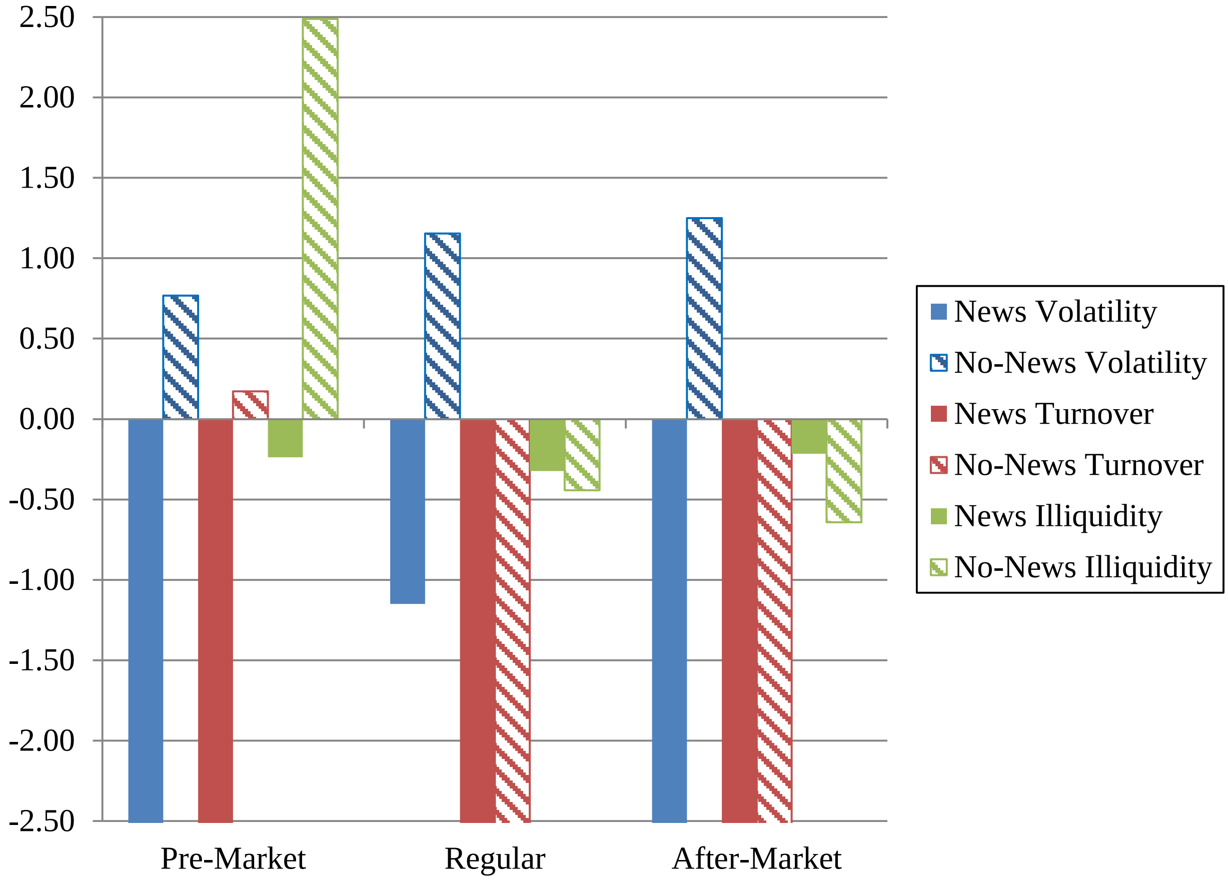

Figure 3, Panel B: Prediction error *t*-statistics for the no overconfidence model.
This figure shows prediction error *t*-statistics for variance, volume, and illiquidity moments in each of three intraday periods (the Regular Market, the Pre-Market, and the After Market) conditional on news or no-news. The rational version of the model is estimated using GMM for the 2001-2010 sample period using as in the right column of Table II.