Testing Factor-Model Explanations of Market Anomalies

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ABSTRACT

A set of recent papers attempts to explain the size and book-to-market anomalies with conditional CAPM or CCAPM models with economically motivated conditioning variables, or with factor models with economically motivated factors. The tests of these models, as presented, fail to reject the proposed model. We argue that these tests fail to reject the null hypothesis because they have very low statistical power against what we call the *characteristics* alternative. Specifically, the low power of these tests arises because they use as test portfolios, characteristic-sorted portfolios that do not have sufficient independent variation in the factor loadings and the characteristics. We propose several methods for constructing more appropriate test portfolios and for designing more powerful tests. We show that with these more powerful tests the models we examine are rejected at high levels of statistical significance.

1   Introduction

The Sharpe (1964) and Lintner (1965) Capital Asset Pricing Model suggests that the expected returns of risky assets should be determined by the covariance of their returns with the returns on the market portfolio. However, studies by Fama and French (1992) and others uncover almost no relationship between market betas and expected returns, and instead find a strong cross-sectional relationship between characteristics like size and book-to-market and returns. The nature of the underlying mechanism responsible for these empirical findings has now been a source of debate for several decades, and there is still no consensus. One hypothesis is that a firm's size and book-to-market serve as proxies for the riskiness of the firm. Another possibility is that these characteristics proxy for mispricing, i.e., high book-to-market stocks have higher expected returns because they are undervalued.

The Fama and French (1993) results appear to support the first hypothesis by showing that a set of factor-mimicking portfolios, RM-RF, SMB, and HML, do a fairly good job of pricing the cross-section of stock returns. However, for several reasons, this evidence is not particularly satisfying. First, the economic motivation of the factors is somewhat loose; a first-order condition for an individual's portfolio to be optimal is that the expected excess return of each asset should be proportional to the covariance of that asset's returns with the agent's marginal utility, and it is not obvious why SMB or HML should be associated with any agent's marginal utility. Moreover, a cynic might argue that the success of the three-factor model only shows that the return of one set of mispriced assets explains the returns of other mispriced assets. Indeed, Daniel and Titman (1997) argue that, even if mispricing is responsible for the cross-sectional differences in characteristic-sorted returns, factor portfolios that are constructed on the basis of those characteristics are still likely to price other characteristic-sorted portfolios.

Largely because of such concerns, a set of more recent papers has attempted to identify the underlying economic risks that might be responsible for the observed return patterns. These papers present models that include macroeconomic rather than just financial market variables as risk proxies, and tend to be focused on what has become known as the *value effect* — the observation that high book-to-market stocks tend to experience relatively high returns.

These models can be divided into two categories: *Conditional (C)CAPM Models* and *Alternative Factor Models*. The conditional versions of the CAPM and Consumption-CAPM (CCAPM) of Breeden (1979) retain the basic structure of the CAPM or CCAPM, but allow for time-variation in the covariation of asset returns with the market return (or consumption growth, in the case of the CCAPM), and time variation in the return premium associated with the measure of systematic risk. These models can be written as unconditional multi-factor models where one factor is the market return, and the second factor is the market return interacted with a conditioning variable (e.g., Harvey, 1989; Cochrane, 2000). Similarly, a conditional-CCAPM model can be expressed as a multi-factor model with factors equal to consumption growth and consumption growth interacted with a conditioning variable.[1] *Alternative Factor Models* propose multi-factor models with a second factor. Intuitively, the proposed alternative factor is supposed to capture variation in the agents' marginal utility that is not captured by the market return or aggregate consumption growth.[2]

A subset of the papers that propose these tests is listed in Table 1. The tests in these papers generally fail to reject the proposed factor models, suggesting that there are a number of plausible economic factors that can explain the value effect.

Although at first glance these results appear promising, there are several reasons to question the findings. The first concern is that these results present a conundrum for anyone attempting to use the models. Which, if any, of these dozen or so models is the correct one to use in determining cost of capital for an individual firm? The results in these papers offer no answer to this question, as each of the proposed models appears to "work" reasonably well, in that the corresponding empirical test fails to reject the model.

To better understand this concern, note that the premise underlying each of the models presented in Table 1 is that by adding an additional economically motivated factor to either the market or to consumption growth, the model will capture marginal utility better than the standard CAPM or

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[1] However, as with tests of any model, a rejection of a conditional factor model with a specification of this kind is a joint rejection of the factor model and the instruments used to capture the time-variation in the conditional covariance. If the instruments do not fully capture the time-variation, the model may be incorrectly rejected.

[2] Note that the models that we discuss here abstract away from the imperfect market considerations, e.g., transaction costs and liquidity, and also away from behavioral factors.

Model	Factor(s)	Cond. Vars.
Conditional (C)CAPM Models		
1. Lettau and Ludvigson (2001)	VW or Cons Growth (Δc)	*cay*
2. Santos and Veronesi (2005)	VW + Labor Income Growth (Δy)	Labor Income to Cons Ratio (s)
3. Petkova and Zhang (2005)	VW Index	$E[R_m]$ based on BC Vars
Alternative-Factor Models		
4. Jagannathan and Wang (1996)	Labor Income Growth (Δy)	DEF
5. Heaton and Lucas (2000)	Proprietary Income Growth ($\Delta(\text{prop})$)	
6. Piazzesi *et al.* (2007)	Cons Growth $+\Delta$ NH Expenditure Ratio ($\Delta \log(\alpha)$)	Non-Housing Expenditure Ratio (α)
7. Lustig and Van Nieuwerburgh (2005)	Scaled Rental Price Change ($A\Delta \log \rho$)	Housing Collateral Ratio
8. Li *et al.* (2006)	Sector Inv. Growth Rates	
9. Parker and Julliard (2005)	Innovations in Future Long Horizon Consumption Growth	
10. Campbell and Vuolteenaho (2004)	CF and DR news	
11. Yogo (2006)*	Durable Consumption Growth	
12. Brennan *et al.* (2004)[†]	Real-interest rate & Max SR	
13. Petkova (2006)	Innovations in DIV, DEF, TERM, and T-Bill	

*In Yogo (2006), utility is non-separable in durable and nondurable consumption. Under his model, an approximate three-factor model obtains with changes in nondurable and changes in durable consumption as priced factors in addition to the return on the market wealth portfolio (see his equation (D5)). Table III presents the tests of the model on the 25 FF portfolios. See also his Table V, which compares this results to those of other models in this Table.

[†]Brennan *et al.* (2004) propose a model in which changes in the investment opportunity set are captured by the real interest rate and the maximum Sharpe ratio, both of which follow O-U processes. This results in an ICAPM-based model with three priced factors: the market return r_M; changes in the real interest rate r; and changes in the maximum Sharpe-Ratio η (see their equations (20) and (21)). With these three factors, they show that they can better price the 25 Fama-French portfolios, and other sets of characteristic-sorted portfolios. (In particular, see their Table II).

Table 1. Proposed factor models.

This table lists a subset of the factor models examined in the finance literature, and the factors and conditioning variables considered in these tests.

	r_m	HML	SMB	cây$\cdot r_m$	cây$\cdot \Delta c$	$-N_{DR}$	$-N_{CF}$	$s\cdot r_m$	Δy	Δ(prop)	$\Delta\log(\alpha)$
r_m	1	−0.38	0.51	−0.06	0.10	0.91	0.34	0.25	−0.24	0.19	0.02
HML	−0.38	1	−0.12	0.06	0.06	−0.51	0.27	−0.05	0.01	0.07	0.11
SMB	0.51	−0.12	1	−0.01	0.19	0.37	0.25	0.24	−0.04	0.11	0.14
cây$\cdot r_m$	−0.06	0.06	−0.01	1	0.12	−0.07	−0.11	0.03	−0.04	−0.15	−0.00
cây$\cdot \Delta c$	0.10	0.06	0.19	0.12	1	−0.08	0.07	0.07	0.13	0.11	−0.06
$-N_{DR}$	0.91	−0.51	0.37	−0.07	−0.08	1	0.02	0.21	−0.27	0.12	0.02
$-N_{CF}$	0.34	0.27	0.25	−0.11	0.07	0.02	1	0.07	−0.10	0.11	0.09
$s\cdot r_m$	0.25	−0.05	0.24	0.03	0.07	0.21	0.07	1	0.03	0.14	−0.07
Δy	−0.24	0.01	−0.04	−0.04	0.13	−0.27	−0.10	0.03	1	0.25	0.15
Δ(prop)	0.19	0.07	0.11	−0.15	0.11	0.12	0.11	0.14	0.25	1	0.28
$\Delta\log(\alpha)$	0.02	0.11	0.14	−0.00	−0.06	0.02	0.09	−0.07	0.15	0.28	1

Table 2. Correlations of candidate factors

This table presents the sample correlation matrix of a subset of the factors that have been proposed in the literature as explanations for the value premium (see Table 1). Each of the conditioning variables ($\widehat{cay}$ and s) is demeaned. The sample correlations are each estimated using quarterly data over the period 1963Q4:1998Q3.

CCAPM. Our concern is that these additional factors are not only distinct in terms of their economic motivations, but as shown in Table 2, the factors are not highly correlated with each other. Thus, given the low correlation between the alternative factors, it is clear that the different models have quite different pricing kernels.

The fact that the models generate different pricing kernels is not necessarily a problem. Since the systematic factor risks proposed in these models cannot necessarily be fully hedged (i.e., since markets are incomplete), two models may each price all assets even if the proposed additional factors are not perfectly correlated. However, a necessary condition for each of the models to be valid is that the projection of each factor into the asset return space be identical.[3] In other words, the portfolio of traded assets that is maximally correlated with any two proposed additional factors must be perfectly correlated if the models are both valid.

We don't directly test this implication for pairs of the proposed models. However, it seems unlikely that it would be satisfied. Table 2 shows that

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[3]   Following Hansen and Richard (1987), in incomplete markets multiple pricing kernels ($\tilde{m}$'s) may exist which price all assets, but there exists a unique projection of each of these pricing kernels onto the space of asset returns $\tilde{m}^*$. Following Hansen and Jagannathan (1991), this is equivalent to the statement that there is a unique mean-variance efficient portfolio that prices all excess returns.

the time series correlations of the proposed factors are low. Moreover, the proposed equity factors are not particularly exotic, meaning that portfolios of equities could be constructed that would comove with the proposed factors.[4]

We argue here that the failure of these tests to reject so many distinct factor models is not because the data are supportive of the models. Instead, we argue, the culprit is the test methodology: the models are tested on portfolios — typically the 25 size and book-to-market portfolios first examined in Fama and French (1993) — using a test methodology like that of Fama and MacBeth (1973) with the factor risk premia as free parameters.[5] As we will show, the puzzling support for this large set of very different models arises from the fact that the 25 Fama-French portfolios lie in an approximate two-dimensional return space, and have close to zero alphas with respect to the Fama and French (1993) three-factor model.[6] Within this very restricted return space, portfolios that are constructed to be maximally correlated with almost any factor that is not highly correlated with the market portfolio will be almost perfectly correlated with each other.

The motivation for the use of size and book-to-market (BM) characteristic sorted portfolios as test portfolios is intuitively appealing, since there is a strong empirical relationship between these characteristics and average returns. All else equal, using test portfolios with a large dispersion in average returns should, *a priori*, generate higher test power. In addition, the returns of size and BM sorted portfolios are likely to be sensitive to a variety of macroeconomic factors. For example, because growth firm values arise more from growth options, BM sorts will capture differential sensitivity to business cycle innovations. Hence, size and book-to-market sorted portfolios are likely to produce variation in expected returns as well as variation

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[4]   Yet another concern is raised by several studies that suggest problems with the proposed conditional (C)CAPM specifications. Lewellen and Nagel (2006) argue that the covariance of the conditional expected return on the market and of the conditional market betas of high and low book-to-market stocks is not high enough to explain the value effect. Also, Hodrick and Zhang (2001) find large specification errors for the Lettau and Ludvigson (2001) conditional CCAPM model. However, tests of these conditional CAPM models fail to reject the models, again suggesting the possibility that the failure to reject is a result of low test power.

[5]   Note that generalized method of moments Hansen (1982, GMM) tests of equivalent moment restrictions across the 25 portfolios (for example) would still be subject to the critique discussed in this paper.

[6]   See Table 6 of Fama and French (1993), which reports the results of a set of time-series regressions of the returns of the 25 portfolios on the three Fama–French factors. Reported time series regression R^2s range from 83% to 97%. Reported loadings on $[RM(t) - RF(t)]$ range from 0.91 to 1.18.

in the loadings on any number of macroeconomic factors. However, as we illustrate in our simulations, by grouping all of the assets with similar size and BM together, any variation in factor loadings that is independent of size or BM is largely eliminated. The end result is that, even if the loadings on a proposed factor are only loosely correlated with the expected returns of the individual assets in the economy, the sorting procedure will result in a set of test portfolios that exhibit a strong relationship between loadings on the proposed factor and expected returns. The problem is that, in grouping all of the assets with similar characteristics together, any variation in factor loading that is independent of these two characteristics is washed out.

Many sources of economic risk can be expected to lie outside the span of the returns of these test assets, even if these sources of risk could be hedged using other portfolios of stocks. Moreover, if the risk-premium associated with each factor is left as a free parameter, as is generally done in the Fama and MacBeth (1973) procedure, any factor that is loosely correlated with the m^* implied by HML and SMB will appear to properly price these test assets, even if this model would not properly price a fuller set of assets.[7]

This means that a powerful test requires that the test assets span a higher dimensional space. Specifically, the test assets should include portfolios with returns that are sensitive to realizations of the proposed factor, but that are not systematically related to the underlying characteristics that — according to the alternative hypothesis — explain returns. To construct such portfolios requires an instrument that is correlated with the loadings on the proposed factor, and that is imperfectly correlated with the characteristics.

We propose two sets of instruments: estimates of betas on the proposed factors based on historical time-series regressions, and industry affiliations. Industry portfolios exhibit variation in factor loadings relative to a number of macroeconomic factors, but this variation is largely unrelated to book-to-market ratios. Using these instruments to form test portfolios, we reexamine three models proposed in the literature. The empirical tests presented in these papers fail to reject the proposed models. We show that by utilizing an expanded asset space and a more powerful test methodology, all three models can be rejected.

The outline of the remainder of the paper is as follows. In Section 2 we present the evidence on the correlation of the candidate factors. Section 3

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[7]   This point is also made by Lewellen *et al.* (2010).

presents the test power results. We present the intuition, some basic analytical results, and a set of simulation results. We show that the standard test methodology yields low power against a simple characteristics alternative, and we propose a basic methodology with higher statistical power against this alternative. These results motivate the empirical tests we carry out in Section 4. Here we apply this new methodology to test several recent alternative factor models, and find that these models are rejected at high levels of significance. Section 5 presents our conclusions.

2   Correlations of Candidate Factors

Table 2 shows the correlations of eight of the alternative factors and scaled factors utilized in the models listed in Table 1. In addition, the first three rows/tables of the matrix are: r_m, the excess return on the CRSP value-weighted index; and the Fama and French (1993) SMB and HML factors. Among the factors and scaled factors, N_{DR} and N_{CF} are the discount-rate and cash-flow news factors from Campbell and Vuolteenaho (2004); Δc is the quarterly change in log nondurable and services consumption; Δy is the quarterly change in log income; $\Delta(\text{prop})$ is the log-change in proprietary income; $\Delta \log(\alpha)$ is equal to consumption growth plus the change in the non-housing expenditure ratio of Piazzesi *et al.* (2007). Among the instruments, $\widehat{cay}$ is from Lettau and Ludvigson (2001) and s is the labor income to consumption ratio of Santos and Veronesi (2005).

The correlations are all calculated on a quarterly basis. Each conditioning variable ($\widehat{cay}$ and s) is demeaned. The sample correlations are each estimated using quarterly data over the period 1963Q4:1998Q3.

Interestingly, the correlation matrix shows that, with the exception of N_{DR} and N_{CF}, the correlations of the factors with HML are generally low. In addition, the correlations between the proposed factors are also for the most part quite small. The maximum correlations between two factors is 28% (between the change in proprietary income and the log growth in the non-housing expenditure ratio). The correlations are mostly less than 20%.

3   The Power of Tests on Characteristic Sorted Portfolios

As discussed in the introduction, there are now more than a dozen factor models that claim to explain the returns of portfolios sorted on size and

book-to-market with economically motivated factor models. Yet, as noted earlier, it is unlikely to be the case that all of these factor models are "correct" in the sense that they all explain the cross-section of individual stock returns. Thus, our first task is to explain how so many different factors, with such a low average correlation, seem to explain the cross-section of returns.

As we discussed in the introduction, our explanation is that these tests are designed in such a way that they lack statistical power. Specifically, we argue that a wide range of two factor models will appear to explain the average returns of size and book-to-market sorted portfolios. As a result, these tests will fail to reject false factor models.

To illustrate this point, this section presents a simulation that demonstrates how the methodology employed in these tests can provide spurious support for a multi-factor model. This simulation is then used to motivate our approach for testing factor models against the characteristic alternative, an approach that we implement in our empirical tests in Section 4.

We note that the simulation here is for illustrative purposes only, and is too simplistic for calibration, *etc.* Even though most of the models and tests to which we apply our methodology are based on two factors and have a two-dimensional asset-return space, our simulation is based on a single factor and a one-dimensional asset return space. This simplification results in a clearer illustration of the problems inherent in this methodology.

To calculate test power, we must specify both a null hypothesis and an alternative hypothesis. The null hypothesis, consistent with the literature in this area, is a linear factor model specifying that a firm's expected return is linearly related to its beta with respect to a specific proposed factor. Under the alternative hypothesis, the return is a linear function of a single characteristic (here the log of the book-to-market ratio).

As we show below, our simulation illustrates that testing a one-factor model with a set of portfolios, based on sorting on a single characteristic, will have low power to reject the characteristic model, which is our alternative-hypothesis. The intuition behind this simulation naturally extends to multiple dimensions, such as the Fama–French size-BM portfolios and two factor models.

3.1 Simulation Results

The simulations presented here consider as a null hypothesis a single factor model that has been proposed to explain the observed cross-sectional

relationship between returns and a single characteristic. For example, innovations in housing price changes have been proposed as a factor that explains the book-to-market effect, which is known to be related to expected returns. To abstract from estimation problems, we assume that we accurately measure both a firm's factor-beta and its expected return. In addition, we assume that factor betas are correlated with the characteristic.

In our simulations, we randomly draw 2500 log book-to-market ratios and the single factor beta from a correlated normal distribution. Specifically, we draw from a multivariate normal distribution such that:

$$bm_i = \log(BM_i) \sim \mathcal{N}(0,1)$$
$$\beta_i \sim \mathcal{N}(0,1)$$
$$\rho(bm_i, \beta_i) = \rho_{bm,\beta}.$$

We assume a relatively weak correlation between the characteristic and the factor loading of $\rho = 0.3$, which is low enough to allow us to distinguish between the two hypotheses in an appropriately designed test. As we mentioned earlier, there are a number of good reasons why factors are likely to be correlated with book-to-market ratios. Theoretically, we know that a firm's book-to-market ratio is a good proxy for a firm's future growth (see, e.g., Fama and French, 1995; Cohen *et al.*, 2003) and high- and low-growth firms are likely to have different sensitivities to a number of economic factors. Empirically, Table 1 shows that this is indeed the case.

Figure 1 illustrates the distribution of characteristics and factor loadings that are generated from the simulation. This figure has two plots. Consider the left panel first. In this panel, the vertical axis is the firm's log book-to-market ratio, and the horizontal axis is the firm's factor beta. Each of the 2500 crosses in this figure represent a single firm or stock. The weak correlation can be seen in the distribution of the crosses: high β firms generally have high BM ratios, but there is considerable variation in βs that is unrelated to BM.

Figure 1 illustrates the null and alternative hypotheses we'll consider. The null hypothesis, that the factor model fully explains the cross-section of returns, is represented by the left plot. Under the null, a stock's expected return increases with β (as you move to the right in the plot), but is unrelated to book-to-market *after controlling for beta.* Of course, there is still an unconditional relationship between book-to-market and expected returns.
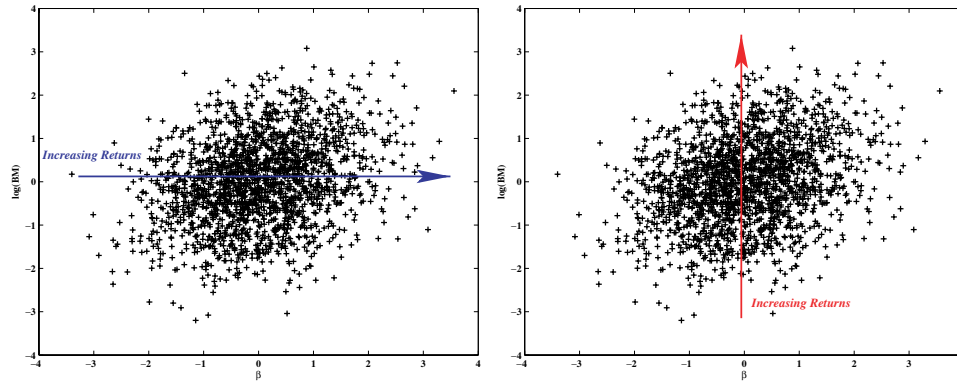
Figure 1. Null and alternative hypotheses.

A test's power is defined as the probability of the test's rejecting the null hypothesis given that the alternative is true. Thus, test power can only be evaluated relative to an alternative hypothesis. The alternative hypothesis we propose is illustrated in the right panel of Figure 1. Under the alternative, the expected return is linearly related to the log book-to-market ratio, but is not directly related to the factor beta. That is, the beta is related to returns only through its correlation with the characteristic.

To evaluate power of the tests, we calculate expected returns under the alternative hypothesis; that is:

$$E[r] = \lambda_0 + \lambda_1 \log (B/M).$$

Following, the test procedures used in the literature, we then sort the 2500 (simulated) firms into 10 portfolios depending on their book-to-market ratios. This sort is illustrated in the left panel of Figure 2. Each horizontal line in the figure represents the cutoff between the BM deciles: the number of firms between any two lines is 250 (one-tenth of the sample).

This figure illustrates the problem that arises when characteristic-sorted portfolios are used to test factor models. Notice that the top decile will have a high average BM ratio, and will therefore have a high expected return. In addition, it will have a high factor beta as a result of the correlation between beta and BM. As we move from the top to the bottom BM decile, we see that the average return and the average (portfolio) beta declines.

Note also that there is very little variation in betas in the different deciles, since differences in the betas that are not correlated with the characteristic
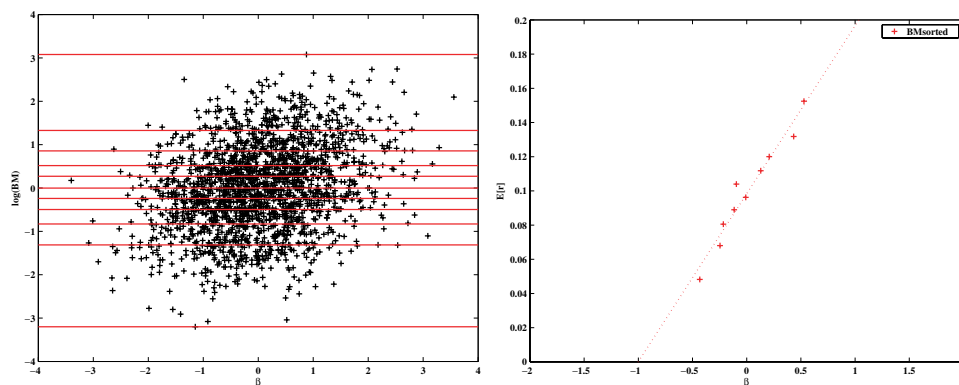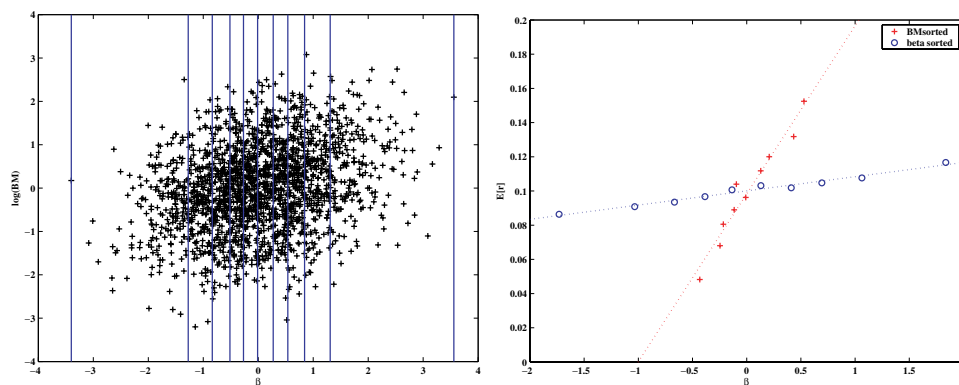
Figure 2. BM-Sorted portfolio formation.

Figure 3. Beta-sorted portfolio and cross-sectional test.

are "diversified away" by the portfolio formation procedure. As a result, as we show in the right panel of Figure 2, which plots the expected returns and betas for these 10 portfolios, these variables are very highly correlated. The regression R^2 here is 94.4%. The reason is that for this set of characteristic-sorted portfolios, there is almost no independent variation in beta.

One can alternatively sort stocks into portfolios based on risk rather than characteristics. Figure 3 illustrates this formation method. In the left panel of the figure, the vertical lines show the cutoffs between beta-sorted deciles. The right panel plots the expected returns and betas of these portfolios, and the regression line relating these two. The corresponding regression line for the BM-sorted portfolios is also shown.

This plot shows that sorting portfolios in this way results in a lower estimated factor risk premium, but still yields a strong estimated relationship between risk and return under the alternative, and again a good model fit, with a regression R^2 of 97.4%. Here, the problem is that high beta portfolios have, on average, high BM ratios and therefore high returns. By sorting in this way, there is almost no independent variation in BM across portfolios so that the betas and the characteristics are again almost perfectly correlated, making it impossible to discriminate between the two hypotheses.

In order to discriminate between the two hypotheses, one must construct test portfolios in a way that provides significant independent variation in betas and BM ratios. Figure 4 shows how this can be done with a multiple sort procedure.

The left panel of Figure 4 is similar to the left panel in Figure 2: the horizontal lines again show the bounds of the 10 BM-sorted portfolios. Now, however, we have additionally superimposed on the plot a set of vertical lines which indicate the result of splitting each of the BM decile portfolios into 10 sub-portfolios based on beta; each of the 100 BM/beta-sorted portfolios contains 25 firms. With these portfolios there is substantial independent variation in beta, and therefore it is possible to discriminate between the null and alternative hypotheses.

The results of the regression tests on the three sets of portfolios are summarized in Table 3. Notice that the R^2s are high in each of the three tests. The R^2 is clearly not a good indicator of model fit. Notice also that, for the test on either the BM-sorted or β-sorted portfolios, the estimated risk
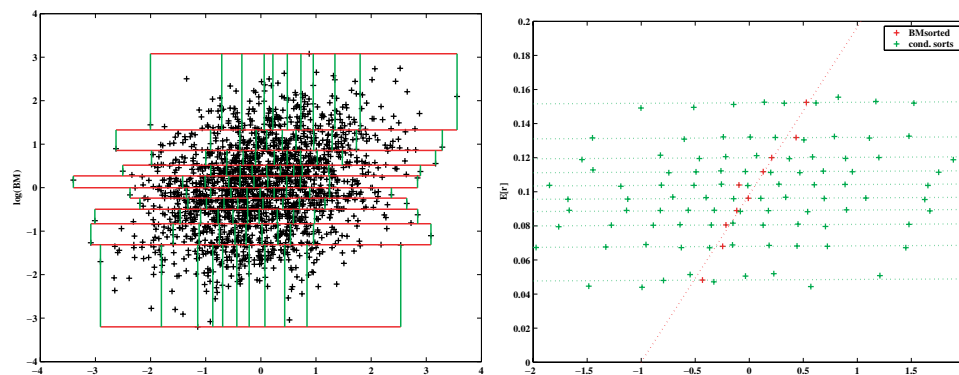

Figure 4. Characteristic/beta sorted portfolio test.

	$\hat{\lambda}_0$	$\hat{\lambda}_1$	$R^2(\%)$
BM Sort	0.1006	0.0946	94.4%
	(11.6)	(39.8)	
β Sort	0.1010	0.0097	97.4%
	(187.0)	(17.4)	
Multiple	0.0463	0.0005	99.8%
Sort*	(92.5)	(3.1)	

*The coefficients and *t*-statistics associated with the 9 indicator variables are not shown for this regression.

Table 3. Simulation test results summary.

The first two rows of this table present the results of OLS regressions of $E[r_i] = \lambda_0 + \lambda_1 \beta_i + u_i$ for the simulated data, sorted into 10 portfolios according to BM (first row), and factor β (second row). The third row of the table shows the results of the OLS regression test for the 100 portfolios sorted first into deciles based on BM, and then into sub-portfolios based on factor β. Here the regression is:

$$E[r_i] = \lambda_0 + \lambda_1 \beta_i + \gamma_2 I(2) + \cdots + \gamma_{10} I(10) + u_i$$

where $I(n)$ is an indicator variables that is 1 if the firm is in BM decile n, and zero otherwise.

The high R^2s in these regressions illustrate our point that, by forming portfolios based on either the characteristic or the estimated betas, we effectively diversify away sources of variation that are not related to the common factor.

premia are large and highly significant. Only when the test is done with the multiple-sort portfolios, and when dummy variables are included for the BM ranking, does the premium get close to the true value of zero. Interestingly, even in the third test, the estimated premium is still significant in the simulation. The reason is that, within each of the 100 portfolios, there is still some correlated variation in expected return and book-to-market.

4 Empirical Tests

In this Section we illustrate the issues raised in Section 3 by re-examining three different models and the corresponding tests that have appeared in the literature. In each case we modify the test by expanding the set of test assets, as discussed in Section 3.

First, in Section 4.1, we revisit two tests that provide support for the single factor CAPM in the pre-1963 period. Here, following the simulation example in Section 3.1, we use historical market betas as an instrument for expanding the asset space. With a powerful test based on the expanded asset space, we find that we can comfortably reject the CAPM in this pre-1963 sample period.

Next, in Section 4.2 we examine a test of a multifactor model in the post-1963 time period. Specifically, we re-examine the findings of Campbell and Vuolteenaho (2004), which argues that although a standard CAPM cannot explain the pattern of average returns in the post-1963 period, their model with distinct cash-flow and discount-rate betas can successfully explain the return patterns observed in this period.

Finally, in Section 4.3, we examine a conditional CCAPM of Lettau and Ludvigson (2001) by augmenting the size-BM sorted portfolios employed in the original test with industry portfolios. The industry portfolios exhibit considerable variation in factor loadings that are independent of the portfolio characteristics. We apply this approach to test and reject the conditional CCAPM model of Lettau and Ludvigson (2001).

4.1   The CAPM in the Pre-1963 Period

Campbell and Vuolteenaho (2004) and Ang and Chen (2007) argue that the standard CAPM is not rejected over the 1929–1963 period. In contrast with the post-1963 period in which the market betas of value stocks are relatively low, in the 1929–1963 period the betas of value stocks are large and consistent with the higher returns earned by value stocks over this period. In formal tests, both papers fail to reject the CAPM over the 1929–1963 time period.

We begin our empirical analysis by re-examining the data from this period, using portfolios that utilize our sort methodology. Table 4 shows the average returns and post-formation betas for our portfolios. Our portfolio formation here is virtually identical to that in Daniel and Titman (1997).[8] We form 45 portfolios using the following procedure: First, we sort all firms

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[8]   See Table VIII of Daniel and Titman (1997), which uses this set of test assets to test and reject the Fama and French (1993) three-factor model. Fama and French (2006) use a similar set of "beta spread" portfolios formed by sorting on Size, B/M, and an *ex-ante* measure of the market beta. Based on this set of test assets, they also conclude that the CAPM is rejected over this period (see their Table VII, and the discussion in their Section III).

into three portfolios according to their market capitalization (i.e., size) as of December of year *t*, on the basis of NYSE breakpoints. Additionally, we sort firms into three portfolios on the basis of the firm's book-to-market ratio. The book-to-market ratio is defined as the ratio of the firm's book value at the firm's fiscal year end in year *t*, divided by the firm's market capitalization as of December of year *t*.

We then sort each of the firms in these nine portfolios into five sub-portfolios based on estimated pre-formation market betas ($\hat{\beta}_{mkt}$). We estimate the pre-formation betas by running regressions of individual firm excess monthly returns on the excess monthly returns of the CRSP value weighted index for 60 months leading up to December of year *t*. Sub-portfolio breakpoints are set so that, across each size-BM portfolio, there are an equal number of firms in each sub-portfolio.[9]

We then construct the realized returns for each of these 45 test portfolios. Even though the portfolios are formed using data up through the end of year *t*, we examine the returns from these portfolios starting in July of year *t* + 1. The reason for this (following Fama and French, 1993) is that the book value data for the firm is unlikely to be publicly available as of January of year *t* + 1, but it is almost certain to be available as of July. All of our portfolio returns are value-weighted. The portfolios are then rebalanced at the start of July of year *t* + 2 using the new firm data from the end of year *t* + 1.

The upper panel of Table 4 gives the average returns and *t*-statistics for each of our 45 portfolios. The lower panel gives the estimated post-formation betas for the realized returns, and the *t*-statistics associated with these betas. The final row of each of the two tables gives the average return/beta and the associated *t*-statistics for the "average portfolio," which is an equal weighted portfolio of each of the nine sub-portfolios in the same pre-formation beta group.

The last two columns of each table give the average return/beta and the associated *t*-statistics for the 5 − 1 difference portfolio: that is a zero investment portfolio which buys one dollar of the high-estimated beta portfolio and shorts one dollar of the low-estimated beta portfolio. Finally, in the the lower right corner of each table, we report the mean return and time-series

9 However, note that the number of firms in the size-BM portfolios will vary because of the use of NYSE breakpoints in the size portfolio sort.

Chr Pt		$\bar{r}$ (%/mo)					$t(\bar{r})$					$\bar{r}$	$t(\bar{r})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5 − 1	
1	1	1.22	1.18	1.19	0.85	0.81	(3.48)	(2.54)	(2.57)	(1.47)	(1.39)	−0.40	(−0.96)
1	2	1.40	1.28	1.41	1.17	1.24	(4.40)	(3.42)	(3.38)	(2.32)	(2.41)	−0.16	(−0.51)
1	3	1.33	1.47	1.68	1.56	1.44	(3.91)	(3.36)	(3.26)	(3.19)	(2.36)	0.11	(0.30)
2	1	1.02	0.98	1.07	0.98	0.94	(4.40)	(3.45)	(3.03)	(2.56)	(2.18)	−0.08	(−0.28)
2	2	1.23	1.20	1.28	1.37	1.18	(5.09)	(4.03)	(3.71)	(3.56)	(2.57)	−0.05	(−0.18)
2	3	1.09	1.30	1.32	1.34	1.28	(3.53)	(3.21)	(3.11)	(2.75)	(2.39)	0.19	(0.54)
3	1	0.70	0.86	0.86	1.07	0.96	(3.46)	(3.75)	(3.16)	(3.50)	(2.77)	0.26	(1.19)
3	2	0.93	1.07	1.27	1.06	0.99	(4.37)	(4.02)	(3.99)	(3.14)	(2.47)	0.06	(0.20)
3	3	0.99	1.11	1.24	1.31	1.26	(3.14)	(2.94)	(2.87)	(2.92)	(2.48)	0.27	(0.88)
avg prt		1.10	1.16	1.26	1.19	1.12	(4.42)	(3.68)	(3.48)	(2.93)	(2.48)	**0.02**	**(0.09)**

Chr Prt		$\hat{\beta}_{Mkt}$					$t(\hat{\beta}_{Mkt})$					$\hat{\beta}_{Mkt}$	$t(\hat{\beta}_M)$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5 − 1	
1	1	1.03	1.40	1.30	1.71	1.76	(22.09)	(23.13)	(19.99)	(22.09)	(22.96)	0.73	(9.50)
1	2	1.06	1.24	1.40	1.66	1.70	(31.88)	(30.99)	(33.38)	(30.01)	(30.81)	0.64	(11.94)
1	3	1.07	1.40	1.66	1.61	1.88	(25.87)	(27.62)	(28.25)	(30.35)	(25.03)	0.82	(14.15)
2	1	0.78	0.99	1.25	1.36	1.53	(32.85)	(39.48)	(42.87)	(42.84)	(41.48)	0.75	(17.82)
2	2	0.82	1.05	1.24	1.40	1.63	(34.11)	(42.33)	(47.79)	(50.05)	(43.75)	0.81	(20.53)
2	3	0.99	1.39	1.47	1.63	1.80	(27.17)	(36.04)	(36.88)	(32.82)	(32.45)	0.81	(14.11)
3	1	0.70	0.82	0.99	1.13	1.29	(36.81)	(48.07)	(53.82)	(58.54)	(59.26)	0.58	(18.06)
3	2	0.70	0.89	1.11	1.21	1.40	(30.50)	(32.76)	(40.13)	(45.52)	(40.22)	0.70	(14.70)
3	3	1.04	1.27	1.44	1.56	1.71	(31.20)	(32.06)	(31.45)	(37.86)	(33.24)	0.66	(12.75)
avg prt		0.91	1.16	1.32	1.47	1.63	(53.20)	(56.75)	(53.61)	(50.75)	(47.68)	**0.72**	**(23.80)**

Table 4. BM/size/pre-β_{Mkt} sorted portfolios — $\bar{r}$s and post-Formation βs — 1933:07-1963:06.

For the 1933:07-1963:06 period, this table presents the average monthly returns (in %/month) and the post-formation market βs and the corresponding *t*-statistics for 45 long-only portfolios formed on the basis of independent sorts into 3 portfolios, each based on size and book-to-market ratio, followed by dependent sorts into 5 sub-portfolios based on pre-formation betas. Size and book-to-market sorts are based on NYSE cutoffs. The final row of the table, labeled "avg prt," gives the statistics for the equal-weighted portfolios of the 9 sub-portfolios listed directly above.

In addition, the final column of the table, labeled 5 − 1, gives the average return and β for the *zero-investment* portfolios formed by buying \$1 of the high β ("5") sub-portfolio, and selling \$1 of the low β ("1") sub-portfolio.

This second panel of the table shows that we can generate a significant spread in betas by forming portfolios based on the estimated betas of individual stocks in a pre-formation period. However the second panel shows that, controlling for size and book-to-market characteristics, the relationship between betas and returns is not reliably different than zero.

t-statistic for the equal-weighted portfolio of the nine 5 − 1 difference portfolios. We note that each of the *t*-statistics reported in this Table, and additionally in Tables 4–8, is based on the time-series standard error for the associated portfolio.

Chr Pt		α̂					t(α̂)				α̂	t(r̄)
SZ	BM	1	2	3	4	5	1	2	3	4	5	5 − 1
1	1	0.24	−0.15	−0.05	−0.78	−0.86	(1.01)	(−0.50)	(−0.17)	(−2.02)	(−2.26)	−1.10 (−2.88)
1	2	0.39	0.10	0.07	−0.41	−0.38	(2.34)	(0.50)	(0.33)	(−1.49)	(−1.39)	−0.77 (−2.87)
1	3	0.32	0.13	0.10	0.02	−0.36	(1.54)	(0.53)	(0.33)	(0.09)	(−0.96)	−0.67 (−2.34)
2	1	0.28	0.03	−0.13	−0.31	−0.51	(2.34)	(0.27)	(−0.87)	(−2.00)	(−2.80)	−0.79 (−3.79)
2	2	0.45	0.19	0.09	0.04	−0.38	(3.75)	(1.57)	(0.73)	(0.27)	(−2.04)	−0.83 (−4.20)
2	3	0.15	−0.03	−0.08	−0.22	−0.43	(0.85)	(−0.14)	(−0.39)	(−0.89)	(−1.57)	−0.59 (−2.05)
3	1	0.03	0.07	−0.09	−0.01	−0.26	(0.36)	(0.81)	(−0.96)	(−0.08)	(−2.43)	−0.30 (−1.84)
3	2	0.26	0.22	0.20	−0.09	−0.35	(2.27)	(1.61)	(1.48)	(−0.71)	(−2.03)	−0.61 (−2.57)
3	3	−0.01	−0.09	−0.13	−0.18	−0.37	(−0.05)	(−0.48)	(−0.58)	(−0.86)	(−1.46)	−0.36 (−1.40)
avg prt		0.23	0.05	−0.00	−0.22	−0.43	(2.75)	(0.52)	(−0.01)	(−1.49)	(−2.55)	−**0.67** **(−4.42)**

Table 5. Time series regression intercepts, 1933:07-1963:06.

The portfolios analyzed in this table are the same as those analyzed in Table 4. Here, for each of the long-only portfolios, we present the estimated intercepts and associated *t*-statistics from the regression:

$$(\tilde{r}_{i,t} - \tilde{r}_{m,t}) = \alpha + \beta_i(\tilde{r}_{m,t} - r_{f,t}) + \tilde{u}_{i,t},$$

while for the zero-investment portfolios in the final column, we estimate the regression:

$$\tilde{r}_{i,t} = \alpha + \beta_i(\tilde{r}_{m,t} - r_{f,t}) + \tilde{u}_{i,t}.$$

The returns and *t*-statistics at the bottom right side of the table shows that a long-short portfolio that is designed to be characteristic-neutral, but with a significantly positive beta, will have a significantly negative alpha in a standard CAPM regression. This indicates that the CAPM does not in fact hold in the pre-1963 period.

A couple of important features of the data are evident in this table. First, in the lower panel, in the two rightmost columns, note that our sort on pre-formation beta produces an economically large and highly significant spread in post-formation (realized) betas. This is important for the power of the test.

In contrast, the rightmost columns in the upper panel of Table 4 show that the sort on pre-formation beta produces little spread in average returns. The average return and *t*-statistic in the lower-right corner of this panel shows that the mean return differential between the high beta and the low beta portfolios is only 0.02% per month. The nine entries directly above this show that the differences in beta do not produce a statistically significant difference in return for any of the nine size/BM portfolios.

It is important to note that the observation of Ang and Chen (2007) and Campbell and Vuolteenaho (2004) that higher book-to-market is associated with higher beta in this early period is confirmed in our test: the lower panel of our table shows that, on average, higher book-to-market firms and smaller firms do indeed have higher betas. Consistent with the reasoning laid out here, this positive correlation implies that market beta "explains"

Chr Pt		$\hat{\beta}_{CF}$						$t(\hat{\beta}_{CF})$			$\hat{\beta}_{CF}$	$t(\hat{\beta}_{CF})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5 − 1
1	1	0.20	0.26	0.26	0.38	0.45	(4.98)	(4.97)	(4.91)	(5.95)	(7.03)	0.26	(5.44)
1	2	0.22	0.28	0.29	0.36	0.42	(6.15)	(6.82)	(6.34)	(6.42)	(7.57)	0.21	(5.94)
1	3	0.31	0.35	0.42	0.41	0.47	(8.49)	(7.39)	(7.54)	(7.75)	(7.07)	0.16	(4.05)
2	1	0.13	0.18	0.24	0.26	0.27	(4.94)	(5.56)	(6.01)	(6.15)	(5.49)	0.14	(4.27)
2	2	0.18	0.24	0.25	0.30	0.34	(6.81)	(7.47)	(6.70)	(7.11)	(6.67)	0.16	(4.84)
2	3	0.28	0.37	0.39	0.43	0.45	(8.45)	(8.53)	(8.68)	(8.24)	(7.75)	0.17	(4.27)
3	1	0.09	0.13	0.16	0.16	0.24	(3.73)	(4.97)	(5.34)	(4.75)	(6.16)	0.15	(6.24)
3	2	0.13	0.21	0.21	0.25	0.29	(5.34)	(7.26)	(5.99)	(6.70)	(6.45)	0.16	(4.70)
3	3	0.22	0.31	0.32	0.34	0.42	(6.28)	(7.67)	(6.75)	(6.96)	(7.66)	0.20	(5.88)
avg prt		0.19	0.26	0.28	0.32	0.37	(7.13)	(7.57)	(7.19)	(7.29)	(7.56)	**0.18**	**(6.77)**

Chr Pt		$\hat{\beta}_{DR}$						$t(\hat{\beta}_{DR})$			$\hat{\beta}_{DR}$	$t(\hat{\beta}_{DR})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5 − 1
1	1	0.84	0.99	1.02	1.13	1.27	(11.91)	(10.07)	(10.57)	(9.04)	(10.37)	0.43	(4.46)
1	2	0.77	0.91	1.02	1.11	1.19	(12.08)	(12.09)	(12.31)	(10.57)	(11.36)	0.42	(6.02)
1	3	0.82	1	1.08	1.15	1.24	(11.91)	(11.06)	(9.91)	(11.48)	(9.55)	0.42	(5.18)
2	1	0.63	0.72	0.91	0.96	1.02	(14.19)	(12.86)	(13.15)	(12.65)	(11.55)	0.39	(6.09)
2	2	0.62	0.75	0.89	0.97	1.13	(13.04)	(12.77)	(13.14)	(12.69)	(12.35)	0.51	(8.22)
2	3	0.68	0.86	0.93	1.04	1.17	(10.60)	(10.06)	(10.51)	(10.17)	(10.43)	0.48	(6.11)
3	1	0.54	0.61	0.69	0.82	0.90	(13.87)	(13.81)	(13.05)	(13.93)	(13.21)	0.36	(7.35)
3	2	0.49	0.52	0.75	0.79	0.91	(11.38)	(9.04)	(11.56)	(11.40)	(11.05)	0.42	(6.21)
3	3	0.65	0.86	0.91	1.01	1.08	(9.73)	(11.08)	(9.96)	(10.90)	(10.15)	0.43	(6.20)
avg prt		0.67	0.80	0.91	1.00	1.10	(14.12)	(12.87)	(12.78)	(12.29)	(12.11)	**0.43**	**(8.25)**

Table 6. Early period post-formation CF and DR betas.

This table presents the estimated cash-flow (CF) and discount-rate (DR) betas from time-series regressions of the realized excess returns and t-statistics of the 45 portfolios on the component of the market return attributable to cash-flow and discount-rate news, as calculated by Campbell and Vuolteenaho (2004), over the 1933:07-1963:06 period.

The portfolios used in this analysis are identical to those described in the caption to Table 4, and in the text.

The left side of each panel reports CF and DR betas, and the right sides of the panels present the t-statistics associated with these betas. The last row of the table gives the betas and t-statistics for the average portfolio, and the last two columns give the estimated betas and t-statistics for the 5 − 1 difference portfolio.

This table shows that we can generate a significant spread in both cash-flow and discount-rate betas by forming portfolios based on the estimated betas of individual stocks in a pre-formation period.

returns fairly well for size/B-M sorted portfolios. However, this positive correlation does not imply that the CAPM explains the returns of the full cross-section of common stocks in this period. When we expand the set of portfolios so as to capture variation in beta unrelated to variation in size or book-to-market, we see that differences in beta that are independent of differences in book-to-market are not associated with average return differences, at least at any statistically significant level.

Chr Pt			$\hat{\beta}_{CF}$					$t(\hat{\beta}_{CF})$			$\hat{\beta}_{CF}$	$t(\hat{\beta}_{CF})$
SZ BM	1	2	3	4	5	1	2	3	4	5		5 − 1
1 1	0.090	0.110	0.119	0.136	0.157	(2.62)	(3.04)	(3.04)	(3.08)	(3.25)	0.067	(2.54)
1 2	0.117	0.127	0.134	0.153	0.181	(4.55)	(4.46)	(4.16)	(4.20)	(4.22)	0.064	(2.63)
1 3	0.133	0.153	0.170	0.160	0.211	(4.89)	(5.25)	(5.24)	(4.47)	(5.12)	0.078	(3.63)
2 1	0.077	0.074	0.091	0.099	0.130	(2.82)	(2.39)	(2.72)	(2.74)	(2.86)	0.053	(1.93)
2 2	0.106	0.110	0.135	0.158	0.157	(4.71)	(4.14)	(4.94)	(5.19)	(4.23)	0.051	(2.10)
2 3	0.115	0.134	0.141	0.173	0.214	(4.83)	(4.99)	(4.71)	(5.41)	(5.49)	0.098	(3.54)
3 1	0.061	0.058	0.068	0.074	0.091	(2.49)	(2.27)	(2.42)	(2.41)	(2.50)	0.031	(1.27)
3 2	0.052	0.075	0.100	0.118	0.146	(2.28)	(3.04)	(3.74)	(4.15)	(4.39)	0.094	(3.70)
3 3	0.084	0.099	0.127	0.123	0.161	(3.50)	(4.00)	(4.48)	(3.95)	(4.67)	0.077	(2.90)
avg prt	0.093	0.104	0.121	0.133	0.161	(4.15)	(4.18)	(4.32)	(4.32)	(4.40)	0.068	(3.54)

Chr Pt			$\hat{\beta}_{DR}$					$t(\hat{\beta}_{DR})$			$\hat{\beta}_{DR}$	$t(\hat{\beta}_{DR})$
SZ BM	1	2	3	4	5	1	2	3	4	5		5 − 1
1 1	1.188	1.267	1.386	1.542	1.658	(17.05)	(17.46)	(17.77)	(17.32)	(16.82)	0.470	(7.28)
1 2	0.859	0.962	1.117	1.237	1.466	(15.90)	(16.07)	(16.92)	(16.37)	(16.55)	0.607	(10.79)
1 3	0.852	0.929	1.037	1.188	1.356	(14.22)	(14.56)	(14.61)	(15.71)	(15.32)	0.504	(9.93)
2 1	0.927	1.108	1.221	1.324	1.642	(16.71)	(18.46)	(18.78)	(18.85)	(18.47)	0.715	(11.33)
2 2	0.669	0.903	0.948	1.030	1.302	(13.35)	(16.44)	(16.63)	(15.91)	(17.34)	0.632	(11.49)
2 3	0.608	0.786	0.952	1.006	1.217	(10.80)	(12.89)	(14.65)	(14.28)	(14.11)	0.609	(9.11)
3 1	0.780	0.877	1.020	1.132	1.304	(15.21)	(17.21)	(18.72)	(19.45)	(18.25)	0.524	(9.16)
3 2	0.551	0.697	0.870	0.878	1.128	(10.41)	(12.69)	(15.27)	(14.07)	(16.35)	0.577	(9.52)
3 3	0.500	0.583	0.759	0.947	1.059	(8.58)	(9.97)	(11.56)	(13.81)	(13.93)	0.560	(8.71)
avg prt	0.770	0.901	1.034	1.143	1.348	(16.80)	(18.15)	(18.90)	(19.05)	(18.79)	0.577	(13.67)

Table 7. Post-formation CF and DR betas, 1963:07-2001:12.

This table presents the estimated cash-flow (CF) and discount-rate (DR) betas from time-series regressions of the realized excess returns and *t*-statistics of the 45 portfolios on the component of the market return attributable to cash-flow and discount-rate news, as calculated by Campbell and Vuolteenaho (2004), over the 1963:07-2001:12 period. The left parts of the two panels report CF and DR betas, and the right parts of each panel present the *t*-statistics associated with these betas.

The portfolios analyzed here are like those described in the caption of Table 4, except that instead of performing the final sort on *ex-ante* market beta, we sort on *ex-ante* cash-flow (CF) beta.

This table shows that we can generate a significant spread in the cash-flow beta by forming portfolios based on the estimated cash-flow betas of individual stocks in the pre-formation period.

We formally test the hypothesis that the CAPM explains the returns of these 45 portfolios by running time series regressions of the realized excess returns of the portfolios on the realized excess returns of the CRSP value-weighted portfolio returns over the 1933:07-1963:06 period. That is, the regressions are of the form:

$$(\tilde{r}_{i,t} - r_{f,t}) = \alpha_i + \beta_i(\tilde{r}_{m,t} - r_{f,t}) + \tilde{\epsilon}_{i,t}$$

Chr Pt		$\hat{r}$ (%/mo)					$t(\hat{r})$					$\hat{r}$	$t(\bar{r})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5 − 1	
1	1	0.53	0.57	0.77	0.53	0.24	(1.81)	(1.87)	(2.30)	(1.40)	(0.59)	−0.29	(−1.28)
1	2	0.71	0.85	0.88	0.76	0.95	(3.23)	(3.45)	(3.18)	(2.44)	(2.57)	0.23	(1.12)
1	3	1.04	0.95	1.07	1.02	1.11	(4.40)	(3.75)	(3.78)	(3.31)	(3.10)	0.07	(0.39)
2	1	0.66	0.64	0.55	0.42	0.49	(2.87)	(2.45)	(1.92)	(1.37)	(1.27)	−0.17	(−0.74)
2	2	0.63	0.62	0.80	0.66	0.93	(3.26)	(2.71)	(3.39)	(2.49)	(2.92)	0.29	(1.44)
2	3	0.67	0.90	1.02	0.95	1.02	(3.25)	(3.88)	(3.97)	(3.43)	(3.02)	0.35	(1.48)
3	1	0.57	0.55	0.54	0.48	0.52	(2.75)	(2.54)	(2.27)	(1.87)	(1.67)	−0.05	(−0.25)
3	2	0.51	0.61	0.48	0.57	0.77	(2.67)	(2.94)	(2.10)	(2.32)	(2.69)	0.25	(1.17)
3	3	0.67	0.57	0.73	0.75	0.79	(3.29)	(2.70)	(3.00)	(2.80)	(2.65)	0.11	(0.50)
avg prt		0.67	0.70	0.76	0.68	0.76	(6.26)	(5.73)	(5.38)	(4.60)	(4.10)	0.09	(0.54)

Table 8. Late-period average portfolio returns.

For the 1963:07-2001:12 period, this table presents the average monthly returns (in %/month) and the corresponding *t*-statistics for portfolios formed on the basis of independent sorts into 3 portfolios each based on size and book-to-market ratio, followed by dependent sorts into 5 sub-portfolios based on pre-formation cash-flow betas. Size and book-to-market sorts are based on NYSE cutoffs. The final column of the table, labeled 5 − 1, gives the average return for the zero-investment portfolio formed by buying $1 of the high β portfolio, and selling $1 of the low β sub-portfolio. The final row of the table, labeled "avg prt," gives the average return and *t*-statistic for the equal-weighted portfolio of the 9 sub-portfolios listed directly above.

This table illustrates that after controlling for characteristics, there is no reliable relationship between cash-flow betas and returns. This means that the characteristics model cannot be rejected in favor of the multi-factor model.

The left side of Table 5 reports the estimated regression intercepts, and the right side presents the *t*-statistics associated with these intercepts. The final row of the table gives the intercepts and *t*-statistics for the average portfolio, and the final two columns give the estimated intercepts and OLS *t*-statistics for the 5 − 1 difference portfolios. Note that, for the zero-investment difference portfolios, as well as for the unit investment portfolios, the reported alphas, betas, and OLS *t*-statistics reported here are from time series regressions.

Consistent with the average returns and estimated betas reported in Table 4, the estimated alphas are negative, economically large, and highly statistically significant. Indeed, for the average 5−1 difference portfolio (the lower-right entry in the table), the *t*-statistic is −4.42, strongly rejecting the single-factor CAPM in this time period.

4.2 A Multi-Factor Model in the Post-1963 Period

Campbell and Vuolteenaho (2004) propose a version of the Merton (1973) Intertemporal CAPM as an alternative to the static CAPM as a way of

explaining the size/book-to-market anomaly. They argue that the realized market return can be decomposed into the conditional expected return ($E_t[r_{t+1}]$) plus the component of the return attributable to news about the level of future cash-flows ($N_{CF,t+1}$), plus the return component attributable to the news about the discount-rates applied to these cash-flows by investors ($-N_{DR,t+1}$):

$$\tilde{r}_{t+1} = E_t[r_{t+1}] + \tilde{N}_{CF,t+1} - \tilde{N}_{DR,t+1}. \tag{1}$$

Based on this decomposition, Campbell and Vuolteenaho (2004) estimate βs with respect to the two return components. To understand the motivation of these empirical tests, it is helpful to start with the representative agent model of Campbell (1993), which expresses expected returns as,

$$E_t[r_{i,t+1}] - r_{r,t+1} + \frac{\sigma_{i,t}^2}{2} = \gamma\,\sigma_{p,t}^2\beta_{i,CF_p,t} + \sigma_{p,t}^2\beta_{i,DR_p,t}$$

where p is the portfolio the representative investor chooses to hold (i.e., the market), γ is the coefficient of relative risk aversion of the representative agent, and $\beta_{i,CF_p,t}$ and $\beta_{i,DR_p,t}$ are the components of portfolio p's return attributable to cash-flow and discount-rate news, respectively. It is clear from this equation that if $\gamma = 1$ (i.e., with a log-utility representative agent) the standard CAPM will obtain, as the covariance with changes in the investment opportunity set don't affect expected returns, consistent with Merton (1973). However, for a non-myopic representative investor, the premium associated with cash-flow and discount-rate betas will differ. Since we generally believe that investor preferences are characterized by $\gamma > 1$, the premium associated with cash-flow risk should be greater than that associated with discount-rate risk.

The insight underlying the Campbell and Vuolteenaho (2004) model is that, since growth stocks have higher duration cash-flows, their returns should be more sensitive to changes in discount-rates. Following Merton (1973), for reasonable preferences, the risk associated with sensitivity to discount-rate shocks is likely to command a far lower return premium than the risk associated with sensitivity to cash-flow shocks. Thus, even though growth stocks have a higher market beta, most of this covariation is due to sensitivity to discount rate news, β_{DR}, which has a low premium, so growth stocks (rationally) require a lower risk premium. In contrast, although value stocks have lower market-beta, most of their beta comes from β_{CF}, which commands a high premium. Hence value stocks (rationally) require a high risk-premium.

As noted above, Campbell and Vuolteenaho (2004) test their model separately in the pre-1963 and post 1963-periods. They find that over the 1929–1963 period, the standard CAPM prices the 25 Fama-French portfolios. Their interpretation of this observation is that value stocks have both a higher β_{DR} and a higher β_{CF} over this sample period, and as a result, there is a high correlation between the CAPM single β and its two components. However, over the 1963–2001 sample, the value stocks have a lower β_{DR}, but a higher β_{CF}. The measured CAPM β, which is the sum of β_{DR} and β_{CF}, is lower for value stocks, but the average returns are higher because of the much higher premium attached to cash-flow risk. This finding leads them to reject the standard CAPM. However, based on their empirical tests they conclude that a model that allows for distinct premia for β_{DR} and β_{CF} is not rejected.

To empirically decompose beta into its cash-flow and discount-rate components, Campbell and Vuolteenaho (2004) first estimate discount-rate shocks. To do this they utilize a number of instruments that forecast future market returns in a vector-autoregression (VAR) framework. One of these variables is their *value spread* — the difference between the average book-to-market ratio of the Fama and French (1993) small-value and small-growth portfolios:

$$VS_t = \log\left(\frac{B_t}{M_t}\right)_{SmVal} - \log\left(\frac{B_t}{M_t}\right)_{SmGro}. \tag{2}$$

Moreover, the one-period change in the value spread will be closely approximated by the difference in the return on the small-growth and small-value portfolios because book values are relatively stable over a single period.

The reason for including the value spread is that it does a good job of forecasting the future market return. However, given the inclusion of this variable in the VAR and the methodology employed, their inferred N_{CF} is strongly positively correlated with the returns to HML. Specifically, given Campbell and Vuolteenaho (2004)'s estimated parameters the inferred innovation in cash-flows is:

$$N_{CF,t} = 0.004 + 0.60\,R_{m,t} + 0.40\,R_{m,t-1}$$
$$+ 0.01\,\Delta PE_t - 0.88\,\Delta TY_t - 0.28\,\Delta VS_t \tag{3}$$

where Δ is the one period change in the variable. As noted just above, the last term (ΔVS_t) is mechanically close to the HML return.

That is, the measured innovations in cash-flow are mechanically linked to the difference between the log-returns of small-value and small-growth firms, and consequently are also strongly correlated with HML. This is important in understanding why β_{CF} "picks up" the value premium.

4.2.1 Pre-1963 Empirical Analysis

Campbell and Vuolteenaho (2004) show that over both their full 1929–2001 period and over two sub-samples (1929–1963 and 1963–2001), their model can explain the returns of the 25 size and value-sorted portfolios. In particular, they show their model is not rejected in the post-1963 period where the CAPM fails. Over this period, value stocks have a lower β_{DR}, but a higher β_{CF}. The measured CAPM β, which is the sum of β_{DR} and β_{CF}, is lower for value stocks, but the average returns are higher because of the much higher premium attached to cash-flow risk. This result leads to a rejection of the standard CAPM. However, they argue, a model that separates out the betas with discount-rate and cash-flow shocks is not rejected for the 25 FF size and book-to-market sorted portfolios.

In Subsection 4.1, we showed that with our 45 portfolios sorted on size, book-to-market, and *ex-ante* market-beta, we were able to reject the CAPM over this sample period. We begin our empirical tests here by examining the ability of the Campbell and Vuolteenaho (2004) model to explain the cross-section of returns in the pre-1963 period, using the same set of portfolios.

The results from these tests of the Campbell and Vuolteenaho (2004) model over this period are reported in Table 6.[10] It is the numbers in the upper panel that are most relevant here. First, notice that there is indeed a large correlation between book-to-market and the cash-flow beta over this period: high BM firms do, on average, have higher cash-flow betas. However, sorting on pre-formation market beta also produces a large spread in cash-flow betas (note the *t*-statistics in the last column of the table), and as we have already seen in Table 4, produces no statistically significant spread in returns. Because we find no statistically significant differences in expected return associated with variation in cash-flow beta, on the basis of this table we fail to reject the characteristics hypothesis that returns are unrelated to cash-flow beta, after controlling for the size and book-to-market characteristics.

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[10] The data on the innovations N_{CF} and N_{DR} used in the empirical tests in this section are courtesy of Tuomo Vuolteenaho.

Of course, the question remains as to whether we can reject the factor model, in favor of the characteristics model, with these portfolios and this test methodology.

It should be noted that the CV cash-flow and discount-rate factors are not associated with traded portfolios. As a result, we cannot test the CV model using the time series regression approach we used to test the CAPM in the pre-1963 period. Instead, we perform cross-sectional regressions of the returns of the 45 Size/BM/ex-ante β_{mkt} sorted portfolios on the CF and DR factor loadings. Specifically, we run two cross-sectional regressions:

$$\bar{r}_i = \lambda_0 + \lambda_{CF}\hat{\beta}_{CF,i} + \lambda_{DR}\hat{\beta}_{DR,i} + u_i \tag{4}$$

$$\bar{r}_i = \lambda_{1,1}I_{1,1} + \lambda_{1,2}I_{1,2} + \cdots + \lambda_{3,3}I_{3,3}$$
$$+ \lambda_{CF}\hat{\beta}_{CF,i} + \lambda_{DR}\hat{\beta}_{DR,i} + u_i \tag{5}$$

where $I_{j,k}$ is equal to 1 if the firm is in the j, k size/BM portfolio, and is zero otherwise.

The result of estimating Equation (4) for the 45 portfolios is

$$\bar{r}_i = \underset{(3.90)}{0.93} + \underset{(1.86)}{1.93} \ \hat{\beta}_{CF,i} \underset{(-0.75)}{-0.38} \ \hat{\beta}_{DR,i} + u_i$$

where *t*-statistics are given below the estimated coefficients. Here, there is an economically strong and marginally significant premium associated with cash-flow beta, consistent with the results of Campbell and Vuolteenaho (2004).

However, we must also ask whether, in our sample of 45 portfolios with independent variation in characteristics and factor loadings, the cash-flow factor loading is priced after controlling for the characteristics. We do this by estimating the cross-sectional regression in Equation (5), which nests the factor model and the characteristics model. The results of this OLS estimation are:

$$\bar{r}_i = \underset{(2.42)}{0.75} \ I_{1,1} + \underset{(3.73)}{1.03} \ I_{1,2} + \underset{(4.68)}{1.34} \ I_{1,3} +$$
$$\underset{(2.66)}{0.71} \ I_{2,1} + \underset{(4.03)}{1.03} \ I_{2,2} + \underset{(4.69)}{1.19} \ I_{2,3} +$$
$$\underset{(2.31)}{0.60} \ I_{3,1} + \underset{(4.16)}{0.89} \ I_{3,2} + \underset{(4.14)}{1.03} \ I_{3,3} +$$
$$\underset{(-1.03)}{-1.49} \ \hat{\beta}_{CF,i} + \underset{(1.12)}{0.70} \ \hat{\beta}_{DR,i} \quad + u_i$$

Note that the estimated premia associated with the characteristics decrease slightly with size (moving vertically), and increase strongly and monotonically in book-to-market (moving horizontally), consistent with the characteristics model. Interestingly, the estimated premium on cash-flow beta switches sign compared to the previous regression and compared to the CV findings. Moreover, the F-statistic that tests whether the nine characteristic premia $\lambda_{j,k}$ are all zero is sufficiently large to reject this hypothesis at the 0.02% level. In contrast, the *p*-value associated with the F-statistic testing whether the premia on cash-flow and discount-rate risk are zero (i.e., that $\lambda_{DR} = \lambda_{CF} = 0$) is 0.207. Thus, based on this test, we can reject the factor model hypothesis at the 0.0002 level, and cannot reject the characteristics hypothesis at conventional levels of significance.

4.2.2 Post-1963 Empirical Results

Following Campbell and Vuolteenaho (2004), who separately analyze their multifactor model in the pre- and post-1963 periods, we now examine the model in the post-1963 period. Our methodology in the pre- and post-1963 periods is similar, with one exception. Recall that in the pre-1963 period we sort our portfolios on the basis of size, book-to-price, and an *ex-ante* estimate of market beta. In contrast to the pre-1963 period sorting on pre-formation market beta does not produce a sufficient spread in cash-flow betas. Given this limitation, we instead sort directly on pre-formation cash-flow betas.

To estimate individual firm cash-flow betas, we regress each firm's excess returns on the CV measure of $N_{CF,t}$ in the 60 months leading up to December of year t (consistent with our method as described in Section 4.1).[11] Other details of the portfolio formation procedure are as described in Section 4.1.

Table 7 presents the post-1963 post-formation CF and DR betas and the associated *t*-statistics. Consistent with the Campbell and Vuolteenaho (2004) results, we find that a higher BM ratio is generally associated with a higher $\hat{\beta}_{CF}$, but with a lower $\hat{beta}_{DR}$. Also, there is variation in both $\hat{beta}_{DR}$ and $\hat{\beta}_{DR}$ (in the same direction) associated with average firm size.

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[11] The measure of N_{CF} is given in Equation (3).

In Table 7 the spread is smaller than what is achieved in the early-period sort on CAPM beta, but it is still statistically significant. Moreover, the CF beta spread in the $5 - 1$ portfolio is about as large as the spread in CF beta achieved via unconditional sorts on book-to-market: the lower-right corner of the upper panel shows a spread of 0.068, with a *t*-statistic of 3.54.

However, while the sort produces a statistically significant difference in post-formation cash-flow betas, it produces no statistically significant difference in average returns. Table 8 reports the average returns of the 45 late-period portfolios. The mean return of the average difference portfolio is 0.09, with a *t*-statistic of 0.054.

As in Section 4.2.1, we again run a set of cross-sectional regressions to determine whether the factor model can be rejected in favor of the characteristics model. The result of estimating Equation (4) for the 45 Size/BM/β_{CF} sorted portfolios over the 1963:07–2001:12 period is:

$$\bar{r}_i = \underset{(0.40)}{0.10} + \underset{(4.06)}{5.37} \ \hat{\beta}_{CF,i} \underset{(0.27)}{-0.08} \ \hat{\beta}_{DR,i} + u_i$$

Here, the premium associated with cash-flow beta is stronger than in the pre-1963 period, and strongly statistically significant as well. So, without controlling for characteristics, it appears that cash-flow beta is strongly priced.

The story changes when we ask whether the factors are priced *after controlling for characteristics*. The regression specification that nests the characteristic and factor model (Equation 5) has estimates for these 45 portfolios over this later time period of:

$$\begin{aligned}
\bar{r}_i = \ & \underset{(1.05)}{0.24} \ I_{1,1} + \underset{(2.63)}{0.59} \ I_{1,2} + \underset{(3.26)}{0.77} \ I_{1,3} + \\
& \underset{(1.24)}{0.27} \ I_{2,1} + \underset{(2.17)}{0.48} \ I_{2,2} + \underset{(2.86)}{0.67} \ I_{2,3} + \\
& \underset{(1.17)}{0.25} \ I_{3,1} + \underset{(1.52)}{0.35} \ I_{3,2} + \underset{(2.11)}{0.48} \ I_{3,3} + \\
& \underset{(0.82)}{1.15} \ \hat{\beta}_{CF,i} + \underset{(0.47)}{0.13} \ \hat{\beta}_{DR,i} + u_i
\end{aligned}$$

Again we see that the estimated premia associated with the characteristics decrease with the size characteristic, and increase monotonically with book-to-market. The estimated cash-flow premium is positive here, but is

not significantly different from zero. Moreover, the *p*-value associated with the F-statistics testing whether $\lambda_{DR} = \lambda_{CF} = 0$ is 0.088. So, in contrast to what we see in the early period, the hypothesis that the premia associated with these two factors is (jointly) zero can be rejected at a 10% level of significance. This evidence suggests that the characteristics model does not fully capture the variation in average returns across the 45 portfolios over this later period.

To test the factor model, we examine the F-statistic to test whether the nine characteristic premia $\lambda_{j,k}$ are jointly zero. This F-statistic rejects this hypothesis at the 0.08% level, suggesting that, even in this later period, the CF and DR covariances alone cannot explain the pattern of average returns across the portfolios.

4.3   A Conditional CCAPM Model

4.3.1 Conditional CAPM and CCAPM Models

A number of (C)CAPM tests have now been proposed in the literature. One that has recently received a good deal of attention is that of Lettau and Ludvigson (2001, henceforth LL). LL argue that, based on the intuition behind the Campbell and Cochrane (1999) model, their *cay* variable should be a good proxy for the market risk premium.

In this paper, LL show that the betas of value/growth stocks are higher/lower when the expected return on the market is high (i.e., when *cây* is low). Even if value stocks have a lower unconditional beta than do growth stocks, their betas are much higher when the expected return on the market is higher. Thus, if one were to test the unconditional CAPM on value/growth stocks, one could reject it. However, LL argue that once the conditional variation in beta and the expected return on the market are taken into account, the *conditional* CAPM and CCAPM do a good job of explaining these returns.

The methodology used by LL to test the conditional CAPM is similar to that used in many recent papers: they test whether the returns of the Fama and French (1993) 25 size/BM sorted portfolios can be explained by their conditional CAPM, using Fama and MacBeth (1973) tests, and with the intercept and market premium as free parameters.

However, the LL conditional CAPM model can also be tested with a Gibbons *et al.* (1989, GRS) like time-series regression (as employed by Fama and French, 1993). For example, one can test whether the return of the

Fama and French (1993) HML portfolio can be explained using the single time-series regression:

$$\text{HML}_t = \alpha + \beta_{vw}(r_{m,t} - r_{f,t}) + \beta_{vwz}\,\widehat{cay}_{t-1}(r_{m,t} - r_{f,t}) + e_t \qquad (6)$$

For a conditional CAPM model such as this one, the interaction term $\widehat{cay}_{t-1}(r_{m,t} - r_{f,t})$ captures the extra return arising from the covariation of the HML beta with the expected return on the market, as discussed in the preceding subsection.

Using quarterly data over the period 1953:01–1998:04, the same period examined by LL, the estimated intercept ($\hat{\alpha}$) for this regression is 1.26%/quarter, ($t = 3.47$), which is both economically and statistically significant. For comparison, without including the $\widehat{cay}_{t-1}(r_{m,t} - r_{f,t})$ interaction term, the α is 1.50% ($t = 4.16$). The difference between the intercept terms in the two regressions is 0.24%/quarter, which is not statistically different from zero. This simple regression dramatically illustrates the point argued by Lewellen and Nagel (2006): although the betas of value stocks do increase in economic downturns, they don't increase anywhere near enough to explain the high returns of the HML portfolio.[12]

This straightforward rejection of the LL model raises the question of why the test reported in Lettau and Ludvigson (2001) appears to offer such strong support for their model. The answer, we again argue, is that their tests fail to reveal evidence inconsistent with the model because their test methodology, which is used throughout this literature, has extremely low power.

As we mentioned in the introduction, researchers test their models with portfolios rather than individual stocks to avoid an errors-in-variables problem and to lower the dimensionality of the covariance matrix of returns. The advantage of using characteristic-sorted portfolios is that their returns exhibit a large spread in both factor loadings and realized returns. Of course, if a proposed factor model is correct, portfolios that generate a large spread in factor loadings should generate a spread in realized returns.

In this section, we re-examine the Conditional Consumption-CAPM (CCAPM) test of Lettau and Ludvigson (2001). First, we reproduce their Fama MacBeth tests (in their Table 3), but use industry-sorted portfolio returns as our test assets rather than the characteristic-sorted portfolio

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[12] A simple test like this cannot be used to test the CCAPM, because the conditional MVE portfolio (which here will be the portfolio maximally correlated with consumption (Breeden, 1979) is not observable.

returns that they use in their paper.[13] The reason for augmenting the test assets with industry portfolios is that, as we demonstrate below, industry-sorted portfolios exhibit a relatively large spread in the loadings on the Lettau and Ludvigson CCAPM factor $\widehat{cay}_t \Delta c_{t+1}$. Recall that what we want, in terms of augmenting the factor space, is a set of assets with disperse loadings on the proposed new factors — here, $\widehat{cay}_t \Delta c_{t+1}$ — which is unrelated to book-to-price. The industry portfolios appear to meet this criterion.

Table 9, which presents the results of these sets of Fama–MacBeth regressions, indicates that the premia on $\widehat{cay}_t \Delta c_{t+1}$ is not significantly different

Ports	Const	$\widehat{cay}_t$	Δc_{t+1}	$\widehat{cay}_t \cdot \Delta c_{t+1}$	R^2
25 FF SZ/BM	4.28	−0.12	0.02	0.0057	0.70
	(11.36)	(−0.66)	(0.23)	(3.10)	
48 FF Indust.	2.94	0.27	−0.10	0.0002	0.30
	(14.49)	(2.99)	(−2.26)	(0.24)	
38 FF Indust.	3.13	0.18	−0.07	0.0003	0.09
	(8.40)	(0.93)	(−0.92)	(0.17)	
11 FF Indust.	2.91	−0.02	0.03	−0.0033	0.51
	(7.08)	(−0.09)	(0.29)	(−1.84)	

Table 9. Tests of the Lettau and Ludvigson (2001) consumption CAPM using size/BM sorted and industry portfolios.

This table presents estimates of the coefficients, *t*-statistics (in parentheses) and R^2s from a second-stage Fama and MacBeth (1973) regression. The data is monthly, and covers the sample period 1963:10–1998:09. The factors $\widehat{cay}_t$, Δc_{t+1}, and $\widehat{cay}_t \cdot \Delta c_{t+1}$ are those used in the corresponding regressions in Lettau and Ludvigson (2001). The test assets are (1) the 25 size-book-to-market sorted portfolios of Fama and French (1993), and (2) the industry portfolios from Fama and French (1997). All of the portfolio return data was obtained from Kenneth French's Web site.

This table illustrates that support for the Lettau and Ludvigson (2001) factor model, which can be seen in the regressions on the Fama and French 25 test portfolios, does not exist when the model is tested on industry portfolios.

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[13] The returns to the sets of industry portfolios are based on the portfolios documented in Fama and French (1997), and are taken from Ken French's web page. Details on the SIC codes associated with each of the industry breakdowns is available there. The factor data — $\widehat{cay}_t$, Δc_{t+1}, and $\widehat{cay}_t \cdot \Delta c_{t+1}$, are courtesy of Martin Lettau, and are the returns examined in Lettau and Ludvigson (2001).

from zero for the industry portfolios. Moreover, although the R^2 is very high for the size/BM sorted portfolios, it is low for the FM regressions using the 38 and 48 industry portfolios. For the regression with the 11 FF industries, the R^2 is 51%, but the sign of the premium on scaled consumption growth is reversed.

In addition, we reproduce Lettau and Ludvigson's Figure 1, panel (d), here for the Fama and French 38 industry-sorted portfolios. The left panel of Figure 5 is done per the LL methodology, and using the 25 Fama and French (1993) test portfolios. Consistent with LL, we find that the conditional consumption CAPM does a good job pricing this set of test assets.

However, the right panel of the figure plots the fitted and realized returns for the 38 Fama and French (1997) industry portfolios. However, in constructing the fitted returns, we use the risk premia as estimated from the Fama-MacBeth regressions with the 25 size/BM-sorted industry portfolios.
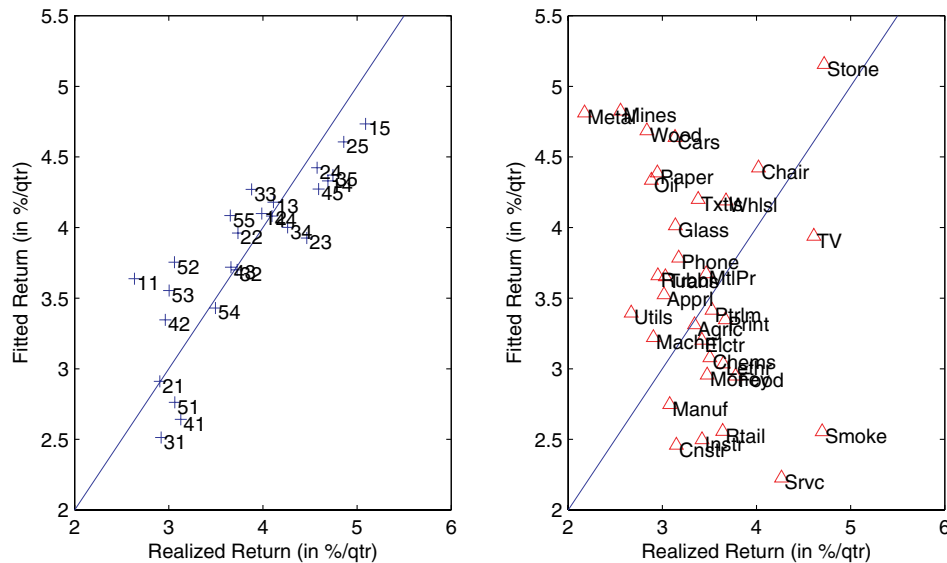
Figure 5. Lettau and Ludvigson (2001) Conditional CCAPM Model — Realized and Fitted Returns with Alternative Test Assets.

This figure presents the fitted and realized returns for (1) the 25 size/book-to-market sorted portfolios of Fama and French (1993) in the left panel, and (2) the industry portfolios from Fama and French (1997) in the right panel. As in Lettau and Ludvigson (2001), the fitted returns are generated from Fama and MacBeth (1973) estimates.

Like the 25 size/BM-sorted portfolios, the industry-sorted portfolios exhibit considerable variation in their loadings on the factors, and consequently very different fitted returns. However, the premia as estimated from the original test assets are not consistent with the pricing of the industry portfolios.

This empirical analysis suggests that the covariation with the proposed factors outside of the original 25-FF portfolio return space is not priced in a manner consistent with the estimates for the original test assets.

5   Conclusions

There is a large recent literature that has proposed a set of conditional versions of the CAPM, conditional versions of the consumption CAPM, and alternative factor models. These models are then tested with size- and book-to-market sorted portfolios. These tests tend to support the proposed models, in the sense that the tests fail to reject the null hypothesis that the return data are consistent with the model. These findings, however, are somewhat disconcerting since, as we show in Section 2, the proposed factors have very low pair wise correlations and generate very different estimates of expected returns of individual assets.

As we argue, the tests do not fail to reject the models because the models are all correct — given the low correlations of the proposed factors they cannot all be correct — but rather because the tests have very little power to reject. More specifically, using logic similar to the arguments in Daniel and Titman (1997), we argue that for the tests to have power, test assets are required that have loadings on the proposed factors that are not highly correlated with the test asset characteristics (size and book-to-market ratios).

To illustrate this point, we examine two specific models: the model of Campbell and Vuolteenaho (2004) and the model of Lettau and Ludvigson (2001). As we show, the empirical tests presented in these papers look very different when the test portfolios are formed either on the basis of predicted factor loadings or industry affiliation.

We tend to agree with Lewellen, Nagel, and Shanken (2010), who argue that these models should ultimately be tested on individual stock returns rather than portfolios. Of course, existing tests examine portfolios rather than individual stocks because of the numerous challenges associated with examining a sample of several thousand stocks over a sample period of

several hundred months. In our future research, we hope to address this challenge.

Appendix A.   Conditional Models and Conditional Tests

This appendix reviews results on conditional and unconditional models and tests of models.

A.1   Conditional and Unconditional Factor Models

In the absence of arbitrage, all assets are priced by a pricing kernel $\tilde{m}$ such that:

$$E_t[\tilde{m}_{t+1}\tilde{R}_{t+1}] = 1.$$

An *unconditional k*-factor model specifies that the pricing kernel is a linear function of a set of factors

$$\tilde{m}_{t+1} = a + \mathbf{b}\,\tilde{\mathbf{f}}_{t+1} \tag{7}$$

where a and $\mathbf{b}$ are time-invariant. In contrast, a *conditional k*-factor model specifies that:

$$\tilde{m}_{t+1} = a_t + \mathbf{b}'_t\tilde{\mathbf{f}}_{t+1}$$

Here, in contrast to the specification in Equation (7), a_t and $\mathbf{b}_t$ are not time invariant, but are adapted to the time t information set.

To test a conditional factor model, we generally specify that a_t and $\mathbf{b}_t$ are linear functions of a $(m \times 1)$ vector of instruments $\mathbf{Z}_t \in \mathcal{F}_t$:

$$a_t = \mathbf{a}'\,\mathbf{Z}_t$$
$$\mathbf{b}_t = \mathbf{b}\mathbf{Z}_t$$

where $\mathbf{a}$ is $(m \times 1)$ and $\mathbf{b}$ is $(k \times m)$. This gives:

$$\tilde{m}_{t+1} = \mathbf{a}'\mathbf{Z}_t + (\mathbf{b}\mathbf{Z}_t)'\,\tilde{\mathbf{f}}_{t+1}$$

A.1.1 Interpreting Conditional Factor Models

As noted by Cochrane (2000), a conditional *k*-factor model with m conditioning variables is equivalent to an unconditional factor model with $(k \cdot m)$ factors.

For example, the unconditional CAPM specifies that:

$$\tilde{m}_{t+1} = a + b\tilde{r}_{m,t+1},$$

where a and b are time invariant. The Lettau and Ludvigson (2001) conditional CAPM specifies that

$$\tilde{m}_{t+1} = (\gamma_0 + \gamma_1 z_t) + (\eta_0 + \eta_1 z_t)\tilde{r}_{m,t+1} \tag{8}$$

where, in their quarterly tests, the instrument z_t is their *cay* variable measured at the start of the quarter. Notice that this model has the implication that, for a $(N \times 1)$ vector of asset returns from t to $t + 1$, and given an observable risk-free rate:

$$(\tilde{\mathbf{r}}_{t+1} - \mathbf{1}r_{f,t+1}) = \text{fi}_m(\tilde{r}_{m,t+1} - r_{f,t+1}) + \text{fi}_{mz}(\tilde{r}_{m,t+1} - r_{f,t+1})z_t + \text{ffi}_t \tag{9}$$

where $\tilde{\mathbf{r}}, \mathbf{1}, \text{fi}_m, \text{fi}_{mz}$, and ffi_t and $(N \times 1)$ vectors, and $r_{f,t+1}$ is the return on an efficient portfolio uncorrelated with the market portfolio return — it is the risk-free rate if it exists, or the (stochastic) return on a minimum-variance zero-beta portfolio.

Either Equation (8) or Equation (9) shows that this conditional CAPM is equivalent to a two factors model with factors equal to:

1. The excess market return, defined as the profit that results from investing $1 in the market portfolio and shorting $1 of the risk-free (or zero-beta) asset.
2. The *scaled excess-market return*, defined as the profit that results from investing z_t in the market portfolio and shorting z_t of the risk-free (or zero-beta) asset.

A.2   Conditional Tests of Factor Models

Any test of a factor model will be a test of the set of moment restrictions:

$$E_t[\tilde{m}_{t+1}\tilde{\mathbf{R}}_{t+1}] = \mathbf{1}. \tag{10}$$

An *unconditional test* examines the moment restriction that results from taking an unconditional expectation of Equation (7):

$$E[\tilde{m}_{t+1}\tilde{\mathbf{R}}_{t+1}] = \mathbf{1}.$$

A *conditional test* examines additional restrictions implied by Equation (10), specifically, that for any set of instruments $\mathbf{Z}_t$ in $\mathcal{F}_t$:

$$E[(\tilde{m}_{t+1}\tilde{\mathbf{R}}_{t+1} - \mathbf{1}) \otimes \mathbf{Z}_t] = \mathbf{0} \tag{11}$$

The set of papers that we consider here perform *unconditional* tests of *conditional* factor models. These papers generally do not test the additional moment restrictions implied by (11). In the language of Cochrane (2000), they don't augment the return space with scaled test assets — but they do augment the set of factors with scaled factors.

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