# Myopic Separability

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August 5, 2013

**Abstract**

In the classic certainty multiperiod, multigood demand problem, suppose preferences for current and past period consumption are separable from consumption in future periods. Then optimal demands can be determined from the standard two stage budgeting process, where optimal current period demands depend only on current and past prices and current period expenditure. Unfortunately this simplification does not significantly reduce the informational requirements for the decision maker since in general the expenditure is a function of future prices. Recent behavioral evidence strongly suggests that frequently individuals significantly simplify or "narrow bracket" intertemporal choice problems. We derive necessary and sufficient conditions such that the current period's expenditure and hence optimal current demands are independent of future prices. Since this preference property is a special case of separability, it is referred to as myopic separability. One well known special case of myopic separability is log additive (or equivalently Cobb-Douglas) utility. However, this form of utility is overly restrictive, especially given the general aversion of Fisher (1930), Hicks (1965) and Lucas (1978) to requiring preferences to be additively separable. Myopic separability requires neither additive separability nor logarithmic period utility. As an application, we derive simple restrictions on equilibrium interest rates which are necessary and sufficient for utility to take the myopic separable form. These conditions are arguably less restrictive than those implied by additively separable preferences.

KEYWORDS. Myopic demand, separable utility, additive separability, myopic separability, equilibrium interest rates

JEL CLASSIFICATION. D01, D11, D53.

# 1 Introduction

Consider a multiperiod, multigood setting in which a well-behaved utility function is maximized subject to a standard intertemporal budget constraint. In order to derive an optimal consumption plan, the consumer needs to know all past, current and future prices. It is natural to ask when these informational requirements can be reduced. Similar questions have been raised in the extensive literature on separable preferences. In his classic paper, Leontief (1947) asks when the complex single period problem of choosing over a large number of commodities can be replaced by the "man-in-the-street" approach of first allocating resources among general categories such as food and clothing and then making within category choices between specific goods such as bread and apples.

This two stage budgeting process was further developed by Strotz (1957), (1959) and Gorman (1959) and it was shown that if preferences are separable then it is possible to choose among commodities in a separable group based on within group prices and the expenditure on the group.[1] Unfortunately this intuitive simplification does not in general result in a reduction in informational requirements for the consumer since to determine the expenditure on a group of goods she needs to know the full set of commodity prices and not just the prices of the goods within the group (see Blackorby, Primont and Russell 2006, p. 6). For example, if food goods form a separable group from all other commodities, although this ensures that the marginal rate of substitution between any pair of food goods is independent of the quantity of non-food items, it does not ensure that the demand for food items does not depend on the price of non-food goods. In this vein Pollak (1971) observes

> It would be nicer if the demand for Swiss cheese depended only on food prices and income, but this is not what separability implies. (Pollak 1971 pp. 246-247)

In a similar spirit for the intertemporal case, Kurz (1987) defines an optimal consumption plan in a given time period as being myopic if and only if that period's demand function depends only on exogenous variables in the current and past periods and not in future periods.[2]

*We thank Herakles Polemarchakis and Faruk Gul for their insightful comments and suggestions. Selden and Wei thank the Sol Snider Research Center – Wharton for support.

[1] Throughout this paper when we say "separable", we will mean "weakly separable" as defined in Goldman and Uzawa (1964). For a thorough discussion of the extensive theoretical and empirical literature associated with separability, see Deaton and Muellbauer (1980), Blundell (1998) and Blackorby, Primont and Russell (1998).

[2] It should be stressed that over the years the term myopia has been used in a number of different ways. The notion of a myopic plan as defined by Kurz and employed throughout this

In this paper, we derive the necessary and sufficient condition such that the current period's expenditure and hence optimal current demands are independent of future prices. We also characterize when demand is myopic in each time period and a subset of periods. Since these preference properties are a special case of separability, we refer to them as myopic separability.

In recent years "a mass of evidence and the ineluctable logic in a complicated world" (Rabin and Weizsacker 2009, p. 1508) strongly suggest that individuals "narrow bracket" decisions by choosing to disregard potentially relevant decisions and information.[3] Specifically in the context of this paper, Read, Lowenstein and Rabin (1999, p. 174) argue that the most common occurrences of narrow bracketing take place in intertemporal settings. These authors also observe that generally narrow bracketing will be suboptimal relative to "broad bracketing" where the full complex decision problem is addressed. Laibson (1999) raises the need for developing a formal model of narrow bracketing. When will narrow bracketing actually be optimal? In the specific context of multiperiod, multigood demand, myopic separability is both necessary and sufficient for narrow bracketing to be optimal for the consumer.

One well known special case of myopic separability is log additive (or equivalently Cobb-Douglas) utility.[4] However this form of utility is overly restrictive, especially given the aversion of Fisher (1930), Hicks (1965) and Lucas (1978), among others, to requiring preferences to be additively separable. More recently Prelec and Loewenstein (1991) provide a very strong case against intertemporal additive separability based on extensive laboratory and empirical studies. Myopic separability does not

paper differs from its use in the changing tastes literature. In Strotz (1956), for example, a myopic plan is used interchangeably with a naive plan where a consumer bases her plan for current and future consumption on current multiperiod preferences ignoring the fact that her preferences in the next period for the then remaining consumption vector may differ. Still another use of myopia was introduced by Brown and Lewis who refer to preferences as being myopic when "present consumption is preferred to future consumption and the taste for future consumption diminishes as the time of consumption recedes into the future" (Brown and Lewis 1981, p. 360).

[3] Although a number of examples of narrow bracketing are couched in certainty settings, it has been most widely discussed in the context decision making under uncertainty (see Rabin and Weizsacker 2009 and the reference cited therein). It is closely related to the discussion of framing in the classic paper of Kahneman and Tversky (1979) on prospect theory. Thaler (1999) introduced the concepts of narrow and broad bracketing in the context of mental accounting.

[4] In a multiperiod uncertainty setting, if intertemporal preferences are represented by log additive Expected Utility, then the consumer's multiperiod investment plan is myopic (see, for example, Rubinstein 1974). In the finance literature, an alternative notion of myopic investment plans is also considered where investors are assumed to maximize Expected Utility of terminal wealth. See, for example, Mossin (1968) and Hakansson (1971). In this paper however, we focus only on the certainty case.

require additive separability, homotheticity or logarithmic period utility. Examples of these more general forms are provided.

As an application, we consider both representative agent and heterogeneous agent exchange equilibrium interest rate settings. Restrictions on interest rates which are necessary and sufficient for utility to take the myopic separable form are derived. And they are compared to the conditions required by other preference restrictions. Interestingly, some may view the implications of myopic separability to be considerably less restrictive than those associated with the commonly assumed additively separable form of utility. This result provides an equilibrium justification supporting the general aversion to additively separable utility cited above.

In the next section, two simple examples are presented contrasting cases where preferences do and do not exhibit myopic separability. In Section 3, we derive our primary preference results. Section 4 considers the application to equilibrium interest rates. The Appendix provides an alternative proof of our primary preference result.

# 2 Motivating Examples

In this section, we assume that preferences over commodities take the standard separable form. In the first of two examples, we illustrate the following two stage budgeting results for separable preferences: (i) the demand for goods in a separable group can be expressed as a function of the prices of goods in that group and the expenditure on the group and (ii) the expenditure on the group depends on the prices of goods outside the group. In the second example where preferences are also separable, we find that the expenditure on the group is independent of prices of goods outside the group and hence the demand for goods in the group depend only on prices in the group and total income. For this second case, the two stage budgeting process is not only economically intuitive but also significantly reduces the informational requirements on the consumer.

Assume preferences over $N$ commodities $\mathbf{c} =_{def} (c_1, ..., c_N)$ are represented by a utility function $U(\mathbf{c})$, where $\mathbf{c} \in \Omega$ and $\Omega$ is a convex subset of the positive orthant. Consider a partition $\{\mathcal{N}^s, \mathcal{N}^c\}$ of the set of commodities $\{1, ..., N\}$, where $\mathcal{N}^s$ is the partition of the separable group and $\mathcal{N}^c$ is the complement group of goods. Let $\Omega^s$ and $\Omega^c$ be a corresponding decomposition of $\Omega$ (i.e., $\Omega^s \times \Omega^c = \Omega$) and let the elements of $\Omega^s$ and $\Omega^c$ be $\mathbf{c}^{(s)} \in \Omega^s$ and $\mathbf{c}^{(c)} \in \Omega^c$, respectively. Then following Russell (1975), $\mathcal{N}^s$ is separable from $\mathcal{N}^c$ if and only if

$$\{\overline{\mathbf{c}}^{(s)} \mid \overline{\mathbf{c}}^{(s)} \in \Omega^s \text{ and } U\left(\overline{\mathbf{c}}^{(s)}, \mathbf{c}^{(c)}\right) > U\left(\mathbf{c}^{(s)}, \mathbf{c}^{(c)}\right)\} \tag{1}$$

is invariant with respect to $\mathbf{c}^{(c)}$. This intuitive definition results in the utility form below following Leontief (1947) and Sono (1961)[5]

$$U(c_1, ..., c_N) = f\left(U^{(2)}\left(\mathbf{c}^{(s)}\right), \mathbf{c}^{(c)}\right), \tag{2}$$

where $f$ and $U^{(2)}$ are strictly increasing, quasiconcave functions. This can be illustrated by the following variation of Example 2.2 in Blackorby, Primont and Russell (1998).[6]

**Example 1** *Consider the following optimization problem*

$$\max_{c_1,c_2,c_3,c_4} U(c_1, c_2, c_3, c_4) = c_1c_2c_3 + (c_1c_2)^{\frac{1}{4}} c_4^{\frac{1}{2}} \;\; S.T. \;\; p_1c_1 + p_2c_2 + p_3c_3 + p_4c_4 = I. \tag{3}$$

*It is clear that* $\mathbf{c}^{(s)} = (c_1, c_2)$ *is separable in the above utility function. Following the two stage budgeting process, first solve*

$$P_1: \;\; \max_{c_1,c_2} U^{(2)}(c_1, c_2) = c_1c_2 \;\; S.T. \;\; p_1c_1 + p_2c_2 = I_2 \tag{4}$$

*and then solve*

$$P_2: \;\; \max_{I_2,c_3,c_4} U(c_1(I_2, p_1, p_2), c_2(I_2, p_1, p_2), c_3, c_4) = c_1c_2c_3 + (c_1c_2)^{\frac{1}{4}} c_4^{\frac{1}{2}} \tag{5}$$

$$S.T. \;\; p_3c_3 + p_4c_4 + I_2 = I. \tag{6}$$

*Solving* $P_1$ *yields*

$$c_1 = \frac{I_2}{2p_1} \;\; \text{and} \;\; c_2 = \frac{I_2}{2p_2}. \tag{7}$$

*Therefore,* $P_2$ *can be rewritten as*

$$P_2: \;\; \max_{I_2,c_3,c_4} U(c_1(I_2, p_1, p_2), c_2(I_2, p_1, p_2), c_3, c_4) = \frac{I_2^2}{4p_1p_2}c_3 + \left(\frac{I_2^2}{4p_1p_2}\right)^{\frac{1}{4}} c_4^{\frac{1}{2}} \tag{8}$$

$$S.T. \;\; p_3c_3 + p_4c_4 + I_2 = I. \tag{9}$$

*The first order condition results in*

$$\frac{\frac{2I_2^2}{4p_1p_2}c_4^{\frac{1}{2}}}{\left(\frac{I_2^2}{4p_1p_2}\right)^{\frac{1}{4}}} = \frac{p_3}{p_4} \Rightarrow c_4 = \frac{2p_3^2}{p_4^2}\left(\frac{p_1p_2}{I_2^2}\right)^{\frac{3}{2}}, \tag{10}$$

*implying that*

$$c_3 = \frac{I - I_2}{p_3} - \frac{2p_3}{p_4}\left(\frac{p_1p_2}{I_2^2}\right)^{\frac{3}{2}}. \tag{11}$$

[5] The notion of separable utility was introduced independently by Sono (1961), where the focus was more on characterizing the substitution and income effects of different functional forms.

[6] We vary the coefficients in Example 2.2 in order to make it analytically solvable, but this does not change the conclusion we draw. The subscripts are also relabeled to be consistent with the discussion in our paper.

*The first order condition for maximizing $U$ with respect to $c_3$ and $I_2$ gives*

$$\frac{\frac{I_2 c_3}{2p_1p_2} + \frac{I_2^{-\frac{1}{2}}}{2}\left(\frac{1}{4p_1p_2}\right)^{\frac{1}{4}} c_4^{\frac{1}{2}}}{\frac{I_2^2}{4p_1p_2}} = \frac{1}{p_3}. \tag{12}$$

*Substituting eqns. (10) and (11) into (12) yields*

$$\frac{2p_3^2\left(p_1p_2\right)^{\frac{3}{2}}}{p_4 I_2^4} - \frac{2I}{I_2} + 3 = 0. \tag{13}$$

*It is clear that $I_2$ depends on $p_3$ and $p_4$. Since optimal $c_1$ and $c_2$ depend on $I_2$, this implies that they depend on prices for commodities outside the $\mathbf{c}^{(s)}$ group.*

It is natural to wonder whether there exist any case such that $I_2$ is independent of $p_3$ and $p_4$. Actually, this can be realized by modifying the utility parameters in the above example.

**Example 2** *Consider the following optimization problem*

$$\max_{c_1,c_2,c_3,c_4} U\left(c_1,c_2,c_3,c_4\right) = c_1c_2c_3 + \sqrt{c_1c_2c_4} \quad S.T. \quad p_1c_1 + p_2c_2 + p_3c_3 + p_4c_4 = I. \tag{14}$$

*It is clear that $\mathbf{c}^{(s)} = (c_1, c_2)$ is separable in the above utility function. Following the two stage budgeting process, first solve*

$$P_1: \quad \max_{c_1,c_2} U^{(2)}\left(c_1,c_2\right) = c_1c_2 \quad S.T. \quad p_1c_1 + p_2c_2 = I_2 \tag{15}$$

*and then solve*

$$P_2: \quad \max_{I_2,c_3,c_4} U\left(c_1\left(I_2,p_1,p_2\right), c_2\left(I_2,p_1,p_2\right), c_3, c_4\right) = c_1c_2c_3 + \sqrt{c_1c_2c_4} \tag{16}$$

$$S.T. \quad p_3c_3 + p_4c_4 + I_2 = I. \tag{17}$$

*Solving $P_1$ yields the same $c_1$ and $c_2$ demand functions as in Example 1*

$$c_1 = \frac{I_2}{2p_1} \quad and \quad c_2 = \frac{I_2}{2p_2}. \tag{18}$$

*Therefore, $P_2$ can be rewritten as*

$$P_2: \quad \max_{I_2,c_3,c_4} U\left(c_1\left(I_2,p_1,p_2\right), c_2\left(I_2,p_1,p_2\right), c_3, c_4\right) = \frac{I_2^2}{4p_1p_2}c_3 + \sqrt{\frac{I_2^2}{4p_1p_2}c_4} \tag{19}$$

$$S.T. \quad p_3c_3 + p_4c_4 + I_2 = I. \tag{20}$$

*The first order condition results in*

$$\frac{\frac{2I_2^2}{4p_1p_2}c_4^{\frac{1}{2}}}{\sqrt{\frac{I_2^2}{4p_1p_2}}} = \frac{p_3}{p_4} \Rightarrow c_4 = \frac{p_3^2}{p_4^2}\frac{p_1p_2}{I_2^2}, \tag{21}$$

*implying that*

$$c_3 = \frac{I - I_2}{p_3} - \frac{p_3}{p_4}\frac{p_1 p_2}{I_2^2}. \tag{22}$$

*The first order condition for maximizing $U$ with respect to $c_3$ and $I_2$ gives*

$$\frac{\frac{I_2 c_3}{2p_1p_2} + \sqrt{\frac{1}{4p_1p_2}c_4}}{\frac{I_2^2}{4p_1p_2}} = \frac{1}{p_3}. \tag{23}$$

*Substituting eqns. (21) and (22) into (23) yields*

$$\frac{2\,(I - I_2)}{I_2} = 1 \Rightarrow I_2 = \frac{2I}{3}, \tag{24}$$

*which unlike the prior example is independent of $p_3$ and $p_4$. Therefore, the optimal demands are given by*

$$c_1 = \frac{I}{3p_1} \quad \textit{and} \quad c_2 = \frac{I}{3p_2}, \tag{25}$$

$$c_3 = \frac{I}{3p_3} - \frac{9p_3}{4p_4}\frac{p_1p_2}{I^2} \tag{26}$$

*and*

$$c_4 = \frac{9p_3^2}{4p_4^2}\frac{p_1p_2}{I^2}. \tag{27}$$

*It is clear from (25) that the goods in $\mathbf{c}^{(s)}$ depend only on the prices of goods in that group and total income.*

**Remark 1** *The utility (14) is of particular interest, since it is neither additively separable nor homothetic but still results in the demand for $c_1$ and $c_2$ being independent of $p_3$ and $p_4$.*

The comparison between these two examples motivates the question of what general form of utility is necessary and sufficient for the optimal demand for goods in a separable group to depend only on the prices within the group and total income. Although in general this question can be addressed in both static and intertemporal settings, the latter seems to offer an especially compelling application.[7] If one can determine current period demands based on past and current prices but not future prices, the informational requirements for the consumer are significantly reduced. For this reason, we will address the question for intertemporal demands in the next section.

[7] The question of investigating the implications of different forms of separable utility in intertemporal settings has been examined in the literature (see, for example, Gorman 1982 where the focus is on simplifying optimal demand in uncertain, dynamic settings and Blundell 1998).

# 3 Myopic Separability: Necessary and Sufficient Conditions

Assume a $T$ period, multigood consumption setting. In each period, one or more goods are consumed. The quantity (purchase) and price of good $i$ in period $t$ are denoted by $c_{ti}$ and $p_{ti}$, respectively.[8] The corresponding consumption and price vectors in period $t$ are denoted by $\mathbf{c}_t$ and $\mathbf{p}_t$. Assume a well-behaved $T$ period utility function $U(\mathbf{c}_1, ..\mathbf{c}_T)$ which is maximized subject to the budget constraint

$$\sum_{t=1}^{T} \mathbf{p}_t \cdot \mathbf{c}_t = I, \tag{28}$$

where $I$ is period one income or wealth. The price and consumption vectors are elements of the positive orthant.

Let the set $\mathcal{U}$ denote the collection of real-valued functions defined on (a subset of) the positive orthant of a Euclidean space, which are $C^3$, strictly increasing in each of their arguments and strictly quasiconcave. Throughout this paper, it will be assumed that the $T$ period utility function $U(\mathbf{c}_1, ..\mathbf{c}_T) \in \mathcal{U}$. (As can be easily verified, these assumptions are satisfied by each of the utility functions employed in each of the examples in this paper.) Unless otherwise stated, we will always assume that $U$ is defined on the whole positive orthant. It should be stressed that our setting is static even though the consumer confronts a multiperiod decision problem, since we only consider her optimal consumption plan as set at the beginning of the initial time period $t = 1$.

Following Kurz (1987), myopic demand is defined as follows.

**Definition 1** *Optimal demand in period $t$, $\mathbf{c}_t$, is said to be myopic if and only if it depends on past and current prices $\{\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_t\}$ but not on future prices $\{\mathbf{p}_{t+1}, ..., \mathbf{p}_T\}$.*

Definition 1 implies that $\mathbf{c}_t$ is myopic if and only if for any $k \in \{1, 2, ..., T-t\}$

$$\frac{\partial \mathbf{c}_t}{\partial p_{t+k,i}} = \mathbf{0} \quad \forall i, \tag{29}$$

where $i$ is the index of good $i$ in period $t+k$. However in contrast to a distribution economy corresponding to (28), in an exchange economy since

$$I = \sum_{t=1}^{T} \mathbf{p}_t \cdot \bar{\mathbf{c}}_t \tag{30}$$

[8] If there is only one good in a period, the subscript $i$ will be ignored.

where $\overline{\mathbf{c}}_t$ denotes the vector of period $t$ endowments, all prices enter into optimal demands through income. We next introduce an alternative definition of myopia so as to make it independent of the form of the economy. It follows from the classic Slutsky demand equation in a distribution economy (see, for example, Mas-Colell, Whinston and Green 1995, p. 71) that

$$\frac{\partial \mathbf{c}_t}{\partial p_{t+k,i}} = \left(\frac{\partial \mathbf{c}_t}{\partial p_{t+k,i}}\right)_{U=const} - c_{t+k,i}\frac{\partial \mathbf{c}_t}{\partial I} \quad \forall i. \tag{31}$$

Combining the above with eqn. (29) we obtain the following which, although equivalent to Definition 1 in a distribution economy, will prove more convenient when discussing the implication of our form of utility for an exchange economy equilibrium in Section 4.

**Definition 2** *Optimal demand in period $t$, $\mathbf{c}_t$, is said to be myopic if and only if $\forall k \in \{1, 2, ..., T-t\}$.*

$$\left(\frac{\partial \mathbf{c}_t}{\partial p_{t+k,i}}\right)_{U=const} = c_{t+k,i}\frac{\partial \mathbf{c}_t}{\partial I} \quad \forall i, \tag{32}$$

*where $i$ is the index of good $i$ in period $t+k$.*

The representation of preferences generally associated with myopic demand is log additive (or equivalently Cobb-Douglas) utility. However, since these preferences are (ordinally) additively separable and homothetic, it is natural to wonder whether these properties are necessary to generate myopic demand.[9] The fact that homotheticity is not required is easily demonstrated by the following non-homothetic utility which generates myopic demands

$$U(c_1, c_2, c_3) = -\exp(-c_1) + \ln c_2 + \ln c_3. \tag{33}$$

The question of whether additive separability is required is more involved since none of the widely used non-additively separable utility functions, of which are we aware (except Example 2 above), results in myopic consumption plans. We return to this issue below.

In this paper we seek to fully characterize the class of preferences which imply and are implied by myopic demand in a multigood, multiperiod setting. We begin by establishing the necessary and sufficient condition for the period one consumption vector to be myopic.[10] This condition is then applied recursively in Result 1 to

[9] Homothetic preferences are characterized as being representable by a homogeneous function. Moreover these preferences give rise to linear Engel curves. See, for example, Chipman (1974).

[10] For the following Proposition, since the distribution of goods within periods does not matter, we use $c_i$ instead of $c_{ti}$ in order to simplify the notation in the proof.

characterize the form of utility associated with the consumption vector being myopic in several (or all) periods. Since the form of utility (35) below generating myopic demands is a special case of the separable form (2), it will be referred to in the rest of the paper as myopic separable utility or corresponding to myopic separable preferences.

**Proposition 1** *Assume that in the first period, there are $m$ goods, where the quantities are denoted by $c_1, c_2, ..., c_m$. In periods 2 to $T$ there are $n$ goods, where the quantities are denoted by $c_{m+1}, c_{m+2}, ..., c_{m+n}$ and the distribution of goods across periods is arbitrary. The utility function $U(c_1, ..., c_{m+n})$ is maximized subject to*

$$\sum_{i=1}^{m+n} p_i c_i = I. \tag{34}$$

*The optimal period one consumption vector $(c_1, ..., c_m)$ is myopic if and only if $U(c_1, ..., c_{m+n})$ takes the form*[11]

$$U(c_1, ..., c_{m+n}) = f(h_1, h_2, ..., h_n), \tag{35}$$

*where*

$$h_i = g(c_1, ..., c_m)\, c_{m+i} \quad (i = 1, 2, ..., n) \tag{36}$$

*$f, g, h_i \in \mathcal{U}$ and $h_i > 0$ $(i = 1, 2, ..., n)$.*[12]

**Proof.** It should be noted that although a $T$ period setting is assumed in Proposition 1, since we are only interested in when the period one consumption vector is myopic, we can combine all of the future periods into one group and the problem effectively becomes a two period problem. First prove sufficiency. Introduce the following notation

$$f_i = \frac{\partial f}{\partial h_i} \quad \text{and} \quad g_j = \frac{\partial g(c_1, ..., c_m)}{\partial c_j}, \tag{37}$$

where $i \in \{1, 2, ..., n\}$ and $j \in \{1, 2, ..., m\}$. The first order conditions are

$$\frac{f_i g(c_1, ..., c_m)}{f_j g(c_1, ..., c_m)} = \frac{f_i}{f_j} = \frac{p_{m+i}}{p_{m+j}} \quad i, j \in \{1, 2, ..., n\}, \tag{38}$$

[11] Given the interest in utility functions with translated origins such as members of the Modified Bergson family and habit formation models (see, for example, Pollak 1970), it is natural to ask whether any of these utilities can exhibit myopic demand. It is clear from the general form (35) that this is not possible where the origins for $c_{m+1}, ..., c_{m+n}$ are translated.

[12] In order to simplify the statement of this result, we follow the convention throughout the paper of assuming without loss of generality that $h_i > 0$. However it should be noted that for the proof of sufficiency, $f, h_i \in \mathcal{U}$ and $h_i > 0$ cannot guarantee that $U \in \mathcal{U}$, which is always assumed in this paper, since the strict quasiconcavity of $f$ and $h_i$ cannot ensure the strict quasiconcavity of $U$. For necessity if $U \in \mathcal{U}$ and $h_i > 0$, we prove that this implies $f, h_i \in \mathcal{U}$. It should be emphasized that when $h_i < 0$, one can always reverse the sign of $h_i$ and the signs of the arguments in $f$ such that the form of $U$ remains the same and $f, h_i \in \mathcal{U}$ where $h_i > 0$.

$$\frac{g_i \sum_{k=1}^{n} f_k c_{m+k}}{g_j \sum_{k=1}^{n} f_k c_{m+k}} = \frac{g_i}{g_j} = \frac{p_i}{p_j} \qquad i, j \in \{1, 2, ..., m\}, \tag{39}$$

and

$$\frac{g_i \sum_{k=1}^{n} f_k c_{m+k}}{f_j g(c_1, ..., c_m)} = \frac{p_i}{p_{m+j}} \qquad i \in \{1, 2, ..., m\}, j \in \{1, 2, ..., n\}. \tag{40}$$

Combining (38) and (40), we have

$$\sum_{k=1}^{n} f_k p_{m+j} c_{m+k} = f_j \sum_{k=1}^{n} p_{m+k} c_{m+k} = \frac{p_i f_j g(c_1, ..., c_m)}{g_i}, \tag{41}$$

which is equivalent to

$$\sum_{k=1}^{n} p_{m+k} c_{m+k} = \frac{p_i g(c_1, ..., c_m)}{g_i}. \tag{42}$$

Substitution of the above equation into the budget constraint, yields

$$\sum_{i=1}^{m} p_i c_i + \frac{p_j g(c_1, ..., c_m)}{g_j} = I \tag{43}$$

for $\forall j \in \{1, 2, ..., m\}$. Choosing for example $j = 1$ in (43) and $i = 1$ in (39), we get the following system of $m$ equations in the $m$ variables $c_1, ..., c_m$:

$$\begin{aligned} \sum_{i=1}^{m} p_i c_i + \frac{p_1 g(c_1, ..., c_m)}{g_1} - I &= 0, \\ p_1 g_2 - p_2 g_1 &= 0, \\ &\vdots \\ p_1 g_m - p_m g_1 &= 0. \end{aligned} \tag{44}$$

This system is functionally independent and can generate a unique solution. In fact, solving the system (44) is equivalent to solving the system of first order conditions associated with the following constrained optimization problem

$$\max_{c_1, c_2, ..., c_{m+1}} g(c_1, ..., c_m) c_{m+1} \quad S.T. \quad \sum_{i=1}^{m+1} p_i c_i = I. \tag{45}$$

The first order condition of the above optimization problem is given by

$$g_i c_{m+1} = \mu p_i \ \ (i = 1, 2, ..., m) \ \text{ and } g = \mu p_{m+1}, \tag{46}$$

where $\mu$ is the Lagrange multiplier. If $\left(c_1^*, ..., c_{m+1}^*, \mu^*\right)$ is a solution to the above equation system, since

$$g_i = \frac{\mu^*}{c_{m+1}^*} p_i \quad (i = 1, 2, ..., m) \tag{47}$$

and

$$\frac{p_1 g\left(c_1^*, ..., c_m^*\right)}{g_1\left(c_1^*, ..., c_m^*\right)} = \frac{p_1 \mu^* p_{m+1} c_{m+1}^*}{p_1 \mu^*} = p_{m+1} c_{m+1}^*, \tag{48}$$

$\left(c_1^*, ..., c_m^*\right)$ is a solution to the equation system (44). If $\left(c_1^*, ..., c_m^*\right)$ is a solution to the equation system (44), then set

$$\mu^* = \frac{g\left(c_1^*, ..., c_m^*\right)}{p_{m+1}} \quad \text{and} \quad c_{m+1}^* = \frac{\mu^* p_1}{g_1\left(c_1^*, ..., c_m^*\right)}. \tag{49}$$

Since

$$g_i c_{m+1}^* = \frac{\mu^* p_1 g_i\left(c_1^*, ..., c_m^*\right)}{g_1\left(c_1^*, ..., c_m^*\right)} = \mu^* p_i \quad (i = 1, 2, ..., m) \tag{50}$$

and

$$p_{m+1} c_{m+1}^* = \frac{g\left(c_1^*, ..., c_m^*\right) p_1}{g_1\left(c_1^*, ..., c_m^*\right)}, \tag{51}$$

$\left(c_1^*, ..., c_{m+1}^*, \mu^*\right)$ is a solution to the equation system (46). Since $h_1 = g\left(c_1, ..., c_m\right) c_{m+1}$ is strictly quasiconcave, the equation system (46) has a unique solution. Therefore, there is a unique solution to the equation system (44). Since the equation system (44) is independent of $p_{m+1}, ..., p_{m+n}$, the optimal period one consumption vector $\left(c_1, ..., c_m\right)$ is myopic.

Next prove necessity.[13] Suppose that there were $n$ copies of all $m$ goods consumed in period one, which are denoted by $\left(c_1^1, ..., c_m^1\right), ..., \left(c_1^n, ..., c_m^n\right)$. Create the following queue of all goods

$$c_1^1, ..., c_m^1, c_{m+1}, c_1^2, ..., c_m^2, c_{m+2}, ..., c_1^n, ..., c_m^n, c_{m+n} \tag{52}$$

and relabel them as $\left(x_1, x_2, ..., x_N\right)$ and the corresponding prices as $\left(p_1, p_2, ..., p_N\right)$, where $N = n \times (m+1)$. Denote $\mathcal{N} = \{1, 2, ..., N\}$ and let $\{\mathcal{N}_1, \mathcal{N}_2, ..., \mathcal{N}_n\}$ be a partition of $\mathcal{N}$, where $\mathcal{N}_1$ consists the first $m+1$ elements in $\mathcal{N}$, $\mathcal{N}_2$ consists the second $m+1$ elements in $\mathcal{N}$ and so on. If $\left(c_1, ..., c_m\right)$ is myopic, then for all $i \in \mathcal{N}_s$, $k \notin \mathcal{N}_s$ $(s \in \{1, 2, ..., n\})$, one must have

$$\frac{\partial x_i}{\partial p_k} = 0, \tag{53}$$

implying that

$$\frac{\partial}{\partial x_k}\left(\frac{\partial U / \partial x_i}{\partial U / \partial x_j}\right) = 0 \tag{54}$$

for all $i, j \in \mathcal{N}_s$, $k \notin \mathcal{N}_s$ $(s \in \{1, 2, ..., n\})$. Therefore, the utility function $U$ is separable with respect to a partition $\{\mathcal{N}_1, \mathcal{N}_2, ..., \mathcal{N}_n\}$. It follows from Pearce (1961) and Theorem 2 in Goldman and Uzawa (1964) that the utility function must take the form

$$U = f\left(u^1\left(\mathbf{x}^{(1)}\right), u^2\left(\mathbf{x}^{(2)}\right), ..., u^n\left(\mathbf{x}^{(n)}\right)\right), \tag{55}$$

[13] We thank Faruk Gul for his very helpful suggestions in simplifying the necessity part of this proof.

where $f$ is a function of $n$ variables and, for each $i \in \{1, 2, ..., n\}$, $u^i\left(\mathbf{x}^{(i)}\right)$ is a function of the subvector $\mathbf{x}^{(i)} = \left(x_{(i-1)\times(m+1)+1}, ..., x_{i\times(m+1)}\right)$ alone, or equivalently,

$$U = f\left(u^1\left(c_1^1, ..., c_m^1, c_{m+1}\right), u^2\left(c_1^2, ..., c_m^2, c_{m+2}\right), ..., u^n\left(c_1^n, ..., c_m^n, c_{m+n}\right)\right). \tag{56}$$

Since for all $i \in \{1, 2, ..., n\}$, $(c_1^i, ..., c_m^i)$ does not depend on the price of $c_{m+i}$, it can be easily verified that

$$u^i = g^i\left(c_1^i, ..., c_m^i\right) c_{m+i}, \tag{57}$$

where $g^i$ is a function of $m$ variables. Since all the replica goods are perfect substitutes and always have the same prices, for any $i \neq j \in \{1, 2, ..., n\}$ and $s, t \in \{1, 2, ..., m\}$, it follows from the first order condition that

$$\frac{\partial g^i\left(c_1, ..., c_m\right)/\partial c_s}{\partial g^i\left(c_1, ..., c_m\right)/\partial c_t} = \frac{\partial g^j\left(c_1, ..., c_m\right)/\partial c_s}{\partial g^j\left(c_1, ..., c_m\right)/\partial c_t} = \frac{p_s}{p_t}, \tag{58}$$

implying that $g^i = g^j = g$. Therefore, the utility function must take the form

$$U\left(c_1, ..., c_{m+n}\right) = f\left(g\left(c_1, ..., c_m\right) c_{m+1}, g\left(c_1, ..., c_m\right) c_{m+2}, ..., g\left(c_1, ..., c_m\right) c_{m+n}\right). \tag{59}$$

Finally, we show that $f, g, h_i \in \mathcal{U}$. Since $h_i > 0$, from the first order condition, it can be easily verified that $U$ being strictly increasing is equivalent to $f$ , $g$ and $h_i$ being strictly increasing. Next we argue that $h_i$ is strictly quasiconcave. Without loss of generality, we only need to prove that $h_1$ is strictly quasiconcave. Set $d_{i-1} = c_{m+i}/c_{m+1}$ $(i = 2, ..., n)$. For any ${}_{m+1}\mathbf{c}' = \left(c_1', c_2', ..., c_{m+1}'\right)$, ${}_{m+1}\mathbf{c}'' = \left(c_1'', c_2'', ..., c_{m+1}''\right)$ and $0 < \alpha < 1$, we want to show that

$$h_1\left(\alpha\left({}_{m+1}\mathbf{c}'\right) + (1-\alpha)_{m+1}\mathbf{c}''\right) > \min\left(h_1\left({}_{m+1}\mathbf{c}'\right), h_1\left({}_{m+1}\mathbf{c}''\right)\right). \tag{60}$$

For any $i \in \{2, ..., n\}$, choose $c_{m+i}'$ and $c_{m+i}''$ such that

$$d_{i-1} = d_{i-1}' = d_{i-1}''. \tag{61}$$

Since $U$ can be viewed as a function of $(h_1, d_1, ..., d_n)$ and $U(h_1, d_1, ..., d_n)$ is strictly quasiconcave,

$$U\left(\alpha\mathbf{c}' + (1-\alpha)\mathbf{c}''\right) > \min\left(U\left(\mathbf{c}'\right), U\left(\mathbf{c}''\right)\right), \tag{62}$$

where $\mathbf{c}' = \left(c_1', c_2', ..., c_{m+n}'\right)$ and $\mathbf{c}'' = \left(c_1'', c_2'', ..., c_{m+n}''\right)$. Noticing that

$$U\left(\alpha\mathbf{c}' + (1-\alpha)\mathbf{c}''\right) = U\left(\alpha h_1\left({}_{m+1}\mathbf{c}'\right) + (1-\alpha) h_1\left({}_{m+1}\mathbf{c}''\right), d_1, ..., d_{n-1}\right), \tag{63}$$

and

$$U\left(\mathbf{c}'\right) = U\left(h_1\left({}_{m+1}\mathbf{c}'\right), d_1, ..., d_{n-1}\right) \text{ and } U\left(\mathbf{c}''\right) = U\left(h_1\left({}_{m+1}\mathbf{c}''\right), d_1, ..., d_{n-1}\right) \tag{64}$$

and $U(h_1, d_1, ..., d_n)$ is strictly increasing in $h_1$, we must have

$$h_1\left(\alpha\left({}_{m+1}\mathbf{c}'\right) + (1-\alpha)_{m+1}\mathbf{c}''\right) > \min\left(h_1\left({}_{m+1}\mathbf{c}'\right), h_1\left({}_{m+1}\mathbf{c}''\right)\right), \tag{65}$$

implying that $h_1$ is strictly quasiconcave. One can prove $f, g \in \mathcal{U}$ similarly. ■

**Remark 2** *The proof of the necessity part of this Proposition builds on results from the literature on separable utility functions (see, for example, Goldman and Uzawa 1964). This proof is a simpler a-posteriori verification of the utility formula (35) obtained using a more complicated (but a more direct) argument in Appendix A.*

**Remark 3** *It is clear that in the utility form (35), $(c_1, ..., c_m)$ is separable. Therefore, the Slutsky component restriction for separability given in Goldman and Uzawa (1964) holds, i.e., $\forall i, j \in \{1, 2, ..., m\}$ and $k \in \{m+1, ..., m+n\}$*[14]

$$\frac{\left(\frac{\partial c_i}{\partial p_k}\right)_{U=const}}{\left(\frac{\partial c_j}{\partial p_k}\right)_{U=const}} = \frac{\frac{\partial c_i}{\partial I}\frac{\partial c_k}{\partial I}}{\frac{\partial c_j}{\partial I}\frac{\partial c_k}{\partial I}} = \frac{\frac{\partial c_i}{\partial I}}{\frac{\partial c_j}{\partial I}}. \tag{66}$$

*For our case, it is easy to see that the above equation holds since it follows from Definition 2 that*

$$\frac{\left(\frac{\partial c_i}{\partial p_k}\right)_{U=const}}{\left(\frac{\partial c_j}{\partial p_k}\right)_{U=const}} = \frac{c_k\frac{\partial c_i}{\partial I}}{c_k\frac{\partial c_j}{\partial I}} = \frac{\frac{\partial c_i}{\partial I}}{\frac{\partial c_j}{\partial I}}. \tag{67}$$

**Remark 4** *Returning to the "narrow bracketing" issue introduced in Section 1, Laibson (1999) points out in commenting on Read, Loewenstein, and Rabin (1999)*

> An individual may overlook complementarities / substitutabilities among different types of consumption. An individual may subdivide his budget into multiple nonfungible accounts. Both of these distortions enable decision-makers to take complex integrated problems and turn them into relatively simple separable problems. (Laibson 1999, p. 201)

*The point of distorting preferences is closely related to separability. If preferences are separable in $(c_1, c_2)$ for example, then the choice of $(c_1, c_2)$ will not be affected by the consumption of other goods and hence the complementarities / substitutabilities between $(c_1, c_2)$ and other goods* can be *ignored. Since myopic separability guarantees separability, this distortion is also satisfied by myopic separable preferences. For the distortion of the budget constraint, Laibson (1999) gives the following example*

> I may decide that I will spend no more than $15 on entertainment this month. Hence, a decision to see the new Star Wars movie this weekend, may imply that I "won't be able" to go to a concert next weekend. Strict budgets create artificial tradeoffs that can potentially lower welfare, but strict budgets also simplify decision problems and facilitate self-regulation. (Laibson 1999, p. 201)

[14] Since here $i$ and $j$ come from the same partition, $\kappa^{st}$ defined in Goldman and Uzawa (1964) in the numerator and denominator of (66) cancel out.

*If preferences exhibit myopic separability in $(c_1, c_2)$, where $(c_1, c_2)$ denotes the choice of this month's entertainment activities, then the portion of the total income spent on current entertainment will be fixed at \$15 and independent of the prices in other segments such as food, clothing, etc. Myopic separability guarantees that the choice made from "distorting" the budget constraint to allocate a fixed portion of income to a certain segment independent of the prices in other segments is actually optimal and not suboptimal.*

There are two points that should be emphasized. First relating to whether additive separability of $U$ is necessary for myopic behavior, it is clear from eqn. (35) that this is not the case.[15] However, the separable form (2) is required. In fact, this was already illustrated in Example 2, where letting $U^{(2)}(c_1, c_2) = g(c_1, c_2) = c_1 c_2$, it is clear that the Example 2 utility is separable. Additionally defining $f(x, y) = x + \sqrt{y}$ in Proposition 1, one immediately obtains the utility function in Example 2, which makes it clear why $c_1$ and $c_2$ are independent of $p_3$ and $p_4$. In contrast, the Example 1 utility, although separable with $U^{(2)}(c_1, c_2) = c_1 c_2$, does not satisfy the requirement in Proposition 1. Second, it should additionally be emphasized that one can also apply Proposition 1 to the case when the demand in period $t$ is allowed to be a function of prices in periods 1 to $t+1$ but independent of subsequent prices. To do this, one only needs to combine periods 1 to $t+1$ to form a new group and then apply Proposition 1.

As summarized next, if there is only one good in certain periods, then the utility function given in Proposition 1 can take simpler forms.

**Remark 5** *For simplicity, assume that there are two periods and at most two goods in each period. Then the following summarizes the forms of utility implied by Proposition 1.*

**i** *Suppose there is only one good in periods one and two. Optimal period one demand $c_1$ is myopic if and only if $U(c_1, c_2)$ takes the form*

$$U(c_1, c_2) = f(g(c_1) c_2), \tag{68}$$

*which is ordinally equivalent to*

$$U(c_1, c_2) = w(c_1) + \ln c_2, \tag{69}$$

*where $f, g(c_1) c_2, w \in \mathcal{U}$ and $g(c_1) > 0$.*

[15]It should be noted that in the special case of two periods with one good per period, (ordinal) additive separability is necessary for demand to be myopic – see eqn. (69) below.

**ii** *Suppose there are two goods in period one and one good in period two. Optimal period one demands* $c_{11}$ *and* $c_{12}$ *are myopic if and only if* $U(c_{11}, c_{12}, c_2)$ *takes the form*

$$U(c_{11}, c_{12}, c_2) = f(g(c_{11}, c_{12})\, c_2), \tag{70}$$

*which is ordinally equivalent to*

$$U(c_{11}, c_{12}, c_2) = w(c_{11}, c_{12}) + \ln c_2, \tag{71}$$

*where* $f, g(c_{11}, c_{12})\, c_2, w \in \mathcal{U}$ *and* $g(c_{11}, c_{12}) > 0$.

**iii** *Suppose there is one good in period one and two goods in period two. Optimal period one demand* $c_1$ *is myopic if and only if* $U(c_1, c_{21}, c_{22})$ *takes the form*

$$U(c_1, c_{21}, c_{22}) = f(h_1, h_2) = f(g(c_1)\, c_{21}, g(c_1)\, c_{22}), \tag{72}$$

*where* $f, h_i \in \mathcal{U}$ *and* $h_i > 0$ $(i = 1, 2)$.

The following Examples illustrate cases (ii) and (iii) above, respectively.

**Example 3** *Assume a two period setting, where* $c_{11}$ *and* $c_{12}$ *are the quantities of the period one goods and* $c_2$ *is the quantity of the period two good. The consumer's utility takes the following special myopic separable form of eqn. (71)*

$$U(c_{11}, c_{12}, c_2) = -\frac{c_{11}^{-\delta}}{\delta} - \frac{c_{12}^{-\delta}}{\delta} + \ln c_2, \tag{73}$$

*which is maximized subject to*

$$p_{11}c_{11} + p_{12}c_{12} + p_2c_2 = I, \tag{74}$$

*where* $\delta > -1$ *and* $\delta \neq 0$. *Combining the first order conditions with the budget constraint yields*

$$p_{11}c_{11} + p_{12}\left(\frac{p_{11}}{p_{12}}\right)^{\frac{1}{1+\delta}} c_{11} + p_{11}c_{11}^{1+\delta} = I, \tag{75}$$

*implying that* $c_{11}$ *depends only on* $p_{11}$ *and* $p_{12}$. *Since*

$$c_{12} = \left(\frac{p_{11}}{p_{12}}\right)^{\frac{1}{1+\delta}} c_{11}, \tag{76}$$

$c_{12}$ *also depends only on* $p_{11}$ *and* $p_{12}$. *Hence the optimal period one consumption vector* $(c_{11}, c_{12})$ *is myopic.*

**Example 4** *Assume a two period setting, where $c_1$ is the quantity of the period one good and $c_{21}$ and $c_{22}$ are the quantities of the period two goods. The consumer's utility takes the following special myopic separable form of eqn. (72)*

$$U(c_1, c_{21}, c_{22}) = (c_1 c_{21})^{\frac{1}{4}} + \sqrt{c_1 c_{22}}, \tag{77}$$

*which is maximized subject to*

$$p_1 c_1 + p_{21} c_{21} + p_{22} c_{22} = I. \tag{78}$$

*Combining the first order conditions with the budget constraint yields*

$$c_1 = \frac{I}{2p_1}, \tag{79}$$

*implying that period one optimal consumption $c_1$ is independent of $p_{21}$ and $p_{22}$.*

Next we derive the necessary and sufficient condition for consumption vectors in multiple periods, including the first, to be myopic. The key tactic is to reformulate the problem so that Proposition 1 can be applied recursively. Therefore, we will refer to this case as recursive myopic separable. Since the notation for the general $T$ period case is quite messy, without loss of generality, we state the following Result for three periods. The argument for more general cases proceeds in a similar manner.

**Result 1** *Assume there are three periods. In periods one, two and three, the quantities of goods are denoted by $(c_{11}, c_{12})$, $(c_{21}, c_{22}, c_{23})$ and $(c_{31}, c_{32})$, respectively. The optimal consumption vector in each period is myopic if and only if $U(c_{11}, c_{12}, ..., c_{32})$ takes the form*

$$U = f^{(1)}\left(h_1^{(2)}, h_2^{(2)}\right), \tag{80}$$

*where*

$$h_i^{(2)} = g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23})\, g^{(1)}(c_{11}, c_{12})\, c_{3i} \quad (i = 1, 2), \tag{81}$$

$$g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23}) = f^{(2)}\left(h_1^{(1)}, h_2^{(1)}, h_3^{(1)}\right), \tag{82}$$

$$h_i^{(1)} = g^{(1)}(c_{11}, c_{12})\, c_{2i} \quad (i = 1, 2, 3),$$

*$f^{(1)}, f^{(2)}, h_i^{(1)}\ (i = 1, 2, 3), h_i^{(2)}\ (i = 1, 2) \in \mathcal{U}$ and $h_i^{(1)}, h_i^{(2)} > 0$.*

**Proof.** To apply Proposition 1, combine the first and second periods into a separable group. Then the necessary and sufficient condition for the optimal consumption vector in this separable group to be myopic is that the utility function $U$ takes the form

$$U = f^{(1)}\left(g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23})\, g^{(1)}(c_{11}, c_{12})\, c_{31}, g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23})\, g^{(1)}(c_{11}, c_{12})\, c_{32}\right). \tag{83}$$

This form of utility ensures that the optimal period two demands $(c_{21}, c_{22}, c_{23})$ are myopic.[16] For the optimal consumption vector in period one to be myopic, Proposition 1 can be applied again to $g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23})$. The necessary and sufficient condition for the optimal period one consumption vector $(c_{11}, c_{12})$ to be myopic is

$$g^{(2)}(c_{11}, c_{12}, c_{21}, c_{22}, c_{23}) = f^{(2)}\left(g^{(1)}(c_{11}, c_{12})\, c_{21}, g^{(1)}(c_{11}, c_{12})\, c_{22}, g^{(1)}(c_{11}, c_{12})\, c_{23}\right). \tag{84}$$

Since $U$ takes the form

$$U = f\left(g^{(1)}(c_{11}, c_{12})\, c_{21}, g^{(1)}(c_{11}, c_{12})\, c_{22}, g^{(1)}(c_{11}, c_{12})\, c_{23}, g^{(1)}(c_{11}, c_{12})\, c_{31}, g^{(1)}(c_{11}, c_{12})\, c_{32}\right), \tag{85}$$

where $f$ is a function determined by $f^{(1)}$ and $f^{(2)}$, it follows from Proposition 1 that the optimal period one consumption vector $(c_{11}, c_{12})$ is myopic. In conclusion, the optimal consumption vector in each period is myopic in this three period setting if and only if the utility function $U$ takes the form (83), where $g^{(2)}$ is defined in (84). Without loss of generality, assume that $h_i^{(1)}\ (i = 1, 2, 3), h_i^{(2)}\ (i = 1, 2) > 0$. Applying the similar argument as in the proof of Proposition 1, it can be shown that $f^{(1)}, f^{(2)}, h_i^{(1)}\ (i = 1, 2, 3), h_i^{(2)}\ (i = 1, 2) \in \mathcal{U}$. ■

If there are three periods and in each period there is only one commodity, then using Remark 5 and applying Proposition 1 recursively (as discussed in Result 1), optimal consumption in every period is myopic if and only if $U$ takes the following form up to an increasing monotone transformation

$$U(c_1, c_2, c_3) = f(h_1) + \ln h_2 = f(g(c_1)\, c_2) + \ln(g(c_1)\, c_3), \tag{86}$$

where $f, h_1, h_2 \in \mathcal{U}$ and $g > 0$.[17] It should be noted that if $f(x) = \ln x$ and $g(x) = \sqrt{x}$, we have

$$U(c_1, c_2, c_3) = \ln c_1 + \ln c_2 + \ln c_3, \tag{87}$$

which is the well-known log additive (or ordinally equivalent Cobb-Douglas) utility. The stronger restriction on preferences (87) guarantees that demand in each period is not only independent of future prices (as in Definition 1), but also of past prices.

[16] Note that, in general, $g^{(1)}$ can be combined with $g^{(2)}$ if one only requires $(c_{21}, c_{22}, c_{23})$ to be myopic. However, in order to ensure the myopia of $(c_{11}, c_{12})$ as well as $(c_{21}, c_{22}, c_{23})$, $g^{(1)}$ and $g^{(2)}$ need to be separated.

[17] More generally in a four period setting (with one good per period), optimal consumption in each period is myopic if and only if $U(c_1, c_2, c_3, c_4)$ takes the form

$$U(c_1, c_2, c_3, c_4) = f^{(1)}\left(f^{(2)}\left(g^{(1)}(c_1)\, c_2\right) g^{(1)}(c_1)\, c_3\right) + \ln\left(f^{(2)}\left(g^{(1)}(c_1)\, c_2\right) g^{(1)}(c_1)\, c_4\right).$$

Thus for instance, optimal $c_2$ will depend on both $p_1$ and $p_2$ in general if $U$ takes the form of (86) but only on $p_2$ if $U$ is given by (87).

On the other hand, in contrast to demand being myopic in each period as in (86), it follows from Remark 5 that optimal consumption will be myopic in period one if and only if $U$ takes the form

$$U(c_1, c_2, c_3) = f(g(c_1)\, c_2, g(c_1)\, c_3), \tag{88}$$

where in general there is no requirement for the functions $f$ and $g$ in (88) to be related to $f$ and $g$ in (86). However, the utility functions (86) and (88) will give the same optimal $c_1$ if and only if $g(c_1)$ is the same. Moreover, it follows from (86) and (88) that given period one demand is myopic, period two demand will also be myopic if and only if the utility $U$ is ordinally additively separable in $c_3$.

**Remark 6** *Although our focus in this paper is on the consumer's period one optimization and not her decisions in subsequent periods, the comparison of utilities (86) and (88) prompts an observation on decisions over time. Whereas the assumption that the consumer is myopic in every period is clearly stronger than assuming myopia in the first period (or some subset of periods), there is a sense in which the former assumption may be viewed as more natural. If the consumer is assumed to be myopic just in the first period, then why when period one ends doesn't she become myopic in the second period – retaining the same intertemporal pattern? If that were the case and her preferences in period one were represented by the utility form in eqn. (88) and not (86), then her utility in period two would have to take the form of eqn. (69),* $U(c_2, c_3) = w(c_2) + \ln c_3$. *In this case the MRS (marginal rate of substitution) between* $c_2$ *and* $c_3$ *would change from period one to period two and the consumer would be inconsistent. On the other hand, if the consumer is myopic in every period, implying that her preferences in the first period can be represented by the utility in eqn. (86), then the same utility with the given* $c_1$ *can be assumed in the second period and her period two demands will still be myopic, i.e., independent of the period three price. Since her MRS between* $c_2$ *and* $c_3$ *remains the same when changing periods, the consumer will not revise her plan and thus is consistent.*[18] *Whereas some may find this argument for assuming the consumer is myopic in every period to be persuasive at the individual demand and preference level, we will see in Examples 5 and 6 below that the equilibrium implications of being myopic in each period are considerably stronger than those associated with the assumption of being myopic in just the first period.*

[18] For a detailed discussion of changing tastes and consistency, see Selden and Wei (2012).

# 4 Interest Rate Implications

In this section, we assume a standard exchange equilibrium interest rate setting and investigate the following two questions. First assuming the existence of a representative agent, what are the different equilibrium consequences of separability, additive separability and myopic separability? Second given an economy of heterogeneous myopic separable consumers each of whom "narrow brackets" the period one demand decision, thereby substantially reducing their informational requirements, what simplification in informational requirements can be achieved for determining equilibrium interest rates? This last question is addressed when an aggregator exists and when it does not.

## 4.1 Representative Agent Economy

A standard certainty representative agent equilibrium model is assumed (see, for example, Kocherlakota 2001).[19] In period one, assume a single good, denoted by $c_1$, and $T-1$ zero coupon bonds, where $b_t$ $(t = 2, 3, ...T)$ denotes the quantity of zero coupon bonds purchased in period one and maturing at the beginning of period $t$ and paying one unit of $c_t$.[20] The period one price of the zero coupon bond is denoted by $q_t$, where subscript $t$ indicates that the bond matures at the beginning of period $t$. And as is standard, the net interest rate $r_{t-1}$ during period $t-1$ associated with the zero coupon bond purchased in period one and maturing at date $t$ is given by

$$q_t = \frac{1}{(1 + r_{t-1})^{t-1}}. \tag{89}$$

The representative agent is endowed in period one with a fixed supply $\bar{c}_1$ of period one consumption and $\bar{b}_2, \bar{b}_3, ..., \bar{b}_T$ zero coupon bonds[21] and has preferences

[19] Since a static setting is assumed, all bond prices are observed at the beginning of the current time period and all interest rates are current spot rates. We do not consider implied forward rates or future spot interest rates.

[20] It should be noted that in order to investigate the impact of myopic separability on the term structure, the choice of a zero coupon bond versus a standard nonzero coupon bond is not innocuous. For example, consider a three period setting in which the two period bond pays a coupon rate of $\xi$ per cent at the end of periods one and two. Then period two consumption is given by $c_2 = b_2 + \xi b_3$, which is a function of both $b_2$ and $b_3$. Thus, it would not be appropriate to say that $c_2$ is myopic if and only if it is independent of the two period bond price $q_3$. The advantage of the zero coupon bond assumption is that all of the information concerning the bond is incorporated in its price and this difficulty can be avoided.

[21] Here we assume that $\bar{b}_2, \bar{b}_3, ..., \bar{b}_T \neq 0$. Such an assumption is not atypical. It could for instance be associated with the debt being issued by a government which is outside the model (see, for example, Parlour, Stanton and Walden 2011 and the literature cited therein). Alternatively, our assumption of nonzero supplies of bonds could be dropped if we were to allow for endowments

over consumption streams $(c_1, ..., c_T)$ represented by $U$.[22] The optimization problem is given by

$$\max_{c_1,b_2,...,b_T} U(c_1, ..., c_T) \qquad S.T.\ c_1 + \sum_{t=2}^{T} q_t c_t = \bar{c}_1 + \sum_{t=2}^{T} q_t \bar{b}_t \tag{90}$$

$$S.T.\ c_t = b_t \quad \forall t \in \{2, 3, ..., T\}. \tag{91}$$

To examine the implications of myopic separable preferences for equilibrium interest rates, assume the first and second time periods are combined to form a separable group with consumption $c_1$ and $c_2$ (or $b_2$). It follows from Proposition 1 that the following form of utility exhibits myopic separability[23]

$$U(c_1, ..., c_T) = f(g(c_1, c_2) c_3, g(c_1, c_2) c_4, ..., g(c_1, c_2) c_T). \tag{92}$$

It would seem natural to conjecture that this myopic separable form is necessary and sufficient for the equilibrium period one interest rate $r_1$ to be independent of the supplies $\bar{b}_t$ $(t = 3, 4, ..., T)$. Indeed it can be verified that for the utility (92), one always has

$$1 + r_1 = \frac{1}{q_2} = \frac{\partial U/\partial c_1}{\partial U/\partial c_2} = \left. \frac{\partial g(c_1, c_2)/\partial c_1}{\partial g(c_1, c_2)/\partial c_2} \right|_{(c_1,c_2)=(\bar{c}_1,\bar{b}_2)}, \tag{93}$$

which is independent of $\bar{b}_t$ $(t = 3, 4, ..., T)$. This is not surprising since following Leontief (1947) and Sono (1961), it is straightforward to obtain the following Proposition.

**Proposition 2** *Assume the representative agent's optimization problem is characterized by (90) and (91). The equilibrium period one interest rate $r_1$ is independent*

in the form of period two and three income (in units of consumption). This would change none of the conclusions, only making the notation more complicated.

[22] It should be noted that because we do not assume additive utility, our analysis will not include the typical period discount factors present in standard equilibrium interest rate models.

[23] It is clear that in the optimization problem (90) - (91), if $U$ takes one of the myopic separable forms of utility derived in the prior section, optimal demands will be myopic in the sense of Definition 2. Because prices will always enter into the demand functions through total income or wealth, i.e.,

$$I = \bar{c}_1 + \sum_{t=2}^{T} q_t \bar{b}_t,$$

the endowments introduce a third term into the classic Slutsky equation typically referred to as the endowment (income) effect (see Arrow and Hahn, 1971, p. 225 and Varian, 1992, p. 145). However, myopic separable utility will ensure that the corresponding demand function income and substitution effects continue to exactly offset each other.

*of the supplies* $\overline{b}_t$ $(t = 3, 4, ..., T)$ *if and only if the agent's preferences is separable in* $(c_1, c_2)$

$$U(c_1, ..., c_T) = f\left(U^{(2)}(c_1, c_2), c_3, ..., c_T\right), \tag{94}$$

*where where* $f, U^{(2)} \in \mathcal{U}$ *and* $U^{(2)} > 0$.

Since separability does not imply myopic separability, $r_1$ being independent of $\overline{b}_t$ $(t = 3, 4, ..., T)$ although necessary is not sufficient for myopic separable utility. Since the most widely assumed form of myopic separable utility the log additive form (87) not only exhibits myopic separability but also additive separability, we first apply a classic result of Samuelson (1947) to the current representative agent equilibrium setting to derive the restrictions on equilibrium interest rates that are equivalent to preferences being additively separable.

**Proposition 3** *Assume the representative agent's optimization problem is characterized by (90) and (91). For all* $t \in \{1, 2, ..., T-1\}$ *and* $T > 2$, $r_t$ *is independent of bond supplies other than* $\overline{b}_{t+1}$ *if and only if the agent's preferences can be represented by an ordinally additively separable utility,*

$$U(c_1, ..., c_T) = \sum_{t=1}^{T} u_t(c_t), \tag{95}$$

*where* $u_t \in \mathcal{U}$.

**Proof.** Observing that in a representative agent exchange economy

$$\frac{1}{(1 + r_{t-1})^{t-1}} = q_t = \frac{\partial U/\partial c_t}{\partial U/\partial c_1} \quad \forall t \in \{2, 3, ..., T\}, \tag{96}$$

the proof of this Proposition directly follows from Samuelson (1947) pp. 176-183.[24] ■

[24] It should be noted that in addition to

$$\frac{\partial q_t}{\partial \bar{b}_i} = 0 \quad (\forall t, i \in \{2, 3, ..., T\}, i \neq t),$$

implied in our Proposition 3, Samuelson also gives the following in his eqns. (33) (Samuelson 1947, p. 179)

$$\frac{\partial}{\partial \bar{c}_1}\left(\frac{q_t}{q_2}\right) = 0 \quad (\forall t \in \{3, 4, ..., T\})$$

for the necessary and sufficient condition such that preferences can be represented by an additively separable utility function. As Samuelson states, his condition implies integrability and if this is postulated as a precondition then eqns. (33) cease to all be independent and can be reduced in number. Since we have assumed the existence of $U$, it is not necessary to include the above set of equations in Proposition 3. To be more explicit, we can show that this set of equations can be

Whereas preferences being separable in $(c_1, c_2)$ guarantees that $r_1$ depends only on $\bar{b}_2$, additive separability ensures that this same result holds for every period. As can be seen from Example 6 below, myopic separability in each period differs from additive separability in allowing the equilibrium interest rates $r_t$ $(t = 2, 3, ..., T-1)$ to depend on the supply of bonds in all prior periods and not just $\bar{b}_{t+1}$. Therefore the equilibrium interest rate restrictions implied by additive separability are clearly stronger which is fully consistent with the preference based reservations of Fisher (1930), Hicks (1965) and Lucas (1978) referenced above in Section 1.

We next characterize the equilibrium interest rate implications which are both necessary and sufficient for preferences to be representable by the myopic separable utility (35) in Proposition 1.

**Proposition 4** *Assume the representative agent's optimization problem is characterized by (90) and (91). The equilibrium interest rates exhibit the property that for any $t \in \{1, 2, ..., T-1\}$, the present value $\sum_{i=t+1}^{T} \frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}$ is independent of $\bar{b}_j$ $(j \in \{t+1, t+2, ..., T\})$ if and only if preferences are representable by the myopic separable utility (35) corresponding to optimal period $t$ consumption $c_t$ being myopic.*

**Proof.** Without loss of generality, we only need to prove the Proposition for optimal period two consumption. First prove sufficiency. It follows from Proposition 1 that period two consumption $c_2$ is myopic if and only if

$$U(c_1, ..., c_T) = f(g(c_1, c_2)\, c_3, g(c_1, c_2)\, c_4, ..., g(c_1, c_2)\, c_T). \tag{97}$$

In equilibrium, the first order conditions are

$$\frac{g f_j}{g_1 \sum_{i=1}^{T-2} f_i \bar{b}_{i+2}} = \frac{1}{(1+r_{j+1})^{j+1}}, \tag{98}$$

---

directly derived from $\frac{\partial q_t}{\partial \bar{b}_i} = 0$. Noticing that

$$q_2 = \frac{\frac{\partial U}{\partial c_2}}{\frac{\partial U}{\partial c_1}} \quad \text{and} \quad q_3 = \frac{\frac{\partial U}{\partial c_3}}{\frac{\partial U}{\partial c_1}},$$

one can obtain

$$\frac{\partial q_2}{\partial c_3} = 0 \Leftrightarrow \frac{\partial^2 U}{\partial c_2 \partial c_3}\frac{\partial U}{\partial c_1} - \frac{\partial^2 U}{\partial c_1 \partial c_3}\frac{\partial U}{\partial c_2} = 0$$

and

$$\frac{\partial q_3}{\partial c_2} = 0 \Leftrightarrow \frac{\partial^2 U}{\partial c_2 \partial c_3}\frac{\partial U}{\partial c_1} - \frac{\partial^2 U}{\partial c_1 \partial c_2}\frac{\partial U}{\partial c_3} = 0,$$

implying that

$$\frac{\partial^2 U}{\partial c_1 \partial c_3}\frac{\partial U}{\partial c_2} - \frac{\partial^2 U}{\partial c_1 \partial c_2}\frac{\partial U}{\partial c_3} = 0 \Leftrightarrow \frac{\partial}{\partial c_1}\left(\frac{\frac{\partial U}{\partial c_3}}{\frac{\partial U}{\partial c_2}}\right) = \frac{\partial}{\partial c_1}\left(\frac{q_3}{q_2}\right) = 0.$$

implying that

$$\sum_{i=3}^{T}\frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}=\sum_{i=1}^{T-2}\frac{\bar{b}_{i+2}}{(1+r_{i+1})^{i+1}}=\frac{g\sum_{i=1}^{T-2}f_i\bar{b}_{i+2}}{g_1\sum_{i=1}^{T-2}f_i\bar{b}_{i+2}}=\left.\frac{g}{g_1}\right|_{(c_1,c_2)=\left(\bar{c}_1,\bar{b}_2\right)} \tag{99}$$

and hence $\sum_{i=3}^{T}\frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}$ is independent of $\bar{b}_j$ $(j\in\{3,4,...,T\})$. Next prove necessity. If $\sum_{i=3}^{T}\frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}$ is independent of $\bar{b}_j$ $(j\in\{3,4,...,T\})$, then we have

$$\frac{\partial}{\partial c_j}\frac{\sum_{i=3}^{T}c_iU_i}{U_1}=0\quad\forall j\in\{3,...,T\}\,, \tag{100}$$

implying that

$$U(c_1,...,c_T)=f\left(g(c_1,c_2)\,c_3,g(c_1,c_2)\,c_4,...,g(c_1,c_2)\,c_T\right). \tag{101}$$

■

**Remark 7** *This Theorem can also be viewed as giving the necessary and sufficient equilibrium condition for the choice of "narrow bracketing" the first t periods to be optimal.*

**Remark 8** *To compare the result in Proposition 4 with the additively separable case, assume the representative agent's utility takes the additively separable CES form*

$$U(c_1,...,c_T)=-\sum_{i=1}^{T}\frac{c_i^{-\delta}}{\delta}\quad(\delta>-1)\,. \tag{102}$$

*Then it can be easily verified that*

$$\sum_{i=2}^{T}\frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}=\bar{c}_1^{1+\delta}\sum_{i=2}^{T}\bar{b}_i^{-\delta}, \tag{103}$$

*implying that* $\forall j\in\{2,3,...,T\}$

$$\frac{\partial\left(\sum_{i=2}^{T}\frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}\right)}{\partial\bar{b}_j}\gtreqqless 0\Leftrightarrow\delta\lesseqqgtr 0. \tag{104}$$

*Therefore, additive separability cannot ensure that* $\sum_{i=2}^{T}\frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}$ *is independent of* $\bar{b}_j$. *The* $\delta=0$ *case corresponds to the log additive utility, which exhibits myopic separability.*

The intuition for why Proposition 4 works is that for myopic separable preferences the set of equilibrium interest rates adjusts so as to keep the present value constant. It should be noted that if preferences take the form associated with consumption being myopic in each period, then we can apply Proposition 4 recursively and conclude that for all $t\in\{1,2,...,T-1\}$, $r_t$ is independent of $\bar{b}_{j+1}$ $(j\in\{t+1,t+2,...,T\})$ and $\sum_{i=t+1}^{T}\frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}$ is independent of $\bar{b}_j$ $(j\in\{t+1,t+2,...,T\})$, where the latter is not only necessary but also sufficient.

**Remark 9** *The Proposition 4 conclusion that the present value of future bond supplies is independent of changes in the supply of bonds in each period may strike the reader as being reminiscent of the irrelevance of government financial policy in the macroeconomics literature (e.g., Wallace 1981, Bryant 1983 and Stiglitz 1984). There it is assumed in an intertemporal setting that a government exists which both collects taxes and issues debt of differing maturities. If for a given supply of bonds a general equilibrium exists, then modifying the supply of bonds will not affect the equilibrium value of real variables such as consumption although equilibrium interest rates may change. (There is a clear analogy of this result to the famous Modigliani and Miller capital structure irrelevance in corporate finance.) However it should be stressed that the source of "independence" in our setting comes from the form of utility since the government sector in our model is not closed as we do not allow for taxes.*

We conclude this subsection with two Examples and a Remark. They illustrate in a four period setting the Proposition 4 implications on equilibrium interest rates of preferences taking the different forms associated with $(c_1, c_2)$ being myopic versus $(c_1, c_2)$ , $c_3$ and $c_4$ being myopic and the implications of log additive utility.

**Example 5** *Assume the representative agent's optimization problem is characterized by (90) and (91), where there are four periods and utility takes the form*

$$U(c_1, c_2, c_3, c_4) = ((\ln c_1 + \ln c_2) c_3)^{\frac{1}{4}} + \sqrt{(\ln c_1 + \ln c_2) c_4}, \tag{105}$$

*where $c_1, c_2 > 1$ and $c_3, c_4 > 0$ are assumed to ensure $U \in \mathcal{U}$. It follows from Proposition 1 that the optimal consumption vector $(c_1, c_2)$ is myopic. Using the representative agent's first order conditions paralleling eqn. (93), straightforward computation results in the following characterization of equilibrium interest rates*

$$\frac{1}{1+r_1} = \frac{\bar{c}_1}{\bar{b}_2}, \tag{106}$$

$$\frac{1}{(1+r_2)^2} = \frac{\bar{c}_1 \left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \left(\left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \bar{b}_3\right)^{-\frac{3}{4}}}{\bar{b}_3 \left(\left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \bar{b}_3\right)^{-\frac{3}{4}} + 2\bar{b}_4 \left(\left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \bar{b}_4\right)^{-\frac{1}{2}}} \tag{107}$$

*and*

$$\frac{1}{(1+r_3)^3} = \frac{2\bar{c}_1 \left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \left(\left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \bar{b}_4\right)^{-\frac{1}{2}}}{\bar{b}_3 \left(\left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \bar{b}_3\right)^{-\frac{3}{4}} + 2\bar{b}_4 \left(\left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \bar{b}_4\right)^{-\frac{1}{2}}}. \tag{108}$$

*First we can see that $r_1$ is independent of $\left(\bar{b}_3, \bar{b}_4\right)$, but $r_2$ and $r_3$ depend on all of the bond supplies. Moreover, using (107) and (108) it follows that the present value of period 3 and period 4 bond supplies*

$$\frac{\bar{b}_3}{(1+r_2)^2} + \frac{\bar{b}_4}{(1+r_3)^3} = \bar{c}_1 \left(\ln \bar{c}_1 + \ln \bar{b}_2\right) \tag{109}$$

*is independent of* $\bar{b}_3$ *and* $\bar{b}_4$.

**Example 6** *Assume the representative agent's optimization problem is characterized by (90) and (91), where there are four periods and utility takes the form*[25]

$$U\left(c_1, c_2, c_3, c_4\right) = \sqrt{\ln\left(c_1 c_2\right) c_1 c_3} + \ln\left(\ln\left(c_1 c_2\right)\right) + \ln c_1 + \ln c_4, \tag{110}$$

*where* $c_1, c_2 > 1$ *and* $c_3, c_4 > 0$ *are assumed to ensure* $U \in \mathcal{U}$. *It follows from Proposition 1 and Result 1 that optimal consumption in each period is myopic. Using the representative agent's first order conditions paralleling eqn. (93), straightforward computation results in the following characterization of equilibrium interest rates*

$$\frac{1}{1+r_1} = \frac{\bar{c}_1}{\bar{b}_2\left(1+\ln\left(\bar{c}_1\bar{b}_2\right)\right)}, \tag{111}$$

$$\frac{1}{(1+r_2)^2} = \frac{\left(\bar{c}_1\ln\left(\bar{c}_1\bar{b}_2\right)\right)^2}{\left(1+\ln\left(\bar{c}_1\bar{b}_2\right)\right)\left(\bar{c}_1\bar{b}_3\ln\left(\bar{c}_1\bar{b}_2\right) + 2\sqrt{\bar{c}_1\bar{b}_3\ln\left(\bar{c}_1\bar{b}_2\right)}\right)} \tag{112}$$

*and*

$$\frac{1}{(1+r_3)^3} = \frac{2\bar{c}_1\ln\left(\bar{c}_1\bar{b}_2\right)}{\bar{b}_4\left(1+\ln\left(\bar{c}_1\bar{b}_2\right)\right)\left(2+\sqrt{\bar{c}_1\bar{b}_3\ln\left(\bar{c}_1\bar{b}_2\right)}\right)}. \tag{113}$$

*It is clear that* $r_1$ *is independent of* $\left(\bar{b}_3, \bar{b}_4\right)$, $r_2$ *is independent of* $\bar{b}_4$ *and* $r_3$ *depends on all bond supplies. Moreover using (111)-(113), it follows that*

**i**

$$\frac{\bar{b}_2}{1+r_1} + \frac{\bar{b}_3}{(1+r_2)^2} + \frac{\bar{b}_4}{(1+r_3)^3} = \bar{c}_1, \tag{114}$$

*where the present value is independent of* $\bar{b}_2$, $\bar{b}_3$ *and* $\bar{b}_4$;

**ii**

$$\frac{\bar{b}_3}{(1+r_2)^2} + \frac{\bar{b}_4}{(1+r_3)^3} = \frac{\bar{c}_1\ln\left(\bar{c}_1\bar{b}_2\right)}{1+\ln\left(\bar{c}_1\bar{b}_2\right)}, \tag{115}$$

*where the present value is independent of* $\bar{b}_3$ *and* $\bar{b}_4$; *and*

[25] The utility (110) can be obtained from the general four good myopic separable utility in footnote 17 by assuming

$$f^{(1)}(x) = \sqrt{x}, \quad f^{(2)}(x) = \ln x \quad \text{and} \quad g^{(1)}(x) = x.$$

It should be noted that if the coefficients in (110) are varied arbitrarily, the resulting utility will not be a special case of the general form and will not result in myopic demand behavior.

**iii**

$$\frac{\bar{b}_4}{(1+r_3)^3} = \frac{2\bar{c}_1 \ln\left(\bar{c}_1\bar{b}_2\right)}{\left(1+\ln\left(\bar{c}_1\bar{b}_2\right)\right)\left(2+\sqrt{\bar{c}_1\bar{b}_3\ln\left(\bar{c}_1\bar{b}_2\right)}\right)}, \tag{116}$$

*where the present value is independent of* $\bar{b}_4$*.*

**Remark 10** *It is interesting to contrast the equilibrium in Example 6 with that resulting from utility taking the log additive form*

$$U(c_1, c_2, c_3, c_4) = \ln c_1 + \ln c_2 + \ln c_3 + \ln c_4, \tag{117}$$

*where utility exhibits not only additive separability but also myopic separability in each period. In this case, it can be verified that*

$$\frac{1}{1+r_1} = \frac{\bar{c}_1}{\bar{b}_2}, \quad \frac{1}{(1+r_2)^2} = \frac{\bar{c}_1}{\bar{b}_3} \quad \text{and} \quad \frac{1}{(1+r_3)^3} = \frac{\bar{c}_1}{\bar{b}_4}, \tag{118}$$

*implying that*

$$\frac{\bar{b}_2}{1+r_1} + \frac{\bar{b}_3}{(1+r_2)^2} + \frac{\bar{b}_4}{(1+r_3)^3} = 3\bar{c}_1, \tag{119}$$

$$\frac{\bar{b}_3}{(1+r_2)^2} + \frac{\bar{b}_4}{(1+r_3)^3} = 2\bar{c}_1 \tag{120}$$

*and*

$$\frac{\bar{b}_4}{(1+r_3)^3} = \bar{c}_1. \tag{121}$$

*It can be seen that a change in the endowment* $\bar{b}_{t+1}$ *exactly cancels out the interest rate change* $(1+r_t)^t$ *and the present value is always a function of only* $\bar{c}_1$*. Whereas myopia in each period ensures that each present value sum is independent of changes in the respective endowment, additive separability ensures that a change in* $\bar{b}_{t+1}$ *only affects* $r_t$ *and hence implies that the changes in the endowment must be canceled out exactly by the interest rate change.*

In summary, in a representative agent economy, if the utility function is separable in $(c_1, c_2)$, then the equilibrium period one interest rate $r_1$ is independent of the supplies $\bar{b}_t$ $(t = 3, 4, ..., T)$ and hence the informational requirement is substantially reduced. If we further assume that the utility function is additively separable, then $\forall t \in \{1, 2, ..., T-1\}$, $r_t$ is independent of bond supplies other than $\bar{b}_{t+1}$. If we further assume that the utility function is myopic separable in $(c_1, c_2)$, then $\forall j \in \{3, ..., T\}$, $\sum_{i=3}^{T} \frac{\bar{b}_i}{(1+r_{i-1})^{i-1}}$ is independent of $\bar{b}_j$.

## 4.2 Heterogeneous Agent Economy

In a heterogeneous economy, suppose there are $N$ $(i = 1, 2, ..., N)$ agents and each agent $i$ considers the following optimization problem

$$\max_{c_1^{(i)}, b_2^{(i)}, ..., b_T^{(i)}} U^{(i)}\left(c_1^{(i)}, ..., c_T^{(i)}\right) \qquad S.T.\ c_1^{(i)} + \sum_{t=2}^{T} q_t c_t^{(i)} = \overline{c}_1^{(i)} + \sum_{t=2}^{T} q_t \overline{b}_t^{(i)} \tag{122}$$

$$S.T.\ \ c_t^{(i)} = b_t^{(i)} \quad \forall t \in \{2, 3, ..., T\}, \tag{123}$$

where $c_1^{(i)}$ denotes consumption in period 1 and $b_t^{(i)}$ $(t = 2, 3, ...T)$ the quantity of zero coupon bonds purchased in period one and maturing at the beginning of period $t$ and paying one unit of $c_t^{(i)}$. Denote

$$I^{(i)} = \overline{c}_1^{(i)} + \sum_{t=2}^{T} q_t \overline{b}_t^{(i)}. \tag{124}$$

As discussed in the prior subsection, separability can substantially reduce the informational requirement in the equilibrium interest rate analysis. Unfortunately, Propositions 2 and 3 cannot be applied to the heterogeneous economy in general even if we assume an aggregator exists. For instance, there is no assurance that the aggregator's utility will be (additively) separable if each agent's preferences are (additively) separable.[26] One known case where the separability of individual agent preferences is inherited by an aggregator is the exact linear aggregation discussed by Gorman (1953, 1961) with the additively separable agent utility. For such exact aggregation, the aggregate demand can be written as a function of prices and aggregate income alone, independent of the income distribution.[27] Gorman shows that the necessary and sufficient condition for the exact linear aggregation is that the Engel curves of all the agents are straight lines and have a common slope. For homothetic preferences, since Engel curves always start from the origin, Gorman's necessary and sufficient condition implies that all agents have the same preferences, which is too restrictive. For the quasihomothetic case where agent $i$'s preferences are representable by

$$U^{(i)}\left(c_1^{(i)}, ..., c_T^{(i)}\right) = -\sum_{j=1}^{T} \frac{\left(c_j^{(i)} - a_i\right)^{-\delta}}{\delta} \quad (\delta > -1), \tag{125}$$

[26] See Kubler, Selden and Wei (2013) for a two agent case in which each agent has constant relative risk aversion (CRRA) Expected Utility preferences which are additively separable across contingent claims. Preferences are characterized by different risk aversion parameters and endowments are assumed to be identical for each agent. In this case an aggregator exists and it is possible to derive the explicit form of the aggregator's utility. However, it fails to be additively separable.

[27] For a nice recent summary, see Cherchye et. al. (2011).

once $\delta$ is the same for all the agents, Gorman's exact linear aggregation condition will be satisfied even if $a_i$ are different across the heterogeneous agents. In this case, the aggregator's utility function is given by

$$U(c_1, ..., c_T) = -\sum_{j=1}^{T} \frac{\left(c_j - \sum_{i=1}^{N} a_i\right)^{-\delta}}{\delta} \quad (\delta > -1), \tag{126}$$

which is still additively separable and hence Propositions 2 and 3 can be applied.

The requirement of the linear Engel curve with the same slope for each agent is clearly extremely restrictive. However, it can easily be seen that myopic separability can be preserved if an aggregator exists. In other words if each agent's preferences are myopic separable in $(c_1, c_2)$ and the aggregator exists, then since the aggregate demand $(c_1, c_2)$ is still myopic Propositions 2 and 4 hold. This is because if a utility function is myopic separable in $(c_1, c_2)$, it is also separable in $(c_1, c_2)$ which implies that $r_1$ is independent of $\left(\bar{b}_3, ..., \bar{b}_T\right)$. Moreover as we will show in the following Proposition, for the myopic separable agents, the conclusion that $r_1$ is independent of $\left(\bar{b}_3, ..., \bar{b}_T\right)$ does not require the assumption of the existence of an aggregator.

**Proposition 5** *Assume a $N$-agent heterogeneous economy with agent $i$'s optimization problem characterized by (122) and (123). If each agent's utility function is myopic separable in $(c_1, c_2)$, i.e., $\forall i \in \{1, 2, ..., N\}$*

$$U^{(i)}\left(c_1^{(i)}, ..., c_T^{(i)}\right) = f^{(i)}\left(g^{(i)}(c_1, c_2)\, c_3, g^{(i)}(c_1, c_2)\, c_4, ..., g^{(i)}(c_1, c_2)\, c_T\right), \tag{127}$$

*and the endowments are collinear,*[28] *then the equilibrium interest rate $r_1$ is independent of $\bar{b}_t$ $(t = 3, 4, ..., T)$.*

**Proof.** Assume each agent's utility function is myopic separable in $(c_1, c_2)$, then we have

$$c_1^{(i)} = c_1^{(i)}\left(q_2, I^{(i)}\right) \quad \text{and} \quad c_2^{(i)} = c_2^{(i)}\left(q_2, I^{(i)}\right), \tag{128}$$

implying that the aggregate demand is given by

$$c_1 = \sum_{i=1}^{N} c_1^{(i)}\left(q_2, I^{(i)}\right) \quad \text{and} \quad c_2 = \sum_{i=1}^{N} c_2^{(i)}\left(q_2, I^{(i)}\right). \tag{129}$$

If the endowments are collinear, then

$$I^{(i)} = \omega_i I, \tag{130}$$

[28] It should be noted that the assumption of the price independent income distribution is made in several general equilibrium analyses (see, for example, Mas-Colell 1991, p. 280). Unlike Chipman (1974), we do not assume homothetic preferences here and hence the aggregator may not exist.

where

$$I^{(i)} = \overline{c}_1 + \sum_{t=2}^{T} q_t \overline{b}_t. \tag{131}$$

Eqn. (129) can be rewritten as

$$c_1 = \sum_{i=1}^{N} c_1^{(i)} \left(q_2, \omega_i I\right) \quad \text{and} \quad c_2 = \sum_{i=1}^{N} c_2^{(i)} \left(q_2, \omega_i I\right). \tag{132}$$

In equilibrium, we have $c_1 = \overline{c}_1$ and $c_2 = \overline{b}_2$, implying that

$$\overline{c}_1 = \sum_{i=1}^{N} c_1^{(i)} \left(q_2, \omega_i I\right) \quad \text{and} \quad \overline{b}_2 = \sum_{i=1}^{N} c_2^{(i)} \left(q_2, \omega_i I\right). \tag{133}$$

Since we have two independent equations for two variables $(q_2, I)$, $r_1 = \frac{1}{q_2}$ is a function of $\left(\overline{c}_1, \overline{b}_2, \boldsymbol{\omega}\right)$, which is independent of $\overline{b}_t$ $(t = 3, 4, ..., T)$. ■

# Appendix

## A Alternative Proof: Necessity Part of Proposition 1

Introduce the following notation

$$U_i = \frac{\partial U}{\partial c_i} \quad \text{and} \quad U_{ij} = \frac{\partial^2 U}{\partial c_i \partial c_j} \qquad i, j \in \{1, 2, ..., m+n\}. \tag{134}$$

The first order conditions give

$$p_i U_1 - p_1 U_i = 0 \quad (i = 2, 3, ..., m+n). \tag{135}$$

Since the optimal $(c_1, ..., c_m)$ depends only on $(p_1, ..., p_m)$, differentiating both sides of the $i^{th}$ first order condition with respect to $p_k$ $(k \in \{m+1, m+2, ..., m+n\})$ we have

$$\sum_{j=m+1}^{m+n} \left(p_1 U_{ij} - p_i U_{1j}\right) \frac{\partial c_j}{\partial p_k} = \delta_{ik} U_1, \tag{136}$$

where $\delta_{ik}$ is the Kronecker $\delta$. Differentiation of the left hand side of the budget constraint with respect to $p_k$, yields

$$\sum_{j=m+1}^{m+n} p_j \frac{\partial c_j}{\partial p_k} = -c_k. \tag{137}$$

We have a system of $n+1$ linear equations with $n$ unknowns $\frac{\partial c_j}{\partial p_k}$ $(j = m+1, m+2, ..., m+n)$.[29] There exists at least one nonzero (non-trivial) solution for this equation system if and only if the augmented coefficient matrix of this system is singular, i.e.,

$$\det M = 0, \tag{138}$$

where

$$M = \begin{pmatrix} p_1 U_{m+1,m+1} - p_{m+1} U_{1,m+1}, & \cdots, & p_1 U_{m+1,m+n,} - p_{m+1} U_{1,m+n}, & 0 \\ \vdots & \vdots & \vdots & \vdots \\ p_1 U_{k,m+1} - p_k U_{1,m+1}, & \cdots, & p_1 U_{k,m+n} - p_k U_{1,m+n}, & U_1 \\ \vdots & \vdots & \vdots & \vdots \\ p_1 U_{m+n,m+1} - p_{m+n} U_{1,m+1}, & \cdots, & p_1 U_{m+n,m+n} - p_{m+n} U_{1,m+n}, & 0 \\ p_{m+1}, & \cdots, & p_{m+n}, & -c_k \end{pmatrix}. \tag{139}$$

Define

$$H = \begin{pmatrix} p_1 U_{m+1,m+1} - p_{m+1} U_{1,m+1}, & \cdots, & p_1 U_{m+1,m+n,} - p_{m+1} U_{1,m+n}, \\ \vdots & \vdots & \vdots \\ p_1 U_{k,m+1} - p_k U_{1,m+1}, & \cdots, & p_1 U_{k,m+n} - p_k U_{1,m+n}, \\ \vdots & \vdots & \vdots \\ p_1 U_{m+n,m+1} - p_{m+n} U_{1,m+1}, & \cdots, & p_1 U_{m+n,m+n} - p_{m+n} U_{1,m+n}, \end{pmatrix}. \tag{140}$$

Using the Laplace Expansion to expand the determinant in eqn. (138) by the last column (and the cofactor of $U_1$ by the last row), yields

$$(-1)^{k-m+n+1} U_1 \sum_{j=m+1}^{m+n} (-1)^{n+j-m} p_j H_{j-m,k-m} - c_k \det H = 0, \tag{141}$$

where $H_{j-m,k-m}$ is the $j-m, k-m$ minor of $H$. Substituting eqn. (135) into (141), we have

$$p_1 \sum_{j=m+1}^{m+n} (-1)^{k+j} U_j H_{j-m,k-m} = -c_k \det H. \tag{142}$$

Assume first that $H$ is non-singular and notice that

$$\frac{(-1)^{k+j} H_{j-m,k-m}}{\det H} \tag{143}$$

is the $(j-m, k-m)$ component of $\left(H^T\right)^{-1}$. Denoting

$$\nabla U = (U_{m+1}, U_{m+2}, ..., U_{m+n}) \quad \text{and} \quad \mathbf{c} = (c_{m+1}, c_{m+2}, ..., c_{m+n}), \tag{144}$$

[29] The $n+1$ equations include eqns. (136) (one equation for each $i \in \{m+1, ..., m+n\}$) and eqn. (137).

it follows from eqn. (142) that

$$p_1 \left(H^T\right)^{-1} (\boldsymbol{\nabla} U) = -\mathbf{c} \Leftrightarrow p_1 \boldsymbol{\nabla} U = -H^T \mathbf{c}, \tag{145}$$

or equivalently

$$\sum_{j=m+1}^{m+n} (p_1 U_{ji} - p_j U_{1i}) c_j = -p_1 U_i \quad (i = m+1, m+2, ..., m+n). \tag{146}$$

The determinant $\det H$ is proportional to the bordered Hessian of $U$ when considered as a function of the last $n$ variables. The strict quasiconcavity of $U$ implies by Theorem VI of Bernstein and Toupin (1962) that $\det H \neq 0$ on a dense set, so that (146) holds on a dense set. By continuity (146) holds everywhere. Notice that for any $i \in \{m+1, m+2, ..., m+n\}$,

$$\frac{\partial}{\partial c_i} \frac{\sum_{j=m+1}^{m+n} c_j U_j}{U_1} = \frac{\left(U_i + \sum_{j=m+1}^{m+n} c_j U_{ij}\right) U_1 - U_{1i} \sum_{j=m+1}^{m+n} c_j U_j}{(U_1)^2}. \tag{147}$$

It follows from eqn. (146) and the first order conditions that

$$\sum_{j=m+1}^{m+n} (U_{ij} U_1 - U_j U_{1i}) c_j = -U_i U_1 \quad (i = m+1, m+2, ..., m+n). \tag{148}$$

Substituting eqn. (148) into (147), it follows that

$$\frac{\partial}{\partial c_i} \frac{\sum_{j=m+1}^{m+n} c_j U_j}{U_1} = 0. \tag{149}$$

Therefore, we have

$$\frac{\sum_{j=m+1}^{m+n} c_j U_j}{U_1} = w^{(1)} (c_1, ...c_m), \tag{150}$$

where $w^{(1)} (c_1, ...c_m)$ is an arbitrary positive function. Similarly, it can be proved that for $\forall i \in \{1, 2, ..., m\}$

$$\frac{\sum_{j=m+1}^{m+n} c_j U_j}{U_i} = w^{(i)} (c_1, ...c_m), \tag{151}$$

for certain positive functions $w^{(i)} (c_1, ...c_m)$. By Frobenius integrability conditions $w^{(i)} \frac{\partial w^{(j)}}{\partial c_i} = w^{(j)} \frac{\partial w^{(i)}}{\partial c_j}$ for $i, j \in \{1, 2, ..., m\}$. Integrating the above over-determined system yields

$$U (c_1, ..., c_{m+n}) = f (g (c_1, ..., c_m) c_{m+1}, g (c_1, ..., c_m) c_{m+2}, ..., g (c_1, ..., c_m) c_{m+n}), \tag{152}$$

and $g$ can be obtained from the following integrable system of equations

$$\frac{g}{g_i} = w^{(i)} (c_1, ...c_m) \qquad (i \in \{1, 2, ..., m\}). \tag{153}$$

Finally, $f, g, h_i = g (c_1, ..., c_m) c_{m+i} \in \mathcal{U}$ is verified as in the proof of Proposition 1 in the main text.

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