

Market Turmoil and Destabilizing Speculation

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Abstract

This paper explores how speculators can destabilize financial markets by amplifying negative shocks in periods of market turmoil. I propose a dynamic trading model with two types of investors – long-term and speculative – who interact in a market with search frictions. During periods of turmoil created by an uncertainty shock, speculators react to declining asset prices by liquidating their holdings in hopes of buying them back later at a gain, despite the asset's cash flows remaining the same throughout. I also show that a reduction in search frictions leads to more severe fluctuations in asset prices. At the root of this result are the strategic complementarities between speculators expected to follow similar strategies in the future.

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1 Introduction

The role played by arbitrageurs in financial markets has been the subject of heated political and academic debate going back to at least [Keynes \(1936\)](#) and [Hayek \(1945\)](#). The view exemplified by standard asset pricing models recognizes that arbitrageurs might be the sole force that ensures market efficiency by pushing prices towards fundamental values ([Friedman \(1953\)](#)). Critics, instead, maintain that arbitrage trading can significantly destabilize prices ([Stein \(1987\)](#)). Moreover, arbitrage trading might have an even more pivotal role when financial markets are in distress and liquidity dries up. For instance, [Ellul et al. \(2013\)](#) consider several episodes of market turmoil dating back to the 1987 market crash and show that speculators amplify the effects of negative market-wide shocks by demanding liquidity at times when other potential buyers' capital is scarce.¹ This suggests that certain investors' behavior in times of high uncertainty determines how severe the effects of negative shocks and their propagation will be.²

In this paper, I theoretically explore the role of speculators during periods of market turmoil and show that, as hinted at by [Ellul et al. \(2013\)](#), instead of stabilizing financial markets, speculators amplify price fluctuations by selling their holdings. I propose a continuous-time model with two types of investors – long-term and speculative – in a market with trading frictions. Long-term investors offer a downward-sloping demand curve to other market participants, whereas speculators trade off the cash flow from holding the asset against the expected capital gains from aggressively exploiting temporary deviations of the price from the long-term value of the asset. A key element in the analysis is the introduction of news or uncertainty shocks, defined as the possibility that at some random time in the future the market will be subject to a negative shock with a small but positive probability. I model these shocks as an increase in supply, which makes the price deviate temporarily from its fundamental value. The initial shock that makes speculators aware of this possibility can be interpreted as an “uncertainty shock” ([Bloom \(2009\)](#)) or an “anxiety shock” ([Fostel et al. \(2010\)](#)), as it increases the uncertainty in the economy. The adverse shocks, which create price pressure, capture the possibility that regulatory constraints or margin requirements may force other investors to sell in the future.

The model reveals a novel mechanism whereby some investors could aggravate market shocks: during high-uncertainty periods, speculators react to declining prices by liquidating

¹To define periods of “market turmoil,” [Ellul et al. \(2013\)](#) use both the S&P500 Index and the VIX over the period 1986-2009. They focus on quarters during which there is a month when the S&P500 returns fall below the 5th percentile and the VIX changes are above the 95th percentile. This procedure identifies three quarters: the fourth quarter of 1987, the third quarter of 1998, and the third quarter of 2008.

²Other related papers include [Brunnermeier and Nagel \(2004\)](#), [Lou and Polk \(2013\)](#), [Gao et al. \(2014\)](#), [Chen et al. \(2008\)](#), and [Griffin et al. \(2011\)](#).

their holdings because they expect further turmoil. This generates *endogenous volatility* by further depressing prices, and leads to price reversals once the shock occurs. Speculators trade off the cash flow from holding against the expected capital gain from selling and buying back the asset when the market bottoms out. The size of the capital gains depends crucially on the behavior of the other speculators who trade in the future. In fact, strategic interactions among speculators are at the root of the amplification of non-fundamental shocks during periods of high uncertainty.

The first finding is that the two opposing views about arbitrageurs might coexist, in fact, two different equilibria can arise: a “*leaning-against-the-wind*” equilibrium and a “*cashing-in-on-the-crash*” equilibrium. In the former, speculators react to the uncertainty shock by buying, because they expect the speculators who trade in the future to buy as well, which leads to an increase in the price. Thus, in this equilibrium investors stabilize the market by absorbing the excess supply, as predicted by the standard arbitrage theory (Friedman (1953)). In the “cashing-in-on-the-crash” equilibrium, however, the speculators behave differently. Because they expect a further price fall due to the potential realization of a shock and, more importantly, due to endogenous price movement driven by the price impact of speculators trading in the future, they start selling until the uncertainty is resolved by the occurrence of the shock. Thus, speculators depress the price *in anticipation* of a negative shock, and to a greater extent than would be the case in the absence of speculators. In this equilibrium the speculators destabilize the market even when the asset’s long-term value is unaffected, as speculators could continue to hold the asset and capture its cash flow.

Moreover, there exists a price at which the *market freezes*. Intuitively, this is the price that makes speculators indifferent between selling and holding the asset. Hence, in my model the market can freeze even in the absence of asymmetric information. In other words, a news or uncertainty shock may not only amplify negative shocks and destabilize financial markets, but also dry up liquidity completely, especially when uncertainty is persistent. Interestingly, *less* trading friction is associated with sharper price decline and faster recovery once the shock hits.

To determine under what conditions such destabilizing trading is most likely to emerge, I investigate the market characteristics that affect investors’ incentives and influence which equilibrium emerges. I derive three main predictions. First, when uncertainty is short-lived speculators have a greater incentive to liquidate before the shock, as the period during which they have to forgo the asset’s cash flow is shorter. Second, it is more likely that speculators will amplify fluctuations in relatively illiquid markets. And third, trading friction has a non-monotonic effect on speculators’ incentives, because it plays two opposing roles: on the one hand, it determines the speed with which speculators can trade and exploit price move-

ments, and on the other it determines the size of the price movement before the shock. The relative absence of trading friction always increases the incentive to profit from the capital gain, because speculators can sell the asset when the price is high, expecting to buy it back immediately after the shock, when the price is lower. But even with significant trading friction speculators have an incentive to trade in the direction of the shock, because the expected price movement until the next trading opportunity is limited. By contrast, for intermediate levels of trading friction, speculators find it optimal to buy and hold to capture dividends, as the expected capital gain from trading is small.

To gain further insight, I relax the assumption that the severity of the future shock is known and constant, posing instead that it follows a Brownian motion that is commonly observed by market participants. This scenario captures the idea that speculators know that a shock may come in the future, but that the situation may either improve or deteriorate over time. In addition to capturing an interesting feature of reality, this extension has the advantage of generating a unique equilibrium: speculators sell if the expected shock is larger than a (unique) threshold, and buy when it is smaller.

Three key properties follow from this characterization. First, this equilibrium shows that a small perturbation to speculators' perception of the severity of the future shock can have discontinuous effects, leading them to liquidate their holdings abruptly and further depress prices. That is, markets become *fragile*. Second, this threshold is decreasing in price, which suggests that in bull markets the expectation of a small future shock is sufficient to generate a sudden wave of selling, causing a price crash. Third, when the severity of the shock is above the threshold magnitude, the price decreases over time, and when it is below it, the price rises towards the fundamental value.

Finally, to show that the speculators' ability to take advantage of expected shocks to the asset supply does not stem from the myopia of their counterparties, I also provide a microfoundation for the downward-sloping demand curve employed in the baseline model. I show that this reduced-form assumption can be rationalized as the outcome of a fully-fledged equilibrium outcome between speculators and profit-maximizing dealers who face inventory risk. In order to do so, I extend the model by assuming that the asset's supply follows a mean-reverting process. The supply volatility makes the price quoted by the dealer sensitive to the speculators' demand, in a manner that resembles the baseline model. Hence, even when speculators trade with forward-looking dealers, they can destabilize the market in response to the fear of a future supply shock. Then, in contrast to the existing literature our mechanism does not hinges upon arbitrageurs' collateral constraints, their myopia or any potential behavioral bias affecting other market participants.

The paper is organized as follows. The remainder of this section discusses the related lit-

erature. Section 2 presents the baseline model and Section 3 characterizes the two equilibria that can arise and the resulting price paths, and analyzes the impact of trading frictions, market liquidity and the persistence of uncertainty on equilibrium behavior. Section 4 presents my uniqueness result and the result on market fragility, whereas Section 5 presents the microfoundation of the downward-sloping demand curve. Section 6 discusses the main empirical implications in light of recent evidence on the role of institutional investors during the financial crisis. Finally, Section 7 discusses the findings and concludes. Appendix 8 contains the proofs. The Supplementary Appendix 9 extends the model to study the effect of the introduction of a transaction cost, such as a Tobin tax, on the equilibrium.

1.1 Related Literature

To my knowledge, this paper is the first to show that, even when assets' fundamentals remain constant, uncertainty about future market shocks in conjunction with trading frictions lead speculators to aggravate the effects of market shocks by liquidating their holdings.

This paper enriches the literature on financial market runs and feedback effects. [Bernardo and Welch \(2004\)](#) study how dealers provide liquidity during a market run in a model in the tradition of [Diamond and Dybvig \(1983\)](#). [Morris and Shin \(2004\)](#) analyze a model in which traders with short horizons and loss limits interact in a market with long-horizon traders. In their model, speculators sell out of fear, while in this paper speculators sell out of greed. [He and Xiong \(2012\)](#) analyze dynamic debt runs in a continuous-time model in which creditors must decide whether to roll over their loans to a bank at discrete points in time and, unlike standard models of runs, derive a unique equilibrium. The focus on the timing of arbitrage trades connects my work to [Abreu and Brunnermeier \(2002\)](#) and [Abreu and Brunnermeier \(2003\)](#). These authors analyze a model in which the emergence of a gap between the price and the fundamental value of an asset is exogenously given and informational asymmetries cause a problem of coordination over the optimal time to exit the market. In my model instead, information is symmetric, the price is *endogenous*, and the focus is on speculators' responses to an increase in uncertainty. Interestingly, in my model trading might come to a complete halt even in absence of any information asymmetry about the asset's value.

The most closely related paper to this is [Diamond and Rajan \(2011\)](#), which shows that to profit from the future fire sale a bank's management might refrain from selling illiquid assets, even though such sales could save the bank. As in my paper, this behavior stems not from uncertain fundamental values but from the potentially low future fire sale prices at which illiquid assets will have to be sold. In my model, however, the strategic interaction among speculators in a market with trading frictions, rather than a risk-shifting motive, drives the

dynamics of the asset price.

My paper is part of a large literature that has identified factors that limit arbitrageurs' ability to prevent mispricing: noise-trading risk (De Long et al. (1990)), fundamental risk (Campbell and Kyle (1993a)), principal-agent problems (Shleifer and Vishny (1997)), coordination risk (Liu and Mello (2011), Carlin et al. (2007)), information barriers (Bolton et al. (2011)), slow-moving capital (Mitchell et al. (2007), Duffie (2010) and Duffie and Strulovici (2011)), and wealth effects (Xiong (2001), Kyle and Xiong (2001), Gromb and Vayanos (2002), and Kondor (2009)). Compared with earlier research, I posit a different source of the limits to arbitrage – namely uncertainty about future market shocks in conjunction with trading frictions.

Fundamentally, it is part of the search literature following Duffie et al. (2005), and Duffie et al. (2007), which treats liquidity shocks (as captured by a shift in the preferences of all market participants in the spirit of Grossman and Miller (1988)) in secondary markets with trading frictions.³ The most closely related papers in this strand of the literature are Weill (2007) and Lagos et al. (2011), which consider out-of-steady-state dynamics and dealers' liquidity provision when dealers can hold inventories *in response to* an aggregate liquidity shock of the same type as in Duffie et al. (2007).⁴ The distinctive feature of my analysis is the focus on investors' trading strategies *in anticipation* of a random negative shock, rather than on the dealers' responses. My first set of results yields new insight into how the interaction between speculators and long-term investors, and the resulting trading dynamics, might accentuate negative shocks by depressing the price even beyond the real effect of the shock. This overshooting of the price is related to the occurrence of predatory trading as shown by Brunnermeier and Pedersen (2005). The main difference is that, in this paper, the key ingredient is the interaction among speculators which gives rise to two different equilibria, one stabilizing and one destabilizing, and potentially to market freeze. Moreover, when the severity of the future shock changes over time, this paper also generates endogenous volatility, i.e. the effects of uncertainty in heightening price volatility are amplified by the behavior of speculators in distressed markets.⁵

³Liquidity provision in normal times has been analyzed in traditional inventory-based models of market-making (see Chapter 2 of O'Hara (1995) for a review).

⁴A related literature employs the techniques developed in the search literature on over-the-counter markets for theoretical analysis of the impact of high-frequency trading. For instance, Pagnotta (2010) shows that traders find it optimal to play a twofold role in the decentralized market, at once demanding and supplying liquidity, whereas Biais et al. (2011) and Jovanovic and Menkveld (2010) show that algorithmic traders –by processing information on stock values faster than other slower traders– generate adverse selection.

⁵Two additional amplification mechanisms have been shown to be important. Kiyotaki and Moore (1997) have highlighted agents' balance sheets as a source of amplification, while Routledge and Zin (2009), Caballero and Krishnamurthy (2008), and Easley and O'Hara (2010) show that investors' Knightian uncertainty about asset values might cause them to disengage from markets and in this way amplify the crisis. The mechanism posited here complements these papers by specifying why and how markets become more fragile when

2 Model

Overview. I propose a continuous-time model in which two types of investors, long-term investors and speculators, participate in a market characterized by trading frictions. That is, speculators cannot change their holdings continuously but only at discrete points in time. Long-term investors provide a downward-sloping demand curve to other market participants, while speculators trade off the cash flow from the asset in hopes of aggressively exploiting temporary deviations from the long-term value. One can interpret these speculators as sophisticated institutional investors, say hedge funds, that have the skills and the capital to provide liquidity in case of negative shocks that drive the price away from fundamentals; while the long-term investors can be interpreted as market makers or uninformed investors. A formal microfoundation of the downward-sloping demand function is provided in Section 5. The key element of my analysis is the introduction of a news or uncertainty shock: the possibility that at some random future time the market will be subject to a negative shock with small, but positive, probability. I model this shock as an increase in asset supply, which causes the price to deviate from the fundamental value. My model aims to capture the speculators' behavior during market turmoil and their role in the destabilization of financial markets.

Environment. Time is continuous, runs forever and is indexed by $t \geq 0$. There is one asset and one perishable good, used as a numéraire. The asset is durable and in supply $S(t) > 0$, and its price is denoted by $p(t)$. The asset generates a constant cash flow stream, i.e. δdt over the interval $[t, t + dt]$. The numéraire good is produced and consumed by all agents. The instantaneous utility flow of a speculator is $\delta a + c$, where a represents the asset's holdings, and $c \in \mathbb{R}$ is the net consumption of the numéraire good ($c < 0$ if the speculator produces more than he consumes). There are two types of infinitely-lived and risk-neutral agents who discount at the same rate $r > 0$: long-term investors and a unit mass of speculators. The analysis focuses on the decisions of the speculators. The drawback to assuming risk-neutrality consists in the possibility that it may be optimal for speculators to submit orders of infinite size. To preclude this, I assume that each speculator can hold at most one unit of the asset and cannot sell short, i.e. $a \in [0, 1]$.⁶ Because agents have linear utility, considering only equilibria in which, at any given time, a speculator holds either 0 or 1 unit of the asset does not undermine generality.

Trading Arrangements. A speculator can trade at random times T_α , distributed according

uncertainty spikes, without the need to rely on collateral constraints or specific assumption about preferences.

⁶Such limits to the positions of arbitrageurs have been rationalized in the literature in a variety of ways, including risk aversion, wealth constraints and asymmetric information. Duffie et al. (2007) show that a risk-neutral investor acts more or less like a risk-averse investor in a search framework. The short-sales constraint can be relaxed, without affecting the main qualitative results, but at the expense of increasing the number of state variables, as well as tractability.

to a Poisson distribution with intensity $\alpha > 0$. The processes are independent across speculators, which means that a fraction αdt of speculators get a chance to trade between t and $t + dt$. This friction has several interpretations.⁷ First, it captures the frequency with which traders can submit orders. The importance of this delay, which is sometimes measured in milliseconds, is shown by the significant investments made by banks and other institutional investors to co-locate their servers next to those of the exchanges' themselves.⁸ Second, it captures the time it takes for a hedge fund to structure a large deal with prime brokers or to consult and re-contract with clients.⁹ Third, it might capture settings in which market monitoring is imperfect and costly so that agents are not always trading as in [Biais and Weill \(2009\)](#). I abstract from all other frictions, such as investors' fees to intermediaries, as considering them would not affect my results and would not add anything to the insights provided on this by [Lagos and Rocheteau \(2009\)](#).¹⁰

Apart from the speculators, the market consists of long-term investors, who form the competitive fringe. The primary difference between these investors and speculators is that the long-term investors are more likely to trade according to fundamentals and to employ less aggressive trading strategies. In other words, they do not attempt to profit from price swings. Such conduct on the part of unsophisticated investors could also follow from the assumption that they lack the information, skills, or time to predict short-term price changes. In particular, my distinction between unsophisticated long-term investors and speculators could be interpreted as a distinction between traders who have the technology to trade at high frequency and take advantage of short-term price fluctuations, such as trading desks at investment banks and hedge funds, and investors such as mutual funds and pension funds, that tend to hold assets to maturity. These long-term investors can be thought of like mutual funds, pension funds or market makers.

The trading mechanism works as follows. The market clearing price $p(t)$ solves $D(p_t) + x(t) = S(t)$, where $D(p_t)$ captures the long-term investors' holdings and $x(t)$ is the fraction

⁷The introduction of small trading frictions also overcomes the technical modeling problems associated with the sequencing of trades when time is continuous. For instance, I want to rule out non-measurable trading strategies, or strategies such as remaining in the market as long as possible, but exiting strictly by some given date t^* . Also, by imposing the assumption that trading opportunities arrive with Poisson rate α , I preclude clustering of trades.

⁸See for example the article “High-Frequency Trading Grows, Shrouded in Secrecy” available at <http://www.time.com/time/business/article/0,8599,1914724,00.html>.

⁹The explicit introduction of dealers into the model would not affect the analysis.

¹⁰As is shown by [Lagos and Rocheteau \(2009\)](#), the dealers' fees only reduce the rate at which orders arrive from α to $(1 - \eta)\alpha$ where η is the dealer's bargaining power. Instead, in the supplementary appendix, I investigate the introduction of small trading costs, such as a “Tobin tax” on financial transactions.

of speculators holding the asset at time t . The asset is traded at the price

$$p(t) = \frac{\delta}{r} - \lambda(S(t) - x(t)). \quad (1)$$

The first term in (1) is the fundamental value $\frac{\delta}{r}$: the net present value of profits to investors when they hold the assets forever and collect the entire future cash flow.¹¹ The parameter λ measures the permanent liquidity effects of trading, and a larger λ captures lower liquidity by long-term investors. As $x(t)$ increases, the price at which the long-term investors can access the asset increases. Grossman and Miller (1988) show that a competitive but risk-averse market-making sector is only willing to absorb the selling pressure at a lower price. Alternatively, if speculators have private information about the fundamental value v , then the long-term investors face an adverse selection problem that naturally leads to a downward-sloping demand curve as in Kyle (1985). Hence, asymmetric information and ownership structure should be among the major determinants of λ , because for an asset with more asymmetric information or more concentrated ownership, the price will more strongly adjust when the net supply of the asset changes. In Section 5, I follow Garleanu et al. (2009) in providing a microfoundation for (1) that stems from the behavior of profit-maximizer dealers, who face inventory risk and a noisy asset supply.

Hence, while in the “long term” the price is expected to be $\frac{\delta}{r}$, in the “medium term” the demand curve is downward sloping as in (1). Campbell et al. (1993) find evidence consistent with this hypothesis by showing that returns accompanied by high volume tend to be reversed more strongly. Pastor and Stambaugh (2003) provide further evidence for this hypothesis by finding a role for a liquidity factor in an empirical asset-pricing model, based on the idea that price reversals are often provoked by liquidity shortages. Interestingly, Lou et al. (2011) show that demand/supply shocks have temporary price effects even in the most liquid market of all, that for Treasuries. The mechanism involves the primary dealers, who are required to participate actively and to submit competitive bids in all auctions but whose risk-bearing capacity is limited, as captured in reduced form by (1).

Negative Shock. Since I am interested in speculators’ response to negative shocks and in particular in how the strategic interactions among them might actually aggravate rather than mitigate these shocks, I assume that at time $t = 0$ the speculators become aware that a supply shock might hit the market sometime in the future. I assume that at random time T_ρ , distributed according to a Poisson distribution with intensity $\rho > 0$, the uncertainty is resolved but with probability $1 - \varepsilon$ the shock does not occur. With complementary probability

¹¹Xiong (2001), Kyle and Xiong (2001) and Brunnermeier and Pedersen (2005), among others, employ a similar formulation for the pricing relationship.

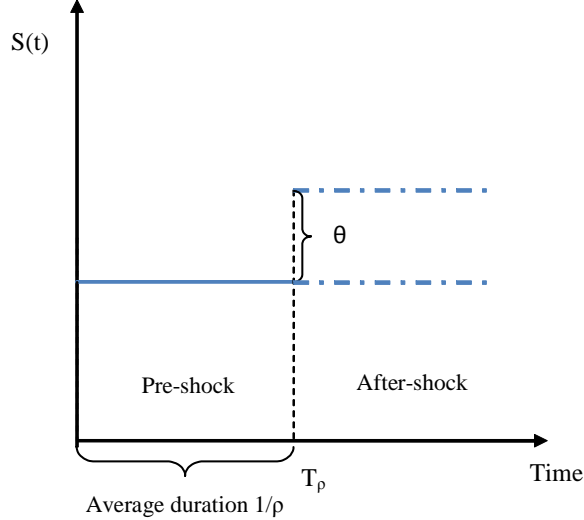


Figure 1: Asset supply might jump at random time T_ρ due to a negative shock.

ε the market is hit by a shock of severity θ that increases supply and accordingly lowers the price. Hence, θ captures the *severity* of the shock, while $1/\rho$ is the *persistence* of uncertainty, namely the average time before the uncertainty is cleared up, and ε can be interpreted as the *level* of uncertainty in the economy.¹² When ε is zero, as it is after T_ρ , speculators do not expect any future shock, but when ε becomes positive, as in the turmoil period $(0, T_\rho)$, investors are uncertain over the future price path, which affects their trading strategies dramatically.

As Figure 1 shows, once the first shock is realized at $t = 0$, speculators are uncertain about future market conditions: with probability $1 - \varepsilon$ the initial uncertainty shock has no consequences on the price, while with probability ε the asset price is depressed by the realization of the shock. I assume that the shock is commonly observed by all market participants, and when it hits the price moves instantaneously as dictated by (1).

Intuitively, negative shocks in the form of an increase in supply could capture a situation in which other traders are forced to liquidate positions owing to margin calls or regulatory requirements. Coval and Stafford (2007) show empirically that funds suffering large investment outflows create price pressure in the securities held in common by distressed funds because they tend to liquidate their positions. Brunnermeier and Pedersen (2009) show that when higher margins and haircuts force de-leveraging and sales, this leads to further increases in margins, forcing still more selling, and Adrian and Shin (2010) find empirical support for this liquidity spiral in data on investment banks. Ellul et al. (2011), instead, investigates fire sales of downgraded corporate bonds in compliance with the regulations on insurance companies.

¹²The assumption that the random time T_ρ is Poisson-distributed simplifies the analysis by guaranteeing that the speculators' problem is time-homogeneous.

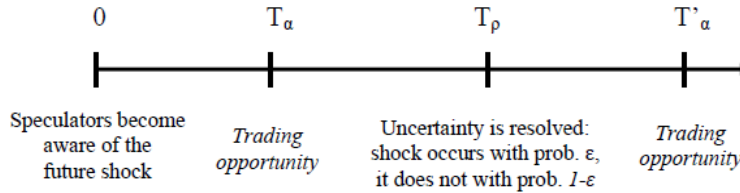


Figure 2: The timeline.

Their collective need to divest downgraded issues may run afoul of a scarcity of counterparts, exacerbating the price fall.

Figure 2 shows the timeline of the model. At time $t = 0$ the speculators become aware that the asset price might be depressed by the realization of the shock at time T_ρ . They can trade the asset before and after the realization of the shock at the random times T_α and/or T'_α .

3 Analysis

In this section I first characterize the speculators' decision problem. As benchmarks I investigate the single-speculator case and the no trading frictions case. Then, I consider how the equilibrium and the price path are affected by strategic interaction among speculators.

3.1 Formulation of the Speculator's Problem

In the case with a single speculator, he merely chooses his optimal trading strategy, buying (or holding) or selling (or not buying) the asset given its fundamental value $\frac{\delta}{r}$ and the expected shock that might lower the price at random time T_ρ .¹³

Let us first consider a speculator who has the opportunity to trade at time $t > T_\rho$: once all uncertainty is resolved. Let $V(a, t)$ denote the maximum expected discounted utility attainable by a speculator who is holding a portfolio $a \in [0, 1]$ at time t . The value function satisfies

$$V(a, t) = \mathbb{E}_t \left[\int_t^{T_\alpha} e^{-r(s-t)} \delta a ds + e^{-r(T_\alpha-t)} \{V(a(T_\alpha), T_\alpha) - p(T_\alpha) [a(T_\alpha) - a]\} \right], \quad (2)$$

where T_α denotes the speculator's next trading opportunity. The expectation operator is taken with respect to the random variable T_α . The first term is the expected discounted

¹³Henceforth I refer simply to buying, implying that a speculator who already owns the asset will hold it, if it is optimal to buy; selling is considered optimal even when the non-owner speculator decides not to trade.

utility flow enjoyed by the investor over the interval $[t, T_\alpha]$: the length of the interval $T_\alpha - t$ is an exponentially distributed random variable with mean $1/\alpha$. The second term is the expected discounted utility of the speculator from the time he next contacts a dealer, T_α , onward. At time T_α , the speculator readjusts his asset holdings from a to $a(T_\alpha)$. In this event he purchases $a(T_\alpha) - a$ in the market (or sells if this quantity is negative) at a price $p(T_\alpha)$. As in [Lagos et al. \(2011\)](#), the following lemma shows how to simplify the speculator's problem:

Lemma 1 (After-shock) *Suppose a speculator can trade at time $t \geq T_\rho$, then his optimal choice of asset holdings is $a(t) = 1$ if and only if*

$$u_a = \frac{\delta}{r + \alpha} > q(t), \quad (3)$$

where

$$q(t) = \left[p(t) - \int_0^\infty \alpha e^{-(r+\alpha)s} p(t+s) ds \right].$$

Intuitively, u_a is the expected discounted value of holding the asset from time t until the next event, i.e. the next opportunity to trade which arrives at rate α ; whereas $q(t)$ is the opportunity cost of holding the asset, i.e. the expected discounted forgone capital gain. By reducing the speculator's problem to a pointwise maximization, this characterization greatly simplifies the analysis. In other words, after the uncertainty in the market is resolved the speculator's decision is driven by a trade-off between the dividends from holding the asset and the potential capital gain from trading.

Now let me analyze the economy during the pre-shock period $[0, T_\rho)$. All the value functions and the prices in this interval before the realization of uncertainty are denoted by the superscript "U" for the uncertainty phase. Following the same steps as in Lemma 1, it can be shown that an investor who accesses the market at time $t < T_\rho$ chooses his asset holdings $a^U \geq 0$ to maximize the following expression

$$\mathbb{E} \left[\int_t^{T_\alpha} e^{-r(s-t)} \delta a^U ds - (p(t) - e^{-r(T_\alpha-t)} p(T_\alpha)) a^U \right], \quad (4)$$

where the price at the next trading time T_α is

$$p(T_\alpha) = \mathbb{I}_{\{T_\alpha < T_\rho\}} p^U(T_\alpha) + \mathbb{I}_{\{T_\alpha > T_\rho\}} p^U(T_\alpha | T_\rho).$$

With some positive probability the speculator will come to the market before the resolution of the uncertainty ($T_\alpha < T_\rho$), while with complementary probability he will be able to trade at the post-shock price $p^U(T_\alpha | T_\rho)$. There is only one difference between expression (4) and

expression (3) for the case in which the speculator trades after the shock: namely, a speculator expects that the shock will have occurred by time T_α , when he is able to retrade the asset. The following lemma provides a simpler formulation of the speculator's problem.

Lemma 2 (Before-shock) *Suppose a speculator can trade at time $t < T_\rho$, then his optimal choice of asset holdings is $a^U(t) = 1$ if and only if*

$$u_a^U = \frac{\delta}{r + \alpha + \rho} > q^U(t), \quad (5)$$

where

$$q^U(t) = [p^U(t) - \int_t^\infty \alpha e^{-(r+\alpha)(\tau_\alpha-t)} (e^{-\rho(\tau_\alpha-t)}) p^U(\tau_\alpha) + \int_t^{\tau_\alpha} \rho e^{-\rho(\tau_\rho-t)} p(\tau_\alpha|\tau_\rho) d\tau_\rho] d\tau_\alpha].$$

Intuitively, the utility flow the speculator captures during the pre-shock interval $[0, T_\rho)$ is also discounted by the average duration of this interval (parameter ρ): the farther in the future the shock is expected (low ρ), the greater the incentive to hold the asset rather than selling in hopes of a capital gain, because the speculator will miss the dividends for a longer period. The price at which the speculator expects to trade takes into account the possibility that if he comes to the market at $T_\alpha < T_\rho$, the speculator has the opportunity to trade before the resolution of the uncertainty, while if he trades after the shock is realized, $T_\alpha > T_\rho$ then the speculator can condition his trading decision on the realization of the shock θ . The probability of being able to trade before or after the random time T_ρ will be influenced by the trading frictions captured by α and by the average length of the pre-shock period $1/\rho$, which explains why the trading frictions appear in the expression for $q^U(t)$.

There are two natural benchmarks for our analysis: the case of a single speculator and the case in which there are no trading frictions. First, to highlight the role played by the interaction with other speculators and the resulting price movement, I characterize the speculator's behavior in the following proposition:

Proposition 1 (Benchmark: Single speculator) *There exists a number $\hat{\theta}$ such that it is strictly optimal for the speculator coming to the market at time $t \in [0, T_\rho)$ to sell or refrain from buying ($a^* = 0$) if and only if $\theta > \hat{\theta}$ and to buy otherwise ($a^* = 1$). As soon as the speculator can rebalance his portfolio at time $t \geq T_\rho$, it is optimal to buy the asset ($a^* = 1$) if the shock is realized.*

Proposition 1 characterizes the optimal trading strategy of a single speculator. He must decide whether to sell or hold the asset, and then decide when to buy it back. When there

is a single speculator, however, the problem is simplified because the greatest capital gain is scored when the repurchase comes after the shock has hit. Buying back earlier would yield no capital gain, paying only the dividend. The only complication is that the speculator does not know when he will have a new trading opportunity, and there is uncertainty about the future price path: the shock might not occur at time T_ρ making it optimal to buy (or hold) the asset before T_ρ . In other words, when he has the opportunity to trade at time $t < T_\rho$, the speculator must decide whether to buy or sell, and forecast when he will have the opportunity to rebalance his portfolio. In particular, the speculator considers the possibility that he can buy back after the shock, but also the risk of loss if the shock does not occur at time T_ρ . The main implication of the benchmark is that with a single speculator only a high expected shock is likely to provoke preemptive selling. Specifically, Proposition 1 states that *only* when the shock is expected to be severe, i.e. greater than the threshold $\hat{\theta}$, will the speculator prefer to sell and repurchase when he gets the opportunity to trade after the shock, because the expected capital gain more than compensates for the forgone dividends.

Notice also that the threshold $\hat{\theta}$ is time-invariant; in particular it does not depend on the current price $p(t)$; whether he is in a bull or a bear market does not influence the speculator's trading strategy. We shall see that this is not so when strategic interaction among speculators is allowed for. The next corollary shows the comparative statics with respect to the main parameters of the model:

Corollary 1 (Comparative statics) *The speculator's net benefit from selling the asset decreases with the persistence of uncertainty $1/\rho$ and increases with the level of uncertainty ε , with market depth λ , and if the dividend δ is sufficiently small it increases with α .*

The parameter ε influences the expected magnitude of the shock and λ its effect on the price. Hence, it is not surprising that higher values of these parameters are associated with a greater likelihood of selling before time T_ρ . Higher ρ means a shorter average duration of the pre-shock period, which increases the speculator's incentive to profit from the price swing because the expected time during which he forgoes dividends is shorter. A similar intuition explains why an increase in the arrival rate of trading opportunity α heightens the speculator's incentive to sell after the initial shock at $t = 0$. In the next section, I show that trading friction has a non-monotonic effect when a number of speculators interact.

To emphasize the role played by the trading frictions in shaping speculators' behavior, let us consider the case in which speculators can continuously trade the asset, i.e. there are no trading frictions $\alpha \rightarrow \infty$. Hence, at any time t before the arrival of the supply shock at T_ρ each speculator faces the same trade-off between dividends and capital gains. As in a rational expectation world, the price will then adjust immediately at time zero so that each speculator

is indifferent between selling ($x_0 = 0$) and buying ($x_0 = 1$) the asset. If the speculator sells the asset, he will forgo the asset dividends until he buys the asset back after the shock:

$$\frac{\delta}{r + \rho} = \int_0^\infty \left(\int_0^\tau e^{-rs} \delta ds \right) \rho e^{-\rho\tau} d\tau.$$

However, speculators are able to capture the following capital gains from selling at the price $P(0)$ and buying it back at the lower price $P(s)$:

$$P(0) - \int_0^\infty \rho e^{-(r+\rho)s} E[P(s)] ds,$$

where $E[P(s)] = \frac{\delta}{r} - \lambda(S + \theta\varepsilon - 1)$, since $x_s = 1$. Then, the speculator will sell at time zero as long as

$$\frac{\delta}{r + \rho} \leq P(0) - \int_0^\infty \rho e^{-(r+\rho)s} E[P(s)] ds.$$

Hence, I can show the following result:

Proposition 2 (Benchmark: No trading frictions) *In absence of trading frictions, the asset price adjusts at $t = 0$ to the following level:*

$$P^{BC} = \frac{\delta}{r + \rho} \left(1 + \frac{\rho}{r} \right) - \frac{\rho\lambda}{r + \rho} (S + \theta\varepsilon - 1)$$

Proposition 2 provides a useful benchmark for the analysis in the next section, in fact, I shall show that the strategic interaction among speculators, generated by the presence of trading frictions, leads to a more severe price decline. Intuitively, in absence of trading frictions, each speculator faces the same time-invariant problem. This means that when considering to sell or buy the asset at time t , each speculator will not have to form beliefs about the behavior of the other speculators trading after time t . This aspect will crucially affect the equilibrium outcome, because the speculator who trades at time t might decide to keep selling even when the price $p_t < P^{BC}$, whenever he expects other speculators to keep selling in the future.

3.2 The Amplification Mechanism

This section analyzes the main mechanism posited in the paper: namely, that the impact of uncertainty may be amplified by the endogenous volatility generated by speculators' trading in the same direction.

As in the previous section, an equilibrium is characterized in two steps: solving first for the equilibrium after uncertainty is resolved for every possible T_ρ and then for the equilibrium

during the pre-shock period, i.e. prior to T_ρ . One simplification of the single-speculator case is that the price path can be computed easily and does not depend on other speculators' dynamic trading strategies. But when interaction among speculators is allowed for, this simplification no longer applies. In other words, asset price volatility is *endogenous*, because it depends as before on the potential shocks to the asset supply, but now, additionally, on speculators' trading strategies. Selling in response to an increase in uncertainty puts pressure on the price and determines the extent of the price swing. Hence, not only are expectations about potential shocks important in predicting the response of speculators to greater uncertainty, so are their beliefs about how the other speculators will respond.

In spite of this difficulty, I can derive clear predictions on equilibrium trading strategies and on price path. I start by defining the equilibrium:

Definition 1 *An equilibrium is a time-path $(\{a^U(t)\}, \{a(t)\}, \{p(t)\})$ that satisfies (3), (5) and market clearing (1) given initial conditions $p(0)$ and $x(0)$. A “leaning-against-the-wind” equilibrium is one in which the price path is increasing over both intervals $[0, T_\rho)$, and $(T_\rho, \infty]$. A “cashing-in-on-the-crash” equilibrium is one whose price path is decreasing over the interval $[0, T_\rho)$ and increasing over $(T_\rho, \infty]$.*

The previous definition identifies two possible types of equilibrium. In the “leaning-against-the-wind” equilibrium, speculators expect others to buy the asset over the interval $[0, T_\rho)$, which drives the price upward and makes it profitable to buy. A “cashing-in-on-the-crash” equilibrium emerges whenever the price swings make it optimal to take a capital gain by selling over $[0, T_\rho)$ and buying back at $T_\alpha > T_\rho$. For concreteness, I assume that if they are indifferent between buying and not trading, the speculators do not trade. I show that, depending on the parameters, either type of equilibrium can emerge when speculators interact and then derive a unique equilibrium in Section 7 by positing that the speculators' perceptions of the severity of the crisis θ fluctuate over time. As we can see in Proposition 4, when the price goes too low to make it profitable to sell and repurchase, they stop trading and the market freezes.

When the speculator interacts with other speculators, the price is endogenously determined by the others' trading strategies. I first compute the price path under the two equilibria. The fraction $x(t)$ of speculators that are long on the asset evolves according to

$$\dot{x}(t) = \begin{cases} -\alpha x(t) & \text{if investors sell} \\ \alpha(1 - x(t)) & \text{if investors buy} \end{cases},$$

that is, the price rises if speculators start buying and falls over time at rate α if they start selling in response to the uncertainty shock. The next lemma shows the price path in the two equilibria.

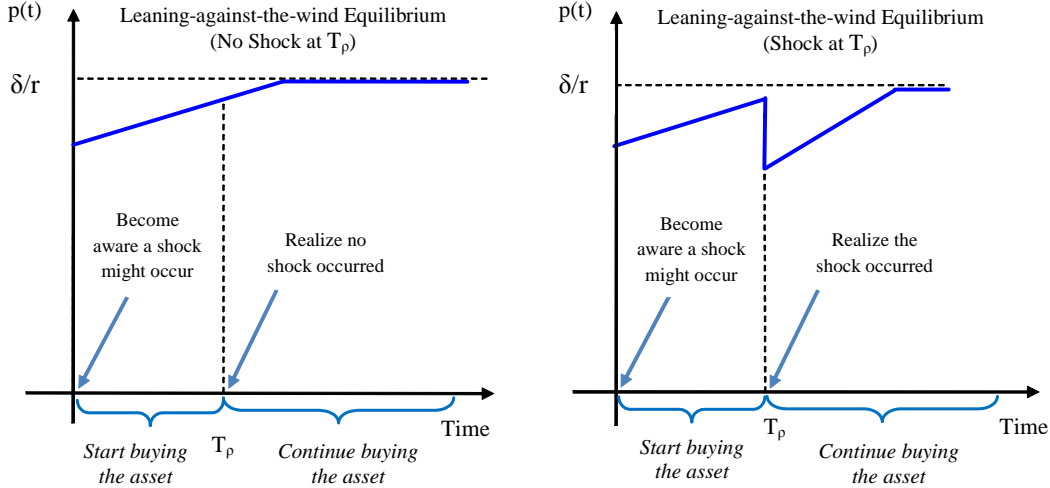


Figure 3: The price path in a “leaning-against-the-wind” equilibrium where the shock does not occur (left panel) and where it does (right panel) at random time T_ρ .

Lemma 3 (Price dynamics) *If $p(t) < F$, in the “leaning-against-the-wind” equilibrium the price rises over time at rate α (at most up to δ/r). In the “cashing-in-on-the-crash” equilibrium, it falls at rate α over the interval $[0, T_\rho)$ as long as trading occurs, and rises at the same rate for $t > T_\rho$. In both equilibria, the price may jump at time T_ρ in the event of a shock.*

Lemma 3 shows that in the first type of equilibrium the price rises over both intervals $[0, T_\rho)$ and (T_ρ, ∞) ; speculators react to the uncertainty shock at $t = 0$ by purchasing the asset, driving the price up. In the “cashing-in-on-the-crash” equilibrium, by contrast, the price declines at a rate α over the interval $[0, T_\rho)$, because speculators sell off their holdings when they get the opportunity to trade, building up selling pressure.¹⁴ The lemma also shows that trading friction affects the speculators’ objective in two ways. It has a direct effect, insofar as α determines the arrival of trading opportunities. And it affects the speed of the price movements, which in turn influence the gains accruing to the speculators.

This sets the stage for the first principal result, characterizing the conditions under which speculators trade in the same direction of the shocks and so amplify their effects.

Proposition 3 (Strategic Interactions) *Consider a speculator who can trade at time $t < T_\rho$:*

- (i) *There exist two severity thresholds $\bar{\theta}(p(t))$ and $\underline{\theta}(p(t))$ such that it is strictly optimal for him to sell when $\theta > \bar{\theta}(p(t))$ and to buy when $\theta < \underline{\theta}(p(t))$, if he expects other speculators trading after time t to buy and sell, respectively.*

¹⁴We shall show that the price cannot fall below a threshold p^* .

(ii) The thresholds are decreasing in the price $p(t)$ and $\underline{\theta}(p(t)) < \bar{\theta}(p(t))$.

Proposition 3 parameterizes the model according to the severity of the shock θ and identifies two dominance regions: one for which it is always optimal to buy ($\theta < \underline{\theta}(p(t))$) and the other in which it is optimal to sell ($\theta > \bar{\theta}(p(t))$), regardless of what other speculators may do in the future. In the intermediate region, namely for $\theta \in [\underline{\theta}(p(t)), \bar{\theta}(p(t))]$, the speculator's response to the increase in uncertainty depends crucially on his expectation of what other speculators will do when they get the opportunity to trade. Intuitively, this proposition highlights the importance of the strategic interactions among speculators. Their strategies depend critically on what other speculators plan to do in the future. When the price is expected to come down in anticipation of the shock, speculators amplify the shock by provoking a fire sale. In short, negative shocks that are only expected for the future could cause a financial run today.

In a *leaning-against-the-wind* equilibrium, speculators start providing liquidity to the market by purchasing the asset, which gradually increases the price, although it may plunge at T_ρ if a shock is realized, as is shown in Figure 3. In expectation of a rising price, speculators stabilize the market with purchases as long as the price is below the fundamental value of the asset ($p(t) < \frac{\delta}{r}$), correcting the temporary mispricing. In the *cashing-in-on-the-crash* equilibrium, the price begins to fall as soon as uncertainty about the future price path emerges. The strategic interactions among speculators also clarify that an identical expected future shock may have very different present implications depending on the speculators' response.

The proposition also brings out another property of the equilibrium. The thresholds $\underline{\theta}(p(t))$ and $\bar{\theta}(p(t))$ that define the intervals in which one equilibrium or both emerge are decreasing in the price. This means that markets where the asset is relatively over-valued are more vulnerable to uncertainty shocks.

The next proposition explicitly characterizes the *cashing-in-on-the-crash* equilibrium by identifying three phases: *crash*, *market freeze*, and *recovery*.

Proposition 4 (Cashing-in-on-the-crash Equilibrium) *Suppose $x(0)$ is sufficiently high and consider a speculator who can trade at time t :*

(Crash) *If $t < T_\rho$ and the speculator expects others to sell the asset in the future, he sells his holdings immediately, provided that $p > p^*$;*

(Market Freeze) *There exists a unique cut-off for the price p^* such that if it reaches this level trading comes to a complete halt, i.e. $\dot{x}(t) = 0$ and $\dot{p}(t) = 0$.*

(Recovery) *If $t \geq T_\rho$, then it is optimal to buy the asset ($a^* = 1$) provided that $p(t) < F$.*

Proposition 4 is represented in Figure 4. The first part of the proposition describes the crash phase, showing that when speculators have the chance to trade prior to the resolution

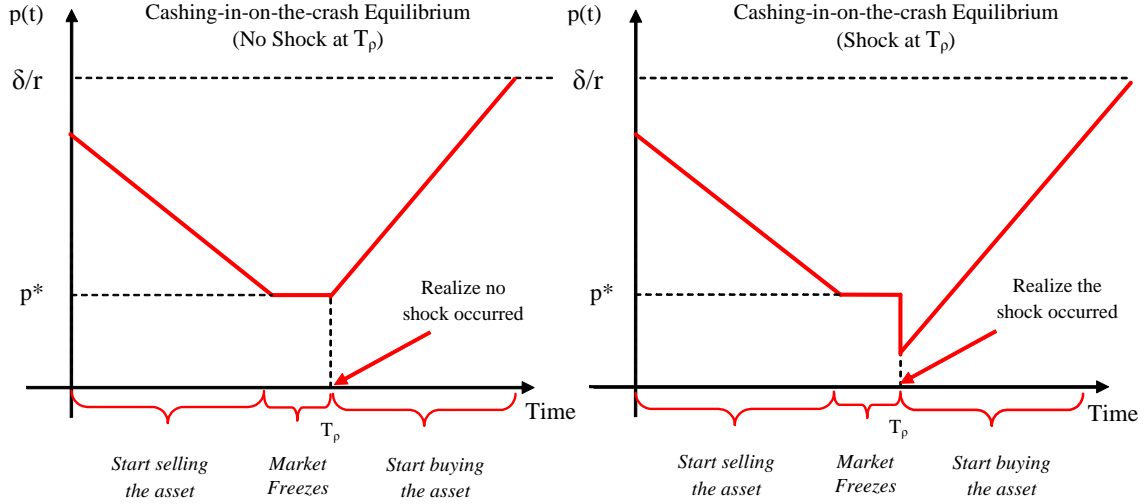


Figure 4: The price path when a “*cashing-in-on-the-crash*” equilibrium emerges and the shock does not occur at random time T_ρ (left panel) and when it does occur (right panel). The flat line in the right panel captures the market freeze when the price reaches the value of p^* .

of uncertainty, they will liquidate their positions if they expect a decreasing price path in the future and if the price is still sufficiently high. The second result of Proposition 4 shows there exists a price p^* at which the market freezes. Intuitively, this is the price that makes speculators indifferent between selling and holding. Interestingly, in my model the market can freeze even in the absence of asymmetric information. In other words, an uncertainty shock may not only amplify negative shocks and destabilize financial markets, but also dry up liquidity completely. As the right panel in Figure 4 illustrates, this is more likely when uncertainty is persistent.

The last part of the proposition shows that once the shock has occurred, it becomes optimal for speculators to purchase the asset as long as its price is below the fundamental value. Equivalently, one could interpret this result as capturing liquidity hoarding by the speculators in order to strategically time the bottom of the market after the shock. The interaction between speculators and long-term investors also generates momentum and reversal in a cashing-in equilibrium. In fact, the price decreases steadily until the shock hits, at which point it jumps down and then reverts toward the fundamental value.

To show that there is price overshooting in equilibrium, Figure 5 depicts the region of parameters for which the decline in price in the cashing-in-on-the-crash equilibrium is greater than in the rational expectations benchmark. Specifically, I compare the price p^* with the benchmark price P^{BC} obtained in Proposition 2 when there are no trading frictions. Obviously, since the arrival of the shock is random, the price might never reach p^* , however, this is a clear way to see under which conditions we observe amplification in equilibrium. Since the price

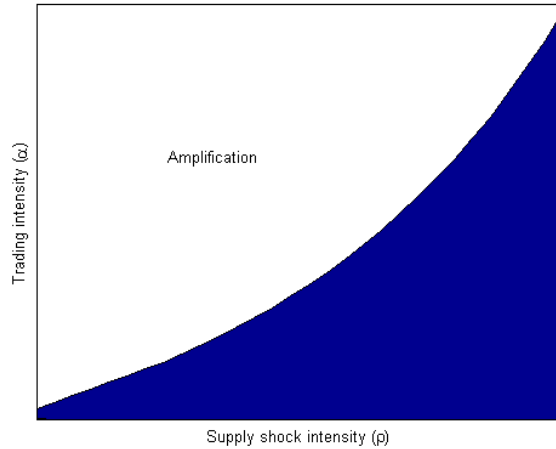


Figure 5: The figure shows when speculators amplify the effect of the shock as a function of the trading intensity α and of the arrival intensity of the supply shock ρ .

movements depend on how many speculators are able to trade before the arrival of the shock, the speculators sell the asset and push the price downward when $\frac{\alpha}{\rho}$ is high, that is, when the shock takes time to materialize or the speculators are able to have access to the market with higher frequency.

Next, I highlight the relationship between α and price dynamics in the following proposition:

Proposition 5 (Trading Frictions and Price Fluctuations) *In a cashing-in-on-the-crash equilibrium less trading friction, i.e. higher α , leads to a greater decline in the asset price for $t < T_\rho$ and a faster recovery towards the fundamental value for $t \geq T_\rho$.*

The previous proposition follows from Lemma 3 and Proposition 4. It implies that we should expect a sharper decline in asset prices when speculators have access to faster trading technology. This is because a larger fraction of speculators will have the chance to sell in hopes of timing the bottom of the market. This result suggests that high-frequency traders can have a very considerable impact on price stability, which may prompt sporadic market crashes like the one that occurred in May 2010. However, in a low-friction market the price recovers faster after the shock at T_ρ . In other words, less trading friction may imply deeper but less persistent price declines in periods of high-uncertainty.

The role of the three main assumptions of the model bears emphasis here. First, in absence of the uncertainty shock, i.e. for $\varepsilon = 0$, the speculators would behave exactly like long-term investors, purchasing the asset as long as the price is below its fundamental value. Second, the downward-sloping demand curve is needed in order to make the price sensitive

to the speculators' trading strategies; without it, the price would not change in response to the supply shock. Note that Section 5 provides a formal microfoundation of the downward-sloping demand function. And lastly, the presence of trading friction allows for a slowly changing price, which results in momentum and reversal.

There are few recent empirical studies which are related to our results. [Ellul et al. \(2013\)](#), for instance, show that investors with short horizons amplify the effects of negative market-wide shocks by demanding liquidity when other potential buyers' capital is scarce, which seems to confirm empirically the trading behavior described in Proposition 4. Relatedly, [Lou and Polk \(2013\)](#) provide a new measure of arbitrage activity and show that during periods of high arbitrage activity momentum strategies tend to crash and revert, reflecting prior overreaction resulting from crowded momentum trading pushing prices away from fundamentals. This evidence is very consistent with our model, because the strategies followed by the speculators in the model can be interpreted as momentum strategies, moreover, the empirical evidence seems to support the main mechanism: it is the arbitrageurs' coordinated activity that might lead to amplification of price movements.

The evaporation of liquidity in the market-freeze phase of the equilibrium is a consequence of the coordinated trading behavior of the speculators highlighted in Proposition 3. The sudden evaporation of liquidity was observed in many sectors of financial markets in 2007-09. [Gorton and Metrick \(2009\)](#) propose the explanation that the crisis amplified asymmetric information problems: that is, several debt instruments became more information-sensitive, which aggravated adverse-selection problems. I propose an alternative, complementary hypothesis: speculators amplified the initial uncertainty shock by reducing liquidity supply because they forecast further shocks and sought to take advantage of them. This also leads to a sudden increase in the returns to liquidity-provision, which follows from the fact that the fundamental value of the asset is unaffected while its price is driven down by speculation. [Nagel \(2012\)](#) provides evidence consistent with this prediction, finding that during the recent crisis, the returns to liquidity-provision increased significantly and were closely correlated with the VIX index.

3.3 The Role of Liquidity, Trading Frictions and Uncertainty

The foregoing investigated two possible equilibria. The present section considers which market and security characteristics make one or the other more likely to emerge. To obtain the predictions, I inquire into the effect of trading frictions and uncertainty on the speculators' incentives, ultimately reaching the following result:

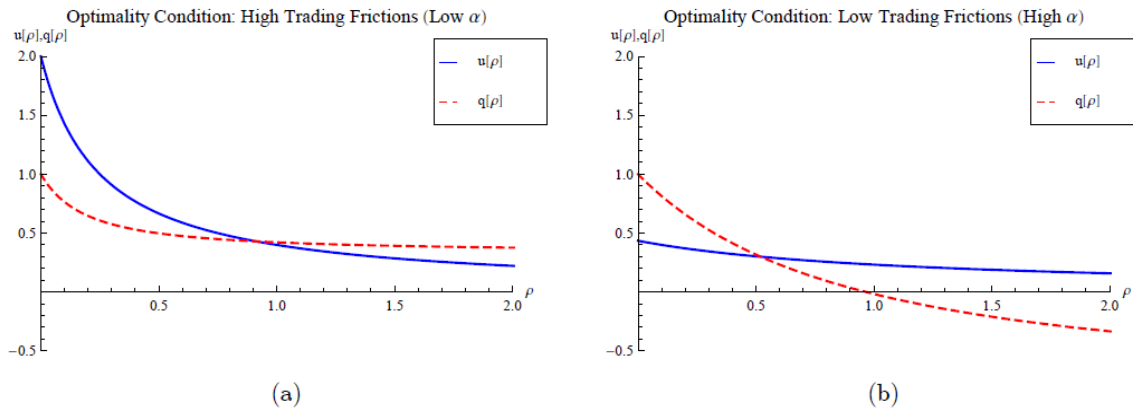


Figure 6: The effect on speculators’ incentives of the persistence of uncertainty ρ for low and high value of the trading frictions α .

Proposition 6 (Comparative Statics) *The amplification of a negative shock is more likely to occur in illiquid markets (high λ). Moreover, there exists an $\underline{\alpha}$ such that the speculators’ incentive to sell at $t < T_\rho$ increases with ρ if $\alpha < \underline{\alpha}$; while there exists a $\bar{\rho}$ such that the effect of trading frictions α is not monotone for $\rho > \bar{\rho}$.*

The effects of market depth λ follows from the fact that in less liquid markets the effect of the shock and of the price pressure exerted by other speculators is expected to be greater. This increases the expected capital gains to speculators from strategically selling their holdings at $t < T_\rho$. Suggestive empirical evidence consistent with this result is provided by [Manconi et al. \(2012\)](#), who show that the investors more exposed to securitized bonds that fell sharply in price, sold more bonds and contributed to the price downswing. They find that the investors who expect liquidity shocks retain liquid assets and sell those that have relatively high temporary price impact.

The intuition for the other results in the previous proposition can be better understood by analyzing Figures 6 and 7. Figure 6 depicts the two curves, $u_a(a)$ and $q(t)$, that determine the investors’ optimality condition found in Lemma 2 and shows an interesting interaction between trading frictions as captured by parameter α and the arrival rate of the shock ρ . Even if an increase in ρ decreases both $u_a(a)$ and $q(t)$, it is not clear at what point it will become optimal to sell rather than buy, as this depends on the interaction between trading frictions α and the arrival rate of the shock ρ . When trading frictions is very great (low α) as pictured in Panel (a) the price is expected to decline very slowly, which means that the expected return to “cashing-in-on-the-crash” is low. This is why it is optimal to sell at $t < T_\rho$ only if ρ is high enough, that is, to the right of the intersection between the two curves. On the other hand, when investors can access the market almost continuously – the scenario in

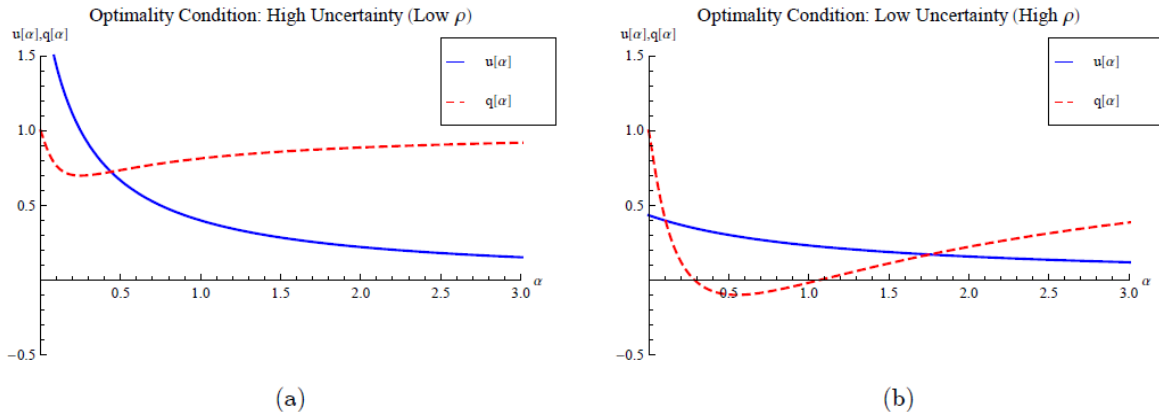


Figure 7: The effect of trading frictions α on speculators' incentives for low and high values of the persistence of uncertainty ρ .

Panel (b) – then the incentive to sell is greater when ρ is low so that the capital gain $q(t)$ is greater than the value $u_a(q)$ obtained by holding the asset.

To illustrate the second part of proposition 6, Figure 7 shows the two curves $u_a(a)$ and $q(t)$ as a function of the trading frictions α in two different scenarios: low uncertainty and high uncertainty (measured by the average time ρ before uncertainty is resolved). Panel (a) shows that if the shock is expected to come very far in the future (low ρ), then only traders with continuous access to the market have an incentive to profit from the price swings, because the speculators will have the opportunity to sell just before the shock and repurchase immediately afterward, forgoing the cash flows for a shorter period of time. Panel (b), instead, shows that speculators' incentives to profit from the crash vary in a non-monotonic fashion with the magnitude of trading frictions: only when α is very high or very low will speculators sell their holdings, amplifying the effect of the initial shock. Alternatively, when α is intermediate, speculators who have the opportunity to readjust their holdings anticipate that they will retain the assets for a longer time (since the average holding period of the asset is $1/\alpha$), which is not fully compensated by the capital gains in a scenario in which ρ is high (because the shock will hit before any significant movement in price). As a consequence, speculators choose to hold rather than sell.

Intuitively, the trading frictions play two opposing roles. First, they determine how fast the speculator can trade and exploit the price swing. Second, they determine how much the price will change before the shock is realized. Low trading friction always heightens the incentive to profit from the capital gain, because the trader can profit the most from the price swing thanks to the opportunity to sell when the price is high and buy back immediately following the shock when the price is at its minimum. This is the case, for instance, of traders who have practically continuous access to the market. At the same time Panel (b) shows

that “cashing-in-on-the-crash” can also occur when traders operate in markets in which it is difficult or costly to submit orders and have a greater incentive to take advantage of the price swing, because even if he may not get the chance to trade right after the shock, in a high-friction market (such as OTC markets) this is less of a concern because the price will recover very slowly, which means that the capital gains opportunity persists for a longer time. This case becomes more important when the uncertainty window is shorter, because the dividends forgone will be less. This no longer holds for intermediate values of α , and speculators will prefer to buy the asset and enjoy its dividend flows rather than selling it.

4 Equilibrium Uniqueness

Overview. The baseline version of the model, which assumes that all speculators have the same information about the severity of the shock θ when they have the opportunity to trade, shows that two types of equilibrium are possible. It also serves to analyze the market characteristics that influence speculators’ strategies. Now, instead, I examine how the results are affected when the severity of the shock changes over time. This section shows that a time-varying shock θ , by introducing some heterogeneity in the speculators’ perception of severity, leads to a unique equilibrium that has several intuitive properties.

Formally, I assume that the perceived severity of the shock θ starts at a value $\theta_0 > 0$ time $t = 0$ and then evolves according to the commonly observed geometric Brownian motion¹⁵

$$\frac{d\theta_t}{\theta_t} = \mu dt + \sigma dW, \tag{6}$$

where μ and σ are constants.¹⁶ The trend μ captures how the mean changes over time; the variance σ measures the size of the random component, specifically, how quickly θ spreads out. This captures the idea that investors expect a shock, but that the situation may either improve ($\mu < 0$) or deteriorate ($\mu > 0$) over time; and market conditions may evolve slowly (low σ) or quickly (high σ). Introducing this dynamic aspect of severity creates heterogeneity in the conditions under which speculators trade. Those trading at time t might observe a different θ than those trading at $t + dt$. In other words, they cannot be sure that others will hold the asset until the next trading opportunity because the expected shock can become

¹⁵Frankel and Burdzy (2005) show that a similar argument can be used when θ_t follows an arbitrary mean-reverting process with time-varying drift μ and volatility σ .

¹⁶A similar approach has been proposed by Frankel and Pauzner (2000) to derive uniqueness in a model of sectorial choice with external increasing returns. In our setting, the speculator’s payoff depends on the other speculators’ behavior through the market-clearing price. The endogeneity of the price provides novel insights absent in Frankel and Pauzner (2000). A related dynamic game is also provided by Dasgupta (2004).

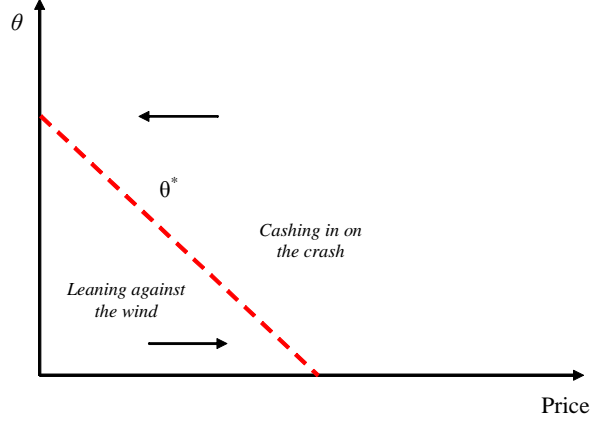


Figure 8: The unique equilibrium

more severe in the future, i.e. θ might increase.

Intuitively, such a set-up captures situations in which there is uncertainty about how severe the shock will be; for instance, it could depend on policy measures to counter the repercussions of the shock; or the uncertainty could simply reflect the fact that speculators do not know the full extent of the crisis at $t = 0$ but gradually discover it. For example, in the European debt crisis speculators expected the Greek economy to suffer large losses, but their extent depended crucially on several factors whose impact was unknown in early 2010, e.g. the ECB interventions and the Greek elections. In other words, a speculator trading in April 2010 (when Papandreou called for a rescue package) had a different information set from one trading in July (when the Greek Parliament passed the pension reform required by the European Union and the IMF) or in June 2011 (when the Greek debt was downgraded by Standard and Poor's from B to CCC). I exploit this heterogeneity to derive a unique equilibrium. In fact, it triggers a contagion argument that leads to equilibrium uniqueness as shown by the next proposition.

Proposition 7 (Unique Equilibrium) *When the severity of the shock θ follows (6) and $\underline{\theta}(p(t)) > 0$, then there exists a unique threshold $\theta^*(p(t))$, such that cashing-in-on-the-crash is optimal if and only if $\theta > \theta^*(p(t))$; while leaning-against-the-wind emerges as an equilibrium for $\theta < \theta^*(p(t))$.*

Figure 8 describes the main intuition behind the proposition. At the right of the threshold θ^* speculators start liquidating their position; while when they expect the severity of the shock to be smaller than θ^* , they find it optimal to buy or hold the asset. Intuitively, if the shock is expected to be sufficiently severe, namely $\theta > \theta^*$, capital gains will be greater and it will be optimal for the speculators to liquidate their positions. But when the shock is expected to be mild, namely $\theta < \theta^*$, they prefer to hold the asset or buy it below the fundamental price in order to capture the dividend flow.

The equilibrium depicted in Figure 8 has three main properties.

Property 1: *Fragility.* Marginal perturbations to speculators' perceptions about the shock's severity may have discontinuous effects.

Property 1 is the main implication of this equilibrium: when traders are uncertain about the future price path, financial markets become more fragile. The model captures fragility in two ways. First, the baseline model exhibits multiple equilibria so that fluctuations in market participants' sentiment can induce drastic changes in the provision of liquidity and in the resulting response to shocks during periods of high uncertainty. Second, the model outlined in this section shows how small changes to the speculators' perception of the severity of future shocks induce large changes in aggregate outcomes. In fact, whereas to the left of the threshold θ^* speculators respond to a decline in price by absorbing the excess supply, small changes to their perceptions of how severe the shock may be are sufficient to shift the parameter θ above the threshold, at which point they start liquidating their long positions and depress the price further by amplifying the initial shock. This also explains why traders may react differently under similar market conditions and why crashes may occur. Greenwood and Thesmar (2011), for instance, analyze the impact of speculative trading and arbitrage capital on stock volatility and find that while for some stocks, hedge funds seem to act as providers of liquidity, by trading in the opposite direction to mutual fund flow-driven trades, they consistently exacerbates fluctuations for other stocks. Our theory provides one plausible explanation for why arbitrageurs might behave differently depending on stock and market characteristics, which is also consistent with the *comomentum* measure of arbitrage activity provided by Lou and Polk (2013).

Property 2: *Dynamics.* The price declines to the right of the threshold θ^* and increases to the left.

Property 2 shows that the equilibrium also has implications for the price path. The arrows in Figure 8 show that to the right of the threshold θ^* the price will decrease over time, because that is the region in which speculators are selling off their holdings, putting pressure on the price. To the left of θ^* , instead, speculators' demand for the asset will push the price up over time. In contrast to the previous subsection, I can now obtain more precise predictions about the price path.

However, the threshold θ^* depends on the equilibrium price and this is highlighted by the next property.

Property 3: *Trend.* Bull markets are more likely than bear markets to undergo waves of selling pressure due to expectations of the same shock in the future.

A further implication of this analysis is that one should expect speculators to react differently depending on market conditions: in a bull market, when the price is high, it is more likely that a small amount of uncertainty will prompt speculators to sell. In other words, amplification of market shock is more likely when the market has just gone through a bout of euphoria, driving prices up, which resembles the mechanism proposed by [Brunnermeier et al. \(2008\)](#) in the analysis of carry trades in exchange markets. Since the price is already high, even a small dose of uncertainty heightens the incentive to sell and realize a capital gain, which also means that a smaller shock will be sufficient to induce speculators to amplify the initial shock. In a bear market, by contrast, the cash flow from holding the asset is more attractive because the price is too low to generate any significant capital gain. This finding is interesting because it implies that the prices of assets with correlated fundamentals could evolve very differently depending on how high each asset price is to begin with.

5 Microfoundation of Long-Term Investors' Demand

This section provides a microfoundation for the downward sloping demand curve employed in the baseline model. This has two objectives. First, it shows that the reduced-form assumption can indeed be rationalized as the outcome of a fully-fledged equilibrium outcome between speculators and dealers. Second, this microfoundation also shows that the speculators' ability to take advantage of expected shocks to the asset supply does not stem from the myopia of their counterparties.

I generalize the baseline model by assuming that the speculators trade with a representative risk-averse dealer who faces inventory risk, and that the asset's supply is noisy, for instance because noise traders might have access to this asset as well. Formally, I follow [Garleanu et al. \(2009\)](#) and [Dasgupta et al. \(2011\)](#) in assuming that if a dealer agrees to buy I shares at price P , then his utility will be given by

$$\mathbb{E}_t \left[\int_t^\infty e^{-q(s-t)} U(C_s) ds \right],$$

where q is the dealer's subjective discount rate and $u(\cdot)$ is his concave utility function. For simplicity, I assume that the dealer has constant absolute-risk aversion γ . The dealer's budget constraint is given by

$$dW_t = (rW_t - C_t) dt + I_t (dP_t + \delta dt - rP_t dt),$$

where I_t is the inventory level at time t , C_t his consumption and, as in the baseline model,

δ is the dividend paid by the asset and P_t its price. We also extend the model by assuming that the asset's supply $S(t)$ follows a mean-reverting process:

$$dS(t) = \kappa (\bar{S} - S(t)) dt + \sigma^s dZ(t), \quad (7)$$

where \bar{S} is the long-run average supply, κ is the mean-reversion speed, and σ^s is its volatility. This assumption is needed in order to induce the optimal pricing function to exhibit demand pressure. In fact, the reason why the dealer needs to be compensated for trading with the speculators derives from his aversion to the risk of future price changes, which results from both θ and σ^s . A similar form for the supply of noise traders (7) has previously been used by Wang (1993), Campbell and Kyle (1993b) and Kyle and Xiong (2001). We can now show the following result:

Proposition 8 (Dealer's Pricing Function) *The dealer sets the price as a function of the asset's supply $S(t)$ and the speculators' demand $x(t)$ according to the following:*

$$P(t) \approx a - c(S(t) - \frac{b}{c}x(t)),$$

where a , b and c are constants whose expressions are provided in the appendix.

Proposition 8 shows that, in equilibrium, the dealer employs a similar pricing function to the one assumed in the baseline model. In fact, we can define $\lambda = 1/c$ and we have in this case exactly the same expression for the price as the one used in the baseline model, except that the speculators' orders are weighted by the coefficient b/c . Intuitively, the dealer quotes a higher price in response to the speculators demand pressure as $c, b > 0$. Moreover, as the dealer's risk aversion γ increases, both b and c decrease, i.e. as the dealer becomes more risk-averse, he will make the price less sensitive to the potential supply shocks in order to minimize his inventory risk.

In sum, given this demand function, we can follow the same steps described in Section 3 to show that the interaction between profit-maximizers dealers and speculators would generate the same equilibrium dynamics as the one presented in the baseline model. We can show the following result:

Proposition 9 *When dealers optimally set the asset's price but face inventory risk, multiple equilibria might arise.*

Proposition 9 shows that the speculators' attempt to profit from the potential crash might lead to multiple equilibria even when dealers set the price optimally. In contrast to the baseline

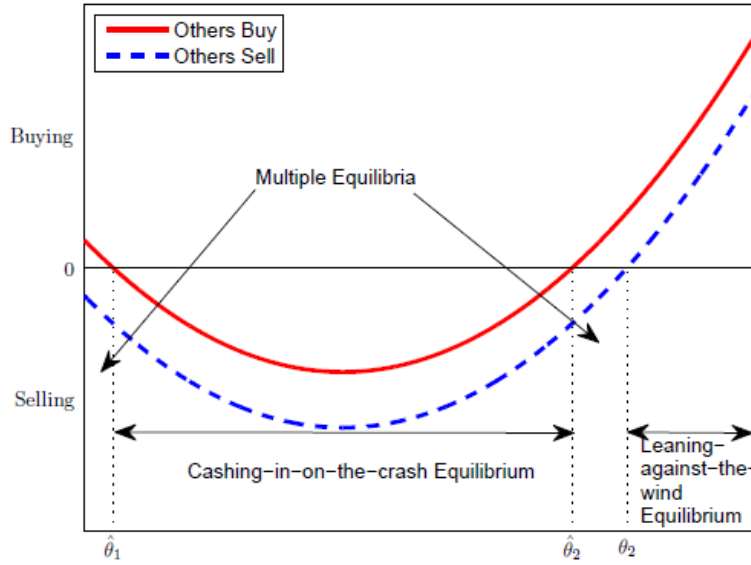


Figure 9: The figure shows the region of parameters for which multiple equilibria arise when dealers optimally set the asset's price.

model, the possibility for the dealers to adjust the price sensitivity to the speculators' demand might lead to two different regions of parameters for which both types of equilibria coexist.

This case is depicted in Figure 9. It plots the speculator's payoffs both when he expects the other arbitrageurs to buy and to sell in the future. When both payoffs are negative (i.e. they lie below the horizontal zero-line in the figure), then it is optimal for the speculator to sell; conversely, when both curves are above zero it is optimal to purchase the asset. However, in contrast to the baseline model, the speculators' response is not a linear function of the shock size θ . The reason is that now, whenever the shock is expected to be more severe, dealers can adjust the price-setting equation so that the price becomes less sensitive to the asset's supply shocks. This means that when the shock is expected to be very high, the price respond less to the investors' demand and so the "leaning against the wind" equilibrium emerges. On the contrary, for more intermediate values of the shock θ , multiple equilibria might arise depending on the expectation about the trading behavior of the other speculators. As before, there is also a region for which the "cash in the crash" equilibrium is the only one that can emerge. For smaller shocks, instead, both types of equilibrium can be justified; since the dealers do not fear a large fluctuation in the price due to the shock being small, and they set the price according to a function that exhibit higher sensitivity to the investors' demand. Overall, these results in addition to showing that the main insights of the baseline model with a reduced-form demand function are valid in a more general setting, also provide new insights due to the presence of market makers.

5.1 Discussion of Other Assumptions

In this section, I discuss the other simplifying assumptions of the model.

Multiple shocks. In the model I have considered the case of a single potential shock. This is clearly an important simplification, as uncertainty may not be suddenly resolved after the random time T_ρ . However, the model could accommodate multiple shocks with exponentially distributed arrival rate. The main difference would be that with multiple shocks the speculators have a greater incentive to sell and wait to repurchase until the last shock, as the price will continue to fall due to multiple shocks. In other words, in the *cashing-in-on-the-crash* equilibrium the recovery phase will start only when the speculators expect no more shocks to occur.

Fundamental shocks. To highlight the speculators' strategic motive, I have focused on shocks that do not affect the cash flow or fundamental value of the asset δ .¹⁷ This allows me to show that the amplification of market shocks does not necessarily depend on the speculators' precautionary motive but can be the result of their desire to profit from the shocks. The model can be extended to allow for a fundamental shock, for instance, by assuming that at some random time in the future the cash flow of the asset, δ , may decrease to a lower value $\underline{\delta}$. Interestingly, while the speculators would have a greater incentive to sell in anticipation of this shock, as now they fear the decline in value, they would not buy it back again if a sufficiently severe shock occurs, e.g. $\underline{\delta} = 0$. In the case of a small shock, e.g. $\delta > \underline{\delta} > 0$, instead, the price after the shock would converge to a different fundamental value $\underline{\delta}/r$. Hence, the empirical predictions of this model are different from those of a model in which the asset's value is affected, which means that the different mechanisms can be empirically disentangled.

Heterogeneous valuation. To simplify the analysis, it is assumed that the only sources of heterogeneity among speculators are the time at which they are able to trade and, as in Section 4, the information they possess when they trade. However, I could assume that speculators value the asset differently, for instance, because they have different hedging motives. The main qualitative result of the paper would still hold, namely that speculators would still find it optimal to react to an expected price fall by selling their holdings; the difference is that there would not exist a unique price p^* at which the trading comes to a complete halt, because if speculators have heterogeneous valuations, there is no unique price at which all are indifferent

¹⁷In the asset pricing literature an increase in uncertainty is usually captured by time-varying second moments of dividend or consumption growth (see for instance, Veronesi (1999), Bansal and Yaron (2004) and Bloom (2009) or, recently, on uncertainty about the impact of government interventions Pastor and Veronesi (2011)). In my model, instead, the fundamental value of the asset does not change, but its price may be severely affected by jumps in supply. This way of modeling uncertainty is not intended to capture uncertainty about the fundamental value, but uncertainty about market conditions, namely the price at which the asset can be traded.

between selling and buying.

State-contingent λ . In the model the price impact is assumed to be constant, in the spirit of Kyle (1985). However, it is plausible that market liquidity might change after T_ρ depending on the realization of the shock. For instance, λ could be higher if the shock is realized than if it is not. This might capture situations in which long-term investors are less willing to provide liquidity to other market participants after a fire sale, for instance, because they fear subsequent shocks. If the price impact λ is greater after time T_ρ when the shock is realized, then the only difference from the current framework is that the price would converge to the fundamental value after T_ρ more rapidly, since speculators would have a greater price impact in the recovery phase.

6 Empirical Implications

My theory yields a wealth of empirical implications. I now discuss some of them in light of recent empirical evidence on the role of institutional investors during the recent financial crisis.

Prediction 1 *Liquidity tends to evaporate abruptly when uncertainty increases. And in such periods the returns to liquidity provision increase.*

Prediction 2 *All else equal, speculators are more likely to amplify fluctuations when the expected future shock is large, more likely and nearer in time.*

Prediction 3 *All else equal, speculators are more likely to amplify fluctuations in a bull than in a bear market; or equivalently, when they are initially more highly exposed.*

Prediction 4 *All else equal, speculators are more likely to amplify negative shocks in markets for less liquid assets.*

The evaporation of liquidity (Prediction 1) follows from the “cashing-in” equilibrium of Proposition 4 and is a consequence of the coordinated trading behavior of the speculators highlighted in Proposition 3. The sudden increase in the returns to liquidity-provision, instead, follows from the fact that the fundamental value of the asset is unaffected while its price is driven down by speculation. The sudden evaporation of liquidity was observed in many sectors of financial markets in 2007-09. Gorton and Metrick (2009) argue that several debt instruments became more information-sensitive, which amplified asymmetric information problems. I propose a complementary hypothesis: speculators reduced the liquidity supply because they sought to take advantage of further shocks, which amplified the initial uncertainty shock.

Nagel (2012) provides evidence consistent with Prediction 1, finding that during the recent crisis, the returns to liquidity-provision increased significantly and were closely correlated with the VIX index. Most likely, the VIX proxies for the underlying state variables that drive the market’s demand for liquidity. Nagel (2012) decomposes the VIX into conditional volatility and a volatility risk premium and shows that both components predict reversal strategy returns, with conditional volatility performing somewhat better. Moreover, several proxies for liquidity supply factors predict positive returns to the reversal strategy. This seems to confirm that what matters is not only news on fundamentals, which could have increased the adverse selection in financial markets, but that some potential arbitrageurs deliberately sought to profit from the potential crash.

Prediction 2 follows from speculators’ trade-off between asset cash-flow and capital gains and is discussed in Section 3.3. The trading behavior hypothesized seems to be confirmed empirically by Ellul et al. (2013). During episodes of market turmoil – dating back to the 1987 market crash, – Ellul et al. (2013) show that investors with short horizons (as proxied by portfolio turnover) sell their stock holdings to a larger extent than those with longer trading horizons. This creates price pressure on the stocks mostly held by short-horizon investors, which therefore experience larger price drops and sharper reversals than stocks mostly held by long-horizon investors. The evidence indicates that investors with short horizons amplify the effects of negative market-wide shocks, as the model predicts, by demanding liquidity when other potential buyers’ capital is scarce. And these effects are larger when the expected shock, or market uncertainty, is greater (as captured by the comparison between the “black Monday” crash of 1987 and the Lehman Brothers’ collapse in 2008). Relatedly, Gao et al. (2014) show that there are hedge fund managers with better skills of exploiting the market’s ex ante rare disaster concerns, which may not realize as disaster shocks ex post, who are able to deliver superior future fund performance. These rare disaster concerns considered by Gao et al. (2014) are captured by the news shock in the model, and their evidence shows that it is possible to profitably take advantage of these shocks.

My results concerning the role of speculators in stabilizing financial markets related more broadly to a strand of the literature that explores institutional investors’ behavior during financial crises. Ben-David et al. (2012), for instance, show that, only part of hedge funds’ substantial reduction of equity holdings during the crisis in 2008 is explained by the need to meet redemptions.¹⁸ This empirical finding suggests the possibility that at least a fraction of the liquidation is driven by strategic motives.

¹⁸Boyson et al. (2014) have analyzed similar data, but with a smaller set of hedge funds; they confirm that during the recent crisis hedge funds sold more equity holdings than required merely to face redemptions.

Suggestive evidence about my mechanism, which relies on the interaction between speculators and long-term investors, comes from the evidence in [He et al. \(2010\)](#). They show that while hedge funds were deleveraging as the financial turbulence mounted, commercial banks significantly increased their asset holdings, absorbing the excess supply generated by the funds' sales. This suggests that in that context the role of long-term investors posited in my model is played by the commercial banking sector.

Prediction 3 follows from the Property 3 of the equilibrium presented in Proposition 7, whereas Prediction 4 follows from the comparative statics results presented in Proposition 6. These predictions suggest that the characteristics of the markets play an important if somewhat surprising role in shaping speculators' incentives. In particular, Prediction 3 posits that amplification of market shock is more likely when the market has just gone through a bout of euphoria, driving prices up. This resembles the mechanism proposed by [Brunnermeier et al. \(2008\)](#) in the analysis of carry trades in exchange markets. Uncertainty shocks have a more severe effect on speculators' incentives to hold an asset when in the aggregate they are more exposed to it. In the model, this corresponds to the fact that since more speculators are holding the asset at $t = 0$ and are not able to liquidate instantaneously due to trading friction, it will take time before they have sold all their holdings. This increases the expected capital gains to speculators from continuing to trade in the same direction as the shock.

Furthermore, Prediction 4 suggests that market illiquidity can be aggravated by speculators' behavior: their incentives to liquidate in the drive for the potential capital gains are greater in illiquid markets. This suggests that the mechanism hypothesized here should be more relevant in less liquid markets, such as those in which long-term investors are reluctant to further absorb excess asset supply without significant price concessions. Suggestive empirical evidence is provided by [Manconi et al. \(2012\)](#) and [Mitchell and Pulvino \(2012\)](#). [Manconi et al. \(2012\)](#) show that the investors more exposed to securitized bonds that fell sharply in price, sold more bonds and contributed to the price downswing. Consistent with Prediction 4, they find that the investors who expect liquidity shocks retain liquid assets and sell those that have relatively high temporary price impact. Similarly, [Mitchell and Pulvino \(2012\)](#) focus on a set of value strategies during the 2008-2009 period, and argue that the disappearance of long-term financing caused arbitrageurs to withdraw liquidity from these markets, generating further price divergence.

7 Conclusion

I propose a model with long-term investors and speculators, who both participate in a market characterized by trading friction. During periods of market turmoil and even though the

long-term value of the asset remains unchanged, speculators react to declining asset prices by liquidating asset holdings, thus amplifying price fluctuations. Moreover, I show that less trading friction is associated with sharper asset price decline and faster recovery after the shock occurs. I provide a framework for formal analysis of the link between periods of high uncertainty and the fragility of financial markets. I show that small shifts in investors' perception of the severity of future negative shocks might have discontinuous effects. The model provides novel results about the role played by arbitrageurs in financial markets, specifically, about under which conditions their strategies might lead to the amplification of market shocks. Moreover, in contrast to most of the existing literature, these limits to arbitrage do not stem from behavioral biases or capital constraints, but are the results of a deliberate attempt of the arbitrageurs to take advantage of potential deviation of prices from fundamentals.

The main insights can be applied to other matters, such as the dynamics of housing prices. Recent papers by [Haughwout et al. \(2011\)](#) and [Bayer et al. \(2011\)](#) identify real-estate investors as a key factor in the house price surge and crash. My main result suggests that when house prices turned down in 2006, real-estate investors did not know whether the decline was temporary or the start of the bust, and they might have reacted to the heightened uncertainty by selling, provoking a more severe bust in the states where these investors were significant participants.

8 Appendix – Proofs

Proof of Lemma 1.

I can rewrite the value function (2) as

$$V(a, t) = \mathbb{E}_t \left[\int_t^{T_\alpha} e^{-r(s-t)} \delta a ds + e^{-r(T_\alpha-t)} \left\{ p(T_\alpha) a + \max_{a'} V(a', T_\alpha) - p(T_\alpha) a' \right\} \right],$$

then subtracting $p(t)a$ and ignoring the terms that do not depend on a the problem of a speculator who gains access to the market at time t is given by

$$\max_{a' \geq 0} \int_t^{T_\alpha} e^{-r(s-t)} \delta a' ds - \{p(t) - \mathbb{E} [e^{-r(T_\alpha-t)} p(T_\alpha)]\} a'. \quad (8)$$

The speculator chooses his asset holdings in order to maximize the expected present discounted value of his utility flow net of the expected present discounted value of the cost of holding the asset from time t until the next time T_α , when he can readjust his holdings. Then, I compute the first term of (8) as

$$u(a) = \mathbb{E} \left[\int_0^{T_\alpha-t} e^{-rs} \delta a ds \right] = a \frac{\delta}{r + \alpha}, \quad (9)$$

where the expectation is over the random variable $T_\alpha - t$. The expected discounted price of the asset at the next time when the speculator gets an opportunity to trade can be written as

$$\mathbb{E} [e^{-r(T_\alpha-t)} p(T_\alpha)] = \alpha \int_0^\infty e^{-(r+\alpha)s} p(t+s) ds. \quad (10)$$

Substitute (9) and (10) into (8) to obtain the formulation of the speculator's problem in the statement of the lemma. ■

Proof of Lemma 2.

I can follow the same steps as in Lemma 1 to simplify the speculators' problem at $t < T_\rho$. I first compute the expected utility flows that the speculator experiences by holding portfolio a . Since the trading opportunities and the arrival of the shock follow independent Poisson processes, $T_\alpha - t$, and $T_\rho - t$ are exponentially distributed random variables with means $1/\alpha$, and $1/\rho$, respectively. Define $T_{\alpha\rho} = \min\{T_\alpha, T_\rho\}$. Then, the utility flows is

$$\begin{aligned} u^U(a) &= \mathbb{E} \left[\int_0^{T_{\alpha\rho}-t} e^{-rs} \delta a^U ds \right] = \mathbb{E} \int_0^\infty \left[\int_0^{\tau_{\alpha\rho}} e^{-rs} \delta a^U ds \right] (\alpha + \rho) e^{-(\alpha+\rho)\tau_{\alpha\rho}} d\tau_{\alpha\rho} \\ &= a^U \left(\frac{\delta}{r + \alpha + \rho} \right) \end{aligned}$$

One sees that this is exactly the same equation as in Lemma 1, except that now α is replaced

by $\alpha + \rho$, which takes into account the possibility to enter the after-shock phase.

To derive the expected value of the price, I use the fact that $T_\alpha - t$ and $T_\rho - t$ are two independent exponentially distributed random variables:

$$\begin{aligned}
& \mathbb{E} \left[e^{-r(T_\alpha - t)} (\mathbb{I}_{\{T_\alpha < T_\rho\}} p^U(T_\alpha) + \mathbb{I}_{\{T_\alpha > T_\rho\}} p^U(T_\alpha | T_\rho)) \right] \\
&= \int_t^\infty \int_t^\infty e^{-r(\tau_\alpha - t)} (\mathbb{I}_{\{\tau_\alpha < \tau_\rho\}} p^U(\tau_\alpha) + \mathbb{I}_{\{\tau_\alpha > \tau_\rho\}} p^U(\tau_\alpha | \tau_\rho)) \alpha e^{-\alpha(\tau_\alpha - t)} \rho e^{-\rho(\tau_\rho - t)} d\tau_\rho d\tau_\alpha \\
&= \int_t^\infty e^{-r(\tau_\alpha - t)} \left[e^{-\rho(\tau_\alpha - t)} p^U(\tau_\alpha) + \int_t^{\tau_\alpha} \rho e^{-\rho(\tau_\rho - t)} p^U(\tau_\alpha | \tau_\rho) \right] \alpha e^{-\alpha(\tau_\alpha - t)} d\tau_\rho d\tau_\alpha
\end{aligned}$$

this completes the proof of lemma 2. ■

Proof of Proposition 1.

Given the results of Lemma 1 and 2 I can compute the optimal trading decision for a speculator who has the opportunity to submit his orders at time t . Let me start with $t > T_\rho$, there are two cases. First, the shock did not occur, in this case the price remains at its initial value $p(0)$, which means that a speculator is indifferent between buying and selling the asset. Second, uncertainty is resolved with the occurrence of the shock lowering the price to $p(T_\rho) = \frac{\delta}{r} - \lambda(S + \theta)$. In this case, it is strictly optimal to buy the asset as soon as he gets the opportunity to do so. In fact, consider a speculator who does not own the asset and contacts the market at time $t > T_\rho$. If he behaves according to the prescribed trading plan of purchasing the asset at time t and afterwards follow the optimal policy of keeping the asset, his value is $\frac{\delta}{r} - p(t)$. To check optimality, by the Bellman principle it suffices to rule out one-stage deviations, whereby a speculator deviates once from the prescribed plan and follows it thereafter. If the speculator deviates once by not purchasing the asset, his value is

$$V(0, t) = E \left[e^{-r(\tau - t)} \left(\frac{\delta}{r} - p(t) \right) \right]$$

where τ is the next time the speculator will have the opportunity to contact the dealers. This is strictly less than $\frac{\delta}{r} - p(t)$ because of discounting. Hence, it is optimal to buy the asset back once uncertainty has been resolved, i.e. after T_ρ .

Next, let me consider what happens when the speculator has the opportunity to trade at $t < T_\rho$. In a market populated by a single speculator, I can easily compute the price path, which in turns determines the capital gains or losses the speculator compares the cash flows with. The expected price is given by

$$p^U(T_\alpha | T_\rho) = (1 - \varepsilon) p^R(T_\alpha | T_\rho) + \varepsilon p^S(T_\alpha | T_\rho)$$

where $p^S(T_\alpha|T_\rho) = \frac{\delta}{r} - \lambda(S + \theta)$ is the price in the case of the realization of the negative shock whereas $p^R(T_\alpha|T_\rho) = p(0)$ is the price if no supply shock occurs. I can then use Lemma 2 to find the value of the severity of the shock θ that makes the investor indifferent between buying and selling the asset. Formally, the threshold $\hat{\theta}$ is given by the following expression

$$\begin{aligned}
\frac{\delta}{r + \alpha + \rho} &= [p^U(t) - \int_t^\infty \alpha e^{-(r+\alpha)(\tau_\alpha-t)} (e^{-\rho(\tau_\alpha-t)}) p^U(\tau_\alpha) + \int_t^{\tau_\alpha} \rho e^{-\rho(\tau_\rho-t)} p(\tau_\alpha|\tau_\rho) d\tau_\rho] \\
&= [p^U(t) - \left\{ \int_t^\infty \alpha e^{-(r+\alpha)(\tau_\alpha-t)} (e^{-\rho(\tau_\alpha-t)}) p^U(t) \right. \\
&\quad \left. + \int_t^{\tau_\alpha} \rho e^{-\rho(\tau_\rho-t)} (\varepsilon(F - \lambda(S + \theta^*)) + (1 - \varepsilon)F) d\tau_\rho \right\}] \\
&= [p^U(t) - \left\{ \int_t^\infty \alpha e^{-(r+\alpha)(\tau_\alpha-t)} (e^{-\rho(\tau_\alpha-t)}) p^U(t) \right. \\
&\quad \left. + \int_t^{\tau_\alpha} \rho e^{-\rho(\tau_\rho-t)} (F - \varepsilon\lambda(S + \theta^*)) d\tau_\rho \right\}] \\
&= p^U(t) - \frac{\alpha p^U(t)}{r + \alpha + \rho} + \frac{\alpha(F - \varepsilon\lambda(S + \hat{\theta}))}{r + \alpha + \rho} - \frac{\alpha(F - \varepsilon\lambda(S + \hat{\theta}))}{r + \alpha} \\
&= \frac{p^U(t)(r + \rho)}{r + \alpha + \rho} - \alpha(F - \varepsilon\lambda(S + \hat{\theta})) \frac{\rho}{(r + \alpha + \rho)(r + \alpha)},
\end{aligned}$$

where I have substituted the prices in the different regions and simplified. Notice that as θ increases, the capital gain $q^c(t)$ increases, which makes more profitable to exploit the price swings rather than holding the asset until maturity. I can further solve to obtain a closed-form expression for the threshold:

$$\hat{\theta} = \left[\frac{\delta(r + \alpha) + \alpha \left(\frac{\delta}{r} - \varepsilon\lambda S \right) \rho - p_0(r + \rho)(r + \alpha)}{\varepsilon\lambda\alpha\rho} \right].$$

Hence, the speculator is going to sell the asset at $t < T_\rho$ if and only if $\theta > \hat{\theta}$. ■

Proof of Corollary 1.

The threshold is increasing in the asset's dividends δ . The effects of the other parameters of interest can be found as follows. First, the effect of the market depth λ on the capital gain in expression (11) is clearly positive as :

$$\frac{\partial q}{\partial \lambda} = \alpha\varepsilon(S + \theta) \frac{\rho}{(r + \alpha + \rho)(r + \alpha)} > 0$$

Hence steeper demand curves of the long-term investors increase the speculator's incentive to sell when he expects negative shocks in the future.

The effect of the persistence of uncertainty is computed as

$$\begin{aligned}\frac{\partial \theta^*}{\partial \rho} &= \frac{[\alpha \left(\frac{\delta}{r} - \varepsilon \lambda S\right) - \left(\frac{\delta}{r} - \lambda S\right) (r + \alpha)] \varepsilon \lambda \alpha \rho - \varepsilon \lambda \alpha [\delta (r + \alpha) + \alpha \left(\frac{\delta}{r} - \varepsilon \lambda S\right) \rho - \left(\frac{\delta}{r} - \lambda S\right) (r + \rho) (r + \alpha)]}{(\varepsilon \lambda \alpha \rho)^2} \\ &= \frac{-\left(\frac{\delta}{r} - \lambda S\right) \rho - \delta + \left(\frac{\delta}{r} - \lambda S\right) (r + \rho)}{(\varepsilon \lambda \alpha \rho)^2} = \frac{-\lambda S r}{(\varepsilon \lambda \alpha \rho)^2} < 0\end{aligned}$$

Hence, higher persistence of uncertainty (low ρ) reduce the speculator's incentive to liquidate his holdings at $t < T_\rho$.

Finally, the effect of the trading frictions α can be computed as follows

$$\begin{aligned}\frac{\partial \theta^*}{\partial \alpha} &= \frac{(\delta + \left(\frac{\delta}{r} - \varepsilon \lambda S\right) \rho - \left(\frac{\delta}{r} - \lambda S\right) (r + \rho)) \varepsilon \lambda \alpha \rho - \varepsilon \lambda \rho [\delta (r + \alpha) + \alpha \left(\frac{\delta}{r} - \varepsilon \lambda S\right) \rho - \left(\frac{\delta}{r} - \lambda S\right) (r + \rho) (r + \alpha)]}{(\varepsilon \lambda \alpha \rho)^2} \\ &= \frac{-\varepsilon \lambda r \rho \delta + \left(\frac{\delta}{r} - \lambda S\right) (r + \rho) \varepsilon \lambda \rho r}{(\varepsilon \lambda \alpha \rho)^2} = \text{sign} \left(\frac{\delta}{r} \rho - \lambda S (r + \rho) \right)\end{aligned}$$

Then, as long as δ is small enough, it is true that $\frac{\partial \theta^*}{\partial \alpha} < 0$. ■

Proof of Lemma 3.

I employ the evolution of the fraction of speculators who own the asset, to derive the evolution of the price path in the two types of equilibrium. In the cashing-in-on-the-crash equilibrium, speculators start selling in the interval $[0, T_\rho)$. Then, we know from (1) that the price evolves according to:

$$\dot{p}(t) = \lambda \dot{x}(t) = -\alpha \lambda x(t),$$

where we can employ the market clearing condition to rewrite the fraction of speculators $x(t)$ as

$$\lambda x(t) = p(t) - \frac{\delta}{r} + \lambda S,$$

which implies

$$\dot{p}(t) = -\alpha \left(p(t) - \frac{\delta}{r} + \lambda S \right).$$

I can solve the differential equation with initial condition $p(0) = p_0$ to obtain:

$$p(t) = \left(\frac{\delta}{r} - \lambda S \right) (1 - e^{-\alpha t}) + e^{-\alpha t} p_0.$$

However, I am interested in computing the price at time $t + s$, which can be obtained by first changing variables $\hat{p}(t) \equiv [p(t) - \left(\frac{\delta}{r} - \lambda S\right) (1 - e^{-\alpha t})] \frac{1}{p_0} = e^{-\alpha t}$ and then by noting that the

price in the next instant can be written as

$$\hat{p}(t+s) = e^{-\alpha t} e^{-\alpha s} = \left[p(t) - \left(\frac{\delta}{r} - \lambda S \right) (1 - e^{-\alpha t}) \right] \frac{1}{p_0} e^{-\alpha s}.$$

I can now revert the change of variables to get

$$p(t+s) = p(t) e^{-\alpha s} + \left(\frac{\delta}{r} - \lambda S \right) (1 - e^{-\alpha s}),$$

which is decreasing over time. Similarly, we can get the other expression in the text of the lemma by starting from $\dot{p}(t) = \lambda \alpha (1 - x(t))$, that is

$$\dot{p}(t) = \lambda \alpha - \lambda \alpha x(t) = \lambda \alpha - \alpha \left(p(t) - \frac{\delta}{r} + \lambda S \right),$$

which gives as a solution

$$p(t) = \left(\frac{\delta}{r} - \lambda S \right) (1 - e^{-\alpha t}) + \lambda (1 - e^{-\alpha t}) + e^{-\alpha t} p_0.$$

I can follow the same steps shown above for the other case to get the result stated in Lemma 3:

$$p(t+s) = p(t) e^{-\alpha s} + \lambda (1 - e^{-\alpha s}) + \left(\frac{\delta}{r} - \lambda S \right) (1 - e^{-\alpha s}),$$

which is increasing over time. ■

Proof of Proposition 3.

I start by conjecturing the price path in the two equilibria, and then I find the optimal trading strategies for the speculators and the conditions under which their trading strategies do indeed generate those price paths.

Consider a speculator who has the opportunity to trade at time $t < T_\rho$. To simplify notation, define $F = \frac{\delta}{r}$.

Part (i). I first find the conditions under which *cashing-in-on-the-crash* is an equilibrium. From Lemma 2, we know that it is optimal to sell the asset if and only if

$$u_a^c(a) = \frac{\delta}{r + \alpha + \rho} < q^c(t) = p^c(t) - \tilde{p}(\tau_\alpha),$$

where

$$\tilde{p}(\tau_\alpha) = \int_t^\infty \alpha e^{-(r+\alpha)(\tau_\alpha-t)} (e^{-\rho(\tau_\alpha-t)}) p^c(\tau_\alpha) + \int_t^{\tau_\alpha} \rho e^{-\rho(\tau_\rho-t)} p(\tau_\alpha | \tau_\rho) d\tau_\rho d\tau_\alpha. \quad (12)$$

I know that $p^c(\tau_\alpha) < p^c(t)$ because in this region the price is strictly decreasing. I start simplifying the terms in expression (12); using Lemma 3 we can rewrite the first term as

$$\begin{aligned}
& \int_t^\infty \alpha e^{-(r+\alpha)(\tau_\alpha-t)} (e^{-\rho(\tau_\alpha-t)} p^c(\tau_\alpha)) d\tau_\alpha \\
&= \int_t^\infty \alpha e^{-(r+\alpha+\rho)(\tau_\alpha-t)} p^c(\tau_\alpha) d\tau_\alpha \\
&= \int_t^\infty \alpha e^{-(r+\alpha+\rho)(\tau_\alpha-t)} (p^c(t) e^{-\alpha(\tau_\alpha-t)} + (F - \lambda S) (1 - e^{-\alpha(\tau_\alpha-t)})) d\tau_\alpha \\
&= \alpha \left(\frac{p^c(t)}{r + 2\alpha + \rho} + \frac{\alpha (F - \lambda S)}{(r + \alpha + \rho)(r + 2\alpha + \rho)} \right).
\end{aligned} \tag{13}$$

The second term in (12) takes into account that the speculator might come into contact with the market after T_ρ , which means that the price is a weighted average of the price after a shock (with weight ε) and of the price once the shock reveals to be temporary (with weight $(1 - \varepsilon)$):

$$\begin{aligned}
p(\tau_\alpha | \tau_\rho) &= \varepsilon (p(t) e^{-\alpha(\tau_\alpha-t)} + (1 - e^{-\alpha(\tau_\alpha-t)}) (F - \lambda(S + \theta) + \lambda)) \\
&\quad + (1 - \varepsilon) (p(t) e^{-\alpha(\tau_\alpha-t)} + (1 - e^{-\alpha(\tau_\alpha-t)}) (F - \lambda S + \lambda)) \\
&= p(t) e^{-\alpha(\tau_\alpha-t)} + (1 - e^{-\alpha(\tau_\alpha-t)}) (F - \lambda S + \lambda) - \varepsilon \lambda \theta (1 - e^{-\alpha(\tau_\alpha-t)}),
\end{aligned}$$

the previous expression also shows that the capital gain $q^c(t)$ is increasing in θ . I can use the previous expression to rewrite the second term in (12) as follows

$$\begin{aligned}
& \int_t^\infty \alpha e^{-(r+\alpha)(\tau_\alpha-t)} \left(\int_t^{\tau_\alpha} \rho e^{-\rho(\tau_\rho-t)} p(\tau_\alpha | \tau_\rho) d\tau_\rho \right) d\tau_\alpha \\
&= \int_t^\infty \alpha e^{-(r+\alpha)(\tau_\alpha-t)} \left(\int_t^{\tau_\alpha} \rho e^{-\rho(\tau_\rho-t)} \left[\begin{array}{c} p(t) e^{-\alpha(\tau_\alpha-t)} + (1 - e^{-\alpha(\tau_\alpha-t)}) (F - \lambda S + \lambda) \\ - \varepsilon \lambda \theta (1 - e^{-\alpha(\tau_\alpha-t)}) \end{array} \right] d\tau_\rho \right) d\tau_\alpha \\
&= \int_t^\infty \alpha e^{-(r+\alpha)(\tau_\alpha-t)} (p(t) e^{-\alpha(\tau_\alpha-t)} (1 - e^{-\rho(\tau_\alpha-t)}) + (1 - e^{-\rho(\tau_\alpha-t)}) (1 - e^{-\alpha(\tau_\alpha-t)}) (F - \lambda S + \lambda) \\
&\quad - \varepsilon \lambda \theta (1 - e^{-\rho(\tau_\alpha-t)}) (1 - e^{-\alpha(\tau_\alpha-t)})) d\tau_\alpha \\
&= (\alpha p^c(t) \left(\frac{\rho}{(r + 2\alpha)(r + 2\alpha + \rho)} \right) + \alpha (F - \lambda S + \lambda) \left(\frac{\alpha}{(r + \alpha)(r + 2\alpha)} - \frac{\alpha}{(r + 2\alpha + \rho)(r + \alpha + \rho)} \right) \\
&\quad - \alpha \varepsilon \lambda \theta \left(\frac{\alpha}{(r + \alpha)(r + 2\alpha)} - \frac{\alpha}{(r + 2\alpha + \rho)(r + \alpha + \rho)} \right)).
\end{aligned} \tag{14}$$

I can now define threshold $\underline{\theta}$ as the one that equates the cash flows with the capital gain:

$$\begin{aligned} \frac{\delta}{r + \alpha + \rho} &= p^c(t) - \tilde{p}(\tau_\alpha) \\ &= p^c(t) - (\alpha p^c(t) \left(\frac{\rho}{(r + 2\alpha)(r + 2\alpha + \rho)} \right) + \alpha(F - \lambda S + \lambda) \mathbb{H} + \alpha \varepsilon \lambda \theta \mathbb{H}), \end{aligned}$$

where, for notational simplicity, I have defined $\mathbb{H} = \left(\frac{\alpha}{(r + \alpha)(r + 2\alpha)} - \frac{\alpha}{(r + 2\alpha + \rho)(r + \alpha + \rho)} \right)$. Finally, the closed form expression for the threshold is

$$\underline{\theta}(p(t)) = \left[\frac{\delta}{r + \alpha + \rho} - p^c(t) \left(1 - \left(\frac{\alpha \rho}{(r + 2\alpha)(r + 2\alpha + \rho)} \right) \right) - \alpha(F - \lambda S + \lambda) \mathbb{H} \right] \frac{1}{\alpha \varepsilon \lambda \mathbb{H}}. \quad (15)$$

Following similar steps to the previous case, I can find the conditions under which buying is optimal by supposing that for $t \in [0, T_\rho)$ the price is expected to rise and check that it is individually optimal to buy the asset rather than selling. Formally, I can substitute in expression (12) the different expected price path:

$$\begin{aligned} &\int_t^\infty \alpha e^{-(r + \alpha)(\tau_\alpha - t)} (e^{-\rho(\tau_\alpha - t)} p^c(\tau_\alpha)) d\tau_\alpha \quad (16a) \\ &= \int_t^\infty \alpha e^{-(r + \alpha + \rho)(\tau_\alpha - t)} p^c(\tau_\alpha) d\tau_\alpha \\ &= \int_t^\infty \alpha e^{-(r + \alpha + \rho)(\tau_\alpha - t)} (\lambda - (\lambda - p(t)) e^{-\alpha(\tau_\alpha - t)} + (F - \lambda S) (1 - e^{-\alpha(\tau_\alpha - t)})) d\tau_\alpha \\ &= \frac{\alpha \lambda}{r + \alpha + \rho} - \frac{\alpha \lambda}{r + 2\alpha + \rho} + \frac{\alpha p(t)}{r + 2\alpha + \rho} + \frac{\alpha(F - \lambda S)}{r + \alpha + \rho} - \frac{\alpha(F - \lambda S)}{r + 2\alpha + \rho} \\ &= \alpha \lambda \left(\frac{\alpha}{(r + \alpha + \rho)(r + 2\alpha + \rho)} \right) + \frac{\alpha p(t)}{r + 2\alpha + \rho} + \alpha(F - \lambda S) \left(\frac{\alpha}{(r + \alpha + \rho)(r + 2\alpha + \rho)} \right). \end{aligned}$$

By substituting (16a) in the optimality condition identified in Lemma 2, I can find a different threshold for the severity of the shock:

$$\bar{\theta}(p(t)) = \left[\frac{\delta}{r + \alpha + \rho} + \alpha \lambda \left(\frac{\alpha}{(r + \alpha + \rho)(r + 2\alpha + \rho)} \right) - p^c(t) \left(1 - \left(\frac{\alpha \rho}{(r + 2\alpha)(r + 2\alpha + \rho)} \right) \right) \right] \frac{1}{\alpha \varepsilon \lambda \mathbb{H}} - \alpha(F - \lambda S + \lambda) \mathbb{H}$$

This completes part (i).

Part (ii). Notice that it is optimal to buy the asset if and only if the following conditions holds

$$\frac{\delta}{r + \alpha + \rho} > p^c(t) - \tilde{p}(\tau_\alpha) \quad (17)$$

Comparing condition (13) with (16a) shows that the threshold $\bar{\theta}(p(t))$ that solves (17) is higher than $\underline{\theta}(p(t))$.

It is now important to notice a key difference of these threshold with the one, $\hat{\theta}$, identified in Proposition 1: when speculators interact with each other in the market, the thresholds depend (negatively) on the price at time t , i.e. $\frac{\partial \bar{\theta}(p_t)}{\partial p_t} < 0$ and $\frac{\partial \underline{\theta}(p_t)}{\partial p_t} < 0$. ■

Proof of Proposition 4.

Part (*Crash*) The result on the crash phase of the equilibrium directly follows from the previous proposition in a cashing-in-on-the-crash equilibrium. Suppose there exists a lower bound for the price $p^* \geq 0$, such that the price never goes below this threshold (next point shows this exists and is unique). Then, a speculator who has the opportunity to trade at $t < T_\rho$ will start selling his holdings when he expects others to do the same in the future and when the price $p(t)$ he obtains by doing so is greater than the lower bound for the price p^* . Both the initial price $p(0)$ and the initial holdings $x(0)$ for the speculators need to be high enough to ensure the existence of this equilibrium.¹⁹ In particular, if $p(0) < p^*$, the only equilibrium that emerges is the leaning-against-the-wind one. If $x(0)$ is close to zero, the speculator who has an opportunity to trade at time t expects the price to move very little before the arrival of the shock at T_ρ , which makes it optimal for him to continue to hold the asset.

Part (*Market Freeze*) Now I show that, in a cashing-in-on-the-crash equilibrium, the price cannot decline indefinitely. The existence of a unique price p^* at which trading activities come to a halt follows from the fact that the speculators' capital gains are decreasing in the price and are compared to the cash flow of the asset which is instead constant and independent of $p(t)$. For $p(t) = 0$ the speculators have no incentive to sell the asset. This means that, at that price, u_a is strictly greater than $q(t)$. By a continuity argument, this is true also for $p(t)$ positive but arbitrarily small. However if, as assumed, $p(0)$ is sufficiently high the potential capital gains are above u_a and are continuously decreasing in $p(t)$. This means that there will exist a price at which the speculator is indifferent between selling and keeping the asset as defined by the price that equates the value and the cost of holding the asset, i.e. $u_a = q(t)$, as $q(t)$ is a function of t only through the price $p(t)$. This threshold is unique because the probability that in the next instant there will be a shock is constant, as T_ρ is distributed according to a Poisson process. Hence, a speculator that can sell at a price just ε above the threshold p^* will do it as he will be compensated by the arrival of the shock θ , whereas once the price reaches the level p^* speculators have no incentive to trade the asset. Specifically, we

¹⁹Notice that the condition on $x(0)$ can be relaxed if I allow the speculators to short-sell the asset.

can explicitly find p^* :

$$p^* = \frac{\delta(r + 2\alpha)}{(r + \alpha)(r + \alpha + \rho)} + (F - \lambda S) \left(\frac{\alpha}{r + \alpha} \right)^2 + \alpha \lambda H(1 - \varepsilon \theta) \frac{r + 2\alpha}{r + \alpha}$$

Part (*Recovery*). As for the proof of Proposition 1, I start with a speculator who meets a dealer at $t > T_\rho$. After T_ρ , if $p(T_\rho) = F$ a speculator is indifferent between buying and selling the asset, then given my tie-breaking rule, they do not buy, and the price stays constant at the fundamental value. If at time T_ρ a shock of severity θ is realized, then it is strictly optimal for the investor to buy the asset as soon as he gets the opportunity to do so (in both equilibria: when the price was increasing at $t < T_\rho$, since it cannot go above F , it will still be below F , or it was decreasing and then after the shock the price is even lower). Consider, in fact, a speculator who does not own the asset and contacts the market at time $t > T_\rho$. If he behaves according to the prescribed trading plan of purchasing the asset at time t and afterwards follow the optimal policy of keeping the asset, his value is $F - p(t)$, which is positive as discussed above. To check optimality, by the Bellman principle it suffices to rule out one-stage deviations, whereby a speculator deviates once from the prescribed plan and follows it thereafter. If the speculator deviates once by not purchasing the asset his value is

$$V(0, t) = E \left[e^{-r(\tau-t)} (F - p(t)) \right]$$

where τ is the next instant at which the speculator has the opportunity to contact the market. This is strictly less than $F - p(t)$ because the price is strictly increasing over $[T_\rho, \infty)$ and because of discounting. Hence, he is not going to deviate from the prescribed strategy.

Consider an investor who owns the asset. The value of following the prescribed plan of holding the asset is F . If the agent deviates once and sells at time $\tau > t$ his value is

$$\begin{aligned} & \mathbb{E}_t \left[\int_t^\tau e^{-r(u-t)} \delta du + e^{-r(\tau-t)} (p(z) + V(0, z)) \right] \\ &= F + \mathbb{E}_t \left[e^{-r(\tau-t)} ((p(z) + V(0, z) - F)) \right]. \end{aligned}$$

This is lower than F because $(p(z) + V(0, z) - F) < 0$ as $V(0, t) < F - p(t)$. Hence, speculators who own the asset are strictly better off by holding their assets for every $t > T_\rho$. This completes the proof of the proposition. ■

Proof of Proposition 5.

This follows from Lemma 3 and the cashing-in-on-the-crash equilibrium of Proposition 3. As the price changes at a rate α , and in a cashing-in-on-the-crash equilibrium it first decreases for $t < T_\rho$ and then reverts toward the fundamental value of the asset, higher α (lower trading

frictions) leads to a faster-changing price. ■

Proof of Proposition 6.

To compute the effects of different parameters on speculators' incentives I separately analyze the effect on the cash flow and on the capital gains terms. This allows me to formally show the results depicted in Figures 6 and 7. The left hand side of the optimality condition in Lemma 2, $\frac{\delta}{r+\alpha+\rho}$ is decreasing in both α and ρ as depicted by the blue curves in the Figures 6 and 7.

The capital gain in 14, instead, is composed of three terms, and I am going to analyze each one of them. First we have the effect of parameters on the coefficient of the price is positive:

$$\frac{\partial \left(\frac{\alpha\rho}{(r+2\alpha)(r+2\alpha+\rho)} \right)}{\partial \rho} = \frac{\alpha(r+2\alpha)(r+2\alpha+\rho) - \alpha\rho(r+2\alpha)}{(r+2\alpha)^2(r+2\alpha+\rho)^2} = \frac{1}{(r+2\alpha+\rho)^2} > 0.$$

The effect of α can be similarly computed as follows:

$$\frac{\partial \left(\frac{\alpha\rho}{(r+2\alpha)(r+2\alpha+\rho)} \right)}{\partial \alpha} = \frac{\rho(r+2\alpha)(r+2\alpha+\rho) - 2\alpha\rho[(r+2\alpha) + (r+2\alpha+\rho)]}{(r+2\alpha)^2(r+2\alpha+\rho)^2} = \frac{r^2 + r\rho - 4\alpha^2}{(r+2\alpha)^2(r+2\alpha+\rho)^2} \leq 0$$

this shows that the effect depends on the value of α and ρ : specifically, it is positive for high value of ρ . Then we have the coefficient \mathbb{H} for the second and third term which is increasing in ρ :

$$\frac{\partial \mathbb{H}}{\partial \rho} = \frac{\partial \left(\frac{\alpha}{(r+\alpha)(r+2\alpha)} - \frac{\alpha}{(r+2\alpha+\rho)(r+\alpha+\rho)} \right)}{\partial \rho} > 0.$$

The effect of α can be found by decomposing \mathbb{H} as

$$\begin{aligned} \frac{\partial \left(\frac{\alpha^2}{(r+2\alpha)(r+\alpha)} \right)}{\partial \alpha} &= \frac{2r^2\alpha + 3\alpha^2r}{(r+2\alpha)^2(r+\alpha)^2} > 0 \\ \frac{\partial \left(\frac{\alpha^2}{(r+2\alpha+\rho)(r+\alpha+\rho)} \right)}{\partial \alpha} &= \frac{2r^2\alpha + 3\alpha^2r + 3\alpha^2\rho + 4\alpha r\rho + 2\alpha\rho^2}{(r+2\alpha+\rho)^2(r+\alpha+\rho)^2} > 0. \end{aligned}$$

This means that for high value of ρ , the capital gains $q(t)$ first decreases as a function of α , but then increases as depicted in panel (b) of figure 7.

Finally, I can investigate the effect of market depth λ . The effect on 14 is positive if $(1 - S) - \varepsilon\theta > 0$, which is true for small enough probability of the shock ε . ■

Proof of Proposition 7.

The proof follows the argument first proposed by Frankel and Pauzner (2000). The idea of the proof is that the existence of dominance regions shown in Proposition 3 starts an

iterative contagion effect that spreads throughout the parameter space. [Frankel and Burdzy \(2005\)](#) show that a similar argument can be used when θ_t follows an arbitrary mean-reverting process, as long as the drift μ is a linear function of the state θ_t .

Let us start with a speculator who has the opportunity to trade when θ is at right of $\bar{\theta}(p(t))$: it is dominant for him to sell. Now consider a speculator slightly to the left of $\bar{\theta}(p(t))$, he will sell his asset holdings too. In fact, if he was observing a shock of size $\bar{\theta}(p(t))$, he was at most indifferent between buying or selling the asset, but now he knows that if θ changes stochastically over time, small perturbations to the severity of the shock θ might approach the dominance boundary, at such time other speculators will find it optimal to sell, which makes the speculator not indifferent anymore. Then, we can now define a new boundary $\bar{\theta}^1$. When speculators believe that the shock hitting the market in the future is exactly $\bar{\theta}^1$, they should be indifferent between buying or selling the asset on the worst-case belief that other speculators choose to sell to the right of $\bar{\theta}$. By repeating this reasoning we obtain the limit boundary $\bar{\theta}^\infty$, which is the limit of the sequence, and is an equilibrium because on each boundary $\bar{\theta}^n$ the speculator was indifferent between buying and selling the asset.

We can now start a similar iteration from the left boundary $\underline{\theta}(p(t))$, but using translations of the boundary $\bar{\theta}^\infty$, the reason for this way of proceeding will be clear soon. We start with a translation where buying is dominant ($\theta < \underline{\theta}(p(t))$). We then iterate constructing the other curves as the right-most translation of $\underline{\theta}'_0$. The limit of these translations being $\underline{\theta}_\infty$. Since we started with translations of $\bar{\theta}^\infty$, it is not necessarily an equilibrium. However, if the speculator contacts the dealer when the perceived shock is on $\underline{\theta}_\infty$, he expects all the other speculators to play according to that, then there must be a point A where he is indifferent, otherwise would strictly prefer buying. Let B the point on $\bar{\theta}^\infty$ at the same height as A .

We need to establish that A and B coincide in order to show that there exists a unique threshold. Let us compare two speculators, one in A and one in B . They expect the state to have the same relative dynamics because $\bar{\theta}^\infty$ and $\underline{\theta}_\infty$ have the same shape. Now consider a given path of changes in θ . Given this path, speculators in A and B expect the same path of θ_t . Suppose by contradiction that A and B were different, then the θ that the speculator in B expects would at all times exceed the θ that A expects by an amount equal to the initial difference in the θ 's. Since the relative payoff to sell is increasing in θ , B 's payoff from choosing selling would be higher than A 's. But this cannot be, since both A and B are indifferent between the two strategies. Therefore, the curves $\bar{\theta}^\infty$ and $\underline{\theta}_\infty$ coincide and the equilibrium is unique. Notice that to apply this procedure, the initial boundaries $\underline{\theta}(p(t))$ and $\bar{\theta}(p(t))$ need to be positive, because they need to be reachable with positive probability. In the case in which they are not, then the equilibrium described in the [Proposition 7](#) is not unique. ■

Proof of Proposition 8. We can consider the case before and after the shock separately. The only thing that changes is the dealer's expectation about the price change. Let us begin by analyzing the case before the shock. We start by conjecturing that the dealer sets the price equal to

$$P(t) = a + bx(t) - cS.$$

Let us define Δ_1 be the price change when there is a supply shock at T_ρ . Then, by the price-setting policy above we have

$$\Delta_1 = -c\theta$$

Let us define Δ_2 be the price change when there is no supply shock at T_ρ . Then, we have $\Delta_2 = 0$. Finally, we can compute the price change when speculators have the opportunity to trade:

$$\begin{aligned} dP(t) &= bdx(t) - cdS(t) = \\ &= \begin{cases} -\alpha bx(t)dt - c\kappa(\bar{S} - S(t))dt - c\sigma dZ(t) & \text{when speculators sell} \\ \alpha b(1-x(t))dt - c\kappa(\bar{S} - S(t))dt - c\sigma dZ(t) & \text{when speculators buy} \end{cases} \end{aligned}$$

Hence, we can derive the dealer's HJB at $\forall t < T_\rho$:

$$\begin{aligned} qJ(W, S) &= \max_{C, I} u(C) + (rW - C + I(-\alpha bx - c\kappa(\bar{S} - S) + \delta - rP)) J_W(W, S) \\ &\quad + \frac{1}{2} I^2 c^2 \sigma^2 J_{WW}(W, S) + \rho\epsilon (J(W - Ic\theta, S) - J(W, S)). \end{aligned}$$

The first order conditions with respect to C and I are

$$\begin{aligned} u'(C) &= J_W(W, S), \\ (-\alpha bx - c\kappa(\bar{S} - S) + \delta - rP) J_W(W, S) &= \rho\epsilon c\theta J_W(W - Ic\theta, S) - Ic^2\sigma^2 J_{WW}(W, S). \end{aligned}$$

Then, we can impose the market clearing condition to obtain the following expression

$$P = \frac{\delta}{r} - \frac{\alpha b}{r} x - \frac{c\kappa}{r} (\bar{S} - S) + \frac{c^2\sigma^2 J_{WW}(W, S)}{r J_W(W, S)} (S - x) - \frac{\rho\epsilon c\theta J_W(W - c\theta(S - x), S)}{r J_W(W, S)}. \quad (18)$$

If the utility function is exponential with risk aversion γ , we can conjecture that

$$J(W, S) = -\frac{1}{\gamma r} e^{-\gamma r(W + G(S))}.$$

Hence, we can simplify the expression (18) as follows

$$\frac{J_W(W - c\theta(S - x), S)}{J_W(W, S)} = e^{\gamma rc\theta(S-x)} \approx 1 + \gamma rc\theta(S - x),$$

and

$$\frac{J_{WW}(W, S)}{J_W(W, S)} = -\gamma r,$$

which results in the following expression for the price:

$$P = \frac{\delta - \rho\epsilon c\theta - c\kappa\bar{S}}{r} - \frac{\alpha b}{r}x + \frac{c\kappa}{r}S - \gamma c^2(\sigma^2 + \rho\epsilon\theta^2)(S - x).$$

Finally, we can now derive the expressions for the coefficients:

$$\begin{aligned} a &= \frac{\delta}{r} - \frac{(r + \kappa)(\rho\epsilon\theta + \kappa\bar{S})}{r\gamma(\sigma^2 + \rho\epsilon\theta^2)}, \\ b &= \frac{r}{r + \alpha\gamma(\sigma^2 + \rho\epsilon\theta^2)} \frac{(r + \kappa)^2}{}, \\ c &= \frac{r + \kappa}{\gamma(\sigma^2 + \rho\epsilon\theta^2)}. \end{aligned}$$

A similar analysis shows the price in the case in which the speculators buy. In this case, the only difference is that a is replaced with

$$a' = \frac{\delta}{r} - \frac{(r + \kappa)(\rho\epsilon\theta + \kappa\bar{S})}{r\gamma(\sigma^2 + \rho\epsilon\theta^2)} + \frac{\alpha}{r + \alpha\gamma(\sigma^2 + \rho\epsilon\theta^2)} \frac{(r + \kappa)^2}{}$$

i.e. the dealer quotes a higher price when the speculators buy than when they sell as $a' > a$.

We can now turn to the after-shock case. For any $t > T_\rho$, the only risk faced by the dealer is given by the volatility σ^s in the noise traders asset supply. We can follow similar steps to the ones above to find that the coefficients are given in this case by the following expressions:

$$\begin{aligned} a &= \frac{\delta}{r} - \frac{\kappa\bar{S}(r + \kappa)}{\gamma r^2 \sigma^2} + \frac{\alpha}{r + \alpha} \frac{(r + \kappa)^2}{\gamma r^2 \sigma^2}, \\ b &= \frac{1}{r + \alpha} \frac{(r + \kappa)^2}{r\gamma\sigma^2}, \\ c &= \frac{r + \kappa}{r\gamma\sigma^2}. \end{aligned}$$

■

Proof of Proposition 9. Let us first derive the price dynamics. When speculators sell, the

price moves according to the following:

$$\begin{aligned}
dP(t) &= bdx(t) - cdS(t) \\
&= -\alpha bx(t)dt - c\kappa (\bar{S} - S(t)) dt - c\sigma dZ(t) \\
&= -\alpha (P(t) - a + cS(t)) dt - c\kappa (\bar{S} - S(t)) dt - c\sigma dZ(t) \\
&= [(\alpha a - c\kappa\bar{S}) - \alpha P(t) + c(\kappa - \alpha) S(t)] dt - c\sigma dZ(t).
\end{aligned}$$

We can derive $E_t [P(t + s)]$ as follows:

$$E_t [P(t + s)] = q + e^{-\alpha s} P(t) + c (e^{-\alpha s} - e^{-\kappa s}) S(t),$$

where q is given by

$$q = a (1 - e^{-\alpha s}) - c\bar{S} (1 - e^{-\kappa s}).$$

Assume $S(t) = \bar{S}$. Then, we have

$$E_t [P(t + s)] = (1 - e^{-\alpha s}) \underline{P} + e^{-\alpha s} P(t)$$

where $\underline{P} = a - c\bar{S}$ is the minimum price level at the stationary state. Hence, the price is decreasing ex ante. Similarly, we can show that the price is increasing ex ante when speculators buy.

Let a_{cb} , a_{lb} , a_a , c_b , and c_a be coefficients on the price when speculators sell / buy, and before / after a jump shock, respectively. Then, we can derive the expression for the lower boundary $\underline{\theta}$ such that buying is optimal for speculator whenever he expects every other speculators to sell:

$$\frac{\delta}{r + \alpha + \rho} \geq P(t) - \tilde{P}(\tau_\alpha)$$

where

$$\begin{aligned}
\tilde{P}(\tau_\alpha) &= \int_t^\infty \alpha e^{-(r+\alpha+\rho)(\tau_\alpha-t)} E_t [P(\tau_\alpha)] d\tau_\alpha \\
&\quad + \int_t^\infty \int_{\tau_\rho}^\infty \alpha \rho e^{-(r+\alpha)(\tau_\alpha-t)} e^{-\rho(\tau_\rho-t)} E_t [P(\tau_\alpha)] d\tau_\alpha d\tau_\rho
\end{aligned}$$

The first part can be easily computed as follows:

$$\alpha \left(\frac{P(t)}{r + 2\alpha + \rho} + \frac{\alpha (a_{cb} - c_b \bar{S})}{(r + \alpha + \rho) (r + 2\alpha + \rho)} \right).$$

Note that the price path is decreasing since every other speculators sell. Next, the inner

integral of the second part is given by ■

$$\int_{\tau_\rho}^{\infty} e^{-(r+\alpha)(\tau_\alpha-\tau_\rho)} E_{\tau_\rho^+} [P(\tau_\alpha)] d\tau_\alpha = \frac{P(\tau_\rho^+)}{r+2\alpha} + \frac{\alpha(a_a + b_a - c_a \bar{S})}{(r+\alpha)(r+2\alpha)}$$

The outer integral of the second part, instead, can be expressed as follows

$$\int_t^{\infty} e^{-(r+\alpha+\rho)(\tau_\rho-t)} \left[\frac{E_t [P(\tau_\rho^-) - \epsilon c_b \theta]}{r+2\alpha} + \frac{\alpha(a_a + b_a - c_a \bar{S})}{(r+\alpha)(r+2\alpha)} \right] d\tau_\rho = \frac{1}{r+2\alpha} \left[\frac{P(t)}{r+2\alpha+\rho} + \frac{\alpha(a_{cb} - c_b \bar{S})}{(r+\alpha+\rho)(r+2\alpha+\rho)} - \frac{\epsilon c_b \theta}{r+\alpha+\rho} + \frac{\alpha(a_a + b_a - c_a \bar{S})}{r+\alpha} \right]$$

Thus, now we can equate both sides to obtain:

$$P(t) - \frac{\delta}{r+\alpha+\rho} \leq \alpha \left(\frac{P(t)}{r+2\alpha+\rho} + \frac{\alpha(a_{cb} - c_b \bar{S})}{(r+\alpha+\rho)(r+2\alpha+\rho)} \right) + \frac{\alpha\rho}{r+2\alpha} \left[\frac{P(t)}{r+2\alpha+\rho} + \frac{\alpha(a_{cb} - c_b \bar{S})}{(r+\alpha+\rho)(r+2\alpha+\rho)} - \frac{\epsilon c_b \theta}{r+\alpha+\rho} + \frac{\alpha(a_a + b_a - c_a \bar{S})}{r+\alpha} \right].$$

This is a quadratic equation of θ which can be rewritten as follows:

$$A \geq \frac{C + D\theta}{\sigma^2 + \rho\epsilon\theta^2} > 0 \tag{19}$$

where

$$\begin{aligned} A &= \frac{\delta}{r+\alpha+\rho} + \frac{\alpha^2\delta}{r(r+2\alpha)(r+\alpha+\rho)} + \frac{\alpha^2\rho(a_a + b_a - c_a \bar{S})}{(r+\alpha)(r+2\alpha)} - BP(t) \\ B &= 1 - \frac{\alpha}{r+2\alpha} \\ C &= \frac{\alpha^2(r+\kappa)^2\bar{S}}{r\gamma(r+\alpha+\rho)(r+2\alpha)} > 0 \\ D &= \frac{\alpha\rho\epsilon(r+\alpha)(r+\kappa)}{r\gamma(r+\alpha+\rho)(r+2\alpha)} > 0 \end{aligned}$$

Hence, θ solves the following quadratic equation

$$\rho\epsilon A\theta^2 - D\theta + \sigma^2 A - C \geq 0.$$

Now, suppose that every other speculators buy. We substitute a_{cb} with $a_{lb} + b_b$. Then, we have

$$A \leq \frac{C' + D\theta}{\sigma^2 + \rho\epsilon\theta^2} \quad (20)$$

and the quadratic equation becomes

$$\rho\epsilon A\theta^2 - D\theta + \sigma^2 A - C' \leq 0$$

where

$$C' = \frac{\alpha^2 (r + \kappa)^2 (\bar{S} - r)}{r\gamma (r + \alpha + \rho) (r + 2\alpha)} < C$$

Figure 9 shows the regions for which the two quadratic equations, i.e. when the other speculators are expected to buy and when they are expected to sell, cross the zero line and give rise to multiple equilibria.

9 Supplementary Appendix – Tobin tax

The recent economic crisis has rekindled interest in James Tobin’s proposal to “throw some sand in the wheels” of speculators. Specifically, on December 3, 2009, the “Let Wall Street Pay for the Restoration of Main Street Bill” to impose a tax on US financial market securities transactions was presented in the House of Representatives. Similarly, in September 2011 the European Commission proposed instituting a financial transaction tax within the 27 EU member states by 2014. Such taxes would be akin to Pigovian taxes, obliging speculators to internalize the costs of the systemic risk they generate. The advocates of the transaction tax believe that it would calm the self-fulfilling financial turmoil we have experienced in recent years; however, many market practitioners have opposed it, maintaining that it would affect market liquidity adversely or cause financial transactions to shift into other jurisdictions.

To study how a transaction cost would affect speculators’ trading strategies and the asset price, I introduce a transaction cost $c > 0$ paid by speculators whenever they modify their portfolio holdings. That is, a speculator who buys the asset pays $p(t) + c$, one who sells receives only $p(t) - c$.

I can extend the analysis proposed in the previous section to analyze the conditions under which speculators decide to liquidate their positions when they are subject to a transaction tax. In the next proposition, I show that the introduction of a tax on financial transactions might reduce the fragility of the market: speculators need to expect a more severe negative shock (higher θ) to induce a cashing-in equilibrium. However, according to Proposition 10 speculators might be induced to refrain from buying if the tax is sufficiently high.

Proposition 10 (Tobin tax) *Suppose a speculator who has the opportunity to trade at time t expects other speculators to sell the asset in the future. A more severe shock θ is needed to induce him to sell as well when $c > 0$. Moreover, there exists a threshold c^* such that if $c > c^*$ the speculators refrain from trading at all.*

Proof. Since the cost lowers the capital gain, it decreases the speculators’ incentive to profit from price swings. In other words, the threshold for the shock θ that makes the traders indifferent between holding the asset and profit from price swings need to increase. In fact, suppose a speculator coming into contact with the market at time $t < T_\rho$, sells the asset if and only if the following condition holds

$$\frac{\delta}{r + \alpha + \rho} > (p(t) - c) - \tilde{p}(\tau_\alpha)$$

which is the same as in Lemma 2, except for the introduction of the transaction cost, which

lowers the price gained by the speculator in the case in which he sells the asset. This means that a transaction cost reduces the speculators' incentives to cash in on the crash.

To show that the introduction of a transaction cost reduces speculators' incentive to profit from fluctuations, we can follow the same procedure as in Proposition 2, but including the tax c in the capital gain $q(t)$. We can rewrite the problem of an investor who gains access to the market at time t who has to pay a transaction cost c as

$$\max_{a' \geq 0} \int_t^{T_{\alpha\rho}} e^{-r(s-t)} \delta a' ds - \left\{ (p(t) + c) - \mathbb{E} \left[e^{-r(T_{\alpha}-t)} \left\{ \mathbb{I}_{\{T_{\alpha} < T_{\rho}\}} p^U(T_{\alpha}) + \mathbb{I}_{\{T_{\alpha} > T_{\rho}\}} p^U(T_{\alpha}|T_{\rho}) \right\} - c \right] \right\} a' \quad (21)$$

The only difference with Lemma 2 is that the investor who buys (sells) the asset will pay (receive) the price $p(t) + c$ ($p(t) - c$). As in Lemma 2 we can compute the expected utility flows that the speculator obtains by holding portfolio a .

$$\begin{aligned} u^U(a) &= \mathbb{E} \left[\int_0^{T_{\alpha\rho}-t} e^{-rs} \delta a ds \right] = \mathbb{E} \int_0^{\infty} \left[\int_0^{\tau_{\alpha\rho}} e^{-rs} \delta a ds \right] (\alpha + \rho) e^{-(\alpha+\rho)\tau_{\alpha\rho}} d\tau_{\alpha\rho} \\ &= a^U \left(\frac{\delta}{r + \alpha + \rho} \right) \end{aligned}$$

To derive the expected value of the price, we use the fact that $T_{\alpha} - t$ and $T_{\rho} - t$ are two independent exponentially distributed random variables:

$$\begin{aligned} &\mathbb{E} \left[e^{-r(T_{\alpha}-t)} \left\{ \mathbb{I}_{\{T_{\alpha} < T_{\rho}\}} (p^U(\tau_{\alpha}) - c) + \mathbb{I}_{\{T_{\alpha} > T_{\rho}\}} (p^U(T_{\alpha}|T_{\rho}) - c) \right\} \right] \\ &= \int_t^{\infty} e^{-r(\tau_{\alpha}-t)} \left[e^{-\rho(\tau_{\alpha}-t)} (p^U(\tau_{\alpha}) - c) + \int_t^{\tau_{\alpha}} \rho e^{-\rho(\tau_{\rho}-t)} (p^U(\tau_{\alpha}|\tau_{\rho}) - c) \right] \alpha e^{-\alpha(\tau_{\alpha}-t)} d\tau_{\rho} d\tau_{\alpha} \\ &= \int_t^{\infty} e^{-r(\tau_{\alpha}-t)} \left[e^{-\rho(\tau_{\alpha}-t)} p^U(\tau_{\alpha}) + \int_t^{\tau_{\alpha}} \rho e^{-\rho(\tau_{\rho}-t)} p^U(\tau_{\alpha}|\tau_{\rho}) \right] \alpha e^{-\alpha(\tau_{\alpha}-t)} d\tau_{\rho} d\tau_{\alpha} - \frac{\alpha c}{r + \alpha} \end{aligned}$$

where the second equality follows from Lemma 2.

The speculators who now need to pay the Tobin tax c and do not own the asset will buy it if and only if:

$$\frac{\delta}{r + \alpha + \rho} - \frac{rc}{\alpha + r} > p(t) - \tilde{p}(\tau_{\alpha}).$$

Hence, the speculators have less incentives to buy the asset. This shows that there exists a high enough transaction cost \bar{c} , above which the speculators find it optimal to abstain from trading outright. ■

Proposition 10 shows that the effect of the tax on financial transactions on speculators' trading strategies is twofold. On the one hand, it can reduce the gain from selling in response to an increase in uncertainty. In fact, by reducing the capital gains speculators expect to

realize from selling and then buying back after the shock, it reduces price volatility. On the other hand, a transaction cost above the threshold c^* induces speculators to refrain from trading until uncertainty is realized, hoping to buy the asset at a lower price in the event of a shock.

A different argument, based on the trade-off between congestion and liquidity, which sets out the rationale for policies to avoid extreme price movements, has been proposed by Afonso (2011). Specifically, she argues in favor of trading halts and circuit breakers that interrupt trading when there is a significant imbalance between buy and sell orders or when asset prices decline beyond trigger levels.²⁰ These instruments, by increasing search frictions during downswings, reduce the effect of congestion, resulting in a lower price discount and in a more liquid market. My result does not depend on negative externalities between speculators – the congestion effect – but follows from the impact of the transaction cost on capital gains: it is more costly for the speculators to exploit temporary price fluctuations. Moreover, Proposition 10 shows that the reduction in fragility might be achieved at the cost of diminishing participation of speculators. This means that even if in equilibrium speculators would have purchased the asset from $t = 0$ onwards, if the tax is excessive they might not.

Up to now I have assumed that only the speculators are affected by the introduction of the Tobin tax. Actually, though, it would affect all market participants and might discourage potential liquidity providers. The model can be extended to accommodate this possibility by assuming that long-term investors' demand function becomes steeper as the transaction cost increases, that is, $\lambda'(c) > 0$. In other words, the transaction cost leads some fraction of long-term investors to leave the market. The next proposition shows that, in this case, the Tobin tax might have a destabilizing effect on financial markets.

Proposition 11 (Tobin tax and liquidity) *There exists a $\bar{\lambda}$ such that if the introduction of a transaction cost c increases λ above $\bar{\lambda}$, then a Tobin tax increases the speculators' incentive to sell at $t < T_\rho$.*

Proof. The result that the introduction of a Tobin tax might actually increase speculators' incentives to sell the asset at time $t < T_\rho$, follows from Proposition 5 and the assumption that the liquidity provided by long-term investors, $\lambda(c)$, is an increasing function of the transaction cost. In fact, Proposition 5 shows that speculators have higher incentives to sell their holdings in less-liquid markets, namely when λ is high. Suppose now that for a given λ , the condition identified in Lemma 2 would predict them to purchase the asset. Consider now the introduction of the tax, which increases λ to $\lambda' > \lambda$; if the market depth function $\lambda(\cdot)$

²⁰For instance, circuit breakers were adopted by the New York Stock Exchange following the 1987 stock market crash to reduce volatility and promote investor confidence.

is sufficiently sensitive to c , such increase might increase the potential capital gains so much that now it is optimal for the speculators to actually sell the asset. Hence, the statement of the proposition follows, namely that when the introduction of transaction costs reduce the liquidity of the market, this might have destabilizing effects. ■

Proposition 11 follows from the comparative statics results in Proposition 5: since speculators have a greater incentive to amplify market shocks in less liquid markets, if the tax on financial transactions reduces participation by long-term investors, speculators expect to make more capital gains by selling in response to the uncertainty shock. Hence, the previous proposition highlights an important factor to weigh in seeking to understand the effectiveness of a Tobin tax: how to target such a measure to speculators and not to potential liquidity providers.

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