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Forward Versus Trailing Earnings in Equity Valuation

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Abstract. This article articulates how forward earnings are more accurate valuation attributes than trailing earnings. First, I show that, while linear accrual rules cannot achieve accurate trailing earnings-value relations in a setting with unobserved information, they can achieve accurate forward earnings-value relations. Second, I prove that, even when accrual rules are restricted so that forward earnings fails to be an exact valuation attribute, more-forward earnings are more accurate valuation attributes than less-forward earnings or trailing earnings. In conclusion, even under deficient accrual policies, more-forward earnings are more accurate valuation attributes—the more-forward, the more accurate.

Keywords: accruals, beliefs, earnings, forecasts, valuation

JEL Classification: M41

Linear earnings-value relations and properties of accrual accounting systems that facilitate them have attracted considerable attention in recent years (Barth et al., 1999; Burgstahler and Dichev, 1997; Easton, 2002; Feltham and Ohlson, 1996; Liu et al., 2002; Ohlson and Juettner-Nauroth, 2000; Ohlson and Zhang, 1998; Penman and Sougiannis, 1998; Yee, 2003). Authors have focused on two types of earnings-value relations as being of particular interest:

- Contemporaneous earnings-value relations, which relate value to contemporaneous reported earnings (and possibly also to other contemporaneous attributes like book value and dividends).

- Forward earnings-value relations, which relate value to earnings forecasts.

I will call accrual systems that accommodate contemporaneous earnings-value relations "contemporaneous accounting" and accrual systems that facilitate forward earnings-value relations "forward accounting."

An example of a contemporaneous earnings-value relation is

$$V_t = \phi ox_t^c - c_t, \tag{C}$$

where V_t is the expected present value at date t of future free cash flow; $\phi \equiv R/(R-1)$; R is the discount factor; c_t is free cash flow; and ox_t denotes operating earnings. The superscript "c" on ox_t^c indicates that accrual accounting has been designed to facilitate equation (C). Analogously, the simplest example of a forward earnings-value relation is

$$V_t = \frac{\phi \overline{ox}^f_{t,1}}{R}, \tag{F}$$

where $\overline{ox}_{t,1} \equiv E_t[ox_{t+1}]$ denotes the one period ahead earnings forecast conditional on date t information. The superscript "f" indicates that ox^f_t is constructed under an accrual accounting set up to facilitate the forward earnings-value relation, equation (F).

Contemporaneous accounting makes more stringent demands on accruals than forward accounting. Ohlson (1991) showed that, while any accrual accounting system that achieves (C) automatically achieves (F), achieving (F) does not automatically achieve (C). In particular, suppose ox^c_t satisfies (C) for all t. Then, it is automatically true[1] that $V_t = \phi E_t[ox^c_{t+1}]/R$, which means that any accrual system that achieves (C) automatically achieves (F) as a fringe benefit. However, the converse is not true: achieving (F) does not automatically achieve (C). For instance, given ox^c_t, consider a new accrual policy that alters ox^c_t with a sequence of unobservable random charges η_t satisfying $E_t[\eta_{t+1}] = 0$. Under the new accrual policy, earnings (after random charges) is

$$y_t \equiv ox^c_t + \eta_t.$$

Each random charge η_t fully reverses in the next period—just like a temporary revenue boost caused by channel stuffing. At date $t+1, \eta_t$ reverses and a brand new zero-mean error η_{t+1} replaces it. Then y_t achieves (F) but fails to achieve (C). y_t achieves the forward earnings-value relation (F) because

$$V_t = \frac{\phi E_t[y_{t+1}]}{R} = \frac{\phi E_t[ox^c_{t+1} + \eta_{t+1}]}{R} = \frac{\phi E_t[ox^c_{t+1}]}{R}.$$

But $V_t \neq \phi y_t - c_t$ because $\phi y_t - c_t = \phi(ox^c_t + \eta_t) - c_t$, which deviates from the expression in (C) whenever[2] $\eta_t \neq 0$.

Ohlson's result suggests that contemporaneous accounting is more demanding on the accrual system than forward accounting. The forward earnings-value relation, (C), is robust to unpredictable zero-mean accounting charges η_t whereas these same charges spoil the contemporaneous relation (C). Thus, whenever there is the possibility of spurious accounting charges such as unusual write-offs, valuation using earnings forecasts is more reliable than valuation using contemporaneous earnings.

This article points out another (independent) reason why forward accounting is more reliable than contemporaneous accounting. For certain important classes of cash flow dynamics (to be described), it is possible to achieve forward accounting with linear accrual policies. In stark contrast, achieving contemporaneous accounting would necessitate nonlinear accrual rules, furthermore, the accrual rules would require the accountant to make Bayesian estimates. Since Bayesian estimates involve subjective initial conditions, contemporaneous accounting would be necessarily subjective. On the other hand, linear accruals can achieve forward accounting because, by requiring analysts to forecast earnings for valuation, forward accounting shifts the Bayesian inference onus away from the accountant to the users of financial statements.

Feltham and Ohlson (1995) and Ohlson and Zhang (1998) proposed a framework for identifying accrual rules that facilitate contemporaneous accounting. Focusing on efficient accounting, under which a weighted average of earnings and book value constitutes a sufficient valuation attribute, they consider linear transaction models such as

$$c_{t+1} = \gamma_{11} c_t + \gamma_{12} v_{t+1} + \varepsilon_{1t+1},$$
$$v_{t+1} = \gamma_{22} v_t + \varepsilon_{2t+1},$$

where c_t is cash flow, v_t are exogenous cash flow shocks, and ε_{1t} and ε_{2t} are unpredictable, mean-zero random variables. They assume the accountant knows the values of $\{\gamma_{11}, \gamma_{12}, \gamma_{22}\}$ *ex ante* and observes all values of $\{c_t, v_t, \varepsilon_t\}$ as they are realized. As a result, the expected present value of future cash flows is completely specified at each point in time. Their models leave no room for rational disagreement about the interpretation of cash flow realizations and their import for expectations about future cash flows. As such, they do not highlight issues associated with subjectivity stemming from incomplete information about a firm and its business environment.

In practice, accountants (and other capital market participants) face much more uncertainty than is granted by the Feltham and Ohlson (1995) and Ohlson and Zhang (1998) models. First, the underlying equations of motion for cash flow shocks are usually unknown. Even if the accountant believes it is $v_{t+1} = \gamma_{22} v_t + \varepsilon_{2t+1}$, the parameter value γ_{22} is not preordained and must be estimated based on historical data, which may be misleading[3] even when the regressions yield a "good fit." Moreover the accountant faces the possibility that the v_t dynamic involves unidentified omitted variables.[4] Such variables are not only difficult to specify and track—they are subject to change over time in unobserved ways. As it is, in any realistic situation, realizations $\{c_t, v_t, \varepsilon_t\}$ provide an imperfect and incomplete proxy for the information set at date t, and any estimation of expected values must rely on additional (subjective) beliefs to supplement correlated omitted and unidentified variables.

Section 1 develops a model of transactions where the cash flow dynamic

$$c_{t+1} = \gamma c_t + v_{t+1} + \varepsilon_{t+1},$$

faces shocks governed by the contingent dynamic

$$v_{t+1} = \begin{cases} +v_t, & \text{probability } \pi(s_t), \\ -v_t, & \text{probability } 1 - \pi(s_t), \end{cases}$$

where s_t is an unobservable stochastic state variable. The probability $\pi(s_t)$ a given shock repeats is contingent on whether the firm's industry is in a "high" $(s_t = H)$ or "low" $(s_t = L)$ state. The new twist here is that realizations of s_t are never directly observed by anyone and, so, must be imperfectly inferred based on realizations of v_t. Assuming[5] $\pi(H) > \pi(L)$, repeating shocks in successive periods suggest that $s_t = H$ and, likewise, flip-flopping shocks in successive periods indicate a higher likelihood that $s_t = L$. Furthermore, instituting the idea that industry condition evolves over time, s_t undergoes stochastic transitions between $s_t = L$ and $s_t = H$ and vice versa. Then, as explained in Section 1, the expected value of next period's shock may be written as

$$E[v_{t+1} \mid \mu_t, v_t] = [\lambda_1 + \lambda_2\mu_t]v_t, \tag{$*$}$$

where μ_t is the Bayesian probability estimate (or "belief") that industry condition is in a high state. Likewise, the expected present value of cash flow is

$$V_t = \left(\frac{\gamma}{R-\gamma}\right)c_t + [\xi_1 + \xi_2\mu_t]v_t. \tag{$**$}$$

The values of constants $\lambda_H, \lambda_L, \xi_1, \xi_2$ are common knowledge by assumption.

While equations (*) and (**) look deceptively linear, they are highly nonlinear because the Bayesian inference function $\mu_t = F(\mu_0; v_t, \ldots, v_1)$ depends nonlinearly on the trailing shock pattern. Nonlinearity arises from the fact that μ_t updates according to Bayes's Rule with each new realization of v_t. Moreover, μ_t also depends on μ_0, a parameter reflecting the initial assessment of the probability that $s_0 = H$. Because μ_0 must be exogenously given as a preconditioning parameter, it may vary from one user to the next. Variations in μ_0 cause rational Bayesian traders to hold heterogeneous beliefs[6] even when they observe the same news.

A difference between contemporaneous accounting and forward accounting arises immediately in this setting: given (*) and (**), one can achieve forward but not contemporaneous accounting with a linear accrual rule. This is because, as elaborated in Section 2, making V_t in equation (C) equal to V_t in (**) requires that

$$ox_t^c = \left(\frac{R-1}{R-\gamma}\right)c_t + \left(\frac{R-1}{R}\right)[\xi_1 + \xi_2\mu_t]v_t. \tag{OXC}$$

In (OXC), ox_t^c is nonlinear in the sequence of trailing shocks by virtue of its dependence on $\mu_t = F(\mu_0; v_t, \ldots, v_1)$. In (OXC), ox_t^c must depend on μ_t because the value function V_t in (**) does. In the reconciliation of (C) to (**), the onus falls entirely on ox_t^c to incorporate μ_t. When the value function depends nonlinearly on shocks $\{\mu_0, v_t, \ldots, v_1\}$, accruals for (C) must incorporate all of the nonlinearity because (C) is strictly linear in ox_t^c. Hence, one cannot make V_t in equation (C) equal to V_t in (**) with a linear accrual rule.

This argument might lead one to (erroneously) conclude that it is impossible to achieve linear earnings-value relations with accruals that are strictly linear in the shock realizations $\{v_t, \ldots, v_1\}$ if the value function is nonlinear like (**). But, surprisingly, this is not true. Given (*) and (**), consider the following strictly linear accrual policy[7]

$$ox_t^f = \frac{R-1}{R-\gamma} \times \{c_t + (\delta_8 - 1)v_t + \delta_9 v_{t-1}\}, \qquad \text{(OXF)}$$

where δ_8 and δ_9 are shock-independent accrual policy parameters. Proposition 1 in Section 3 shows that (OXF) achieves the linear forward earnings-value relation, equation (F). That is, (OXF) achieves (F) free of any references to the nonlinear inference function μ_t. This is possible because (F) does not equate V_t to ox_t^f; rather, (F) equates V_t to capitalized $E[ox_{t+1}^f \mid c_t, \mu_t, v_t]$, and the expectation value $E[ox_{t+1}^f \mid c_t, \mu_t, v_t]$ processes ox_t^f in a way that incorporates μ_t. Mathematically, applying (*) to (OXF) yields

$$E\left[ox_{t+1}^f \mid c_t, \mu_t, v_t\right] = (R-1) \times \left\{\frac{\gamma}{R-\gamma} c_t + \frac{1}{R-\gamma}[(\delta_8\lambda_1 + \delta_9) + \delta_8\lambda_2\mu_t]v_t\right\},$$

which implies that the accrual policy

$$\delta_8 = (R-\gamma)\left[\frac{\xi_2}{\lambda_2}\right] \quad \text{and} \quad \delta_9 = (R-\gamma)\left[\xi_1 - \left(\frac{\lambda_1}{\lambda_2}\right)\xi_2\right],$$

achieves (F). Therefore, in stark contrast to (OXC), the accrual policy (OXF) that achieves forward-earnings accounting does not need to refer at all to μ_t and, thus, it avoids nonlinearity.

The broader point being made is that forward earnings accounting demands less of accruals than contemporaneous accounting. In a contemporaneous accounting regime, analysts have no opportunity to contribute Bayesian inferences μ_t or make (nonlinear) Bayesian updates on μ_t. This means the onus of supplying μ_t falls completely on the accountant, who must Bayesian update μ_t when she calculates accruals as in equation (OXC). Since Bayesian beliefs update in a nonlinear fashion, accruals for contemporaneous accounting must be nonlinear. In contrast, financial statement users who value a firm using earnings forecasts must actively forecast earnings conditional on μ_t. In doing so, users of forward valuation attributes impound their private μ_t into the earnings-value relation via equation (*). As a result, the use of forward valuation attributes shifts the onus of supplying μ_t away from the accountant to the users of financial statements.

Giving users the onus of supplying μ_t is appealing because, if users hold heterogeneous beliefs, they would prefer to have accounting numbers that are not distorted by accountants' beliefs they may not share. Forward-earnings accounting also provides analysts with a natural opportunity to convey private information about future cash flows when they disclose their earnings forecasts.[8]

Section 4 makes the second main point of this article. As implied by Proposition 2, even when accruals rules are restricted so that forward earnings is an inaccurate valuation attribute, more-forward earnings tends to be a more accurate valuation attribute than less-forward earnings or trailing earnings. As a rule of thumb, more-forward earnings is a more accurate valuation attribute—the more-forward the more accurate.

Section 5 revisits efficient accounting and shows that efficient accounting is possible if, and only if, the accountant revises her accrual rule based on Bayesian updating of her beliefs with each new realization of cash flows. It confirms that book value and earnings cannot be sufficient valuation attributes unless accountants are permitted to incorporate Bayesian beliefs into accruals.

Cash flow forecasts may proxy for μ_t. Hence, it is not surprising to find—as in loss reserve accounting for insurance policy obligations (Beaver and McNichols, 1998)—accrual rules anchoring to cash flow forecasts. Section 6 shows that accrual rules based on cash flow forecasts are equivalent to μ_t-dependent accruals. Since cash flow forecasts proxy for μ_t, accruals anchored to cash flow forecasts can achieve contemporaneous accounting as well as μ_t-dependent accruals can. However, cash flow forecasts suffer from the same subjectivity issues that beliefs do, and they are also nonlinear functions of the cash flow shocks.

1. Unobservable Information Variable

This section introduces a model of transactions that contains an unobservable information variable. In the model, free cash flow is subject to shocks, v_t, that fluctuate stochastically from period to period depending on the value of an unobservable state variable s_t. State s_t may be thought of as an industry condition that affects the stability of market demand for the firm's output. When the industry is in a "high" persistence state, demand is more predictable. When the industry state is "low" persistence, demand is more erratic. Since s_t cannot be directly observed, capital market participants (including accountants) are left to infer s_t based on past realizations of the shocks v_t. If same-sign shocks repeat over several periods, they infer a greater chance that current demand will persist into the future. If shocks reverse sign erratically over several periods, they infer a greater chance that demand will be more erratic in the near future.[9]

Formally, characterize transactions by the following variables for all periods $t \geq 0$:

- $c_t \in (-\infty, \infty)$: free cash flow;
- $v_t \in \{-1, 1\}$: shock to free cash flow;
- $s_t \in \{H, L\}$: unobservable variable that governs the expected persistence of the shock sequence; $s_t = H$ is the high persistence and $s_t = L$ is the low persistence state;
- ε_t: unpredictable (but observable) mean zero random noise.

Free cash flow evolves according to the dynamic

$$c_{t+1} = \gamma c_t + v_{t+1} + \varepsilon_{t+1}, \tag{1}$$

where $0 \leq \gamma < R$. The first term on the right-hand side of equation (1) is the predictable component of cash flow. The shock term is semi-persistent, and the ε_t term is purely transitory. At each date $t \geq 0, \{c_t, v_t, \varepsilon_t\}$ are realized, observed, and recorded.

The persistence of shocks v_t from one date to the next depends on whether the firm's industry is in a "low" ($s_t = L$) or "high" ($s_t = H$) state. State s_t evolves stochastically over time and its realizations are unobservable. Shock v_t evolves according to a dynamic that depends on s_t:

$$\begin{aligned} v_{t+1} &= \begin{cases} +v_t & \text{pr} = \pi_H, \\ -v_t & \text{pr} = 1-\pi_H, \end{cases} \quad \text{if } s_{t+1} = H \quad \text{and} \\ v_{t+1} &= \begin{cases} +v_t & \text{pr} = \pi_L, \\ -v_t & \text{pr} = 1-\pi_L, \end{cases} \quad \text{if } s_{t+1} = L, \end{aligned} \tag{2}$$

where $\pi_H > 1/2$ and $\pi_L < 1/2$. In the high state, v_t repeats with high probability $\pi_H > 1/2$. In the low state, v_t repeats with small probability $\pi_L < 1/2$ and reverts with large probability $1 - \pi_L > 1/2$. Since s_t is not directly observed, observers rely on the shock realizations to form Bayesian beliefs about the underlying value of s_t. A sequence of same-sign shocks leads Bayesian observers to raise their estimate that the economy is in an $s_t = H$ state; a sequence of flip-flop shocks causes Bayesian observers to believe $s_t = L$ is more likely. However, observers can never be 100% certain whether the industry is in a high or low state at any given time.

What makes this Bayesian inference problem realistic is that, not only is s_t never directly observed, it changes over time according to the Markov transition rule

$$\begin{aligned} s_{t+1} &= \begin{cases} H & \text{pr} = 1-\theta_{HL}, \\ L & \text{pr} = \theta_{HL}, \end{cases} \quad \text{if } s_t = H \quad \text{and} \\ s_{t+1} &= \begin{cases} L & \text{pr} = 1-\theta_{LH}, \\ H & \text{pr} = \theta_{LH}, \end{cases} \quad \text{if } s_t = L, \end{aligned} \tag{3}$$

θ_{LH} is the probability, per period, for state L to turn into state H. Likewise, θ_{HL} is the per period probability for a state H to transform into state L. It is helpful to think of θ_{LH} and θ_{HL} as being small enough to avoid excessive jumping between states; that is, $0 < \theta_{LH} + \theta_{HL} < 1$. The analysis in this paper goes through whether or not this inequality is satisfied.

Equations (1)–(3) and the probability values $\{\pi_L, \pi_H, \theta_{LH}, \theta_{HL}\}$ are common knowledge. Moreover, at each date $t \geq 0$ all realized values of $\{c_t, c_{t-1}, \ldots, c_0\}$, $\{\varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_0\}$, and $\Omega_t \equiv \{v_t, v_{t-1}, \ldots, v_0\}$ are common knowledge.

The only variables that are not common knowledge are the unobservable state realizations $\{s_t, s_{t-1}, \ldots, s_0\}$. In this sense, all capital market participants have an incomplete specification of the information set that would help them to forecast future cash flows. Because of this incomplete information, a capital market participant cannot interpret cash flow realizations without adopting (subjective) suppositions—"beliefs"—to supplement her lack of knowledge about the current state s_t. Although s_t is not observable and may vary over time, Bayesian market participants have a noisy estimate of its value based on past realizations of v_t because v_t is more persistent when $s_t = H$. Formally, define a market participant's belief μ_t as her Bayesian probability assessment at date $t \geq 0$ of the likelihood that $s_t = H$ given all realized information:

$$\mu_t \equiv pr(s_t = H \mid \mu_0, \Omega_t).$$

μ_t depends on the initial belief μ_0 as well as past information Ω_t. Bayesian updating only stipulates how a rational market participant should modify her belief—it does not stipulate the participant's original belief. Because μ_0 is exogenously given, it may vary across different capital market participants. Whether it is caused by "genetics" or information asymmetry at $t = 0$, μ_0 variation across participants provides one source of heterogeneous beliefs in a population of Bayesians.

Given μ_0, Bayes's Rule determines how each market participant updates her belief every period. While not saying anything new about Bayesian updating, I state the formula here for future reference:

Lemma 1 *At each date $t > 0$, after observing the realization of v_t, a Bayesian market participant updates her belief according to the rule*

$$\mu_t = \begin{cases} \dfrac{[(1-\theta_{HL})\mu_{t-1} + \theta_{LH}(1-\mu_{t-1})]\pi_H}{[(1-\theta_{HL})\mu_{t-1} + \theta_{LH}(1-\mu_{t-1})]\pi_H + [\theta_{HL}\mu_{t-1} + (1-\theta_{LH})(1-\mu_{t-1})]\pi_L}, & \text{if } v_t = v_{t-1}; \\[2ex] \dfrac{[(1-\theta_{HL})\mu_{t-1} + \theta_{LH}(1-\mu_{t-1})](1-\pi_H)}{[(1-\theta_{HL})\mu_{t-1} + \theta_{LH}(1-\mu_{t-1})](1-\pi_H) + [\theta_{HL}\mu_{t-1} + (1-\theta_{LH})(1-\mu_{t-1})](1-\pi_L)}, & \text{if } v_t = -v_{t-1}. \end{cases}$$

Proof: All proofs are in the Appendix. ■

According to Lemma 1, belief updating is Markov; that is, $\mu_t = pr(s_t = H \mid \mu_{t-1}, v_t)$, which means belief μ_t depends only on prior-period belief μ_{t-1} and the contemporaneous realization of the information variable v_t. This is because, by construction, the values of s_t and v_t depend only on $\{s_{t-1}, v_{t-1}\}$.

Since no market participant is ever 100% certain about whether $s_t = H$ or $s_t = L$, in general $0 < \mu_t < 1$. Figure 1 depicts an example of how belief evolves over time when $\theta_{HL} = \theta_{LH} = 0.05$, $\pi_H = 0.85$, and $\pi_L = 0.15$. The bold line (whose value is either 1 or 0) indicates the unobservable true state s_t. The other three lines track the evolution of Bayesian beliefs updated according to Lemma 1 starting from, respectively, $\mu_0 = 0.1$, $\mu_0 = 1/2$, and $\mu_0 = 0.9$. As shown, the three belief trajectories do not converge until after about $t = 7$ periods of Bayesian updating. Even after they converge to a common value, μ_t does not equal the true state s_t—in fact it fluctuates perilously—because the true state is unobservable and also changes stochastically (as it does at $t = 10$ and $t = 17$ here). Note that even if the true state does not change,

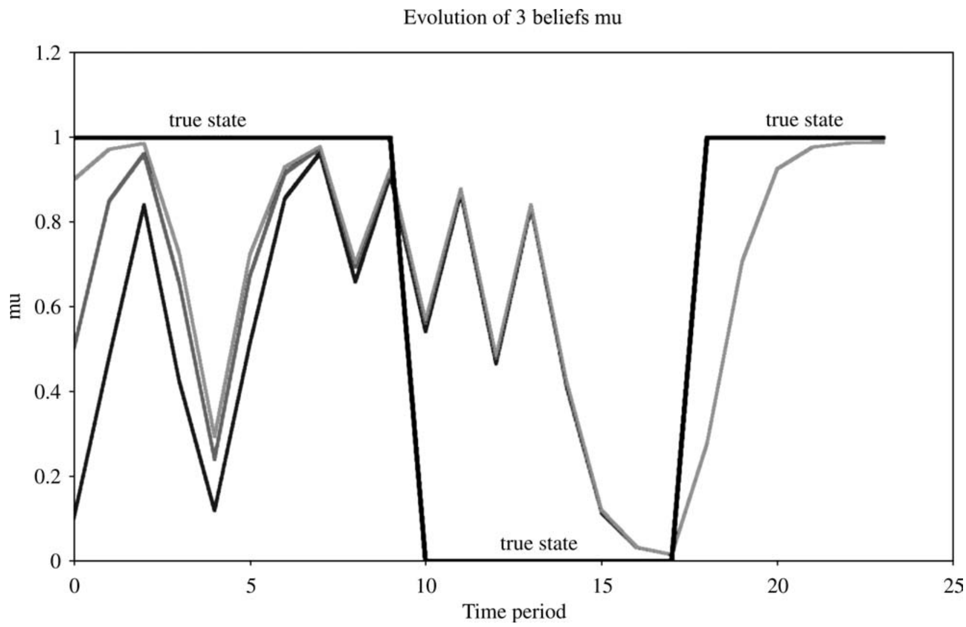

Figure 1. Bayesian beliefs and their relation to the true (unobservable) state.

beliefs may undergo wild fluctuations caused by random flip flops of the observable shock v_t, which is essentially a noisy signal of s_t.

At any date t, the expected present value of cash flows depends on belief μ_t:

Lemma 2 *The expected present value of free cash flow characterized by equations* (1)–(3) *is*

$$V_t = \left(\frac{\gamma}{R-\gamma}\right)c_t + [\xi_1 + \xi_2\mu_t]v_t,$$

where

$$\xi_1 = \frac{2R(\pi_H - \pi_L)\theta_{LH} - (1-2\pi_L)[R-(2\pi_H-1)(1-\theta_{HL}-\theta_{LH})]}{(R+1-2\pi_L)[R-(2\pi_H-1)(1-\theta_{HL})] - (1-2\pi_L)(R+1-2\pi_H)\theta_{LH}} \times \left(\frac{R}{R-\gamma}\right),$$

$$\xi_2 = \frac{2(\pi_H - \pi_L)(1-\theta_{HL}-\theta_{LH})R}{(R+1-2\pi_L)[R-(2\pi_H-1)(1-\theta_{HL})] - (1-2\pi_L)(R+1-2\pi_H)\theta_{LH}} \times \left(\frac{R}{R-\gamma}\right).$$

In Lemma 2, coefficient ξ_2 is always positive while ξ_1 may be positive or negative. When $\theta_{HL} + \theta_{LH} < 1$, the denominators of ξ_1 and ξ_2 are both positive. The numerator of ξ_2 is always positive. The numerator of ξ_1 is negative when θ_{LH} or π_L is sufficiently small. Otherwise, ξ_1 is positive. For example, suppose $R = 1.1$, $\theta_{HL} = 0.1$, and $\pi_H = 0.75$. Then $\xi_1 = -0.24$ and $\xi_2 = 0.87$ if $\theta_{LH} = 0.1$, and $\pi_L = 0.25$. On the other hand, $\xi_1 = +0.043$ and $\xi_2 = 0.64$ if $\theta_{LH} = 0.1$, and $\pi_L = 0.48$.

Note that, unless $\xi_2 = 0$, the weight of v_t in V_t varies over time because μ_t is updated with each new realization of v_t. In other words, despite its deceptively linear appearance, V_t is a highly nonlinear function of v_t with the nonlinearity buried in the belief function μ_t.

Since

$$[\xi_1 + \xi_2\mu_t]v_t = \{(\xi_1 + \xi_2)\mu_t + \xi_1(1 - \mu_t)\}v_t,$$

the second term of V_t in Lemma 2 allows the market to capitalize shock realization v_t with weights $(\xi_1 + \xi_2)$ or ξ_1 depending on how much it believes $s_t = H$. If the market believes the underlying state is $s_t = H$ with certainty, it sets $\mu_t = 1$ and uses the capitalization factor $(\xi_1 + \xi_2)$; if it believes $s_t = L$ with certainty, it sets $\mu_t = 0$ and capitalizes with ξ_1. If the market is uncertain about s_t, it capitalizes with a weighted average of $(\xi_1 + \xi_2)$ and ξ_1 weighted according to the value of μ_t.

2. Bayesian Beliefs and Accounting Policy

Since V_t depends on μ_t by Lemma 2, if contemporaneous book value and contemporaneous earnings are to be sufficient valuation attributes, accruals must incorporate beliefs—otherwise, there is no other way for μ_t to enter into the earnings-value relation.

Is it possible to avoid nonlinear accruals and still achieve linear earnings-value relations when V_t depends on beliefs? To address this question, introduce the following notation:

- $ox_t \in (-\infty, \infty)$: reported operating earnings at date $t \geq 0$;
- $oa_t \in (-\infty, \infty)$: operating assets, which evolves according to

$$oa_t = oa_{t-1} + ox_t - c_t. \tag{4}$$

I will also refer to the following accounting concepts:

- *Forward-Earnings Accounting Policy:* a construction of ox_t such that

$$V_t = \phi\overline{ox}_{t,1}/R, \quad \text{where } \overline{ox}_{t,1} \equiv E[ox_{t+1} \mid c_t, oa_t, v_t, \mu_t] \quad \text{and} \quad \phi \equiv R/(R-1).$$

- *Efficient Accounting Policy:* a construction of ox_t such that, for some real number κ, $V_t = (1-\kappa)oa_t + \kappa\phi[ox_t - c_t]$.
- *Belief-Free Accounting Policy:* a construction of ox_t that does not condition on Bayesian updated beliefs $\{\mu_t, \ldots, \mu_0\}$ or a subset thereof.
- *Belief-Dependent Accounting Policy:* any accounting policy that is not belief-free.

By virtue of Lemma 1, any belief-dependent accounting policy is a nonlinear function of the trailing shock sequence $\{v_t, \ldots, v_0\}$. Hence, a linear accrual rule must be belief-free.

As explained in the Introduction, Ohlson (1991) showed that any $\kappa = 1$ efficient accounting policy (permanent earnings) is automatically a forward-earnings policy. However, $\kappa \neq 1$ efficient policies are not equivalent to forward-earnings policies (Ohlson, 1995). In general, forward-earnings policies are not efficient and efficient policies are not forward-earnings. Feltham and Ohlson (1996) and Ohlson and Zhang (1998) studied the most general, linear accounting policies and showed that efficient accounting can always be achieved with belief-free accounting policies provided the cash flow dynamic is linear and the accountant observes all variable realizations as they occur. They are silent about what happens if the cash flow dynamic contains an unobservable information variable such as s_t.

I will now show that efficient accounting cannot be achieved with a belief-free accounting policy when s_t is unobservable. For the variables here, the Ohlson and Zhang (1998) policies correspond to

$$ox_t = (1 + \delta_1)c_t - (1 - \delta_0)oa_{t-1} + \delta_2 v_t, \tag{5}$$

where $\{\delta_0, \delta_1, \delta_2\}$ are accounting policy parameters. The policy $\delta_0 = 1$ and $\delta_1 = \delta_2 = 0$ corresponds to cash-basis accounting. A deviation of δ_0 from unity reflects depreciation. A deviation of δ_2 from zero allows for recognition of a v_t-dependent accrual. Such an accrual would be, for example, an incremental provision for bad debts corresponding to each shock to cash flow. Since equation (5) does not refer to $\{\mu_t, \ldots, \mu_0\}$, policies of this form are belief-free. Ohlson and Zhang (1998) showed that, for a broad class of linear cash flow dynamics not encompassing the model defined by equations (1)–(3), there is always a linear accounting policy like equation (5) that achieves efficient accounting.

Does Ohlson and Zhang's (1998) result prevail if the cash flow dynamic involves an unobservable variable? Is it possible to achieve efficient accounting with a belief-free and, hence, linear accounting policy if the transactions are characterized by equations (1)–(3)? It turns out that the answer is No:

Lemma 3 *No choice of equation* (5) *accounting policy parameters* $\{\delta_0, \delta_1, \delta_2\}$ *achieves efficient accounting if cash flows are characterized by equations* (1)–(3).

Lemma 3 shows that under belief-free accounting, it is impossible to have operating book value and earnings as sufficient valuation attributes even if accruals take into account v_t with a linear $\delta_2 \neq 0$ term. The reason is because, when the underlying cash flow dynamic contains an unobservable variable s_t, accruals that condition only on $\{c_t, ox_t, v_t\}$ do not capture all value-relevant information since V_t also depends on belief μ_t about s_t. Extending the accounting policy to incorporate all historical observations so that

$$ox_t = (1 + \delta_1)c_t - (1 - \delta_0)oa_{t-1} + \sum_{s=0}^{t} \delta_{2,s} v_{t-s},$$

does not fix this problem because, according to Lemma 2, μ_t affects value and, by Lemma 1, Bayesian belief updates in a nonlinear way. Hence, no linear accrual policy can achieve efficient accounting unless it also depends on μ_t. Moreover, Bayesian belief necessarily depends on an exogenous parameter μ_0, the initial belief, which means μ_0 also affects value and must be incorporated into ox_t to achieve efficiency. Based on these fundamental considerations, it is clear that no earnings construction, whether linear or not, that depends on only $\{v_t, v_{t-1}, \ldots, v_0\}$ can achieve efficient accounting.

3. Belief-Free Accrual Policy

Is it possible to circumvent Lemma 3? Can a linear accounting rule achieve a linear earnings-value relation when cash flows dynamics depend on an unobservable variable?

Lemmas 1 and 2 demonstrate that the value of a cash flow stream under incomplete information depends on the initial belief parameter, μ_0, and, therefore, is inherently subjective. If two accountants (or market participants) happen to start with different values of μ_0, at any given time, they may hold different beliefs μ_t even though both accountants update their beliefs in a completely rational, Bayesian way. Their disagreement is due not to information asymmetry[10]—everyone observes the same set of $\{v_t, \ldots, v_0\}$ realizations; it is due entirely to their initial beliefs stemming from their subjective assessments of the initial situation. In other words, when information is incomplete, there is room for rational disagreement between equally informed accountants and other market participants.

In this setting, Section 2 introduced the concept of a belief-free accrual policy. An accrual policy is belief-free if the size of accruals do not condition on the accountant's belief. Belief-free accounting policies are appealing because their accruals avoid the subjectivity that belief-dependency introduces. From a valuation perspective, an additional feature one would like to demand of earnings-value relations is linearity. However, Lemma 3 states that it is impossible to achieve linear contemporaneous accounting with a belief-free accrual policy for cash flow evolving according to equations (1)–(3). To achieve efficient accounting, accruals must incorporate the accountant's beliefs. Since belief and cash flow forecasts necessarily require subjective input μ_0, efficient accounting is necessarily subjective. Two accountants, following the same accrual policy with the same precise information about $\{v_t, \ldots, v_0\}$, can rationally disagree about the accrual to recognize if they hold different values of μ_0. Furthermore, a financial statement user with a third, independent value of μ_0 would not accept either accountants' earnings report at face value. This problem is due to incomplete information, and sustains without any agency conflicts, information asymmetry, or earnings manipulation.

It would seem, then, that achieving a linear earnings-value relation necessitates a tradeoff of linearity against subjectivity; to achieve linear efficient accounting, it is necessary to ask the accountant to interject subjectivity and nonlinearity into accruals. Can this tradeoff be circumvented? Is it possible to achieve linear earnings-value relations with belief-free accounting?

While Lemma 3 is a no-go theorem for contemporaneous earnings-value relations, it is silent with regard to forward earnings-value relations. An obvious candidate is forward-earnings accounting, according to which $V_t = \phi\overline{ox}_{t,1}/R$. Forward-earnings accounting has attracted much attention because it is known to be the most effective value screen when consensus analysts' forecasts are used as proxies for expected earnings (Liu et al., 2002). Moreover, earnings forecasts have a central role in representations of the discount dividend formula (Ohlson and Juettner-Nauroth, 2000).

Here, we focus on forward-earnings accounting for a different reason. The key difference between forward-earnings accounting and efficient accounting is that efficient accounting stipulates that value is a weighted average of the accountant's book value and operating earnings report, whereas forward-earnings accounting stipulates value is proportional to the financial statement user's earnings forecast. Since book value and earnings realizations condition on the accountant's beliefs, efficient accounting leaves no opportunity for financial statement users to interject their own beliefs into the valuation process. In contrast, a user's earnings forecast $\overline{ox}_{t,1} \equiv E[ox_{t+1} \mid c_t, oa_t, v_t, \mu_t]$ conditions on his own belief. Accordingly, forward-earnings accounting requires the user to interject his own belief μ_t into the earnings-value relation. Thus, it is plausible that forward-earnings accounting can be achieved with belief-free accounting. Indeed, this is true:

Proposition 1 *If*

$$\delta_8 \equiv \frac{R^2}{(R+1-2\pi_L)[R-(1-\theta_{HL})(2\pi_H-1)]-(R+1-2\pi_H)(1-2\pi_L)\theta_{LH}},$$

$$\delta_9 \equiv \frac{(2\pi_H-1)(1-2\pi_L)(1-\theta_{HL}-\theta_{LH})R}{(R+1-2\pi_L)[R-(1-\theta_{HL})(2\pi_H-1)]-(R+1-2\pi_H)(1-2\pi_L)\theta_{LH}},$$

then any linear accrual policy that constructs earnings as

$$ox_t = \frac{R-1}{R-\gamma} \times \{c_t + (\delta_8 - 1)v_t + \delta_9 v_{t-1}\} + \eta_t, \tag{6}$$

where η_t *is any variable that satisfies* $E[\eta_{t+1} \mid c_t, \ldots; v_t, \ldots; \eta_t, \ldots; \mu_0] = 0$, *achieves forward-earnings accounting.*

Proposition 1 expresses a fundamental advantage of forward-earnings accounting over efficient accounting: the belief-free nature of accruals in forward-earnings accounting. By asking users of financial statements to make their own earnings-forecasts, forward-earnings accounting separates beliefs from the accrual accounting process and, thus, achieves a linear earnings-value relation with a linear accrual rule even when cash flow dynamics involve an unobservable variable. The forward-earnings accrual policy, equation (6), anchors only to observable realizations and does not condition on beliefs about unobservable degrees of freedom. As such, it does not allow subjectivity to drive a wedge between the accountant and the user of financial reports. These benefits, however, do not come without a tradeoff. The tradeoff is that, under forward-earnings accounting, users of financial statements must make earnings forecasts to obtain a valuation. Although this entails additional work for users, if users hold heterogeneous beliefs, they would prefer to make subjective earnings forecasts conditional on their own beliefs rather than to adopt an accountant's belief, with which they may disagree.

An earnings forecast under accrual policy equation (6) conveys information[11] about the forecaster's belief about the unobservable state variable s_t. Following Lemma 2, an observer can completely back out the forecaster's μ_t value from his earning forecast $\overline{ox}_{t,1}$ based on the linear formula

$$\overline{ox}_{t,1} = (R-1) \times \left\{ \left(\frac{\gamma}{R-\gamma} \right) c_t + [\xi_1 + \xi_2 \mu_t] v_t \right\}.$$

This means that, in a forward-earnings accounting regime, the information reflected in analysts' earnings forecasts is simply the analysts' private beliefs about the unknown.

Finally, note that, even though ox_t is belief-free in equation (6), it correlates to future cash flows. This feature highlights the fact that belief dependency and association with future cash flows are distinct issues. Here, cash flows are auto-correlated according to equation (1). Any earnings construction that anchors to trailing cash flows, including equation (6), will correlate accruals to future cash flows by virtue of the autocorrelation of cash flows independent of whether accountants incorporate additional beliefs in their construction of earnings. Hence, one cannot draw any conclusions about the role of beliefs or private information in accrual accounting from the mere fact that accruals predict future cash flows.

4. Generality of Proposition 1

Proposition 1 has broader implications and is more amenable to generalization than first impression might lead one to believe. To see this, let V_t denote the exact expected value of discounted cash flows and consider the following set of trial valuation functions:[12]

$$\hat{V}_t(F) \equiv \begin{cases} \phi o x_t - c_t, & F = 0; \\ \dfrac{\phi E_t[ox_{t+1}]}{R}, & F = 1; \\ \dfrac{\phi E_t\left[ox_{t+F} + \phi^{-1} \times \sum_{f=1}^{F-1} R^{F-f} c_{t+f}\right]}{R^F}, & F \geq 2. \end{cases}$$

As defined, $\hat{V}_t(F)$ equals the F period-ahead earnings forecast suitably capitalized, discounted, and adjusted for lost interest income due to prior cash outflow. Absent a specification of accruals like equation (6), there is no reason to expect that $V_t = \hat{V}_t(F)$ for any finite F.

Nonetheless, one can make two broad statements about the accuracy of these trial valuation functions without an accruals assumption. First, provided earnings is bounded so that $\lim_{F\to\infty} (E_t[ox_{t+F})]/R^F) = 0$,

$$\lim_{F\to\infty} \hat{V}_t(F) = \sum_{f=1}^{\infty} R^{-f} E_t[c_{t+f}],$$

which implies that capitalized infinitely forward earnings is always an accurate valuation attribute. Second, any earnings construction ox_t that achieves $\hat{V}_t(F) = V_t$ for all dates t automatically achieves[13]

$$\hat{V}_t(F') = V_t, \quad \forall F' \geq F.$$

This implies Proposition 1 has the following corollary: the linear accrual policy equation (6) achieves not just $V_t = \phi E_t[ox_{t+1}]/R$—this accrual policy also achieves *all* of the more-forward earnings-value relations like $V_t = \phi E_t[ox_{t+2} + \phi^{-1} R c_{t+1}]/R^2$, $V_t = \phi E_t[ox_{t+3} + \phi^{-1}\{R^2 c_{t+1} + R c_{t+2}\}]/R^3$, and so on.

On the other hand, achieving more-forward earnings-value relations does not guarantee the accuracy of less-forward earnings-value relations. Accrual rule equation (6) achieves $\hat{V}_t(1) = V_t$, but it does not achieve $\hat{V}_t(0) = V_t$. Likewise, an accrual rule may achieve an $F \geq 2$ forward earning-value relation without achieving[14] $V_t = \phi E_t[ox_{t+1}]/R$. Generally, $\hat{V}_t(F) = V_t$ for all dates t does not imply $\hat{V}_t(F') = V_t$ for any $F' < F$. In summary, Proposition 1 implies the linear accrual policy equation (6) achieves $\hat{V}_t(F') = V_t$ for all $F' \geq 1$ since achieving $V_t = \phi E_t[ox_{t+1}]/R$ makes stronger demands on accruals than achieving any of the $F' \geq 2$ earnings-value relations. But Proposition 1 does not say that accrual policy equation (6) achieves $\hat{V}_t(0) = V_t$. Indeed, a Lemma 3-type analysis quickly shows that no linear accrual policy can achieve $\hat{V}_t(0) = V_t$.

Another issue concerns the range of the unobservable information variable. Proposition 1 focuses on a scenario where the unobservable variable $s_t \in \{H, L\}$ ranges only over two states. But, in principle, the unobservable variable may range over several more states or a continuum of states. What happens when the number S of states is large?

Formally, Proposition 1 extends straightforwardly to cases where s_t ranges over $S > 2$ states as follows. When $S > 2$, the net present value of cash flows is of the form

$$V_t = \left(\frac{\gamma}{R-\gamma}\right)c_t + \sum_{u=1}^{S} \xi_u \mu_{u,t} v_t,$$

where $\mu_{u,t}$ is the Bayesian belief that s_t is in state u at date t; $\sum_{u=1}^{S} \mu_{u,t} = 1$; and $\{\xi_1, \ldots, \xi_S\}$ are S common knowledge constants. The accrual policy equation (6)—because it has only two policy parameters, δ_8 and δ_9—does not have enough degrees of freedom to achieve $V_t = \phi E_t[ox_{t+1}]/R$. However, the linear extension of equation (6) to

$$ox_t = \frac{R-1}{R-\gamma} \times \left\{ c_t + \sum_{u=0}^{S-1} \sigma_u v_{t-u} \right\} + \eta_t, \tag{7}$$

where $E_t[\eta_{t+S-1}] = 0$, has S policy parameters $\{\sigma_0, \ldots, \sigma_{S-1}\}$. The S policy parameters provide enough degrees of freedom to achieve any $F \geq S-1$ forward earnings-value relation. The lesson here is that a linear accrual policy can achieve linear earnings-value relations even when the space of what is unobservable is large. The caveat is that, the larger this space is, the further forward the earnings-value relation must be. If s_t ranges over S states, then a linear accrual policy can achieve only the $F \geq S-1$ forward earnings-value relations.

This raises an important issue. What if the number S of states is large (e.g., $S = 100$) and, instead of a 99-years-ahead forecast (which, under an appropriate accrual policy, would be an accurate valuation attribute), one prefers to use $F = 1$ or $F = 2$ earnings forecasts as approximate valuation attributes? Analysts often use $F = 1$ or $F = 2$ years ahead earnings forecasts even though the underlying number of states associated with unobserved information variables must be huge. It would be appealing if one could prove a proposition that ranks the valuation accuracy of near- and intermediate-term earnings forecasts. In particular, can one be certain that the two period ahead earnings forecast is a more accurate valuation attribute than the one period ahead earnings forecast when S is a lot greater than 3? Are more-forward earnings forecasts generally better approximate valuation attributes than less-forward ones?

An affirmative answer to these questions would be very appealing; otherwise, it would be difficult to understand why analysts are willing to spend effort making $F \geq 2$ earnings forecasts if the one year-ahead earnings forecast is a better valuation attribute.

An extension of the Ohlson (1991) model described in the introduction helps to motivate why it is plausible that more-forward earnings forecasts are generally better approximate valuation attributes than less-forward ones. Suppose

$$V_t \equiv \frac{\phi E_t[ox_{t+T} + \phi^{-1} \times \sum_{\tau=1}^{T-1} R^{T-\tau} c_{t+\tau}]}{R^T}$$

for all t. However, suppose traders do not observe earnings ox_t. Instead, they must rely on a noisy earnings report

$$y_t = ox_t + \eta_t + \eta_{t-1} + \cdots + \eta_{t-(T-1)},$$

to make their valuation estimates. This noisy earnings report y_t suffers i.i.d. random accounting charges $\eta_\tau \sim N(0, \sigma_\eta^2)$, each of which persists for T periods before completely reversing. Trial valuation estimates based on forecasts of reported earnings are

$$\hat{V}_t(F; y) \equiv \frac{\phi E_t\left[y_{t+F} + \phi^{-1} \times \sum_{f=1}^{F-1} R^{F-f} c_{t+f}\right]}{R^F}.$$

A measure of the inaccuracy of these trial valuation estimates is

$$\text{error}(F) \equiv \text{var}\left[\hat{V}_t(F; y) - V_t\right].$$

In this special setting, one can prove that $\hat{V}_t(F'; y)$ has smaller valuation error than $\hat{V}_t(F; y)$ for all $F' > F$. This means that more-forward earnings forecasts are more accurate approximate valuation attributes than less-forward earnings forecasts when reported earnings have persistent noise. Formally:

Observation 1# In the extended Ohlson model setting, valuation error strictly decreases with increasing forecast horizon F until $F = T$, upon and after which $\hat{V}_t(F; y)$ is an exact valuation function. In particular, error(F) decreases monotonically with increasing F as follows:

$$\text{error}(F) = \begin{cases} \left(\dfrac{\phi\sigma_\eta}{R^F}\right)^2 (T - F), & F < T; \\ 0, & F \geq T. \end{cases}$$

Hence, if valuation error is caused by persistent random accounting charges, more-forward earnings forecasts are guaranteed to be more accurate valuation attributes than less-forward forecasts.

Does this rule extend to the setting where unobservable variable s_t ranges over more states than the length of the forecast horizon (e.g., $S > F + 1$)? As explained above, in this case the valuation error is caused by deficiencies in linear accruals. To the extent that these deficiencies manifest only as i.i.d. random errors in earnings, Observation 1 says the answer is yes. However, unfortunately, these deficiencies also cause intertemporal correlations in the earnings errors and, so, Observation 1 does not apply. Hence, one needs a new theorem when there are unobservable information variables.

Ideally, one would like to prove the following (very general) theorem. Let Π denote a pre-specified space of accounting policies of interest. For each accounting policy $p \in \Pi$, let $ox_\tau(p)$ denote earnings constructed according to that accounting policy. Let V_t denote the exact value of PVED, and define trial valuation functions

$$\hat{V}_t(F) \equiv \frac{\phi E_t\left[ox_{t+F}(p) + \phi^{-1} \times \sum_{f=1}^{F-1} R^{F-f} c_{t+f}\right]}{R^F}, \quad \forall F \in \{1, \ldots, S\}.$$

Then an appealing theorem would be:

Conjecture A *For all accounting policies*[15] $p \in \Pi$, $\|V_t - \hat{V}_t(F')\| \leq \|V_t - \hat{V}_t(F)\|$ *for all* $F' > F$.

If true, Conjecture A would guarantee that more-forward earnings is always a more accurate valuation attribute than less-forward earnings for all accounting policies (including all those of practical interest). Unfortunately, I have not been able to prove Conjecture A, or even convince myself that it is plausible.[16]

A somewhat less general theorem along the same lines is:

Conjecture B *There exists at least one accounting policy* $p \in \Pi$ *such that for all,* $\|V_t - \hat{V}_t(F')\| \leq \|V_t - \hat{V}_t(F)\|$ *for all* $F' > F$.

Conjecture B is weaker than Conjecture A because B requires only that more-forward earnings be a more accurate valuation attribute for some accounting policy. Conjecture B is almost true, but first it is necessary to relax its monotonicity prediction. As stated, Conjecture B predicts that valuation errors, $\|V_t - \hat{V}_t(F)\|$, decrease monotonically with increasing forecast horizon F. This prediction is too strong, because it rules out scenarios where valuation errors tend to decrease with increasing forecast horizon without decreasing in a strictly monotonic fashion.

Proposition 2, proved in the appendix, relaxes the monotonicity prediction of Conjecture B but otherwise reflects the spirit of this conjecture. Proposition 2 guarantees that "more-forward earnings forecasts are more accurate approximate valuation attributes" in the presence of deficient accounting for an unobservable variable in the following sense:

Proposition 2 *For any desired bound* $E > 0$ *on the valuation error, there exist a linear accrual policy and a threshold* F_* *such that*

$$\left\|V_t - \hat{V}_t(F)\right\| < E, \quad \forall F \geq F_*.$$

Proposition 2 says that, given a desired level of valuation accuracy and a forecast horizon F_*, one can always set up an accrual rule so that $\hat{V}_t(F)$ achieves the level of desired accuracy for all forecast horizons $F \geq F_*$. Proposition 2 also implies that, once $\hat{V}_t(F_*)$ is an accurate enough valuation estimate, $\hat{V}_t(F)$ for any $F \geq F_*$ will be also accurate enough—for some linear accrual policy. In this sense, more-forward earnings forecasts are always at least as good valuation attributes as less-forward ones under linear accounting policies.

Propositions 1 and 2 are the main results of this article. The next three sections, Sections 5, 6, and the Conclusion, address peripheral issues and summarize.

5. Contrast to Efficient Accounting

Based on Lemma 3 and the discussion at the end of Section 2, one expects to achieve efficient accounting if, and only if, accounting policy incorporates beliefs. To this end, consider the belief-dependent accounting policy

$$ox_t = (1 + \delta_1)c_t - (1 - \delta_0)oa_{t-1} + [\delta_2 + \delta_3\mu_t]v_t, \tag{8}$$

where $\{\delta_0, \delta_1, \delta_2, \delta_3\}$ are accounting policy parameters. The difference between equations (5) and (8) is the additional belief term, $\mu_t v_t$. While the $\mu_t v_t$ accrual may appear innocuous, remember that it is highly nonlinear in $\{v_t, v_{t-1}, \dots, v_0\}$ due to the Bayesian way μ_t updates.

The accounting policy equation (8) achieves efficient accounting as follows:

Observation 2 *The accounting policy equation (8) with*

$$\delta_0 = \frac{R\kappa}{R - 1 + \kappa},$$

$$\delta_1 = \frac{(R-1)\gamma - (R-\gamma)\kappa}{(R-\gamma)(R-1+\kappa)},$$

$$\delta_2 = \frac{(R-1)}{R-1+\kappa} \times \xi_1,$$

$$\delta_3 = \frac{(R-1)}{R-1+\kappa} \times \xi_2,$$

where ξ_1 and ξ_2 are as given in Lemma 2, implies the net present value of free cash flows characterized by equations (1)–(3) is $V_t = (1-\kappa)oa_t + \kappa\phi[ox_t - c_t]$.

Lemma 3 and Observation 2 imply that it is possible to achieve efficient accounting if, and only if, one allows the accounting policy to depend on belief. But GAAP rules for expensing bad debts and other anticipated losses refer, not directly to beliefs, but to cash flow forecasts. The next section turns to the use of cash flow forecasts as belief proxies.

6. Cash Flow Forecasts as Belief Proxies

In practice, accrual accounting often disguises the use of beliefs by referring to cash flow forecasts rather than beliefs. The Statement of Financial Accounting Standards No. 60, "Accounting and Reporting by Insurance Companies," guides insurers to match policy revenues to servicing costs by using a loss reserve account. To value the loss reserve, insurers are required to make (and disclose) itemized forecasts of future cash payouts on policy claims, and update these detailed forecasts on a period-by-period basis. The loss reserve expense for each period is determined in accordance with each period's revised future cash flow projections. Benchmarking to cash flow projections allows insurers to incorporate their beliefs in the loss accruals without explicitly referring to beliefs.

Efficient accounting can be achieved by using $\overline{c}_{t,1}$ to proxy for beliefs because

$$\overline{c}_{t,1} \equiv E[c_{t+1} \mid c_t, v_t, \mu_t],$$

is a function of $\{c_t, v_t, \mu_t\}$. Inverting this function, one obtains an expression for μ_t in terms of $\{c_t, v_t, \overline{c}_{t,1}\}$, which can be used to replace μ_t in equation (8) with $\overline{c}_{t,1}$. The result is an equivalent efficient accounting policy that conditions on $\{c_t, oa_{t-1}, v_t, \overline{c}_{t,1}\}$:

Observation 3 *The accounting policy*

$$ox_t = (1 + \delta_1 - \gamma\delta_5)c_t - (1 - \delta_0)oa_{t-1} + \delta_4 v_t + \delta_5 \overline{c}_{t,1}, \tag{9}$$

with $\{\delta_0, \delta_1\}$ *as given in Observation 2 and*

$$\delta_4 = \frac{(R-1)R}{(R-1+\kappa)(R-\gamma)} \times \frac{(1-\theta_{HL}-\theta_{LH})(2\pi_H-1)(1-2\pi_L) - 2R[1-2\pi_L-2(\pi_H-\pi_L)\theta_{LH}]}{(R+1-2\pi_L)[R-(1-\theta_{HL})(2\pi_H-1)] - (R+1-2\pi_H)(1-2\pi_L)\theta_{LH}},$$

$$\delta_5 = \frac{(R-1)R}{(R-1+\kappa)(R-\gamma)} \times \frac{R}{(R+1-2\pi_L)[R-(1-\theta_{HL})(2\pi_H-1)] - (R+1-2\pi_H)(1-2\pi_L)\theta_{LH}},$$

implies the net present value of free cash flows characterized by equations (1)–(3) is $V_t = (1-\kappa)oa_t + \kappa\phi[ox_t - c_t]$.

According to Observation 3, efficient accounting is achieved if each realization of v_t causes the accountant to revise her period-ahead cash flow forecast, $\overline{c}_{t,1}$ and, then, to recognize accruals based on accounting policy equation (9).

The forward cash flow term in equation (9), $\delta_5\overline{c}_{t,1}$, is in addition to the $\delta_4 v_t$ term. $\delta_4 \neq 0$ in equation (9) because the one-period-ahead cash flow forecast provides incremental information that proxies for $\mu_t v_t$ alone, not for both v_t and $\mu_t v_t$. Adequately proxying for $[\delta_2 + \delta_3\mu_t]v_t$ in equation (8) requires a linear combination of period-ahead and two-periods-ahead cash flow forecasts. In particular, the accounting policy

$$ox_t = (1 + \delta_1 - \gamma\delta_6)c_t - (1 - \delta_0)oa_{t-1} + [\delta_6 - \gamma\delta_7]\overline{c}_{t,1} + \delta_7\overline{c}_{t,2}, \tag{10}$$

with $\{\delta_0, \delta_1\}$ as given in Observation 2 and

$$\delta_6 = \frac{(R-1)R}{(R-1+\kappa)(R-\gamma)} \times \frac{R + 2[1 - (1-\theta_{HL})\pi_H - (1-\theta_{LH})\pi_L] - \theta_{HL} - \theta_{LH}}{(R+1-2\pi_L)[R - (1-\theta_{HL})(2\pi_H - 1)] - (R+1-2\pi_H)(1-2\pi_L)\theta_{LH}},$$

$$\delta_7 = \frac{(R-1)R}{(R-1+\kappa)(R-\gamma)} \times \frac{1}{(R+1-2\pi_L)[R - (1-\theta_{HL})(2\pi_H - 1)] - (R+1-2\pi_H)(1-2\pi_L)\theta_{LH}},$$

is equivalent to equations (8) or (9). Like them, equation (10) achieves efficient accounting.

Equation (10) relies on two cash flow forecasts in addition to contemporaneous cash flows. This is because the system defined by equations (1)–(3) has an additional (unobservable) degree of freedom, s_t. From this perspective, the extensive supplementary forecast disclosures required for the loss reserve accrual in insurance accounting makes sense if one accepts that future policy claims depend on multitudinous degrees of freedom.

7. Conclusion

Ohlson (1991) observed that, because forward earnings-value relations naturally overlook non-recurring accounting charges, they are less demanding on the accrual accounting system than trailing earnings-value relations. Feltham and Ohlson (1996) and Ohlson and Zhang (1998) showed that one can always achieve linear earnings-value relations with linear accrual rules when all information variables are observed. But what if accountants and capital market participants cannot observe all information variables? Is it still possible to achieve linear earnings-value relations with linear accrual rules? This article demonstrates that the answer, yes, but with a caveat: Linear accruals rules can achieve only forward earnings-value relations, under which value is proportional to earnings forecasts. In stark contrast, linear accrual rules cannot achieve contemporaneous earnings-value relations without observing all the information variables. Linear accrual rules are able to achieve forward but not contemporaneous linear earnings-value relations because requiring analysts to forecast earnings shifts the onus of making Bayesian inferences about unobserved variables—the source of nonlinearities—away from the accountant to the users of financial statements.

A simple model serves to illustrate how forward earnings-value relations demand less of accrual accounting when there are unobservable information variables. In particular, when cash flow dynamics depend on an unobservable state variable s_t, nonlinearities arise because market participants must make Bayesian inferences about the value of s_t in order to forecast cash flows. The net present value of cash flows, V_t, depends on a Bayesian "belief" function μ_t, which updates in a nonlinear way according to Bayes's Rule. Because V_t depends on μ_t and μ_t is a nonlinear function of cash flow shocks, achieving a linear earnings-value relation requires either that (i) accruals be nonlinear or (ii) users of financial statements insert μ_t after the accrual accounting process. As it is, accruals must depend on μ_t to achieve contemporaneous accounting because users have no opportunity to contribute μ_t when the earnings-value relation relies only on contemporaneous or trailing accounting numbers. On the other hand, linear, μ_t-free accruals may achieve forward-earnings accounting because users input μ_t when they make their earnings forecasts. Therefore, at the cost of imposing on users the burden of making earnings forecasts, forward-earnings accounting achieves linear earnings-value relations with linear, μ_t-free accruals even when some information variables are unobservable.

These results articulate two conceptual points. First, achieving accurate forward earnings-value relations demands less of accrual accounting than achieving trailing earnings-value relations. In particular, linear accounting rules cannot achieve accurate trailing earnings-value relations in the presence of unobservable variables because Bayesian updating is nonlinear. In contrast, linear accounting rules can achieve linear forward earnings-value relations because requiring analysts to forecast earnings shifts the onus of making Bayesian inferences to the users of financial statements. Hence, accurate valuation using earnings forecasts demands less of accrual accounting than valuation using trailing or contemporaneous accounting numbers. This is the first point. The second point is that, even when accruals rules are deficient so that forward earnings is an inaccurate valuation attribute, more-forward earnings is still a more accurate valuation attribute than less-forward earnings or trailing earnings. In conclusion, more-forward earnings is invariably a more accurate valuation attribute—the more-forward, the more accurate.

Appendix

Proof of Lemma 1: By Bayes's rule, $pr(A \mid B, C) = (pr(B \mid A, C)\ pr(A \mid C))/pr(B \mid C)$ (see Greene, 2003, pp. 429–430). Identifying A with $s_t = H$, B with v_t, and C with $\{\mu_0, \Omega_{t-1}\}$ yields for all $t > 0$ that

$$\mu_t \equiv pr(s_t = H \mid \mu_0, \Omega_t) = \frac{pr(v_t \mid s_t = H, \mu_0, \Omega_{t-1})\ pr(s_t = H \mid \mu_0, \Omega_{t-1})}{pr(v_t \mid \mu_0, \Omega_{t-1})}.$$

I will evaluate the probabilities on the right-hand side assuming $v_t = v_{t-1}$; the

$v_t = -v_{t-1}$ case proceeds analogously. If $v_t = v_{t-1}$, then

$$pr(v_t \mid s_t = H, \mu_0, \Omega_{t-1}) = pr(v_t \mid s_t = H, v_{t-1}) = \pi_H,$$
$$pr(s_t = H \mid \mu_0, \Omega_{t-1}) = pr(s_t = H \mid \mu_{t-1}) = (1-\theta_{HL})\mu_{t-1} + \theta_{LH}(1-\mu_{t-1}),$$

and

$$pr(v_t \mid \mu_0, \Omega_{t-1}) = \sum_{s_t \in \{H,L\}} pr(v_t \mid s_t, \mu_0, \Omega_{t-1})\; pr(s_t \mid \mu_0, \Omega_{t-1}),$$

where

$$pr(v_t \mid s_t = L, \mu_0, \Omega_{t-1}) = pr(v_t \mid s_t = L, v_{t-1}) = \pi_L,$$

and

$$pr(s_t = L \mid \mu_0, \Omega_{t-1}) = pr(s_t = L \mid \mu_{t-1}) = \theta_{HL}\mu_{t-1} + (1-\theta_{LH})(1-\mu_{t-1}).$$

Combining these expressions recovers the expression for μ_t when $v_t = v_{t-1}$. ■

Proof of Lemma 2: Taking the discounted expectation value $R^{-t}E[\,\cdot \mid \mu_0, v_0]$ of both sides of equation (1), and then summing from $t = 0$ to $t = \infty$ yields $E[\sum_{t=0}^{\infty} R^{-t}(c_{t+1} - \gamma c_t) \mid c_0, \mu_0, v_0] = E[\sum_{t=0}^{\infty} R^{-t} v_{t+1} \mid c_0, \mu_0, v_0]$. Rearranging the left-hand side and identifying $V_0 \equiv E[\sum_{t=1}^{\infty} R^{-t} c_t \mid c_0, \mu_0, v_0]$ yields

$$V_0 = \frac{\gamma}{R-\gamma} c_0 + \frac{R}{R-\gamma} E\left[\sum_{t=1}^{\infty} R^{-t} v_t \mid \mu_0, v_0\right]. \tag{11}$$

It is necessary to evaluate $E[\sum_{t=1}^{\infty} R^{-t} v_t \mid \mu_0, v_0]$. To this end, the transition matrix

$$Q \equiv \begin{pmatrix} (1-\theta_{HL})\pi_H & (1-\theta_{HL})(1-\pi_H) & \theta_{LH}\pi_H & \theta_{LH}(1-\pi_H) \\ (1-\theta_{HL})(1-\pi_H) & (1-\theta_{HL})\pi_H & \theta_{LH}(1-\pi_H) & \theta_{LH}\pi_H \\ \theta_{HL}\pi_L & \theta_{HL}(1-\pi_L) & (1-\theta_{LH})\pi_L & (1-\theta_{LH})(1-\pi_L) \\ \theta_{HL}(1-\pi_L) & \theta_{HL}\pi_L & (1-\theta_{LH})(1-\pi_L) & (1-\theta_{LH})\pi_L \end{pmatrix},$$

and the probability vector

$$\Xi_t \equiv \begin{pmatrix} pr(s_t = H, v_t = v_0 \mid v_0) \\ pr(s_t = H, v_t = -v_0 \mid v_0) \\ pr(s_t = L, v_t = v_0 \mid v_0) \\ pr(s_t = L, v_t = -v_0 \mid v_0) \end{pmatrix} \forall t > 0, \qquad \Xi_0 \equiv \begin{pmatrix} \mu_0 \\ 0 \\ 1-\mu_0 \\ 0 \end{pmatrix},$$

are useful. First, if observe that $\forall t > 0$

$$\Xi_t = Q\Xi_{t-1}.$$

Also, since $pr(v_t = \pm v_0 \mid v_0) = pr(s_t = H, v_t = \pm v_0 \mid v_0) + pr(s_t = L, v_t = \pm v_0 \mid v_0)$,

$$pr(v_t = v_0 \mid v_0) = \Xi_t' \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad pr(v_t = -v_0 \mid v_0) = \Xi_t' \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}.$$

Putting all this together yields

$$\begin{aligned} E[v_t \mid \mu_0, v_0] &= v_0 pr(v_t = v_0 \mid v_0) + (-v_0) pr(v_t = -v_0 \mid v_0) \\ &= v_0 \times (1, -1, 1, -1) Q^t \Xi_0. \end{aligned}$$

Summing $E[v_t \mid \mu_0, v_0] = v_0 \times (1, -1, 1, -1) Q^t \Xi_0$ yields

$$\begin{aligned} E\left[\sum_{t=1}^{\infty} R^{-t} v_t \mid \mu_0, v_0\right] &= v_0 \times (1, -1, 1, -1) \sum_{t=1}^{\infty} (QR^{-1})^t \Xi_0 \\ &= v_0 \times (1, -1, 1, -1)[R - Q]^{-1} Q \Xi_0. \end{aligned}$$

Inverting the matrix $[R - Q]$ and multiply out the matrices yields $E[\sum_{t=1}^{\infty} R^{-t} v_t | \mu_0, v_0]$ and V_0. V_t has the same functional form as V_0 with $\{c_0, v_0, \mu_0\}$ replaced by $\{c_t, v_t, \mu_t\}$ because the system of equations (1)–(3) is stationary. ■

Proof of Lemma 3: The idea of the proof borrows from Ohlson and Zhang (1998). For V_t to be expressible as a weighted average of operating assets and earnings, plus perhaps a $\mu_t v_t$ term, it is necessary and sufficient that $\{\delta_0, \delta_1, \delta_2\}$ and η are such that

$$(1 - \kappa) oa_t + \kappa \phi [ox_t - c_t] + \eta \mu_t v_t = \left(\frac{\gamma}{R - \gamma}\right) c_t + [\xi_1 + \xi_2 \mu_t] v_t. \tag{12}$$

To determine the required values of $\{\delta_0, \delta_1, \delta_2\}$ and η, express the left-hand side of equation (12) in terms of $\{c_t, oa_{t-1}, v_t, \mu_t v_t\}$ using equations (4) and (5). It is necessary to rely on oa_{t-1} rather than oa_t because, by equation (4), oa_t is not independent of c_t. Then equating the constant coefficients of the four variables $\{c_t, oa_{t-1}, v_t, \mu_t v_t\}$ in the re-expressed version of equation (12) yields four equations for the four unknowns $\{\delta_0, \delta_1, \delta_2\}$ and η. Solving these four equations yields the unique values of $\{\delta_0, \delta_1, \delta_2\}$ and η that satisfies equation (12). Since it is impossible to set $\eta = 0$ and still satisfy equation (12), the accounting policy equation (5) is not efficient. ■

Proof of Proposition 1: Forward-earnings accounting is achieved if, and only if, ox_t satisfies

$$\frac{\overline{ox}_{t,1}}{R - 1} = V_t = \left(\frac{\gamma}{R - \gamma}\right) c_t + [\xi_1 + \xi_2 \mu_t] v_t. \tag{13}$$

Since $\overline{c}_{t,1} - \gamma c_t = \overline{v}_{t,1}$ by equation (1) and $\overline{v}_{t,1} = v_t \times (1, -1, 1, -1)Q\Xi_t \equiv Av_t + B\mu_t v_t$ in the notation of the proof of Lemma 2, $\mu_t v_t$ in equation (13) can be eliminated from equation (13) and replaced by $\overline{c}_{t,1}$. This yields

$$\overline{ox}_{t,1} = (R-1) \times \left\{ \frac{\xi_2}{B}\overline{c}_{t,1} + \left[\frac{1}{R-\gamma} - \frac{\xi_2}{B} \right] \gamma c_t + \left[\xi_1 - \frac{\xi_2}{B} A \right] v_t \right\}. \tag{14}$$

Equation (14) defines ox_t in expectation. The general solution to equation (14) is

$$ox_t = (R-1) \times \left\{ \frac{\xi_2}{B} c_t + \left[\frac{1}{R-\gamma} - \frac{\xi_2}{B} \right] \gamma c_{t-1} + \left[\xi_1 - \frac{\xi_2}{B} A \right] v_{t-1} \right\} + \eta_t,$$

where η_t is any zero-mean random variable. Plugging in the expressions for A, B, ξ_1, and ξ_2, and using equation (1) (again) to replace γc_{t-1} with $c_t - v_t - \varepsilon_t$ recovers the expressions for δ_8 and δ_9 stated in the Proposition. ■

Proof of Observation 1: When $F \geq T, \hat{V}_t(F; y) = V_t$ so there is no valuation error. When $F < T, \hat{V}_t(F; y) - V_t = -(\phi/R^F) \sum_{w=F}^{T-1} \eta_{t+F-w}$, which implies

$$\text{error}(F) = \left(\frac{\phi}{R^F} \right)^2 \text{var} \left[\sum_{w=F}^{T-1} \eta_{t+F-w} \right] = \left(\frac{\phi \sigma_\eta}{R^F} \right)^2 (T-F). \quad ■$$

Proof of Proposition 2: First, it is necessary to specify the linear accounting policy. For $\hat{V}_t(F)$, define $ox_t(p) = (R-1)/(R-\gamma) \times \{c_t + \sum_{u=0}^{F} \sigma_u v_{t-u}\} + \eta_t$ where η_t is any random variable satisfying $E_t[\eta_{t+F}] = 0$. The accrual policy parameters $\{\sigma_0, \ldots, \sigma_F\}$ are specified as follows. Recall that $V_t = (\gamma/(R-\gamma))c_t + \sum_{u=1}^{S} \xi_u \mu_{u,t} v_t$ when the unobservable variable ranges over S states. Without loss of generality, one can always re-cast V_t in the form

$$V_t = \left(\frac{\gamma}{R-\gamma} \right) c_t + \sum_{u=0}^{S-1} \psi_u E_t[v_{t+u} \mid \mu_{1,t}, \ldots, \mu_{S,t}, v_t],$$

since $\{\psi_0, \ldots, \psi_{S-1}\}$ can be chosen to make $\sum_{u=1}^{S} \xi_u \mu_{u,t} v_t = \sum_{u=0}^{S-1} \psi_u E_t[v_{t+u} \mid \mu_{1,t}, \ldots, \mu_{S,t}, v_t]$. Also without loss of generality, suppose the states are enumerated so that the capitalization factors are conveniently ordered:

$$\|\psi_1\| \geq \|\psi_2\| \geq \cdots \geq \|\psi_S\|.$$

Then specify the accrual policy parameters $\{\sigma_0, \ldots, \sigma_F\}$ by requiring that

$$\hat{V}_t(F) = \left(\frac{\gamma}{R-\gamma} \right) c_t + \sum_{u=0}^{F} \psi_u E_t[v_{t+u} \mid \mu_{1,t}, \ldots, \mu_{S,t}, v_t].$$

This requirement provides $F+1$ linear equations, which determine uniquely the $F+1$ accrual policy parameters. Under this accounting policy, the absolute value of the valuation error is

$$\|\hat{V}_t(F) - V_t\| = \left\| \sum_{u=F+1}^{S-1} \psi_u E_t[v_{t+u} \mid \mu_{1,t}, \ldots, \mu_{S,t}, v_t] \right\|.$$

The Proposition follows from the observation that

$$\begin{aligned}\left\| \sum_{u=F+1}^{S-1} \psi_u E_t[v_{t+u} \mid \mu_{1,t}, \ldots, \mu_{S,t}, v_t] \right\| &\leq \sum_{u=F+1}^{S-1} \|\psi_u\| \, \|E_t[v_{t+u} \mid \mu_{1,t}, \ldots, \mu_{S,t}, v_t]\| \\ &\leq \sum_{u=F+1}^{S-1} \|\psi_u\|,\end{aligned}$$

since $v_{t+u} \in \{+1, -1\}$ by construction. This implies

$$\|\hat{V}_t(F) - V_t\| \leq \sum_{u=F+1}^{S-1} \|\psi_u\|,$$

which implies that the upper bound on the absolute value of the valuation error (i) decreases with increasing F and (ii) decreases all the way to zero when $F = S - 1$. Hence, one can always make $\|\hat{V}_t(F) - V_t\| \leq E$ for any $E > 0$ by choosing F to be bigger than some $F_* \in \{0, \ldots, S-1\}$. Moreover, since the upper bound on $\|\hat{V}_t(F) - V_t\|$ never increases with F, $\|\hat{V}_t(F) - V_t\| \leq E$ implies $\|\hat{V}_t(F') - V_t\| \leq E$ for all $F' > F$. ■

Proof of Observation 2: This proof proceeds in the same way as the proof of Lemma 3 except that ox_t now has the extra $\delta_3 \mu_t v_t$ term. This extra term allows equation (12) to hold with $\eta = 0$. ■

Proof of Observation 3: Following the notation in the proof of Lemma 2,

$$\begin{aligned}\overline{v}_{0,1} &= v_0 \times (1, -1, 1, -1) Q^1 \Xi_0, \\ \overline{v}_{0,2} &= v_0 \times (1, -1, 1, -1) Q^2 \Xi_0,\end{aligned} \tag{15}$$

where, without loss of generality, I assume the current time is $t = 0$. Moreover, by equation (1),

$$\begin{aligned}\overline{v}_{0,1} &= \overline{c}_{0,1} - \gamma c_0, \\ \overline{v}_{0,2} &= \overline{c}_{0,2} - \gamma \overline{c}_{0,1}.\end{aligned} \tag{16}$$

Combining equations (15) and (16) yields two linear equations relating $\{c_0, \overline{c}_{0,1}, \overline{c}_{0,2}\}$ to $v_0 \mu_0$ and $v_0(1 - \mu_0)$. Inverting these two equations yields the linear expression for μ_0 and $\mu_0 v_0$ in terms of linear combinations of $\{c_0, \overline{c}_{0,1}, \overline{c}_{0,2}\}$. Plugging these expressions for μ_0 and $\mu_0 v_0$ into the formula, equation (8), for the efficient accounting policy of Observation 2 yields Observation 3. The algebra is complicated but the argument is simple. ■

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Notes

1. Plugging (C) into the no arbitrage relation $RV_t = E_t[V_{t+1} + c_{t+1}]$ yields $R(\phi ox_t^c - c_t) = \phi E_t[ox_{t+1}]$, which implies $V_t = \phi ox_t^c - c_t = \phi E_t[ox_{t+1}^c]/R$.
2. If market participants know that $V_t = \phi ox_t^c - c_t$ but do not observe η_t and ox_t^c directly, they would estimate $V_t = \phi E_t[ox_t^c] - c_t$ conditional on their information set $\Omega_t = \{y_{t-s}, c_{t-s}\}_{s=0}^t$. Bayesian theory (Yee, 2004) asserts that $E_t[ox_t^c]$ would be a weighted average over all of the independent (noisy) estimators of ox_t^c one can construct from the variables of Ω_t. Two such estimators are y_t and $Ry_{t-1} - (R-1)c_{t-1}$ but there are more using additional lagged variables.
3. There is no reason why future shocks must perpetuate the historical pattern.
4. Barth et al. (1998) describe some problems of trying to identify v_t empirically.
5. In other words, $s_t = H$ is the high persistence and $s_t = L$ is the low persistence state.
6. That Bayesian estimates have a subjective component is perfectly consistent with mainstream accounting wisdom. Beaver (1991) observed that traditional "accruals can be viewed as a form of forecast about the future based on current and past events, and accrual accounting can be viewed as a cost-effective way of conveying expectations about future benefits or sacrifices." At the same time, Beaver cautions, "probabilities are generally regarded as subjective or judgmental in nature. People will differ in their probability assessment according to what information they use to condition their beliefs. For example, two persons could examine the same aging schedule for receivables and yet, perhaps because of differing credit management experience, could arrive at different probabilities of collection of the currently held receivables."
7. In fact, any accrual policy of the form $ox_t^f = (R-1)/(R-\gamma) \times \{c_t + (\delta_8 - 1)v_t + \delta_9 v_{t-1}\} + \eta_t$, where $E_t[\eta_{t+1}] = 0$, also achieves (F). This is because, as explained earlier, the forward-earnings relation is unaffected by random zero-mean charges.
8. The debate over the relative merits of shifting the Bayesian estimation onus away from accountants has a long history. Beaver (1991, p.125) observed that "bank analysts routinely state that they expect loan loss allowances and provisions to increase further in the next quarter.... What information are the analysts reflecting in their expectations that is not reflected in the current loan loss allowance? Is this information 'out-of-bounds' (to accountants) for the purposes of estimated uncollectible accounts? If so, why?"
9. The model of cash flows posited here has been suggested to underlie behavioral over-reaction to the dividend growth rate or post-earnings-announcement drift (Barksy and De Long, 1993; Barberis et al., 1998). In the BSV interpretation, the market believes cash flow contains a persistent v_t-like component whereas, in truth, v_t is completely transitory. Hence, market price overreacts to accidentally persistent-looking occurrences of v_t. In contrast to BSV, the interpretation in this article does not posit any incorrect or behavioral misperceptions by accountants or market participants. Here, equations (1)–(3) is the true model of cash flow dynamics and all market participants, who are strictly rational and Bayesian, agree that this is the model.

10. This model can be extended to incorporate information asymmetry in a Kyle setting (Kwon, 2001) or in a Grossman–Stiglitz setting (Yee, 2003).
11. Begley and Feltham (2002) exploit the fact that earnings forecasts contain private information in their research design.
12. In this section, I will use the shorthand $E_t[\cdot]$ to mean $E_t[\cdot \mid \Omega_t]$, where Ω_t is the appropriate information set. Ω_t may vary across different examples.
13. Plugging $V_t = \phi E_t[ox_{t+1}]/R$ into the no arbitrage relation, $RV_t = E_t[V_{t+1} + c_{t+1}]$, and applying the law of iterated expectations yields $\phi E_t[ox_{t+1}] = \phi(E_t[ox_{t+2} + \phi^{-1} Rc_{t+1}]/R)$, which implies $V_t = \phi E_t[ox_{t+1}]/R = \phi E_t[ox_{t+2} + \phi^{-1} Rc_{t+1}]/R^2$. Iterating this argument forward $F' - 1$ times implies $V_t = \hat{V}_t(F')$ for all $F' \geq 1$ whenever $V_t = \hat{V}_t(1)$.
14. For example, if ox_t satisfies $V_t = \phi E_t[ox_{t+1}]/R$, then $y \equiv ox_t + \eta_t$ where $E_t[\eta_{t+2}] = 0$ satisfies $V_t = \phi E_t[y_{t+2} + \phi^{-1} Rc_{t+1}]/R^2$ but $V_t \neq \phi E_t[y_{t+1}]/R$ if $E_t[\eta_{t+1}] \neq 0$.
15. $\| \cdot \|$ is the absolute value function.
16. Proving Conjecture A requires first identifying the space Π of relevant accounting policies, which in general may be nonlinear. In my mind, this by itself is a formidable task.

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